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Undead argument: the truth-functionality objection to fuzzy theories of vagueness

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Abstract From Fine and Kamp in the 70's—through Osherson and Smith in the 80's, Williamson, Kamp and Partee in the 90's and Keefe in the 00's—up to Sauerland in the present decade, the objection continues to be run that fuzzy logic based theories of vagueness are incompatible with ordinary usage of compound propositions in the presence of borderline cases. These arguments against fuzzy theories have been rebutted several times but evidently not put to rest. I attempt to do so in this paper.

Keywords Vagueness · Fuzzy logic · Truth-functionality

1 Zadeh Logic

We begin by considering one particular version of fuzzy logic: the version against which many (but not all) of the objections to be considered below are directed.[1] Recall the classical truth tables:

α	β	$\neg\alpha$	$\alpha \wedge \beta$	$\alpha \vee \beta$	$\alpha \rightarrow \beta$
1	1	0	1	1	1
1	0		0	1	0
0	1	1	0	1	1
0	0		0	0	1

[1] As we shall see later, however, the resources of fuzzy logic far outrun those introduced in this section.

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Another way of expressing these truth conditions is as follows, where $[\alpha]$ is the truth value of α:

$$\begin{aligned}
[\neg\alpha] &= 1 - [\alpha] \\
[\alpha \wedge \beta] &= \min\{[\alpha], [\beta]\} \\
[\alpha \vee \beta] &= \max\{[\alpha], [\beta]\} \\
[\alpha \rightarrow \beta] &= [\neg\alpha \vee \beta] = [\neg(\alpha \wedge \neg\beta)]
\end{aligned}$$

In Zadeh logic the truth values are the members of the real closed interval [0,1]—all the real numbers between 0 and 1 inclusive—and the truth conditions for compound propositions are defined in the second way above. We can define a logical consequence relation via choosing a set of designated truth values (or in other ways) and this can all be extended in a straightforward manner to quantificational logic.[2]

2 The objections: a sample

There have been many different kinds of objections to theories of vagueness based on fuzzy logics. This paper is concerned with a particular class of objections: those which claim that fuzzy logic based theories of vagueness are incompatible with ordinary usage of compound propositions in the presence of borderline cases. In this section I give a sample of objections from this class.

Many (but not all) of the objectors assume (falsely, as we shall see) that fuzzy logic is simply Zadeh logic. Some of the objections are directed against specific fuzzy theories (principally Zadeh logic). Other objections are more general: they are directed against any truth(-degree)-functional account (that is, any account that holds that the degree of truth of a compound proposition—for example $\alpha \wedge \beta$—is a function of the degrees of truth of its component propositions—in this example α and β).

2.1 Fine I and Osherson & Smith I

Suppose that a certain blob is on the border of pink and red and let P be the sentence 'the blob is pink' and R the sentence 'the blob is red'—so P and R are neither clearly true nor clearly false. Fine thinks that $P \vee R$ is clearly true and that $P \wedge R$ is clearly false. This is not predicted by a fuzzy account based on Zadeh logic.[3]

On a related note, Osherson & Smith think that where Ax means that x is an apple, $Aa \wedge \neg Aa$ should be true to degree 0 and $Aa \vee \neg Aa$ should be true to degree 1, whatever a is. This conflicts with Zadeh logic.[4]

2 See Smith (2008, Sect. 2.2.1) for further details.

3 See Fine (1975, pp. 269–270).

4 See Osherson and Smith (1981, pp. 45–46). They present their argument in terms of degrees of membership of objects in sets rather than degrees of truth of statements.

2.2 Kamp

Kamp thinks that the following is clearly true to degree 0, even when $[\alpha] = 0.5$:

- $\alpha \wedge \neg\alpha$

and that the following is clearly true to a degree strictly greater than 0, when $[\alpha] = 0.5$:

- $\alpha \wedge \alpha$

Assuming $[\neg\alpha] = 1 - [\alpha]$, no truth function for $\wedge$ can predict this. So this is an argument not just against Zadeh logic but against any degree-functional account that agrees with Zadeh logic about negation.[5]

2.3 Fine II

With P and R as in Sect. 2.1, Fine claims that $P \wedge P$ is equivalent to P and hence is neither clearly true nor clearly false, while (as already discussed) $P \wedge R$ is clearly false. Given that P and R have the same degree of truth, this is an argument against any degree-functional account of conjunction.[6]

2.4 Osherson & Smith II

Consider an apple (a), illustrated thus:

(a)

Osherson & Smith claim that (a) is psychologically less prototypical of the concept ‘apple’ than of the concept ‘striped apple’. If we equate prototypicality with degree

[5] See Kamp (1975, p. 546).

[6] See Fine (1975, p. 269).

of membership/truth and take ‘striped apple’ to be formed from the two components ‘striped’ and ‘apple’ by intersection/conjunction, then this is an objection to the Zadeh rule for conjunction—and more generally to any account according to which $[\alpha \wedge \beta]$ can never be strictly greater than $[\alpha]$ (or $[\beta]$).[7]

2.5 Sauerland

Consider the following two claims:

$$A : \text{A } 5'10'' \text{ guy is tall.} \qquad B : \text{A guy with \$100,000 is rich.}$$

Sauerland (2011) obtained the following experimental data:

Proposition	A	$\neg A$	B	$\neg B$	$A \wedge \neg A$	$B \wedge \neg B$	$A \wedge \neg B$	$B \wedge \neg A$
Mean agreement (%)	45	42	45.75	47.25	48.15	46.5	43.2	25.65

He notes that while the levels of agreement with A, $\neg A$, B and $\neg B$ are similar, the average levels of agreement with $A \wedge \neg A$ and $B \wedge \neg B$ are significantly higher than the average levels of agreement with $A \wedge \neg B$ and $B \wedge \neg A$. Sauerland claims that no truth function for $\wedge$ can predict this.

3 Responses

In the following sections we shall consider (possible) responses on the part of advocates of fuzzy theories of vagueness to objections of the kind illustrated in the previous section. I shall distinguish three different lines of response:

(1) Disputing the data (Sect. 4)
(2) Accommodating the data (Sect. 5)
(3) Questioning the relevance of data (Sect. 6)

Within these broad categories I shall further distinguish particular moves available to the defender of fuzzy approaches to vagueness.

Note that what follows is a menu of responses. One cannot adopt them all at the same time: one picks and chooses a coherent set—a meal, as it were. The aim of this paper is to dispel the idea that fuzzy theories are refuted by considerations of ordinary usage of compound propositions in the presence of borderline cases. What we shall see is that—on the contrary—there are many ways in which fuzzy theorists can proceed in the face of such considerations.

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[7] See Osherson and Smith (1981, pp. 43–45). Essentially the same objection (using the example of *pussy willow* and *willow*) was made earlier by Kay (1975, p. 153). For subsequent empirical work on this kind of case see Storms et al. (1998). Osherson and Smith (1981, pp. 46–48) also present a dual objection concerning alleged disjunctions that are apparently less true than either disjunct.

4 Disputing the data

It is characteristic of the kind of objection we are considering that it involves data: either intuitive or experimental. The claim is then made that fuzzy theories cannot predict or accommodate this data. One obvious strategy for responding to the objections is to dispute the data on which they trade. Within this general strategy, we can distinguish various different ways of disputing the data.

