Chapter 3
Hofweber's Nominalist Naturalism

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Abstract In this paper, we outline and critically evaluate Thomas Hofweber's solution to a semantic puzzle he calls Frege's Other Puzzle. After sketching the Puzzle and two traditional responses to it—the Substantival Strategy and the Adjectival Strategy—we outline Hofweber's proposed version of Adjectivalism. We argue that two key components—the syntactic and semantic components—of Hofweber's analysis both suffer from serious empirical difficulties. Ultimately, this suggests that an altogether different solution to Frege's Other Puzzle is required.

Keywords Thomas Hofweber · Number words · Frege's other puzzle · The substantival strategy and the adjectival strategy

3.1 Introduction

This paper is part of a larger project in which we develop an empirically informed, methodologically naturalistic philosophy of mathematics. Our primary concern is with the natural numbers of basic arithmetic, and the idea that empirical results from linguistics, psychology, and cognitive neuroscience may shed light on their nature and our knowledge of them. Our basic conviction is that such a methodologically naturalistic approach can help illuminate traditional core questions that preoccupy

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philosophers of mathematics: Do numbers exist, and if so, what are they like? Can we have mathematical knowledge, and if so, how? Indeed, we maintain that such methods can help where more traditional, a priori methodologies cannot.

Against the backdrop of this larger project, the present paper serves two main purposes. First, we provide a detailed exploration of one influential, methodologically naturalistic project of the sort that we seek to pursue: Thomas Hofweber's (2005, 2007, 2016) analysis of number talk and thought, and his defense of nominalism based on that analysis. In doing so, we pay special attention to the manner in which Hofweber's account – in contrast to more traditional, a priori approaches – relies heavily on linguistic considerations in support of this controversial ontological thesis.

Second, we argue that Hofweber's attempt to recruit such considerations to this end is unsuccessful. Hofweber's account is presented as a series of empirical hypotheses, regarding ordinary number-related talk and thought. As such, it is both appropriate – and charitable – to assess his view by the same standards operative in empirical research more generally, including linguistics. Like all local empirical hypotheses, Hofweber's hypotheses should be assessed (among other things) in terms of their ability to generate accurate predictions, and the extent to which they cohere with more basic background theory. By these standards, however, Hofweber's proposal performs poorly. Specifically, insofar as those hypotheses make concrete empirical predictions, they appear to be largely incorrect. In other cases, however, it is not clear whether Hofweber's hypotheses make concrete predictions, or how they may cohere with contemporary linguistic theory. Ultimately, the upshot will be that the best available linguistic evidence does not support nominalism, contra Hofweber.

The rest of the paper proceeds as follows. In Sect. 3.2, we sketch a linguistic puzzle, known as Frege's Other Puzzle, which Hofweber's analysis is primarily designed to solve. We explain how the Puzzle arises, along with two popular philosophical strategies for solving it. In Sect. 3.3, we outline Hofweber's solution, breaking the analysis developed into two components: a syntactic component and a semantic component. In Sect. 3.4, we criticize both components, arguing that neither stand up to empirical scrutiny. We conclude the paper in Sect. 3.5, where we summarize our conclusions and tease out some broader implications for methodologically naturalistic approaches to the philosophy of mathematics.

3.2   Frege's Other Puzzle

Hofweber's analysis is framed largely around a certain linguistic puzzle. In the *Grundlagen*, Frege (1884, §57) notes that number expressions such as 'four' are used in two importantly different ways:

> Since what concerns us here is to define a concept of number that is useful for science, we should not be put off by the attributive form in which number also appears in our everyday use of language. This can always be avoided. For example, the proposition 'Jupiter has four

> moons' can be converted into 'The number of Jupiter's moons is four'. Here the 'is' should not be taken as a mere copula ... Here 'is' has the sense of 'is equal to', 'is the same as' ... We thus have an equation that asserts that the expression 'the number of Jupiter's moons' designates the same object as the word 'four'.

Specifically, 'four' has an "attributive form" witnessed in (1a), and an apparently referential form witnessed in (1b).

(1)   a. Jupiter has four moons.
       b. The number of Jupiter's moons is four.

On its face, the function of 'four' in (1a) is to count the collection of moons belonging to Jupiter. In this respect, 'four' resembles non-referential expressions like the adjective 'large' or the determiner 'no' in (2):

(2)   Jupiter has large/no moons.

On the other hand, (1b) looks like a prototypical identity statement. As such, it apparently involves singular terms, namely 'the number of Jupiter's moons' and 'four'. In this respect, 'four' in (1b) resembles the name 'Wagner' in (3), due to Hofweber (2007).

(3)   The composer of *Tannhäuser* is Wagner.

This suggests that 'four' in (1b) is a numeral, or a name of a number.
    At the same time, there are clear semantic differences between attributive 'four' and the numeral 'four'. For example, numerals require singular morphology, whereas attributive 'four' requires plural morphology.

(4)   a. Which one of these three numbers is even? [Let's see. Three isn't, and five isn't ...] Four {is/??are}.
       b. How many of these eight numbers are even? [Let's see. Two is, and six is ...] Four {??is/are}.

Also, while attributive 'four' is typically acceptable with modifiers like 'exactly' and 'almost', the numeral 'four' is not.

(5)   a. {Four/??Almost four} is an even number.
       b. {Four/Almost four} children clapped.

Furthermore, numerals license entailments like (6a), but attributive 'four' does not.[1]

(6)   a. Mary divided four by two yesterday morning ⊨ Mary divided four by two yesterday
       b. Mary cooked exactly four eggs yesterday morning ⊭ Mary cooked exactly four eggs yesterday

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[1] This follows a referentiality test due originally to Kratzer and Heim (1998).

All of this suggests that 'four' serves different, and indeed incompatible, semantic functions: counting collections and naming numbers.

The above is puzzling because, as Felka (2014) notes, it seems that different occurrences of the same expression in semantically equivalent statements ought to serve the same semantic function. Consider the names 'John' and 'Mary' in (7a) and (7b), for instance.

(7)   a. John saw Mary at the mall.
       b. Mary was seen by John at the mall.
       c. John saw a Mary at the mall.

Plausibly, (7a) and (7b) are equivalent because the names serve the same semantic function in those examples, namely to refer. On the other hand, neither (7a) nor (7b) is equivalent to (7c), where 'Mary' is being used instead as a predicate.

This leads to what Hofweber (2005) calls *Frege's Other Puzzle*, which consists of the following four seemingly plausible, but jointly inconsistent, premises.

(FOP1)   (1a) and (1b) are semantically equivalent.
(FOP2)   The different occurrences of 'four' in (1a) and (1b) are witness to the same expression, namely 'four'.
(FOP3)   The different occurrences of 'four' in (1a) and (1b) serve different semantic functions.
(FOP4)   Different occurrences of an expression occurring in semantically equivalent statements serve the same semantic function.

FOP1 is taken for granted by everyone in the relevant debate. It is possible, though ultimately unsatisfactory, to deny this premise, however. For example, one could point out that while the follow up to (8a) is perfectly consistent, the follow up to (8b) seems contradictory.

(8)   a. Jupiter has four moons. In fact, the number of Jupiter's moons is *sixty-two*.
       b. ?? The number of Jupiter's moons is four. In fact, the number of Jupiter's moons is *sixty-two*.

Based on facts like (8a), Horn (1972) argues that attributive uses have *lower-bounded* truth-conditions, so that (1a) is true if Jupiter has at least four moons. Conversely, (8b) might be taken to show that (1b) has *two-sided* truth-conditions, and so is true if instead Jupiter has exactly four moons. If so, then FOP1 would be false. Call this *the Non-Equivalence Strategy*.

The obvious problem with the Non-Equivalence Strategy is that even if we grant that facts like (8) demonstrate the non-equivalence of (1a,b), we can easily reformulate the Puzzle by substituting (9) for (1b) in the original formulation.

(9)   Jupiter has exactly four moons.

The follow up to (10) sounds just as contradictory as that of (8b),

(10)   ?? Jupiter has exactly four moons. In fact, Jupiter has sixty-two moons.

and yet the Non-Equivalence Strategy would be inapplicable to this new formulation.

It is also possible, though ultimately untenable, to deny FOP2, thus resulting in what we might call *the Homonym Strategy*. According to it, the different occurrences of 'four' in (1a) and (1b) are witnesses to altogether different expressions, ones which just happen to be spelled and pronounced alike. In general, we do not expect homonyms like the noun 'fire' and the verb 'fire' to be acceptably intersubstitutable.

(11)   a. The rapid oxidation of combustible materials is fire.
        b. Let's {fire/??the rapid oxidation of combustible materials} John.

