Dialectics of Difference and Negation:
The Responses of Deleuze and Hegel to Representation

by

Henry Somers-Hall

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Declaration of Inclusion of Published Work

An abbreviated version of chapter one was published under the title, *Sartre and the Virtual*, in *Philosophy Today*, 2006 supplemental volume.

This thesis has not been submitted for a degree at another university.

Abstract

This thesis has the following aims. First, to show that Deleuze can be situated clearly within the post-Kantian tradition. This is achieved through an analysis of the relations between Kant's transcendental idealism and Deleuze's transcendental empiricism. Second, to explore the criticisms of representational theories of difference which can be found in the work of Deleuze and Hegel. Representational theories are best understood as theories which rely on a logic which is governed by relations between entities which pre-exist those relations. Deleuze argues that these logics presuppose the formal equivalent of a homogeneous space within which these relations can be construed. Hegel similarly understands representation as the utilisation of finite categories which rely on the fixity of the subject of predication. The third aim is to provide a rigorous explication of some of the key themes of Deleuzian ontology, particularly in relation to the problem of representation. This will involve looking at the logic of multiplicities, which attempts to provide a theory of difference that is non-oppositional. This logic will be clarified through a discussion of Deleuze's use of modern geometry, and his analysis of the foundations of the calculus. The fourth aim will be to contrast Deleuze's solution with that of Hegel, particularly with respect to their relationships to Kant and the calculus. This is achieved through the Deleuzian distinction between finite and infinite representation, the latter in Deleuze's view characterising the Hegelian attempt to bring the idea of transition into representation itself. Finally, having shown where Deleuze and Hegel differ in their respective projects, the thesis will explore whether either of these philosophies has the resources to provide a refutation of the other with reference to the dialectic of force and the understanding in the *Phenomenology of Spirit*, and the problem of the one and the many.

List of Abbreviations

In citing works in the notes and text, short titles have generally been used. Works frequently cited, or works by Deleuze or Hegel, have been identified by the following abbreviations:

Works by Deleuze

B	Deleuze, Gilles. *Bergsonism*. (New York: London : Zone, 1988).
BCD	Deleuze, Gilles. "Bergson's Conception of Difference," in Deleuze, Gilles, *Desert Islands and Other Texts 1953-1974*. Edited by David Lapoujade and translated by Daniel W. Smith. (Los Angeles, Calif.: Semiotext(e), 2004).
DR	Deleuze, Gilles. *Difference and Repetition*. Translated by Paul Patton. (New York: Columbia University Press, 1994).
LS	Deleuze, Gilles. *The Logic of Sense*. Translated by Constantin V. Boundas. (London: Continuum, 2001).
N	Deleuze, Gilles. *Negotiations, 1972-1990*. Translated by Martin Joughin. (New York ; Chichester: Columbia University Press, 1995).
NP	Deleuze, Gilles. *Nietzsche and Philosophy*. Translated by Hugh Tomlinson. (London: Athlone Press, 1983).
PrS	Deleuze, Gilles. *Proust and Signs*. Translated by R. Howard. (London: Athlone, 2000).
S	Deleuze, Gilles. "Review of Simondon (1966)." in *Pli – The Warwick Journal of Philosophy*, 12:43-49. Translated by Ivan Ramirez.
SPP	Deleuze, Gilles. *Spinoza, Practical Philosophy*. Translated by R. Hurley. (San Francisco: City Lights Books, 1988).
WP	Deleuze, Gilles and Felix Guattari. *What Is Philosophy?* Translated by Graham Birchill and Hugh Tomlinson. (London: Verso, 1994).

Works by Hegel

EL	Hegel, Georg Wilhelm Friedrich. *The Encyclopaedia Logic, with the Zusätze*. Translated by Theodore F. Geraets, Wallis Arthur Suchting, and Henry Silton Harris. (Indianapolis: Hackett, 1991).
FK	Hegel, Georg Wilhelm Friedrich. *Faith & Knowledge*. Translated by Walter Cerf and H. S. Harris. (Albany: State University of New York Press, 1977).

JL	Hegel, Georg Wilhelm Friedrich. *The Jena System, 1804-5 : Logic and Metaphysics*. Translated by H. S. Harris, John W. Burbidge, and George Di Giovanni. (Kingston: McGill-Queen's University Press, 1986).
LHP	Hegel, Georg Wilhelm Friedrich. *Lectures on the History of Philosophy. 3 vols*. Translated by Elizabeth Sanderson Haldane and Frances H. Simson. (Lincoln: University of Nebraska Press, 1995), cited by volume then page.
PS	Hegel, Georg Wilhelm Friedrich. *Phenomenology of Spirit*, Translated by J. N. Findlay and Arnold V. Miller. (Oxford: Clarendon Press, 1977).
RSP	Hegel, Georg Wilhelm Friedrich. "On the Relation of Skepticism to Philosophy," in Di Giovanni, George, and H. S. Harris. *Between Kant and Hegel : Texts in the Development of Post-Kantian Idealism.* (Indianapolis: Hackett Pub. Co., 2000).
SL	Hegel, Georg Wilhelm Friedrich. *Hegel's Science of Logic*. Translated by Arnold V. Miller. (Atlantic Highlands, NJ: Humanities Press International, 1989)

Other Frequently Cited Works

CE	Bergson, Henri. *Creative Evolution*. Translated by Arthur Miller. (Mineola, N.Y.: Dover Publications, 1998)
CM	Bergson, Henri. *The Creative Mind : An Introduction to Metaphysics.* Translated by M. L. Andison. (New York: Wisdom Library, 1946)
CPR	Kant, Immanuel. *Critique of Pure Reason*. Translated by Norman Kemp Smith. (London: New York : Macmillan ; St. Martin's Press, 1929)
EM	Merleau-Ponty, Maurice, "Eye and Mind," in *The Primacy of Perception.* Edited by James M. Edie. (Evanston, Ill: Northwestern University Press, 1964)
ISA	Porphyry. *Isagoge*. Translated by Edward W. Warren. (Toronto: Pontifical Institute of Mediaeval Studies, 1975)
MP	Aristotle. "Metaphysics," in *The Complete Works of Aristotle*. Edited by Jonathan Barnes and translated by W. D. Ross. (USA: Princeton University Press, 1984)
TE	Sartre, Jean-Paul. *The Transcendence of the Ego : An Existentialist Theory of Consciousness*. Translated by Forrest Williams. (New York: Noonday Press, 1972)

All references to Aristotle are to *The Complete Works of Aristotle*, edited by Jonathan Barnes and translated by J. L. Ackrill, (USA: Princeton University Press, 1984), unless otherwise stated.

Introduction to the Thesis

This thesis will attempt to articulate and evaluate the responses to what Deleuze calls finite representation put forward by Hegel and Deleuze. The meaning of the term finite representation should become clear as we explore the problematic in detail in the first couple of chapters of the thesis, but in Deleuzian terms, it is the problem generated by a logic of discrete multiplicities, that is, a multiplicity made up of elements which remain indifferent to their relations, or, at the least, pre-exist the relations between them. This notion of a discrete multiplicity, which Deleuze derives from Bergson's work on time, is primarily characterised by Deleuze in terms of Aristotelian logic, but as we shall see, it also permeates more modern logical systems, and we shall also find it at play in the work of Russell, and something similar operating in Kant's philosophy. We will see that the same problematic emerges in Hegel's philosophy, where it is characterised not in terms of representation but instead in terms of the finite thought of the understanding. The operation of the finite understanding turns out to be much the same as that of finite representation, however, and just as Deleuze criticises propositional thinking for instantiating fixed relations between elements in a homogeneous space, Hegel also criticises the finite understanding of the proposition for the fixity of its parts, replacing it with the speculative proposition. As we shall see, finite representation entails certain difficulties, such as a necessarily equivocal conception of being, since it is *a priori* impossible to specify a concept of totality (what Deleuze calls the problem of the large), and the difficulty in specifying that which falls beneath the level of the species, the individual. Combined with these are the inevitable difficulties in explaining temporality within a system of finite categories.

In exploring the two responses to these problems, we will begin by taking up Hegel's claim that philosophy must take Kant into account, and his statement that "there

are two ways of going further [than Kant] …: one can go forward or backward. Looked at in the clear light of day, many of our philosophical endeavours are nothing but the (mistaken) procedure of the older metaphysics, an uncritical thinking on and on, of the kind that anyone can do" (*EL*, § 41, Add. 1). Deleuze explicitly relates his own project to Kant's by characterising himself as a transcendental empiricist. The first chapter will therefore deal with Deleuze's relation to Kant, in order to position Deleuze within the post-Kantian tradition. This will allow us to more easily differentiate the approach of Deleuze from that of Hegel, but also to relate them to each other through their shared concerns with the Kantian project. For both Deleuze and Hegel, the difficulty with the Kantian project is in large measure its inability to provide genetic explanations. For Hegel, this amounts to a failure to carry through a proper metaphysical deduction of the categories, which can only be remedied by showing how the categories develop out of each other. Thus Hegel moves to a philosophy of what he calls infinite thought, or Reason, whereby contradictions inherent in categories lead not to scepticism, but instead to further, more adequate categories. Deleuze instead argues that the difficulty is that the categorical system developed by Kant cannot explain the genesis of experience, but only its conditioning. In taking this line, Deleuze posits a fundamental difference between the empirical and the transcendental, which allows us to understand the transcendental as truly generative.

In exploring these differences, we will look at the uses made of the differential calculus by both Deleuze and Hegel. The calculus provides, for Hegel, an important antinomy of thought, which he takes to show that its foundations must fall outside of the finite categories of mathematics. Indeed, the calculus provides an illustration of the structure of Reason as a whole. The calculus has an even more central role for Deleuze. As he writes, "modern finitist interpretations [of the calculus] betray the nature of the calculus no less than the former infinitist interpretations, because both fail to capture the extra-propositional or sub-representational source – in other words the 'problem' from

which the calculus draws its power" (*DR*, 264). The approaches of Deleuze and Hegel are differentiated by their interpretations of the calculus. In Deleuze's terms, Hegel takes finite representation, and infinitises it. That is, he resolves the problems found in finite representation by moving to a position of infinite representation. Deleuze's solution is instead to understand representation as grounded in that which is both non-representational but still determinable. In highlighting these interrelations, I have largely followed the terminology of Deleuze, particularly in associating Hegel with representation, even if of an infinitised kind. I have done this purely insofar as these characterisations seem to reasonably accurately capture the relation of Deleuze to Hegel, and have tried to avoid imbuing them with normative associations. Thus, whilst I believe Deleuze is right to characterise Hegel as a philosopher of identity, what is at issue are the differences between the kind of identity philosophy put forward by Hegel and those of his predecessors. It would therefore be a mistake to take this interpretation as explicitly providing grounds for a rejection of Hegel. Once we have an idea of how these positions are structured, we will then explore how they relate to one another. The approaches of Deleuze and Hegel are not compatible with one another, and so we will also have to look at whether either of these two thinkers has the resources to show the inadequacy of the other's approach. We will do this by returning to the *Phenomenology of Spirit*, in order to see how Hegel's dialectic of 'Force and the Understanding' relates to Deleuze's project, which maintains a transcendental approach similar to that criticised by Hegel.

In approaching the relations between Deleuze and Hegel, I have concentrated for the most part on two works, Deleuze's *Difference and Repetition*, and Hegel's *Science of Logic*. The primary reason for this is that my concern in this thesis has been the structure of dialectic, in both its Hegelian and Deleuzian varieties. In the case of the *Science of Logic*, its subject matter, the movement of thought itself, makes it ideal for this task. *Difference and Repetition* provides the most detailed explication of Deleuze's

conception of dialectic in his early work, and is also considered to be his most 'philosophical' work in a classical sense. For this reason, it provides the best place to look for a parallel account of the structure of Deleuzian logic. In explicating the positions put forward in these texts, I have also relied on other works, particularly Deleuze's earlier work on Bergson, and Hegel's *Jena Logic* in order to clarify them. Following Badiou's influential work, *Deleuze and the Clamour of Being*, it has become customary to posit a break in Deleuze's writings between *Difference and Repetition* and *The Logic of Sense*, on the one hand, and *Capitalism and Schizophrenia*, on the other. This allows Badiou, and derivative works by Žižek and Hallward, to dismiss the later work as lacking in rigour, and then to go on to provide more engaged criticism of the early work. The typical response to this 'divide and conquer' approach has been to attempt to reinstate the later work. I do not want to take a position on whether this division of Deleuze's work is valid, but will nonetheless for the most part be engaging with the early work, and in doing so will implicitly oppose the Badiouian interpretation of this part of Deleuze's system.[1]

In terms of the interpretations of Deleuze and Hegel themselves, I have tried with Deleuze to provide an interpretation which gives appropriate weight to both the philosophical and scientific sides of his thinking, as it strikes me that his work cannot be understood adequately from either position in its own right. Deleuze's use of science should be seen as providing a further level of determinacy to a metaphysics largely derived from Bergson. In interpreting the work of Hegel, I have argued for an ontological rather than transcendental interpretation. The ontological reading is closest to that put forward by Deleuze, and I think, is well justified by the text. I hope this will become clear in my explication of Hegelian dialectic. In regard to work encompassing both Hegel and Deleuze, it is remarkably scarce given Deleuze's own orientation of his philosophy in the field of a "generalised anti-Hegelianism" (*DR*, xix), and is generally schematic in character, such as the treatments found in the works of Daniel Smith,

Michael Hardt, and Nathan Widder.[2] On the Hegelian side, the situation generally seems one of ignorance, with, to my knowledge, only Stephen Houlgate providing any engagement with Deleuze's work. Houlgate attempts to show that Deleuze's philosophy does not move beyond the doctrine of essence, which Hegel claims in the *Science of Logic* is only a partial view of the world. I have not dealt with Houlgate's interpretation directly, but in chapter seven where I look at Hegel's resources to deal with a transcendental empiricism, I have raised a similar Hegelian objection based on Hegel's dialectic of force. The first of the two texts which deal with Deleuze's relation to Hegel in any detail is Miguel Beistegui's *Truth and Genesis*, which provides a dense but balanced analysis of Hegel's relation to Deleuze, Aristotle, and Hegel. The aim of *Truth and Genesis* is, however, to provide a dual ontology of Heideggerian *Truth* and Deleuzian *Genesis*, and to this extent, direct engagement with the relation between Hegel and Deleuze is limited. The other text, Simon Duffy's *The Logic of Expression: Quality, Quantity and Intensity in Spinoza, Hegel and Deleuze*, does attempt to explicitly engage with the relationship between the philosophies of Deleuze and Hegel, mediated by their interpretations of Spinoza. Whilst it provides many interesting analyses, the work as a whole suffers from a major interpretative error, positioning the axis of division between Hegel and Deleuze along the line of finite/infinite, rather than the propositional/extra-propositional. Thus, Deleuze is taken to use modern mathematical interpretations of the calculus to show the infinite thought of Hegel to be redundant. In actual fact, as I show in chapter six, Deleuze is opposed to both philosophies of the finite and of the infinite, as he takes both terms to be irredeemably tied to representation. Difference therefore falls outside of both categories. In the rest of this introduction, I wish to give a brief outline of the thesis as a whole.

Structure of the Thesis

Chapter One: Deleuze and Transcendental Empiricism

The aim of this chapter is to provide a basic understanding of the structure of Deleuze's philosophy through an exploration of his relation with Kant. In doing so, some of the general problems to do with the representational approach are brought out in contrast with Deleuze's own philosophy. The chapter begins with an exposition of the transcendental deduction, before moving on to the critique within *The Transcendence of the Ego* of the notion of the transcendental subject. Arguments are put forward for the applicability of Sartre's central argument to Kant, as well as Husserl, his explicit target. The conditions for a transcendental philosophy which does not rest upon the idea of a transcendental subject are then derived, followed by a discussion of the use to which they are put by Deleuze himself. The chapter concludes by showing how the rejection of the subject and the move to a new transcendental logic are interderivable for Deleuze. This allows the remainder of the thesis to focus on the problem of the structure of the dialectics of Deleuze and Hegel.

Chapter Two: Difference and Identity

This chapter looks at the role of difference in classical theories of logic, thereby expanding the specific worries of chapter one. Beginning with Aristotle, I show how his formulation of the hierarchy of species leads to difficulties both in terms of the largest element, Being (the problem of equivocity), and the smallest elements, (the atemporal nature of essence). Analogous conclusions are uncovered for Russell's philosophy in the *Principia Mathematica* and the theory of types. The root of the difficulty in both systems is traced to the logic of opposition which lies at their foundation. Both Deleuze's and Hegel's approaches to these difficulties are then sketched out; Deleuze in

terms of the development of a non-oppositional logic of difference, and Hegel in terms of the move to a logic of contradiction.

Chapter Three: Bergsonism

Chapter three amplifies the critique of Aristotle and Russell from the previous chapter, as well as relating these to difficulties highlighted in the first chapter with the Kantian system. Whereas the first chapter gave the conditions for a transcendental empiricism, this chapter provides a detailed exposition of Bergson's logic of multiplicities. This will open the way for our discussion of Deleuze's use of Bergson's logic. The difficulties with Russell and Aristotle are re-examined in the light of Bergson's conceptions of space and duration.

Chapter Four: The Virtual and the Actual

This provides the opening to an analysis of Deleuze's philosophy in terms of complexity theory, followed by a discussion of his theory of depth and its origins in the aesthetics of Merleau-Ponty. Finally, I explore Deleuze's interpretation of the problematic, and of the proposition.

Chapter Five: Infinite Thought

This chapter begins by returning to Kant's first *Critique*, and exploring his interpretation of the concepts of finitude and infinity in relation to discursivity. The aim is to show that, from a Hegelian perspective, Kant presupposes a certain interpretation of these concepts from the outset. I then show how a Hegelian dialectical approach to these ideas offers the possibility of genetic interpretations of them which show each to be unthinkable without the other. By drawing parallels between Hegel's dialectic of

difference in the doctrine of essence and the dialectic of the infinite in the doctrine of being, I in turn show that the application of the genetic concept of the infinite allows Hegel to overturn the Kantian position by moving to an understanding of philosophy as governed by infinite thought. This is then used to clarify Hegel's concept of the speculative proposition.

Chapter Six: Hegel and Deleuze on Ontology and the Calculus

This chapter attempts to relate Deleuze and Hegel through exploring two concrete areas of disagreement between the two thinkers, i.e. their interpretations of the differential calculus and the Kantian antinomies. I then show how both of these examples function as images of the systems as a whole. From here we can see that the difference between the two thinkers revolves around their approaches to the categories of the finite and infinite, which Hegel attempts to resolve through showing their reciprocal implication whilst Deleuze instead tries to introduce an element of a-finite difference which serves as a precondition for both categories.

Chapter Seven: Force, Difference, and Opposition

This chapter will extend the discussion to Hegel's concepts of force and the understanding, and to the inverted world. The idea is to bring out Hegel's consideration of the possibility of an understanding of the world in terms of structures different in kind from the actual, and to see to what extent Hegel can produce a critique of transcendental empiricism. From here, we will see how Deleuze can answer these possible criticisms. This will lead us to a discussion of determination, and the problem of the one and the many.

Chapter One – Deleuze and Transcendental Empiricism

Introduction

In the opening chapter of this thesis, my aim will be to begin to explicate what Deleuze means by his claim that he is attempting to create a form of transcendental empiricism. In doing so, we will see that Deleuze's philosophy can be placed firmly in the post-Kantian tradition. The radical difference between Deleuze's system and those of the post-Kantians who precede him is the attempt to construct a theory of the transcendental which maintains the differentiated structure of the transcendental field whilst removing the subject as the synthesising agent. We can see that in the framework of the Kantian system, the transcendental unity of apperception plays a central role in structuring the empirical world for consciousness. This is made clear in the transcendental deduction, which sets up a relation of structural parallelism between the transcendental categories and the functions of judgement. For Kant, the categories provide the form of the empirical world, and judgement provides the form of our knowledge of the empirical world. Deleuze will argue that, traditionally, transcendental philosophy has been founded on this claim that "the conditions of the object of knowledge must be the same as the conditions of knowledge" (*LS*, 105). Without the identity of these conditions, the subject is no longer able to account for its ability to understand the nature of the object as it appears to it. The thesis of the identity of conditions allows us to explain our ability to make statements about the nature of the world, as the synthesis of the empirical world is now a function of the subject, and secures a direct correspondence between the structure of knowledge and the structure of the world.[1] The alternative to this thesis would be to posit the synthesis of the world outside of the subject. It is the difficulties raised with just such a notion of an external synthesis which led Kant to construct the transcendental idealist framework in the first place.[2] If the synthesis of the world takes place outside of the subject, we have to

account for the fact that the predicates which we use to describe the world correspond to the properties of the object within the world itself. The most obvious ways of doing this are either through a metaphysics of essences and pre-established harmonies, returning to the notion of God as guarantor of the isomorphism of the two structures, or through the rejection of essences and metaphysics and a move towards a raw empiricism. In both these cases, the formulation of synthetic *a priori* propositions becomes problematic, as they either rest on the presupposition of a benevolent God as guarantor, or are simply put out of play by empiricist scepticism. For Deleuze, the difficulty with the debate between the metaphysical thinkers and those of a Kantian persuasion is that, for both, the necessity of an isomorphism between the two structures has been presupposed, whether through the Kantian notion of synthesis, or the metaphysical notion of essence.[3] Deleuze instead will posit a difference in kind between the transcendental and empirical.

As we shall see, Deleuze argues that this presupposition is that either Being is seen as differentiated, in which case it maintains the predicable structure, or else it is undifferentiated, in which case nothing can be said of it.[4] Deleuze instead proposes a third alternative, which is that the structure of the transcendental field is different in kind from the empirical. The result of this would be that the transcendental field would become entirely pre-individual, but still differentiated, removing the subject from the role of synthesising agent, and thus splitting the conditions of *knowledge* of the object, in the sense that our knowledge of the object is understood propositionally, or in terms of the structure of judgement, from the conditions of the *object*, which will now be given by what Deleuze calls a 'sub-representational' transcendental field. This will mean that whilst conditions of the object will be formulated in terms of the *difference* between the transcendental and empirical, conditions for knowledge of the object will be formulated in terms of a structural *identity* between the constituted object and judgement. In fact, knowledge of the object will require both sets of conditions to be fulfilled for Deleuze, since the first constitute the subject and the object (and hence the sphere of

representation). Knowledge of the object requires in excess of the conditions of the object a further set of conditions – an isomorphism between judgement, as subject-predicate based, and the object, as substance-property based. It is in this sense that Deleuze's rejection of the identity of conditions of objects and conditions of knowledge of objects is to be understood. For Deleuze, this difference in kind between the empirical, which is governed by the structure of judgement and the transcendental allows the transcendental to be seen as properly generative. That is, rather than merely conditioning the object, it actually generates the objectival structure of the empirical without itself possessing that structure. This is in contrast to Kant, who assumes the identity of the condition and the conditioned (that is, of the transcendental and the empirical). Beginning to clarify the grounds as well as the structure of this move will be the main task of this chapter, although this clarification will necessarily be schematic until we have looked at the material in chapters four and six. In outlining the structure of Deleuze's development of this primary thesis, I will provide an analysis of the relevant sections of the *Critique of Pure Reason*, before evaluating Sartre's critique of the role of the subject within transcendental philosophy. This will allow us to see why Deleuze feels the necessity to move to a theory of the virtual and the actual, and to highlight what he considers to be the two fundamental misunderstandings of the transcendental field: the "dogmatic confusion between event and essence" and the "empiricist confusion between event and accident" (*LS*, 54). I will conclude with some comments about the validity of this Deleuzian deduction of transcendental empiricism, given his reliance on Sartre's notion of the transcendental field, which turns out to be not so different from Kant's conception of the transcendental. By the conclusion of the chapter, we should, therefore, be in a position to understand Deleuze as attempting both to engage with and overcome the limitations of Kant's philosophy. This will form the groundwork for the comparison of Deleuze's approach with Hegel's similar (at least in respect of the problematic from which their thought arises) project.

Kant and *The Critique of Pure Reason*

Deleuze's break with Kant concerns the nature of the transcendental field, and the isomorphism between the functions of judgement, which allow us to make judgements about the empirical world,[5] and the categories, which as transcendental allow the synthesis of the empirical manifold. For Kant, this parallelism of the operations of the understanding is essential, as it is this which allows us to attribute to the subject the power to condition the empirical manifold, and therefore to know with certainty that the understanding is able to apply its concepts to this manifold. As our consciousness of the empirical manifold is generated through categories of the understanding, the same understanding which employs the functions in order to form judgements, then the structure of judgement will mirror that of the world, thus allowing certain synthetic propositions about that world to be guaranteed valid. By showing that the categories apply to the world, Kant is able to follow Hume in granting that all knowledge begins with experience, whilst at the same time allowing contentful propositions about the structure of that experience to remain necessarily true, since these fundamental structures are imposed on experience by the subject as the conditions under which experience is possible at all. The transcendental for Kant therefore contains those structures which concern the non-empirical determinations of the object, those which make experience of the object possible. For Deleuze, as a transcendental empiricist, what is of interest about the transcendental field is not its ability to guarantee knowledge, but rather the *generative* principles that it provides for the empirical world. In this change of emphasis, what is at stake is both the structure of the transcendental field, and the rules which govern this structure and through this the structure of empirical experience, these rules being what Kant would call transcendental logic.

Kant specifies two conditions for a transcendental logic. First, that it must "contain solely the pure thought of an object, [and] exclude only those modes of

knowledge which have an empirical content" (*CPR*, A55/B80). Second, that it must "also treat of the origin of the modes in which we know objects, in so far as this origin cannot be attributed to the objects" (*CPR*, A55/B80). Despite the differences of purpose in the employment of the transcendental by Kant and Deleuze, virtually identical conditions are accepted by both these thinkers as necessary conditions that must be fulfilled by any transcendental logic. For Deleuze to accept these conditions, however, the references to the subject need to be removed: he will argue that Kant has not shown the necessity of the transcendental field being generated in relation to a subject. Thus transcendental logic for Deleuze would firstly concern purely the Idea of an object, whilst still excluding those structures of the object as an empirical (actual) manifestation of that Idea, and secondly must concern itself with the origins of the object as experienced, insofar as this origin is not attributed to the object (or even of an objectival[6] nature). In order to see why Deleuze feels that this revision is necessary, we need to look primarily at two sections within the *Critique of Pure Reason*: 'The Clue to the Discovery of all Pure Concepts of the Understanding', or metaphysical deduction, where Kant lays out the relations between the functions of judgement and the categories (the pure concepts referred to in the section title), and 'The Deduction of the Pure Concepts of the Understanding', or transcendental deduction, where Kant attempts to show the necessity of the subject in the structuration of experience. My aim in this section will therefore be to outline the arguments which Kant puts forward in support of the notion of the subject. This will allow us to deal with the objections which Sartre will put to Kant. Whilst the aim of this chapter is to outline the path from transcendental idealism to transcendental empiricism, this should not blind us to the fact that the insights provided by Kant within the transcendental deduction have been taken up even by his adversaries, and so, Deleuze believes, by attacking the transcendental subject as found in the Kantian system, he is able to show the limitations of this notion of the subject in general.

The 'Clue to the Discovery of all Pure Concepts of the Understanding' (otherwise known as the 'metaphysical deduction') opens the 'Transcendental Analytic', and deals with the second of the faculties, the understanding. Kant has attempted to show in the Aesthetic that the sensibility deals with *a priori* forms of intuition, i.e. that space and time are ideal forms through which objects are presented to the subject. By attempting to show that intuition is a function of sensibility, Kant has laid the foundation for transcendental idealism, as these intuitions have been shown to have an *a priori* basis in the subject's cognition. The sensibility may show how objects are presented to consciousness, but Kant has not yet established how objects can be thought by consciousness. Showing that the manifold can be conceptualised proves a far more complicated task than showing its *a priori* nature, as it requires showing both the *a priori* nature of concepts and their application to the manifold. Kant's problem is therefore the problem of showing how it is possible for two faculties, the faculty of sensibility, and the faculty of the understanding, to interact with one another. Whilst it is in the transcendental deduction that the attempt to justify the application of categories to intuition is put forward, the metaphysical deduction sets forth the relation between the ordinary functions of the understanding and the categories. For Deleuze, this setting forth of the relation to the categories, even though the categories' legitimate application has not yet been shown, is one of the most important sections of the first *Critique*, as it is here that the conditions for the possibility of objects are first equated with the conditions for the possibility of knowledge of objects through their joint origin in the understanding.

The metaphysical deduction aims to analyse the structure of the understanding, and in doing so to determine possible candidates for pure concepts which as categories will relate the understanding to intuition. It proceeds by establishing an isomorphism between the functions of the understanding and the categories, as shown in the tables of functions and categories. Kant begins his analysis by showing that the faculty of the

understanding deals essentially with representations. Through a particular function, it is able to bring these representations into relation with one another, and through these relations, representations are subsumed under other representations in order to create unities. As this act is performed by the faculty of the understanding, it is natural to equate this act with that of judgement. Following the prevailing understanding of logic at the time, Kant assumes that judgements are always of the subject-predicate form and equate to the relation of a general representation, that may range over a large domain of entities, such as that of divisibility, and a specific representation, which relates itself directly to an object, such as that of a body. The unity formed by the operations of the understanding on these concepts is one of subsumption, and thus we can form the classical propositions that are the raw materials of syllogisms, as in for example the judgement, 'all bodies are divisible', which operates through the subsumption of the specific term, body, under the general concept, divisibility. Kant will argue that all thought can be reduced to variations on this structure, and thus that all thinking is minimally judging. Thus, all thought requires synthesis, understood loosely,[7] since all judgements require the construction of a subject-predicate structure. Just as there are a variety of syllogisms, so there are a variety of possible functions through which a judgement can be formed. In the above case, the concept of 'body' and of 'divisibility' could be synthesised into other unities, such as 'some bodies are divisible', or 'this body is divisible'. When we abstract from particular judgements, we find that all judgements must deploy certain functions, or rather a particular function from each of four groups of functions. These four heads (quantity, quality, relation and modality) each contain three moments, one of which will be present in every judgement. Therefore, all judgements must, for instance, either relate to all, some, or one of a particular class of subjects; thus one of these moments of quantity, must be present in every judgement. The four heads of judgement, as they stand, are purely formal, however, as they define solely the possible relations between representations, and if we are to understand the nature of the pure *concepts* or categories of the understanding, as opposed to the functions, which

Kant claims to have thus far discovered, we have to consider these functions in terms of transcendental logic.

The categories turn out to be these functions considered from this standpoint of transcendental logic. The ground for their discovery is the synthetic nature of judgement, its ability to form unities, and the necessity of a synthetic function within the transcendental field in order to synthesise the sensuous manifold. The architectonics of *The Critique of Pure Reason* require that if intuition is to be understood, it must take on an objectival form, since judgements relate to objects, understood as substances with properties. Thinking cannot take place within intuition, as Kant claims to have proved in the transcendental aesthetic that pure space and time are not conceptual, and therefore non-objectival. Whilst the attempted proof of the thesis that objectival judgements can be related to the empirical manifold must wait until the transcendental deduction, the metaphysical deduction gives us the structural relation that will hold between the transcendental and the empirical, providing this proof is carried through successfully. As the understanding is capable of the subsumption of representations according to a subject-predicate structure into judgements, it becomes necessary that a prior synthesis generate the objectival structure which provides the foundation for these judgements, that is, to synthesise intuition so as to allow it to become the subject of judgements. The functions of judgement, being purely logical forms will, however, prove to be insufficient for the unification of the manifold, as they deal only with judgements. This unity must therefore be provided by concepts other than, but compatible with the functions of judgement, if judgement is to be validly applicable to intuition at all. Therefore, the unifications of the manifold will be carried out by categories, which correspond to the functions, whilst also containing a *conceptual*[8] reference to intuition. Thus, for instance, the hypothetical function, if *A* then *B*, will be mirrored in the category of causality and dependence, which takes the form, *if* ... *then*, and relates it to a manifold ("The schema of cause or causality of a thing in general, is the real upon

which, whenever posited, something else always follows. It consists, therefore, in the succession of the manifold, in so far as that succession is subject to a rule" [*CPR*, A144/B183]). Thus we develop the second table, the table of the categories, divided into four heads of three moments, paralleling the table of functions. This of course has to be the case, as the functions constitute the primitive rules by which unities of judgement are synthesised. To each of these primitive functions, there must be a corresponding category in order that the possibility that the object may be fully comprehended is realised. More categories would make the object incomprehensible, as it would contain determinations not capable of being captured by judgement, and fewer would show that some of the functions were not in themselves primitive, but in turn derived from a subset of more powerful functions.

The metaphysical deduction presents a series of problems. The standard criticism of Kant is that he employs a method of deriving the categories just as arbitrary as that which he criticises Aristotle for using.[9] Whilst this objection may call for a restructuring of the table of categories, our purpose in outlining the metaphysical deduction is rather to illustrate the main Kantian thesis that two of the functions that the understanding fulfils are the synthesis of objects and the subsumption of representations of objects.[10] This connection extends to the fact that the table of judgements is already fundamentally intertwined with the transcendental functions which are derived from it.[11] We can now see why for Kant the transcendental field is structured according to the model of classical logic, as this will allow Kant to explain the synthetic nature of the transcendental, and to provide it with a differentiated structure. Deleuze's own metaphysical deduction will rest not on classical logic, but on the differential calculus, and will thus attempt to overthrow the double nature of the understanding as both synthesising the manifold and formulating judgements, upon which the Kantian model relies. We must now move on to the transcendental deduction itself, as the metaphysical deduction merely shows the conditions which Kant believes must be fulfilled by the

transcendental category in order that it may fulfil its role. The metaphysical deduction is not a proof of the categories' actual application to the manifold, but rather of their relation to the understanding. It is in the transcendental deduction that we will see that this isomorphism rests on the presupposition of an 'I' which provides a point in the transcendental field around which the empirical field is unified. Kant believes himself to have shown "the subjective conditions of thought" (*CPR*, A89/B122), and must now show how these conditions have objective validity, in other words, show how these conditions are also the conditions of objects of experience.

The transcendental deduction extends the results of the metaphysical deduction, which claims to have shown the structure of the understanding, by attempting to show the application of the understanding to objects. This new move amounts to attempting to meet the Humean challenge; one may be willing to accept that the concept of causality is indeed present within the understanding, but given that concepts such as causality are "not grounded in experience," that is, cannot themselves be discovered within the manifold of intuition, they must "arouse suspicion" (*CPR*, A88/B120). Thus Hume will argue that the concept of causality is not justified by our actual experience of nature, which only warrants the much weaker idea of *constant conjunction*. The actual concept of causality is instead a function of habit, which makes the inductive leap from particular instances of concordance to a general law-governed conception of the relations between objects. Thus Hume's deduction shows the pragmatic, rather than logical grounds for our application of the category of causality. While Hume's notion may explain how the concept comes to be recognised by the understanding, such a derivation obviously cannot show us whether its application is justified (hence Hume's scepticism). The alternative to this form of empirical deduction, which would necessarily only provide Kant with contingent truths, is a transcendental deduction, which moves to the necessary preconditions of experience. If we are to attempt this for the understanding, we must ask whether there are any conditions that need to be met in

order to "*know* anything *as an object*" (*CPR*, A92/B125). Whilst intuition is necessary for the experience of an object, it is not sufficient. Experience also requires the concept of an object in general, and intuition itself cannot furnish this notion, as the transcendental aesthetic has already shown that intuition is non-conceptual. Thus, the task which Kant faces is to show whether the concept of an object in general itself requires certain other *a priori* concepts in order to be comprehensible. As the concept of an object in general is a necessary (and therefore transcendental) condition for experience, any concepts which it itself presupposes must also be of a transcendental nature, as any contingency in these conditions would infect the concept of an object itself. We are therefore to search for concepts which themselves determine the concept of an object. Here we see why the metaphysical deduction is a clue, as the analysis of the faculty of judgement gave us a complete table of the functions necessary for any judgement to be formed. Kant claims that the categories, as deduced from the functions of judgement and related to intuition, provide just those concepts for which we are now searching: the necessary and sufficient condition for the determination of an object in general.[12]

In the B deduction, Kant begins his analysis of the conditions for the application of the categories with the premise that "it must be possible for the 'I think' to accompany all my representations" (*CPR*, B131). For my representations to belong to me, it is a minimal requirement that I be able to assert of them that it is I who thinks them. If this were not the case, there would be thoughts which both belonged to me (as they are my representations), but which did not belong to me (I could not lay claim to them). This 'I' has the further function of unifying my experience, for it allows perceptions at various moments to be integrated together, as the unity of the 'I' grants a unity to the various moments of experience, tying them together as they share a relation to the self-identical structure of the ego. This allows the subject to conceive of the manifold as *a* manifold. As everything empirical is itself within time, and thus is also

affected by change, an empirical entity is unable to provide the identity which is required to effect this unification of the manifold. For Kant, this therefore excludes the empirical ego, which is the self of which the subject is conscious, from fulfilling this function. "No fixed and abiding self can present itself in this flux of inner appearances" (*CPR*, A107). Kant believes, therefore, that the unity of apperception must be instead a transcendental structure, the source of the opposition of Sartre and Deleuze. Such a self cannot itself be intuited, but instead must be posited as a presupposition of our having successive representations. We should note that this ego which is presupposed is a mere formal unity, and cannot be thought (justifiably) as substantial. Substance is a category, and therefore, since this unity is the foundation of the categories, it must itself be pre-categorial. It amounts simply to the correlate of the unity of experience to which it must be possible to attach the 'I think.' It is known simply as a necessary identity, a 'that' which, as it precedes the categories, cannot admit of further determinations (as it is the categories which are at the root of the determination of objects). Although my knowledge of this unity (the "I think" with which Kant begins his deduction) is of the unity as analytic, "the analytical unity of apperception is only possible under the presupposition of some synthetic one" (*CPR* B133-4). This is because experience, at its simplest, must necessarily consist of a multiplicity of elements. Even the simplest perception, such as the perception of a coloured dot, can only show itself against a differently coloured background. In view of this complexity, the manifold is represented as a single complex thought, which in turn requires a singular subject, as "a set of distinct thoughts of the elements of the whole can never be equivalent to the thought of the whole itself."[13] Through this recognition of the unity of the manifold, the subject also comes to know the manifold as a synthetic unity. That is, regardless of whether the manifold is already structured, in order to be taken up by the subject, the subject must be able to compose the manifold himself to recognise this structure. This in turn requires that the subject bring these different elements of the manifold together as a synthetic unity, and that he is conscious of this synthesis.

As we have seen, as well as the concept of a subject, Kant argues that we also need a concept of an object as the foundation of the categories. Such a concept has been shown to be necessary for experience, as the manifold of intuition cannot itself be presented in such a way as to allow us to know it without it. As with the categories, this object is a precondition for experience, and as such cannot show up within experience without generating an infinite regress. Like the transcendental unity of apperception, it is the foundation of the categories, and therefore cannot itself be categorial for the same reason. In fact the nature of the object at this stage of the deduction is left indeterminate; it is simply the correlate of the subject. The function it fulfils is related to the faculty of judgement, to the extent that it is simply a posited unity which allows judgements to be formed, and perceptions to hold to a unified structure. Such a concept of an object, which makes all judgement possible, must, for this reason, be completely free from all content. It is the concept of an object insofar as judgements require a relation to an objectival structure. As we have seen, Kant claims that this concept of an object in general must be free from the manifold in any case, and so cannot contain anything given by experience. The transcendental object must therefore be conceived, like the transcendental unity of apperception, as a self-identical, singular transcendental condition for the unity of the manifold for the understanding. As it does not have its origin in intuition, the transcendental object cannot be conceived of as temporal. The freedom from content must mean that its function is purely one of providing a unity, and this it does, unifying representations in such a way as to guarantee a distance between our representations of objects and the transcendental unity of the object itself which underlies these representations.