4.1 Other intuitions

Some of the data on which the objections trade is simply intuitive—for example:[8]

> we would have $[\phi \wedge \neg\phi] = \frac{1}{2}$, which seems absurd. For how could a logical contradiction be true to *any* degree? (Kamp 1975, p. 546)[9]
> We are ready to demonstrate that prototype theory in conjunction with fuzzy-set theory contradicts strong intuitions we have about concepts. (Osherson and Smith 1981, p. 43)[10]
> intuition suggests that r is a better example of a round block than of a round ball…intuition reveals that h is at least as good an example of *red flower* as it is of *flower* (Osherson and Smith 1982, pp. 306, 309)
> …the conjunction of the two is greater than zero. This consequence is absurd, because a self-contradiction surely merits a truth value of zero. (Johnson-Laird 1983, p. 199)
> a given object x may be a triangle (say) to degree 0.9; $f_\Delta(x) = 0.9$. If the complement of f_Δ represents 'is not a triangle' and union disjunction, then $\max(f_\Delta, 1 - f_\Delta)$ should represent 'is a triangle or isn't a triangle' *and should be the constant 1 function*; but it isn't. (Urquhart 1986, p. 108; my emphasis)[11]
> At some point 'He is awake' is supposed to be half-true, so 'He is not awake' will be half-true too. Then 'He is awake and he is not awake' will count as half-true. How can an explicit contradiction be true to any degree other than 0? (Williamson 1994, p. 136)
> Evidently [the concept *apple which is not an apple*] unequivocally excludes everything.…According to the most familiar versions of fuzzy logic the degree to which *a* satisfies the conjunctive concept *apple which is not an apple* is…greater than 0. Clearly this is not the right result. (Kamp and Partee 1995, p. 134)
> *apple that is not an apple* is a self-contradictory predicate and thus should have a characteristic function that always gives the value 0. Also, *fruit that either is*

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[8] Note that some authors are explicit about the intuitive basis of their claims while others simply assert what they take to be obvious without explicitly noting that intuition is the sole support for their assertions.

[9] I have omitted a superscript and a subscript from Kamp's notation because they add complexity that is irrelevant in the present context. Cf. Fine (1975, p. 270): "Surely P & $-$ P is false even though P is indefinite."

[10] See also Osherson and Smith (1981, pp. 46, 55), Jones (1982, p. 283) and Osherson and Smith (1982, p. 299).

[11] Bonini et al. (1999, pp. 389–390) express a similar intuition—and notably (given that their paper includes empirical work) do not subject this intuition to experimental testing.

> *or is not an apple* should have the same characteristic function as the simple *fruit*. …there is no way that [the fuzzy] conception can meet even such minimal requirements as the simultaneous satisfaction of the following:
> (i) $\phi \& \neg\phi$ should always have truth value 0.
> (ii) $\phi \& \phi$ should always have the same truth value as ϕ. (Kamp and Partee 1995, pp. 146–147)
>
> the same value must be given to (e) 'if Tek is tall then Tek is not tall' as to (f) 'if Tek is tall then Tek is tall' since the respective values of the antecedent and consequent of these two conditionals are the same. But (f) is intuitively true and (e) is not, so again no choice of value will capture our intuitions about both of these cases (Keefe 2000, p. 97)
>
> In Zadeh's fuzzy logic…if for some instance x and class A, [x's degree of membership in A] is 0.5, both $A(x)$ and $\neg A(x)$ will have values of 0.5, and [the self-contradictory $A(x)\&\neg A(x)$…and the tautological $A(x) OR \neg A(x)$] will both be evaluated as 0.5 true, an obviously counterintuitive conclusion to reach in each case. (Hampton 2007, pp. 366–367)

In response to objections based on intuition, one line of response is to profess different intuitions—as numerous authors have done. For example:[12]

> [Fine] claims that 'red' and 'pink', even though vague and admitting of borderline cases of applicability, are nevertheless logically connected so that to say of some color shade that it is both red and pink is obviously to say something false. I must confess being completely insensitive to that intuition of a penumbral connection. (Machina 1976, p. 77, n. 2)
>
> Support for the present position is provided by the observation that, as Osherson and Smith themselves remark in a footnote (p. 45), people do in fact frequently use locutions such as (of tomatoes) 'They are both fruit and not fruit'.…contrary to traditional binary taxonomy but in accordance with the present view, a concept such as *ape that is not an ape* is indeed not an empty one. (Jones 1982, pp. 287–288)
>
> An aspect of the theory of fuzzy sets which Osherson and Smith find objectionable is that, in the theory, the union of A and its complement, A′, is not, in general, the whole universe of discourse. This relates, of course, to the long-standing controversy regarding the validity of the *principle of the excluded middle*…The principle of the excluded middle is not accepted as a valid axiom in the theory of fuzzy sets *because it does not apply to situations in which one deals with classes which do not have sharply defined boundaries*. (Zadeh 1982, p. 292; second emphasis mine)

[12] Cf. also Lakoff (1987, p. 141), Fuhrmann (1988, pp. 323–324], Keefe (2000, p. 164) (who unlike fellow supervaluationists Fine and Kamp & Partee does not take $Fa \vee \neg Fa$ to be assertible when a is a borderline case of F), Belohlavek et al. (2002, p. 578), Belohlavek et al. (2009, p. 31), Smith (2008, p. 86) and Ripley (2013, p. 341). Some of the objectors have at least noted the existence of differing intuitions—and yet this appears to take no wind from their sails. See e.g. Osherson and Smith (1982, p. 313), Williamson (1994, p. 293 n. 47), Kamp and Partee (1995, pp. 149 n. 13, 179 n. 33), Osherson and Smith (1997, p. 201), and Bonini et al. (1999, p. 390).

> the failure of contradiction and excluded-middle laws is typical of fuzzy logic as emphasized by many authors. This is natural with gradual properties like 'tall'. (Dubois and Prade 1994, p. 152)

4.2 Other data

A second line of response is to turn from intuitions to experimental data. Psychologists have been gathering relevant data since theories of vagueness based on fuzzy set theory began to appear in the 1960's; more recently, philosophers with an interest in vagueness have also become involved in empirical work in this area.[13] In the previous section we looked at some intuitions of opponents and defenders of fuzzy approaches to vagueness. Some of the data clearly support some of the fuzzy intuitions. For example, studies by Alxatib and Pelletier (2011a, b), Ripley (2011a) and Sauerland (2011) show a significant willingness of subjects to agree with statements such as:

- X is tall and not tall.
- The circle is near the square and it is not near the square.
- The circle is and is not near the square.

Furthermore, since the early 1970s there has been a significant amount of data in the psychological literature supporting the idea that concepts have a graded rather than binary structure: that is, an object need not simply fall under a concept or else fail to do so—objects can fall under concepts to greater or lesser degrees.[14] Whether or not fuzzy theories have trouble with complex claims such as 'This leaf is red and orange' (said of a borderline red-orange leaf)—that is the topic of the present paper—it should not be forgotten that fuzzy theories are at a significant advantage over rival theories that do not countenance degrees of membership/truth, when it comes to explaining the data that support the idea that concepts such as 'red' have a graded structure.[15]

4.3 Questionable data

Some of the data that appear to pose problems for fuzzy accounts appear suspect on closer inspection. For example, consider Sauerland (2011). Unlike Fine and Kamp, for example, Sauerland does not assume that statements of the form $\alpha \wedge \neg\alpha$ are definitely false. His point is that when there is equal agreement with the components (A, B, $\neg A$ and $\neg B$), truth-functional accounts cannot predict lower agreement with conjunctions ($A \wedge \neg B$ and $B \wedge \neg A$) than with contradictions ($A \wedge \neg A$ and $B \wedge \neg B$). Yet, he claims, his data show precisely such a pattern of agreement:

[13] See e.g. Bonini et al. (1999), Alxatib and Pelletier (2011a, b), Ripley (2011a), Sauerland (2011) and Serchuk et al. (2011).

[14] For an introduction to this research in psychology see Rosch (2011). See also Kalish (1995) and Hampton (2007, p. 377).

[15] Furthermore, moving from the empirical to the theoretical literature, Smith (2008) presents a sustained argument that the correct account of vagueness must involve degrees of truth.

Proposition	A	$\neg A$	B	$\neg B$	$A \wedge \neg A$	$B \wedge \neg B$	$A \wedge \neg B$	$B \wedge \neg A$
Mean agreement (%)	45	42	45.75	47.25	48.15	46.5	43.2	25.65

He notes that while the levels of agreement with A, $\neg A$, B and $\neg B$ are similar, the average levels of agreement with $A \wedge \neg A$ and $B \wedge \neg B$ are significantly higher than the average levels of agreement with $A \wedge \neg B$ and $B \wedge \neg A$.