Similarly, Hofweber notes that substituting 'the number of Jupiter's moons' for 'four' in (1a) leads to unacceptability despite (1b) appearing to establish their coreferentiality.

(12)   Jupiter has {four/??the number of Jupiter's moons} moons.

However, because homonyms are typically spelled and pronounced alike as a matter of historical accident, we do not expect their meanings to be related. Thus, the problem with the Homonym Strategy is that the occurrences of 'four' in (1a) and (1b) are clearly semantically related; both tell us something about how many moons belong to Jupiter.

3.2.1   Two Strategies of Analysis

Given the failures of the Non-Equivalence Strategy and the Homonym Strategy, it appears that we must reject either FOP3 or FOP4. It turns out that nearly all approaches within the philosophical literature deny the former, including Frege (1884), Wright (1983), Hodes (1984), Hofweber (2005, 2007, 2016), Moltmann (2013), and Felka (2014). In fact, denying FOP3 is the hallmark of two opposing positions Dummett (1991, p. 99) dubs *the Substantival Strategy* and *the Adjectival Strategy*:

> Number-words occur in two forms: as adjectives, as in ascriptions of number, and as nouns, as in most number-theoretic propositions. When they function as nouns, they are singular terms, not admitting of the plural; Frege tacitly assumes that any sentence in which they occur as adjectives may be transformed either into an ascription of number ... or into a more complex sentence containing an ascription of number as a constituent part. Plainly, any analysis must display the connection between these two uses ... Evidently, there are two strategies. We may first explain the adjectival use of number-words, and then explain the corresponding numerical terms by reference to it: this we may call the adjectival strategy. Or, conversely, we may explain the use of numerals as singular terms, and then explain the corresponding number-adjectives by reference to it; this we may call the substantival strategy.

According to the Substantival Strategy, or *Substantivalism*, both occurrences of 'four' in (1a,b) are in fact numerals, and the apparently non-referential use witnessed in (1a) is to be explained in terms of the genuinely referential use witnessed in (1b). In contrast, according to the Adjectival Strategy, or *Adjectivalism*, both occurrences of 'four' in (1a,b) are either adjectives or determiners,[2] and the apparently referential use witnessed in (1b) is to be explained in terms of the genuinely non-referential use witnessed in (1a).

The most well-known defender of Substantivalism was, of course, Frege (1884). His primary interest was in developing an ideal logical language suitable for science. In such a language, the sole semantic function of a number expression would be to refer to numbers. Thus, non-referential uses of number expressions in natural language are misleading with respect to their ideal semantic function. Consequently, Frege proposes "converting" the attributive form witnessed in (1a) into the singular term witnessed in (1b). To do so, he first proposes paraphrasing (1a) as (1b), and then (equivalently) analyzes the latter as (13).

(13) $\#[\lambda x. \text{moon-of-Jupiter}(x)] = 4$

Here, '#' is a cardinality-function mapping a concept Φ to a natural number n representing how many objects fall under Φ. Thus, (13) is an identity statement: it equates a certain number, namely the number of moons belonging to Juptier, with the natural number referenced by the numeral '4', namely four. Thus, on Frege's proposal, (1a) and (1b) should both be analyzed as identity statements, at least for the purposes of an ideal logical language.

Now, as stated, Frege's Other Puzzle is a puzzle about *natural language*: How can one and the same expression serve seemingly different semantic functions in equivalent statements? On the other hand, Frege's analysis was not intended to be a piece of natural language semantics. Rather, as the above quotation from *Grundlagen* §57 makes clear, his primary objective was to "define a concept of number that is useful for science". Thus, the question arises as to whether Substantivalism might be viewed as an independently viable strategy for rejecting FOP3.

Indeed, something like this appears to be rhetorically suggested by Crispin Wright (1983). Speaking of Frege's example *abstraction principle* in (14),

(14) $\forall l_1. \forall l_2. D(l_1) = D(l_2) \leftrightarrow l_1 \sim l_2$
        (For any lines l_1 and l_2, the direction of l_1 is identical to the direction of l_2 just in case l_1 and l_2 are parallel)

Wright (1983, p. 31–32) says the following:

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[2] The label "Adjectivalism" is due to Dummett (1991). It is somewhat unfortunate, however, because it suggests that what Frege calls "attributive uses" like (1a) must be adjectives. However, the intended view is that "attributive uses" are *non-referential* expressions, and this is consistent with 'four' in (1a) being an adjective or a determiner. Despite this, we follow the literature in retaining the label "Adjectivalism".

> The reductionist idea was that since the right-hand contains no apparent direction-denoting singular term, we can take it that the apparent reference to a direction on the left-hand side is mere surface grammar, a misleading nuance. But why should we not turn that way of looking at things on its head? What is there to prevent us saying that, since the left-hand side does contain an expression referring to a direction, it is the apparent lack of reference to a direction on the right-hand side which is potentially misleading, or 'mere surface grammar'? ... Why should it not be possible for a sentence containing no isolatable part which refers to a particular object nevertheless achieve, as a whole, a reference to that object, as is attested by the fact that it is equivalent to a sentence in which such a reference is explicit?

Wright's suggestion appears to be that although (15b) but not (15a) contains explicit singular terms referring to directions, because those statements are *equivalent*, we may nevertheless analyze 'line l_1' and 'line l_2' in (15a) as singular terms referring to directions.

(15)   a. Line l_1 is parallel to line l_2.
        b. The direction of l_1 is identical to the direction of l_2.

Applying similar reasoning to (1a,b), although 'four' in (1a) appears to serve a non-referential semantic function, because (1a,b) are *equivalent*, we may nevertheless analyze 'four' in (1a) as a singular term referring to a number.

However, the obvious problem with this proposal is that because equivalence is *symmetric*, the equivalence of (1a,b) will not alone substantiate Substantivalism. Indeed, a similar point is made by Dummett (1991, p. 109), while criticizing Frege's Substantivalism:

> If it is legitimate for analysis so to violate surface appearance as to find in sentences containing a number-adjective a disguised reference to a number considered as an object, it would necessarily be equally legitimate, if it were possible, to construe number-theoretic sentences as only appearing to contain singular terms for numbers, but as representable, under a correct analysis of their hidden underlying structure, by sentences in which number-words occurred adjectivally... If the appeal to surface form, in sentences of natural language, is not decisive, then it cannot be decisive, either, when applied to sentences of number theory. Frege has merely expressed a preference for the substantival strategy, and indicated a means of carrying it out.

The same criticism applies to Wright's rhetorical suggestion: if we are allowed to ignore surface syntax and analyze the apparent adjective or determiner 'four' in (1a) as a genuine singular term in virtue of the equivalence of (1a,b), then it should be equally legitimate to ignore surface syntax and analyze the apparent numeral in 'four' in (1b) as a non-referential adjective or determiner, thus vindicating Adjectivalism.

In more recent times, Adjectivalism has become by far the more popular solution to Frege's Other Puzzle. Although there are different versions of the strategy available, perhaps the most influential is the one articulated and defended by Hofweber. As we will see, Hofweber's solution has potentially far reaching consequences not merely for the meanings of number expressions, but also for ontology. In the next section, we will outline Hofweber's Adjectivalism, along with its significance for issues central to the philosophy of mathematics.

3.3   Hofweber's Adjectivalism

The version of Adjectivalism defended by Hofweber (2005, 2007, 2016) is complex, consisting of several (controversial) theses. For exegetical clarity, it can be factored it into three major components: a syntactic component, a semantic component, and a cognitive component. Since our primary concern here is with the linguistic aspects of Hofweber's analysis, we will be less concerned overall with the cognitive theses Hofweber puts forward. In what follows, we will sketch these linguistic theses, the solution they recommend to Frege's Other Puzzle, and its implications for the ontology of numbers.

3.3.1   The Syntactic Component: Determiners, Extraction, and Focus Effects

The key semantic fact about natural language determiners is that they cannot function referentially. No empirically respectable semantics would claim that 'no', for instance, can refer to an object. Rather, determiners combine with nouns like 'moon(s)' to form quanticational phrases such as 'no moons', denoting second-order properties (or *generalized quantifiers*).

(16)   Jupiter has no/some moons.

   So, if 'four' in (1a) is a determiner,

(1)   a. Jupiter has four moons.

then it too must function non-referentially. Thus, Hofweber's first key linguistic contention is that 'four' in (1a) is in fact a determiner, one which has a meaning given within *Generalized Quantifier Theory* (GQT; Barwise and Cooper (1981)). On one variation, this is given in (17):[3]

(17) $[[four]] = \{<S,S'>: S,S' \subseteq U$ and $|S \cap S| = 4\}$
        ('four' denotes pairs of sets S and S' such that S and S' are subsets of the domain U and the cardinality of the intersection of S and S' is exactly four)

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[3] (17) is in fact the denotation of 'four' assumed by Breheny (2008). In GQT, cardinal determiners are actually given *lower-bounded* truth conditions, so that 'four' denotes a relation between sets whose intersection has a cardinality of *at least* four:

(i) $[[four]] = \{<S,S'>: S,S' \subseteq U$ and $|S \cap S| \geq 4\}$

The reason for adopting a two-sided analysis instead will become apparent in the next section, when we consider paraphrases of basic arithmetic equations like 'three and two is five'.