In reliance on the notions of a transcendental subject and a transcendental object as the grounds for the categories, Kant has not at this point in the transcendental deduction succeeded in providing a basis for the application of the categories. The

conditions for the possibility of the subject and the conditions for the possibility of the object need to be given in such a way as to avoid a potentially infinite explanatory regress, as each precondition in turn requires its own preconditions. Furthermore, Kant has not yet provided a justification as to *why* the thought of these transcendental structures allows the categories to relate to intuition. We shall begin by exploring the conditions for the possibility of the subject and the object, before moving on to their connection with the categories. It turns out that the subject and the object determine each other reciprocally. First, the subject makes the object possible. For representations to stand in relation to objects, it is necessary that the representations themselves have a certain unity. This unity is provided by the transcendental unity of apperception, which allows the 'I think' to accompany all of our representations. As subjects unify representations, they consequently ground the transcendental object, which is simply this formal unity of representations. The subject in turn is grounded by the object, since through the synthetic nature of the manifold it comes to know itself as a subject, and as that which synthesises the manifold. As we have argued, Kant cannot know the self as substantive, since it is not given in intuition, being a bare unity. Therefore, it is necessary for the subject to ground itself through some other means. In this case, the manifold, which is a synthetic unity, gives us this grounding, since it appears as the result of an act of the subject. If the subject were passive in relation to the representations which come before it, the subject would find itself unable to draw apart from those representations. Without the notion of an object, there can be no distinction between a representation and an object, and without this distinction, the subject would be unable to know representations as representations. They would simply "crowd in upon the soul" (*CPR*, A111). The concept of an object allows the subject to recognise representations as representations of the object, and thus to distinguish itself from them. Thus the subject becomes aware of himself through the unification of representations into an object, through his recognition of himself as a spontaneous consciousness. The subject therefore makes the object possible for Kant, and the object makes the subject

possible. This means that the subject necessarily relates to something beyond its own empirical representations, to a world of objects, even though the form of these objects must be generated by the subject itself. The challenge for Deleuze will therefore be to provide an analysis of the transcendental field which contains neither subjectival, nor objectival elements, as the intimacy of these two terms means that the presence of one leads inexorably to the other. This in turn will allow Deleuze to conceive of the transcendental field as generative, rather than merely conditioning, as the transcendental will no longer be structured analogously to the empirical, but will instead give rise to the objectival from a non-objectival field. Instead of the transcendental having the form of the empirical, therefore providing the merely *formal* conditions of the empirical, the structure of the object in general, we will be able to understand the transcendental as actually generating the empirical. This is the ground for Deleuze's ultimate rejection of Sartre, who does not go far enough with the pure rejection of the transcendental subject. We can furthermore see that the rejection of the transcendental subject may have implications for the transcendental logic, as it is the concept of an object which makes the classical forms of judgement possible.

The question of the legitimation of the categories can now be answered. We can see that it is the subject that makes the object possible through a process of synthesis, and that this synthesis must be conducted according to *a priori* rules, for otherwise, it would rely on empirical concepts, thereby begging the question at issue, namely the pre-empirical conditions for the possibility of experience (the use of empirical concepts would give us a purely physiological derivation).[14] The categories, as concepts which apply to an object in general, and are transcendental, seem like the only choice for the rules of this synthesis. The categories are therefore legitimated through the role they play in allowing the subject to actively synthesise the object. In actual fact, it is not the categories alone which play the role of conditioning intuition, as the deduction attempts to show in general terms the relation of the faculty of sensibility to the understanding,

thus allowing for the possibility of the conceptualisation of intuitions unlike our own. The manifold is in fact synthesised by the schemata, which play the role of intermediaries between the conceptual and the intuitional, thus allowing the two heterogeneous matters to be brought into relation. They perform this role by sharing characteristics of both, and since time governs all empirical phenomena (whereas space only governs those external to the subject), the schemata can be considered as temporalised categories. For the purposes of explicating Deleuze's move to a transcendental empiricism, these final moves, which concern the specific implementation of the results of the two deductions within the Kantian system of transcendental idealism, can be left to one side.

We now see how Kant conceives of the understanding as both being responsible for synthesising objects, through the categories, and uniting representations, through judgements. We can also see how the transcendental subject is thus generated through the synthesis of objects, and is also its precondition, through the reciprocal determination of the subject and the object. The subject's role in actively synthesising the object is not given directly, but rather is established through our knowledge of the subject's spontaneity, gathered through its ability to perform analogous acts of unification in the domain of judgement. These two threads reinforce each other, as the isomorphism of the categories and judgements guarantees the subject-object structure of the transcendental, because the categories extend the objectival logic of judgement into the transcendental domain, and the subject-object structure makes possible the isomorphism between transcendental logic and the logic of judgement, as it allows the analogous structures to operate in both domains. As we shall see, Deleuze will attempt to refute both theses simultaneously in order to move away from Kantianism whilst maintaining the concepts of the transcendental and the empirical, redesignated as the virtual and actual. This would allow him to propose a transcendental philosophy which was generative, rather than just conditioning. We are now ready to analyse Sartre's

critique of the transcendental ego, which is a transitional point on the journey to transcendental empiricism.

Sartre and *The Transcendence of the Ego*

Sartre's early essay, *The Transcendence of the Ego*, provides the raw material for Deleuze's critique of the transcendental subject as the constituting principle which allows the isomorphism between the transcendental categories and empirical judgements. It is Sartre who provides the new direction for philosophy in the face of the established philosophical thinkers, and was "the breath of fresh air from the backyard" (*N*, 12). In reconstructing a move from transcendental idealism to transcendental empiricism through Sartre's critique of the transcendental ego,[15] there are three difficulties which must be overcome. First, there is the schematic nature of the references to Sartre in Deleuze's writings. Whilst Deleuze credits Sartre with developing the notion of an "impersonal transcendental field, not having the form of a synthetic personal consciousness or a subjective identity," (*LS*, 98) and with providing a "decisive" (*LS*, 103) critique of the notion of a transcendental subject, Deleuze provides almost no commentary on the text itself. This presents difficulties as the text, as it stands, is not overtly critical of Kant, its target rather being the transcendental ego of Husserl. Second, although Sartre's essay is clearly aimed at a Husserlian conception of the ego, Sartre's intention in this essay is not to criticise phenomenology itself, but rather to save phenomenology from certain notions which in fact place the phenomenological project in jeopardy through their illegitimate application in the post-phenomenological world. His statement that "all the results of phenomenology begin to crumble if the *I* is not, by the same title as the world, a relative existent" (*TE*, 42), is followed by an attempt to reconfigure the ego in order to give it such a nature, and in the process to preserve the results in question. Given Deleuze's hostility to the

phenomenological project in general (*WP*, 145-50), we must ask how he is able to make use of this essay by the self-proclaimed saviour of the phenomenological tradition. Third, Deleuze takes it for granted that Sartre's essay ends in failure, that "it is no more possible to preserve for [the transcendental field] the form of consciousness" (*LS*, 105) than it is to preserve the *I*. These difficulties can be resolved if we assume that the argument which Sartre deploys against the transcendental ego in fact cuts deeper than he had anticipated, and finally undermines the foundations of even the reinforced phenomenology he himself proposes. We thereby accept Sartre's argument for the revised specification of the transcendental field without accepting the formulation developed by Sartre to meet this specification. We accept the schematics provided by Sartre whilst arguing that phenomenology cannot itself provide a solution to the difficulties raised. Thus Sartre's argument would provide the negative critique which leads positively to Deleuze's transcendental empiricism. In order for Deleuze to put the argument to his own use it must also, therefore, be the case that what Sartre discovers is not a problem specific to the structure of phenomenology, but a general problem, or at least a problem which is applicable to other systems containing certain functional analogues.

From a purely phenomenological angle, Kant cannot be the target of Sartre's critique, as the fundamental structures discovered by Kant in the deduction have no place in a philosophy of description such as phenomenology, and it is for this reason that Kant is excluded from Sartre's criticism. Indeed, at various points in the essay, he seems to turn to Kant to provide the resources to oppose Husserl. While Sartre may claim that the standard interpretation of Kant, as positing an existent transcendental ego, comes from the failure of the neo-Kantian movement to separate questions of validity from those of fact (*TE*, 34), claims that the transcendental ego "does not bind up the unity of phenomena" (*TE*, 100) would be very difficult to reconcile with any reading of the transcendental deduction.[16] Here Kant seems to suggest that the formality of the

transcendental ego stems not from its purely hypothetical nature, but rather from the fact that its position as pre-categorial means that it must be *understood* as an indeterminate existent. Whilst the argument itself stands or falls regardless of these phenomenological considerations, and thus is equally applicable to Kant, I will discuss briefly the differences between the transcendental ego and transcendental apperception. The radicalised argument will therefore rely more upon Sartre's Bergsonian heritage than his phenomenological roots, as in fact it is probably Sartre's proximity to Bergson on many points which draws Deleuze to him.[17]

Whilst Husserl on occasion even labelled his philosophy a transcendental idealism, there are fundamental differences between the two philosophical positions which are pertinent to our inquiry. While both systems attempt to provide a description of the *a priori* laws of objects, the emphasis on the intentional character of consciousness within Husserlian phenomenology creates a radical divergence between the domains of what counts as thought within these two systems. For Kant, as we have seen, in the end, the search for the conditions for the possibility of experience leads to the identity of the conditions for the object and the conditions for knowledge of the object. The identity of these two conditions is guaranteed by the fact that it is the transcendental unity of apperception which allows the categories to condition the object such that the understanding can know it *as* an object. Thus, the rules governing consciousness necessarily also cover the objects for consciousness. From a transcendental perspective, what consciousness 'knows' is already within consciousness. Intentionality changes this situation, as intentionality is the property of being towards the world. As Ricoeur notes,[18] the fundamental preoccupation of the *Critique* is with the epistemological question, "How are synthetic *a priori* propositions possible?" This means that Kant's emphasis is on the representation of objects, and in particular, the representation of objects given to us by already constituted sciences. Kant's main pre-occupation is therefore with the validity of propositions given in advance of our enquiry,

rather than with a genuine description of subjective life. Once intentionality is seen as one of the primary characteristics of consciousness, consciousness becomes essentially "consciousness of" the object, rather than the Kantian consciousness which deals with representations of objects. The focus instead on intentionality allows us to explore not simply our representation of the object, but also our mode of relation to it. "Knowledge, or pure 'representation', is only one of the possible forms of my consciousness 'of' this tree; I can also love it, fear it, hate it, and this surpassing of consciousness by itself that is called 'intentionality' finds itself again in fear, hatred, and love."[19] One of the central ideas of moving to this conception of consciousness is that if consciousness refers to an object outside of itself, then the question of how the elements are to be synthesised together into a representation within consciousness does not arise. "It is possible those believing 'two and two make four' to be the *content* of my representation may be obliged to appeal to a transcendental and subjective principle of unification, which will then be the *I*" (*TE*, 38). On this conception of consciousness, it is therefore possible to study the object in its own right. The object stands transcendent to consciousness, and is thereby governed by its own conditions, which are the subject matter of the phenomenological method.[20] A further implication of this is that the methodology by which these phenomena are analysed also requires alteration. Kantian philosophy proceeds by a critical method, using transcendental arguments to clarify the preconditions underlying a particular existent state (thus space is an *a priori* condition for experience in general because it is impossible to conceive of an object outside of a spatial milieu). As Husserlian phenomenology does not presuppose that the object is a function of the understanding, the transcendental analysis is replaced by a pure description, free from all assumptions, of the object. The setting aside of assumptions also includes the assumption of the existence or non-existence of the object, thus allowing phenomenology to deal with all intentional objects, including phantasms, and safeguarding its role as the foundation for the sciences, which can proceed to an analysis of the actual facts of the world once the structure of objects has been disclosed. The

point at issue between Sartre and Husserl in this essay is whether a Husserlian phenomenology presupposes the presence of the transcendental ego for the same reasons that it is required within the Kantian system, namely to create a point from which various consciousnesses can engage in various acts of apprehension, yet still maintain a coherent unity. If this function is necessary, then phenomenology once again returns to the situation of an internal synthesis. If consciousness emanated from a transcendental ego, we would need to explain how it would be possible for such a consciousness to make contact with the object, which is fundamentally transcendent to it. Husserl's solution to this seemingly intractable problem is to presuppose a medium which shares the properties of both consciousness and the object, which can thereby communicate between the object and the consciousness. Such a medium, or *hyle*, according to Sartre, undercuts the fundamental doctrine of phenomenology, "to the things themselves", as now consciousness is consciousness not of an object, but instead of the representation of the object through the *hyle*. Furthermore, in the work of Husserl, the *hyle* is a function of consciousness, returning us precisely to the theory of contained representations as put forward by Kant.

Despite this final convergence between Husserl and Kant, we must recognise a difference which results from the difference of methodology referred to earlier. Whereas Kant's critical philosophy raises the question of the transcendental ego as a question of validity, for Husserl, this is a question of fact. The Kantian transcendental ego is a critical precondition for the possibility of knowledge of objects. It is not as such actually given to empirical consciousness. For Husserl, the project of philosophy is fundamentally descriptive, however, and as such, the notion of a transcendental ego as presupposed, but not actually present to empirical consciousness, would be invalid, and would necessarily fall before the *epoché* which suspends all such theoretical considerations. The difficulty is that it seems possible on the face of it that empirical consciousness could exist without an *I*, whilst still requiring the transcendental field to

be organised by a transcendental ego. Sartre seems to support something like this position at various points within the essay.[21] We should bear in mind that for Sartre, the status of the transcendental ego within critical philosophy is to a large degree made irrelevant by considerations which phenomenology brings to bear which seem to rule out the possibility of the Kantian system *tout court*. For our purposes, there is no need to determine an answer to the question of Sartre's intended target. Given that we are concerned with the logical structure of the argument, and not its specific relation to phenomenology, the use Sartre makes of it is of minor importance. The distinction between the empirical and the transcendental must still be borne in mind, however, in order to test the applicability of the argument to a more generally Kantian framework.

Despite the emphasis on phenomenology in Sartre's paper, his argument still begins with a reference to the transcendental deduction. Kant's statement that "the I think must *be able* to accompany all my representations" (*CPR*, B131-2) raises the question of whether the *I think* actually does accompany all of our representations, or on the contrary, whether in fact, many of our representations occur without the presence of an *I*. This in turn raises two further questions. "First, does the movement from a representation not being accompanied by an *I* to being accompanied by an *I* lead to a change in the structure of that representation? And second, whilst the *I* must be able to accompany our representations, is this because the *I* makes possible the unity of our representations, or rather is it the case that our representations are structured in such a way that it is always possible to prefix an *I think* to them?" (*TE*, 34) The structure of Sartre's argument follows these three questions, with his answer to the first attacking the necessity of the transcendental ego, the second presenting an alternative theory of the unification of consciousness, and the third showing the impossibility of a transcendental ego. It is important to note that Sartre is here not attacking what is one of the fundamental premises of the transcendental deduction, but rather seeking a reinterpretation of the significance of this axiom. In this sense, he fully recognises that

for Kant, it is possible for the *I* to be lacking from empirical consciousness. It is for this reason that Sartre is happy to accept Kant's claim that the premise only requires a formal unity to find its fulfilment. For Kant, this analytic unity will in fact turn out to be grounded in a synthetic unity, which is the ground for the representations themselves. What Sartre is instead considering is the possibility that the unity of our representations is not caused by the transcendental ego, but that, if this unity can be grounded by some other means, this does not exclude the possibility of the 'I think' accompanying all of our representations. In fact, it would make it possible, as it would form the set of representations to which the 'I think' is applied. Whilst on a logical level, showing that the transcendental unity of apperception is not necessarily responsible for the unity of representations may not be too problematic, the difficulties emerge through the multiple roles that it plays throughout the Kantian system. As well as unifying consciousness, the transcendental ego also allows consciousness to separate itself from our representations, and provides the feeling of spontaneity which characterises the rational subject. Sartre must also attempt to explain why the idea of a subject cannot be legitimately held, given that it has previously been characterised as a self evident truth. First then, we shall look at Sartre's account of the unity of consciousness.

Within phenomenology, there is a distinction between consciousness and the transcendental ego which, for Sartre, is vital. As we have said, consciousness is always consciousness *of* an object, and thus is a relation to a particular object, as well as a particular mode of thinking of this object. An individual will naturally be conscious of a variety of different objects, states of affairs and events, which raises an important problem for the Husserlian phenomenologist, namely how these fragmentary experiences in disparate locations and at disparate times can be attributed to the same individual. This problem is compounded by the fact that the intentional acts of consciousness are intended to purely illuminate the essence of the object under consideration. This is after all the fundamental aim of phenomenology in grounding the

sciences – the revelation of the pure essences of its objects of inquiry. The implication of this purity is, however, also the exchangeability of acts of consciousness, such that any idea of the individual itself dissolves. There is no way of differentiating one individual from another on the basis of consciousness, as its import comes from the object, which is public. The transcendental ego is introduced for these two reasons, as a structure from which acts of consciousness emanate both guarantees the unity of these acts, as continuity is provided from their common source, and also provides personality, since these acts, though individually replaceable, form a coherent whole nonetheless within the transcendental ego. Thus, the role of the transcendental ego, as unifying consciousness, plays the same role as the transcendental unity of apperception within the Kantian system. The primary difference being that the transcendental ego for Husserl must be accepted as a factual existent, whereas for Kant it is instead a necessary posit, a formal unity. Sartre rejects these two reasons for the transcendental ego on the grounds that both can be supplied by consciousness itself, and that neither, therefore, provide a necessary reason for its existence. Individuality is firstly guaranteed to consciousness by Sartre through his recognition that consciousness exists much like a Spinozistic substance. That is, that because consciousnesses share nothing in common, interaction is impossible. There is no danger therefore of a confusion between consciousnesses as each is both unlimited, and necessarily separated from all others.

We can answer the question of the unity of consciousness by pointing to the unity of the object, which does not itself require a subject to make its unity possible. Here is what is for Sartre a fundamental tenet of phenomenology, but what will for Deleuze be transposed into an empirical thesis about duration derived from Bergson. For Sartre then, the claim will be that Kant has misconstrued a fundamental phenomenological fact, namely the durational experience of time, and so has derived conditions for the possibility of experience which do not relate to actual empirical experience. Sartre observes the fact that time appears to us not as a series of instants, but

rather primarily as a continuum, through which the past and present are not separated from one another, but rather are undifferentiated. Consciousness "unifies itself through escaping itself" (*TE*, 38). That is, the unity comes from the order present in the object which is transcendent to consciousness. Thus the roll of a dice unifies consciousness through the necessary relations between its faces as it progressively gives itself to consciousness. "The object is transcendent to the consciousness which grasps it, and it is in the object that the unity of the consciousness is found" (*TE*, 38). Sartre's central point is that once consciousness is seen as intending towards an object, rather than just synthesising representations, the object itself can take on the role of providing identity for the subject. The flux of consciousness itself also participates in this unity through the retention of previous experiences. "It is consciousness which unifies itself, concretely, by a play of 'transversal' intentionalities which are concrete and real retentions of past consciousness" (*TE*, 39).[22] The ego is not needed to unify consciousness as consciousnesses themselves traverse one another in such a way as to provide a decentred unity. Rather than emanating from a central point, they are interwoven in such a way as to make this central spoke redundant. Sartre further argues that rather than unifying the phenomenal world, transcendental consciousness would instead lead to its fragmentation, as the transcendental functions slice through the temporally unified field. Such a function would in fact destroy the unity, rather than being its precondition.

The transcendental subject is further necessary on the Kantian account in order to provide a distance between the subject and the world. For Kant, regardless of whether the subject synthesises the world or merely recognises it, an act of synthesis is necessary for a relation to the world to be formed. As Sartre has removed what for Kant is the centre of syntheses, Sartre must explain how it is possible that this relation to the world takes place. In fact, Kant himself provides an alternative. Without the synthetic activity of the subject, representations would "crowd in upon the soul". The distance between self and world would be lost as it is the active taking up of the world which allows the

subject to conceive of himself as separate from it. It is this option which Sartre will himself accept, noting, with Kant, that the 'I think' can, but need not accompany all of our representations. In these cases, representations do indeed crowd in upon the soul. This is the state of being enthralled by the world, which itself calls for us to act. In this state, the world seems to occupy us with tasks, events themselves dictating our actions. Whilst in this everyday mode of existence we frequently do make reference to ourselves, Sartre claims that many of these references are simply called forth by the grammar of our language. Here Sartre argues that there are limitations to doing philosophy with grammar. This does not mean that we cannot reflect on ourselves, but for Sartre, this reflection merely relates to an empirical self, which is itself constructed from the history of our relations to the world. This self for Sartre is real, but is not in itself generative.

For a phenomenological account of consciousness to dispense with the transcendental ego, it is furthermore necessary for it to explain how the concept of the transcendental ego emerges in the first place. This stems from the fact that if phenomenology is to proceed on the level of pure description, it cannot rely on inference to explain the presence or absence of any particular entity. Sartre must therefore explain why the transcendental ego appears in our accounts of the life of consciousness; why in other words we are led to draw this false picture of consciousness. For Husserl, the transcendental ego is a phenomenological fact, for Kant a presupposition. It is a hypothesis that seems to draw us irresistibly towards it in whatever form it takes. In fact, Sartre will claim that the transcendental ego does fulfil a function, but one which is practical, rather than transcendental. Once we accept the disavowal of the ego, we see consciousness itself as the foundation of the psychic life. Consciousness as creative spontaneity (in what he describes as a Spinozan sense[23]) overflows any unity that could be given to it through the presence of a unified personality. Without the ego, consciousness becomes equal to the transcendental field. When consciousness is revealed to itself in this respect, as utterly unbounded and ungoverned, it is struck by

dread. Perceptions crowd in upon the soul, and consciousness becomes lost within the world. Without the background of a unity, consciousness is now without concepts such as passion and will, appearance and reality. Thus the transcendental ego becomes equated with the desire to become being-in-and-for-itself in the later terminology of *Being and Nothingness*, that is, to become a mixture of passivity and spontaneity. The idea of a unified ego allows acts to be referred to a set of states and dispositions which are their cause, thus making any experience of angst such as that described by Sartre in his chapter on the origin of negation impossible.[24] Action is now determined by a kind of *logique du coeur*. With the addition of the subject to the transcendental field, consciousness itself becomes justified by something outside of itself, and this grounding both allows consciousness to set up terms through which its actions can be understood, as well as to displace responsibility to the transcendental subject which, for Kant, remains indeterminable for consciousness itself. Deleuze himself, when he goes beyond Sartre in removing consciousness from the transcendental field, will come to a similar conclusion regarding the relation between responsibility and transparency, as his ontology of forces will ultimately dispense with the concept of a responsible agent.[25]

As we have seen, for Kant, whilst the transcendental unity of apperception is a formal unity, as Sartre well recognises, this formal unity is in fact the foundation for the synthesis of experience, and for judgements of experience. As the act of judging requires a unification on the part of the subject, the relation of concepts presupposes the transcendental unity of apperception. For Sartre, using Husserl's notion of consciousness as an intending towards the concept, the transcendental ego is no longer necessary as that which grounds the act of judging. Concepts are intended towards, and justified, not by the *a priori* synthesis of the ego, but by this spontaneous act of consciousness itself. With this recognition, it is possible to reinterpret the nature of this formal requirement put forward by Kant. Kant's concept of synthesis as being enacted by the subject

validates the notion of a subject. For Sartre, once he has shown the possibility of consciousness unifying itself without the necessity of a subject, this formal condition can be seen not as a transcendental presupposition of consciousness, but rather as an implication of the unity of consciousness. The transversal strands of consciousness provide this unity, and it is this unity of consciousness which permits the attribution of the 'I'. Thus, for Sartre, it is the unity of consciousness which forms the transcendental field, a consciousness which is impersonal through the removal of the concept of the 'I' from its foundational role. Sartre opposes Kant's unity of apperception to the inherent unity of the object itself and the transversal interrelations between specific acts of consciousness provided by protention and retention which no longer require the centralised synthesis of the Kantian system. The ego still exists, but is now a unity on the same level as any other object to which consciousness relates. It is a formal unity produced by the unity of consciousness. For Sartre, the spontaneity which is the key to Kant's transcendental deduction has also been passed on to consciousness itself, and derives from the essentially non-objectival nature of consciousness itself. As consciousness takes on the role of the transcendental unity of apperception, it also gains the indeterminable nature of the former, as it becomes the foundation of the unity of experience. It is this lack of determination, which Sartre will unfortunately describe through the language of nothingness, which provides the feeling of spontaneity which Kant discovers. In eliminating the subject from the transcendental field, Sartre's conception of phenomenology comes close to Deleuze's of philosophy in general. The empirical subject now remains, but cannot, if Sartre's arguments are accepted, be considered as a valid foundation for philosophy.

Sartre lays out the implications of the rejection of the transcendental ego as a series of four consequences. These consequences form the conditions for the transcendental field alluded to by Deleuze, and are thus the conditions for a transcendentally structured empiricism:[26]

First, the transcendental field becomes impersonal, or, if you like, pre-personal, *without an I.*

Second, the *I* only appears at the level of humanity, and is only one aspect of the *me*, the active aspect.

Third, the *I Think* can accompany our representations because it appears as the foundation of unity which it did not help to create; rather, this prior unity makes the *I Think* possible.

Fourth, one may ask if personality (even the abstract personality of an *I*) is a necessary accompaniment of consciousness, and if one cannot conceive of absolutely impersonal consciousnesses (*TE*, 37).

These premises are almost sufficient to outline the major structures of the Deleuzian transcendental system. The conditions for objects and the conditions for the knowledge of objects can no longer be identical precisely because the transcendental field is not quantitatively identical with the field of empirical states of affairs. As the transcendental field no longer has an objectival structure, knowledge of objects is now a matter of empirical correspondence of propositions to determinate objects, whereas the conditions of objects themselves rely on the non-objectival transcendental field. This is because the *I*, which exists at the level of the empirical, or the human, no longer finds a transcendental correlate. Thus, whilst the structure of the actual world still can be captured by judgements, the conditions which generate actual objects are no longer derivative of the structure of judgement itself. Once the responsibility for the synthesis of the empirical is taken out of the hands of the subject himself, it becomes possible that the functions generating this synthesis are completely alien to the understanding of the subject. The difference in structure between the two fields is also made apparent by Sartre's notion of the personal. The personal does not apply simply to a certain level of acculturation at the level of humanity, or a level of differentiation between different *I*s

within the empirical world, but instead to a form of structure. The *I* has its personality "however formal, however abstract one may suppose it to be" (*TE*, 41). Thus, the personal becomes a signifier for the structures of the empirical world, or, in the language of phenomenology, of the natural attitude. Sartre is here prefiguring Deleuze's later assertion that "Kant's dream was not to abolish the distinction between two worlds (sensible and super-sensible) but to secure *the unity of the* personal in the two worlds" (*NP*, 93). We must also note that it is not just the *I* which does not occur on the transcendental field, but in fact all objectival properties fall away. "The ego is a noematic rather than noetic entity. A tree or a chair exist no differently" (*TE*, 88). From this statement, we can universalise the result of Sartre's analysis of the ego on this point, as we may have suspected from the reliance of the concept of the transcendental object on the subject revealed by the transcendental deduction. These implications of course find their phenomenological description in *La Nausée*, and we can see here the Bergsonian influence.[27] Whilst the empirical field may be structured in accordance with a transitional logic of states of affairs, the transcendental field is now non-objectival. There can be no direct correlation between what synthesises and what are the results of these syntheses.[28] As Deleuze will say in relation to his own project, "it does not suffice to say of the foundation that it is another matter – it is also another geography, without being another world" (*LS*, 99). The first three implications of Sartre's critique define the general conditions which Deleuze claims a transcendental empiricism must meet. The fourth, however, is rather a question, as to whether consciousness itself is impersonal or personal, or rather, whether it exists at the level of the transcendental, or the empirical. To this question, Sartre will answer that it is impersonal, as the noematic existence of the *I* allows all content to be placed within this personalised structure, allowing consciousness to become fully impersonal. If consciousness is without content, Husserl's problem of how the transcendental ego could come into contact with the things themselves is dissolved, as we no longer have the interaction of two substances, since consciousness is now entirely empty. Consciousness is now free to return once

again to the things themselves. For Deleuze, the answer to this question will be that it is still personal, as what is important for him is not that the transcendental field is pre-personal, but that it is pre-individual. That is, that what is logically prior to all individualised states of affairs is not itself individuated. Consciousness, whilst lacking content, is still an individuality. It is like a point in a geometrical space that automatically brings with it the axes through which it is specified. Thus, Deleuze's critique of Sartre will invert the normal line of attack. Whereas Merleau-Ponty criticises Sartre's concept of consciousness as being too minimal to allow any relation to Being to be established, Deleuze argues that the concept of consciousness is already too defined, thus imparting an illegitimate structure to the transcendental field.

Sartre's aims in *The Transcendence of the Ego* are, in the end, not met due to the strength of its assertions regarding the synthetic powers of objects themselves. For Sartre, it is this series of objects within the world which allows consciousness to synthesise itself, thus removing the need for a transcendental ego standing behind consciousness. Consciousness is individuated and unified by this series of objects within the world. Sartre believed by this that he had also solved some ancillary problems relating to the transcendental ego, namely the problems of concrete action within the world, and of solipsism created by the transcendental ego's necessary withdrawal from the world. In fact the presence of others within the world will ultimately destroy the innocence of the transcendental field for Sartre, through the discovery of the synthetic role of consciousness itself, as we can see in Sartre's discussion of our encounters with others in the section of *Being and Nothingness* entitled "The Look". Thus, although the two consciousnesses share the same transcendental field, each one asserts his own right to be an individual, and this struggle between individuals itself takes place within the transcendental field. Each has the potential to "disintegrate" the other's relations to the world through "the unfolding about itself of its own distances."[29] This means that I see the other as a part of the transcendental field, but as a part which individuates itself

through the particular relations it holds to the world about it. The look is the attempt by each consciousness to subsume the other within its own synthesis of the transcendental field. The disintegration is literally the failure of this synthesis. The transcendental field therefore allows of the possibility of multiple different syntheses. This is to say that consciousness is not merely an individuating, but also a personalised presence within the transcendental field (syntheses are multiple and depend on the particular consciousness). As this contradicts the first implication of the removal of the transcendental ego, we can now see that what guarantees the pre-personal nature of the transcendental field is precisely that it is also pre-individual. The possibility of different syntheses implies that consciousness itself in some sense 'personalises' the empirical in its synthesis of it. In effect, while the specific structures developed by Kant are removed, the underlying logic remains the same within the Sartrean system, with the more developed conception of consciousness finally taking the place of the transcendental unity of apperception found in Kant's philosophy. This is perhaps not so surprising when we consider Sartre's fundamental axiom, that consciousness is always consciousness of *x*. The implication of this is that neither consciousness nor its correlate can exist independently of the other, paralleling the celebrated result of Kant himself. The attempt to do away with the transcendental ego whilst preserving consciousness is therefore doomed to failure. This result is fully recognised by Sartre, for whom this dynamic personalisation of the transcendental field answers questions left open within the *Transcendence of the Ego*, but still involves a radical move away from the project Deleuze is proposing.[30] We have therefore seen the reasons Sartre wishes to remove the notion of the transcendental Ego. Deleuze is much more of a classical metaphysician than Sartre, and therefore the implications he takes from the depersonalisation of the transcendental field are very different from those of Sartre.

Deleuze and *The Logic of Sense*

From our reading of Kant, we have shown that the theses of the transcendental unity of apperception and the identity of the conditions for knowledge of an object and the conditions of an object themselves imply one another. What Deleuze takes from Sartre's analysis of the transcendental ego is that its position within the transcendental field can no longer be upheld. "The task of a philosophy which does not wish to fall into the traps of consciousness and the *cogito* is to purge the transcendental field of all resemblance" (*LS*, 123). Following from Kant's theory of judgement, we can state that the rejection of the subject means that the logic of the transcendental field cannot be that of a subject-predicate structure. For Kant, in order for a judgement to be made of an object, what is required is for the representations of the object to be synthesised into a judgement; thus, the statement, 'All metals are heavy' requires the subsumption of the representation of 'heaviness' under that of 'metal'. Whilst the judgement itself is based on the reciprocal determination of these terms through the structure of the subordination of the predicate to the subject, the two terms, predicate and subject, are still, in themselves, fully determined.[31] Thus, in order for them to be united, it is necessary that they be held together through a function that remains outside of them. This function, for Kant, is the synthetic unity of apperception, the condition of the possibility of the 'I think', which remains constant through its application to different elements, thus allowing, through its attachment to these concepts, an element of homogeneity to enter into them, thus overcoming their intrinsic heterogeneity.[32] Removing the transcendental ego from the process of synthesis therefore will require a new model of the way in which the elements of a representation can both exist prior to the subject whilst retaining the possibility of their synthesis.

Deleuze argues that without the possibility of a new logic, philosophy has been led to consider the foundation of the world in terms of two alternatives. On the one hand

the "supreme *I*", which relates either to a transcendental subject, or to an absolute Being, and on the other, the undifferentiated abyss. That is to say, in explaining the existence of a subject-predicate structure of states of affairs, it has generally been considered that either this subject-predicate structure must prefigure that of the empirical world, or that the ground must itself be indeterminate. Thus for Kant, the parallelism between the transcendental and empirical is justified through the necessity of a transcendental field, and at the same time the belief that if this field is to be structured, it must be structured analogously to the empirical. The lack of a belief in an alternative form of differentiation therefore justifies this parallelism by ruling out any other possibilities. In transcendental idealism, the finite subject determines the complete set of possible subject-predicate relations through the table of judgements. Whilst the subject may be finite, his understanding is synthetic, which allows all possible permutations to be related to the pure subject-object structure of the transcendental field. This is for Deleuze merely a variation on the traditional metaphysical concept of God, who as a perfect being contains all possible predicates. In this case, the supreme subject is infinite, and so it forms an analytic unity, possibility already being encompassed in its perfection. In this model, the subject predicate structure of God leads to a parallel structure in states of affairs as their properties are derivative of those of God. Whilst Deleuze recognises that some philosophers have chosen the model of an undifferentiated abyss over that of the transcendental/metaphysical models, this abyss has always been expressed in terms of subject-predicate structure. Hence Schopenhauer, for instance, although recognising the will as existing prior to the categories in an undifferentiated form, only allows it to find coherent expression through the world of representation. Even Nietzsche, in *The Birth of Tragedy*, requires Dionysus to speak through Apollo.[33] In contrast to both these approaches, Deleuze is looking for a conception of the transcendental field as both pre-individual, in contrast to the transcendental and metaphysical philosophers, for whom being is already individuated at its origin – i.e. already possesses a subject-predicate structure[34] – and as differentiated, in contrast to the thinkers of the abyss, for whom

nothing can be said of being, except as it presents itself in the schematised forms of the phenomenal world. Such a logic moves beyond the conditioning of the Kantian transcendental method by explaining the origin, not just of individual states of affairs, but of the possibility of any state of affairs whatsoever. This is because it is an investigation of the possibility of subject-property structures in general, rather than just showing how empirical structures are conditioned by the transcendental field. Deleuze will say of this logic that it is no longer "of the form, but neither of the formless: it is rather of the pure unformed" (*LS*, 107). Form and formlessness cover the two traditional options provided by philosophy. The third option represents the Deleuzian alternative: that which is unformed in itself, but which is still determinable. As the unformed will generate the formed, we can see that there is a fundamental difference in kind between the transcendental and the empirical.

Deleuze calls the elements of his particular brand of transcendental philosophy singularities, and in line with his critique of the Kantian subject, these possess several important properties. Firstly, they are non-objectival. In Deleuzian terms, this is designated by the fact that they are pre-individual, as well as pre-personal. As we shall see, in order to provide a characterisation of these singularities, Deleuze will turn to the differential calculus. Secondly, they are capable of self-organisation. In doing so, he will try to develop a concept of difference to replace the role of the self-identical subject in Kant's philosophy. One of the functions of the transcendental ego, on the Kantian model, is to provide homogeneity to representations, which in themselves are heterogeneous. Without the transcendental ego, the parts of a judgement, or the representations of the object would have no common properties by which to form a unity. Thus representations on their own are incapable of synthesis. On the Deleuzian interpretation, this is due to the fact that these representations are already individuated, and thus, needing no relations in order to subsist, require the agency of a subject in order to effect their combination. As Deleuze sees singularities as pre-individual, this

constraint no longer holds, and, it will turn out, singularities will always already be found in relations to one another. Thirdly, singularities do not bear a direct relation to the entities which they form. This follows from the fact that singularities determine entities through reciprocal relations, meaning that singularities can never individually determine a state of affairs. We will have an opportunity to return to the exact structure of the transcendental field when we discuss Deleuze's logic, but for now, this schematic analysis still leaves the question of how Deleuze believes the subject is constituted.

For Deleuze, as we have seen, the transcendental field is radically distinct from the empirical. This does not mean, however, that we are talking about two distinct ontological realms. Instead, the transcendental reflects the configuration of singularities which are *expressed* in empirical states of affairs. We can therefore make a first approximation to the distinction between the virtual and the actual by stating that this is an epistemological division. For Deleuze, the actual is the expression of the transcendental field. The first level of actualisation of the transcendental field produces the subject-predicate structure which characterises states of affairs in general. Here bare individuals and their properties are expressed free from any form of hierarchy. As the transcendental field is constituted by the relations between singularities, it is the case that these singularities present many different sets of possible relations that could be actualised. Deleuze refers here to Leibniz (*DR*, 45), claiming that the transcendental field contains many different and incompossible worlds, only one of which is really actualised. The brute actualisation of the first level cannot give us an adequate account of consciousness, as it does not contain the materials to create anything like the spontaneity which is an integral quality of consciousness. Instead, we require a second level of actualisation. This comes about, according to Deleuze, through the recognition that certain features remain stable throughout different incompossible worlds. While it is possible that the state of affairs obtains in which Adam eats the apple, there is another possible state of affairs in which Adam does not eat the apple. In both of these

propositions, there is a determinate subject, Adam, who is individuated by his relations to certain predicates, such as in this case to the predicate of "sinner." These two states of affairs are two possible solutions to the problem of how the set of singularities can be actualised simultaneously. Through this, we can see that the structure Adam itself, aside from its relation to the predicate in question, has a correlate in the transcendental field, to the degree that it is partially independent of the precise state of affairs. It is this fixed point, with its ties to the transcendental field, which forms the ego within the Deleuzian system. This ego which is common to several worlds contains within itself a certain ambiguity, as it is not strictly determined by any particular world. It is this ambiguity, this recognition of other possible ways of being, which provides the feeling of spontaneity that is characteristic of the ego. Through this process of actualisation, the two principles which Deleuze claims are responsible for the Kantian system come to light. Firstly, the first stage of actualisation produces the principle of 'good sense', that is, the principled organisation of differences according to subject-predicate structure. It is this principle that, when applied beyond its proper domain, leads to the positing of the subject-predicate structure within the transcendental field. The second, 'common sense', is the principle of identification. It is this that, when applied to the transcendental field, generates the structures, object = x, and the transcendental unity of apperception. In both these cases, Deleuze claims that it is the illegitimate application of these principles which lead to an erroneous conception of the transcendental field.