Look closely at the data however. Why is the figure for $B \wedge \neg A$ so much lower than the figure for $A \wedge \neg B$? Sauerland does not say: he simply reports the *average* of the two, and compares it to the average of the figures for $A \wedge \neg A$ and $B \wedge \neg B$. Of course the former is much lower—but without some explanation of the discrepancy between the figures for $A \wedge \neg B$ and $B \wedge \neg A$, this data looks noisy and unreliable.

Serchuk et al. (2011) present a devastating critique of the design, implementation and methodology of the empirical work in Bonini et al. (1999), concluding:

> Each of the arguments we gave in this section is, in our view, sufficient to show that the experiment done by Bonini *et al.* is methodologically unsound and that its results should not be given any philosophical weight by philosophers of vagueness. (p. 555)

They also criticise Alxatib and Pelletier (2011b) for failing to test most of their data for statistical significance (p. 572). These criticisms are convincing. Yet there are also problems with some of their own experiments. For example, they wish to test the 'confusion hypothesis': the idea that speakers hear 'x is F' (where F is a vague predicate) as claiming that x is definitely F. They therefore asked subjects for their reactions to the following two sentences:

(4) Susan is rich.
(5) Susan is definitely rich.

They write:

> We test our hypothesis by comparing the responses of each participant to (4) and (5) …The distribution of responses was statistically significant for both groups…The data do not support a generalized confusion hypothesis. Only 24 % of participants in [the first group] and 39 % of those in [the second group] answered (4) and (5) identically. (p. 558)

There is a serious problem here. We generally try our best to make sense of what others say. Faced with (4) and (5), a subject might try to understand why the experimenter would say the same thing twice in a row—or she might try to hear (4) and (5) as saying different things. The latter is much easier in this case: for example, one could interpret (4) as saying that Susan is rich and (5) as saying that Susan is *very* rich. The experimental design takes no account of the possibility that presenting two different sentences side by side encourages speakers to find a way to see them as saying different things. The fact that speakers respond differently to (4) and (5) when presented together is compatible with the hypothesis that, had either been presented alone, they would have prompted the same reaction. Furthermore, the more plausible versions of the confusion hypothesis are those according to which ordinary speakers hear ‘x is F’ as having assertibility conditions which correspond to the truth conditions assigned by the *theorist* of vagueness to some theoretical claim ‘x is definitely F’—where the latter is cashed out in different ways in different theories of vagueness. It is of no relevance to *this* hypothesis to ask ordinary speakers for their reactions to ordinary sentences incorporating the word ‘definitely’.[16]

5 Accommodating the data

In the face of intuitions and/or experimental data that appear to pose problems for fuzzy approaches, one line of response is to question the data or intuitions and/or to present different intuitions or data that are more friendly to fuzzy theories. That was the line examined in Sect. 4. A second line of response is to accept the intuitions or data that apparently pose problems for fuzzy approaches and argue that in fact they do not pose problems. One way of doing this is to explain the data using fuzzy tools. Another way is to note that the data are not of a kind that fuzzy theories are in the business of explaining and so can simply be accepted without having to be explained.[17] We explore the latter idea in Sect. 5.1 and then move on in Sects. 5.2 and 5.3 to the former kind of approach: the idea of accommodating apparently recalcitrant data using other resources in the fuzzy toolbox, beyond those of Zadeh logic.

5.1 Careful use of resources

Fuzzy logics tell us how the degrees of *truth* of statements such as $\alpha \wedge \beta$ relate to the degrees of truth of their components α and β (and fuzzy set theories tell us how the degrees of *membership* of objects in sets such as $A \cap B$ relate to their degrees of membership in the sets A and B). But some of the data that are supposed to pose problems for fuzzy theories do not seem to be about degrees of truth (or membership) at all.

For example, as discussed in Sect. 2.4, Osherson and Smith (1981, pp. 43–45) take it as a datum that “There can be no doubt that [(a)] is psychologically less prototypical of an apple…than of an apple-with-stripes” and then take this to mean that (a)’s degree

[16] Serchuk et al. (2011, p. 561) subsequently raise the worry themselves that “any confusion between (4) and (5) was neutralized by their apparent juxtaposition on the survey instrument: each participant was asked about both ‘ϕ’ and ‘definitely ϕ’.” Their response is that “This worry can be set aside by considering Experiment #2, where participants were asked for the boundaries for either ‘ϕ’ or ‘definitely ϕ’.” But this does not answer the worry about their experiment: it in effect concedes that the experiment is fundamentally flawed and hence directs our attention to a different experiment. (Also, we are told that the first experiment involved 350 undergraduates at the University of Calgary and was conducted in 2005, and that the second experiment involved 164 undergraduates at the University of Calgary and was conducted in 2005. We are not told whether the group of 164 was a subset of or overlapped with the group of 350. If there was overlap, then the second experiment *does* face a version of the problem raised above for the first experiment.) Furthermore, the second experiment still faces the second worry raised in the text above: that we should not be asking ordinary speakers for their reactions to ordinary sentences incorporating the word ‘definitely’.

[17] This is not to say that the data have no explanation at all: just that it is not the job of fuzzy theories of vagueness to provide the explanation.

of membership in the set of striped apples is greater than its degree of membership in the set of apples. But one could accept the former datum about *prototypicality* and yet maintain that when it comes to degrees of *membership* (and *truth*), (a) is a degree 1 member of 'striped apple' and of 'apple' ('(a) is a striped apple' and '(a) is an apple' are both quite simply true, to degree 1).[18]

For a second example, Serchuk et al. (2011, p. 563) ask subjects:[19]

> Consider the following sentences, where X stands for an arbitrary number. We'd like to know which, in your opinion, express the vagueness of 'heap' most persuasively and which ones the least. Please rank them in order of persuasiveness on the table below. Please break ties.
> (B) The following statement is false: X grains of sand are a heap, but $X - 1$ grains of sand are not a heap.
> (D) Either X grains of sand are not a heap or $X - 1$ grains of sand are a heap.

They found that (B) is more persuasive than (D) (p. 564). So for so good—but they then take this to be a problem for any logic in which De Morgan's laws hold and hence (B) and (D) are equivalent (p. 565). This is a non sequitur. Surely no-one seriously believes that *persuasiveness* is a function (solely) of truth: that statements that do not differ with respect to truth value cannot differ with respect to how persuasive they are (as an expression of such and such). If we wish to draw conclusions about truth (or logical equivalence, etc.) we probably should not ask subjects about persuasiveness: certainly there is no direct route from the latter to the former.[20]

The examples just discussed point to a general issue: *how* to gather data that shed light (favourable or unfavourable) on fuzzy theories of vagueness (as opposed to data that simply pass by fuzzy theories—because they have no evident relevance to claims about degrees of truth/membership)? Even many theorists of vagueness are confused about the subtle relationships between notions such as degree of truth, degree of distance from the truth and degree of belief.[21] How can we pose questions to ordinary speakers—or in other ways investigate their usage—in such a way that we elicit information that has clear implications concerning degrees of truth (as opposed to degrees of belief, for example)? Procedures used in the literature can be classified along three dimensions:

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[18] The point that degrees of membership and truth on the one hand and degrees of typicality on the other hand need to be carefully distinguished has been made by numerous authors including Zadeh (1982, p. 293), Smith and Osherson (1988, pp. 51–52), Kalish (1995), Kamp and Partee (1995, pp. 131, 133), Osherson and Smith (1997, p. 191), Belohlavek et al. (2002, p. 578) and Belohlavek and Klir (2011b, pp. 132–133). Hampton (2007, Sect. 2) agrees that membership and typicality are distinct functions but argues that both are determined by a single underlying psychological process of measuring the resemblance between an object and the prototypes for a concept.

[19] Sentences (A) and (C) are omitted from the quotation because they are irrelevant to the present discussion.

[20] Setting aside the problem noted in the text above, a further issue with this experiment is that subjects are forced to break ties: this would seem to build a bias against views according to which (B) and (D) are equivalent into the very design of the experiment.

[21] For discussion of the difference between degree of truth and degree of distance from the truth see Smith (2008, pp. 264–265); for discussion of the relationships between degree of truth and degree of belief see e.g. Smith (2014).