According to (17), the determiner 'four' denotes a relation between sets whose intersection has a cardinality of exactly four. As such, 'four' in (1a) is thus a prototypical *non*-referential expression. Indeed, as noted just above, expressions of this type not only typically fail to function referentially, they *cannot* function referentially.[4]

Hofweber's primary reason for thinking that 'four' in (1a) is a determiner, as opposed to an adjective, appears to be the undoubted success of GQT. GQT is the predominant analysis of natural language quantification within linguistic semantics, thanks in large part to its ability to state and predict various linguistic universals, specifically generalizations about possible determiner meanings across languages. Thus, Hofweber (2007, p. 3–4) says:

> In contemporary natural-language semantics the uses of 'four' as in [(1a)] are pretty well understood, and 'four' is usually considered to be a determiner, an expression of the same kind as 'some', 'many', and 'all'. Such expressions are not disguised referring terms.

Indeed, if 'four' in (1a) is a determiner, then GQT's success provides an excellent reason for thinking that it expresses a relation between sets.

Hofweber's second key linguistic contention is that 'four' in (1b) *is the very same* quantificational determiner witnessed in (1a).

(1) b. The number of Jupiter's moons is four.

According to Hofweber (2005, p. 211), this is due to what he calls *extraction*.

> In Hofweber [2007], I argue that this focus effect can't be explained if one thinks that [(1b)] is both syntactically and semantically an identity statement with two (semantically) singular terms. But it can be explained if [(1b)] has a different syntactic structure, one that results from extracting the determiner and placing it in an unusual position that has a focus effect as a result. Thus, in [(1b)] 'four' is a determiner that has been "moved" out of its usual position.

The idea appears to be that through "extraction", 'four' in (1a) gets "moved" from its "usual [determiner] position", thereby "placing it in an unusual [post-copular] position" in (1b). Crucially, and despite this, 'four' in (1b) retains its semantic function as a non-referential *determiner*. To quote Hofweber (2005, p. 211): "The word 'four' is the same in [(1a)]) and [(1b)]." Consequently, 'four' in both (1a,b) denotes a property of sets, *not* a number.

Hofweber's main source of evidence for "extraction" concerns so-called *focus effects* witnessed in examples like (18).

(18) a. Johan likes soccer.
      b. What Johan likes is soccer.
      c. It is Johan who likes soccer.

Whereas (18a) is acceptable in response to both 'Who likes soccer?' and 'Which sport does Johan like?', (18b) is only acceptable in response to the latter, while

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[4] See Landman (2003).

(18c) is only acceptable in response to the former. Contrast this with prototypical identity statements like (19), which apparently do not give rise to focus effects.

(19)   Cicero is Tully.

Indeed, (19) is perfectly fine in response to both 'Who is Tully?' and 'Who is Cicero?'.

In contrast, (1b) does apparently display focus effects: while (1a) is acceptable in response to both 'Which planet has four moons?' and 'What belongs to Jupiter?', (1b) is only acceptable in response to the former. What this shows, according to Hofweber, is that (1b) is not a genuine identity statement, contra Frege (1884). Moreover, it is indirect evidence that (1b) results from "extraction" since, if it were an identity statement, we would expect to see no focus effects.

3.3.2   The Semantic Component: Numerals and Semantically Bare Determiners

To summarize, according to Hofweber, 'four' in (1b) is the same non-referential determiner witnessed in (1a), thanks to "extraction". As such, the truth of neither (1a) nor (1b) implies the existence of a number, no more so than (16) does.

(16)   Jupiter has no/some moons.

However, "extraction" is a construction-specific syntactic operation, presumably: it applies to sentences broadly having the structure of (1a), and returns sentences broadly having the structure of (1b). As such, it is not operative in numerical equations like (20).

(20)   Three and two is five.

After all, it is hard to see how (20) could result from anything similar to (1a), where the numerals 'three', 'two', and 'five' feature originally as determiners. But then there is no obvious reason for thinking that the numerals in (20) are non-referential expressions. In other words, it would appear that (20) straightforwardly entails the existence of numbers.

To this end, Hofweber distinguishes between two kinds of *bare determiners*, or determiners occurring without overt accompanying nouns, such as 'most' in (21).

(21)   How many boys kicked the ball? Most kicked the ball.

Although 'most' does not occur explicitly restricted by the noun 'boys' in (21), it is implicitly understood that way. In other words, the continuation of (21) is interpreted as 'Most boys . . .'. Contrast this with 'most' in (22), where there is no antecedent noun available.

(22)   Most is/are more than none.

Rather than claiming something about most boys, or most people, or whatever, (22) is intended to be *generic*: whatever it is that we're talking about, most is more than none. Thus, Hofweber calls determiners like 'most' in (22) *semantically bare determiners*.

Hofweber's third key linguistic contention is that the number expressions in (22) are really semantically bare determiners, not genuine names of numbers. Put differently, (22) has something like the logical form informally suggested in (23), where *X* is a noun phrase restricting the determiners 'three', 'two', and 'five', and 'GEN' is a genericity operation.

(23)   GEN: [three *X* and two (more) *X* are five *X*]
        (In general, three things and two (more) things are five things)

Hofweber's primary piece of linguistic evidence for (23) is that arithmetic equations can be parsed in two ways, namely in the singular or in the plural, similar to (22).

(24)   Three and two is/are five.

Furthermore, (24) also resembles (22) in that both are entirely general: no matter what we are talking about, three and two are five, just as most are more than none. As a result, despite surface syntactic appearances, (24) does not involve relating two first-order objects (3 and 2) to a third first-order object (5), through a first-order operation ($+$). Rather, it actually involves *counting* objects, though in an entirely general way.

This raises another question, however: What guarantees that things (the X's) being counted in (24) do not overlap. This is crucial to getting the truth-conditions for (24) correct, of course: if A = {a,b,c}, B = {a,b}, and C = {d,e}, then $|A \cap B| = 2$, $|A \cup B| = 3$, and $|A \cup C| = 5$. So, what guarantees that (22) behaves like $|A \cup C|$, rather than $|A \cap B|$ or $|A \cup B|$? To this end, Hofweber appeals to a well known distinction between *cumulative* (or 'non-boolean') conjunction and *propositional* (or 'boolean') conjunction. Examples of the latter include (25a-c), while examples of the former include (26a-c), due to Krifka (1999).

(25)   a. John and Mary slept.
        b. Mary sang and danced.
        c. This cocktail is cheap and refreshing.

(26)   a. John and Mary met at the mall.
        b. This concoction is beer and lemonade.
        c. That flag is entirely green and white.

(25a–c) can all be paraphrased as the conjunction of two propositions. For example, (25a) can be paraphrased as 'John slept and Mary slept'. In contrast (26a) cannot mean that John met at the mall, and also Mary met at the mall, just as (26b) cannot

mean that this concoction is beer, and also this concoction is lemonade. What (26a–c) show is that cumulative conjunction can coordinate expressions of different semantic types – names, predicates, and modifiers.

Thus, Hofweber's fourth linguistic contention is that 'and' in (22) and (24) is *cumulative* conjunction involving semantically bare determiners, where non-overlap is guaranteed through "ellipsis, or a pragmatic mechanism, or a form of "free enrichment," or something else" (Hofweber (2005, p. 193)). Thus, Hofweber likens (24) to (27).

(27)   She only had an apple and dessert.

Normally, an utterance of (27) would be judged misleading if she happened to have only an apple, even though apples may serve as perfectly fine desserts. Presumably, according to Hofweber, this too is a function of "ellipsis, or a pragmatic mechanism, or a form of "free enrichment," or something else". The important point is that just as an utterance of (27) apparently presupposes non-overlapping extensions for 'apple' and 'dessert', an utterance of (24) apparently presupposes non-overlapping extensions of 'three Xs' and 'two Xs'.

3.3.3 Frege's Other Puzzle and the Consequences for Ontology

Given that Hofweber defends a version of Adjectivalism, it is hardly surprising that the premise he denies in Frege's Other Puzzle is FOP3.

> The different occurrences of 'four' in (1a) and (1b) serve different semantic functions.

Specifically, despite 'four' occurring as a determiner in (1a) and as a name in (1b), it serves the same non-referential semantic function of a determiner in both.