Conclusion

Transcendental empiricism emerges directly from this rejection of the transcendental subject. If the subject is no longer responsible for the synthesis of the manifold to which judgement relates, then the ground of the synthesis must be discovered in the object itself. We should remember that Deleuze still retains the notion

of the transcendental field, so the form of empiricism practiced cannot be that of naïve empiricism of impressions. Instead, transcendental empiricism still searches for the conditions of experience. Unlike transcendental idealism, what are sought are not the conditions of possible experience, however, as the analysis begins with states of affairs within the world. In following empiricism in dealing with the actual world, Deleuze claims that what are sought are the conditions of real experiences. That is, what are sought are the conditions for the *generation* of specific phenomena, rather than the formal conditions for the possibility of phenomena in general. A further implication is that whereas for Kant problems are defined through the idea of a judgement, for Deleuze problems instead become objective, as the synthesis no longer takes place in the subject, a feature which we will return to in looking at Deleuze's positive philosophy in more detail in chapters four and six. With the rejection of the transcendental subject, we necessarily move to a transcendental empiricism, as the conditions for the knowledge of the object are no longer identical with the conditions of the object themselves, to the extent that we see knowledge of the object in terms of our ability to make judgements about it. That is, whereas it is integral to the conditions of the object that there is a difference between the empirical and the transcendental, it is integral to the conditions of knowledge of the object that there is a structural similarity between the structure of the object (substance and properties), and the structure of judgement (subject and predicate). Transcendental empiricism can therefore be established either through the rejection of the *cogito*, or the proof of the necessity of Deleuze's alternative logic. The difficulty is that, as it stands, and regardless of Deleuze's claims for his proof, Deleuze's arguments have only established the *possibility* of transcendental empiricism. The structure of his argument runs from the transcendental deduction of Kant, through the rejection of its fundamental axiom, the transcendental unity of apperception in Sartre's work, to the implications of this rejection, as drawn out by Deleuze himself. Sartre does indeed reject the transcendental ego, and while this may be a necessary condition for the move to transcendental empiricism, it proves not to be sufficient. As Deleuze himself will argue,

Sartre replaces the unifying function of the transcendental ego with the individuating function of consciousness. The functional structure of the transcendental deduction is therefore maintained, although the entities performing these functions may have been replaced. In order for Deleuze's deduction to be successful, what needs to be dealt with are not the fundamental elements, but rather the relations between the fundamental elements. In this regard, the replacement of the transcendental ego with consciousness cannot be sufficient if there is no change in this element's relation with the object. Thus, Sartre's arguments do not provide the force which Deleuze needs. We will return to the question of the grounds of Deleuze's metaphysics when we discuss his system in detail in chapters three and four. For now we should note that Deleuze's philosophy is intimately engaged with the work of Kant, and therefore must be understood in relation to the post-Kantian tradition. In our next chapter we will turn to the philosophy of representation, specifically the philosophies of Aristotle and Russell, in order to clarify the problems which both Deleuze and Hegel will engage with. In doing so, we will get a better idea of the issues prompting Deleuze and Hegel to make their respective moves away from Kant.

Chapter Two – Difference and Identity

Introduction

In the last chapter we explored the extent to which Deleuze tries to overcome what he considers to be the limitations of the Kantian approach to philosophy, which essentially required predication to take place through a third term, the understanding, that provides a common ground upon which the autonomous elements of the judgement can take a common form. In highlighting Deleuze's criticisms of the Kantian model, we saw how these very limitations provide a direct path to the central axioms of his philosophy. It also allowed us to see that Deleuze can be situated firmly in the post-Kantian tradition. We concluded by seeing that the success of Deleuze's deduction was limited, in that Sartre's 'decisive' new concept of the transcendental field itself presupposed a structure analogous to those employed by Kant. In this chapter, we will concern ourselves with the more general problem of representation by looking at the logics of Aristotle and Russell. We saw on page 28 how Kant was unable to go beyond an account of the transcendental as formally conditioning the empirical. By analysing the logics of Aristotle and Russell, we will extend this result by looking at the limitations of representation more generally. After having done so, we will sketch the general approaches of both Hegel and Deleuze to the problems thrown up by the representational approach to philosophy. The key issues we will address are the problems of transition, and of univocity.

At issue for Deleuze are two interrelated questions. First, there is the question of the position of the concept of difference itself within the tradition. For Deleuze, difference itself has always been understood by thinkers as a difference between concepts. As we shall see, this understanding of difference is particularly clear in the case of Aristotle. This has prevented the formulation of a concept of difference itself, as

difference has always been subordinated to identity: "difference must leave its cave and cease to be a monster" (*DR*, 29). Second, there is the more general problem of the homonymy of being. As Heidegger argues, the question of the sense of Being is covered over by the metaphysical tradition from Aristotle, as "'being' is used in many ways."[1] Thus the unity of phenomena is lost within Aristotle's analysis. We will begin from the analysis of being in terms of species and genera put forward by Aristotle, as it is here that the problem of the univocity of being is first discovered in the form of the problematic of homonymy, synonymy and paronymy. These questions are related by the fact that for Deleuze, it is an illegitimate concept of difference which is at play in the work of Aristotle and his successors. This concept effectively prevents an understanding of several phenomena, from temporality to unity (there can be no determinate concept of being for Aristotle). These points of silence Deleuze will call the catastrophes of representational thought – the points where difference shows through in spite of the limitations placed on it by identity. After dealing with Deleuze's analysis and criticisms of the form of difference at play in Aristotle, we will move through Porphyry the Phoenician's formalised Aristotelianism to the problem of analogy in Aquinas. At this stage a problem presents itself, in that the systems of Aristotle and Aquinas both have no clear separation between metaphysics and logic (the problem of analogy in Aquinas, for instance, comes to light in an attempt to differentiate the Being of God and man; in Aristotle, the problem of definition is tied to his theory of essences). A final analysis of the problem of univocity will therefore be made, this time in terms of the calculus of the *Principia Mathematica*. As modern symbolic logic attempts to provide a purely formal description of deduction, it can be seen as providing the pure schematic of representational thought. We will see that this purely formal description cannot, however, avoid a fall into an equivocal concept of Being. Formal logic, therefore, and those schools of philosophy which define themselves in terms of the rigour of formal deduction that it provides, are themselves fundamentally tied to a metaphysical interpretation of Being from which they cannot escape, since this interpretation is not

even an axiom of the system, but, and prior to this, a function of the very laws through which the axioms themselves are interpreted. This analysis will centre on Russell's theory of types, as well as the paradox which is named after him. The conclusion of the chapter will deal with the connection of the problem of the univocity of being to Hegel's philosophy. The problem will be to reconcile Hegel's often stated love of Aristotle with the difference between Aristotle's dialectic and his own. Important in this regard is the opening to the *Science of Logic* where the concept of Being is introduced, and Hegel's understanding of contradiction, which differs both from the concept of contradiction as conceived of by Russell, and the concept of difference thought by Deleuze.

Aristotle

The founder of the systematic study of logic is generally considered to be Aristotle. This logic, syllogistic logic, advanced from the conversational style of the Platonic dialogues, which, whilst relying on logically structured arguments, rarely reflect on the nature of logic itself,[2] to the reflexive study of the form of logical arguments.[3] Rather than focus on syllogistic reasoning, the process of making truth-preserving inferences from premises to conclusions, what is of concern to us here is the system of organisation of genera, species, and individuals which underlies this syllogistic reasoning. For Aristotle, the terms operated on by logic have an essentially hierarchical organisation, with the lowest term, the individual, being subsumed under an intermediate term, the species, which is itself subsumed under a genus. The genus is "what is predicated in the category of essence of a number of things exhibiting differences in kind."[4] What this means is that the genus is a part of the definition of whatever falls under it, that is, it is *essential*[5] to those things of which it is predicated. The second part of this statement refers to that of which the genus is predicated. While it is the case that for a given genus, say, animal, it is predicated of an individual, for

instance Socrates, since it is clear that Socrates is an animal, the reference to a difference in kind in Aristotle's definition means that there must be an intermediary between the genus (animal) and the individual (Socrates). This is because if the genus were related directly to the individual, the genus would be the only function which was *essential* to each individual. This would mean that in essence each individual would be different only in number, whereas the definition of genus requires that what it is predicated of also differs in kind. The necessity of this requirement will become clear shortly, but for now it is enough to recognise that between the genus and the individual is an intermediary category. This category is the species. In the simplest case, we can define the species as "that which is predicated of many things which differ in number" (*ISA*, 35). This case would be the one reached so far, where we have one genus, one group of individuals, and one level of species (a genus cannot simply have one species as in this case we could not meet the definition of a genus as applying to a number of things differing in kind). In this case, we can see that a given genus can be predicated of a species, and of an individual can be predicated both the species and the genus. Thus, of Socrates, we can say that he is both animal (according to his genus) and man (according to his species). Clearly, however, this classification can be expanded, for in between the genera and species, and perhaps in other cases as well as the species and individuals, we can see that other classes also exist; for instance, we could qualify further the nature of Socrates the 'animal man' by recognising that before these classifications Socrates is also a substance. Now animal itself will appear to act like a species to the genus substance, but like a genus to the species man. As Porphyry writes, "the species intermediate between the extremes they call subordinate genera and species, and they regard each of them as a species and a genus, since each is comprehended in relation to the highest genus and the lowest species" (*ISA*, 38). A corollary of this is that we now need to define the species in terms of something other than the individual, as only the lowest species relates directly to a field of numerical identity. The species will now therefore become defined in terms of the genus above it. Thus we now have a hierarchy,

reaching from the highest genera to the individual, through which the individual is specified by a process of division from the genus through the various species, gaining determinations as it goes, since each genus will determine the essence of that below it.[6] As such, what we have is in essence the first theory of classes, albeit one which is founded on a particular metaphysical analysis of the world. We will return to this idea that Aristotle's philosophy is a precursor of the theory of classes when we discuss Whitehead and Russell.

What is key to recognise in this theory of classes is that it makes possible the definition of the lowest species purely in terms of universals. From the highest genus, we can move through a process of division, whereby at each stage there is a branching of the higher concept into multiple options, allowing the particular entity to be more and more closely specified. Thus Socrates is the final point of an arborescent structure, characterised by a perpetual process of branching in terms of universal properties. The use of universals in this way is in fact essential to the Aristotelian project, as the universal performs two functions within his system. First, it provides the necessary linguistic economy to allow an infinite number of things to be described using a purely finite number of terms. "Names are finite and so is the sum total of accounts, while things are infinite in number. Inevitably, then, the same account and single name qualify several things."[7] Obviously, a language made up simply of singular terms would be incapable of capturing the world itself, as the world contains an infinite multiplicity of things. For this reason, words must necessarily have a universal element to range over classes of things. Second, and really a corollary of the first reason, if logic is to match up with the world, it must be the case that it mirrors structures to be found within the world itself. As Aristotle believes that we can truly talk about things within the world, it must be the case that terms such as properties, which have universal application, can find a place within the world. Thus, for Aristotle, the genera and species pick out real features of the world. "In the case of all things which have several parts and in which the whole

is not, as it were, a mere heap, but the totality is something besides the parts, there is a cause of unity" (*MP*, 1045a). The fact that species and genera exist within the world means that inferences about the world can be valid. This use of the universal can be traced back to Plato, who "says to leave the infinite individuals alone, for we have no knowledge of them" (*ISA*, 40).[8] We will return to the role of the universal in representational logic in the conclusion of this chapter, but for now we should note that a problem presents itself for Aristotle's system. If we are to be able to talk meaningfully about the world, it is necessary that for Aristotle, species and genera don't merely define general 'heaps' of things, but group things together according to criteria which capture something common to their essence. It is for this reason that right at the opening of Aristotle's *Organon*, we have a discussion of three terms, homonymy, synonymy, and paronymy.[9]

Aristotle defines these various terms in the following ways:

> When things have only a name in common and the definition of being which corresponds to that name is different, they are called *homonymous*. Thus, for example, both a man and a picture are animals.
>
> When things have a name in common and the definition of being which corresponds to the name is called the same, they are called *synonymous*. Thus, for example, both a man and an ox are animals.
>
> When things get their name from something, with a difference in ending, they are called *paronymous*. Thus, for example, the grammarian gets his name from grammar, the brave get theirs from bravery.[10]

What is important in these definitions is the recognition that certain forms of differentiation of species may not capture what is essential to the species itself. As terms range over different objects, it is possible that, if we rely on the fact that the same term is used to designate different entities, we may be forced into a definition of a species which does not accurately capture what it is to be that particular thing. Thus, in the case above, the species, animal, may refer both to the man and the picture of a man, despite the fact that it is clear that in these cases the term animal is being used in substantially different ways. Whilst it is clear from the comments made earlier about the universal nature of language that homonymy cannot be avoided at some level, much of Aristotle's logical work consists in a struggle against the possible encroachment of homonymy within the logical system.[11] Instead, we need to look for synonymous expressions, as it is these that capture something essential about the thing in question. The definition of the thing in question will be in terms of this essence.

Once we have a set of terms which relate to something essential about the thing in question, we are in a position to move onto the question of definition. Given the hierarchical model put forward by Aristotle, the question of definition becomes remarkably straightforward. As we saw on page 55, the genus is predicated of the species, so, for instance, in the case of man, part of the definition of man is the genus to which he belongs. In this case, the genus is animal. Therefore man is an animal. Second, we can note that the genus ranges over things which are different in kind. Thus to the species will be attributed a difference which allows it to differ both from the genus, and from other species which are subsumed under the genus. Thus, the species to which man belongs is a species of animal characterised by its rationality. Man therefore becomes a rational animal. If we further recognise that a particular level of the hierarchy can be both a species and a genus, such as in the case of animal, which is a species of substance, and the genus of man, we can see how the hierarchical definition allows a complete specification of the essence. Man is defined by the combination of its genus

with a difference, the animal that is rational, and the genus is defined similarly, hence animal is further clarified as the substance with animality. Thus man is specified through rationality, animality, and substance. The concept of difference plays an important role, therefore, as difference in kind is essential to Aristotle's system.

As it is important to recognise that the specific difference from the genus must be essential, rather than merely accidentally inhering in a substance, we now need to look at the whole cluster of concepts which would today just be considered as predicates, but which for Aristotle play this special role. First, dealing with properties, a property differs from the kind of difference we have been talking about in that it only occurs within a particular species (although not every individual within a particular species needs to exhibit it), rather than also occurring in derivative species. Examples of properties include "the capacity to laugh in man" or "becoming grey in old age" (*ISA*, 48). These properties do not define the essence of a particular thing, although they do, at least in the first case, extensionally define a species, so that that which can neigh is a horse, and a horse is that which can neigh. Accidents, though similar to properties, "come into being and pass away apart from the substratum" so that "it is possible to conceive of a white crow and of an Ethiopian who has lost his colour apart from the destruction of the substratum" (*ISA*, 49). True Aristotelian difference therefore differs from properties in that it applies to all derivative species and from accidents in that it cannot be removed from a subject without that subject's destruction. The concept of difference itself has three common uses for Aristotle. First, common difference defines difference as commonly understood. That is, difference in any way, as, for instance, Socrates differs from Plato (*ISA*, 42). Proper difference is difference in terms of an accident. Specific difference finally characterises the difference in species. Therefore whilst the first two lead to a difference in quality, the third leads to a difference in essence.

We have now designated the features of Aristotle's system central to our account of Deleuze's argument. Aristotle's system provides a hierarchical arborescent structure through which the essence of a species can be specified through a process of division, from the most general to the specific. Central to this hierarchy is the concept of difference. It is difference which differentiates the different species from the genera, through this process of division. As we saw in the last chapter on page 48, providing a concept of difference is one of the key aims of the Deleuzian system. Our first aim will therefore be to look at Deleuze's evaluation of Aristotle's concept of difference. Whilst Deleuze will be critical of Aristotle's concept of difference, the failure to provide a concept which meets Deleuze's criteria cannot itself be seen as a failure on Aristotle's part. Deleuze does, however, provide a more definitive critique of Aristotle's philosophy, although it is yet to be seen whether this critique is directly reliant on an incorrect analysis of difference itself in the philosophy of Aristotle.

Aristotle's Concept of Difference

So far we have been looking at three levels in Aristotle's system, namely, the genus, the species and the individual. Deleuze begins by noting that to only one of these terms can be ascribed a form of difference which actually plays a part in generating the hierarchy of terms responsible for differentiation in Aristotle's system. Difference at the level of the individual, "in so far as it pertains to matter, will always be accidental" (*DR*, 30). That is, for Aristotle, if difference is to characterise the essence of a being, accidental difference will not capture this essence, instead merely reflecting the 'accidental' deviation from the proper form which the essence will exhibit within the particular individual. In terms of genera, the difference is too great. As we have seen, the function of difference is to differentiate species from other species and from genera. Genera, as indivisible objects, "have no contrariety either" (*DR*, 31). That is, difference

between genera can also not be definitive for Aristotle, as the difference between genera cannot define their difference in terms of a higher, common source. Therefore specific difference becomes the greatest difference for Aristotle.[12]

As we saw in the first chapter, Deleuze's ontology moves away from a Kantian conception of the transcendental by trying to consider the transcendental as generative, as opposed to merely conditioning. Aristotle's conception of difference at first seems to fulfil the criteria of a properly productive conception, which would allow this ideal to be realised (*DR*, 31). Deleuze explains this first conception of the relation of Aristotelian difference to Deleuzian difference as follows. As essence is a formal characteristic of a thing for Aristotle, and difference is generative of essence, difference is a formal characteristic, and as such presents itself as, in Deleuze's words, pure difference. In its relation to essence, it also has a qualitative characteristic, as it defines differences in kind, rather than numerical differences, which, as we saw, for Aristotle cannot be considered to constitute the essence of a thing. Deleuze considers it to be synthetic, since the species is composed of both a difference and a genus, and is also productive, as difference itself is a part of essence, meaning that difference itself divides the genus, rather than the genus simply being divided into differences. That is, difference plays an active role in defining the species. Difference thus has all of the characteristics which were attributed to the concept of difference which Deleuze himself employs, the difference which was outlined in the first chapter. Without the concept of difference, it would be impossible to move from the highest genus to any of its intermediaries. Aristotle's concept of difference is thus, like Deleuze's, directly responsible for the generation of the individual. Division thus appears as differenciation. Below this apparent surface, however, the concept of difference which Aristotle relies on becomes more problematic. First, it is clear that in the final analysis, the concept of difference in Aristotle's system is parasitic on a higher identity. Differentiae are only specified under the higher identity of the genus. Genera "destroy the differences but are not themselves

destroyed" (*ISA*, 42), meaning that genera are logically prior to differences, and are not logically dependant on them, whereas differences are logically dependant on genera (that within which they are a difference). A further question arises about the relation between difference and the genus. The highest genera form the top of the hierarchy of species, from which the species are differenciated by difference. This highest point in the hierarchy, however, cannot itself be the result of differenciation, as, if it was, this would presuppose a higher genus, from which this genus would itself be divided. Given that species and their differentiae are reliant on the preceding genus, the question of the status of difference at the top of the hierarchy now becomes problematic. A similar problem arises at the lowest point, where we have the problem of common difference. To quote Porphyry once again, "commonly one thing is said to differ from another when by otherness it differs in any way at all from either itself, or from another, for by otherness, Socrates differs from Plato and, indeed, from himself: he was a child and became a man" (*ISA*, 42). The second problem is to explain in this Aristotelian system how Socrates can differ from himself, and yet remain under the identity of the same concept, given that differentiation takes place in relation to the species and essence, through Aristotle's focus on specific difference, and not in relation to the individual, which is understood through the contingencies of accidental difference. In particular, the question which will arise is, how are we to differentiate essential from accidental properties of the individual? Aristotle's concept of difference therefore seems to be problematic on both sides. As Deleuze puts it, "specific difference refers only to an entirely relative maximum, a point of accommodation for the Greek eye which sees the mean, and has lost the sense of Dionysian transports and metamorphoses" (*DR*, 32). The failure to find a truly Deleuzian concept of difference in Aristotle's logic may be indicative of problems, but it is not in its own right problematic itself. We therefore need to look at these two problems in further detail, that of the genus, and that of the individual.

The Genus and Equivocity in Aristotle

The problem of the relation of the genera and the differentiae was recognised by Aristotle. He formulates it in the following way:

> It is not possible that either unity or being should be a single genus of things; for the differentiae of any genus must each of them have both being and be one, but it is not possible for the genus taken apart from its species (any more than for the species of the genus) to be predicated of its proper differentiae; so that if either unity or being is a genus, no differentiae will either have being or be one (*MP*, 998b).

This argument is fairly condensed, and so will require some explication.[13] First, as was stated earlier, a genus is "what is predicated in the category of essence of a number of things exhibiting differences in kind."[14] Therefore, to be a genus, something must possess differentiae. These differentiae must furthermore be different from the genus itself, as "genera are prior to the differences under themselves" (*ISA*, 51). Now, as the genera are organised in a hierarchy, taking the all-encompassing genus, it would be the case that this genus would be predicated of all genera. This leads to a contradiction, however, since, as a genus, it would also have to be predicated of itself, whereas following from the definition of a genus, it must not be predicated of itself. The highest genus, therefore, that of being or one, becomes contradictory. If we are to maintain the stability of the system, this highest genus must therefore be left undefined.[15] For this reason, the ultimate categories through which being is understood must be multiple, as they themselves are species in relation to the undefined genus. Aristotle lists ten in total.[16] This means that the terms in the hierarchy are now to be characterised in two divergent ways. In the intermediate terms, difference will descend from the identity of the genus, whereas for the highest genus, difference itself will reign, as it does not itself

partake in a higher identity. As Deleuze puts it, "it is as though there were two 'Logoi', differing in nature, but intermingled with one another: the logos of species, … which rests upon the condition of the identity or univocity of concepts in general taken as genera; and the logos of Genera … which is free of that condition and operates both in the equivocity of Being and in the diversity of the most general concepts" (*DR*, 33).

The difficulty with the present situation is the problematic nature of a metaphysics that would stand above the particular sciences. Aristotle makes three statements on the possibility of a metaphysics that need to be reconciled:

> (A) "There is a science of being as being and the attributes which belong to this in virtue of its own nature" (*MP*, 1003a).
>
> (B) "For every single class of things, as there is one perception, so there is one science, as for instance grammar, being one science, investigates all articulate sounds. Therefore to investigate all species of being *qua* being, is the work of a science which is generically one …" (*MP*, 1003b)
>
> (C) "There are many senses in which a thing can be said to 'be'" (*MP*, 1003b).

Clearly, any two of these statements can be asserted together, so we could have by (A) and (B) a science of the single sense of being, if this hadn't been ruled out by our previous arguments relating to the highest genus. Likewise (A) and (C) together allow the possibility of a science of being which ranges over multiple classes. (B) and (C) together would assert that although science in general is possible, a science of being is not. Taking all three statements together, however, would assert the existence of a science of that which is multiple, as we have shown that being is not a synonymous concept, which is also a science of the one, as this follows from statement (B). Aristotle's claim, however, that being is not homonymous (*MP*, 1003b), draws us to the

only remaining solution. Being must be paronymous. Going back to our initial definition, paronymy is a word which relates to words which differ in their endings. So, for example, in the case of the words health and healthily, although they differ in their meanings, these meanings are clearly related one to another by some kind of central meaning which is common to both terms, albeit actualised in different ways. This central term is what Aristotle calls a focal meaning. We can say therefore that whereas for synonymy, the term and its focal meaning coincide, and with homonymy, there is no focal meaning for the different terms, paronymy provides a situation where there is a focal meaning, but one which does not coincide directly with any of the terms. Thus, Aristotle will claim that what is really at issue in the definition of a science is not the identity of the sense through which the class is spoken, but rather the identity of the focal meaning which underlies the differing senses. This seems to close off the possibility which Deleuze has highlighted, that of difference becoming an essential moment of the system in its own right, as, once again, the concept of a self-identical concept stands at the centre of Aristotle's ontology, albeit one which must be said in many senses.

Whilst on the surface, this solves the problem of providing a ground to the system, it cannot be said to be ideal. Following Bencivenga,[17] we can see that the problem of the determination of the focal meaning now becomes serious. The purpose of the concept of paronymy is to move us from a conception of being as a 'heap' to a concept of an organised constellation of coordinated meanings. Whilst Aristotle believes that throughout the multitude of categories, an underlying focal concept of being shines through, Deleuze's opening up of the mere *possibility* of the concept of an ontology of difference throws the certainty of this concept into doubt. Now the onus falls on Aristotle to show that the concept of being, or unity, really is itself a unified concept. The difficulty is that this appears to be an empirical issue. If being is to be a universal concept, Aristotle has to show the universality of its application across the multitude of

seemingly different domains. Given that Aristotle requires that there is in fact a definitive central meaning of being,[18] the empirical account must explain how these various divergent meanings of being come to both be separated from the central meaning and yet remain semantically related. Beyond Bencivenga's analysis of this point, a further problem emerges of the radical ontological difference between the term being and the species. This emerges because even if a focal meaning for the concept of being could be established, this focal meaning could not be integrated into the hierarchy, as if it became the highest genus in a formal sense, we would revert to the previous problem of the highest genus. Being must therefore remain outside of the world as described by the hierarchy, and merely be referred to indirectly through the categories, as opposed to the categories themselves, of which we can speak. This throws into doubt the possibility of using an empirical concept of paronymy to describe that which exists outside of the hierarchy. This latter problem, which leads to the development of the technical use of the term, 'analogy', in Aquinas' work will be discussed later in the chapter, as it has implications for the symbolic approach expounded by Russell and Whitehead in the *Principia Mathematica.* The problem of paronymy, however is tied to the problem of individuals, and of essence in the Aristotelian system, as it is the question of how the meanings of terms can 'drift' from a central concept. Thus the problem of how various categories can maintain difference from one another, whilst remaining the same, is related to the question of difference brought up earlier, namely how a thing can differ from itself whilst retaining the same essence; the case of Socrates the man who differs from Socrates the boy. In both cases, what is at issue is the question of transition, the importance of which should become clearer when we sketch Hegel's response to Aristotle at the end of the chapter.

Change and the Individual

As we have seen, Aristotle's logic is an attempt to capture the essence of a thing in terms of language, that is, in terms of universal concepts. This presents a problem when dealing with individuals, as these are potentially infinite in number. A straightforward definition of the individual in terms of universal concepts cannot be achieved, as this would rob the individual of his individuality within the world. In discussing the concept of difference within Aristotle's system, we mentioned a distinction which here will come into play in explaining the individuality of the individual, that between form and matter. In Aristotle's hierarchy of genera and species, the genus was considered to be matter and the difference form. This meant that, proceeding through the hierarchy, prior differences as differences of a genus shift from form to matter as the difference itself generates a new genus. Thus whilst difference provides a formal distinction of the material of the genus when generating species, since these species can themselves be considered as genera of lower species, the formal difference gets incorporated into the matter of the genus. Similarly, in the case of the young boy Socrates and the old man Socrates, the former can be considered as the matter through which the form of the latter is actualised. At each stage in this process, the movement from matter to form is therefore characterised as a process of actualisation of a potentiality. Thus throughout the movement between the potentiality and actuality Socrates is asserted as a unity – potentiality is analogous to actuality. While the young Socrates differs from the old Socrates, these differences are purely accidental – they do not affect the actual essence of Socrates himself. Two related questions now assert themselves. First, how are we to differentiate between the accidental properties of Socrates and the essential properties of Socrates, and second, how are we to understand the process whereby we move from potentiality to actuality?

To answer the first question, we need to return to the Greek term for essence itself, *ti ēn einai*,[19] which can be translated as 'the what it was to be that thing'. What is important to recognize about this definition of essence is that it provides a self-identical underpinning for the thing whose essence we are searching for.[20] Through this self-identity, the underlying essence allows the same individual to be re-identified, even though the individual in question may have altered in his passage through time. Thus essence provides a tool for the selection of properties which are properly definitive of an individual, as opposed to those which are merely accidental to the individual. The fact that Socrates may be sitting is not constitutive of the essence of Socrates himself, although it does represent a state that he is in. If accidental states such as this were taken to be definitive, the identity of Socrates himself would break down through the collapse of all continuity. Whilst Socrates is becoming, the development of a fixed definition of his essence is problematic, as there is no way to differentiate accident and essence. It is only, however, once Socrates has ceased to be that the fixity we require for a determination of essence comes into being. Thus essence, as the "what it was to be a thing" is essentially retrospective.[21] This means that the becoming of Socrates is related entirely to an atemporal state of being.[22] When we ask by what process the identity of the individual is maintained in the movement from potentiality to actuality, we are told that "the proximate matter and the form are one and the same thing, the one potentially, the other actually. Therefore to ask the cause of their being one is like asking the cause of unity in general; for each thing is a unity, and the potential and the actual are somehow one" (*MP*, 1045b). In discussing the genera, we came across the difficulty of explaining the existence of the paronymous terms which orbit the concept of being. When we tried to elucidate the concept of unity in our discussion of genera, we found the concept undefined, and were referred to empirical notions of unity. When we ask about particular cases of unity, however, we are referred back to the generic concept of unity. Thus, the concept of unity has a circular definition in Aristotle's philosophy. This question of unity relates to the problem of transition in two ways. First, we have the

problem that we have no way of explaining how the categories can be the same, yet different. In other words, how they can be related to one another, but yet remain distinct. That is, Aristotle has no way of justifying his claim that the categories are paronymous. Second, we cannot understand the process of change in the individual as we have no way of understanding the unity of the individual through the various changes. In both cases, Aristotle lacks the ability to explain change or transition, in the first, through semantic drift, and in the second, temporal change. The concept of analogy, whilst it appears to solve the problem of transition within the Aristotelian system, instead merely serves as a marker for a problem which lacks any possible solution using the tools available to Aristotle. This analysis accords with Deleuze's comment that difference for Aristotle merely allows the "extraction or cutting out of generic identities from the flux of a continuous perceptible series" (*DR*, 34). These identities are extracted at the cost of rendering incomprehensible a certain mode of being, that of the transitional state. Whilst this seems quite clear in the case of temporal entities, we can also find cases where the difficulty of transition emerges in other situations. Given the notion of species, we can see that within a given species there will be individuals which do not completely conform to the specification of that species. Thus, while man may be a biped, it is possible to conceive of a man without legs. For him, lacking legs would be a genuine lack, whereas for fishes the absence of legs would not be a lack, as this does not form a part of the essence of a fish.[23] The man without legs would therefore still orbit the concept of a man, as his lack of one of the qualities of man would still be related to his underlying essence of being a man, whereas for another creature, lacking this same quality would be entirely unrelated to its essence. The difficulty arises when a particular individual is caught between two particular essences.[24]Aristotle gives the example of sea-anemones, which fall between plant and animal classifications. In this case, it is fairly simple to contemplate there being a tighter specification of species which would force the anemone into one category or another.[25] The cases of ring species, however, are more problematic. A ring species is a species which contains a series of natural

populations, each of which is able to interbreed with its neighbour. As the distance between these various populations increases, genetic differences accumulate, however, to the point where the first species in the sequence cannot interbreed with a species a certain distance away, even though all intermediaries are able to still produce viable offspring.[26] The classic example of this kind of relation is found with the gulls found encircling the higher latitudes of the northern hemisphere. In this case, the Herring gull, which breeds throughout Western Europe and Northern America, is able to interbreed with the Vega Herring gull, which inhabits North-Eastern Russia. This species is in turn able to interbreed with Birula's gull, and so forth until we reach the lesser black-backed gull, which is also an inhabitant of Northern Europe. Gulls from this particular community cannot, however, interbreed with our original gull, the Herring gull, even though a continuous path of genetic compatibility around the northern hemisphere can be traced. Thus the problem emerges, one which has not been solved by the current biological classification system, of whether we are here dealing with one or two species. Given the strict method of division which forms the heart of the Aristotelian system, it is clear that the intermediate species must either fall outside of the system of classification, or must be incorporated at the cost of the integrity of the categories of species themselves. In examples of this kind, what we seem to be faced with is an objective kind of ambiguity which cannot be solved by simply applying our method more rigorously, an approach we might take in other cases such as the ambiguity in the word 'sharp' in the phrases 'a sharp knife' and 'a sharp taste'. The heart of the problem would therefore seem to be the method of division itself. As we have seen, division for Aristotle is accomplished by the differentiae, and so it seems that the problem is once again the concept of difference at play in Aristotle's work. This is the ground for both the problem of ambiguity in classification, and of the ambiguity of becoming. Before returning to this theme, we need to see how these difficulties play themselves out in more modern representationalist systems which do not rely on the peculiarities of the Aristotelian metaphysic.

Aquinas

We shall move quickly through Aquinas, primarily in order to show how the terminology at play in Aristotle's account changes, and to show how the form of analogy at play becomes refined. The process of movement from Aristotle to Aquinas comes through Porphyry's text, *Isagoge*. Porphyry, a student of Plotinus, writing in the second and third centuries A.D., wrote the *Isagoge* for the Roman senator Chrysaorius, as an aid to the study of Aristotle's *Categories*.[27] *Isagoge* was taken up by the scholastic philosophers of the middle ages, and became one of the key introductory texts ('isagoge' is a transliteration of the Greek for 'introduction'). As such, it provides a bridge for much of the terminology of Aristotle's logic, as well as many of its problems. The two primary effects of this adoption were the recognition of the fact that homonymy and paronymy were in fact related. As Porphyry writes of Aristotle, "if, then, anyone names all things beings, he will name them homonymously, he says, but not synonymously; for if being were one genus of all things, all things would be called beings synonymously" (*ISA*, 39). This comment can be explained easily by the fact that if we apply Aristotle's notion of definition by division to the three categories of homonymy, synonymy, and paronymy, then homonymy and synonymy become the genera (in terms of a common name), whilst paronymous and non-paronymous homonymy become species according to the presence of a focal meaning. Second, the reliance on Porphyry's text as a source for the study of Aristotle led to the development of standard translations of formal terms into Latin. It was Boethius who produced the earliest and most fundamental translation in the late fourth or early fifth centuries (*ISA*, 21), which led to the standard translations of *aequivoce* for homonymously, and *univoce* for synonymously. From these two terms we derive the English terms, equivocity and univocity.[28] Thus the problem of the homonymy of being can be related directly to the problem of the equivocity of being

which Deleuze raises.[29] The problem of Aristotle's highest genus is taken up by Aquinas:

> Nothing can be predicated univocally of God and creatures; for in all univocal predication the sense of the name is common to both things of which the name is univocally predicated … and yet one cannot say that what is predicated of God and creatures is predicated purely equivocally.[30]

The solution to this dilemma is to use the notion of analogy. What is novel in Aquinas' approach is the formulation of two separate concepts of analogy: proportion and proportionality. Proportion is when "we find something predicated analogically of two things of which one has a relation to another,"[31] and corresponds to the relations of the categories and being in Aristotle's system. This holds that there is a relation between two things. Aquinas' second concept of analogy, proportionality, goes further, holding that there is a relation between two relations: "but sometimes predication is made according to the second kind of conformity, e.g. the name of sight is predicated of corporeal sight and of intellect, because as sight is in the eye, so intellect is in the mind."[32] The main point of interest in this advance is that it allows the specification of the properties of relations between different categories and their species without a knowledge of the nature of the terms which stand in the relations. Thus we can talk about the nature of God's wisdom without having to specify the nature of the being of God himself. In this way, scholastic logic allows the discussion of things which derive from different categories in the same terms. This is in effect the discovery of an isomorphism across the different senses of being. We should recognise, however, that this is a clarification of the notion of analogy, and its reliance on the concept of an indeterminate identity, rather than a solution to the problem of the fracture of being itself.[33] We are now ready to see how the same problems and solutions also occur

through the notion of systematic ambiguity employed by Russell and Whitehead within the *Principia Mathematica.*

Symbolic Logic

Whilst symbolic logic in its modern form begins with Frege (or perhaps with Boole), the system we shall concentrate on will be that of Whitehead and Russell, as it is here that the degree of sophistication of the formal system allows its inherent contradictions to surface. As a formal discipline, symbolic logic provides a purely syntactical description of the relations between entities, and with this creates a sharp divide between the form of thought and its subject matter. It governs the purely formal relations of arbitrary terms, thus allowing the hope of escaping from any particular interpretation of the reasoning process which would force on us a particular metaphysical outlook. It also allows the characterisation of phenomena in an artificial language, formulated to allow precision in the expression of propositions, and to allow a purely mechanical movement between propositions, allowing the rules of derivation to be formally specified. Whilst the process of moving to an artificial language begins with Aristotle and his use of variables, formal logic also allows the introduction of signs for constants, such as and (&), or (v), and not (-). While symbolic logic contains many divisions, and formalisations, the most fundamental formalisation is the logic of classes. The logic of classes deals with the relations between sets of things (in the loosest possible terms), such as the inclusion or exclusion of both individuals and classes of things within classes of things. It should be clear from this definition that Aristotle's hierarchy is itself essentially a theory of classes, providing rules whereby different individuals and species can count as being included in or excluded from genera, which in turn are either included or excluded under their aspect as species. Returning to Aristotle, we may recall that he defined a species through a difference, which

determined the particular objects which fell under it. The modern theory of classes provides two methods of defining membership of a class, the first through extension, by giving a list of membership of the particular class, and the second, intensionally, that is, by giving a particular property by which members of the class can be identified, or in other words a criterion for membership. Whilst it may be theoretically possible to define a class by the first method, practically all classes are defined in terms of a common property (were we to attempt defining an infinite class by extension, "Death would cut short our laudable endeavour before it had attained its goal."[34]). Russell therefore originally put forward the intuitive axiom that given a class of things with a property *F*, *x* will be a member of this class if and only if it has the property *F*. On this basis, we are therefore able to formalise aspects of the hierarchy of Aristotle. Thus all mammals are animals is rendered as (x)(xεa → xεb). Literally, this means that for any arbitrary object (x), if that object is a member of the class of mammals (a), then it is a member of the class of animals (b). We could further define the set of men as a subset of mammals, and in this way allow the mechanical derivation of the animality of man from his status as a mammal. This process could be repeated to give a complete set-theoretic representation of Aristotle's arborescent hierarchy. Given that we are able to represent the hierarchy of Aristotle within set theory, the question naturally arises as to the status of Aristotle's problem of the equivocity of being within the theory of classes. The question, translated into set theory is therefore whether it is possible to specify the class of all classes, i.e., that to which all classes belong, equivalent of Aristotle's genus, being, of which everything partakes. On the face of it, this could be achieved simply by making it definitional that if something is a class, then it is a member of the class of all classes. This form of self-referentiality was excluded from Aristotle's system by the requirement that differentiae differ from genera. In the case of Russell's system, however, at first glance it is possible to specify a class which is a member of itself, as, given the class of things which have the property of being classes, x will be a member of this class only if

it is a class. The class of all classes meets this definition, and is therefore a member of itself.

Before moving on to the paradox generated by this intuitive use of the term class, let us first consider one of Russell's earlier paradoxes. Suppose there is a certain town, and in this town lives a barber. As a part of his job, the barber shaves the men of the town, but only those men who do not shave themselves. The paradox arises when we ask, who, in that case, shaves the barber? If we say that the barber shaves himself, then it is also the case that the barber does not shave himself, for the barber shaves those men who do not shave themselves. Likewise, if the barber does not shave himself, then, as his role as a barber is to shave those who do not shave themselves, he must shave himself. In this case, therefore, the barber is both in a state and not in that same state simultaneously, which, within a representational system, will lead to the collapse of all states within that system.[35] In the case of this early formulation of the paradox, the solution is clear, however. Implicit in such a paradox is the proposition that there can be such a barber. The solution, therefore, is to deny the possibility of the existence of the barber. By doing this, we can preserve the consistency of the logical system, and prevent the possibility of contradiction emerging. An analogous paradox can be raised in the theory of classes, however, which cannot be so easily resolved. One of the fundamental principles of the theory of classes is that a class can be defined intensionally by any clearly defined property which belongs to all members. Such a definition of class carries with it no existential statements, so we could define the class of, for example, the descendents of the last Tsar of Russia, without having to affirm that this class contains any members. It could instead simply be an empty class.[36] Given this, we can specify the following class:

> Let *w* be the class of all those classes which are not members of themselves.[37]

If we take a class, *x*, and say of it that it is a (member of) *w*, then this is equivalent of stating that "*x* is not an *x*" (as we might say that the class of all chairs is not itself a chair). If we give *x* the value of *w*, however, then the statement, "*w* is a *w*" becomes equivalent to the statement, "*w* is not a *w*". This is because if *w* is a member of itself (*w* is a *w*), then according to the definition of what it is to *be* a *w*, *w* is not a member of itself. On the other hand, if it is not a member of itself, then it meets the definition for membership of itself. It is therefore the case that *w* both is and is not a *w*. Clearly this particular case parallels the result of the barber's paradox, but whereas in that case, the paradox relied on the subject matter being discussed, the barber, this second paradox instead is derived from the fundamental rules of the logical system itself. The paradox is therefore implicit in the very structure of thought.