(1) type of response requested
For example, subjects may be asked to judge the *truthfulness* of a statement (Oden 1977, p. 568), to give the degree to which an exemplar is *typical* of a category (McCloskey and Glucksberg 1978, p. 464), or to indicate their level of *agreement* with a sentence (Ripley 2011a, p. 173).

(2) number of possible responses
For example, subjects may be asked for a true/false response (Alxatib and Pelletier 2011b, p. 306), given a greater but still limited range of responses (say, seven or ten) (Ripley 2011a, p. 173) (McCloskey and Glucksberg 1978, p. 464), or allowed potentially infinitely many responses (Oden 1977, p. 568).[22]

(3) processing of responses
For example, in experiments with multiple possible responses, responses might be interpreted more or less directly as measures of degrees of truth or membership (McCloskey and Glucksberg 1978, p. 464), while in experiments with only two possible responses (positive and negative), *probability* of a positive response might be taken as a measure of degree of truth or membership (Hampton 2007, p. 361).

There have been some discussions of the relevance of various procedures to the issue of testing fuzzy theories of vagueness.[23] My point here is not to contribute but simply to say that there needs to be more such discussion.[24] Before we even consider the problems to be raised in Sect. 6, it is already far from clear that simply gathering data about usage of vague language in the presence of borderline cases—without very careful thought about how the data is to be gathered and processed—will automatically contribute to the assessment of fuzzy theories of vagueness.

5.2 Logico-semantic resources

One could easily get the impression from much of the philosophical literature on vagueness that fuzzy logic is simply Zadeh logic. If one turns to the fuzzy logic literature, however, one quickly realises that this is not so. There are many systems of fuzzy logic, and there are many additional resources available in fuzzy logics, beyond degree-functional definitions of truth for conjunctions, disjunctions and so on. Data that cannot be accommodated within Zadeh logic can be accommodated quite readily when we make use of the full contents of the fuzzy toolbox. Here we look at some examples of other logics and additional resources—and how they may be applied to the objections of Sect. 2.[25]

[22] Oden (1977, p. 568) asks subjects to indicate their judgements "by placing a pin in a 200-mm cork-topped board so that the position of the pin corresponded to the judged truthfulness, with the right end labeled "absolutely true" and the left end "absolutely false." The position of the pin was measured using a ruler attached to the back of the board."

[23] See e.g. Hampton (2007, pp. 364, 366, 379, 381 n. 1), Alxatib and Pelletier (2011b, pp. 293–294) and Ripley (2011a, p. 173 n. 8). Cf. also Serchuk et al. (2011, p. 558 n. 13).

[24] Cf. Smith (2011, p. 61).

[25] The fuzzy toolbox is vast and ever growing and what follows is certainly not a complete presentation of its contents.

Fig. 1 Conditions on t-norms

$$\begin{array}{rcl} x \wedge y & = & y \wedge x \\ (x \wedge y) \wedge z & = & x \wedge (y \wedge z) \\ x_1 \leq x_2 & \Rightarrow & x_1 \wedge y \leq x_2 \wedge y \\ y_1 \leq y_2 & \Rightarrow & x \wedge y_1 \leq x \wedge y_2 \\ 1 \wedge x & = & x \\ 0 \wedge x & = & 0 \end{array}$$

5.2.1 Other connectives

As examples of systems of fuzzy logic distinct from Zadeh logic, let us consider *t-norm fuzzy logics*.[26] A t-norm is a binary function $\wedge$ on [0, 1] satisfying the conditions shown in Fig. 1. A t-norm logic is specified by picking a t-norm and taking it to be the conjunction operation, and then defining the other operations (conditional, negation and so on) in certain specific ways. Notably, the conditional is taken to be the residuum of the t-norm:[27]

$$x \to y = \max\{z : x \wedge z \leq y\}$$

and the negation the precomplement of the conditional:

$$\neg x = x \to 0$$

Figure 2 shows the conjunctions, conditionals and negations in three prominent t-norm logics.[28]

It is common in these logics to define a second, ‘weak’ (or ‘lattice’) conjunction (with the t-norm conjunction then termed ‘strong’). In all these logics, the weak conjunction is the same as the min operation used to define conjunction in Zadeh logic.[29]

In light of this, let’s return to the objections presented in Sect. 2.

Fine I and Osherson and Smith I In Łukasiewicz logic, when $[P] = [R] = 0.5$, $[P \vee R] = 1$ and $[P \wedge R] = 0$.[30] This meets Fine’s desiderata. Likewise, in Łukasiewicz logic, *whatever* the degree of truth of α, $[\alpha \wedge \neg\alpha] = 0$ and $[\alpha \vee \neg\alpha] = 1$.[31] This meets Osherson and Smith’s desiderata.

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26 For a slightly longer brief introduction see Smith (2012) and for full details see Hájek (1998).

27 The residuum exists iff the t-norm is left-continuous.

28 Contrast the methodology here with that of supervaluationists such as Kamp and Partee (1995), who focus on a relatively small number of isolated data points—e.g. $[\alpha \vee \neg\alpha] = 1$, $[\alpha \wedge \neg\alpha] = 0$ and $[\alpha \wedge \alpha] = [\alpha]$—and then try to hit them. In t-norm fuzzy logics, by contrast, a broad system of constraints that anything worthy of the name ‘conjunction’ should satisfy is outlined (i.e. the t-norm conditions) and operations satisfying these constraints are then investigated—together with other connectives defined so that they all fit together in ways that are important in logic.

29 So in Gödel logic, there is no difference between the strong and weak conjunction.

30 Where $\wedge$ is the Łukasiewicz t-norm and $\vee$ is its dual: $x \vee y = 1 - ((1 - x) \wedge (1 - y))$.

31 Where $\wedge$ is the Łukasiewicz t-norm and $\vee$ is its dual. See Belohlavek et al. (2009, p. 31), Belohlavek and Klir (2011b, p. 138) and Paoli (forthcoming).

Fig. 2 Three prominent t-norm logics

Łukasiewicz logic :

$$x \wedge y = \max(0, x + y - 1)$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ 1 - x + y & \text{if } x > y \end{cases}$$

$$\neg x = 1 - x$$

Gödel logic :

$$x \wedge y = \min(x, y)$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{if } x > y \end{cases}$$

$$\neg x = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

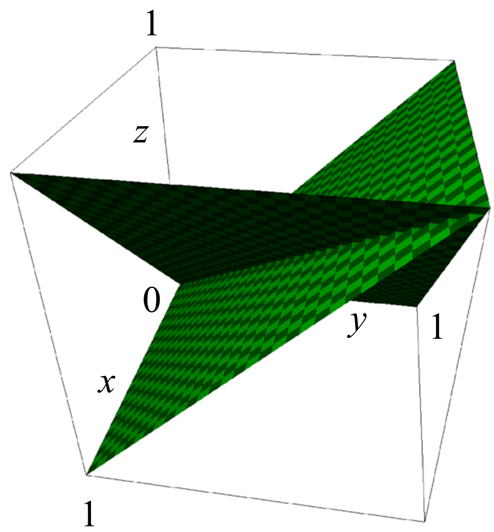
Product logic :

$$x \wedge y = x \cdot y$$

$$x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y/x & \text{if } x > y \end{cases}$$

$$\neg x = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Fig. 3 Min and max



Kamp We do not have to define $[\neg\alpha] = 1 - [\alpha]$. In Gödel logic, $\alpha \wedge \neg\alpha$ is 0 true and $\alpha \wedge \alpha$ is 0.5 true when $[\alpha] = 0.5$. In product logic, $\alpha \wedge \neg\alpha$ is 0 true and $\alpha \wedge \alpha$ is 0.25 true when $[\alpha] = 0.5$. This meets Kamp's desiderata.

Fine II In Łukasiewicz logic, when $[P] = [R] = 0.5$, 'P and P' is true to degree 0.5 where 'and' is read as weak conjunction and 'P and R' is true to degree 0 where 'and' is read as strong conjunction (Paoli forthcoming). This meets Fine's desiderata.