(1)   a. Jupiter has four moons.
       b. The number of Jupiter's moons is four.

As a result, (1b) does not entail the existence of a number.

Contrast this with Frege's analysis,[5] where (1b) immediately implies the existence of a number thanks to the referential function of the numeral 'four'. That is, (28a) entails (28b), paraphrased in English as (28c).

(28)   a. $\#[\lambda x.\ \text{moon-of-Jupiter}(x)] = 4$
        b. $\exists n.\ \#[\lambda x.\ \text{moon-of-Jupiter}(x)] = n \land n = 4$
        c. There is a number which is the number of Jupiter's moons, namely four.

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[5] The same applies to Wright (1983) and Hale (1987).

Since (28c) seemingly wears its ontological commitment to numbers on its sleeves, and since (1a) entails (1b), Frege's analysis apparently implies that in virtue of successfully counting some moons, thereby establishing the truth of (28a), we can validly infer that numbers exist, i.e. (28b). This is puzzling, as the question of whether numbers exist is a longstanding, difficult question central to the philosophy of mathematics. Thus, it would be surprising if an answer to that question could be so easily obtained. Accordingly, this is known in the literature as *the Easy Argument for Numbers*.[6]

Hofweber ([2007](#))'s solution appeals to the same Adjectivalist analysis responsible for debunking Frege's Other Puzzle: because 'four' in (1b) is a non-referential determiner, we cannot infer from it that a number exists, at least not in a substantial sense relevant to ontology. Ultimately, and more generally, Hofweber's view is that *no* apparently referential use of number expression is in fact referential, including their use in arithmetic statements, such as (24).

(24)   Three and two is/are five.

Thus, in the end, Hofweber defends a version of what Dummett ([1991](#)) calls *the Radical Adjectival Strategy*: *no* occurrence of e.g. 'four' or '4' is a genuine singular term.[7] Consequently, not only does our ordinary talk of counting moons fail to entail an ontology of numbers, but so also does the mathematician's talk of the number four being even.

In summary, Hofweber's Adjectivalism may thus be viewed as a sustained defense of nominalism with respect to the natural numbers. All apparent reference to numbers is just that – apparent. Upon further linguistic investigation, we discover that explaining arithmetic truths does not require positing numbers. That's because, despite surface appearances, arithmetic discourse is not about numbers as abstract objects and the various properties those objects may have, but rather an elaborate form of counting, something we all learned to do as children. What's more, arithmetic discourse is *true* – indeed objectively and necessarily so – in virtue of the meanings of the bare numerical determiners involved. Thus, unlike with various versions of *fictionalism*,[8] Hofweber's Adjectivalism is not an error theory with respect to number talk.

Of course, the strength of Hofweber's defense of nominalism depends wholly on the empirical adequacy of the analysis proposed. In the next section, we will sketch objections to the two components of the analysis considered here – the syntactic component and the semantic component. Ultimately, we will argue that neither survives empirical scrutiny.

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[6] See e.g. Balcerak-Jackson ([2013](#)) and Snyder ([2017](#)).

[7] The details are complex and beyond the scope of a single paper. But see Hofweber ([2016](#)).

[8] Roughly, fictionalism is the view that numerals in arithmetic discourse genuinely have the function of naming numbers, but since numbers do not exist, all arithmetic discourse is either false or else involves widespread presupposition failure. See e.g. Hodes ([1984](#)), Yablo ([2005](#)), and Leng ([2010](#)).

3.4 Problems with Hofweber's Adjectivalism

Despite its influence, Hofweber's Adjectivalism has received a fair amount of criticism in the philosophical literature. This has focused largely on the syntactic component of Hofweber's analysis, specifically "extraction" and the evidence purporting to motivate it. In this section, we will consider those objections, while also developing novel objections to the comparatively neglected semantic component of Hofweber's analysis. The upshot will be that Hofweber's key linguistic theses highlighted in Sect. 3.3 are empirically problematic.

3.4.1 Problems with Extraction

Much of the extant criticism of Hofweber's Adjectivalism revolves around "extraction", i.e. the linguistic mechanism responsible for "moving" 'four' from its position in (1a) to its position in (1b). Recall the quote from Hofweber (2005, p. 211):

> [(1b)] has a different syntactic structure, one that results from extracting the determiner and placing it in an unusual position that has a focus effect as a result. Thus, in [(1b)] 'four' is a determiner that has been "moved" out of its usual position.

It is natural to interpret this talk of "movement" as an instance of the same kind of "movement" familiar from transformational theories of syntax (e.g. transformational grammar, government and binding theory, and minimalist syntax). (29) provides a prototypical illustratration, known as "extraposition", where underlining indicates the expression "moved", and the blank indicates the position out of which "movement" is assumed to occur.

(29) a. Something <u>that we weren't expecting</u> happened.
      b. Something ___ happened <u>that we weren't expecting</u>.

As a result, 'that we weren't expecting' in (29b) becomes *focused*, much like post-copular 'four' in (1b) on Hofweber's analysis. It should thus be unsurprising that some of Hofweber's detractors, chiefly Brendan Balcerak-Jackson (2013) and Friederike Moltmann (2013), have interpreted "extraction" as a transformational mechanism responsible for "rearranging" the syntactic material in (1a), ultimately resulting in (1b).

The problem, according to these detractors, is that the actual syntactic principles or operations that would be required to do this kind of "rearranging" would not be recognized by contemporary transformational theories, and their postulation would be highly dubious. For one thing, unlike with (29), (1b) clearly contains material missing in (1a): 'the', 'number', 'of', and '-'s'. Conversely, there is material contained in (1a) that is missing in (1b): 'has'. Even if there were "movement" of the parts of (1a), no other known transformational mechanism would also delete and add material in the manner required. Rather, as Balcerak-Jackson notes, it would seem far more plausible to hold that (1a,b) are simply *different sentences*, with

(1b) attempting to paraphrase (1a). Yet without (1b) resulting from some kind of "rearrangement" of (1a), there would be no guarantee that post-copular 'four' in (1b) is *the same* non-referential determiner (1a), thereby undermining Hofweber's case for some version of Adjectivalism.

In response, Hofweber (2014) accuses these detractors of misinterpreting "extraction", by assuming that it must involve some kind of transformational "movement". To quote Hofweber (2014, p. 264):

> But I made no such proposal. I never talk about transformation rules, or deriving [(1b)] from [(1a)] via some mysterious sentence level transformation. In fact, 'transform' or 'transformation' don't even appear in my article.

To help clarify, Hofweber further distinguishes between two possible interpretations of "extraction", one involving what he calls "displacement", and the other involving what he calls "transformation". To continue the quote:

> To bring out the difference, we can say that 'extraction' could be understood either as displacement or as transformation. Displacement occurs when a phrase appears in a position contrary to where it naturally belongs, that is, contrary to its canonical position. This is still metaphorical, of course, but at least talk of displacement rather than extraction might suggest less that this is to be understood as sentence level transformation. Transformation occurs when one sentence gets turned into another, via some syntactic rules. I proposed that in [(1b)], but not in [(1a)], 'four' is displaced and as a result we can see, in outline, why [(1b)] has a focus effect, while [(1a)] does not. Balcerak Jackson instead takes me to propose that [(1a)] gets transformed into [(1b)]. All that is needed for the argument, however, is displacement, not transformation.

Thus, according to Hofweber, the Balcerak-Jackson/Moltmann criticism is ultimately a straw man, requiring a "transformation"-based interpretation of "extraction", rather than a "displacement"-based interpretation.

Suppose so. The obvious question now becomes: What exactly is "displacement", and what does it have to do with 'four' in (1a) getting "moved" into post-copular position in (1b)? Unfortunately, Hofweber has little to offer in response to these questions. To quote Hofweber (2014, p. 265) again:

> In Hofweber (2007) I did not propose any particular view of how the syntax for the relevant examples was supposed to work more precisely. I made no proposal about the precise syntactic structure of [(1b)], nor about the relationship between focus and syntax in general, nor did I endorse a particular framework in syntactic theory. I don't say this proudly, I wish I had such views to offer. But the argument that [(1b)] is not an identity statement is rather neutral with respect to the more precise syntactic mechanisms that underlie all this. It is motivated more by the data for a theory than the theory itself. It relied on a notion of extraction/displacement that was metaphorical, but clear for many cases, its connection to syntactic focus, and the relationship between focus and identity statements, but not any particular syntactic theory, certainly not transformational grammar.

A similar sentiment is expressed in Hofweber (2016, p. 41):

> Talk of "extraction" or "displacement" or "movement" is a theory-neutral metaphor that we don't need to spell out now. What is crucial for us instead is that constructions of this kind give rise to a syntactic focus effect, not how precisely the syntactic connection to focus is to be understood.