The only response is therefore to limit the scope of thought through artificial rules governing its application. Russell formulates this limitation as the rule, "whatever contains an apparent variable must not be a possible value of that variable,"[38] which as we will see below will prevent classes from referring to themselves. Already, we can see how the problem which dogged Aristotle's system of the determination of the highest genus is mirrored by a similar problem in the formalisation of Russell. This principle is analogous to Aristotle's requirement that the differentiae must differ from the genus under consideration, which also prevented genera from referring to themselves. In Russell's implementation of this principle, further parallels to Aristotelian and scholastic procedure will become apparent. [39] Russell's own implementation of this principle is to propose a hierarchy of types, meaning that an apparent variable must be of a different type to its possible values, known as Russell's theory of types.[40] This works by constructing a hierarchy from the most basic individuals to the highest classes. The individual in Russell's system functions as the anchor for the whole, and is considered to be absolutely simple. By means of this, Russell hopes to remove the possibility of any self-reflexivity at the base of the hierarchy.[41] Taking absolutely simple, non-reflexive

elements as our starting point means that, provided we are rigorous in our development of the system from these points, the problem of self-reflexivity may not arise. Designating these simple individuals using the subscript 1, as they form the base of the hierarchy, we can formulate simple propositions of the form, a_1 is F, or, this individual has a specified property (F). This proposition, as the highest type it contains is of the first order, we can call a first order proposition. We can then generalise this proposition, into the form, All a_1's are F. Here, a_1 is no longer taken as an individual, but instead as a variable ranging over all individuals. The statement, "All a_1*'s* are F", will itself, however be a second order proposition; it will be of a different *type* to the previous statement, a_1 is F. This process can be repeated indefinitely, thus generating a hierarchy of classes, each level of which is of a different type. As statements refer to the level below themselves, reflexivity is blocked. If we return to the issue of Russell's paradox, it is now clear how this difficulty has been solved. The class w now must be of a different level to the classes over which it ranges, so that if its members are classes of type n, w will be itself a class of type n+1. This means that the question of w's membership of itself becomes not even false, but meaningless, as it is impossible within the system to formulate the function w being applied to itself.[42] Instead, w is a type n+1 class which refers only to type n classes which are not a member of themselves. Types thus parallel the function of the differentiae in Aristotle's system by creating a hierarchy of terms, and the problem which emerges is in turn similar. Although Russell managed this way to solve the problem of self-reflexivity, it was at the cost of the univocity of the system.[43] Clearly, any conceptualisation of the highest type is meaningless, as this would have to take place in terms of an even higher type, but more than this, simple universal statements are also outlawed. It is clear that universal statements are not possible within this system, but also central notions, such as truth, being normally considered to apply universally throughout a system, are ambiguous, as they cannot be allowed to refer beyond a particular type.[44] We likewise cannot make statements including terms such as, "all properties of a", as these imply properties of different types, but we are instead

forced to make statements using the form, "all *n*-th level properties of *a*". In situations where the proposition holds for properties of *a* regardless of the value of *n*, however, "no harm results from regarding the statement as being about 'all properties of *a*', provided we remember that it is really a number of statements, and not a single statement, and not a single statement which could be regarded as assigning another property to *a*, over and above all properties."[45] As "it will be possible, sometimes, to combine into a single verbal statement what are really a number of different statements, corresponding to different orders in the hierarchy"[46], it becomes clear that what is being used to overcome the limitations of the symbolic system is once again a form of analogy, and in particular a form of isomorphy, as that which grounds the single verbal usage are the structural parallels between the various statements. Thus, the *Principia Mathematica* ultimately suffers from the same limitations as the Scholastic system, systematic ambiguity being synonymous with analogy.

There are also parallels between Russell's analysis of individuals and that of Aristotle. Whereas for Aristotle, essence allows the abstraction of a stable character from a continuous flux, Russell instead takes the route of simply denying the existence of that flux, instead arguing "that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest."[47] The arrow in question is, of course, Zeno's arrow, which occurs in the paradox that "if everything is in rest or in motion in a space equal to itself, and if what moves is always in the instant, the arrow in its flight is immovable."[48] The point of the paradox is that at every point that we wish to inspect the arrow, it appears to be at a fixed place, and that at no point is the arrow seen to be moving. Zeno takes this statement to refute the idea of motion within the world. For Russell, this "seems a very plain statement of an elementary fact,"[49] and this view derives from his belief that a stretch of time merely involves a particular kind of infinity, namely that between any two points of time, there is another point of time. This means that time appears as a continuum, while each moment is instead a fixed state. The

stability of the system is therefore once again purchased at the cost of excising all traces of motion. The problem of the nature of change itself is still present, however, as is clear from Russell's definition:

> Change is the difference, in respect to truth or falsehood, between a proposition concerning an entity and a time *T* and a proposition concerning the same entity and another time *T'*, provided that the propositions differ only by the fact that *T* occurs in the one where *T'* occurs in the other.[50]

This definition suffers from the difficulty that although it gives us a criterion to show that a change has taken place, it in no way defines the nature of that change itself. The same explanatory gap which occurred through Aristotle's use of analogy again occurs here.

Preliminary Conclusions

What can we conclude from this survey of representational logic? As we saw on page 65, in Aristotle's system, there is no possibility of developing a univocal conception of being, due to the impossibility of determining the highest genus. This led to Aristotle defining being paronymously. Whilst Aristotle's logic may prevent the determination of the highest genus, our discussion of the *Principia* showed that this difficulty is in fact far more serious than it appears in Aristotle's work, as if this highest genus could be specified, then the concept of the totality of the system could only be achieved at the cost of consistency. Thus, within representational logic from Aristotle to Russell and Whitehead, totality and consistency remain mutually exclusive. As both Aquinas and Russell make clear, and without Aquinas having any direct influence on

Russell, the concept of a totalised thinking of being remains a necessity, and so in both cases the notion of analogy is brought in to provide the same kind of pseudo-totalising effect on the systems. For Aquinas, this allows us to talk of God and man in the same terms, regardless of their differences in being. For Russell, systematic ambiguity allowed us to refer to a series of statements of different types as if they were one universal statement through structural analogies between them. Likewise, at the level of the individual, we find for both Aristotle and Russell that the concept at issue, namely the transition between states, is presupposed, rather than explained. For Deleuze, these issues develop from what he calls the "four shackles of mediation:" (*DR*, 29) identity, analogy, opposition, and resemblance. Identity in the form of the highest genus, which unifies the system, but remains an undetermined concept. This is present for Russell as the universal statement which cannot be more than a verbal pronouncement, one that is comprehensible, but cannot be explained through the system itself. Analogy is a relation between the ultimate determinable concepts, that is the categories for Aristotle, or the isomorphic statements of variable type for Russell. Opposition deals with the relations between determinations within concepts, and thus refers to the differentiae, which relate the determinations of the genus to each other through a process of exclusion, and to the clear specification of the predicate for Russell, which delimits a class from other classes through purely bivalent criteria. Finally resemblance, in relation to the determined object of the concept itself allows the selection of individuals who differ but are subsumed under the same species. Essence clearly fills this role for Aristotle, and a similar function is required for Russell in his theory of change, where identical things which differ are required to show that a change has happened.

For Deleuze, the source of the problems which occur as a result of these four functions is the faculty of judgement, and in particular the forms of common sense and good sense which we discussed in relation to Kant in the first chapter. Here, common sense once again refers to the partition of concepts, whereas good sense now refers,

beyond the attachment of a predicate to a subject (subsumption in Kant's sense) to the construction of hierarchy itself. These two functions are both present in Aristotle and Russell. What is at issue in both Russell and Aristotle is what is absent from their systems. Identity provides a concept which is inexplicable, analogy provides an isomorphism between elements within a totality that cannot be defined, opposition provides a stability which is contradicted by phenomena such as ring distributions of species, and resemblance merely shows the gap between differing temporal individuals rather than explaining it. What, according to Deleuze, is being covered over in all of these cases is the concept of difference. "On the basis of a first impression (difference is evil), it is proposed to 'save' difference by representing it, and to represent it by relating it to the requirement of the concept in general" (*DR*, 29). The main thrust of Deleuze's argument is that the concept in general, as presented by common sense, requires the clear demarcation of concepts. This leads to a concept of difference which is a form of maximal difference, represented by the law of the excluded middle ($P \vee -P$). This makes difference essentially oppositional, and prevents the understanding of temporality, as well as the diversity present in indeterminate species (the issue of ring distributions applies just as much to Russell's system as to Aristotle's). Static states form the entirety of the system, leaving no space for either generation or transition. The points where true difference shows itself are in the places where representational logic breaks down, in the movement between these oppositional points, in the chasms which appear within the system. "Difference ceases to be reflexive and recovers an effectively real concept only to the extent that it designates catastrophes: either breaks of continuity of the series of resemblances or impassable fissures between analogical structures" (*DR*, 35). What Deleuze is therefore seeking is a concept of difference which is not oppositional in the sense described, one that does not have to be a maximal difference in the sense of Aristotle. We have already seen something like this through the categories, before they were once again subsumed under the undetermined highest genus. This conception of Deleuze's will be dealt with in chapter six. Now, however, there is another possible

response to the systems of identity of Aristotle and Russell that we must consider, that of Hegel.

Hegel and Aristotle

The relationship between Hegel and Aristotle is complex. Given the breadth of subject matter dealt with by both Hegel and Aristotle, comparisons between the two are common. Indeed, Hegel, in his analysis of Aristotle, forms an image of Aristotle which is very close to that which one might draw of Hegel himself. Whilst Aristotle's system "does not give the impression of its being in construction a self-systemised whole" (*LHP*, 2, 118), its parts "still form a totality of truly speculative philosophy." Aristotle is "the perfect empiricist" (*LHP*, 2, 133), in that through an absolutely thorough analysis of the empirical world, we are led back to the idea of the speculative notion. The Aristotle of Hegel is therefore an empiricist who pushes empiricism so far as to arrive at a truly speculative form of knowledge. The fact that Hegel held Aristotle to be the greatest of all his predecessors is beyond question,[51] but there are also structural similarities between the two systems that should be of interest to us here. For both Hegel and Aristotle, speculative thought begins with the concept of being. We have already seen how for Aristotle, this concept is given to us as necessarily indeterminate, and this is also the case for Hegel. The *Science of Logic* begins its deduction with the concept of "*Being, pure being*" (*SL*, 82) which, like that of Aristotle is, for thought, "pure indeterminateness and emptiness" (*SL*, 82). We saw how the dialectic of Aristotle was a process of increasing determination, through the addition of differentiae, to the point at which the lowest species can be specified in its essence. This too is the movement of the *Science of Logic*. The progressive accumulation of determinations allows us to move from the purest, but also the emptiest, of notions, to one which is adequate to the conception of being. Does this mean that Hegel too has an equivocal concept of being?

To answer this, we should first note that nowhere in Hegel's writings on Aristotle is the problem of the highest genus recognised.[52] Indeed, the central concept of Aristotle's theology, the unmoved mover, is very far removed from the self-movement of being which Hegel posits. If we look at the *Science of Logic*, the difference between the two systems becomes clear. We have emphasised how Aristotle's system eliminates the possibility of movement through what Deleuze will call a "mediated concept of difference", but reading the problem of the highest genus in the light of the *Principia Mathematica*, we also see that this move eliminates the possibility of contradiction. Hegel, on the contrary, is willing to push the question of the indeterminate notion of being to the point where this very notion itself breaks down. Thus, if being is such that "there is nothing intuited in it", this very concept itself will vanish into nothingness: being, as indeterminate, is impossible to differentiate from nothingness. The instability of being, however, is paralleled by a similar indeterminacy of nothingness, which in turn vanishes into being. Thus, in this process of vanishing, a further concept is developed, that of becoming, the movement between the two prior concepts, which shows that the provisional meaning of being and of nothing is for both to be simply this vanishing of one into the other. This provisional result will itself be qualified, as becoming, the unstable unity of being and nothing, gives way to determinate being, which is a resolved unity of both. Now, what has been presented here is a certain movement between concepts. Of course, in the logics of Aristotle and Russell, movement between concepts is possible, but this movement must purely be one of implication, that is, nothing not already present in the concept can emerge from it. Hegel himself recognises this distinction between his own system and that of Aristotle's. Whilst for Hegel, the movement of the system is a progressive determination of the concept of being, for Aristotle, the process by which the indeterminate concept is reached is one of enumeration and abstraction. "If, for example, we take away from space all its empirical determinations, the result will be in its highest degree speculative, for the empirical,

comprehended in its synthesis, is the speculative notion" (*LHP*, 2, 133). It is as if Aristotle simulates movement through a dialectic that works backwards. We should note further that the movement Hegel is making is not a temporal movement. In fact, it cannot be, as the notion of pure being with which the *Science of Logic* begins is logically prior to temporality. Similarly, whilst the inability of either Russell or Aristotle to deal adequately with the notion of temporality is an indication for Deleuze of the problematic nature of their systems, we saw that the real root of the problem was not itself temporal. What exactly is driving the Hegelian dialectic becomes clear when we look at Hegel's comments on Zeno's dialectic, and compare them with those of Russell.

Zeno

We can begin with Hegel's assertion that, "Zeno's dialectic of matter has not been refuted to this day" (*LHP*, 1, 265). Thus Hegel, like Russell, takes seriously the problem which Zeno presents, namely that there is no truth in movement. What is of interest is the way in which Zeno arrives at this result. Hegel draws the distinction between two forms of dialectic, external dialectic, "in which the movement is different from the comprehension of the movement" (*LHP*, 1, 264) and internal dialectic, which is "not a movement of our intelligence, but what proceeds from the nature of the thing itself, i.e. from the pure Notion of the content." The first form of dialectic is that practiced by Parmenides, in which he proves from certain axioms that "All is One". The limitation of this form of dialectic is that it relies on a set of assumed axioms which need not be accepted by the interlocutor. The result of this is that even if the system derived from these axioms is coherent, the system itself will always depend on my axiom, which to the other will "always seem to be false and external" (*LHP*, 1, 264). Thus the debate moves in the wrong direction. I try to prove my case by unfolding the implications of my position, whilst it is the grounds of the position itself which are the issue. This does

not mean that external dialectic is entirely without benefits, as through it, "reasons are revealed, and new light thrown" (*LHP*, 1, 264). The result of this is that the object of the debate is criticised, but only from one side, at best merely placing the held assumptions in question. Internal dialectic, on the contrary does not reason from alien premises, but is itself the movement of the object under discussion. This is the dialectic of Zeno, who *contra* Parmenides argues that "the Many cannot be." This is true dialectic, which "leaves nothing whatever to its object," leading it instead to "disintegrate itself in the entirety of its nature" (*LHP*, 1, 265). Thus Hegel follows Zeno in arguing that the motor of the philosophical process is contradiction. Zeno's dialectic itself cannot move beyond the destruction of the object, however. "The result of this dialectic is null, the negative; the affirmative in it does not yet appear" (*LHP*, 1, 265). The negative which Hegel is referring to is the fact that a contradiction leads to the negation of its object, as in Zeno's case above, where the many is negated. In talking of an 'affirmative' within contradiction, Hegel is going beyond the structures of classical logic. Russell's paranoia about the possibility that universality would lead to contradiction was grounded in the fact that the existence of a contradiction makes all propositions within a system simultaneously true and false. Thus, what Hegel is seeking is something between the mad proliferation of propositions within a system and the scepticism which "ends up with the bare abstraction of nothingness or emptiness and cannot get any further than there, but must wait to see if something new comes along and what it is, in order to throw it too into the same empty abyss" (*PS*, 51). The answer to this is to see the negation of the concept that has proved to be contradictory as a determinate negation, meaning that in showing a particular object to be contradictory, we do not simply reject it, but trace out the path that its own rejection forces us to take. In this way, Hegel moves beyond a purely formal logic to one in which the *content* itself opens up a determinate movement beyond the impasse that contradiction leads to in earlier systems. Thus, in the case of the problematic concepts of being and nothingness, we are led not to a scepticism concerning these concepts, but rather to a further concept of becoming.

Furthermore, the concept of becoming does not result from the destruction of these prior categories, but is instead the result of the resolution of this contradiction at a higher level. The contradiction is *aufgehoben*, that is, simultaneously surpassed and preserved. In this movement, the force of Russell's anxiety in the face of contradiction is removed. Instead of leading to an indeterminate proliferation of concepts, the unfolding of the contradiction is determinate, and productive. Through this moment of *Aufhebung*, the problems which occurred within the earlier representational systems are resolved. The contradictory nature of the highest genus, being, becomes instead positive and productive. The further lesson which Hegel draws from Zeno is that this dialectical process is not an imposition on the object (object in the sense of that which is analysed – dialectic does not presume that the analysand must always be objectival), but rather the natural development of the object itself. For Hegel, this means that the method of dialectic itself is a product of the dialectic of the object itself, rather than being a presupposition of the analysis. Further, as we saw on page 80, for Russell, unity and contradiction imply one another, hence the need to reject the former to safeguard against the latter. For Hegel, accepting the possibility of contradiction means opening the possibility of a unified, total system. The question of the semantic drift of concepts is likewise resolved, as the dialectic allows different concepts to be united through a genetic method, the tracing back of these differing concepts to a common root. Motion can also be explained through the contradiction of difference and identity highlighted earlier. This discussion of Hegel's response has been somewhat schematic, and we discuss Hegel's philosophy in more detail in chapter five.

Conclusion

We have shown how representational logic presents, structurally, limitations in the degree to which the world can be thought. Following Deleuze's analysis, this

difficulty seems to emerge from the concept of difference at play within the structure of these systems. Deleuze and Hegel offer us two alternatives in dealing with these problems: the non-oppositional difference of Deleuze, and the radical concept of opposition proposed by Hegel. Now that we have laid out the difficulties that are to be dealt with by these two different approaches to the limitations of representational logic, we can open the discussion of the logics themselves, before dealing with Deleuze's critique of Hegel's approach. To put it simply, Deleuze will argue that in extending the idea of difference to its absolute limit, that of contradiction, Hegel has not truly escaped from the limitations of the representationalist paradigm. Deleuze, on the contrary, by moving to a transcendental notion of difference, hopes to produce a difference which differs in kind from that of Aristotle and Russell. The extent to which this is problematic will be examined in the final chapter. In the next chapter, we will begin to add content to Deleuze's response to representation by looking at Bergson's philosophy of duration, which is a central, perhaps the central, influence on the development of Deleuze's thought.

Chapter Three – Bergsonism

Introduction

In the first chapter we looked at the general conditions of a transcendental empiricism through an argument traced from Sartre's rejection of Kant's transcendental ego. After, in the last chapter, exploring the limitations of classical logic, we are now ready to consider Deleuze's diagnosis and proposed remedy for the problems that we uncovered. Of all Deleuze's precursors, it is perhaps Bergson who makes the most significant contribution to Deleuze's philosophy, and so in this chapter we will focus on some of the Bergsonian themes which will become central to Deleuze's own thinking.[1] As Deleuze makes clear in his study of Bergson, what is needed is "a renewal or return to his project today" (*B*, 115). In making this claim, however, Deleuze is arguing for the selection of certain concepts elaborated by Bergson which may be preserved, and, together with elements gleaned from other sources, revitalised within the context of a new philosophical system. In particular, it is the method of intuition and the theory of multiplicities which Deleuze will take up, removing them from what he calls the "French-style history of philosophy" (*B*, 8). In making a sharp distinction between space and duration, Bergson was able to provide a theory of the foundations of classical logic, as well as explaining why classical logic fails to explain large groups of systems, such as living systems. In this chapter, we will begin by turning our attention to Bergson and his criticisms of the approaches of both evolutionary theory and Kant. Through this, we will see that the concept of space plays a key role in classical thinking's understanding of the world and our representations of it. We shall also see that Bergson's understanding of Kant on this point shows itself to involve certain important simplifications. Moving on to Bergson's own response to the difficulties which emerge within the paradigm of representation, we will isolate two kinds of organisation which are at play in Bergson's philosophy: the spatial, and the durational. This will allow us to see just how these two

kinds of multiplicity are reworked by Deleuze in *Difference and Repetition* into the notions of the virtual and actual.

Bergson's Account of Kant and Classical Logic

Much of the work of Bergson, and also Deleuze, can be seen as an attempt to come to terms with the intuitions and results of evolutionary theorists such as Herbert Spencer, whilst simultaneously trying to show how the formulations of these theories, as they stood in the nineteenth century, were problematic. In particular, in this chapter, we will look at one point which relates the work of Bergson, Russell, Kant, and evolutionary theory, namely the relation of the structure of knowledge to the structure of the object. Kant's project of providing a ground for the sciences provides one solution to this problem, in the form of conditioning the object by the categories of our thought, thereby escaping the inherent problems of both metaphysical and empirical theories of the correspondence of the understanding and the world.[2] In moving to a theory of transcendental categories, Kant attempts to show that these categories cannot have an empirical origin, as they introduce necessity and universality into our judgements. From this, as we saw in the first chapter, a certain isomorphism between the understanding and the object under consideration emerges. For Russell, likewise, the object under consideration, and the system used to examine it, must be seen as possessing certain similarities. It is axiomatic in his approach to philosophy that "in a logically correct symbolism there will always be a fundamental identity of structure between a fact and a symbol."[3] Thus, as we saw in the previous chapter, the analytic conception of problems tends to lead to a parallel atomistic conception of the world, hence Russell's solution of Zeno's paradoxes through the idea of an infinite discrete series. The resolution of a paradox of thought therefore leads to an alteration of the conception of the object. We will consider in this part of the chapter briefly how this conception of an isomorphism

between representations of the world and the world itself was taken up by one of the leading representatives of evolutionary theory. This will be done through examining the idea of a certain kind of multiplicity, which results in the concept of a homogeneous space employed by Russell to provide a place within which the process of inference and the correlation of inferences and the world could take place. This space will be seen to have similar properties to Kant's conception of the structure of the understanding.

Kant describes his project as an attempt to answer the question "what makes synthetic *a priori* judgement possible?" This possibility, once accepted, is explained by Kant through the idea of the application of the categories of the subject to intuition, taking on the role of *conditioning* experience. As the same categories play a role in both conditioning experience and conceptualising it, their application to experience is vouchsafed. We have dealt with some of the difficulties of and responses to this approach in the first chapter, where we showed how Deleuze, using Sartre, wants to break this parallelism between the empirical and the transcendental. Before moving on to Bergson, we need to look briefly at the *milieu* within which Bergson developed his philosophy, and in particular, the influence of Spencer's philosophy. As we shall see, Spencer attempts to provide a genetic interpretation of the development of our categories of thought which would allow us to see exactly why space appears to us in the particular *a priori* form which it has. We shall further see that this argument proves to be problematic, relying on the notion of space it seeks to explain in order to generate its explanation. Whilst Bergson will reject Spencer's argument, we shall see that its attempt to explain the genesis of space opens the way to a more fruitful line of argument which Bergson himself will pursue. In contrast to Kant's project, Spencer's philosophy, and in particular the intuition captured in the maxim of "the survival of the fittest," argues that our modes of understanding of the world, as well as their fit with the world, have an empirical origin. On a Spencerian reading, such as that provided by Čapek, Kant's proof for the *a priori* nature of space rests on the ability of the subject to represent to itself, or

to conceive of, the world in certain ways. The first argument states that "in order that I may be able to represent [the referents of sensations] outside and alongside one another, and accordingly, as not only different but as in different places, the representation of space must be presupposed" (*CPR*, A23/B38).[4] That is to say, relations between empirical representations are inconceivable without the representation of space. The second, that "we can never represent to ourselves the absence of space, though we can quite well think of it as empty of objects" (*CPR*, A23/B38) follows a similar structure, placing the weight of the proof on the subject's capacity to conceive of certain relations between space and objects. In other words, what we cannot conceive of, in terms of empirical phenomena, cannot be the case. It is this reliance on the possibilities of representation which at first appears to be the target of evolutionary theory. Rather than holding that we can only conceive of space in a certain manner due to the subject's role in conditioning the empirical, evolutionary theory instead argues that our cognitive capacities are based on the fitness of the organism to the environment. Thus, alternative, partial conceptions of space lead to an organism which is not optimally attuned to its environment and therefore has a lower chance of survival than one with a more practical conceptual schema. As natural selection eliminates those organisms with suboptimal representational schemata, a more optimal scheme becomes sedimented in the organism, so that what is *a posteriori* for the species becomes *a priori* for the individual. Conceivability therefore *seems* to fail for the evolutionary theorist as a criterion for the correspondence of intuition and the world, as conceivability now becomes a function of fitness, although, as we shall see, Spencer's approach ultimately leads to a reversal of this result.

For Spencer, life "in its simplest form is the correspondence of certain inner physico-chemical actions with certain outer physico-chemical actions," so that "each advance to a higher form of life consists in a better preservation of this primary correspondence by the establishment of other correspondences."[5] Evolution is thus

characterised by the gradual broadening of the correspondence between the world itself and the set of physico-chemical reactions which lie at the heart of our interactions with the world. As the number of correspondences and the number of relations of correspondences increase, so the *milieu* within which the organism situates itself expands. Whilst this model is clearly founded on the pragmatic value of this expansion of the *milieu* for the organism, as this *milieu* widens, the subjective representation of the world approaches coextensity with the world itself. That is, through the gradual development of intelligence through small differences in degree, a difference in kind is finally encountered, whereby the pragmatic utility falls away in the face of the relation to the whole of the external. As the *milieu* is now the whole of the world, it is no longer a partial view governed by the concerns of the organism, or rather, these concerns are now extensionally equivalent to an objective understanding of the world. "What we call truth, guiding us to successful action and the consequent maintenance of life, is simply the accurate correspondence of subjective to objective relations; while error, leading to failure, and therefore towards death, is the absence of such correspondence."[6] Thus, finally, pragmatic truth, when it is applied to a *milieu* which encompasses the entirety of the real, becomes objective truth. As Spencer considered the evolution of humanity to have reached this endpoint, primarily through the successes of the scientific enterprise, for him, the validity of the *a priori* categories is restored, as they now once again correspond to reality. As Ernst Mach wrote, "we may, indeed, say that our sense of causality is not acquired by the individual, but has been perfected in the development of the race."[7] Since, during the phylogeny of the organism, it has developed a representation of the world, and this representation is now adequate to the world, the inability to conceive of, say, an object without an underlying intuition of space, represents the history of the evolution of humanity struggling to consider a representation which finds no place in nature. Ultimately, therefore, evolutionary theory, at least insofar as it is represented by the work of Spencer, rejoins Kantianism in positing the fundamentally *a priori* nature of our representation of space and the

categories. The *a priori* nature of the categories is now grounded in a pragmatic correspondence between the organism and its environment rather than through the conditioning of the object by the subject.

For Bergson, Spencer opens up a potential direction for philosophy whereby it can "continue without laying itself open to the Kantian criticism" (*CE*, 364), but in fact, his approach fails to provide a real alternative to the Kant's position. As we have just seen, the end result of the Spencerian picture resembles Kant, with the resurrection of an *a priori* interpretation of space. This, however, is not what Bergson is referring to. Instead, the difficulty is that Spencer, in attempting to explain the genesis of space has resorted to "*reconstructing evolution with fragments of the evolved*" (*CE*, 364). That is, in attempting to explain the genesis of space, Spencer has instead merely presupposed the result of his investigation at its outset by relying on a process of evolution which takes place *in space*. "The diffused elements which he integrates into visible and tangible bodies have all the air of being the very particles of the simple bodies, which he first supposes disseminated through space" (*CE*, 365). Spencer therefore fails to provide an adequate response to Kant, as his explanation of the genesis of space presupposes an account of a space such as that which Kant gives wherein this genesis takes place. Despite this limitation of Spencer's account, Bergson takes several important themes from Spencer's metaphysics. First is the need to give an account of space which retraces its genesis, rather than accepting "space ready-made as given" (*CE*, 205), as he takes Kant to do. Second, in his positive theory of the nature of space, Bergson will argue that Kant has been guilty of an error in the construction of his theory, namely in the exclusion of a possibility in his account of the nature of space. In enumerating the possible relations between the subject and the world, Bergson claims Kant considers three possibilities: "either the mind is determined by things, or things are determined by the mind, or between the mind and things we must suppose a mysterious agreement" (*CE*, 205). Bergson instead posits a fourth possibility, that "intellect and matter have

progressively adapted themselves one to the other in order to attain at last a common form" (*CE*, 206). As we saw in the case of Spencer, this process cannot be seen in terms of a preceding space, as this would lead to us merely presupposing what we are trying to show. In discussing Kant, Bergson finds himself in agreement with the result that space is to be taken to be an ideal feature of the world, and also with his refutation of any empiricist attempt to explain our conception of space as a result of our perception of the external world, which he calls "definitive in what it denies" (*CE*, 205). Bergson avoids the dilemma of thinking the genesis of space spatially by noting that Kant has not considered the possibility of "degrees in spatiality" (*CE*, 205), rather than its simply being given or not given. Of course, arguing that Kant has made an unwarranted assumption in assuming that space does not admit of degrees, Bergson also must posit that the understanding is wider that Kant's conception of it, in order to explain the possibility of its connection not just to pure space, but also to these intermediate forms. What Bergson is trying to do is to give an explanation of the genesis of space, thus highlighting an important limitation in Kant's account – his inability to explain how space arises as the form of our perceptive faculty. Before moving on to Bergson's positive philosophy, we will therefore look at what Bergson takes to be the connections between homogenous space and the understanding for Kant and Russell.

We noted in the first chapter how judgement for Kant involves the subsumption of two terms which, in themselves, are already fully determined. Thus, in order for a relation to be formed between the two parts of the judgement, something external to the elements of the judgement, namely the understanding, is required, which allows these two terms to be brought into relation with one another. Underlying the unity of the judgement is therefore a third term. For Bergson, this feature of the understanding can be generalised to our reflections on our own consciousness. Once the elements of consciousness are conceived of as fundamentally objectival, the intellect "is obliged next to reunite them by an artificial bond. It imagines therefore, a formless ego, indifferent

and unchangeable, on which it threads the psychic states which it has set up as separate entities" (*CE*, 3). According to Bergson, for Kant, this conception of the objectivity of the elements of judgement is also reflected in his conception of the elements of the world. Whilst this interpretation of Kant as having a similarly atomistic conception of space is problematic, we will follow Bergson's analysis, as it opens out onto the most important result from Deleuze's point of view: the positing of two forms of multiplicity.

In the transcendental aesthetic, we find a distinction between the object as it appears to us, and the space which makes possible its appearance. According to Bergson, it is space which functions as a necessary *a priori* representation, which is (logically) prior to the objects that occupy it ("we can never represent to ourselves the absence of space, though we can quite well think of it as empty of objects" [*CPR*, A24/B38]). In the cases of both the mind and the understanding of space therefore, we find the model of a medium through which the elements can interact. The ego enables judgement and space, in coordination with the categories of the understanding, enables perception of the object. We can see a similar situation emerging out of the set theory practised by Russell. We saw that set theory provides a hierarchical model of relations, sets being related to others by the relative domains of objects over which they range. Cantor's definition of the set as "a collection of definite, well discernible objects"[8] emphasises the use of spatial metaphors at the foundation of the discipline of classical logic, which allows the graphical representation of logical results through, for instance, Venn diagrams.[9] This process of viewing the elements of propositions as both being discrete, and inhering in a space which remains inert to their interactions allows us to see the relations between members of a set as purely external to those objects themselves. Furthermore, the relation of sets to each other becomes purely external. It was this feature that allowed the arbitrary definition of sets which both allowed the extensive definition of a set for Russell (that is, the definition of a set by the pure enumeration of its elements), and generated the paradoxes (the arbitrary definition of sets allows certain

forms of pernicious self-referentiality). It also, however, provides certain advantages for the method of analysis. Given that the relations between elements are purely external to one another, it becomes possible to bracket these relations for the purpose of the study of these phenomena. Bergson claims that a similar situation emerges in Kant's philosophy, where it is the categories of the understanding which impose a field of relations on the given.[10] Thus, the spatial model allows the method of analysis to develop, where a complex phenomenon can be broken down into its component parts in order to understand the whole through a later process of synthesis. As Russell puts it, it allows us to "tackle problems one at a time, instead of having to invent at one stroke a block theory of the whole universe."[11] For Bergson, the understanding and space are considered to be identical, as he argues that not only is space a homogenous medium, but, by the principle of indiscernibles, every homogenous medium is a space.[12] In chapter one, we saw that insofar as Kant takes the empirical to have the same form as the transcendental, he is unable to take account of the *generation* of the empirical. This led us so see that by privileging the structure of judgement, Kant belonged to the tradition of representation. We should note here, however, that Bergson's account of Kant's concept of space is rather simplistic. As Deleuze notes in relation to Kant's argument from incongruous counterparts (*DR*, 13 & 26), Kant recognises an "*inner* difference"[13] within space that escapes the understanding. We can further note that spatio-temporal objects for Kant cannot be described 'atomically', as "all substances, in so far as they can be perceived to coexist in space, are in thoroughgoing reciprocity" (*CPR*, B256). Whilst these simplifications limit the scope of Bergson's criticisms, as the analogy between space and the understanding for Kant no longer holds, we should note that it does not affect his argument against the limitations of the understanding, which as we saw in chapter one, page 47, is central to Deleuze's attack on finite representation. We saw there that the finite understanding for Kant produced judgements by subsuming representations under other representations. As each of these terms was in itself self-sufficient, the 'I think' was necessary to bring these various elements into relation with

one another. Whilst Bergson's attack on the Kantian conception of space as containing atomic elements within a homogenous field may fail, therefore, the Bergsonian criticism *does* apply to the 'I think' of the understanding. This is because it provides the element of homogeneity which allows the terms to relate *externally* to one another. The difficulties which we encountered with Kant's philosophy in chapter one would, therefore, for Bergson, be characterised in terms of a spatial conception of the 'I think', even if, ironically, the Kantian conception of space itself escapes Bergson's criticisms. Leaving aside his argument for this point, given the apparent differences between the structure of the understanding and the structure of space, what will actually be important to Deleuze is the multiplicity which underlies this conception of space, and the recognition that it is possible to draw a distinction between extensity and space.

If we are to understand the importance of space for Bergson, we need to note that our understanding of geometry now allows for the formulation of different kinds of geometry. Consequently, classical geometry has become known as Euclidean geometry, after Euclid, who was first to systematise its fundamental theorems and axioms, and later geometries as non-Euclidean geometries. In this chapter, we will focus on Euclidean geometry before looking in more detail at Riemannian geometry in the next chapter. All of the theorems of Euclidean geometry can be derived from 5 postulates or axioms, although it is the fifth axiom which will interest us here.[14] With the fifth axiom, which can be restated as asserting that "through a point not on a given straight line, one and only one line can be drawn that never meets that given line," we arrive at a conception of space as fundamentally homogenous. This means that a particular metric applied at one point within a Euclidean space can equally be applied at any other point. Euclidean space therefore has the fundamental property of measurability, in that we can compare the objects within it by their superposition upon one another. A consequence of this is that an object within a Euclidean space is invariant to transformation by displacement, or in other words, that the space of Euclidean geometry functions as a homogenous

medium where position does not affect the constitution of objects within it. Euclidean geometry therefore provides the ideal model of how we are to understand something like the ego as that which allows the relation of already determined concepts.

The idea of using Euclidean geometry to understand the interaction of concepts finds its parallel in the use of Euclidean geometry in the physics of Kant. Already with Descartes, analytic geometry opened the way to the algebraic representation of geometry, further allowing the conception of physics as the interaction of quantitatively characterised matter within the field of homogenous space defined by this geometry. Descartes' conception of matter as fundamentally inert can be seen as an attempt to reduce the study of physics to the study of geometry (aside from the property of impermeability given to matter[15]). By moving to a purely quantitative definition of matter, Descartes allowed for the application of mathematical concepts to the world, which in turn was to open up the possibility of the mechanics of Newton. Within this mechanics, the interpretation of consciousness which is found in Kant and Russell finds its parallel in an understanding of the world in terms of the relations of discrete objects. As we saw on page 92, Spencer's evolutionary theory develops through a process of closer and closer approximation to the structure of the world, guided by the force of natural selection. The final stage of Spencer's phylogenic account is the mirroring of an internal world grounded on the invisible thread of consciousness, and the external world grounded in the understanding's relations to the homogenous field of space. For Spencer, Newtonian physics therefore represents the final *milieu* of the development of the organism, one which allows the complete representation of the world to the organism. As we should expect from a physics that takes the truth of Euclidean geometry for granted, Newtonian physics contained its own equivalent of the corollary of Euclid's fifth postulate, the axiom of free mobility, which asserts that neither the size nor shape of bodies is affected by their displacement.[16]

The mechanistic model of the universe opens the way to several useful procedures. First, given that the mathematical concept of space provided by Euclid is conceived to be directly applicable to the universe, the logic which is intimately bound up with it is also applicable. The concept of space at the heart of Euclidean geometry is a metric space, and so the representation of the world also becomes absolutely quantifiable. Russell's approach to the question of time in chapter two exemplifies the possibilities opened up by this metric understanding of the world, seeing it as a discrete, infinite series of unextended points forming an imaginary line. Such a system fulfils the Laplacean dream that given an unlimited intelligence and a knowledge of the complete state of the universe at any given moment, "nothing would be uncertain, and the future, as the past, would be present to its eyes."[17] The mathematico-analytic approach therefore allows the analysis of any closed system in a similar fashion, that is, the quantitative analysis of the external interactions of discrete atoms of matter.

Whilst the analytic approach provides a powerful method of analysis, as we saw in the last chapter, this method also leads to substantial difficulties, as appeared in the logics of both Russell and Aristotle. These difficulties are at the root of the Bergsonian method of intuition, and its Deleuzian derivative. Before moving on to discuss the meaning of these difficulties for Bergson, we will briefly consider the difficulties which emerged for the proponents of the first quantitative laws, the Pythagoreans. The fundamental axiom which grounded these laws, which originated in the fifth century BC,[18] was Philolaus' assertion that "all things which can be known have number; for it is not possible that without number anything can be conceived or known." Although the Pythagoreans combined a belief in the numerical understanding of the world with a form of number mysticism, the relation between the intellect and number, positing number as the limit point at which the understanding becomes acquainted with the world, mirrors more modern conceptions.[19] A revolution in this classical conception of number took place, however, with the discovery of the incommensurable. When Pythagoras' theorem

is applied to the square, we find that the length of the diagonal of the square is √2 times the length of the side. As the square root of two is irrational, the ratio of the length of the side to the diagonal is irresolvable into an integral ratio.[20] The Greek concept of number, built on the idea of the integral numerical progression, was unable to incorporate the idea of a number which could not be reduced to integers or their relations. With the discovery of incommensurable numbers, Pythagoreanism collapsed, the man who disclosed the difficulty being said to have died in a shipwreck as a result.[21] What therefore defeated the Pythagorean model was the discovery of a mismatch between the world and the subject's ability to conceptualise the world. For Greek thought, the relation between logic and the physical world can be seen more clearly than in modern thought due to the restricted definition of number, although similar restrictions on the notion of negative numbers in algebra persisted (at least) until the Renaissance.[22] In a similar way to Pythagoreanism, Spencer's model of the gradual adequation of the mind to the world produced the corollary that the structure of the mind was isomorphic with the structure of the world. It is in respect to this relation that Bergson developed the method of intuition as a method of recognition of the mismatch between our representation of the world and the structure of the world itself. It is this method, and the implications of its results for the notion of space which we will explore in the next section.

Bergson's Method of Intuition

The method which Bergson uses to develop his alternative understanding of the world to that which we have outlined above is the method of intuition. Such a method is not to be confused with the Kantian faculty of the same name, and instead is the name of a two stage process whereby a particular conceptual scheme is at first recognised to be inadequate, and then effectively bracketed in order to return to an understanding of the

phenomenon itself (for Bergson, the phenomenon of duration). As Deleuze points out in his discussion of Bergson, however, "intuition, as [Bergson] understands it methodically, already presupposes duration" (*B*, 13). Thus, whilst intuition may be the name for the method Bergson employs, this method itself presupposes an acquaintance with the object to be disclosed. Whilst this may seem paradoxical on the surface, returning to the exact nature of the problems which we found with the classical theories of Russell and Aristotle will help to clarify how this approach works. When we analysed the theories of Aristotle and Russell, we came across what Deleuze calls points of catastrophe, where the logical scheme of the system as a whole appeared to break down. In particular, we noted how in both cases, the very large (the highest genus, and the set of all sets), and the very small (the individual, motion), showed at best incomplete, and at worst paradoxical, characterisations of the genus and individual. We also saw this in the opening chapter, page 38, where we looked at Sartre's argument that the transcendental ego in fact leads to the fragmentation of consciousness. It is these moments of catastrophe which are the material for the first half of Bergson's method of intuition, as can be seen by returning to Russell on time. What we found problematic about Russell's account of temporality is not the structure of his representation itself, which at least appears to be consistent, but rather the intuition on our part that something is missing from this representation, namely the actual movement of time itself, or, in Bergson's language, its duration. For this reason, whilst Bergson's method of intuition may open out onto a rigorous conception of duration, it requires at its root an intuitive grasp that there is something which is captured by Deleuze's descriptions of catastrophic breaks within the structure of the representation as a whole. In order to give content to this intuition, we need to "make a strenuous effort to put aside some of the artificial schemata we impose unknowingly between reality and ourselves. What is required is that we should break with certain habits of thinking and perceiving which have become natural to us."[23] Deleuze's comment on the relation of intuition to duration therefore arises from the fact that this act itself presupposes in some sense the conclusion of the

act, as it is duration which for Bergson opens up to the subject the possibility that the present schemata do not adequately capture the nature of the world. Thus, it is the 'counter-intuitive' nature of Russell's assertion that the arrow does not move in Russell's interpretation of Zeno which leads to the possibility of a thought outside of the conceptual scheme which generates such an assertion. Once this framework of conceptual reasoning has been put to one side, the second half of the method of intuition, the rigorous reflexive observation of pure duration can take place. Thus Bergson's method is circular, but not viciously so, since the idea of duration presupposed is pre-philosophically intuited, but only as providing the opening, and the motivation, for a more rigorous account. We can now move on to Bergson's own critique of classical forms of representation. In doing so, we will bring into play the two types of multiplicity which are central to Deleuze's account of the actual and virtual.