Osherson and Smith II Consider Fig. 3. The lower 'paper aeroplane' is the graph of $z = \min(x, y)$; the upper 'paper aeroplane' is the graph of $z = \max(x, y)$. Fuzzy conjunctions/intersections (almost) fill the space below min; fuzzy disjunctions/unions (almost) fill the space above max; and fuzzy *averaging operations* fill the space in between min and max.[32] So—setting aside the point made in Sect. 5.1 and taking it as a datum that (a)'s degree of membership in 'striped apple' is greater than its degree of membership in 'apple'—the combination of 'striped' and 'apple' to form 'striped apple' could be modelled by an averaging operation. Not only are fuzzy theorists not committed to Zadeh intersection and union as the only binary operations on fuzzy sets—they are not even committed to intersections and unions in general: they can also make use of averaging operations.[33]

5.2.2 Determinately operator/truth predicates/degreed predicates

It is straightforward to introduce a 'determinately' operator Δ where:[34]

$$[\Delta\alpha] = \begin{cases} 1 & \text{if } [\alpha] = 1 \\ 0 & \text{if } [\alpha] \neq 1 \end{cases}$$

When $[\alpha] = 0.5$, then even in Zadeh logic:

$$[\Delta\alpha \wedge \Delta\neg\alpha] = [\Delta\alpha \wedge \neg\Delta\alpha] = 0$$

Similar remarks apply to truth predicates[35] and to degreed versions P_n ($n \in [0, 1]$) of predicates P where:

$$[P_n a] = \begin{cases} 1 & \text{if } [Pa] = n \\ 0 & \text{if } [Pa] \neq n \end{cases}$$

For example, when $[Pa] = 0.5$, then even in Zadeh logic $[P_1 a \wedge P_0 a] = 0$. Thus the fuzzy theorist could accommodate the (alleged) datum that 'Bob is bald and Bob is not bald' is definitely false when Bob is borderline bald by saying that speakers interpret this sentence as (for example) $\Delta Bb \wedge \Delta\neg Bb$ or $B_1 b \wedge B_0 b$ rather than as $Bb \wedge \neg Bb$.

5.3 Pragmatic resources

While I do not share the intuition that 'x is red or orange' is always definitely true when x is borderline red/orange, I do think that one can tell stories in which such a

32 For details see Belohlavek and Klir (2011a, pp. 57–60).

33 This point is made by Belohlavek et al. (2002, p. 578). (Note that this also works as a response to the dual disjunction objection mentioned in n. 7 above; cf. Belohlavek et al. (2002, p. 580).) There are other ways of accommodating Osherson & Smith's data within a fuzzy framework—see e.g. Zadeh (1978) and Zadeh (1982, p. 291) (and Osherson and Smith (1982, Sect. 4.1) for criticism—and Belohlavek et al. (2009, p. 33) for counter-criticism).

34 See Takeuti and Titani (1987) and Baaz (1996).

35 For details see Smith (2008, Sect. 5.5).

claim is fully assertible. For example, Tappenden (1993, p. 565) presents a case in which

> you have the job of sorting color samples on an assembly line. The samples come along the line in varying shades of red or orange. No other colors are sent rolling out. You are to drop the orange samples into one bin and the red ones into another. Every so often an indeterminate case comes along and you cannot make up your mind about it, so you set it aside.

Each time you do this, the foreman comes along and says, pointing to the sample: "That is red or that is orange."[36] Eventually, we may suppose, you get the message and classify all the samples one way or the other, rather than setting some aside. Now even a proponent of Zadeh logic can explain why we find 'That is red or that is orange' fully assertible in this context: by appealing to pragmatics. This context is rather special in that each sample must be put into a bin and there are only two bins available. It is very natural to hear the foreman's utterance of 'That is red or that is orange' as a reminder of precisely these two points. That is, the information conveyed by the utterance in this context is the same as would be conveyed by the (more unwieldy) utterance 'Each sample must be put into this bin or that bin'—and there is no reason why a proponent of Zadeh logic should regard the latter as true to any degree other than 1.

Other examples in the literature in which a particular context is described in which certain sentences do indeed seem fully assertible—and yet this apparently conflicts with fuzzy theory (which seems to accord them an intermediate degree of truth)—can be handled in the same way: in these special contexts, uttering the sentences serves as a convenient way of conveying information that (even according to the fuzzy theorist) is clearly true.[37]

6 Questioning the relevance of data

One response to the suggestion that data about ordinary usage of vague language in the presence of borderline cases refute (or even pose difficulties for) theories of vagueness based on fuzzy logic would be that this is impossible, because fuzzy logic is a normative theory. The thought might be expressed as follows: "Fuzzy logic tells us how people *should* reason in the face of vagueness; if people do not in fact reason in such ways, that simply shows that their reasoning is defective. As Frege taught us, the laws of logic are not like the laws of physics. If a moving body violates your proposed laws of motion, this shows that you got your laws wrong. If an agent violates the laws of logic, this shows that her reasoning is incorrect."

This kind of response would be too strong. In cases such as fuzzy theories of vagueness, where logics are investigated in connection with natural language—whether as

[36] In Tappenden's example the foreman waits until you have a pile of samples, and then says "Every one of these samples is either red or orange".

[37] For further details and examples, see Smith (2008, pp. 260–262). An example not discussed there—but which can be handled in the same way—is given by Kamp and Partee (1995, pp. 156–157). Cf. also the example at Bonini et al. (1999, p. 390).

part of giving a formal semantics of natural language or as part of codifying correct reasoning as carried out in such language—there is two-way interaction between normative and descriptive aspects. We cannot ignore actual usage altogether and simply claim that logic is a normative enterprise not a descriptive one. At the very least there has to be enough connection between the logic and the empirical facts to make it the case that it is the correct logic *for this language*. On the other hand, the enterprise is importantly different from a purely descriptive one. The option is indeed open of saying—at least in some cases—that ordinary speakers are confused or mistaken or in some other way not behaving as they should.

Many contributors to this literature do not discuss these issues at all. One author whose discussion is admirably detailed, however, is Ripley (2011a). Here is some of what he says:

> We have supervaluationist and contextualist and fuzzy theories of vagueness, and we can take these theories to be formal semantic theories, answerable to speaker intuitions in just the same way that other semantic theories—about gradable adjectives, or quantifier inferences, say—are.
> …there is of course much more to participant responses than simply their competence; any number of performance factors may intervene. While there is no direct inference to be made from data about participants' responses to conclusions about their competence, the two are still related. The connection is provided by theories of the intervening performance factors.
> …different logical theories accord different status to borderline contradictions—some predict them to be fully true, some predict them to be at best half-true, and some predict them to never be true at all. I'll present and consider some evidence about which of these predictions seems to accord best with speakers' intuitions. Where predictions seem to come apart from participants' intuitions, I'll consider various performance-based explanations that might be offered. (p. 172)

As Ripley is aware, an assumption behind the modus operandi of formal semantics is that theories are answerable to the intuitions of *competent* speakers: they are not answerable to any and all linguistic behaviour. Thus there is a crucial assumption at work when theories of vagueness are tested against the intuitions of ordinary speakers: that these speakers are *competent*.

Now of course the existing studies take competent English speakers as subjects and involve words with which those speakers are presumably familiar (e.g. 'red' and 'near')—so what's the issue? Well, this is not enough—a further assumption is required: that the performance with particular words that is being measured in a study does indeed flow from competence with those words. (In the quotation above, this key assumption is expressed in the phrase "the two are still related". Note that Ripley simply assumes this—i.e. that performance is related to competence). Now the fact that this extra assumption is a substantial one can be hard to see. If a speaker is a competent user of 'red' and I measure her responses to sentences involving the term 'red' (and no other terms with which she is unfamiliar) then how could it be that her performance was in no way related to her competence? Well, consider the following case. Suppose that subjects are given spoken words and asked to write them down. The only materials made available to them are coloured pencils. We record which colour each subject uses for each word. Now even assuming that the subjects are competent users of the given words, their performance (choice of colour) probably does not *in any way* flow from that competence: not only not directly (i.e. because it is filtered by performance factors)—rather, not at all. But then their performance sheds no light on the *semantics* of the words involved in the study: even though (to repeat) the subjects are competent users of the words involved.[38]

The crucial point is this: the thing that the imagined study is asking subjects to do with words is something about which competence *doesn't care*. That is, it is no part of gaining competence with these words that one be disposed to write them in certain colours. Now the possibility I want to raise is that subjects' responses to compound statements involving vague predicates in the presence of borderline cases do not flow from their competence with those predicates. That is, it is no part of gaining competence with vague predicates that one use compound sentences involving these predicates in particular ways in the presence of borderline cases.