So, "displacement", and thus "extraction", is only intended to be a metaphor, not a fleshed out syntactic mechanism situated within the background of a particular syntactic theory, including transformational theories.

In some ways, it is understandable that Hofweber might want to back off from making any specific proposals regarding how exactly "extraction" is to be understood. After all, doing so might lead to potentially falsifiable empirical predictions, and it might also hold his analysis hostage to cohering with other elements of a background syntactic theory. On the other hand, because Hofweber's solution to Frege's Other Puzzle rests wholly on the *empirical viability* of "extraction", what's required, minimally, is some assurance that this syntactic mechanism, whatever it is, is empirically motivated.

There are two points we'd like to emphasize here in this connection. First, it should be stressed that "movement" of any sort is highly controversial within contemporary syntactic theory. That's because there are *numerous* mainstream syntactic theories whose formulation is grounded principally upon the explicit rejection of "movement", notably "representational theories", including head-driven phrase structure grammars, lexical functional grammars, construction grammars, and most dependency grammars. The latter attempt to explain the same phenomena covered by the postulation of "movement" within transformational theories, but via other means, e.g. feature passing.

Secondly, and largely for this reason, talk of "movement" is typically understood as presupposing some version of transformational syntax. This includes "displacement". To illustrate, consider the following example from Abels (2017), where again underlining indicates the expression "moved", and gaps indicate the "canonical position" from which it is "displaced":

(30)   a. (I know that) John will drink absinthe.

        b. I know <u>what</u> John will drink ___.

        c. <u>Absinthe</u>, John will drink ___.

        d. the beverage <u>which</u> John will drink ___

As Abels explains, the "canonical" word order for English sentences, as illustrated in (30a), is subject-auxiliary-verb-object. In (30b-d), the underlined expression is the object, thus revealing that it does not occur within its "canonical" position – it has been "displaced". This is also presumably the notion of "displacement" Hofweber (2014) has in mind: "a phrase appears in a position contrary to where it naturally belongs, that is, contrary to its canonical position."

If so, then it is very difficult to see how Hofweber's distinction between "displacement" and "transformation" addresses the crux of the Balcerak-Jackson/Moltmann criticism. Hofweber's central thesis is that 'four' in (1b) results from "displacement", and as such is the very same determiner witnessed in (1a). For this to make sense, 'four' needs to be "moved" out of its "canonical position" in (1a) , presumably as the head of a determiner phrase, to post-copular position in (1b). Minimally, then, as with typical cases of "displacement" like (30b-d), we should expect (1a) to *share* much of its syntactic material with (1b), contrary to fact. Thus, independent of any particular version of transformational syntax and

corresponding syntactic principles or operations which might underlie this kind of "movement", it would appear that "displacement" cannot do what Hofweber's Adjectivalism requires of it.

In any case, there would appear to be more direct evidence against "extraction", independent of how it might be spelled out. Generally speaking, we expect syntactic operations to apply to expressions of the same categories. For example, we should presumably be able to replace 'absinthe' in (30a) with any other mass noun, thus leading to a grammatical sentence of the form in (30c). So, if "extraction" is a syntactic operation, then it should apply to all determiners, not just numerical determiners. For example, it should apply to 'no' and 'some' in (31).

(31)   Jupiter has {no/some/four} moons.

Yet, as Balcerak-Jackson also points out, the result of applying "extraction" to 'no' and 'some' would be clearly unacceptable:

(32)   The number of Jupiter's moons is {??no/??some/four}.

More generally, it appears that *no* uncontroversial determiner can occur in post-copular position of constructions like (1b). Why is this?

In response, Hofweber (2014, p. 266) says the following:

> Balcerak Jackson contends that my account does not explain why similar constructions do not seem to work with other determiners. That is true, my account does not explain this, and neither does, I may add, Balcerak Jackson's own account outlined at the end of his paper. But it is an overstatement that my account makes this "mysterious" (p. 451). The account simply leaves this open, but it is certainly compatible with an explanation that comes form a difference in the syntactic behavior among determiners or adjectives in general. That not all determiners behave the same syntactically is a well-known fact.

Thus, Hofweber's response to this objection looks similar to his response to the previous objection: because "extraction" is not intended to be situated within any particular syntactic theory, it is not intended to offer an explanation of contrasts like (32). Rather, (32) is apparently witness to a more general phenomenon, of which Hofweber unfortunately offers no specific examples, that "not all determiners behave the same syntactically".

The problem with this response, as with the first, is that it does not actually address the argument at issue. The concern is not with whether Hofweber's analysis can explain contrasts like (32), but rather with what it apparently predicts. Specifically, the claim is that Hofweber's analysis seemingly makes a false empirical prediction: all determiners should be subject to "displacement", and yet *no* uncontroversial determiners can acceptably occur in post-copular position, similar to 'four' in (1b). Now, Hofweber's response could be that his analysis does not make this prediction, because it makes *no predictions*, in virtue of not being situated within any particular syntactic theory. But this would ignore the crucial fact about syntactic operations more generally: they apply to expressions of the same category, *independent* of whichever syntactic theory they happen to be embedded within. Thus, independent of which specific syntactic operation is assumed to be responsible

for "displacement" (e.g. "Inner Merge"), Hofweber's analysis appears to make an important, demonstrably false prediction.

Furthermore, note that Balcerak-Jackson's original observation readily extends to numerous further constructions. For example, uncontroversial determiners cannot appear bare in predicative positions more generally:

(33)   Jupiter's moons are {??no/??some/four} (in number).

Nor can they occur as the complement of the verb 'number':

(34)   Jupiter's moons number {??no/??some/four}.

Nor can they generally be "stacked", i.e. co-occur bare.

(35)   All {??no/??some/four} moons of Jupiter are large.

Finally, and perhaps most significantly, determiners cannot occur as *names*.

In contrast, color expressions such as 'green' can occupy these various positions, and they can also apparently function as names.

(36)   a. Jupiter has green moons.
        b. The color of Jupiter's moons is green.
        c. Jupiter's moons are green (in hue).
        d. Jupiter's moons are colored green.
        e. All green moons of Jupiter are large.
        f. Green is a color.
        g. The color green is Mary's favorite.

What's more, such expressions are standardly assumed within linguistic semantics to be *adjectives*, and, in any case, they are certainly *not* determiners. Furthermore, their use in constructions like (36f) is also standardly assumed to be referential, so that 'green' in (36f) is a genuine singular term.[9]

Here's a simple argument, then, building on Balcerak-Jackson's original observation. On the one hand, number expressions differ from other uncontroversial determiners in numerous important respects. On the other hand, they pattern exactly like certain adjectives in those same respects. Furthermore, merely announcing that "not all determiners behave the same syntactically" will not suffice to explain these similarities and differences, given their breadth. Rather, it is tempting to conclude based on these observations that Hofweber's analysis rests principally on a syntactic misclassification – number expressions, at least in their "attributive" use, are *adjectives*, not determiners.

It is thus worth noting that according to Hofweber (2016, p. 124), the issue of how exactly number expressions should be syntactically classified is ultimately irrelevant to the success of his solution to Frege's Other Puzzle.

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[9] See e.g. Kennedy and McNally (2010), McNally (2011), and McNally and de Swart (2011).

> There is some controversy about whether number words in the relevant uses are determiners, modifiers, or adjectives. This is also an issue which is insignificant for us here... What ultimately matters for our discussion is that number words in their determiner use can form complexes... and that they are not themselves referring expressions in this use. Whether they are in the end adjectives, determiners, or form a separate class of their own, is secondary.

Presumably, the thought is that because both determiners and adjectives (used attributively or predicatively) function *non*-referentially, so long as 'four' (1b) also functions that way, Hofweber's proposed solution to Frege's Other Puzzle will go through.

However, we have just seen at least some (apparent) adjectives have genuinely referential uses – again, witness (36f). In fact, there are analyses on which 'four' in (1b) arguably has this same referential semantic function, in virtue of the same general semantic operations responsible for rendering 'green' in (36f) a singular term.[10] What's more, (36) is standardly taken to show that one and the same expression ('green') can perform different semantic functions – it can function e.g. as a predicate, a modifier, and as a singular term. Moreover, all "extraction" apparently guarantees is that, to quote Hofweber (2005, p. 211) again, "the word 'four' is the same in both [(1b)] and [(1a)]". If so, then this alone will not guarantee that 'four' in (1b) has the *same semantic function* as four in (1a). Rather, if 'four' in (1a,b) is an adjective, then nothing obviously precludes the possibility that 'four' in (1b) is a genuine singular term, despite being "displaced". In contrast, because it is a *distinguishing feature* of determiners that they cannot function referentially, no such possibility arises if 'four' in (1a) is instead a determiner. It thus appears that the classification of number expressions in their "attributive" use is far more empirically significant than Hofweber recognizes.