The opening of Bergson's critique of the classical mechanistic model of space and time comes through his re-evaluation of the model of the ego as an invisible thread which we discussed earlier. Bergson begins with the observation that our thoughts are not related to one another "as beads on a necklace", but instead form "a fluid mass of our whole psychic existence" (*CE*, 3) in a move which parallels the one we saw Sartre making in the first chapter on page 38. With this move, Bergson opposes the model of judgement which was outlined in relation to Kant in that chapter, and instead posits the possibility of a mode of thought which is inherently durational. For Bergson, the origin of the atomistic model of thought is not to be found in an act of introspection, which would instead show a far more fluid nature of thought, but rather a transposition of the physical model of space to consciousness itself. Thus, Bergson echoes the central moves of Spencer, positing a correlation between our representations of the external world and those of our own thoughts. The divergence between Spencer and Bergson comes about, however, through Bergson's belief that the application of a spatial model to consciousness exhibits a structure still founded upon pragmatism. Bergson thus opposes

Spencer's assessment that in the final stage of our evolution, pragmatism collapses into a genuine understanding of the world. As we saw on page 97, the advantage of the analytic method is that it provides a great deal of utility in terms of our ability to understand components of a system separately from the whole within which they participate. What Bergson believes he has shown through the introspective moment is a clear situation in which this utility has been purchased at the cost of a genuine understanding of the phenomena themselves. Although Bergson's philosophy of duration predates the arrival of Einsteinian physics, Einstein's developments reverse the Spencerian schema, with the complexities of our view of space-time now mirroring those of consciousness itself. The principle of correspondence itself overturns Spencer's account, as Einstein brings into play a new theory of space which corresponds to Bergson's theory of mind. The principle which Russell cites, that displacement does not affect the size or shape of bodies, is now rejected in modern physics, as Einstein's general theory of relativity reduces spatial entities to convolutions of space itself. Whilst the axiom of free mobility is violated by the move to Einstein's conception of physics, the fact that the evidence for this violation only becomes apparent at astronomical scales is important, as it is precisely at these scales that the human organism finds itself in a situation where the *a posteriori* development of the race cannot provide adequate *a priori* principles. That is to say, both internally and externally, the model of a homogeneous space central to both the models of Russell and to Kant breaks down. For Bergson, what is at issue is not the nature of Einsteinian space, but instead the understanding of the relation between entities which becomes radically transformed between an ontology which is based on a Euclidean notion of multiplicity (Newton) and one which instead relies on Riemannian multiplicities (Einstein). In terms of Bergson's philosophy, the results of Einstein should be read purely in terms of the possibility that they open up (and indeed Bergson rejected Einstein's interpretation of his world). In rejecting the Newtonian conception of space, Einstein also rejected a Euclidean notion of *multiplicity*. In fact, as we shall see, it is the idea of a multiplicity, which is intimately

tied to the idea of a certain kind of space, which is key for Bergson. Bergson, in arguing that analytic thought does not correspond to the *milieu* is therefore arguing that the process of evolution, which Spencer took to be at an end, is not completed by a purely intellectual form of thought. It is important to note that whilst the development of Riemannian geometry is important for Bergson, this importance is not in the results of these developments, such as Einstein's physics, but rather the new possibilities of thought that are opened up by the new multiplicities. Thus, in talking of mathematics, what will be revolutionary for Bergson is the differential calculus, not because of the advances in mathematics which it provides, but rather the possibility of a new way of thinking of motion which is now opened up outside of the field of mathematics itself. This new possibility for thought will help to overcome the problems of transition we encountered in the work of Aristotle and Russell.

Having looked briefly at the negative aspects of Bergson's theory of evolution, we can now turn to what are the key features of his positive theory from a Deleuzian standpoint. For Deleuze, one of the keys to the project of a renewed Bergsonism is to recognise the two concepts of multiplicities at play in the work of Bergson. The first, that of traditional geometry, we have already considered. This spatial form of multiplicity is to be contrasted with another form of multiplicity grounded in the idea of duration. Bergson's own conception of the state of consciousness saw it as "a moving zone which comprises all that we think or will" of which the "states" of the atomistic view comprised only the "best illuminated points" (*CE*, 3). This distinction has its origin in his conception of organisation. If we take a quantity of inert matter, we can, on the classical model, break it down through a process of division until we reach a point at which no further division is possible. This is equivalent to the methods of logical analysis of Russell or Descartes.[24] Given the fact that we have reached the simplest elements, we can now define change in the state of the original matter through displacement, or the alteration of the external relations between these parts. Through this

method of analysis, however, the state of the system remains, in a sense, outside of time: "A group of elements which has gone through a state can always find its way back to that state, if not by itself, at least by means of an external cause able to restore everything to its place" (*CE*, 8). Such a concept of space also implies a parallel concept of time, one that, as well as being in principle absolutely reversible, is also absolutely without duration. The concept of time is abstract, reduced to another dimension of space, so that we can conceive of time passing at whatever rate we fancy, in that it merely provides another axis along which events happen. In contradistinction to this conception, Bergson turns to systems which have a different kind of organisation. "If I want to mix a glass of sugar and water, I must, willy nilly, wait until the sugar melts" (*CE*, 9). In this case, the time of the situation is not felt in the manner of an additional axis to the normal dimensions of space. Instead, my own reaction to the situation, my impatience, opens out onto a conception of time which holds out to me the experience of duration. The situation endures for me. Such a state does not have the same analytic structure. That it cannot be measured, or broken down in the same way as the prior conception implies that a different form of metric is at work, and with this comes a conception of continuity opposed to the Russellian idea put forward in the last chapter.

This move away from atomism has five main consequences for our understanding of time, which Čapek highlights.[25] First, and as we have already seen from Bergson's interpretation of introspection, processes in time cannot be seen as being made up of clearly defined external elements. Within such models we always find the catastrophes highlighted by Deleuze – the gaps between structures within the system. Second, if we are to reject the atomistic model, we can no longer see a process as complete. For Bergson, the completion of a phenomenon can only make sense within the world of discrete objects and states. If a state is defined by discrete components, then there can also be no novelty, merely rearrangements of pre-existing entities. Given a continuous becoming, novelty is a constant feature. If we consider a musical tone as

reducible to a composite of discrete temporal intervals, then we are just presented with a brute repetition. If we see it as unfolding within time, each 'instant' will be different from the last, in that it will carry this past with it. Third, this notion of time will be heterogeneous. Following Deleuze, if we take the mixture of the sugar and water, and speed it up by stirring it with a spoon, the duration of the event will change, but this is only through making it the duration of a different event. In adding the movement of the spoon, we have changed the duration, but the extra movement itself changes the nature of the event as well. Thus, by changing the duration of the event, we change the event itself. Fourth, and following the third point, if duration changes the event, divisibility of the time of the event is impossible without changing the nature of the event itself. And fifth, as duration is this unfolding of events, the model of time of Newtonian physics must be rejected. Time can no longer be seen as a container of events. Instead, duration is simply the unfolding of events. Hence, the change in the form of the event means a change in the event itself. We therefore have two conceptions of time at play, one for systems which contain merely inert matter, and the other, so far, for consciousness. The idea of duration, in actual fact, is broader than consciousness for Bergson, and has an interpretation within the wider world. We shall now turn to look at the fundamental characteristics of some systems which embody this idea.

The second law of thermodynamics states that "the entropy of an isolated system not at equilibrium will tend to increase over time, approaching a maximum value." It is this law which is responsible for the idea of heat death, the hypothesis that the end of the universe will come about when all disequilibrium states of energy have been cancelled, leading to the universe becoming an unstructured field of constant temperature. The law can be illustrated by the example of a room containing two gasses, for example, nitrogen and oxygen, each separated from the other by a central barrier.[26] We can see such a system as presenting a high level of order, as each segment of the room contains just one kind of molecule. When the barrier between the two sections is

removed, the free movement of molecules from one section to the other leads to a gradual mixing of the elements. Eventually, the system will reach a point of equilibrium, where the mixture of the molecules is relatively complete, meaning that the gas in the room has become homogenous. We can further represent this process as an increase in the symmetry of the system. The initial state of the system, differentiated into two defined areas, has a low level of symmetry, whereas, when the gas becomes mixed and reaches its equilibrium point, the amount of symmetry within the system has increased. Entropy highlights the configurational approach of classical physics, as here the notion of disorder is given purely in terms of the relative positions of molecules, and their external relations. If we look at living systems, however, we can see that the reverse trend is apparent. Rather than an increase in the degree of symmetry of the system, we instead have a process of differentiation, leading from the simple egg to the manifoldly complex organism. The second law of thermodynamics does not contradict the development of order within systems on their way to disorder. In fact, it is possible that structures may exhibit ordered properties when there is a differential between two levels of energy, or if order is added from outside of the system.[27] Thus, we can imagine a process whereby some external force separates the two gasses back into their initial states (much as heating a mixture of water and alcohol at the correct temperature will separate the two liquids through distillation). This is indeed Bergson's view of life:

> Let us imagine a vessel full of steam at a high pressure, and here and there in its sides a crack through which the steam is escaping in a jet. The steam thrown into the air is nearly all condensed, and this fall represents a loss of something, an interruption, a deficit. But a small part of the jet of steam subsists, uncondensed, for some seconds; it is making an effort to raise drops which are falling; it succeeds at most in retarding their fall. (*CE*, 247)

The example, as it stands, however, does not explain what is special about life. This is because the level of complexity of this system does not make necessary the transformation to the second notion of multiplicity present in Bergson's work. Such a system can be explained purely by the interaction of the elements which compose it. Such a system is "determined necessarily" (*CE*, 247) – it does not exhibit the kind of novel creation of new forms which was one of the main characteristics of our second kind of multiplicity. Returning to a more complicated system, the egg, we find, following Deleuze, "that the division of an egg into parts is secondary in relation to more significant morphogenetic movements: the augmentation of free surfaces, stretching of cellular layers, invagination by folding, regional displacement of groups" (*DR*, 214). In this case, what is required is not a metric understanding of the space, but a topological understanding.[28] What underlies the development of the embryo is not the arrangement of different molecules, but rather the folding of planes. Whilst this can always be understood as the interaction of elements within a three dimensional Euclidean space, what such an approach cannot grasp is the fact that the motions which occur in the development of the embryo are not reliant on precise metric relations between discrete elements, but instead on relations of stretching and folding of a surface. Whilst the mathematics of topology, which uses continuous rather than discrete functions, has been immensely successful in our understanding of the development of living systems, what is important is not the actual models themselves, but rather the fact that thinking in terms of Riemannian multiplicities, which underlie the topological approach, captures the sense of the transformations of the embryo in a way not possible through a metric analysis. What is of key importance in understanding these systems is therefore their process of generation, as it shows itself through these series of transformations. The embryo cannot be understood through the parts, but rather must be seen through the process by which it has developed. "Take a division into 24 cellular elements endowed with similar characteristics: nothing yet tells us the dynamic process by which it was obtained – 2 x 12, (2 x 2) + (2 x 10), or (2 x 4) + (2 x 8)…" (*DR*, 216).

In this case, therefore, the model of the mechanistic multiplicity has broken down. Other examples also show the advantages of understanding these phenomena in terms of duration. If we look at the leaves on a plant, whilst all higher plants may distribute their leaves in three basic patterns, this cannot be explained by a pre-established plan present within the genetic material of the plant. Instead we have a complex interaction of dynamic processes between the cells within the tip of the branches, and the surface of the tip itself. It is the dynamic interaction which generates the budding pattern of the leaves. Similar processes occur with the growth of limbs.[29] In fact, much modern work in embryology is arguing that many of the important processes in embryo development are the result of processes relying on differential speeds of development.[30] The duration of the events is thus integral to their development.

If we look back at the five characteristics given before for the new concept of multiplicity, we can see that the topological multiplicity which defines the development of the embryo possesses each of these characteristics. It is non-atomistic, process based, heterogeneous, continuous, and also as the development of the organism takes place at a rate given by the differential relations of the various processes which form it, it no longer treats time as external. These examples do not eliminate the possibility of understanding these processes in terms of purely metric conceptions of space, but, for Bergson, to understand the phenomena in this way would be to impose on the phenomena a logic which is not the logic of the phenomena themselves, leading to results such as those we tried to show in the previous chapter. At this stage, it is important to note that whilst Hegel also has a logic of relations, in setting out a logic of contradiction, he is also purposefully attempting to overcome the limitations of the static conception put forward in the previous chapter, and so the question of the relation of Bergson's criticisms of classical logic to Hegel must remain open. Given that we now have two different concepts of multiplicity, we need to look at how these concepts are to be related by Bergson, as the relation between the two will help us to illuminate the

relation between the two fields in Deleuze's philosophy, the virtual and actual, as well as to deal with some potential objections to the idea of what is potentially a dualistic ontology.

Bergson and the Two Kinds of Multiplicity

We have two different multiplicities at work in Bergson. These present the two foundational principles of what Deleuze calls "one of the least appreciated aspects of Bergson's thought – the constitution of multiplicities" (*B*,117). What needs to be explained, however, is how these two elements interact with one another. Bergson claims that with the concept of life, we have "a reality which is making itself in a reality which is unmaking itself" (*CE*, 248). We must be careful not to understand this in dualistic terms, even if Bergson may encourage this thinking with his talk of the *élan vital* as the driving principle of life. Clearly, if we have two different kinds of ontological principle at work, we could end up with something analogous to Descartes' dualism. The Cartesian problem of the relation between extended and unextended substance would be mirrored in Bergson by the relation of the metric to the non-metric. In fact, these two principles turn out to be intertwined. We should note first that for Bergson the mechanistic mode of understanding has some foundation in the world, as can be shown by the Spencerian arguments highlighted above. Likewise, returning to Bergson's example taken from consciousness, we can see some intimation of the connection itself. Bergson's claim that the atomistic conception of consciousness focuses only on the "best illuminated point of a moving zone which comprises all that we feel or think or will" (*CE*, 3) supplies us with the idea that the mechanistic model is in fact something like the limit case of the durational multiplicity. In fact, the solution to the difficulty comes from a reflection on the origin of the Cartesian problem. If mental phenomena are seen as unextended, then "how should they find their way back to space,

choose a locality within it, and coordinate themselves so as to build up an experience that is common to all men?"[31] Instead, we must regard "all sensations as primarily extensive."[32] As we saw on page 95, Bergson criticises Kant for not recognising that there could be varying degrees of spatiality. Likewise, in Bergson's discussion of consciousness, we saw that although the clearest mental states appeared to be discrete from one another, most states instead interpenetrated one another. The degree of extensity of a sensation or event is, for Bergson, the degree to which these events approach the ideal point of being totally spatial. For Bergson, therefore, mental life exhibits extensity just as much as physical objects do. Whilst this may appear to break down the divide between our two multiplicities, as the first was defined in terms of space, and the second came to light through introspection which has now also turned out to be extensive, this similarity between the two multiplicities only becomes an identity if we first equate spatiality with extensity, and second take extensity to be an absolute characteristic, either present, or absent. For Bergson, extensity is not the same as spatiality, and it is only this equation of the one with the other which leads to the difficulty of relating the two multiplicities.

Whilst the level of extensity may vary between sensations, or events within the world, it does not have the characteristics associated with the purely spatial. Therefore, extensity itself cannot be divided in the same way that the spatial can be. Instead, extensity turns out to be inversely proportional to the durational quality of an event. As thoughts become sharper, they also begin to lose their continuity with the past. They approach the point of becoming discontinuous, and by implication, the point where their explanation can be given purely in terms of the preceding event (just as the entire future of the universe can be explained from the position and velocity of all bodies at a single time). We might consider the limit of this sequence to be a point of maximal extension where matter becomes entirely spatial, and duration entirely falls away, and it is true for Bergson that we do imagine this possibility. "Matter, at first, aided the mind to run down

its own incline [towards extension]; it gave the impulsion. But, the impulsion once received, mind continues its course" (*CE*, 202). Thus it is mind that takes the extension of matter to the limit. In this, the mind creates a schema of the idealised notion of space which allows the kind of logical analysis used by Kant and Russell to be permitted. Pure spatiality is, however, just a tendency of matter, which is different in kind from the tendency towards pure duration, but expressed through differences in degree in matter itself. Thus Kant, in the transcendental aesthetic, is right to call space an *a priori* intuition on Bergson's reading, but wrong to take this intuition as fully characterising the way in which objects can be presented to the subject. In allowing the complete separation of all entities from all others, we open the way to the analytic tools which are the foundation of science. This comes at the price of developing a scientific theory which will always be marginally out of step with reality itself. This fact was best highlighted by the collapse of the Newtonian system with its Euclidean foundations.

Underlying this property of extension of varying in degree is a theory of vibrations which allows us to explain why exactly extension never reaches the limit point of pure spatiality, and how this variation in extensity is possible. Once again, Bergson's theory begins with the notion of psychological duration, although, as we have seen, he wishes the results to be generalised to the world itself. For Deleuze, this project will instead be carried out through the critique of the classical representationalist paradigm which we saw him developing in the previous chapter. As Bergson's deduction provides some insights into the relations of the two multiplicities, we will look briefly at its structure. On Bergson's psychological deduction, we begin by seeking "that which we have that is at the same time the most removed from externality and the least penetrated with intellectuality" (*CE*, 199). This is the moment where we are furthest removed from the idea of the mathematical instant, therefore where we are closest to a communion with pure duration. To think outside of the pragmatic paradigm of the intellect requires great effort, and this is the case here, as the past must be

gathered up into the present as an undivided whole. In doing so, the past, therefore, forms an interpenetrating mass. Bergson describes this moment as a moment when we are "truly free" (*CE*, 200), as it is at this moment that we are furthest removed from the mechanistic model which sees action as the pure reconfiguration of pre-existing elements. Thus, the past, instead of being seen as a series of discrete events, is seen as a continuum, incorporating these different moments into one whole. As the past is contracted into the present as an interpenetrating multiplicity, action becomes creative, since we are no longer dealing with the (reversible) configuration of discrete elements. Such an effort is essentially a contraction of duration. When we relax the effort to contract the past into the present, we find that gradually the past coalesces into "a thousand recollections made external to one another" (*CE*, 201). The present instead of being the continuation of the past becomes a series of instants which emerge only to immediately disappear once again. Thus as interpenetration lessens, external relations begin to become possible. The limit at which genuine externality would be reached, however, is an illusion. Extensity is never equivalent to matter. Instead, at bottom, we have "elementary vibrations, the shortest of which are of very slight duration, almost vanishing, but not nothing" (*CE*, 201). Bergson is thus proposing a vibratory model of extension. In doing so, Bergson is trying to overturn one of the key dogmas of the mechanistic view, that the interval of time over which a body is observed does not change its material characteristics, or, as Whitehead writes, "the lapse of time is an accident, rather than of the essence of the material."[33] Instead, if we view matter in terms of a waveform, or vibration, it becomes clear that extensity, as a movement, carries temporality with it as a fundamental property. Extensity, under a pulsational theory of extension, requires a time for matter to 'pulse'. It should be noted that Bergson here is not talking of the vibration of an atom, but rather a vibratory notion of duration itself. Thus, the act of contraction leads to the drawing together of a larger part of these interpenetrating vibrations, leading to greater novelty. If we instead focus on the individual vibrations, we discover the simple repetition much like that found in a solid

musical tone. Just like a musical tone, however, the durational character cannot be completely eliminated without eliminating the event itself. For similar reasons, the idea of pure duration, free from all extension would seem to be impossible. What we have instead is a mixture. Bergson's theory thus represents both a dualism and a monism. It is dualistic in the sense that it generates two principles which are different in kind – the tendency toward pure matter and the tendency toward pure duration – but it is monistic to the extent that in actual duration, there is only a difference in degree between the maximally extensive and the maximally durational. At this point, we can also therefore qualify the standard characterisation of Bergson as a vitalist. Insofar as Bergson opposes life to matter, Bergson is indeed a vitalist. If we look at what is opposed by these two terms, however, we find that what Bergson has given us is an ontology of extensive duration where the living organism is merely different in degree from the inorganic. In this sense, Deleuze's assertion that "the entire world is an egg" (*DR*, 216) is anticipated by Bergson.

Before moving on to look at Deleuze, we need to look at one more feature of Bergson's analysis: the nature of order and disorder, and its relation to the concept of negation. As we saw in the first chapter, on page 47, Deleuze argues that traditionally, the idea of a logic of phenomena has been opposed to the notion of the abyss. Schopenhauer provides perhaps the best example of this idea, the world as will being characterised through the absence of the categories which govern phenomenal experience. As we also saw in the first chapter, Deleuze is opposed to this interpretation, but through the analysis of Bergson's two concepts of multiplicity, we now have some idea as to what will replace the purely negative account of Deleuze given before. For Bergson, on Deleuze's reading, the conception of order which philosophy normally deals with is in fact a problem "whose very terms contain a confusion of the 'more' and the 'less'" (*B*, 17). That is, whilst we normally take the idea of order to contain more than the concept of disorder, the inverse is in fact the case. Whilst order may appear to

be an objective category, reflecting reality, Bergson will argue instead that "reality is ordered exactly to the degree in which it satisfies our thought" (*CE*, 223). Bergson gives the example of opening a book on my shelf at random, glancing at it, and uttering, "this is not prose", a statement which does not refer to the present absence of prose on the page, but rather a failure of the book to meet the expectations which we have of it. In a similar way we may have picked up a book of prose, and said, "this is not poetry", having had a similar disappointment. In this example, it is clear that there are two varieties of structure at play, and that it would be foolish to assert that underlying the two structures of prose and poetry there was a language, itself without structure, to which one or other of the structures could be added. To argue in this way would be to construct a false problem. The same holds true for the idea of order itself, which is posited as organising a previously neutral field. In the idea of this field, we once again find the notion of a homogenous space which forms the ground for the structure of the system. We also find the Aristotelian idea of negation as exclusion, in that disorder is defined as the absence of being. Instead, what Bergson wishes to put forward is the idea of a difference of orders. This idea, which departs from the rigid exclusivity of Bergson's example, comes about through the belief that both kinds of order coexist as tendencies within the same system.

Conclusion

We have now looked at some of the most important features of Bergson's metaphysics, at least as far as Deleuze is concerned. These are the positing of two kinds of multiplicities which are to be understood as tendencies rather than states, and the critique of the spatial, atomistic conception of the world. In the next chapter we will see how these ideas are taken up by Deleuze in his own transcendental empiricism, in order to give a more rigorous conception of the structure of these two multiplicities. We will

also finally see how Deleuze aims to provide an account of the determinate structure of the world without relying on the categories of representation.

Chapter Four – The Virtual and the Actual

Introduction

In an afterword written in 1988 to his study, *Bergsonism*, Deleuze calls attention to the three themes of Bergson's work which he believes must be pursued if we are to continue to practice metaphysics. These are intuition, the relation of science and metaphysics, and the theory of multiplicities. These three themes will be the central focus of this chapter on the structure of Deleuze's ontology, although we will also briefly look at some areas of Deleuze's thought on aesthetics. Taking the themes in reverse order, it is the theme of the two multiplicities in Bergson's thought which is most fundamental to Deleuze's project of renewing metaphysics. In the last chapter, we looked at the differences Bergson posits between space on the one hand and duration on the other. In making this distinction, Deleuze claims that "Bergson moves toward a distinction between two major types of multiplicities, the one discrete or discontinuous, the other continuous, the one spatial and the other temporal, the one actual, the other virtual" (*B*, 117). The first of these multiplicities is that of Aristotle and Russell, and operates with a logic of identity, negation and opposition, the second, which corresponds to duration in Bergson's system, instead uses the principles of difference. In exploring the interrelations of these two multiplicities, Deleuze highlights Bergson's "constitution of a logic of multiplicities" (*B*, 117). Extending and clarifying this logic will be fundamental to Deleuze's own project. The second aspect of Bergson's thought taken up by Deleuze is the constitution of a new relation between science and metaphysics. "Bergson did not merely criticise science as if it went no further than space, the solid, the immobile. Rather, he thought the Absolute has two 'halves,' to which science and metaphysics correspond" (*B*, 116). To properly take up the insights of Bergson therefore means, for Deleuze, not to consider science simply to be a reductionist mode of understanding the world, but to recognise that without a metaphysics it remains abstract

and without meaning. Thus, modern metaphysics must make an understanding of modern science's achievements possible, as well as building on these advances. Returning to the question of the two multiplicities, Deleuze's project will involve fulfilling Bergson's intention to "[give] multiplicities the metaphysics which their scientific treatment demands" (*B*, 117). Given that contemporary science, particularly the fields of chaos and complexity theory, has taken up the idea of non-classical geometrical space as a vital tool in understanding the behaviour of systems, Deleuze will frequently refer to the scientific (and in particular, mathematical) treatment of continuous multiplicities in order to provide an explanation of them which gives a level of rigour unavailable at Bergson's time of writing. We should be careful to recognise, however, that Deleuze is not himself engaged in a scientific enterprise, and whilst mathematics provides perhaps the easiest way to understand his project, it simply provides the paradigm case for understanding terms such as actualisation, the relations of problems and solutions, and the structures of the virtual and the actual. This distinction is especially clear in his discussion of differential calculus, which will become a central tool in developing his theory of difference: "Differential calculus in the most precise sense is only a mathematical instrument which, even in its own domain, does not necessarily represent the most complete form of expression of problems and the constitution of their solution" (*DR*, 181). There is a wider sense to the differential calculus, however, as a calculus of difference, which will apply to all domains. "Each engendered domain, in which dialectical Ideas of this or that order are incarnated, possesses its own calculus" (*DR*, 181).[1] The analysis of the differential calculus, and its application to non-standard geometries should therefore be seen as the application in one domain of a calculus of difference which in the end will go beyond purely mathematical results ("*mathesis universalis* but also universal physics, universal psychology and universal sociology" [*DR*, 190]). Deleuze's philosophy thus involves neither a reduction of science to metaphysics, nor a reduction of metaphysics to science.

The third aspect which Deleuze will take from Bergson is the method of intuition. Deleuze emphasises the two functions of intuition as "a cutting up or division of reality in a given domain, according to lines of different natures and, on the other hand, an intersection of lines which are taken from various domains and converge" (*B*, 117). As we saw on page 105, for Bergson, the two multiplicities represent two tendencies of a system. In fact, Deleuze writes, "things, products, results are always *composite* ... and one must understand that a composite is undoubtedly a blending of tendencies that differ in nature, but as such is a state of things in which it is impossible to make out any differences in nature" (*BCD*, 35). This means that the logic of multiplicities must be able to deal with that which exists between the two tendencies, or in Deleuze's terms, that which is the result of the differenciation of difference.

The Two Multiplicities

We have already looked at one of the multiplicities which Deleuze is concerned with in his logic in the form of the structure provided by set theory. As we saw, this structure relied on the category of negation and worked by analogy with a certain conception of geometrical space. The second form of multiplicity which Deleuze considers also has its origins in a geometrical conception of space, albeit one which emerged through the rejection of standard, Euclidean geometry. The term multiplicity in fact originates in the work of the mathematician, Riemann, and Deleuze goes as far as claiming that Bergson must have been aware "of the scientific origin of the term and its novel metaphysical use" (*B*, 117).

Great advances were made in geometry in the nineteenth and early twentieth centuries. Kline writes that "the importance of non-Euclidean geometry in the general history of thought cannot be exaggerated. Like Copernicus' heliocentric theory,

Newton's law of gravitation, and Darwin's theory of evolution, non-Euclidean geometry has radically affected science, philosophy, and religion. It is fair to say that no more cataclysmic event has ever taken place in the history of all thought."[2] For Deleuze, non-Euclidean geometry opens up the possibility of new modes of thought outside of mathematics itself. Euclid's geometry provides the mathematical model of our intuitions about the structure of space, and, in Euclid's *Elements*, rests on five axioms. Whilst the first four were relatively uncontroversial, the fifth presented more difficulties. It can be stated as follows: "through a point not on a given straight line, one and only one line can be drawn that never meets the given line."[3] Beginning with the Greeks, this axiom was seen as problematic, as it relies on a concept of infinity (as we have to show that the two lines never meet), which falls outside of our spatial intuition. Furthermore, despite repeated efforts, this axiom could not be derived from the other axioms. An alternative approach to proving the correctness of the fifth postulate and therefore of Euclidean geometry was developed independently by the mathematicians Lobatchevsky and Bolyai[4] in the nineteenth century. Their approach was instead to try to show that if one rejected Euclid's fifth postulate, the resulting geometry would contain inconsistencies. The rejection of the fifth postulate did not lead to inconsistency, however, and instead led to the development of a new form of geometry. Rather than focussing on their work, we will instead attempt to see, informally, how the mathematician, Riemann, developed his own geometry by rejecting some of the fundamental axioms of Euclidean geometry.

For Riemann, the opening for his own enquiries into non-Euclidean geometry was not the fifth postulate itself, but rather another assumption of Euclid's. In order to formulate the postulate that two parallel lines never meet, we require the assumption that it is possible to extend a straight line infinitely. Instead of this assumption, Riemann put forward a distinction between the endless and the infinite. Thus, for instance, the circumference of a circle can be seen to be endless, as unbounded, but not infinite in length. He applied this axiom to geometry by postulating, *contra* Euclid, that all straight

lines are finite, yet endless. In doing so, Riemann's geometry departs from our intuitive conception of space, leading to an important difference between physical and mathematical geometry. To see how this affects the nature of space which Riemann develops, we shall return to the question of the parallel line in Euclid's geometry. As we have just seen, Euclid's axiom can be stated as "through a point not on a given straight line, one and only one line can be drawn that never meets the given line." We can explore some of the implications of this through the following diagram:

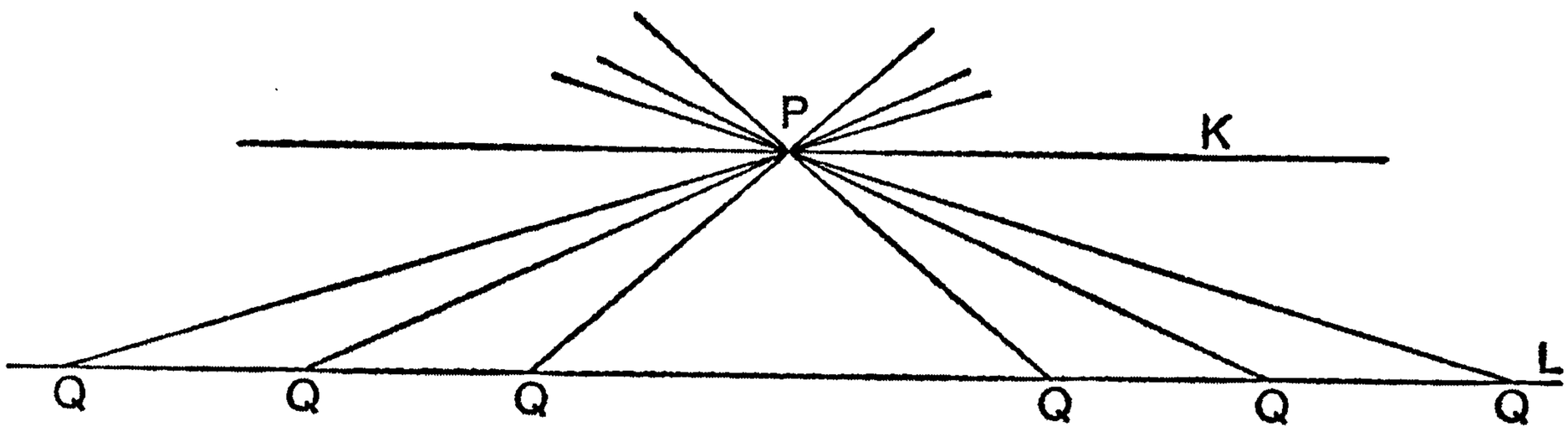


Figure 1. Euclid's parallel as a unique limiting line.

Here we have two parallel lines, K and L, of which line K passes through point P. Euclid's axiom states that K is the only line which can pass through P without meeting L. Furthermore, we can see that it is the limit case for the lines passing through P and Q. As the point Q moves further from the point P along L, it rotates to become closer and closer to K. At the limit case of a point infinitely far from P, therefore, the line passing through P and Q will equal K. We can also see that it is irrelevant in which direction along the line Q is moved. Both lead to the same limit. We can therefore say that parallel lines are lines which meet at infinity. How is this affected by Riemann's alternative geometry based on the idea of the endless? Another diagram will be helpful here:

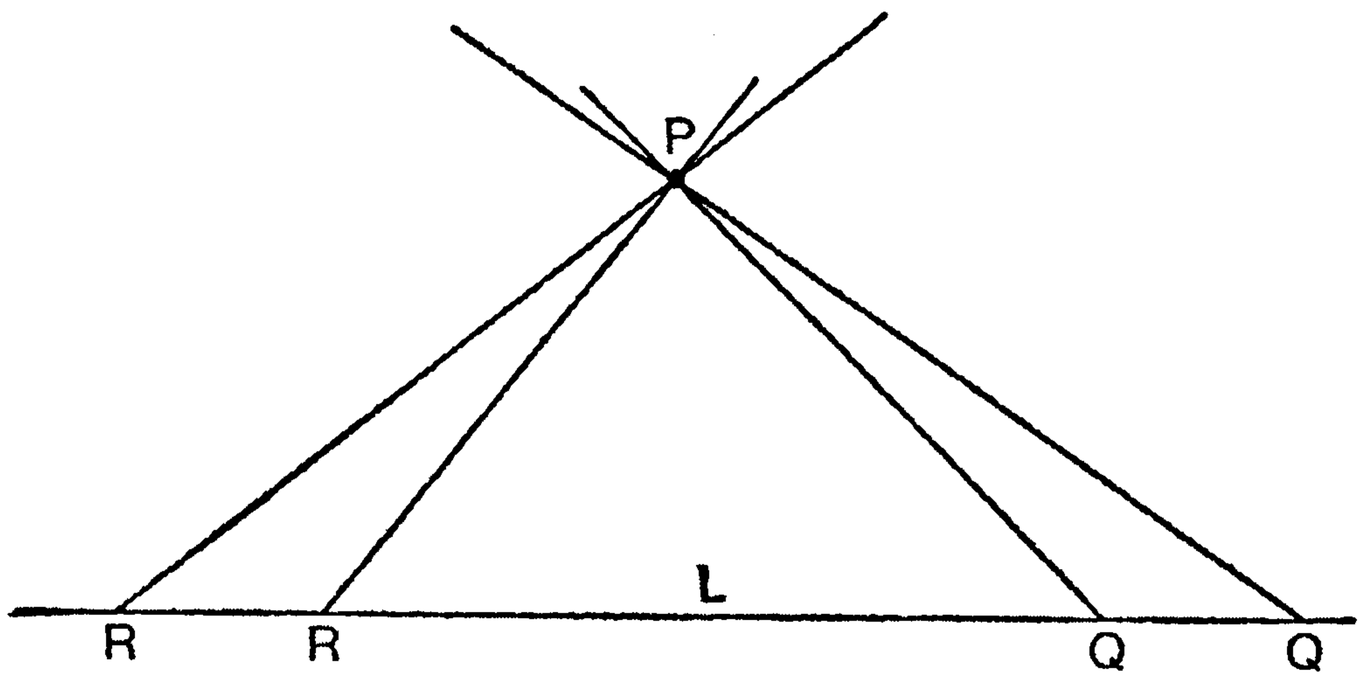


Figure 2. Geometrical basis for Riemann's parallel axiom.

This diagram represents the Riemannian equivalent to the one presented for Euclid. The line L cannot here be extended to infinity, however, as L represents a finite, though endless line. This means therefore, *ex hypothesi*, that Q cannot be moved infinitely far from P along L. Instead, as L is of finite length, but unbounded, when Q is moved far enough to the right, it will instead coincide with R. If the parallel line is the limit case of the infinite distance of Q from P, therefore, it means that in Riemannian geometry, there are no parallel lines. Riemann also chose a third axiom at odds with Euclid's approach. For Euclid, two points uniquely specify one straight line, which is the shortest distance between those points. Riemann instead assumed for his geometry that two points may determine more than one line. With this further axiom, a number of theorems can be produced, such that all perpendicular lines to a straight line meet in one point:

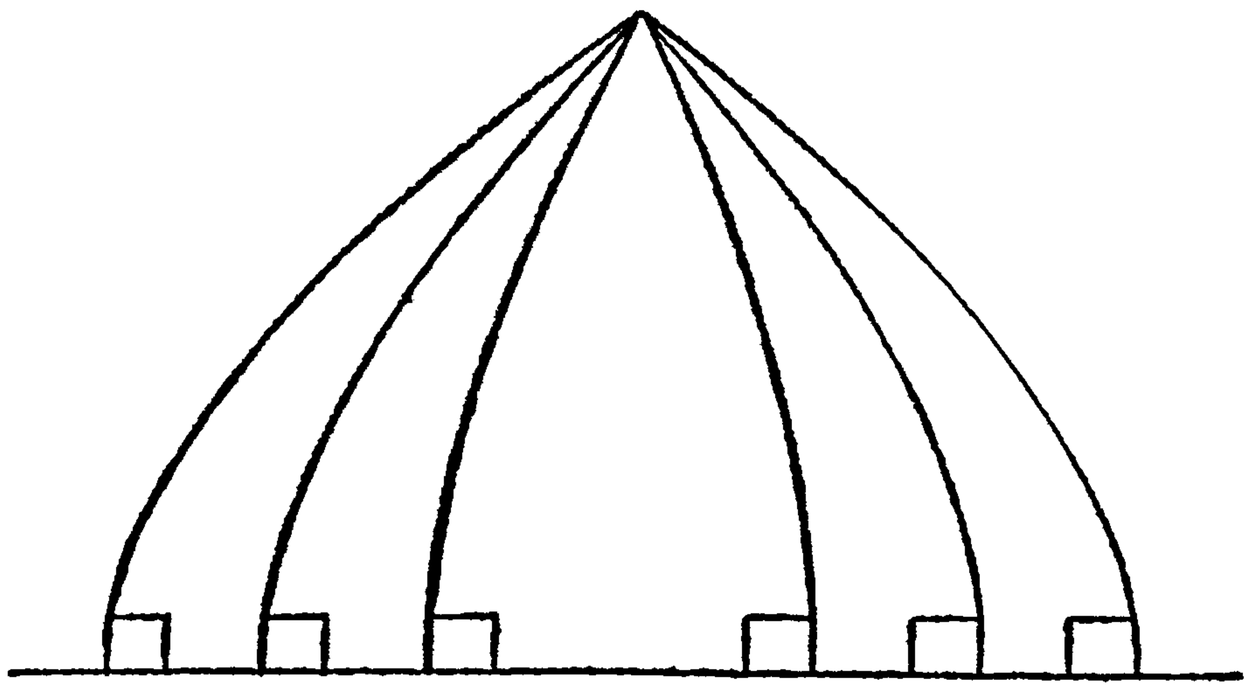


Figure 3. All perpendiculars to a straight line meet in a point.