Let me clarify what I'm saying here. First, I am distinguishing simple predications such as 'x is red' and 'y is heavy' from *compound* sentences involving vague predicates such as 'x is red and y is heavy', 'x is red or x is heavy', 'x is red and y is not red' and so on. Second, I am distinguishing situations in which only clear cases of predicates are involved (either because the predicates are precise, or because they are vague but the only objects in question are all definite cases or definite noncases of each predicate) from cases involving borderline cases of predicates (e.g. a case involving Bill and Ben and the predicates 'tall' and 'heavy', where Bill is borderline tall and Ben is borderline heavy).

Obviously competence with vague predicates such as 'tall' and 'heavy' involves using them in certain ways in simple predications, whether or not any borderline cases are present: roughly speaking (one might debate the details here—the point is just that competence requires *something* in these cases) one should agree with simple predications applied to clear cases, disagree with simple predications applied to clear noncases, and hedge over simple predications applied to borderline cases. Equally obviously, competence with vague predicates such as 'tall' and 'heavy' involves using compound sentences involving them in certain ways when no borderline cases are involved: one should reject 'x is red and y is heavy' when x is a clear noncase of 'red' or y is a clear noncase of 'heavy'; one should accept 'x is red or y is heavy' when x is a clear case of 'red' or y is a clear case of 'heavy'; and so on.

The possibility I'm raising is that when it comes to compound sentences involving vague predicates in contexts involving borderline cases of those predicates, competence imposes no requirements at all. For example, it is no part of being competent with 'tall' and 'heavy' that one react in *any* particular way to 'Bill is tall and Ben is

38 Of course this is just an example, to make clear the *possibility* that one might be competent with a word, and yet one's particular performance with that word (on some occasion) might not in any way flow from that competence. For all I know it may be that one can gain useful information about the semantics of words by seeing in what colours subjects choose to write them. My point is just that for this to be so, their performance would indeed have to flow from their semantic competence—and this is a substantive assumption: it is not automatic.

heavy' (or 'Bill is tall or Ben is not heavy', etc.) when Bill is borderline tall and Ben is borderline heavy.

I'll say below why I think this is a possibility that needs to be taken seriously. For the moment, note that this possibility casts grave doubt on the relevance of much empirical work to the logic or semantics of vagueness. Empirical work that tests behaviour involving simple predications in the presence of borderline cases is unaffected. But as has often been pointed out, the results of this kind of work have left open numerous different theories of vagueness. The hope surrounding recent empirical work has been that we can differentiate these theories by testing behaviour involving compound sentences in the presence of borderline cases (because, for example, one kind of theory says that 'Bill is tall and not tall' is 0.5 true when 'Bill is tall' is 0.5 true, while another kind of theory says that it is 0 true—and so, the thought goes, the former kind of theory predicts greater agreement with 'Bill is tall and not tall' when Bill is borderline tall). It is this hope that would be dashed if competence in fact imposes no requirements on speakers' reactions to compound sentences when borderline cases are in play.

So, why should we think that competence might be silent in this way? One reason is that if competence did require particular kinds of behaviour in relation to compound sentences involving vague predicates in the presence of borderline cases of those predicates, then it would be extremely hard to see how any of us could have acquired such competence. Our training with vague predicates consists in learning to recognise their clear cases and clear noncases—and in some cases perhaps also their clear borderline cases. We learn to accept/assert in the presence of clear cases, reject/deny in the presence of noncases, and hedge in the presence of borderline cases. At the same time, our training with connectives such as 'and', 'or' and 'not' involves statements which are clearly true or clearly false. For example, ordinary speakers are typically taught (whether explicitly or implicitly) that 'Pa and Qb' is true when Pa and Qb are both true and false when one or both of the components is false—but they are not taught what to say about 'Pa and Qb' when a is a borderline case of P and b is a borderline case of Q. Nowhere, then, do we seem to acquire any training on how to handle compound sentences involving vague predicates in the presence of borderline cases. So unless knowledge of how to handle them is innate—which is absurd—it seems that no particular way of handling them could be required by competence with the terms in which they are expressed.

There are certain isolated exceptions where speakers do receive training in how to handle complex claims in the presence of borderline cases—but reflecting on them only confirms how different they are from the normal case, in which we receive no such training. Consider a doctor learning to diagnose a condition that has multiple symptoms, for example depression.[39] Patients will typically be borderline cases of one or more of these symptoms, and doctors need to learn to weigh the importance of the different symptoms and to aggregate the degrees to which a patient exhibits each symptom in order to reach an overall diagnosis. One might wish to say that 'depression' is a complex concept—a conjunction of its symptoms—and that the

[39] See Verkuilen et al. (2011, Sect. 6.4) for further discussion of this case, including a list of the nine major symptoms.

doctor is learning precisely how to wield such a concept in the presence of borderline cases of its components. Say that if you wish: but note how specific this case is. Even once the doctor is competent at judging the presence of each individual symptom, she still has to learn how to aggregate those judgements: the method does not follow simply from her competence with the word ‘and’. In the case of another condition, which has the same number of symptoms, the aggregation method might be different. If we define a new term using the symptoms of depression and some familiar connectives—e.g. symptom 1 and (symptom 2 or symptom 3) and not (symptom 4 and symptom 5)—we should not automatically expect doctors well trained in diagnosing depression to agree in diagnosing the presence of this new condition. Overall, then, the point is that training in how to wield complex statements or concepts in the presence of borderline cases of their components is the exception, not the norm; when speakers do receive such training, it is specific to the case in hand and not automatically generalisable.

A second reason for thinking that competence imposes no requirements on speakers’ reactions to compound sentences when borderline cases are in play emerges from the clash of intuitions documented in Sect. 4.1. Fine, Kamp and others think that ‘Bill is tall and not tall’ is obviously false when Bill is borderline tall; Zadeh, Dubois & Prade and others have a completely different intuition. What can we say about this situation? One option is that Kamp et al and Zadeh et al mean different things by ‘and’ or ‘not’ or ‘tall’—and each camp is using its own terms correctly. But this is extremely far fetched. How would these speakers have come by different meanings for these terms? It is fanciful to suggest that they are embedded in different speech communities in which these terms are used differently (say in the manner of Burge 1979). A second option is to say that one camp lacks competence with ‘and’ or ‘not’ or ‘tall’—but again, this is bordering on absurd. But then if both camps are using the same terms with the same meanings and are doing so competently—and yet they have this disagreement—it must be that competence imposes no requirement either way here. As far as competence is concerned, whether to say that ‘Bill is tall and not tall’ is false or middling true when Bill is borderline tall is on a par with the question whether to write it in blue or green: competence doesn’t care either way.

A third reason for thinking that competence imposes no requirements on speakers’ reactions to compound sentences when borderline cases are in play emerges from empirical work. I have argued that it is hard to see how competence could require any particular behaviour in such cases because ordinary speakers apparently receive no training in such matters. It might be countered that the training is subtle and indirect and therefore goes unnoticed. But in that case we should at least expect the requirements of competence to manifest themselves as regularities in the behaviour of ordinary speakers. But such regularities have not emerged from the empirical work: the picture that emerges is as diverse and chaotic as the clash of intuitions just considered.[40] Here are some examples:

[40] Even if the data were highly regular (which, as we shall see, they are not) this would not show that the regularly observed behaviour flows from competence. Compare: it might be that for some reason connected with their upbringing—but not connected to their competence with the words involved—all subjects write certain sentences in green and certain other sentences in red. Not all regularly observed behaviours are results of competence: there are also other kinds of regularities (e.g. widespread systematic biases).