3.4.2 Problems with Focus Effects

In addition to "extraction", Hofweber's analysis relies crucially on a number of dubious semantic assumptions. One concerns the role of so-called focus effects. Hofweber's argument, recall, is that because genuine identity statements do not exhibit focus effects, but (1b) does, (1b) cannot be a genuine identity statement. However, Brogaard (2007) points out that (3), which Hofweber (2007) claims to be a genuine identity statement, exhibits similar focus effects.

(3)   The composer of *Tannhäuser* is Wagner.

In particular, (3) would be an appropriate answer to the question 'Who composed *Tannhäuser*?' but not 'Who is Wagner?' or 'What did Wagner do?'. Thus, it appears that exhibiting focus effects is insufficient to show that (1b) is not an identity

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[10] See Snyder (2017).

statement. In that case, it could well be that (1b) entails the existence of a number, just as Frege (1884)'s analysis suggests.

Indeed, it is worth emphasizing in this connection that even if constructions like (1b) and (3) are not identity sentences, this alone will not establish that post-copular 'four' in (1b) functions non-referentially. In fact, it has been argued by many that constructions like (1b) and (3) are better understood as *specificational sentences*, in the sense of Higgins (1973).[11] Higgins originally distinguished between at least three forms of the English copula, including:

(37)   a. Cicero is Tully.   (equative)
        b. Cicero is bald.   (predicational)
        c. The most famous Roman orator is Cicero.   (specificational)

Equative sentences are prototypical identity statements like (37a), equating the referents of two singular terms. Predicational sentences such as (37b) predicate a property such as being bald of the subject. Finally, specificational sentences such as (37c) specify an individual under a certain description, e.g. the most famous Roman orator.

The semantic motivations for this taxonomy are well known.[12] Moreover, it might be reasonably thought that Hofweber could appeal to that taxonomy not only to explain the apparent focus effects in (1b) and (3), but also to establish a different version of Adjectivalism which does not rely on the dubious syntactic operation of "extraction". In fact, this is broadly the strategy pursued by other Adjectivalists, including Moltmann (2013) and Felka (2014). On these analyses, specificational sentences more generally express question-answer pairs, via ellipsis.[13] For example, the pre-copular material in (37c) expresses an indirect question corresponding to 'Who is the most famous Roman orator?', while the post-copular material expresses an answer to that question, namely 'Cicero is the most famous Roman orator', all through ellipsis.

Similarly, it has been argued that the pre-copular material in (1b) expresses an indirect question corresponding to 'What is the number of Jupiter's moons?', an answer to which is expressed by the post-copular material, namely 'Jupiter has four moons', again through ellipsis. Since the pre-copular material expresses a question, it is little wonder that we see focus effects. After all, by hypothesis the question expressed concerns the cardinality of Jupiter's moons, not what belongs to Jupiter more generally. Better yet, because post-copular 'four' has the same non-referential function witnessed in (1a), (1b) would not entail an ontology of numbers.

However, for this suggestion to succeed, it needs to be that specificational sentences really do express question-answer pairs in virtue of ellipsis. Yet this is not the *only* analysis of specificational sentences available. In fact, there is an alternative analysis on which the copula is systematically ambiguous between equa-

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[11] See Moltmann (2013), Felka (2014), and Snyder (2017).

[12] See Mikkelsen (2005).

[13] See Schlenker (2003).

tive, predicational, and specificational meanings.[14] In particular, the specificational copula receives the meaning in (38), where '$\underline{y}$' ranges over *individual concepts*, i.e. functions from worlds to individuals.

(38) $\lambda x.\lambda \underline{y}_{<s,e>}.\lambda w.\ \underline{y}(w) = x$

On this analysis, the pre-copular material in (37c) expresses an individual concept, namely a function from worlds w to whoever is the most famous Roman orator in w, while the post-copular name 'Cicero' is a genuine singular term. Thus, (37c) will be actually true if Cicero is in fact the most famous Roman orator.

Applying the same analysis to (1b) would suggest that the pre-copular material expresses an individual concept, i.e. a function from worlds w to the maximal number of Jupiter's moons in w, while post-copular 'four' functions as a genuine *singular term*. Hence, even if (1b) is not a genuine identity statement, i.e. a copular sentence involving the equative copula, it needn't follow that post-copular 'four' functions non-referentially. In other words, it needn't follow that some version of Adjectivalism is correct. Ultimately, then, focus effects lend no direct support for Adjectivalism of any kind.

3.4.3 Problems with Numerals

Consider (39), where 'four' apparently functions as a numeral, i.e. a name:

(39)   Four is an even number.

Clearly, 'is an even number' is a predicate. Given standard semantic assumptions, it should thus be something which either takes 'four' as an argument and returns a truth-value, or else is taken by 'four' as an argument and returns a truth-value. In the first case, 'four' would be a referential-type expression, presumably referring to a number. In the second case, it would function as a generalized quantifier, denoting a set of sets, one of which would include the even numbers. In either case, it would appear that making semantic sense of the truth of (39) requires acknowledging the existence of numbers.

Of course, this realist conclusion might be avoided if 'four' functions instead as a semantically bare determiner, in which case 'four' and 'is an even number' would have different semantic types than their surface syntax suggests. The problem, however, is that determiners without accompanying nouns generally have the wrong semantic type to occupy argument positions, as witnessed by the unacceptability of 'every' in (40a,b).

(40)   a. {??Every/Everyone/Every person/Mary} is happy.
        b. John loves {??every/everyone/every person/Mary}.

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[14] See Partee (1986b) and Romero (2005).

Rather, in order to occupy argument positions, determiners need to combine with a noun like 'person' to form a generalized quantifier like 'every person'.

Hofweber is seemingly aware of this issue. Indeed, speaking about an example similar to (39), Hofweber (2005, p. 209–210) says:

> I will cover only the case of the relationship between sentences like [(1a,b)]. It will not be a general account of the singular-term use of numerals. It will still leave open what is going on in certain other uses of number words as singular terms in statements that are neither singular basic arithmetical equations nor of the same kind as [(1b)].

This is rather surprising, given that Hofweber's analysis is designed to handle cases like (24), which also apparently involve numerals.

(24)   Three and two is/are five.

The latter, recall, is analyzed as (23), so that the apparent numerals in (24) are in fact generically quantified semantically bare determiners.

(23)   GEN: [three X and two (more) X are five X]
        [In general, three things and two (more) things are five things]

Hofweber's contention is that because determiners more generally are non-referential expressions, (24) does not entail commitment to numbers. Thus, one might reasonably think that something similar could be said for cases like (39), so that 'four' similarly functions as a semantically bare determiner.

However, as Rothstein (2017) observes, this suggestion would make numerous incorrect predictions. First, 'count' is ambiguous between two senses, roughly corresponding to what Benacerraf (1965) calls *intransitive counting* and *transitive counting*.[15] These are witnessed respectively in (40a,b), due to Rothstein.

(40)   a. I counted to thirteen (??things/??people/??books).
        b. I counted thirteen (things/people/books).

Thus, as the labels suggest, transitive 'count' requires a direct object, where intransitive 'count' does not. Semantically, this suggests that while transitive 'count' has an essentially *relational* meaning, intransitive 'count' does not. Thus, consider Rothstein's (41a,b):

(41)   a. I counted thirteen. – Thirteen what?
        b.?? I counted to thirteen. – Thirteen what?

Secondly, (42a) and (42b) are clearly not synonymous, as (42a) is true but not (42b).

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[15] To a first approximation, intransitive counting consists in reciting the numerals in their canonical order –"1, 2, 3,..." In contrast, transitive counting consists in the counting *of* things. That is, when transitively counting we use the numerals to answer 'how many'-questions, roughly by establishing a one-to-one correspondence between an initial segment of those numerals and a collection of objects being counted.

(42)   a. Two is an even prime.
        b. Two things are even primes.

Third, numerals and bare determiners differ in their agreement features. Specifically, whereas numerals require singular morphology, bare determiners require plural morphology.

(43)   a. Which one of these three numbers is Mary's favorite? Four {is/??are}.
        b. How many people are coming to the party? Four {??is/are}.

And the same holds for numerals in comparative constructions, as Rothstein points out.

(44)   Four {is/??are} bigger than three.

Finally, Rothstein observes examples like (45), where one number expression clearly modifies another.

(45)   Two twos are four, three twos are six.

None of this is to be expected, however, if all numerals are really semantically bare determiners. In that case, for instance, (42a) would entail (42b), contrary to fact. Rothstein (2017, p. 28) thus reasonably concludes: "Together, these data show that... there are cases where a bare cardinal numerical must be a singular term."