Moreover, in this Riemannian geometry, two straight lines enclose an area, and, as can be seen from the diagram above, the sum of angles in any triangle is now more than that of two right angles. Riemann's geometry clearly differs in many ways from Euclid's but as should have already become clear, it is capable of an interpretation in Euclidean terms. Namely, it gives us a geometrical interpretation of the surface of a sphere. If we give Riemannian geometry this interpretation, the meaning of the various axioms now becomes clear. A straight line is the shortest distance between two points, and the shortest distance between two points on the surface of a sphere is the circumference, or great circle, which passes through the two points. Therefore, the notion of a straight line is to be interpreted in terms of a great circle. Just as all great circles intersect, all straight lines intersect in Riemannian geometry, meaning that there are no parallel lines. Bearing this in mind, the following diagram should make it clear how, on this interpretation the sum of angles in a triangle can be more than two right angles, and how the perpendicular lines meet at a point:

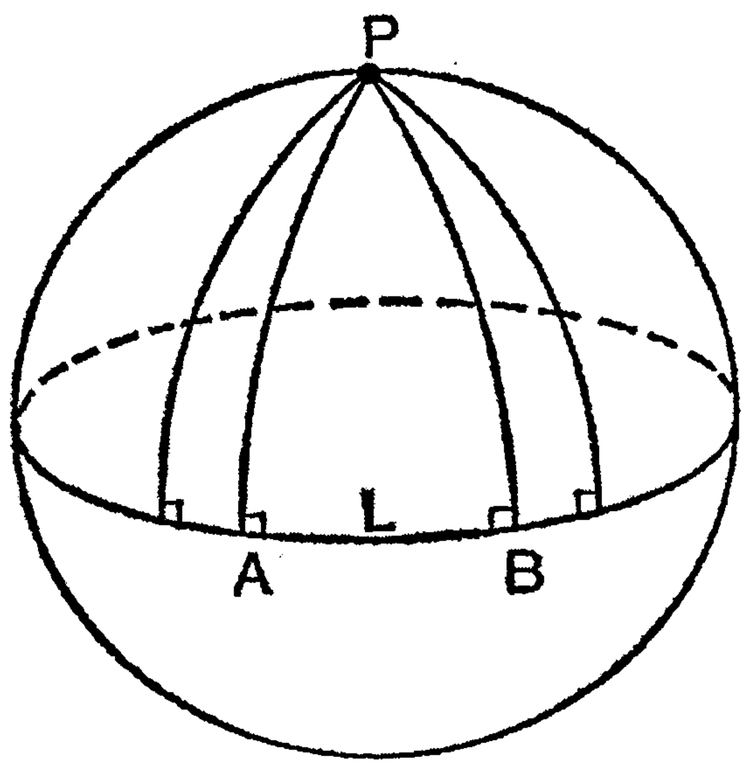


Figure 4. Pictorial representation of Riemann's geometry.

Several important results follow from this. As the geometry of a two dimensional Riemannian plane can be described in terms of three dimensional Euclidean space, Riemannian geometry can be shown to be consistent, or at least as consistent as

Euclidean geometry. Moreover, an important distinction is now made between mathematical space and real space, as different forms of geometry are now possible. Perhaps most importantly, Riemannian geometry allows one to describe the topology of the plane in its own terms. That is, the two dimensional plane of the Riemannian space contains within itself a description of its own curvature. This curvature can indeed be represented in a Euclidean space, but only by adding an extra dimension, through which we can add information about the dips and rises in the surface. In this case, this extra dimension is apparent in the fact that the two dimensional plane can be interpreted as the surface of a three dimensional sphere. From here, Riemann, building on the work of Gauss, developed three further novel features to his geometry which are relevant to our enquiry. As we saw, Riemannian geometry was able to characterise certain features of the surface intrinsically, such as its curvature, whereas Euclidean geometry needed to understand the surface in terms of another dimension, thereby treating the plane as the surface of a body in three dimensional space. In the geometry which we have looked at so far, the space represented is one of constant curvature, the surface of a sphere. Such a geometry is for obvious reasons, called spherical geometry. In fact, we can generalise from the spherical geometry outlined above to cases in what is known as differential geometry where the space instead has a variable curvature. In order to achieve this, Gauss leveraged the power of the differential calculus, which allows one to consider not only the curvature of a line, but also the rate of change of that curvature. As we can use the differential calculus to describe any curve, and not just the constant curvature of a sphere, so, when this is incorporated into the geometry, we are able to describe a complex surface through geometry, without having to add an extra, extrinsic, dimension. Such a surface would not have the fixed curvature of the surface of a sphere, but would instead contain peaks and dips more akin to the topology one would find on a map. Just as in the case of spherical geometry, however, this space can be defined *intrinsically*, not requiring an extra dimension to capture these changes in curvature. In this sense, it is as if the contours on a map were directly present in the paper without the need for them to

either be drawn on, or for the map to extend into a third dimension. Second, as we are dealing with mathematical geometry, there is no reason to limit the dimensionality of the geometry to three dimensions. Instead, we can use Riemannian geometry to describe *n*-dimensional spaces, where *n* is any integer. Third, and perhaps most significantly, Riemann showed that if we move to a geometry involving a space without constant curvature, absolute metrics no longer hold. To explain this point, we can consider the case of an ordinary triangle in Euclidean space. It should be intuitively clear that, as we discussed in chapter three, the triangle will retain the same shape regardless of where it is situated within the space. The triangle is composed of three straight lines. As the shortest distance between two points defines the straight line, and Euclidean geometry is essentially 'flat', these lines will remain the same regardless of their positions on the plane. The same is true on the surface of the sphere, where the shortest distance will always follow the path of a great circle. In both of these cases, the space has a constant curvature – in the case of the Euclidean space, this curvature is 0. If the curvature is not constant, however, then the shortest distance between two points, which Gauss called the geodesic, will be different at different points in the space. Following from this, the nature of the straight line will also vary at different points. If we add a triangle to such as space, therefore, the size and shape of the triangle will vary depending on its position within the space.

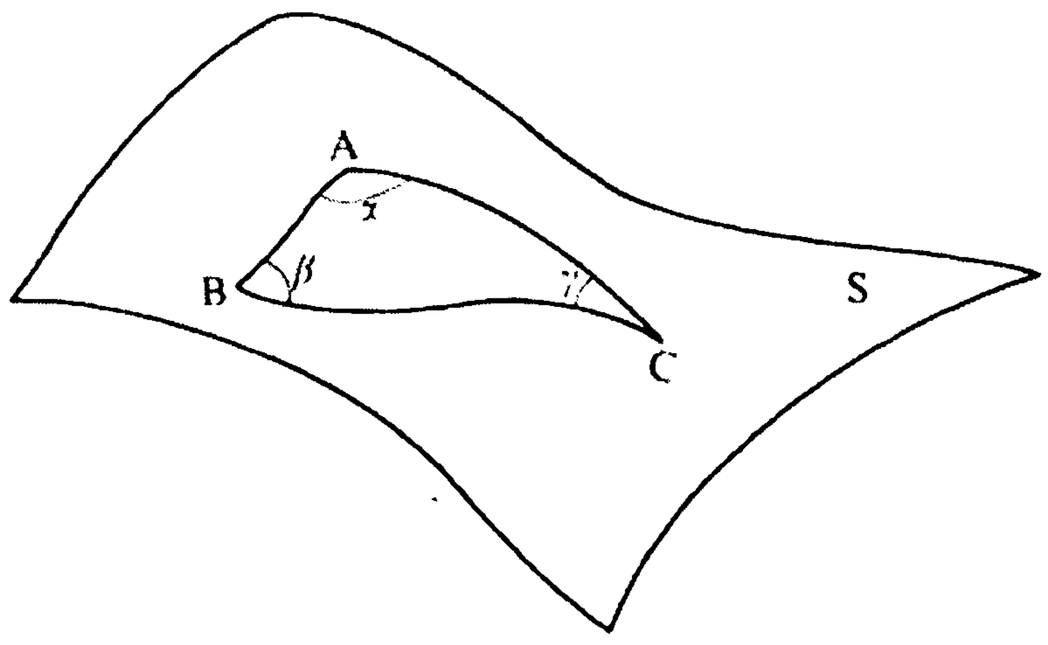


Figure 5. A triangle ABC composed of geodesics on the surface S.

It is this feature which leads to Einstein's renunciation of the notion of absolute length in favour of distance being a function of the structure of space-time.[5] This feature also makes non-Euclidean geometry ideal for analysing the kind of non-metric transformations which are best described topologically, such as those of the embryo discussed in the last chapter. What was important in the examples discussed were not the metric properties of the systems, but rather the way in which forms could be created by operations such as folding. It is a short step from Riemannian geometry to the field of topology. When we looked at Euclidean space, we noted that it provided an extrinsic way of defining bodies, such as the surface of a sphere. The extrinsic structure of Euclidean space is known as a metric space, in the sense that the relation between any two points is defined in terms of the distance between them. As we have just seen, for a non-Euclidean geometry, this distance does not have to be constant, and so the relations between points cannot be defined in terms of distance. If we do not define the relations between points in terms of distance, however, it becomes the case that the space can be stretched, changing the *distance* between the points, but no longer changing their relations.[6] This idea is captured by the idea that the triangle will appear 'distorted' if it is moved in a space without constant curvature. As such spaces are not defined by metrically, they are known as non-metric, or continuous, spaces. This leads us to topology is the branch of mathematics concerned with the relations of figures apart from their specific metric relations (what Gleick calls "geometry on rubber sheets"[7]).

The implications of this result paved the way for Einstein's characterisation of space itself as curved, rather than the curvature of trajectories within space (a motion for Einstein effectively takes place in a 'straight' line through a space curved by the effects of mass, rather than in a 'curved' trajectory through a homogenous space as the moving body under the influence of gravity as it is for Newton). It also paved the way for Einstein by breaking the link between Euclidean geometry and actual space. Before non-Euclidean geometry, it had been inconceivable that other geometries with different

axioms could be specified. Whilst Riemannian geometry thus at first appeared to be the most abstract result of mathematics, relying for its validity on counter-intuitive axioms, it led to very concrete revisions of the foundations of the sciences. Just as the surface of a sphere looks like a flat plane over short distances, so Riemannian geometry looks like Euclidean geometry over short distances. This similarity has parallels with the fact that the breakdown of Newtonian physics is only apparent at extreme velocities. The scope of geometry is wider than providing a model for physical space. Fluctuations in share prices, for instance, can be plotted on a graph, which is in effect a two dimensional Euclidean plane. Such a graph of price to time does not claim to model actual space, but rather the abstract space of the market. In a similar way, Riemannian geometry can be used to provide a model, without any reference being made to the nature of physical space. What is most important for Deleuze is not the application of this geometry to physical space, as in Einstein, but rather the use that is made of it to model the behaviour of systems in dynamic systems theory. In the context of dynamic systems theory, this second notion of space is not used to replace the geometry of the world, but instead to provide a model for the dynamics of the system which are then actualised in the system itself. With some qualifications, we will see that this move to a dynamic characterisation of the system will provide the basis for a Deleuzian notion of virtuality which will fulfil something like the role of essence in classical systems:

> Thus, from antiquity, just as physics related movement to privileged positions and moments, metaphysics constituted eternal forms from which these positions derive. But 'modern' science begins, on the contrary, when movement is related to 'any instant whatever': it demands a new metaphysics which takes into account immanent and constantly varying durations (*B*, 117).

The aim of systems theory is to provide a model of the dynamics of a system, or in other words, the different states of a particular system, as well as the relation between these states, that is, the way in which the system moves between these states. The systems addressed by this type of theory are very broad, as we are primarily interested in their dynamics, and includes simple physical systems such as pendulums, as well as more complex systems, such as the relations between the number of predators and prey in a particular environment. We begin to model the system by choosing the variables which will describe the state of the system. In the case of a pendulum, every state of the system can be described by two variables, namely, the angular velocity and the angle of the pendulum itself.[8] For a predator-prey system, the variables we choose would be the number of predators, and the number of prey. In each case, these variables represent the degrees of freedom of the system. If we take each of the previous examples, we can therefore say that any state of the system (for our purposes) can be represented by two values. These two values together can be used to describe points on a two dimensional plane, with each axis representing one of these two variables. This plane, known as a phase space, therefore contains all possible states of the system, in other words, each possible combination of states of the different variables. The plane does not directly represent a physical space, as is clearly the case in the predator-prey system. In these cases, we want to use the system to determine its long term behaviour. In order to do this, we need to keep in mind two things. First, we are not interested in any particular state of the system, but its general behaviour. Second, and following from this, we are not so interested in the states of the system as the movement between states. We therefore begin by checking the relations of the variables at various points whilst the system is in motion. As well as registering these points, we are interested in the vectors present at these points, that is, the direction in which the variables are moving. In the case of the predator-prey model, this would be found by determining the population levels which follow a certain relation of populations. By determining the change in values which follow from a particular point in the phase space, we get what is known as

a vector. To determine the general tendencies of the system, we therefore begin by populating this space with vectors:

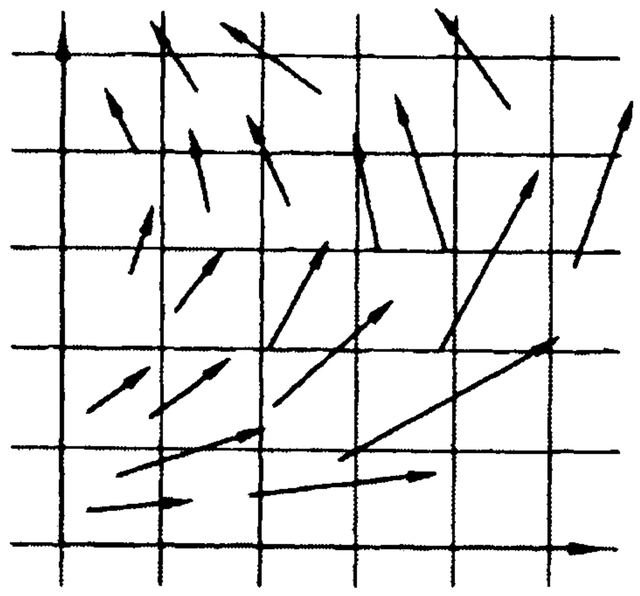


Figure 6. A phase space with two degrees of freedom populated with vectors.

Once we have populated the phase space with enough vectors, we have what is known as a vector field, which will contain the totality of the experimental data. We can take this vector field, and using the operation of integration, one of the functions of the differential calculus, we can create what is called a phase portrait of the system. A phase portrait reintegrates these individual vectors into paths through the phase space, such that if we know the state of the system at a given time, we can trace the path of the system into the future. If we therefore imagine a pendulum unaffected by friction, we will find that the paths drawn through the phase space will form circuits, as the motion of the pendulum repeats itself. In fact, a series of paths through the phase space will be shown on the phase portrait, in the form of a 'dart board' pattern of circles with the same centre (although with enough energy, a different path will emerge as the pendulum completes a full swing). This is because the phase portrait shows the behaviour of the system given a certain relation of the variables, and as the initial state of the system (the amount of energy in the system) may differ, so the movement of the pendulum will differ (i.e. if there is less energy in the system, the circle on the phase portrait will be smaller). Thus, the system will return to its initial conditions with the frictionless pendulum, as no energy is lost from the system, with the circumference of the path through the phase space determined by the initial amount of energy since the more

energy that is put into the system, the higher the angular velocity the pendulum will attain. We could also, for instance, use a phase portrait to examine the interactions of predators and prey within a system,[9] correlating the population of each to one dimension of the phase portrait. Here, we would find that the trajectories of the system tended towards a stable circle, as too many predators leads to a drop in the population of the prey, which in turn leads to a drop in the population of predators, as they do not have enough food to support themselves, and a consequent rise in prey, as there are now fewer predators around. Regardless of the initial conditions of the system, it will tend to settle down into this state, as the number of predators to prey reaches an optimum value. Likewise, if we add friction to the model of the pendulum, we will find that it settles into a particular state, but in this case, not a circuit, but instead a point, as the pendulum eventually stops moving:

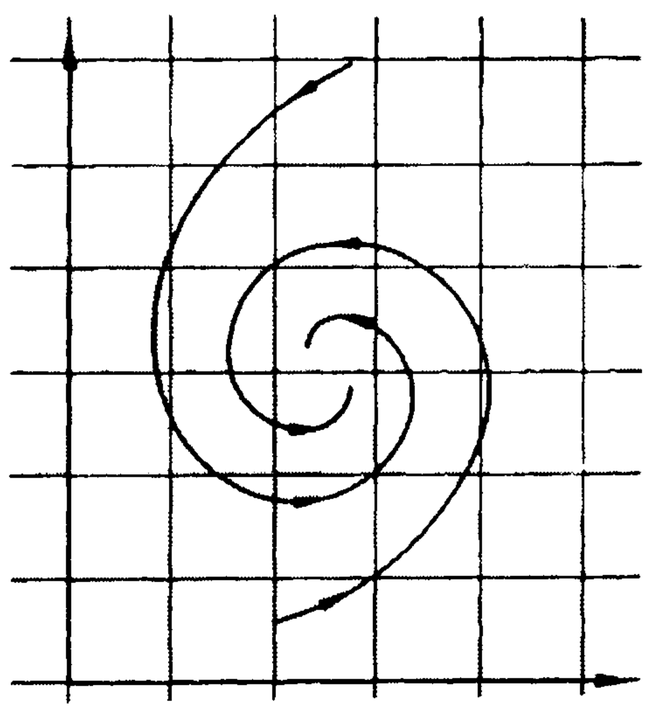


Figure 7. Phase portrait, arrived at through the integration of vectors in the vector field.

The phase portrait therefore allows us to specify the state of the system with a single point.

We have now explored the two central mathematical techniques at work in dynamic systems theory. In the models above, we were principally concerned with the question of individual trajectories through the space of the phase portrait (the phase space). We could find these by plotting a single path, or set of paths, through the space,

leading to phase portraits such as the one above. If we are concerned, however, with the tendencies of the system in general, however, this approach has certain limitations. The diagrams can tell us what happens to a particular trajectory, but to understand the general properties, we would have to draw a line for each possible trajectory within the phase space. A better solution to this problem is to manipulate the phase space itself. This is the approach taken by the mathematician, Stephen Smale, who combined the approaches of dynamic systems theory and topology.[10] In order to do so, he conceived of the phase portrait as being situated, not in the discrete, metric space of Euclidean geometry, but instead in the smooth, non-metric space of a non-Euclidean geometry. We will now see how the three features of Riemannian geometry which we highlighted earlier in this chapter affect the understanding of the phase portrait put forward so far. First, since we are no longer understanding the phase portrait in terms of metric, Euclidean geometry, we will no longer be concerned with the particular trajectory, but rather with the space itself. This is because, as we shall see when we consider the third feature, with the move away from understanding the space in terms of distance functions, we are no longer concerned with *specific* paths through the phase portrait. In diagrams of phase portraits, such as the one above, the tendencies of the system are represented by lines which move towards a central point representing the stopped pendulum. These lines function something like contours of the system, which show the way in which the system flows from state to state. If we consider the various phase portraits which we have described so far, we can see that certain topological features appear within their spaces. Thus, in the case of the entropic pendulum, all trajectories move towards a single point (the pendulum at rest). For the non-entropic pendulum, however, the trajectories instead proved to be cyclical. These features, which describe the tendencies of trajectories, are called singularities. They function like dips and peaks in the phase space, giving the phase space something like the structure of a 'landscape'. That is, in the example of the pendulum, the point at the centre, the attractor, is like a dip in the landscape, towards which any point in the phase space naturally moves, much like

water tends to move from the high ground to the low. Similarly, repellers operate in the phase portrait like points of high ground which the system tends to move away from. Just as in the case with Riemannian geometry, these structures are intrinsic to the structure of the plane, unlike the surface of the sphere in Euclidean geometry. Returning to the attractors themselves, as we can see in the following diagram of the predator-prey system, the singularities define the normal behaviour of the system.

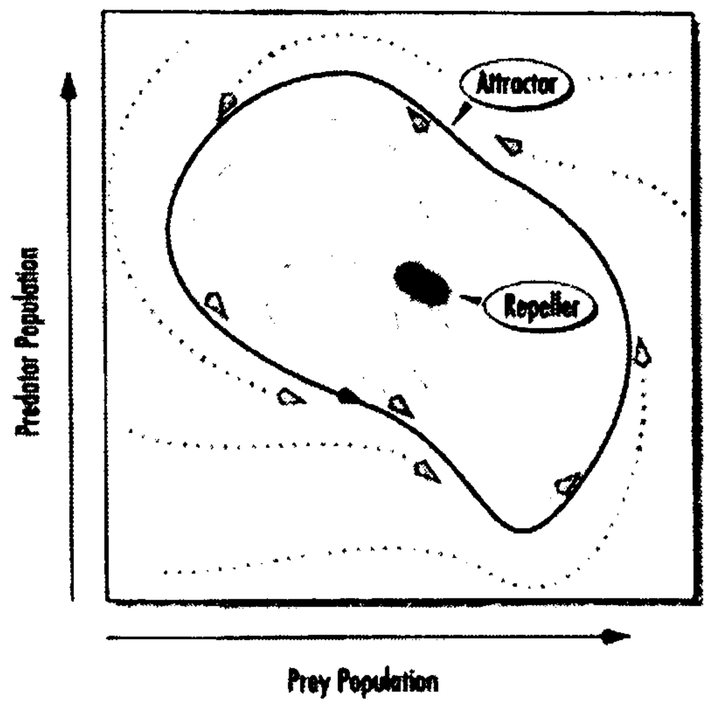


Figure 8. Phase portrait of a predator-prey system.

Second, the phase portrait doesn't have to be limited to two dimensions. In the examples of the pendulum and the predator-prey system, there were two degrees of freedom for the systems, and so only two dimensions were needed to map the totality of possible states. If we were to model a more complex situation, for example the interaction of two billiard balls, we would instead need four dimensions for each of the billiard balls, as each will have a location specified by two axes, and a velocity along those axes. Thus, in an eight dimensional space, a single point could represent any possible state of the system. Similarly, the action of two atoms in a gas would require a twelve dimensional space. Third, as we saw, the space is defined in terms of continuous, non-metric, geometry. This means that the topological features of the space (the singularities) can be understood apart from the precise metrics of the system in which they are embedded. We saw how the structure of the triangle could be altered by changing the structure of the space in which it lay. If the triangle is not defined in terms of distance functions, however, then these various 'distorted' triangles can be seen as equivalent. We can

extend this equivalence intuitively by saying that shapes are equivalent topologically if we can move from one to the other through deformations which do not involve cutting or gluing. Thus, topologically, a circle and a square are equivalent, as one can be deformed to look like the other. Since a figure of eight has two holes, however, it cannot be deformed to look like a square or circle. These structures are not, therefore, topologically equivalent. If we view the structure of phase space purely in terms of this topological understanding, we can see that different systems will have topologically equivalent phase portraits – that is, they are governed by the same singularities.

Deleuze's statement that "an Idea is an *n*-dimensional, continuous, defined multiplicity" (*DR*, 183) is to be understood in the light of this kind of model of the dynamic properties of the system. In talking of the Idea, by which Deleuze alludes to Kant, he tells us that "no doubt, if one insists, the word 'essence' may be preserved" (*DR*, 191), but only at the cost of distorting its meaning so radically as to make its retention redundant. As we shall see at the end of the chapter, the Idea is Deleuze's replacement for the problematic notion of essence in Aristotle, and he also calls it a concrete universal (the significance of this last claim will be dealt with in chapter seven). The *n*-dimensional Idea represents the various dynamics which together generate the system itself, much as the phase portrait represents the dynamics of the system. The continuity refers to the fact that the space we are dealing with is non-metric. By 'defined', Deleuze means that the *n*-dimensional space is governed by the interrelations between the different dimensions. It is these interrelations which create the topology of the space. These three features define the structure of the Idea, but Deleuze also gives three criteria by which its application to the world is shown. First, "the elements of the multiplicity must have neither sensible form nor conceptual signification, nor, therefore, any assignable function" (*DR*, 183). This follows from our conclusions from the first chapter. As we saw on page 28, if the transcendental field is to be conceived of as different in kind from the empirical, it will have more than a merely conditioning role. It

cannot merely replicate it at another level. As a follower of Bergson, Deleuze also does not want to separate the genesis of space from the genesis of the object itself, as this would be to reintroduce the kind of dualism which we saw was at the root of the problems thrown up by the logical systems of Aristotle and Russell. As Deleuze says, the Idea is "not even actually existent, but inseparable from a certain virtuality or potentiality" (*DR*, 183). In positing a difference in kind between the Idea and the actual spatiotemporal world, we should therefore note that there is a further difference between the Deleuzian model and the classical model of the relation of the Idea as possibility to the actual. In the work of Kant, for instance, the idea of the possible was seen as structurally identical with that of the real, the difference between possibility and reality being defined purely in terms of existence. Once the transcendental has been posited as having a different structure, however, a different concept of modality enters. Returning to the idea of the phase portrait of the system, we can ask what, in terms of the possibilities of the system, the phase state represents. As we saw, a particular point in the phase portrait represents the complete state of the system at any one time. A trajectory through the system shows the state of the system as it evolves through time. The space itself forms a certain topology, which determines the trajectory which the point will take through the system, however. We have already seen how many systems will settle down into certain kinds of behaviour over a long enough period of time, so the pendulum will settle to a point (in entropic systems), or else a cycle (in non-entropic systems). Predator-prey systems also tend towards a cycle. More complex topologies can also be developed, as, for instance in the case of the interactions of two pendulums (a cycle within a cycle), which takes the form of a doughnut-shape, or torus, interaction as the pendulums are brought into relation with one another.[11] These attractors, towards which the system tends, represent the equivalent of basins within the topology of the phase space which trajectories close enough to the edges will tend towards. Thus, in the case of the pendulum modelled with friction, trajectories in the phase space will be attracted towards a single point, as all initial conditions lead to the same final state (the

stationary pendulum). Furthermore, as the pendulum swings, each oscillation will move a shorter distance than the previous one, as the path in the phase portrait will be a spiral towards the single point, rather than the circle around the point for the frictionless pendulum. It is the point in the case of the pendulum with friction, or the circle in the case of the non-entropic pendulum, which interests Deleuze, rather than any specific path taken. It is therefore to the space itself, not the trajectories which occupy it, that Deleuze is drawing attention to with his model of the Idea. The phase portrait, rather than being a representation of the possibilities of the system is instead what Deleuze calls, alluding to Leibniz, a field of incompossibles out of which the actions of the system are generated. Out of the phase space of the system, only one trajectory of the system is actualised, as the actual state of a particular system, but all potential trajectories are virtually present. As the phase portrait refers to this particular system, it does not provide a merely formal condition, but captures the particularity of the system in question. The phase portrait, indeed, cannot be seen as separable from the system itself, as it specifies the dynamic tendencies of the system (as it specifies how we move from each state to the next), which cannot be divorced from its actual state. This means that whilst the phase portrait may function in an analogous way to the idea of possibility, it remains different in kind to that which is actualised. This difference is not in terms of reality, as the phase portrait captures the tendencies, or long term behavioural trends, of the system and is therefore real, but rather in terms of the distinction between virtuality and actuality. It is able to capture all potential states of the system without these states needing to be understood as opposed to one another, as they are all present in the structure of the space itself. Further, whilst the attractor may determine the behaviour of the system, it does not fulfil a teleological function. The system itself is not goal orientated in following the trajectory of the Idea, as the phase portrait itself is a result of the interactions of the system, rather than the cause.[12] There is also a structural difference, to the extent that in moving from virtuality to actuality, we are moving from a non-Euclidean multiplicity to a Euclidean multiplicity. By fulfilling this first

requirement, Deleuze has therefore provided an alternative to the bare repetition of the empirical at the transcendental level which occurred with Kant.

Second, the "elements must in effect be determined reciprocally" (*DR*, 183), meaning that the space is not determined from the outside through the imposition of another space. This opposes the kind of multiplicity being spoken of here to a Euclidean spatial multiplicity, in that as we saw on page 125, to capture the structure of the plane of a sphere, for instance, we needed to consider it within an extrinsically defined set of three axes specifying a three dimensional space. Third, this multiplicity "must be actualised in diverse spatio-temporal relationships" (*DR*, 183). This means that the virtual Idea does not simply describe a singular situation, but instead has a kind of generality. The idea of the vector field in fact shows this characteristic. If we take the vector field for the non-entropic pendulum (i.e. a model of a pendulum where the effects of friction have been ignored), for instance, we find a model of a simple oscillator, and so a model which can be applied to a multitude of common sinusoidal wave functions found throughout physics and nature.[13] Dynamic systems show more complex similarities, however. If we take the case of two different systems such as soap bubbles and crystals, we find that although spatially the two structures differ, the phase portraits of the two systems are identical, in that the dynamics by which both systems attempt to move to the lowest energy state, in the first case by minimising surface area, and in the second, bonding energy are the same.[14] In this case, a generality in the dynamic process that generates the system leads to systems with two very different material structures. Whereas the essence of the soap bubble might be traditionally considered as a sphere, and that of a crystal a cube, the dynamic approach, in identifying systems with their process of genesis, allows the same underlying virtual Idea to generate both forms. In that the vector field maps the generation of the system over time, we also have the model of the temporal morphogenesis of the system.

One of the key features of the Bergsonian analysis was the recognition that as well as a process of entropy, through which systems lose structure over time, there was also another tendency than that of thermodynamic systems, archetypally in the form of life. The vector field provides another parallel with Bergson when we turn to non-linear systems. So far, we have dealt with simple, deterministic systems, such as pendulums, and salt crystals. These systems, linear systems, are the mainstay of mechanical analysis. Linear systems can be defined as systems without exponential functions, that is, systems where a small change in the initial conditions of the system lead to small changes in the final conditions.[15] Such systems are classically deterministic systems, such as the pendulum example mentioned before. Another classical example of such a system would be the rotation of the Earth around the sun. In this case, the problem can be analysed as if it were the problem of the motion of a single body,[16] as we can treat the motion of the two bodies as if it were the motion of one body moving around the combined centre of mass of the sun-Earth system. Once a third body is added, however, the problem becomes irreducible to a simpler form, and becomes unintegrable, that is, insoluble (except in certain special cases) with the techniques available to linear analysis. At this point, the central premise of linear systems analysis breaks down, as small changes in initial conditions can lead to large changes in the resultant behaviour of the system. This possibility of large disparities resulting from initial conditions breaks the linearity of the relations between the initial state and the final state, and so such systems are called non-linear. While non-linear systems are not a major part of the field of mathematical analysis, most systems in fact turn out to be non-linear. As the example of the Newtonian three body problem shows, however, such systems are entirely deterministic, in that each state follows from the previous state, but at the same time, the systems are also non-computable, meaning that it is impossible to predict accurately the behaviour of the system over the long term. Non-linear systems therefore fall outside of the model put forward by Laplace of a completely determinable universe by driving a wedge between determinism and predictability. The classical approach to such systems has therefore

been either to ignore them, or else to attempt to reduce them to linear systems. For example, in the case of the three body problem as it occurs within the solar system, the impact of the other planets on the movements of the Earth and the sun is taken to be inconsequential for the purpose of calculation. It is worth noting at this point that the kind of deterministic chaos shown by three body systems does not just emerge with the advances in understanding provided by modern physics, and applies just as easily to Newtonian as to more recent systems. With the increasing availability of computational power, however, it became possible to plot the phase portraits of simplified non-linear systems. The first model, created by Lorenz, was a simplified model of a convection system.[17] A real world example of this would be a small cell of fluid, within which the bottom, which is smooth, can be heated and the top, also smooth, can be cooled. When the bottom of the cell is gently heated, heat moves through the box by conduction, and the fluid remains stable. Such a system would have a phase portrait similar to the entropic pendulum, as any external force acting on the system tends to be dampened, returning the state to inertia. If the temperature differential is increased, however, a motion starts to appear in the box. Fluid at the bottom is heated, and so becomes less dense. It therefore begins to rise. At the same time, cool fluid from the top of the cell begins to fall, and the fluid moves in a cylindrical motion. This would correspond to the phase portrait of the non-entropic pendulum. If the cell is heated further, however, then, the motion of the fluid begins to change. At first, the motion becomes faster until, at a particular point, the hot fluid does not have enough time to dissipate its heat before it reaches the top of the cell. It therefore starts to resist the motion of the fluid, leading to a wobbling motion as the cylindrical motion switches direction unpredictably. The equations governing convection are non-linear in nature, and so Lorenz's model remained a non-linear model, although he reduced the number of variables to three. With three variables, the phase portrait of the system can be constructed within a three dimensional Riemannian space. Within this system we find that despite the absence of predictability, a definite structure emerges. This structure, called a Lorenz attractor after

its discoverer, looks like the wings of a butterfly (the two 'wings' in this case the two directions of flow of the system), and although the trajectory through the space forms a clear pattern, at no point does this trajectory intersect with itself:

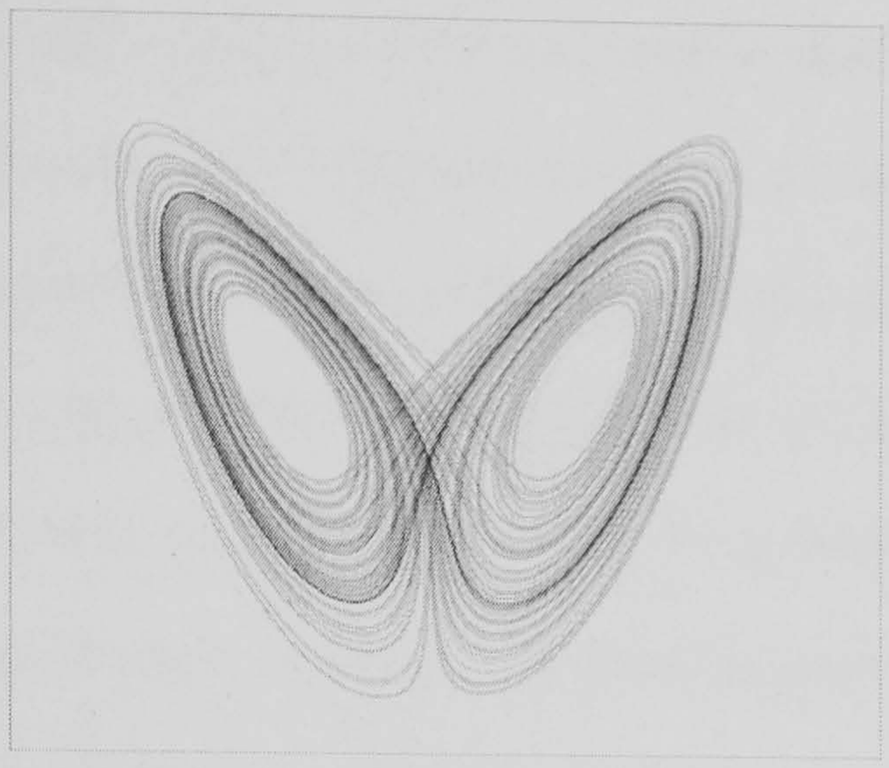


Figure 9. A Lorenz attractor.

The phase portrait of the system is therefore infinitely complex, yet ordered. As such, the phase portrait of the complex system shows an underlying order within the system which is absent from the mechanical interpretation. Following Bergson's intuition that the durational multiplicity's archetypal instantiation is in living systems, we find that non-linear dynamics are the key to understanding organisms. First, we need to note that whilst systems of a high degree of complexity will have an enormous number of degrees of freedom, it is the case that within complex systems, the number of degrees of freedom actually reduces as the amount of non-linearity within the system increases. This is because the linear functions of the system fall into line with the non-linear functions. Thus, as the chaos of the system increases, so does the simplicity. This is known as the slaving principle.[18] If we return to the example of entropy put forward by Bergson, and the opposition to it in the case of life illustrated by the decrease in the amount of symmetry of the system, we find that this is also modelled in phase spaces. Just as in embryology, the organism develops through a process of symmetry-breaking transformations, so the convection system also develops through the reduction of symmetry. At these points the attractor field changes, and we have what is known as a phase transition. Different phases of the system exhibit different tendencies, and so, their

phase portraits, which describe these changes, will also differ. In the case of the convection system, this is seen in the form of changes in the motion of the fluid, but it is also visible throughout the natural world in cases such as that of water, which at a certain temperature will radically change its structure, transforming itself from ice into water into steam. Around these points of transition, we have a maximal amount of non-linearity, such that a slight change in conditions will lead to a large change in the results. In the case of water, this is visible in the fact that both water and ice can exist at 0°C, meaning that a slight perturbation in the energy level of the system will lead to a large change in structure. Cell membranes are found to use this property, inhabiting a state poised between solid and liquid states on the edge of a phase transition which allows them to regulate the movement of materials into and out of the cell.[19] Similar results can be found for the propagation of electrical activity through the heart. Whilst such systems may be able to perform radical changes in behaviour on the basis of small changes of conditions, these systems also display an ability not to fall into periodical cycles, and so are resistant to becoming dominated by other systems.[20] Such systems maintain themselves at far from equilibrium states, as equilibrium for an organism means to be no longer able to moderate the impacts of the environment on the organism, or to be dead. In this sense, non-linearity, or deterministic chaos, has become associated with health, whereas order, or linearity, represents disease. If we follow the dynamical systems approach, it therefore becomes clear that we move away from the classical Aristotelian model of the world. Whilst minimisation of energy in salt crystals and bubbles shows the possibility of a new model of essence, Deleuze also wants to apply this approach to life itself. Such an approach would not define like organisms by their properties, but rather by the dynamic tendencies which form them. "There are greater differences between a plough horse or draft horse and a race horse than between an ox and a plough horse" (*SPP*, 124). This amounts to understanding the organism in terms of its dynamics, or how it is able to interact with the world, rather than according to the actual hierarchy of genus and species which is central for, for instance, Aristotle's approach.

Depth in Deleuze and Merleau-Ponty

We have so far looked at two of the aspects of Deleuze's call for a return to Bergsonism: the logic of multiplicities and the new relation to science. The third important moment is the method of intuition, as that which allows us to see what happens between the two kinds of multiplicity. In Deleuze's system, everything in actual fact takes place 'in the middle', even though it is to be understood in terms of the two tendencies. In this case, Deleuze takes up the concept of depth as key to understanding this middle ground. In doing so, he draws on Merleau-Ponty's later work on aesthetics. In this work, Merleau-Ponty brings in the concept of depth in order to create an essentially Bergsonian theory of space. This involves moving away from the view of space as something which is purely fixed and external to that which we find within it, a view which suggests a purely Euclidean interpretation of space, and moving towards a theory of space as essentially generative. In this sense, Merleau-Ponty's goal is the same as Bergson's as we saw it on page 94 in chapter three. Thus Merleau-Ponty wishes to show that space and the object are generated at the same time, characterising depth as that which brings the other dimensions into being. This insight is taken up by Deleuze largely wholesale. He writes, "everywhere, the depth of difference is primary. It is no use rediscovering depth as a third dimension unless it has already been installed at the beginning" (*DR*, 51). In characterising this second space, Deleuze defines it as an intensive space. As Delanda points out, intensity differs from other properties in that whilst one can divide a non-intensive property without changing the property itself, the division of an intensive property can lead to a difference in kind.[21] If we return to the example of the convection cell mentioned before, it is clear that by dividing the non-intensive property of the quantity of the water, there is no serious alteration in the water itself. Dividing it in half leads to two quantities of water which behave the similarly to

the original quantity. Dividing the temperature of the system can change the nature of the system itself, however, as the behaviour of the system becomes radically altered once it passes through the various phase transitions. This non-metric property of intensity mirrors that of Bergson's duration, which if we recall, also could not be divided without the nature of the system becoming different in kind, as in the example of the sugar water. Intensity therefore is not a kind of property defined through Euclidean space, and instead is related to the non-Euclidean space of the virtual. It is intensity which is responsible for the actualisation of the virtual, that is, the bringing of the virtual into extensity, as it is intensity which allows the structure inherent in the virtual idea of the system to become an actual geometrical structure. In the case of the egg, we can see that although the egg has extensive symmetry, it is through the alteration in intensity, i.e. through phase transitions, that the structure of the embryo becomes actualised in a spatio-temporal framework. In keeping with the critique of Kant put forward in the first chapter, it is the case that the intensive fulfils the function of that which is different in kind from the Euclidean geometrical, and yet provides the conditions for its coming into being. In this section we will therefore turn to Merleau-Ponty's aesthetics to clarify the role of the intensive. In order to incorporate it into our reading of Deleuze, we will first deal with two points. First, we will need to show that Merleau-Ponty's analysis is an ontological analysis, and second, that this ontology is fundamentally compatible with Deleuze's.