- On two separate occasions about one month apart, McCloskey and Glucksberg (1978) presented subjects with pairs of category names and things and asked whether the thing falls under the category (Yes or No). They found high levels of both between-subjects disagreement and within-subjects inconsistency. That is, different subjects often rated the same pairs in different ways—and the same subject often rated the same pair in different ways on the two occasions.[41]
- Oden (1977) presented pairs of simple predications and had subjects perform three tasks: judging the average degree of truthfulness of the two statements; judging the degree to which one statement or the other was true; and judging the degree to which both statements were true. Concerning the last of these tasks, Oden writes: "of the 32 subjects, 23 were fit best by the multiplying rule, whereas only 8 were fit best by the averaging rule and only 1 was fit best by the minimum rule" (p. 571). Oden takes this as strong support for a treatment of conjunction wherein the degree of truth of 'α and β' is obtained by multiplying the degrees of truth of α and β. What is really striking about the data however is that while 23 out of 32 subjects responded in accordance with this multiplicative rule, a full quarter of respondents responded in accordance with a different rule: an averaging rule according to which the degree of truth of 'α and β' is obtained by averaging the degrees of truth of α and β. There are then two quite distinct patterns of response here: and a significant number of participants follows each pattern.
- Ripley (2011a) projected a slide with seven circle/square pairs on it and asked subjects to indicate their level of agreement with a given compound sentence as applied to each pair. Ripley noted four distinct patterns of response:

  Flat (24/149 participants): A flat response gives the same number for every question.

  Slope up (22/149 participants): A slope up response is not a flat response, and it never goes down from question to question.

  Slope down (18/149 participants): A slope down response is not a flat response, and it never goes up from question to question.

  Hump (76/149 participants): A hump response is not a flat response or a slope response, and it has a peak somewhere between the first and last question; before the peak, responses never go down from question to question, and after the peak, responses never go up from question to question (pp. 174–176).

  9 participants fitted none of these four patterns. Ripley focusses his discussion on the hump responses, which he notes form a majority of the responses. But it is the barest of majorities. What is really striking about these data is the great variety they exhibit—not only of responses but of response patterns. There are four quite distinct patterns of response here: and a significant number of participants follows each pattern.
- Alxatib and Pelletier (2011b) presented subjects with an image of five men of different heights standing against a height scale (as in a police line-up). The men are identified by numbers, 1–5; their heights appear to be about 5′4″, 5′11″, 6′6″,

[41] Note that these studies concern simple predications, not compound sentences. For further results showing persistent disagreement and inconsistency amongst responses see Parikh (1994, p. 524) (and further references there), Hampton (2011, Sect. 9.4) and Egré et al. (2013) (and further references there).

5′7″ and 6′2″ respectively. Subjects were asked to respond—True, False or Can't Tell—to sentences about each man. Out of the five sentences of the form '#*x* is tall and not tall' (i.e. one sentence for each value of *x* between 1 and 5) the second received the greatest number of True responses: 44.7 % of subjects responded True to '#2 is tall and not tall'. In a separate study, Serchuk et al. (2011) asked subjects to imagine that "on the spectrum of rich women, Susan is somewhere between women who are clearly rich and women who are clearly non-rich". They then asked subjects for their response—'true', 'not true, but also not false', 'partially true and partially false', 'false', 'both true and false' or 'true or false, but I don't know which'—to sentences including:

(10) Susan is rich and Susan is not rich.

(11) Susan is rich and it is not the case that Susan is rich.

19 % of subjects responded 'true' to (10) and 7 % responded 'true' to (11). As Serchuk et al. (2011, p. 571) note: "These results differ greatly from those found by Alxatib and Pelletier".[42]

Let's take stock. The hypothesis that I have put on the table is this: competence with connectives and vague predicates does not require responding in any particular way to compound sentences involving vague predicates in contexts involving borderline cases of those predicates. This is a possibility that has not hitherto been noticed. I am not claiming that this hypothesis is true. I am claiming that it is a live option: the reasons just presented establish that it at least needs to be seriously considered. But once we do consider it, the role of empirical data about linguistic behaviour involving compound sentences with vague predicates in contexts including borderline cases is thrown into serious doubt. We can no longer assume that such behaviour flows from competence—not only not directly, because it is filtered by performance factors: there may be no connection at all. In that case, such data cannot play the role of deciding between theories of vagueness that differ over the truth conditions of compound sentences about borderline cases.[43]

Before moving on, three further issues need to be discussed. First, it might be argued that the hypothesis just mooted must be false, because if it were true then—given the ubiquity of vague predicates and borderline cases thereof—chaos would reign (we would all be doing different things): whereas in fact we communicate smoothly using

42 Ripley (2011a) asks subjects for their responses to the sentences 'The circle is near the square and it isn't near the square' and 'The circle both is and isn't near the square'. He says (p. 174) that his results are similar to those reported by Alxatib and Pelletier (2011b). However this is not clear, because Ripley asks subjects for their level of *agreement* (he gives subjects seven possible responses, with 1 labelled 'Disagree' and 7 labelled 'Agree') whereas Alxatib and Pelletier (like Serchuk et al.) ask subjects for responses framed in terms of *truth*. Cf. Sect. 5.1 above.

43 I raised a version of this hypothesis in Smith (2011, p. 61) but my remarks there were necessarily brief. Ripley (2011b, pp. 63–64) responds, concluding: "Since the goal is to learn about borderline cases, diving in and asking participants about borderline cases is an important source of data. It cannot be dismissed as unreliable on the grounds of participants' discomfort; that discomfort is itself part of the phenomenon to be studied. This sort of methodology has resulted in considerable success when it comes to simple categorization judgments, and there is no reason to expect it to be less reliable when it comes to compound judgments." My point, however—as I hope is clear from my longer discussion here—is not simply that subjects are *uncomfortable* around borderline cases: it is that competence may well *impose no requirements* here.

vague language all the time. Of course I agree that vagueness is ubiquitous, that borderline cases abound (e.g. there are people of all ages and heights about—hence plenty of borderline cases of ‘young’, ‘old’, ‘short’ and ‘tall’) and that we nevertheless manage to communicate smoothly using vague language all the time. I deny that this casts doubt on the hypothesis that competence with connectives and vague predicates does not require responding in any particular way to compound sentences involving vague predicates in contexts involving borderline cases of those predicates. I think that if you were to get into a situation in which (a) it is important how people react to a *compound* sentence involving vague predicates—that is, successful communication depends on predictable reactions, on speakers taking the sentence in the same way, and (b) borderline cases of these predicates are under consideration—then you could *not* in fact expect that communication would proceed smoothly. (Imagine that a little old lady needs someone to help her. You send out some boy scouts to find someone ‘big and strong’. Suppose also that you know full well that every person in the vicinity is either a borderline case of ‘big’ or of ‘strong’: then you should *not* expect predictable results.) This must mean that we generally avoid such situations—and I think this is precisely how we do in fact proceed. There are at least three methods by which we achieve this result:

(1) precisifying vague predicates so as to eliminate borderline cases (think of legal definitions of ‘adult’ or definitions of certain medical conditions in diagnostic manuals)
(2) coining new vague predicates whose clear cases are precisely the borderline cases of our existing vague predicates (think of ‘tween’, whose central region covers the borderline between ‘child’ and ‘teenager’, or ‘balding’, whose central region covers the borderline cases of ‘bald’)
(3) giving special training (think of ‘depression’ as discussed above).

To pose a problem for my hypothesis, someone would need to show that we regularly and successfully communicate using compound sentences involving vague predicates in contexts in which borderline cases of those predicates are in play (in a way that matters: i.e., successful communication depends on different speakers assessing complex claims about borderline objects in the same way). However (with certain isolated exceptions, e.g. the case of doctors communicating using the term ‘depression’) this seems not to be the case.