If so, and if their most plausible candidate referents are *numbers*, as argued by Hale (1987), then it would appear that the truth of e.g. (39) straightforwardly entails the existence of a number. In other words, despite Hofweber's proposed solution to the Easy Argument involving (1b), which crucially relies on 'four' functioning as a semantically bare determiner, there would appear to be an equally "easy argument" involving (39), for which that solution does not apply.

To be fair, Hofweber does offer an explanation as to why (apparent) numerals like 'four' in (38) at least *appear* to function as genuine singular terms. The explanation appeals to what he dubs *cognitive type-coercion*. In essence, cognitive type-coercion is the cognitive analog of type-shifting. However, whereas type-shifting is typically taken to "coerce" the meanings of natural language expressions, "shifting" their lexical meanings (at least) in the presence of type-mismatches, cognitive type-coercion instead operates exclusively on *mental representations*, within the language of thought. To quote Hofweber (2016, p. 137):

> The process of cognitive type coercion forces a representation to take on a certain form so that a certain cognitive process can operate with this representation. Systematically lowering the type of all expressions (or the mental analogue thereof) is a way of doing this, and the difference between our ability to reason with representations involving low types rather than high types explains why this type lowering occurs in the case of arithmetic.

The basic idea appears to be that because number expressions occurring within arithmetic statements have the complex semantic type of a determiner, they are difficult to semantically process. Consequently, we are forced to "coerce" the

corresponding mental representations in such a way that our reasoning mechanisms can "get a grip".

It is in virtue of this kind of cognitive coercion, apparently, that numerals in arithmetic statements seemingly function as singular terms. To quote Hofweber (2016, p. 137) again:

> Note that according to the cognitive type coercion account we merely change the form of the representation. We do not replace one representation with another one that has a different content. We take the same representation and change its syntactic form so that our reasoning mechanism can operate on it. The content of what is represented remains untouched by this. To put it in terms of the language of thought, we change the syntax of a representation so that our reasoning mechanism can get a grip on these representations. Other than that we leave it the same. And what holds good for mental representations will hold good, *mutatis mutandis*, for their linguistic expression in language. The singular arithmetical statements are the linguistic expression of thoughts involving type lowered mental representations.

It is crucial to recognize that the kind of "coercion" being alluded to here is *not* type-shifting of the more familiar semantic variety.[16] If it were, then numerals occurring within arithmetic statements would need to function as genuine singular terms, thus resulting in a different version of the Easy Argument. Presumably, this is why, according to Hofweber (2016, p. 141), "semantic type coercion [i.e. type-shifting] is the second best attempt to solve Frege's Other Puzzle." Regardless, the claim appears to be that *within the language of thought*, numerals occurring in arithmetic statements like (24), or their cognitive analogs, do function referentially, and this presumably explains why they appear to function referentially in English as well. If so, then perhaps this explanation can be extended to numerals as they figure in arithmetic statements like (39) as well.

We have criticized the notion of cognitive type-coercion and its role within Hofweber's larger nominalist program at length elsewhere.[17] Here, we will limit our discussion to how this might help explain contrasts like (40)–(45). The latter are presented (by a linguist) as *semantic* contrasts, intended to reveal a difference in the semantic function of numerals (referential) and numerical modifiers (non-referential), in English. However, by hypothesis, cognitive type-coercion operates on mental representations within the language of thought, *not* the meanings of English expressions. The claim appears to be that because we cognitively lower the "types" of corresponding representations at least when dealing with arithmetic statements, this explains why (apparent) numerals in English seem to function referentially.

The question here is: How, exactly? As far as we can tell, Hofweber offers no concrete answer. However, perhaps the most obvious answer is that the judgments reported in (40)–(45) do not reflect anything about the *meanings* of the English expressions at all, but rather their cognitive analogs within Mentalese. If so, then a primary question for Hofweber's account, as we see it, is this: What prevents

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[16] See especially Partee (1986a).

[17] See Snyder et al. (2021).

all (purported) semantic judgments from likewise reflecting something about Mentalese? After all, presenting contrasts like (40)–(45) is a primary empirical tool available to semanticists, with the presumption being that such contrasts report native speakers' intuitions about the meanings of natural language expressions. So, if such contrasts actually fail to reveal what linguists standardly take them to reveal, then why should this response, if correct, not rob linguistic semantics of its very empirical foundations?

3.4.4 *Problems with Semantically Bare Determiners*

We have seen that there are problems with analyzing numerals as semantically bare determiners. To this end, it is worth reexamining Hofweber's original justification for claiming that 'four' in (1a) is a determiner in the first place. It relies crucially, recall, on the observation that GQT is the predominant analysis of natural language determiners. Thus, if 'four' in (1a) is also a determiner, then all other things being equal, we ought to assume that it has the non-referential meaning GQT attributes to it. Specifically, we should assume that it has something like the meaning in (17).

(17) $[[four]] = \{<S,S'>: S,S' \subseteq U \text{ and } |S \cap S| = 4\}$
        ('four' denotes pairs of sets S and S' such that S and S' are subsets of the
        domain U and the cardinality of the intersection of S and S' is exactly
        four)

There are two kinds of problems with this reasoning, however. First, it does not follow that just because GQT analyzes 'four' in (1a) as a determiner having a meaning like that in (17), the *lexical* meaning of 'four' must be as specified in (17). Secondly, even if 'four' in (1a) – or indeed *every* occurrence of 'four' – had the meaning suggested in (17), it would not follow that the semantic evidence best supports nominalism.

As for the first problem, it turns out that there are good, independent reasons for thinking that the GQT analysis in (17) is independently flawed. Consider the following example from Krifka (1999), which is ambiguous between at least a *distributive interpretation* given in (46b), and *cumulative interpretation* given in (46c):

(46)   a. Three boys ate seven apples.
        b. Three boys each ate seven apples, so that twenty one total apples were eaten.
        c. Three boys together ate seven apples, so that seven total apples were eaten.

The problem, as Krifka explains, is that because (17) only predicts distributive interpretations, it cannot capture the cumulative interpretation. This suggests that even if GQT is the predominant analysis for natural language quantificational determiners, this provides no compelling reason for thinking that we should adopt that same analysis for 'four' in (1a), let alone (1b).

In fact, in order to explain how cumulative interpretations are possible, Krifka argues that "attributive" uses of 'four' must be understood as *adjectives* expressing cardinal properties of sums of countable individuals, or "atoms" in the sense of Link (1983). On this analysis, "attributive" 'four' in (1a) has something like the meaning suggested in (47), where '#' is a cardinality function mapping sums to numbers representing their atomic parts.

(47) $\lambda P.\lambda x.\ \#(x) = 4 \wedge P(x)$

Ultimately, this affords the following analysis of (46a):[18]

(48) $\exists x.\exists y.\ \#(x) = 3 \wedge \text{boys}(x) \wedge \#(y) = 7 \wedge \text{apples}(y) \wedge \text{ate}(x,y)$

The cumulative interpretation paraphrased in (46c) then arises if the predicate is interpreted *collectively*, so that three boys *together* ate seven apples.

To be clear, the claim is not that "attributive" uses of number expressions must be adjectives rather than determiners because no version of the GQT analysis could, in principle, capture cumulative interpretations. Rather, the semantic case for "attributive" 'four' being an adjective is far more comprehensive in scope. Specifically, as many have noted, 'four' has *many* interrelated uses apart from the "attributive" use witnessed in (1a), including e.g. those in (49).

(49)   a. Jupiter's moons are four (in number).
        b. No four moons of Jupiter orbit Saturn.

Thus, a desideratum on any empirically adequate semantics for number expressions is that it should not only provide meanings appropriate for all of these uses, but also explain how those meanings are *related*.[19]

Thus, the widespread assumption within linguistic semantics is that number expressions are *polymorphic*, taking on different semantic types in different syntactic environments, thanks to type-shifting (see e.g. Partee (1986a), Landman (2003, 2004), Geurts (2006), Scontras (2014), Kennedy (2015), Rothstein (2013, 2017), and Snyder (2017)). What's more, on all such analyses, 'four' in (1a) and (52a,b) is an *adjective*, and for good reason. On its face, 'four' in (49a) is a predicate, a seemingly appropriate meaning for which is given in (50).

(50) $[[\text{four}]] = \lambda x.\ \#(x) = 4$

The meaning suggested in (47) – appropriate for (49b) – is then derivable from (50) via an independently motivated type-shifting principle, as are meanings potentially appropriate for (1a) and (1b).[20] Crucially, however, determiners *cannot* function as predicates or modifiers – cf. Sect. 3.4.1. This would be entirely mysterious if the

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[18] This presupposes type-shifting. See e.g. Rothstein (2017), and Snyder (2017).