Merleau-Ponty's last published work, *Eye and Mind*, concentrates on the role of the artist as providing an alternative way of disclosing the world from that of science. In that the artist attempts to capture some aspect of the world, Merleau-Ponty writes that "any theory of painting is a metaphysics" (*EM*, 171), as such a theory will have to explain how the artist's practice becomes possible. Just as with Bergson, Merleau-Ponty in this essay is keen to create a distinction between two types of relation to the world,

and just as with Bergson, these two types of relation are defined in terms of, on the one hand, our pragmatic relation that admits "only the most 'worked-out' phenomena" (*EM*, 160) and, on the other, the vision of the artist, who "is entitled to look at everything without being obliged to appraise what he sees" (*EM*, 161). As we will see, the scientist and the artist operate on something akin to the two kinds of multiplicities opened up by Bergson. Merleau-Ponty points out that the scientist's approach, called operationalism, avoids dealing with the properties of the world directly. Instead, operationalism aims to define concepts purely through our operations of measuring them, making science in this sense autonomous, since it deals simply with the correlation and analysis of data. In this approach, according to Merleau-Ponty, there is no longer any contact with the 'opacity of being', which was present even in the early moments of modern science. Mirroring Deleuze's own views on the nature of science, we can see that for Merleau-Ponty, science seen in this way is abstract, to the extent that it renounces metaphysical questions in order instead to characterise its success in terms of the manipulability of the object. In opposing this viewpoint, Merleau-Ponty wishes to show the artist as having a non-manipulative relation to the world, but simultaneously to provide an account of such an attitude which does not separate the mind from the world: "it is by lending his body to the world that the artist changes the world into paintings" (*EM*, 162). The main body of the essay is therefore an attempt to explore the world as opened up by the artist, particularly by showing the movement from Renaissance painting, focussed on geometrical perspective, to modern artists, such as Klee, Cézanne and Rodin. The focus of the essay can be said to be Cézanne's turn away from the techniques of geometrical perspective, his attempt to paint the world as it appears without the methods of geometry. In the opening quotation from Gasquet's study of Cézanne, Gasquet claims that "what I am trying to translate to you is more mysterious; it is entwined in the very roots of being, in the impalpable source of sensations" (*EM*, 159), and as we shall see, it will be the field of depth which Merleau-Ponty believes is shown to us in the work of Cézanne. As the correlation between perspectivalism and Descartes on the one hand, and

Klee and modern geometry on the other, will further show, the aim of the enquiry is not to renounce science, but to give it the metaphysics that it lacks. This enquiry, furthermore, will be a transcendental enquiry into the grounds of perception itself, or the invisible in the visible. Klee's slogan, "not to render the visible, but to render visible" (*EM*, 183), brings this movement out clearly.

We can now look at some of the parallels between the structures which Merleau-Ponty brings out and those of Deleuze. First, although painting for Merleau-Ponty clearly involves an action within space, this action does not define what is painted. "Anyone who thinks about the matter finds it astonishing that a good painter can also make good drawings or good sculpture" (*EM*, 182). The lack of focus on the particular method of actualisation of the artwork, however, derives from the fact that what is being painted, drawn, or sculpted is not in itself a thing, but rather an Idea. The effort of modern artists has been toward "severing their adherence to the envelope of things" (*EM*, 182). This means to move away from the spatio-temporal, as is exemplified for Merleau-Ponty by the work of Klee, whose line paintings "subtend the spatiality of the thing" (*EM*, 183). Within such a painting, each inflection of the line forms "another aspect of the line's relationship with itself, will form an adventure, a history, a meaning of the line" (*EM*, 183). Furthermore, when we ask what the nature of this line is, we find that it "is no longer the apparition of an entity upon a vacant background, as it was in classical geometry. It is, as in modern geometries, the restriction, segregation, or modulation of a pre-given spatiality" (*EM*, 184). This essence of the visible, this "layer of invisibility" (*EM*, 187), which is painted by the artist matches Deleuze's criteria for an Idea. Whilst the elements of the painting have sensible form, what is painted does not. Likewise, for Merleau-Ponty, in abstract art, the painting does not have an external significance; it does not have to take its meaning from directly representing a particular state of affairs. Instead, it is the giving of a name to the painting which defines its relation to this or that particular object or event. Thus, its actualisation is possible in a

variety of contexts, depending on the relation the artist gives it to the visible. It is also not a work within space, properly seen, but instead the work itself is a convolution of space, paralleling Riemann's approach to geometry. Merleau-Ponty's reference to non-Euclidean geometry makes clear that there are strong parallels between his conception of the line and that of Deleuze. Instead of being a work in an n+1 dimensional space, it defines itself internally, and, in Deleuze's terms, reciprocally, through the relation of different dimensions. Klee's painting therefore meets the three criteria for the Idea. The line does not capture a spatial intuition of the object, but instead aims at "the blueprint of the genesis of things" (*EM*, 183). "It is the line's relation to itself … [that] will form the meaning of a line" (*EM*, 183). The fact that the line operates in an intrinsic space means that both it and the space itself are reciprocally determined, meeting the second criterion. Third, the importance that Klee gives to the name makes clear that what we are referring to here is something which can be actualised in a number of different contexts. We must "leave it up to the title to designate by its prosaic name the entity thus constituted" (*EM*, 184).

Merleau-Ponty's concept of the work of art therefore meets Deleuze's concept of the Idea in general, albeit in the field of aesthetics, rather than mathematics. We can now move on to look at the concept of depth in Merleau-Ponty's work. Beginning with the alternative approach, Descartes relates the concept of depth to representation. The Cartesian subject seeing himself in a mirror, according to Merleau-Ponty's interpretation, does not see himself, but rather an image impressed on the retina. With painting, this view reaches its most extreme point. The figure represented in a painting is a necessary deformation of that which it is supposed to stand for. "It is only a bit of ink put down here and there on the paper" (*EM*, 170). In order for the painting to represent the object, it is necessary that square shapes take on the form of rectangles, circles take on the form of ovals. In this case, the operation of seeing a painting is conceived of as an intellectual operation, that of reading the work as if it were a text. The painting provides

the cues with which to reconstruct a three-dimensional representation from the lines of projection and relation of forms present within the image.[22] Thus, by recognising that certain forms are both present, but obscured by one another within the picture, we are able to "see a space where there is none" (*EM*, 172). Depth therefore for Descartes is a third dimension which is generated from those present.[23] Going further, for the Cartesian, there is no true concept of depth as "another man, situated elsewhere – or better, God, who is everywhere – could penetrate [the objects'] hiding place and see them openly deployed" (*EM*, 173). Depth is in this sense a relative concept. Such an approach leads to the perspectival painting techniques of the Renaissance, taking their cue from Euclidean geometry, with its attempt to artificially generate perspective. As Merleau-Ponty points out, the focus on brass etchings in Descartes' work on optics is indicative of this approach. In order for depth to be considered in this way, objects have to be conceived of as outside one another, with solid boundaries. The Cartesian approach therefore mirrors the approach of atomism in science which Bergson attacked. As Merleau-Ponty makes clear, it is a conception of space which makes Descartes' approach possible. "Space remains absolutely in itself, everywhere equal to itself, homogenous; its dimensions, for example, are interchangeable" (*EM*, 173). As with Newtonian physics, the perspectivalism of the Renaissance attempted to "bring an end to painting" (*EM*, 174), regardless of the fact that even with the geometry used by the perspectival school, there was no absolute answer to how the structure of the field of vision was to be constructed.[24]

Whilst the Cartesian approach attempts to return the field of aesthetics to a branch of physics, it does represent an advance, in that for Descartes, the work of art relies on the *production* of space. This idea of the production of space is taken up by artists such as Klee, albeit with the idea of space at play no longer the Euclidean space of Cartesian (and Newtonian) physics, but rather the space of the "lines of generation" which designate something analogous to the essence of a thing. Deleuze will later refer

to Klee's "non-conceptual concept" (*WP*, 218) as that which indicates the 'nonthinking thought' of the "people to come," where "philosophy, art, and science become indiscernible, as if they shared the same shadow that extends itself across their different nature and constantly accompanies them" (*WP*, 218). The strands of Sartre's critique of Kant, Klee's reworking of the concept of depth, and the work of Gauss and Riemann, all therefore represent a move away from the classical paradigm of a homogeneous space to that of a Bergsonian interpenetrative multiplicity. For Merleau-Ponty, this new concept of depth is summed up by Klee's aphorism that it is colour that is the "place where our brain and the universe meet" (*EM*, 180), as the property of colour does not rely on the solid boundaries found in the brass etchings of Descartes. Whereas the solid boundaries of the brass etching give rise to the classical conception of space, that of colour gives rise to an interpenetrative space, exemplified by the late works of Cézanne. The idea of depth created by the colour field differs from that of the brass etching in that the brass etching specifies depth as a third dimension, whereas for Cézanne and Klee, depth is that which binds objects to one another, as the ground through which they interpenetrate. Thus it is instead the first dimension which generates the others. Even the depth of the Cartesian geometry follows from this elementary depth. What is important to note, according to Merleau-Ponty, is that it is not colour which is the dimension of depth, but depth which is the dimension of colour. That is, it is the dimension of depth which "creates identities, differences, a texture, a materiality, a something – creates them from itself, for itself" (*EM*, 181). In the context of Klee he argues that "sometimes Klee's colours seem to have been born slowly upon the canvas, to have emanated from some primordial ground" (*EM*, 182). What is important about colour is that it acts for Merleau-Ponty like one of the intensive qualities which Deleuze associates with the virtual. That is, colour cannot be divided without being changed in nature. Thus, the primordial dimension of depth for Merleau-Ponty is an intensive dimension. It is not colour, which is an intensity, but that which is more general than colour, but can be captured by colour's intensive nature.

This intensive field which Merleau-Ponty reaches through his analysis of painting is at the root of his ontology; indeed, as we saw on page 143, painting for Merleau-Ponty is a form of ontological enquiry. For Deleuze, the field of intensity is described in terms of the mathematics of Riemann, leading to a description of physical systems also in terms of intensive qualities. Whilst we have described the nature of this field, our detour through the work of Merleau-Ponty has allowed us to begin to see how this field of depth can be actualised. Just as for Merleau-Ponty, depth generates the other dimensions, including Cartesian depth, for Deleuze, "extensity does not account for the individuations which occur within it" (*DR*, 229). That is, returning to Merleau-Ponty's analysis of the brass etchings of Descartes, extension itself both implies the already individuated in order to be generated, as Descartes sees the phenomenon of depth to be generated from the relations of bodies, and precludes the movement of individuation itself, since, from Bergson, the notion of a homogenous space renders an interpenetrative analysis impossible, as shown by the replacement of interpenetration with action at a distance in the work of Newton. Instead, Deleuze argues, "individuation and extensity must flow from a "'deeper' instance" (*DR*, 229). If this notion of depth is to be generative, it cannot be assimilated to the other dimensions, as an already present mode of extensity, as what we are trying to capture is the moment where extensity itself is generated. Following Merleau-Ponty once again, we can say that "a first dimension which contains all the others is no longer a dimension" (*EM*, 180), or in Deleuze's words, "once depth is grasped as an extensive quantity, it belongs to engendered extensity, and ceases to include in itself its own heterogeneity in relation to the other two" (*DR*, 229). Depth therefore forms the generative space from which extensive space is generated. For Deleuze, this originary space is referred to as the *spatium*, which is for Merleau-Ponty, an intensive space.[25] Returning to Bergson, we can see that the *spatium*, as an interpenetrative multiplicity, functions for Deleuze like the multiplicity of duration. Extensity, furthermore, as the space of individuated bodies, is likewise

Deleuze's equivalent of the notion of space within Bergson's system. We can now finally clarify the notion of difference at play in the work of Deleuze. For Deleuze, "difference is that by which the given is given" (*DR*, 222). That is, difference exists within the intensive field of depth which is generative of extensity. In this sense, Deleuze calls it "the closest noumenon" (*DR*, 222). Thus, difference, as the differential form of intensity, gives rise to diversity by a process of explication. That is, as intensive force finds itself actualised within an extensive system, the unindividuated differential relations give way to relations between bodies external to one another. As we saw in the first chapter, there is a difference in kind between the transcendental and empirical, and it is this process which is generative of it. We still need to look at the process which governs this movement between the pure virtual and the pure actual, which for Deleuze is to be understood through the framework of the differential calculus, but we can see here that for Deleuze, as for Bergson, the true logic of the system will not be understood in terms of negation. Negation and opposition govern the state of the system as actualised, and therefore, for Deleuze, do indeed provide the limit case of the world in terms of the fully actual. "The negative is difference seen from below" (*DR*, 235). Likewise, the classical conception of identity, which requires individuation, is the result of the explication of differential relations into a spatio-temporal context. In making difference the transcendental principle of his system, Deleuze has moved away from the principle of the equivocity of being put forward by Aristotle. The differential field of depth which forms the closest noumenon for Deleuze instead allows us to conceive of being as univocal. As the virtual is not defined in terms of opposition, it also allows the possibility of a non-hierarchical understanding of being, as the virtual is not to be understood in terms of opposition, which led to the theory of species and genera for Aristotle.

Whilst the virtual and actual represent the two key terms within Deleuze's philosophy, it is important to note that pure virtuality or pure actuality cannot be used to

explain the world. We can see by now that the object for Deleuze cannot be completely specified, except under the dual descriptions of the virtual and the actual. We require both the given, and that which gives the given in order to explain an object, as the given and that which gives the given differ in kind. As we have seen from Merleau-Ponty, however, some objects, in this case, the work of art, hold themselves between the envelope of things and the dimension of depth. This is the case with the work of Klee, and that of Cézanne. For Deleuze also, it is possible for an art work to be "immersed in a virtuality" (*DR*, 209), that is to say, to display a "completely determined structure formed by its genetic differential elements" (*DR*, 209). Whilst the work of art comes closest to virtuality for Deleuze, it is the case that "every object is double without it being the case that the two halves resemble one another" (*DR*, 209). Following Bergson once again, we can note that what we are dealing with are two properly determinate forms of order of the object. Whereas for Descartes, the clarity and distinctness of an object are in proportion to one another, Deleuze's conception instead follows from an example from Leibniz (*DR*, 213). Taking the example of the murmuring of the sea, we can see that two interpretations of the sound are possible. On the one hand, we can focus on the 'white noise' of the waves as a whole. In this case, the apperception of the noise will be clear, but at the same time confused, as our focus on the white noise precludes a focus on the noise of the particular waves which together make up the noise as a whole. If we, on the contrary, focus on the noise of the waves themselves, the waves are perceived distinctly, as we grasp the differential relations which make up the noise as a whole, but also obscurely, as our focus on these particular relations precludes our comprehension of the 'white noise' as a whole. Thus the understanding of one kind of order within the murmuring of the sea covers over the other. For Deleuze, it is the Apolline of Nietzsche's *Birth of Tragedy* which provides the best expression of the clear-confused, as that tied to representation, whereas the distinct-obscure is the Dionysian, and properly philosophical order of the object. It is thus Apollo who gives the power of representation, of actuality to the object, while Dionysus provides the

obscure genetic, and virtual conditions of the object, and of its representation. For Deleuze, "the two never unite in order to reconstitute a natural light. Rather, they compose two languages which are encoded in the language of philosophy and directed at the divergent exercises of the faculties: the disparity of style" (*DR*, 214). In spite of this, however, an object can never achieve pure actuality, as this would mean that it would no longer be capable of being further differenciated, nor can it be pure virtuality, as pure virtuality would be without the grain of actuality around which the order of the actualised system can crystallise. Instead, we need to recognise that just as for Bergson, pure space and pure duration were impossibilities, merely representing the tendencies of a system, so for Deleuze, everything occurs between pure virtuality and actuality. An idea "is confused *in so far as it is clear*" (*DR*, 213). We will return to this idea in chapter seven. What we have, therefore, is a difference in kind between the two languages of philosophy, which is only ever found as a difference of degree. Everything occurs in the space between the two tendencies of virtuality and actuality.

Deleuze and the Structure of the Problem

For Aristotle, "the difference between a problem and a proposition is a difference in the turn of phrase" (*DR*, 158). The number of propositions and the number of problems is equal, as for any proposition, we can formulate a question, such that 'Animal is the genus of man' can be reformulated as the problem, 'Is animal the genus of man?' Thus, for Aristotle, the nature of the problem is derived from the proposition. This is also the case with Russell's axiomatic approach to philosophy, where the problem itself is defined as a set of propositions or axioms from which a conclusion is derived. In both these cases, therefore, the emphasis is on the proposition, or the search for the solution to the problem. As truth or falsity apply to propositions, it is natural that these form the basis for the method. Indeed, for Deleuze, whilst one is concerned purely

with the actual, the resources for a proper examination of the problematic itself are not present. Instead, it is only possible to evaluate the problem in terms of the possible solubility of the problem itself. The result of this is that the value of the problem itself cannot be adequately determined, leading to a philosophy built on "puerile examples taken out of context and erected into models" (*DR*, 158). Deleuze's conclusion here can be seen as originating directly from Bergson's recognition of the two forms of multiplicity. If the proposition is made up of self-sufficient entities arbitrarily conjoined through their subsistence within a homogeneous space, then the possibility of judging the adequacy of the problematic itself is impossible. In the model taken up by Russell and Aristotle, determination is not made possible by the reciprocity of interacting elements, but rather by inherence in a logical space, meaning that the context of a proposition does not determine its content. This "infantile" focus on the ability to respond to the problematic, rather than on its formulation, leads to "the grotesque image of culture that we find in examinations and government referenda as well as in newspaper competitions" (*DR*, 158), and presumably, from a Deleuzian point of view, analytic philosophy. Deleuze heralds Kant as introducing the concern with problems through adding to the propositional "the pure thought of another element which differs in kind from representation (noumena)" (*DR*, 178), and Deleuze too follows this trajectory through the introduction of the differential in the understanding of problematics. In doing so, Deleuze follows Kant in claiming that, *contra* Descartes, thought itself can fall into error not simply by a mechanical failure in our deductions (as when Descartes argues in the *Regulae* that it is the introduction of the faculty of memory which leads to our missteps in following a series of deductions), but that thought itself can be led astray by its own nature. We shall return to this point in chapter six when we look at the Kantian antinomies. If we return to the idea of the phase portrait of the system, we have seen how the structure of its space differs in kind from the structure of the space of that which it models, and it is this difference which provides the foundation for his philosophy. As we saw on page 131, the vector field of a system is generated

through the repeated integration of the actual vectors of the system, generating a topological phase portrait, which can be used to reconstruct actual trajectories of the system. The phase portrait thus provides the model for the idea of a space differing in kind from that of the system itself; it is defined through differential geometry. The phase portrait thereby represents the general tendencies of the system. Through integration of this field, however, we are able to generate a particular, actual trajectory of a set of states through the system. This difference is essential to the problematic, and gives rise to the result that "the solution is at once both transcendent and immanent in relation to its solutions" (*DR*, 163). It is transcendent in that it represents an entirely different geography to that of the actualised states of affairs, but at the same time is immanent due to the fact that the operations of differentiation and integration are reversible. Thus, the nature of the problematic is not defined in terms of the proposition. Indeed, "an organism is nothing if not the solution to a problem, as are each of its differenciated organs, such as the eye which solves a 'light' problem" (*DR*, 211). In this sense, the problematic is another name for the transcendental field. The example of the eye in particular illustrates the advantages of moving to a morphogenetic conception of the problematic, that is, an understanding of the problematic as the emergence of form. Whilst understanding the transformations necessary for the eye to evolve through the standard metric account require a seemingly insurmountable number of modifications which would have to be carried out virtually simultaneously in order not to prejudice the organism's chances of survival through being sub-optimally adapted to the environment, an analysis in terms of the morphogenetic processes of folding which generate the structure reveal that the eye can be generated through the iteration of a low number of basic topological transformations.[26] Thus, viewing the development of the eye in terms of the virtual field of singularities which generate it reveals a large attractor capable of explaining the eye's seemingly improbable actual evolution. Further, now that the problematic itself can be specified, it becomes possible to understand how the problematic itself can be 'noble' or 'base', as our aim in generating the problematic is to

show the tendencies inherent within a system. "A solution always has the truth it deserves according to the problem to which it is a response" (*DR*, 159). If we are trying to understand the behaviour of a system, it is important not simply to provide an analysis of it, but to recognise those aspects of the system which characterise that behaviour. Analysing the behaviour of an organism on a cellular level will provide a problematic which, whilst true, will be inappropriate if we wish to understand the interactions of populations of the organism. Thus, the context of the system once again becomes important for its characterisation. The process of selecting problems, Deleuze calls dialectics "the combinatory calculus of problems as such" (*DR*, 157), but it should be clear that the form of dialectics Deleuze is proposing differs from any system of extrapolation of problems from propositions. Instead, Deleuze is proposing a mode of dialectics which relates to the generation of structure itself, rather than its transformation. The problem is thus related to the space of the virtual, which is structured along the lines of a Riemannian multiplicity. It is here that the concept of difference comes into play. As we saw in chapters two and three, there is a strong correlation between the structure of Euclidean space and the rules of classical logic. Riemannian space, however, is constructed using the differential calculus, which, Deleuze holds, opens the way for a different logic, which is based on difference rather than identity. An understanding of the differential as it occurs in the calculus will be key to finally getting to the bottom of Deleuze's notion of difference, but we can already see why Deleuze claims that virtuality is to be understood as pure affirmation. The virtual itself does not give us an understanding of systems in terms of their objectival properties, but instead according to their dynamic tendencies. The virtual Idea of a system is a differential field which contains every possible trajectory of the system. All of these trajectories maintain themselves as equally real, although it will only be one such trajectory which is actualised.[27] It the non-objectival nature of this field of incompossible trajectories which is at the root of Deleuze's new concept of difference, where the differences are maintained without the necessity of the 'this-not-that' logic

which occurs in actuality. We will return to this point in chapter six when we discuss the differential calculus, but for now, we need to look at Bergson's example of colour in order to clarify how exactly Deleuze's approach differs from Aristotle's.

Bergson on Ravaisson

Deleuze brings up Bergson's treatment of colour in his 1956 essay, *Bergson's Conception of Difference*, and the treatment itself appears first in Bergson's essay on Ravaisson in *The Creative Mind*. The aim of Bergson's example is to show that there are two different ways of determining what colours have in common "and consequently, two ways of philosophising on them" (*CM*, 225). The first, primarily Aristotelian method, consists in generating the general idea by "removing from the red that which makes it red, from the blue what makes it blue, from the green what makes it green" (*CM*, 225). What is achieved is a universal concept which specifies exactly what is common to different colours. We have effectively a relation of subsumption, where each colour is a species of the genus colour. This is Aristotle's approach, which is, for Bergson, "an affirmation made up of negations" (*CM*, 225). Whilst this approach appears to capture the general nature of colour, it in fact remains abstract, and only achieves generality through the gradual extinction of the term under consideration. The alternative approach, for Bergson is to take the "thousand and one different shades of blue, violet, green, yellow and red, and, by having them pass through a convergent lens, bring them to a single point" (*CM*, 225). This generates a "white light in which [each shade] participates, the common illumination from which it draws its colouring" (*CM*, 225). What is central to this approach to the universal is that the different shades are unified in the white light. "The different colours are no longer objects *under* a concept, but nuances and degrees of the concept itself" (*BCD*, 43). The concept contains the different colours as degrees of difference in such a way that they are no longer opposed to each

other, but instead interpenetrate. The example itself gives some insight into how Deleuze wants us to think a differential understanding of the general, although in the case of Bergson's example, we are still operating at the level of sensuous intuition. It is with the differential Idea, which we have understood in terms of Riemannian space, that Bergson's example is given a non-sensuous interpretation, through a multiplicity where each virtual trajectory of the system is understood through its participation in the same multiplicity, and in fact, immediately after defining the Idea as an *n*-dimensional continuous defined multiplicity, he goes on that "Colour – or rather, the Idea of colour – is a three-dimensional multiplicity" (*DR*, 182).[28] Thus, twelve years after his essay on Bergson's conception of difference, he will remain true to this intuition of Bergson's claiming that "the Idea of colour…is like white light which perplicates in itself the genetic elements and relations of all the colours, but is actualised in the diverse colours with their respective spaces"[29] (*DR*, 206). Here, virtual understanding is contrasted with an actual understanding where each trajectory or difference would be actual in a different space, each excluding the other. As we have seen, for Deleuze, there are two multiplicities, and so, with the actualisation of the virtual idea, we will see the generation of hierarchies as the virtual multiplicity finds actual expression. Thus, there is a sense in which Deleuze's aim is not opposed to representation, but rather attempts to supplement it. These two moments provide us with an analysis of the object which contains both its actual structure, and its virtual dynamic tendencies, thus reincorporating the moments which were excluded from the Aristotelian and Russellian models.

Conclusion

In this chapter, we have looked more deeply at the ontological structures which Deleuze uses to characterise the world, in particular in the spheres of art and science. As

we can see, Deleuze's philosophy grows out of the tradition of Bergson, but seeks to expand it, both in terms of its spheres of application, as well as through the technical rigour through which its concepts are grounded. The virtual, which operates according to a different logic to actual states of affairs, allows Deleuze to explain the genesis of form itself. There is still one important area which we will need to discuss, namely Deleuze's interpretation of the differential calculus. We have alluded to the fact that the calculus is essential to the specification of Riemannian multiplicities, and thus, the nature of the operations of the calculus is of great importance to Deleuze. In particular, we will need to look at the foundations of the calculus, as Deleuze's use of Riemannian geometry is grounded on the fact that it escapes from the limitations of representation. For this reason, Deleuze's own approach will have to be seen as opposed to modern interpretations of the calculus which aim to ground the calculus through set theory. In this attempt to find a metaphysics of the calculus, Deleuze will be joined by Hegel, and we will therefore look at their two theories together. First, however, we need to look at Hegel's own approach to the problems of finite representation, through his move to a philosophy of infinite thought, and the development of a new concept of contradiction.

Chapter Five – Infinite Thought

Introduction

In the previous two chapters, we looked at Deleuze's conception of difference, and the role it plays in resolving some of the problems encountered in the systems of Aristotle and Russell. This was achieved through a conception of the object as essentially doubled, understood as both virtual and actual. In this chapter, we will move on to Hegel's solution as presented primarily in the *Science of Logic*. Rather than moving to a quasi-transcendental theory to attempt to supplement what is missing from classical logic, Hegel's strategy is instead to try to set the categories of thought themselves in motion through recognising that as well as the finite aspect of thought of the understanding, thought also has an infinite aspect which can only be understood as movement. This chapter will be divided into three parts. First, we will explore the basic aims of the *Science of Logic*, as well as the similarities and differences between Hegel's dialectic and Kantian transcendental logic. Second, we will explore how the dialectic functions in the doctrine of essence. This will enable us to gain an understanding of Hegel's conception of contradiction, as well as opening up discussions of essence and difference as they appear in the dialectical movement. These last two concepts will prove important in our analysis of Deleuze's criticisms of Hegel, and will also allow us to open up a Hegelian line of attack against Deleuze's philosophy. Last, after having developed a positive understanding of Hegel's concept of contradiction, we will return to the problems of Kant and Aristotle to show how these are resolved through reconceptions of the infinite and of the structure of the proposition. We will therefore begin by looking at the logical relations which hold between Kant's project and Hegel's alternative deduction of the categories of thought.

Kant and Hegel

We began our study of the relations between Hegel and Deleuze by looking at Deleuze's relation to Kant, in order on the one hand to show that Deleuze belongs within the post-Kantian tradition, and on the other to show how his own philosophy attempts to move beyond the limitations of transcendental idealism towards what he characterises as a transcendental empiricism. In exploring Hegel's project, we will similarly open with a survey of some of the key differences between the project of Hegel and that of Kant. In criticising Kant, Hegel takes a similar point of departure to Deleuze, namely, the problematic nature of Kant's move from the structures of logical judgement to the categories of transcendental logic in the metaphysical deduction. Kant argues in the metaphysical deduction that "the same function which gives unity to the various representations *in a judgement* also gives unity to the mere synthesis of various representations *in an intuition*; and this unity, in its most general expression, we entitle the pure concepts of the understanding" (*CPR* B105). The metaphysical deduction attempts both to explain why this is the case, and to trace exactly how the functions of judgement are related to the categories. This move will prove essential to the Kantian project, as once it has been established, Kant is able to go further in the transcendental deduction to show that the categories, which are derived from the functions of judgement can apply to objects of empirical experience, as these objects have been conditioned by the isomorphic transcendental categories. Thus, Kant considers himself to have moved beyond bare empiricism on the one hand, and naïve metaphysics, which does not raise this question of the applicability of the categories to their object, on the other. Although Hegel presents several criticisms of Kant, in this chapter, we will focus on those of the metaphysical deduction, both because these appear to be the strongest criticisms which Hegel has to offer of transcendental philosophy,[1] and also because in focussing on the limitations Kant places on the metaphysical deduction, it allows us to see how Hegel positively develops his own logic. Hegel's relation to the metaphysical

deduction is in fact quite ambivalent, and whilst he praises Kant for raising the question of the categories by "investigating just how far the forms of thinking are capable of helping us reach cognition of the truth" (*EL*, §41, Add. 1), ultimately Kant's procedure proves problematic. Rather than focussing on the finite determinations of the categories which Kant's deduction tries to uncover, Hegel wants instead to introduce movement into the process of the deduction of the categories by a renewed understanding of the role of reason in philosophy. "The struggle of reason consists precisely in overcoming what the understanding has made rigid" (*EL*, §32, Add. 1). His approach is not to replace Kant's assumption of the understanding as the origin of the categories, however, but instead to remove all assumptions from our enquiry altogether. These assumptions will fall into two categories. First, assumptions about the type of thing cognition is and so forth (for example, Kant's assumption *that* thought is finite). These assumptions we will deal with at the beginning of the chapter. Second, Kant also makes assumptions about the nature of the possible types of cognition themselves: "Such presuppositions that infinity is different from finitude, that content is other than form, that the inner is other than the outer, also that mediation is not immediacy (as if anyone did not know such things), are brought forward by way of information and narrated and asserted rather than proved" (*SL*, 41). It is important to note that Hegel does not believe that we cannot draw such distinctions, but argues that the meanings of the distinctions themselves must be disclosed dialectically. This second set of assumptions will be dealt with after we have looked at the positive results of Hegel's alternative approach.

Returning to the first set of assumptions, these assumptions, according to Hegel, invalidate Kant's metaphysical deduction as they show neither the immanent development nor the necessity needed to provide a ground for the kind of transcendental logic Kant requires. The first of these assumptions is that "the only use which the understanding can make of … concepts is to judge by means of them" (*CPR*, B93). In conceiving of thought in this way, Kant limits in advance the scope of his enquiry so that

all categories will be considered to have a form analogous to those of judgements. Second, whilst berating Aristotle for his loose derivation of the categories, Kant relies on functions which "spring pure and unmixed, out of the understanding" (*CPR*, B92), relying essentially on the results of classical logic rather than providing an *a priori* derivation of these concepts. Hegel sees Kant's reversion to the rules of classical logic as an unwarranted approach to deriving the fundamental categories of thought. [1]Third, Kant sees thinking, when it is operating legitimately, as requiring a connection to a sensible manifold. "Without sensibility no object would be given to us, without understanding no object would be thought. Thoughts without content are empty, intuitions without concepts are blind" (*CPR*, B75). As we shall see, in fact, all of these assumptions will turn out to be interrelated. In exploring what happens as Hegel strips away these assumptions, we will see how Hegel's project can be seen as a completion of the Kantian enterprise, as well as a criticism of it. Once these assumptions are removed, however, we will also see that the implications of the metaphysical deduction will go beyond the restrictions placed on thought by transcendental philosophy.

As we saw in the first chapter, on page 47, Deleuze takes issue with Kant's reliance on subject-predicate structures in his analysis of the fundamental categories of thought. For Deleuze, this led to the transcendental merely repeating the structure of the empirical, providing merely a conditioning role, rather than one that was truly generative of the object. In attempting to overcome this limitation, Deleuze instead conceived of the transcendental as composed of reciprocally determining entities which when brought together generated both form and content. Hegel's rejection of Kant follows a similar line, arguing that he relies on the assumption that "the form of judgement could be the form of truth" (*EL*, §28). Relying on the categories of formal logic leads to certain difficulties. First, the structure of judgement relates essentially inert elements to one another. This means that judgement has a tendency to be reduced to the simple activity of classifying particulars under pre-established universals, the kind of activity permitted

by the taxonomic system of Aristotle. "Cognition is compounded from [form and inert content] in a mechanical or at best chemical fashion" (*SL*, 44). Second, the structure of statements becomes arbitrary; whilst judgement provides a certain form of negative necessity, in that they require the terms to be understood as subject and predicate, they do not show that the predicate in question is in any sense essential to the subject. "The universal (the genus, etc.) contained in [this kind of science] is not determined on its own account, nor is it intrinsically connected with what is particular; but universal and particular are mutually external and contingent" (*EL*, §9). These limitations can present difficulties when judgment is taken as the paradigm of thought within a philosophical system, though the use of judgement in this way is not itself objectionable. If we are, then, to understand why Hegel finds this approach problematic, we will have to look deeper. In fact, what proves to be key in the structure of judgement, and which is still integral to modern formal logics, is discursivity. By this is meant the fact that judgements, as well as the propositions of classical logic, are *about* something. It is this aboutness which leads to the judgement being forced to relate pre-formed elements, but it is also this factor which Kant takes to be essential in explaining the proper connection between concepts and intuition. According to Allison, there are three "bedrock" assumptions involved in the Kantian enterprise:

> (1) That cognition of any kind requires an object to be given (this applies even to the problematic intellectual or archetypal intuition).
>
> (2) That since the finite mind like ours is receptive rather than creative, its intuition must be sensible, resting on an affection by objects.
>
> (3) That sensible intuition is of itself insufficient to yield cognition of objects and requires the cooperation of the spontaneity of the understanding.[2]

Whilst Hegel would agree with the third of these assumptions, at least insofar as it presents an attack on a base empiricism, the first two are more problematic. The effect of the first two assumptions of the Kantian project is to make sure that the structure of transcendental logic is tied to the structure of the object, as it now deals with "the rules of the pure thought of the object" (*CPR*, A55/B79). By looking at the second of these assumptions, we will see why Hegel parts company with Kant. This second assumption has the form of a conditional statement, with the antecedent term asserted. As such, it actually constitutes two assumptions. First, that the mind is finite, and second, that, if the mind is finite, intuition must be sensible. Now that we have recognised that in fact two assumptions are in play, we can examine whether both parts are in fact supportable. If we look for evidence of the first of these assertions, that the mind is finite, we find that, following Michael Rosen, the evidence for this premise itself rests on the sensibility of intuition. As Kant writes in the *Critique of Judgement*, "establishing that our concepts have reality always requires intuitions. If the concepts are empirical, the intuitions are called *examples*. If they are pure concepts of the understanding, the intuitions are called *schemata*."[3] Whilst it is true that Kant allows thought to operate beyond the sphere of the spatio-temporal, its operations in this sphere do not lead to objective truths, but rather to transcendental illusions,[4] or at best, regulative concepts. This is precisely because categories which are supposed to be applicable to objects are now being applied beyond their legitimate domain. The legitimate application of the categories is therefore limited to the spatio-temporal. It is this process of limiting the application of the categories which further leads to the impossibility of the presentation of infinite concepts. This is most clearly seen in Kant's treatment of the sublime, which can only be presented negatively:

> We need not worry that the feeling of the sublime will lose [something] if it is exhibited in such an abstract way as this, which is wholly negative in respect of what is sensible. For though the imagination finds nothing

beyond the sensible that could support it, this very removal of its boundaries also makes it feel unbounded, so this very separation [from the sensible] is an exhibition of the infinite; and though an exhibition of the infinite can as such never be more than merely negative, it still expands the soul. Perhaps the most sublime passage in the Jewish law is the commandment: 'Thou shalt not make unto thee any graven image, or any likeness of any thing that is in heaven or on earth, or under the earth, etc.'[5]

Here we see that for Kant, the sublime only provides a negative presentation of the infinite, as infinity as such cannot be presented in sensuous intuition. Thus we end up with an abstract presentation of the infinite, which depends upon the absence of limitations.[6] Combining the necessity of the possibility of the presentation of concepts within space and time with the impossibility of presenting the infinite within space and time gives us the following result: if thought is related to sensible intuition, it must be finite. Once we recognise this, we can see that Kant's assumption that thinking must be finite is unwarranted. The necessary relation to a spatio-temporal manifold is justified by the finitude of the subject, but the finitude of the subject is in turn grounded in its relation to the manifold. Thus there is an essential (and vicious) circularity in Kant's assertion of the finitude of the subject.

Whilst Kant's argument at first seemed to be the brute assertion of the connection between functions of judgement and the categories, it turned out on further analysis that it was the finitude of the subject as requiring spatio-temporal intuition which seemed to be the ground of his argument, as it was this connection of thought with a manifold which was supposed to parallel the connection of a judgement with its content. Finally, we showed how each of these terms in fact required the assumption of the other. The fact that these two conditions are interrelated is further emphasised by

Kant's characterisation of intellectual intuition: "in natural theology, in thinking an object [God], who not only can never be an object of intuition to us but cannot be an object of sensible intuition to himself, we are careful to remove the conditions of space and time from his intuition" (*CPR*, B70). What we should notice here, however, is that it is not God's thinking which has become infinite, but rather his intuition, as "all his knowledge must be intuition, and not thought, which always involves limitations" (*CPR*, B70). This restriction on thought, however, rests on the discursivity thesis, as thought was shown there to relate to intuition, and therefore to have a finite structure.

As we saw on page 160, Hegel is not opposed to the idea of a metaphysical deduction, and in fact saw it as a necessary move to overcome the limitations of pre-critical metaphysics, but is, rather, just opposed to the arbitrary nature of Kant's procedure. Whereas Kant adopts a purely taxonomic procedure, deriving the categories from the contemporary interpretation of formal logic, Hegel instead proposes to provide a genetic derivation of the categories, whereby thought, by thinking itself, generates its own structure. Such a procedure is ruled out by the Kantian understanding of the categories, since for Kant, it is judgement which brings concepts into relation with each other, rather than concepts themselves inherently implying a relation to something outside of themselves.[7] This means that the content itself cannot determine itself, as it has no implicit relations to explicate. This limitation, however, is based on the above series of assumptions Kant makes about the nature of thinking. In assuming that thought is finite, related to a sensible manifold,[8] and premised on a distinction between thought and content, Kant has, according to Hegel, excluded the possibility of producing anything more than an arbitrary deduction of the categories. This difficulty is particularly evident if we look at Strawson's study of Kant's *Critique of Pure Reason*, as he shows here not only that modern logic presents a different set of categories to those assumed by Kant, but also that, in modern logic itself, the specific categories which are taken as fundamental are necessarily arbitrary, as each can be defined in terms of the

others. If we return to Hegel's comments on Parmenides and Zeno which we discussed from page 85 of chapter two, we can see that Hegel's criticism of philosophising with assumptions goes beyond a reprimand to Kant. What Hegel praised in the work of Zeno was not the conclusions drawn from his paradoxes, which he saw as matching those of Parmenides, but rather the way in which, by working from the assumptions of his interlocutors, Zeno was able to show that the dissolution of the thought of the interlocutor proceeded immanently from this thought itself. In doing so, Zeno moved beyond Parmenides' position which required the acceptance of assumptions which could simply be denied by another thinker. What Zeno presents is therefore the first move towards an understanding of immanent dialectic, although this dialectic was purely negative in his case, as it merely invalidated the interlocutor's assumptions. What we should take from this is that Hegel's criticisms of Kant's unwarranted assumptions is not an *ad hominem* attack, but rather the appeal to a methodological procedure which involves relying on the immanent movement of the subject matter rather than external assumptions.