The second issue is compositionality. Given the hypothesis under consideration, two competent speakers might, in the same context, disagree in their assessment of some compound sentence while agreeing in their assessments of its components. Indeed—but compositionality is not sacrosanct. It standardly forms part of the explanation of why speakers can interpret new utterances in a systematic, regular way. Where speakers’ interpretations are not systematic or regular, compositionality need not be maintained. As Kamp and Partee (1995, p. 185) note: “compositionality in some form appears to be a crucial part of any account of *semantic competence*” [my emphasis]. Where we exceed the bounds of semantic competence, there we are free to abandon compositionality.

The third issue is whether this hypothesis helps fuzzy theories in the long run. In the short term it rules out as irrelevant certain kinds of evidence that some authors have tried to use against fuzzy theories—but then doesn't it also end up making it impossible to argue that fuzzy theories are better than their rivals? For now no matter how a speaker assesses 'Bill is tall and not tall' (say) when Bill is borderline tall, we cannot say that she is or is not behaving as competence dictates. So how can we argue for a fuzzy logical treatment of conjunction over, say, a supervaluationist treatment? Well, we can't (or at least, not on the basis of competent usage). But we can nevertheless argue in a different way that fuzzy theories hold a special place amongst current theories of vagueness: they alone can yield a coherent overall description of how competent speakers use vague language. In order to see this, consider classical, three-valued and fuzzy valuations (say for a standard first order language—but the point generalises).[44]

In a classical valuation, the extensions of predicates are crisp sets. There is then only one natural way of defining truth conditions for compound sentences: using the classical truth tables.[45]

In three-valued models, the extensions of predicates are three-valued sets.[46] There are then several kinds of way open for defining the truth conditions of compound sentences—and several specific ways within each kind. For example:

- The recursive kind of way, where we give (three-valued) truth tables for the connectives. Within this kind: the three-valued logics of Bočvar, Kleene, Łukasiewicz, Post and so on.[47]
- The 'possible classical extensions' kind of way, where we go via a consideration of *classical* valuations that extend the given three-valued valuation.[48] Within this kind: supervaluationism (and subvaluationism) and the degree-theoretic form of supervaluationism.[49]

In fuzzy models, the extensions of predicates are fuzzy sets. The standard way of defining truth conditions for compound sentences is then the recursive way: for each connective, we specify a corresponding function on fuzzy truth values. As we saw in Sect. 5.2.1, there are many possible choices here. Note however that we do not have to go the recursive way: it is also possible to consider classical valuations that extend a fuzzy valuation and to proceed in, for example, a supervaluationist way.[50]

Now let's return to the issue of giving a coherent overall description of how competent speakers use vague language, in light of the hypothesis that competence with

44 By a 'valuation' I mean the part of a model that assigns values to primitive nonlogical symbols—as distinct from the part that says how values are assigned to complex expressions, given a valuation.

45 Here and in the following, for ease of presentation I explicitly mention only connectives when considering complex expressions—but the points made are general.

46 A three-valued set is a total function from the domain to a set of three truth values, say $\{0, *, 1\}$. Objects mapped to 1 are thought of as definite cases of the predicate, objects mapped to 0 as definite noncases of the predicate and the remaining objects as borderline cases. The points made here about three-valued valuations apply equally to partial valuations, in which the extensions of predicates are partial or gappy sets: partial functions from the domain to the classical truth values $\{0, 1\}$.

47 See Smith (2012) for an introduction to such logics.

48 Here 'extend' means that they retain all mappings to 1 and 0.

49 See Smith (2008, Sect. 2.4) for an introduction to these views.

50 See Smith (2008, Sect. 2.4.1) for details. Cf. also Fermüller and Kosik (2006).

connectives and vague predicates does not require responding in any particular way to compound sentences involving vague predicates in contexts involving borderline cases of those predicates. Suppose we model predicates and concepts as crisp or three-valued sets. Suppose we then encounter a speaker who appears, for all the world, to treat objects as falling under certain concepts to greater or lesser degrees, and to process compound claims roughly in accordance with, say, Product logic. We could make no sense of such a speaker—and yet we can indeed expect to encounter such speakers! (Recall the experiments of Oden (1977) discussed earlier in this section.) So we should not expect a coherent overall description of how competent speakers use vague language to be achievable if we start by representing predicates and concepts as crisp or three-valued sets.

The problem does *not* arise in the opposite direction. We can move from a fuzzy valuation to a classical one by imposing a threshold $x \in [0, 1]$ (and then setting all lower values to 0 and all higher values to 1) and we can move to a three-valued valuation by imposing two thresholds (and then setting all values below the lower threshold to 0, all values above the upper threshold to 1, and all values in between to the third value). Thus, starting with a fuzzy valuation, we can easily recover three-valued, super- and sub-valuationist and even classical logics (as well as different fuzzy logics). So now suppose we model predicates and concepts as fuzzy sets. Suppose we then encounter a speaker who appears, for all the world, to treat objects as having one of only three statuses (Yes, No and Maybe) relative to certain concepts and to process compound claims roughly in accordance with, say, supervaluationist logic. No problem—we can make perfect sense of this! The speaker is simply—by imposing thresholds—moving from her initial fuzzy representation of concepts to a three-valued representation, and then proceeding along supervaluationist lines. As we noted, this is a perfectly legitimate way of assessing compound statements *within* an overall view according to which predicates and basic concepts are originally represented as fuzzy sets.

Thus, only theories that use fuzzy sets (or something relevantly like them) to model predicates and concepts can hope to rationalise the full range of legitimate behaviour with vague language that we have seen and should expect to see.[51]

7 Conclusion

Several views are open at this point. First, suppose we do not take seriously the hypothesis about competence raised in Sect. 6. Then we will want a systematic story that explains all the facts about ordinary usage. Of course we do not at present have all those facts at hand: there is a growing number of relevant empirical studies—but as yet, no complete picture of exactly what ordinary speakers say under what conditions. So we cannot hope at this point to construct a full theory. We can however distinguish possible views by the logical resources on which they think a complete theory would

[51] A related point concerns the enormous success of fuzzy logics in applications (e.g. in engineering and computer science). (In this connection, Serchuk et al. (2011, p. 561 n. 14) state that "degree theory is the only theory of vagueness that has been put to use".) This uptake and success is itself an empirical fact that demands explanation. From a point of view according to which, say, classical or supervaluationist logic provides the (only) correct treatment of vagueness, this fact is incomprehensible.

need to draw (of course they might also differ over whether pragmatic resources would also be required):

(1) a single set of fuzzy logical operations (i.e. one conjunction, one disjunction etc.)
(2) multiple fuzzy logical operations (each appropriate to different domains)
(3) multiple fuzzy logical operations *and* non-truthfunctional connectives (such as in supervaluationism)

The first option is not common. The second option is popular.[52] We also should not overlook the possibility of the third option: as we have noted, super- and sub-valuationist logics can be generated from *fuzzy* valuations and so appealing to them in certain situations is quite compatible with maintaining the fundamental importance of fuzzy models.[53]

Second, suppose we accept the hypothesis about competence raised in Sect. 6. In that case we will not seek a systematic story that *explains* all the facts about ordinary usage (as flowing from competence). However, we will still want a story that rationalises the range of behaviours with vague language: and as we have seen, this will need to involve a base level at which predicates and/or concepts are represented in terms of fuzzy sets, together with an account of how this or that kind of behaviour arises either by using certain fuzzy operations, or by moving from a fuzzy set representation of predicates to a more coarse-grained representation (e.g. by imposing thresholds) and then using the resources of, for example, classical or supervaluationist logic.

According to the truth-functionality objections, fuzzy theories of vagueness are incompatible with ordinary usage of compound propositions in the presence of borderline cases. So do these objections have any force? No, they do not. On any of the views just considered, fuzzy theories play a central role.[54]

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[52] See Oden (1977, p. 572), Zadeh (1982, p. 292), Kamp and Partee (1995, p. 148 n. 11), Belohlavek et al. (2002, p. 578), Belohlavek et al. (2009, p. 31), Fermüller (2011, pp. 200–201) and Paoli (forthcoming).

[53] This point tends to be overlooked. For example, Osherson and Smith (1982, p. 303) simply *assume* that the "gradient theorist"—i.e. the theorist who takes concepts to have degrees of membership—will want to take a degree-functional approach to conjunctive concepts.

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