[19] Cf. Geurts (2006) and Rothstein (2013).

[20] See Snyder (2017).

lexical meaning of 'four' were that of a determiner since, in that case, the type-shifting principles responsible for generating meanings appropriate for 'four' in (49a,b) would likewise generate meanings appropriate for *all* determiners. In other words, this would incorrectly predict that all determiners can in fact function as predicates and modifiers, contrary to fact.

In short, the problem is not that the GQT analysis provides the wrong *meaning* for 'four' in (1a) – in fact, a meaning equivalent to (17) can be generated from (47) or (50) via commonly accepted type-shifting principles. Rather, the problem is with the inference potentially drawn based on (17): because GQT analyses 'four' in (1a) as denoting a relation between sets, *lexically* 'four' must be a determiner having that same meaning. This is a non-sequitur, as should now hopefully be clear. Yet without some such assumption in place, it simply does not follow that 'four' in (1b) must also have this meaning, even if, to repeat the quote from Hofweber (2005, p. 211) again, "the word 'four' is the same in [(1a)] and [(1a)]".

All of this points towards two important observations relevant to Hofweber's nominalist program. First and foremost, contrary to what Hofweber apparently assumes, it is *not* uncontroversial that "attributive" uses of number expressions, such as 'four' in (1a), are quantificational determiners to be analyzed on the model of GQT. Recall the quote from Hofweber (2007, p. 3–4):

> In contemporary natural-language semantics the uses of 'four' as in [(1a)] are pretty well understood, and 'four' is usually considered to be a determiner, an expression of the same kind as 'some', 'many', and 'all'.

A similar sentiment is expressed in Hofweber (2016, p. 123):

> As it turns out, [GQT] works perfectly well, at least for the cases we are considering here, and it is widely accepted.

It is not clear on what empirical grounds Hofweber could justifiably make either of these assertions. In fact, the first, also endorsed in Hofweber (2016), seemingly belies an understanding of the current state of research within contemporary linguistic theory: the best, most current available evidence points towards 'many' being an *adjective*, not a determiner (see e.g. Rett (2008), Solt (2009), Wellwood (2018), and Snyder (2020)).[21] Again, just because 'many' was analyzed as denoting a relation between sets in Barwise and Cooper (1981), it does not follow that it must be a determiner having that lexical meaning.

More to the point, as the citations above indicate, GQT was not the only analysis of number expressions available at the time of publishing Hofweber (2005), and there was already ample evidence available suggesting that number expressions are better understood as adjectives. Furthermore, while there has been a growing consensus among linguists towards that conclusion ever since, virtually all of this research presupposes, contra Hofweber, that number expressions can function

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[21] For one thing, unlike all prototypical determiners, 'many' has a comparative and superlative form – 'more' and 'most', respectively – and is gradable – cf. 'very/so/how many'.

referentially.[22] Thus, it would appear that Hofweber's pronouncements regarding the current state of research within linguistic semantics are at best misleading.

A second significant fact about the polymorphic analyses mentioned above is that they presuppose an independent domain of numbers, to serve as the range of the cardinality function '#'. Specifically, '#' is a *measure function*, or function from entities to numbers. What's more, this features in all meanings relevant to cardinal uses of 'four', including (1a) (cf. (48)). The implication is that even if "attributive" uses do not overtly reference numbers, the metalanguage in which the semantics is formulated is clearly committed to their existence.

However, the same can be said for the GQT analysis. Specifically, (17) contains a numeral ('4'), the referent of which assumed to be a number. In fact, GQT makes rampant use of such numbers to provide a unified analysis of determiner meanings. These include e.g. 'at least four', 'between four and six', and 'four out of five', which explicitly involve number expressions, as well as e.g. 'many', 'most', and 'infinitely many', which do not.[23] Thus, even if 'four' in (1a) had the non-referential meaning attributed in (17), since the metatheory of GQT is committed to numbers, any theorist evoking GQT is also committed to their existence.[24] What's more, those numbers are not obviously eliminable in favor of something else, e.g. the language of first-order quantificational logic. After all, one of the original motivations for GQT was to provide a *uniform* analysis of quantificational expressions, including those which are known to be unanalyzable in terms of first-order quantificational logic, e.g. 'infinitely many'.

In short, even if the lexical meaning of 'four' were that provided in (17), and even if this was the meaning witnessed in (1a,b), it still would not follow that our ordinary number-talk does not involve a commitment to numbers, at least at the metasemantic level. More generally, even if *all* occurrences of number expressions, including their apparent use as numerals in arithmetic statements, were quantificational determiners to be analyzed on the model of GQT, it would not follow that making semantic sense of number talk more generally supports nominalism.

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[22] In fact, the only counterexample we are aware of, which happens to be directly informed by, and formulated partially in response to, Hofweber (2005), is Ionin and Matushansky (2006). Incidentally, this also happens to be the target of the semantic arguments mentioned in Sect. 3.4.3.

[23] See Barwise and Cooper (1981).

[24] An anonymous reviewer observes that the same argument would extend to sets, which should be just as objectionable from a nominalist perspective, but that this kind of commitment might be avoided by appealing to a pluralist metalanguage, perhaps following Boolos (1985). As far as we know, whether all of GQT can be recovered within a pluralist metalanguage is an open question, though McKay (2006) makes progress in this direction. Even so, the question would remain as to whether an empirically adequate, nominalist-friendly pluralist semantics for number expressions could be formulated, something which some of us have cast doubt on in other work (e.g Snyder and Shapiro 2021).

3.5 Conclusion

We have argued that neither linguistic component of Hofweber's analysis of ordinary number talk survives empirical scrutiny. Specifically, the syntactic component fails in virtue of the empirical implausibility of the operation posited ("extraction"), while many of the distinctive semantic theses put forward by Hofweber are empirically problematic. These include:

(ST1) 'four' in (1a) is a non-referential determiner, to be analyzed on the model of GQT.

(ST2) 'four' in (1b) has the same non-referential meaning witnessed in (1a).

(ST3) Numerals, or at least those occurring in arithmetic statements, are semantically bare determiners.

Some of these problems arguably stem from an initial syntactic misclassification encoded in (ST1), namely that 'four' in Frege's (1a) is a determiner, rather than an adjective. Without that initial assumption in place, (ST2) clearly doesn't follow, even if we grant "extraction". On the other hand, we have seen that it is potentially important for Hofweber's larger program that 'four' in (1a) be seen as a determiner, as at least certain adjectives appear to have genuinely referential uses, unlike all known determiners.

Furthermore, in addition to the numerous problems noted for analyzing numerals in arithmetic statements as semantically bare determiners, the empirical motivation for (ST3) is further weakened once we recognize that a variety of expressions can be coordinated in a manner seemingly resembling (51).

(51)   Three and two is five.

Consider the examples in (52), for instance, respectively involving color expressions, measure phrases, and bare nouns.

(52)   a. Red and blue is purple.
        b. Two feet and twelve inches is one foot.
        c. Horseradish and ketchup is cocktail sauce.

Naively, all four examples have a distinctly "combinatory" feel: the result of combining the pre-copular things results in the post-copular thing. Seen this way, nothing about (51) itself forces the conclusion that the number expressions involved are (semantically bare) determiners, and thus non-referential expressions. In fact, the expressions in (52a–c) are commonly assumed within linguistic semantics to have genuinely referential uses.[25] What's more, it has been argued, notably by Rothstein (2013, 2017), that the same semantic operation responsible for the referentiality of the expressions in (52) – *nominalization* – is also responsible for the referentiality

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[25] See e.g. Scontras (2014) for measure phrases, and Chierchia (1998) for bare nouns.

of numerals. Thus, given that a *uniform*, compositional analysis of (51) and (52a-c) is independently desirable, providing an empirically adequate semantics for (51) might well require that the apparent numerals involved are genuine singular terms.[26]

All of this casts significant doubt on the empirical motivations for Hofweber's Adjectivalism, and with it the proposed resolutions of Frege's Other Puzzle and the Easy Argument. Ultimately, this highlights the difficulties inherent in the sort of empirically informed methodological naturalism that Hofweber's project intends to engage in.

In our view, Hofweber's analysis is thus perhaps best viewed as an impressive exploration of an intriguing linguistic hypothesis that, if true, could have significant ontological consequences for the philosophy of mathematics. Specifically, if all uses of number expressions could be viewed as non-referential determiners, then making semantic sense of number talk more generally might not require an ontology of natural numbers. It's just that, given the best available linguistic evidence, the antecedent of this conditional is highly implausible. The takeaway lesson is that insofar as one seeks to engage in this sort of methodological naturalism, as we intend to do, one must ignore prior metaphysical predilections and let the empirical chips fall where they may.

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[26] Contra Moltmann (2013).

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