The structure of the *Science of Logic* differs in two fundamental ways from the paradoxes of Zeno. First, Zeno's paradoxes do begin with assumptions, namely those of the interlocutor. As the metaphysical deduction will be a deduction of the categories of thought, Hegel cannot begin with any assumptions as to the nature of thought. "With regard to its beginning and advance, [philosophy] cannot *presuppose* the *method* of cognition as one that is already accepted" (*EL*, §1). The enquiry must therefore be presuppositionless. Secondly, Zeno's paradox leads simply to the dissolution of the object. If a deduction of the categories of thought is to be successful, it must instead generate new content through its movement.[9] As is well known, Hegel attempts to solve the first problem by beginning with a concept of being stripped of all determinations, being purely as being, before asking us simply to follow its own development. Once we have discarded the supposition that the understanding is responsible for the connections

between concepts, it now becomes possible for the concepts to explicate the connections between them themselves. As we proceed from the indeterminate concept of being through more and more determinate concepts, we will find that each concept immanently shows itself to be unsustainable without some kind of reference to another concept. The first difficulty which must be dealt with is the fact that within a classical logic, when we show a concept to be unsustainable, the unsustainability of this concept does not lead us to a new concept, but rather to the negation of the original concept, which is not another determinate entity, but everything which is not the determination in question, as when we say "the rose is not red," we do not assert that the rose is a specific colour, but rather that it does not hold a particular determination.[10] Classical logic therefore provides a theory of indeterminate (or in Hegel's terms, abstract) negation. We should note, however, that the ground for the classical concept of negation is a system whereby content is separated from form.[11] Within such a system, it becomes impossible to provide a determinate movement, as the process of inference is separated from the material that the inference relates to. This is because inference is reduced to a purely mechanical operation that is performed *on* the content. As such, it is impossible for the content itself to add anything productive to the movement of thought. For Hegel, therefore, we once again have a situation where the finite understanding is being employed beyond its proper domain. "When [the high scepticism of antiquity] was applied to the forms of reason also, it first foisted something finite onto them in order to have something to lay hold of" (*EL*, § 24, Add. 3). If we do not accept this limitation, however, thought becomes free to recognise in negating the object the specific movement of the object itself, and thus to move beyond a purely mechanical external conception of inference. This means that a contradiction such as Zeno's can indeed produce positive content. Thus, Hegel will assert that, "even in the most general way, … apart from scepticism and dogmatism there [is] still a third possibility, to wit, philosophy" (*RSP*, 325),[12] thereby rejecting the alternatives offered by both Zeno and Parmenides. This third possibility comes from the recognition that whilst contradiction

is necessary, so long as it isn't understood in purely formal terms, it contains the possibility of a movement which is productively determinate. The aim of the later sections of this chapter, which deal with contradiction and the speculative proposition, will be to explain this approach.

For Hegel, once again, it is Kant's move to a formalist conception of the understanding that prevents a proper understanding of how one overcomes the limitations of scepticism. Whilst Kant uses contradiction in the transcendental dialectic to show the limitations of pure reason when divorced from sensible intuition, Kant like Zeno concludes by producing purely negative results. For Hegel, the reason for this is that Kant exhibited a misplaced "tenderness for the things of the world" (*EL*, §48 Rem.) which had its origin in his belief that reason could only function by using the categories of the (finite) understanding (we will return to the Kantian antinomies in chapter six). If we are to successfully resolve the contradiction, we need to recognise a new relation between the contradictory propositions of the antinomy:

> The so called 'principle of contradiction' is thus so far from possessing even formal truth for Reason, that on the contrary every proposition of Reason must in respect of concepts contain a violation of it. To say that a proposition is merely formal, means for Reason, that it is posited alone and on its own account, without equal affirmation of the contradictory that is opposed to it; and for that reason is false (*RSP*, 325).

Whilst this assessment may seem obscure now, we will return to look at the implications of this new understanding of contradiction when we have had a chance to explore in detail what it amounts to in the dialectics of the determinations of essence.

Once we recognise that it is the content which determines the process of movement within the dialectic, we encounter a problem regarding the structure of this movement itself. Given that it is the content which motivates the transitions between different categories, can we discern a method within the *Science of Logic*? The answer to this question must clearly be no, with some caveats, as what defines a method is the possibility of its application in different contexts.[13] As the nature of the content forms an integral part of the dialectic, it should be clear that such a move is impossible. This does not mean, however, that the dialectic does not proceed necessarily, but merely that the means for determining the necessity of the dialectical movements is not transcendent to the content itself. The procedure at play in the dialectic is methodical, however, as it is grounded in the precept that we allow the object to determine itself without intervention. In this sense, dialectic claims to be more methodical that the procedures of mathematics and classical logic, within which the movement from term to term is arbitrary, in the sense that whilst each deduction may be negatively constrained by the laws of inference, these laws do not determinately specify which term follows from the preceding one in the deduction. We can further say that the dialectic exhibits certain classes of structures in terms of which one can differentiate transitions of the understanding, dialectical reason and speculative reason.[14] Whilst these patterns cannot be imposed on the object to provide a form of teleology, they can be discerned in retrospect. Ultimately, as the structure of cognition is what we are investigating, to begin with a method would be to repeat the Kantian mistake of presupposing the nature of thinking prior to its investigation. Hence, we "cannot *presuppose* the *method* of cognition as one that is already accepted" (*EL*, § 1).

As we have seen so far, Hegel's *Science of Logic* seeks to accomplish what Kant set out to do with the metaphysical deduction – namely, to derive the fundamental categories of thought – but without the presuppositions which were implicit within the Kantian system. Principally, Kant was shown to have presupposed both the idea that the

subject has a finite understanding, and that thought remains separated from its content. These presuppositions turned out in fact to be interrelated, as the finite understanding of thought arose because it merely conditioned the sensuous manifold, and hence was distinct from it. We will return to Hegel's alternative account of the relation of the finite to the infinite at the end of the chapter. Before then, however, we need to see the extent to which Hegel actually moves beyond the metaphysical deduction by arguing against the incongruence of thought and being.

The Metaphysical Deduction and Metaphysics

So far in this explication of the *Science of Logic*, we have focussed on its aim of replacing Kant's metaphysical deduction with a deduction with resolves the shortcomings which we discovered in Kant's own account. Hegel's actual aims in making this move are in fact broader than those of Kant, however. With his movement to a transcendental idealism, Kant replaced metaphysics with transcendental logic, meaning that what we were concerned with now were the structures which condition experience, rather than the kind of truths which would transcend experience. This replacement of metaphysics with transcendental logic, however, does not only lead to the recognition of the importance of the categories – a recognition supposedly missing from pre-critical metaphysics – but also to the reduction in scope of the enterprise of philosophy so that "the most the understanding can achieve *a priori* is to anticipate the form of a possible experience in general" (*CPR*, B303). This means that:

> The proud name of an Ontology that presumptuously claims to supply, in systematic doctrinal form, synthetic *a priori* knowledge of things in general (for instance, the principle of causality) must, therefore, give

> place to the modest title of a mere Analytic of pure understanding (*CPR*, B303).

As we have seen, Hegel accepts the idea that the aim of philosophy is to provide a deduction of the categories of thought. In dealing with the implications of this for the possibility of a metaphysics, however, Hegel does not accept that taking up this project means that one is debarred from simultaneously analysing the structure of reality itself. According to Hegel, Kant's philosophy has meant that, "the forms of objective thinking … have been removed by this criticism only from the thing; but they have been left in the subject as they were originally" (*SL*, 47). In place of this, Hegel conceives of these structures as "straddling the opposition between subject and object."[15] Hegel is thus arguing not just for a determination of the categories of thought, but also of being. In making this move, we need to remember that what Hegel is proposing is still a deduction of categories. As such, the equation of thought and being does not allow us to derive without presuppositions the nature of specific objects, but rather the structure of the object in general. Likewise, in equating thought and being, Hegel is not arguing for a form of panpsychism, but rather that reality itself is structured according to the same categories as thought.[16] If we do not assume that the structures of thought are different from the structures of being, it means we can incorporate into the Kantian project of a metaphysical deduction the pre-Kantian project of the delineation of the structure of the world itself. Thus, thinking once again becomes ontological. As the aim of this study is to contrast the philosophy of Hegel with that of Deleuze, and Deleuze himself accepts the possibility of a metaphysical philosophy, albeit a philosophy of difference, we will just sketch the structure of Hegel's arguments in support of this conclusion. The first argument, which occurs in the *Science of Logic* itself, stems once again from a desire to avoid all presuppositions within our metaphysical deduction of the categories. Kant's restriction of the scope of our enquiry is grounded in the distinction of the phenomenal realm from the noumenal realm. That is, Kant assumes that what is the object of

experience is not the object in-itself. If our enquiry is not to contain presuppositions, however, we cannot begin by assuming such a separation. Instead, we must simply take pure being, that which thought discloses, at face value, and allow being itself to determine whether it is merely the appearing of something else.

If we do not accept this first argument, Hegel presents another in the form of the *Phenomenology of Spirit*.[17] Hegel notes in the *Science of Logic* that, "it was remarked that the phenomenology of spirit is the science of consciousness, the exposition of it, and that consciousness has for its result the *Notion* of science, i.e. pure knowing" (*SL*, 68). The result of the *Phenomenology* is therefore to bring consciousness to the position whereby it is ready to conduct the enterprise of the *Science of Logic*. Instead of requiring us to accept that thought is identical to being at the beginning of our enquiry the *Phenomenology* provides a bridge from our conception of ordinary consciousness to absolute knowing. It proceeds from the assumption of a pre-critical consciousness that thought stands over against a world of things to which it relates. Beginning with the simplest conception of this relation, sense certainty, Hegel shows how each form of consciousness surpasses itself in showing that its understanding of its relation to the object is inadequate. The inadequacy of its relation to the object is not shown by applying external criteria to the object, but rather by showing immanently that consciousness's formal understanding of its relation is one sided, and in this regard false. Thus, whilst we begin with the simplest possible conception of a relation between a subject and an object, that of simply knowing *that* the object is, we discover immanently that in characterising its object in this singular, immediate way, consciousness has in fact related to the object under its most universal aspect, as it lacks the capacity with this immediate form of understanding to differentiate the various moments of the object, reducing the object to a simple this, here, now. From the collapse of sense certainty, we are led to a new form of consciousness which seeks to overcome the limitations of its forerunner. The ultimate stage of the *Phenomenology*, absolute knowing, represents the

moment where the opposition between subject and object, which has driven the process of generation of new forms of consciousness collapses, and so we are left simply with the identity of thought and being. We are therefore left with a negative result (as William Maker characterises it[18]), or a bracketing of consciousness (in Hyppolite's terms[19]), whereby we are free to begin the positive project of science without having to assume that thought is separated from the object of thought. This therefore provides a deconstructive argument against the distinction of thought and being.

Having analysed the basic requirements of dialectic, we are now in a position to look at the dialectical sequence which encompasses the determinations of reflection, including both difference and contradiction. One of the key characteristics of the dialectic is that it develops concepts immanently, and for this reason any attempt to study a part of the dialectic in isolation is problematic. Short of writing a commentary on the entire process of the *Science of Logic*, the best we can do is to mitigate this violence to the dialectic by attempting to provide at least a summary of the important preceding elements of the dialectic. We will therefore begin with such a summary of the doctrine of being before moving on to more detailed analyses of the dialectic of seeming and essence, and reflection itself.

From Being to Essence

We will begin by considering briefly the doctrine of being in order to understand exactly why Hegel believes reflection, and with it the determinations of reflection, arise. We will be returning to some of the determinations of being in the next two chapters, but for now we will concern ourselves simply with why the sphere of being is sublated into the sphere of essence. As our focus in this chapter is on the doctrine of essence, this account will be of a cursory nature. Given that the structure of Hegel's logic is one of

immanent determination, it must be the sphere of being itself which reveals itself to be in and of itself inadequate, thus opening the way to a transition beyond the sphere of being.[20] As we know, the logic proper begins simply with "Being, pure being, without any further determination", being "equal only to itself" (*SL*, 82). In being equal only to itself, and lacking any relation beyond itself, it is seen as a pure immediacy. Such a pure immediacy, however, collapses quickly into nothing, as, lacking any determinations, there is nothing by which one could determine its difference from its opposite. Nothing, in turn, as it is also pure immediacy, collapses back into being. In the movement between the two, however, we find that a new concept emerges, that of becoming, as the vanishing of each into the other.[21] In this becoming, however, we have a pure movement back and forth between being and nothing, which cannot be resolved apart through the collapse of the contradictory set of concepts into determinate being, in which being and nothing are presented as a unity. With the rejection of the constant vacillation of the categories of being, nothing, and becoming, however, we are presented once again with a concept in its immediacy, albeit one which contains both being and nothingness. The movement therefore returns us to an immediacy, and indeed, immediacy is a recurring theme which runs through the doctrine of being as it reoccurs with the rejection of previously unsustainable categories. Determinate being in turn becomes qualified as something, which is once again immediately what it is. Something cannot remain determinate without some other over and against which it determines itself, however. What we took to be immediately just something thus turns out to rely on another for its determination. Each thing relates to another, but is simultaneously limited by it. The limit defines what a thing isn't, but also, in that the limit provides a determination of the thing, the limit defines what it is, since if something were no longer limited, it would fall back into the indeterminacy of being. Immediacy therefore passes into mediation. As we pass further through the doctrine of being, we discover furthermore that neither quantity in the form of the quantum, nor quality itself can hold themselves to a pure immediacy. As quantity develops into a ratio of powers, it becomes qualitatively differentiated from

the simple quanta, as it is now pure self relation, since each element in the ratio only gets its meaning through the other, which in turn gets its meaning from the first term. Each term therefore relates to itself through the other. Equally, qualities themselves turn out to be mediated by state transitions brought on by quantitative changes, as in the case of, for example, water, whose qualities change completely depending on temperature and pressure.

Throughout the doctrine of being therefore, we have a movement whereby we begin with pure immediacy, and passing through mediation, arrive at a mediated immediacy. As each term passes over into another which is also a simple immediacy, however, mediation is always understood as the mediation by another. Whilst each stage of the doctrine of being provides an implicit connectedness with the terms preceding and succeeding it, being itself is not able to explicate the fact that immediacy is always mediated, as the mediation of one concept is always grounded in the *prima facia* immediacy of its successor. In Burbidge's words, it is only by a "meta-logical reflection"[22] that this connectedness is made apparent outside of the dialectical movement itself. As such, whilst these discoveries are certainly true of the dialectical process, as they are not explicit immanently within the dialectic, they exert no pressure on the characterisation of the terms themselves. That is, they are apparent from the outside of the dialectical movement, rather than being thematised within the movement of thought itself. We therefore have a structure much like that of the *Phenomenology of Spirit*, where each category "goes beyond its limits, and since these limits are its own, it is something that goes beyond itself" (*PS*, 51). Each category suffers a 'death' in its passing into otherness, which leads to the new category appearing as immediacy. With the movement towards measure, however, we find that there begins to be an explicit relation between terms, as to measure is to bring two different entities into relation with one another. Moreover, a more developed conception of measure will show us that the entities brought into relation cannot be simply arbitrarily connected, but must instead on

some level solicit the relation to other entities, as we find, for instance with chemical valency bond theory, where "two salts, both resulting from the union of an acid with an alkali, exchange radicals once they are placed in the same solution."[23] Measure thus begins to make explicit the inherent mediation between the categories, meaning that what was before a meta-logical analysis of the interrelations of the categories now becomes incorporated into the movement of Reason itself. The terms are now interrelated explicitly, as "each is only *through the mediation* of the other" (*EL*, § 111), so that the terms themselves can no longer even apparently be seen as bare immediacies. The recognition that being cannot simply be seen as immediacy leads us to essence as the realm of non-immediacy. "Essence was already implicit within measure, and its process consists simply in positing itself as what it is in-itself" (*EL*, § 111, Add.).

For Hegel, "essence is past – but timelessly past – being" (*SL*, 389), a pun on the German terms for essence (*Wesen*), and the past participle of 'to be' (*gewesen*). We can leave aside the word-play, however, as the underlying logical structure of essence at this point in the doctrine of essence is that of a recollection, and internalisation (*Erinnerung*) of those dialectical movements which led to the positing of each concept within the concept itself. What we saw on a meta-dialectical level as the dialectical conditions for the emergence of a particular term are now taken up into each term. In doing so, however, the characterisation of these terms as purely immediate can no longer be supported. Thus, the meta-dialectical analysis must now be incorporated into the dialectic itself, or in other words, the dialectic must reflect on its own movement by thematising the connections between categories themselves.

Before moving on to discuss essence, reflection, and the determinations of reflection, we should note some results of the foregoing analysis. First, essence is a concept which comes after being, since it emerges precisely as a response to the limitations of pure immediacy. Second, this response will take the form of a negation of

immediacy, since essence emerges in opposition to the simple immediacy of being. Third, as the doctrine of essence involves the collapse of the distinction between logical and meta-logical results, we will have to retrace many of the dialectical movements which occurred in the doctrine of being, although this time the logic will deal with the fact that the dialectical thought is explicitly relating to thought itself. This means that many of the structures of the doctrine of essence will be analogous to those uncovered in the doctrine of being. The new reflective structures will not replace the earlier structures, however, as the two differ, at least insofar as the former are reflective in a way in which the categories of being are not.

It should be clear now why the doctrine of essence will provide the focus for our discussion of difference and contradiction within Hegel's logic. Whilst contradictions may have emerged during the doctrine of being, it is within the doctrine of essence that these concepts of difference and contradiction themselves become explicated by the dialectic itself. Also, as we saw in our discussion of dialectic and univocity in chapter two, it is through self-referentiality, and as a consequence, contradiction, that Hegel is able to overcome the equivocal conception of being which is found in classical logic. We need to also recognise a fourth point, that the rejection of immediacy is not the rejection of it in all forms, but only as bare immediacy. Instead it is the claim that immediacy can only be understood as constituted by mediation. We can now move on to the first characterisation of essence.

The Essential and the Inessential

The doctrine of essence begins with the assertion that "essence is the truth of being" (*SL*, 389), and indeed, essence is sublated being; it is the negation of being, insofar as being is characterised in terms of pure immediacy. Essence, as the result of the

dialectic of being, also preserves being. This first characterisation of the relation of being to essence, however, although in the doctrine of essence, does not move beyond the sphere of being. In setting up essence as the negation of being, essence becomes simply directly opposed to being. As we saw on page 86, for Hegel negation isn't to be understood as purely indeterminate, and so essence as not being immediacy is still something; essence is *determined* as not-immediacy. Essence, as the moment of the dissolution of being, however, is thereby granted a degree of immediacy itself. The essence of being has a determinate character as essence itself ('*This* is the essence of being'). Whilst it appears that the relation of being to essence is one where being is inessential in the face of its sublation, essence, once these two terms both develop their own immediacy, each stands on its own, in themselves: "these are indifferent to each other, and with respect to this being, being and essence are equal in value" (*SL*, 394). As these two concepts are held before us, it is only our own intervention that characterises one as essential and the other as inessential, since in themselves, they lack relation to each other. "Insofar as the distinction is made of an essential and an unessential side in something, this distinction is *externally* posited, … a division which has its origin in a *third*" (*SL*, 395). In holding these two terms over and against one another, we find confirmation of the fact that the move to essence has at this stage not yet moved beyond immediacy.

Once being and essence are conceived of without relation to one another, the idea that essence can somehow be the essential in relation to being, its foundation, dissolves. The two categories are understood through a form of picture thinking which allows the relations between the terms to be understood purely externally, and so the relative positions of the terms can be interchanged without altering their contents. In actual fact, however, essence cannot be so simply conceived of as different from being, as essence is simply the activity of being's dissolution of itself. This means that being cannot be seen as standing opposed to essence, precisely because being cannot itself *be*.

By recognising this inherent connection between being and essence, we remove the need for a third outside of the two terms to unite them, but also, in the process, we transform being. Being, insofar as it is immediacy, is now a mere seeming, a *Schein*, as it is only the seeming of immediacy which remains distinct from essence. As essence shows the mediated nature of being, the immediacy it presents cannot be more than this seeming. "The being of *Schein* consists solely in the sublatedness of being, in its nothingness" (*SL*, 395). With this move, the relation of being to the movement of its development is now thematised as essence within the dialectic itself.

Schein and Essence

The relation of seeming to essence at this stage of the dialectical process is unclear. Whilst they have been shown to be interconnected, the nature of these connections is still unstable. Although the focus of this chapter is on the concept of reflection and its determinations, we should take note of the transpositions which Hegel asserts govern the relations of seeming and essence, since it is here that Hegel's dialectic moves away from the kind of transcendental position one might ascribe to Kant.[24] Given Deleuze's place in the post-Kantian tradition, and his self-description as a transcendental empiricist, we will have cause to return to the dialectic of *Schein* and essence in chapter seven.

The difficulty which we have come across in our study of essence and seeming is that these two terms appear both to have a form of self-sufficiency. "Illusory being, therefore, contains an immediate presupposition, a side that is independent from essence" (*SL*, 397). In order to show that this separation of seeming from essence breaks down, we need to show that "the determinations which distinguish it from essence are

determinations of essence itself' (*SL*, 397). The key determinations which essence must therefore be shown to have are immediacy and non-being.

First, we need to consider the nature of essence. Essence first of all is negativity, as it is the negation of immediate being. As such, essence is not immediate. This does not give us a complete understanding of essence, however, as essence emerged immanently from the movement of being itself. Essence is the truth of being. It is not, therefore, simply not immediacy, as it is the true character of immediacy, or what immediacy has turned out to be.

Essence is therefore the negation of a negation, which will lead to an inversion in our conception of essence. When we looked at the structure of immediacy in the sphere of being, we characterised it through its apparent independence of any grounds. We found, however, that in referring outside of itself, both in terms of its implicit history (which was to become the being-past of essence), and also in terms of the collapse of immediacy through the immanent movement forward of the dialectic itself (as when being and nothing prove to be unsustainable), each term proved to be in fact mediated. If self-sufficiency is to be taken as the defining characteristic of immediacy, however, then we have with the negation of the negation a movement where the negation (essence) is not supported by anything external, but instead refers to itself. Such a moment of self-relation is therefore a moment where essence is *immediately* itself. It is, through its persistence through the activity of dissolution, identical with itself. At this moment, we see essence as that which is the real immediate 'beneath' the apparent immediate of being. Paradoxically, however, when we look at the structure of this movement of double negation, we find that it is also a mediated structure. This is because in moving beyond the simple negation to a structure where negation negates itself (what Hegel calls negativity), negativity is no longer a simple immediacy, but instead involves a complex structure which takes it beyond simply being what it is. Negation cannot remain

immediately what it is under the pressure of the second negation, and so becomes mediated.

This means, however, that essence only seems to be immediate being. With this move, essence is now seen as that which is itself appearance, through its own self-relation, that which generates the immediacy which we take to be illusory being. The dialectic draws us further, however, once we recognise that our conception of essence as projecting seeming relies on their separation, which is opposed to the conjunction of the two which we have just posited. The root of this is the immediacy which we supposed essence to hold. Essence has shown itself to be mediation; in assuming essence to be separate from its projections, we are still implicitly holding to the view that it has a level of immediacy, as self-sufficiency, which allows it to sustain itself. Recalling the opening of the dialectic, we should note, however, that essence was defined as mediation. Once this fact is recognised, we can further note that just as immediate being is the seeming of essence, so essence as separated from this immediacy is once again an immediacy, and therefore once again a seeming of essence. We can now see that essence precisely is nothing other than what it seems to be. "Essence contains the illusory being within itself as the infinite immanent movement that determines its immediacy as negativity and its negativity as immediacy, and is thus the reflection of itself within itself" (*SL*, 399).

We can sum up the interrelations of essence to seeming by following the course that seeming takes through the dialectic. At first, being is conceived of as a determinate being that is merely inessential in relation to essence. Through our recognition that essence is the truth of being, we were brought to the understanding of being as mere seeming. With the recognition of self-reflection, we at first move to a view that seeming is a mere projection of essence, but once we reject the immediacy implicit in this view, we cannot any longer separate seeming from essence even as a projection. At this point, essence itself, in its own reflexivity, is the process of seeming. What this means is that

essence constitutes seeming through a process of reflection (as negativity). It is in this movement of reflection that the ideas of difference and contradiction emerge, as determinations of reflection. That is, these concepts play a role in the very determination of seeming constituted through the reflection of essence. Once the immediacy of seeming is recognised to be constituted by essence itself, however, seeming is now understood as *constituted* immediacy.

The Structure of Reflection

As we saw on page 181, the structure of essence was negativity – the negation of a negation. Furthermore, this structure actually turned out to be a process. "Essence is reflection, the movement of becoming and transition that remains internal to it" (*SL*, 399). We can contrast the idea of becoming as it appears here briefly with that in the doctrine of being. In the doctrine of being, becoming emerges through the immanent instability of the thought of pure being. It becomes apparent that in attempting to think pure being without any determinations, we lack any way to differentiate it from pure nothingness. Without differentiation, pure being in fact becomes nothingness, and in the process there emerge two separate becomings, the coming to be and the ceasing to be of being. Becoming in the sphere of being occurs between two terms. When we look at the process of reflection, however, what we have is instead a movement from "nothing to nothing, and so back to itself" (*SL*, 400). Thus, reflection cannot be seen as a process of coming to be and ceasing to be, as there is no origin or destination between which the becoming can move. The movement of disappearing itself is therefore checked by negativity, as it cannot *arrive* at a state of having disappeared. As such, this movement persists, and is, albeit as a movement constantly in tension. Equally, however, as a pure movement, we can say that reflection is not, as it lacks any stability.

From this analysis, we can see two facets of reflection emerging. First, we have the movement from nothing to nothing itself. In being simply the moment of transition, it appears in its simple immediacy; it can only be understood as a transition without an origin or destination. This aspect of reflection therefore shows both of the characteristics of *Schein*, as an immediacy which is dependent on something else. Reflection is also, however, essence, as the reflection back into itself, or the return to itself. Therefore, reflection encompasses both moments of the object, its inner essence and its outward showing. Indeed, the difference between these elements is now not one of two things which can be brought into relation with one another, but instead a question of emphasis. As reflection has no beginning or end, we can characterise its essentiality by focussing on its nature as a persistence of motion. If we focus instead on its status as a becoming, we see the constant motion of appearance. This structure is at the heart of the later collapse of the distinction between the thing-in-itself and the phenomenon, when both are shown to simply be aspects of the phenomenal itself. We can now see why this analysis should be important for Deleuze's treatment of the real, as the question of the possibility of considering virtuality and actuality as independent terms is in doubt, provided we can assimilate these two categories into the dialectical structure. For now, we need to look further at these relations between seeming and essence as they play themselves out through the dialectic of reflection. The three stages which reflection itself moves through, positing reflection, external reflection, and determining reflection, will set the stage for looking at exactly how the determinations of reflection are related to phenomena.

Positing Reflection

When we look at the structure of reflection, we see that it is a turning back on itself. We can therefore picture reflection as analogous to a circle. With this return, it has

the structure of an immediacy, since this return to itself, as the negation of the negation, is self-sufficient. In this sense, reflection, in its turning back on itself, *posits* an immediate being. Immediate being emerges as the result of the movement of reflection. Since it is now a *posit* of reflection, immediate being becomes simply a moment that is mediated by reflection. Reflection is a movement, however, and as such, it requires a beginning. It therefore *presupposes* a point from which it can begin, as this beginning is essential for its return into self. This point of beginning is therefore an immediacy which precedes reflection. Such a presupposition, however, as it is taken up into the movement of reflection, is mediated by reflection. This is because in order for the presupposition to be a *presupposition* of reflection, the movement of reflection as a return to itself must take place. It is dependent on that which presupposes it for its status as a presupposition. This opens up a third moment of positing reflection, whereby reflection shows itself as the process of *positing* its own presupposition. This is because what is the result of reflection is also its beginning, and *vice versa*. Positing reflection thus exhibits a circular structure, an "absolute recoil" (*SL*, 402). The difficulty is that this movement runs the risk of producing a mere static tautology, *A=A*. Without the other, the movement back into itself is in a sense barren, as reflection cannot return to itself without a moment to return from. If reflection is to be genuine negativity, and so not just to be itself, as a tautology, it must relate to a presupposition that is not simply posited by reflection itself, or rather, is posited by reflection as *external* to reflection.

External Reflection

Positing reflection has therefore shown how *Schein* and essence are parts of the same movement: "reflection, as absolute reflection, is essence that reflects its illusory being within itself and presupposes for itself only an illusory being" (*SL*, 402). We noted, however, that in order for reflection to not simply fall into barren tautology, it

requires a moment *external* to reflection. External reflection therefore takes the posited immediacy of positing reflection, and sublates its act of positing. That is, in order to preserve the external nature of the immediate, it takes all determinations of the moment of immediacy to be determinations which reside within the moment of reflection itself. In doing so, reflection falls apart into two moments. On the one hand, we have the pure positivity of the moment of immediacy, which reflection no longer takes to be determined by itself. On the other, we have reflection, which in order to maintain the externality of the immediate moment, considers all transition to be introduced purely from its own movement. It is thus the inessential in relation to the essential of the presupposition. These two separate moments immanently collapse into one another, however. The moment of immediacy is taken by reflection to be a given, but taking it as a given involves *positing* it as such. This means that the immediate is in fact *determined* as immediate by reflection. This determination of the immediate is, however, a negation of the immediacy of the immediate. What was originally seen as simple positive immediacy thus turns out to be negatively determined. There is a second moment in external reflection, however, in that this initial moment of positing must be negated in order that the external moment be seen as genuinely external to reflection. Thus, reflection, in trying to remain separate from the immediacy, in fact determines the presupposition as the negation of a negation, once again the structure of negativity. Thus the immediacy which was supposed to be external to reflection is now itself seen as negativity, or self-relating negation, which was the structure of essence. Now that the external is seen to be mediated by reflection, the two moments collapse back into one another.

Determining Reflection

Whilst external reflection determines immediate being externally through reflecting on it, this externality must necessarily collapse back into an immanent relation between immediate being and reflection. This is because as we have seen, each term is in fact only constituted through its other. External reflection takes immediate being as a presupposition, leaving immediate being as external. Once we recognise that immediate being only makes sense as a term constituted by reflection, as we saw in the dialectic of positing reflection, however, the presupposition of immediate being becomes a positing, through which immediate being is determined as such. Reflection thus becomes constitutive of genuine immediacy. Whereas immediacy was posited by positing reflection, it was no more than that, a *posited* immediacy, and thus not genuinely immediate. External reflection instead took immediacy to fall outside of reflection, and thus to be genuinely non-reflexive and immediate. As we saw, however, this externality proved in fact to be posited by reflection itself. Now that the sheer externality of external reflection has been overcome, we can say that "immediate being is the *being of* reflection. Or vice versa, reflection is the immanent essence of immediacy."[25] Determining reflection recognises that reflection itself posits non-reflexive immediacy. This is not, however the immediacy of positing reflection itself, as this was sublated by external reflection. Instead, we recognise that it is reflection which both constitutes *immediacy* (positing reflection), and constitutes immediacy as genuinely *free-standing* (external reflection). This leads the way to determining reflection, which "is in general the unity of positing and external reflection" (*SL*, 405). In positing reflection, the unity of being and reflection was apparent. Indeed, as we saw in the dialectic, their separation was purely a formal matter. This pure formalism proved to be unsustainable, however, and the two terms collapsed into externality towards one another, as we saw in external reflection. This separation of the two terms allowed them to become determinate in

opposition to one another. Ultimately, however, external reflection proved unable to support the externality of these terms, as each could only *be* in relation to the other. Whilst determining reflection appears simply to repeat the movement of positing reflection by reincorporating immediate being into reflection, this movement must take into account the discovery of external reflection, that these moments are not simply formal, but have a determinacy to them. The act of positing is a positing of both being and reflection, as these have been shown to be the same, but yet each remains free-standing, as external reflection has shown that the terms differ in their identity. The positing therefore has a certain independence beyond the movement of reflection itself. As such, it represents essential determinations of the object (although, as we shall see, these 'determinations' are at first indeterminate). Since these determinations result from determining reflection, they mirror its structure in each holding a certain indifference to the others, whilst at the same time relating the object as a whole. Each determination takes from positing reflection the fact that it encompasses the whole of the object, and from external reflection that it is able to stand on its own. Determining reflection retains two features of its determinations from external reflection. On the one hand, these determinations retain their indifference to one another, and on the other, each still represents the whole of the object, leading to its status as essential.

The Determinations of Reflection

Identity

We can now move on to discuss Hegel's determinations of reflection. The first of these determinations comes from what it is to be an essential determination of reflection. This first determination, identity, arises from the immediacy of reflection. Identity emerges as the first determination as it captures the self-identical nature of essence in its immediacy. It at first appears as the $A = A$ of the understanding, which

"grasps everything finite as *something identical with itself and not inwardly contradicting itself*" (*EL*, §113 Rem.). We saw, however, that once we had reached the level of reflection, the conception of the relations between essence and appearance as essentially static entities held apart from one another had dissolved. Identity "is not that equality-with-self that *being* or even *nothing* is, but the equality-with-self that has brought itself to unity" (*SL*, 411). Reflection showed the object to be a constant movement back into itself. If identity is to be conceived of as "in the first instance, essence itself" (*SL*, 412), then just as absolute negation contains a moment of positing of the non-being of the other, so identity requires, in its return into self as $A = A$, a further moment of absolute difference. Without this moment, the identity itself lacks all determination as the two terms collapse into one another. In order for identity to maintain itself as identity, therefore, it must exclude this second moment which differs from it. On this basis, the meaning of identity now rests on difference. Identity differs from difference by excluding it. In making this move, however, identity turns difference itself into a determination of essence, as now difference is given a persistence as that which seems to fall outside of identity.

Difference

Difference therefore holds the same position which seeming held in relation to essence within reflection. That is, difference is the movement of identity returning into itself, just as seeming turned out to be an integral movement of essence itself. As such, it is to be conceived of as the pure returning which had been excluded from the concept of identity. Identity, however, represented the beginning and end of differing, in the form of the points of departure and return through the structure of the $A = A$. This means that if we are going to attempt to characterise difference, we cannot conceive of it in terms of these points. "In the absolute difference of *A* and *not-A* from each other, it is the *simple*

not which, as such, constitutes it" (*SL*, 417). Difference thus becomes a pure differing from itself, "not difference resulting from anything external, but self-related, therefore simple difference" (*SL*, 417). What Hegel is pointing to here is that we cannot see difference as the difference between two terms, as such a difference would rely on a prior identity, that is, they would differ in respect to something which was an identical determination, as we saw on page 62 was the case for Aristotle, where all differences in species were to be related to identity in the genus. Difference rather simply differs from itself; it is not a relation between two beings, but a relation purely to itself. Through this self-relation, however, difference turns out to be identity, as in differing from itself, difference differs from difference, and what differs from difference is identity. "Difference as thus unity of itself and identity, is *in its own self determinate* difference" (*SL*, 418). In incorporating identity within itself, difference doesn't cease to be difference, however, once again much as seeming turned out to be essence without ceasing to be seeming.

Diversity

Identity showed itself, in order to assert itself as pure identity, to incorporate the concept of difference within itself. In a similar manner, difference was seen to include a moment of identity in order to complete the movement of differing from itself. With this result, therefore, we have shown that the concept of difference includes identity, and *vice-versa*. The difference between them now appears purely as a matter of emphasis. Just as we saw with seeming and essence, however, this opens the way for the two terms to collapse into a new, external relationship. As each contains the whole within it, neither one requires the other for its existence. The two terms are now seen to be equivalent to one another. As both are capable of supporting themselves, we now have a situation whereby the two hold purely external relations with one another. The

connection between these two terms now requires a moment of external reflection in order to generate these external relations. Whilst reflection compares these two terms, we should note that these comparisons cannot here take the form of relations of identity or difference. In the preceding analysis, both identity and difference showed themselves to have reflective structures. We were dealing with self-identity and self-difference. As such, neither of these terms is applicable to the activities of external reflection. Instead, this relation is carried out through the categories of likeness and unlikeness, which are identity and difference's external (both to the diverse and to one another) determinations.

Likeness and unlikeness are indifferent to each other to the extent that likeness involves no reference to unlikeness, as one can simply call two things alike without having to bring in the concept of unlikeness. In themselves, these two moments maintain a degree of separation from one another; that which is alike is alike, and that which is unlike is unlike. If comparison is to properly relate both objects to one another, however, it must take up both of these moments simultaneously. It is this movement which leads to the transition from diversity to opposition.

Opposition

When we looked at the concepts of identity and difference, we noted that both categories entailed and contained the other. As such, we were led to a purely formal difference between difference and identity. With the development of the determination of diversity, however, these two categories showed themselves to be determinate, but only related purely externally. These first two movements show parallels with the process of reflection itself, with the dialectic of difference and identity mirroring the dialectic of positing reflection. Similarly, diversity mirrored external reflection. Now,

however, with opposition, these two terms will become related directly to one another, leading to the third, the movement of comparison, falling away. Whilst the likeness and unlikeness of two objects may be a matter of indifference, when we look at the concepts of likeness and unlikeness themselves, we see that they can only be understood in opposition to one another. Once this explicit relation to the other is made clear, we move beyond the like and unlike to another set of categories: the positive and the negative. These categories differ from likeness and unlikeness in that the positive is both alike and differentiated from the unlike, and the negative is differentiated from the like. As we now have an inherent relation, the third drops out of our analysis, and the two terms are now directly related. As they are directly related, and in fact, within the context of the moment of opposition, determine one another, we can move to consider the positive and negative as one single concept of 'positive-and-negative,' in which each term continues to oppose its other. Tracing back through the dialectic, we can see that likeness derived from identity, but allowed a relation between indifferent objects, and similarly, unlikeness took on board the *not* which was basic to the concept of difference. As these terms further developed into the positive and negative, we can now see that once the two terms are connected immanently rather than externally, each contains the other, as each is internally differentiated (negatively) from the other, but maintains its own identity (positively) through this process of differentiation. Furthermore, a parallel emerges with the movement from the essential and inessential to essence and seeming. With those two terms, it appeared at first that each term was interchangeable with the other as they were indifferent to each other. If we reflect on the fact that as the dialectic progressed it became clear that each of these terms was in fact determined by its relations to the other, it should come as no surprise that prior to this, when these relations themselves were considered to be external (and hence in a sense arbitrary) and generated by a third, the relative meanings of these terms also became arbitrary. Once the immanent relations of these terms to one another became clear, however, we were able to distinguish them from one another. Likewise, we can now understand the positive and negative as

differing from one another, even whilst they include one another. Thus, whilst the positive includes the negative, it includes it inessentially as that which is negatively excluded, thus preserving its essential positivity. Likewise, the negative, in order to be self-contained, essentially excludes its relation to the positive. In doing so, however, it excludes itself as self-contained being, as the positive is itself self-contained being.

Contradiction

We saw in opposition that each term contained the other. As well as necessarily containing their other, each term also necessarily excludes its other. We see further in contradiction that by excluding what is included, the term therefore excludes itself. The positive can only gain its meaning through the exclusion of the negative. This very process of exclusion, however, is negative. Therefore, in the very moment of its coming to be pure positivity, the positive includes a moment of negativity which itself excludes the positive. The positive, in being negative, thus excludes itself. For the negative, likewise, we have a structure that both includes and excludes itself. We can see this equally with the negative, as the negative has been shown to have a reflexive character which allows its identification as a category. This reflexivity, however, means that the negative relies on a moment of self-identity in order to perpetuate itself. As such, however, the negative must exclude itself. Thus each both includes and excludes its other, and itself, leading to contradiction. This contradiction is resolved by the movement of ground, which is "the unity but equally the distinction of identity and distinction" (*EL*, §121 Add. 1).

Contradiction and Movement

We can now move on to discuss the significance of the result that reflection leads through dialectical necessity to the concept of contradiction. As we saw, the determination of contradiction emerged out of the contradictory nature of the subject matter itself, where the positive and the negative excluded what had to be included. When we ask what it is that leads us to contradiction, it is the fact that the structure of reflection is the structure of a movement, whereby nothing returns to itself through a self-negation, whereby it is not what it is. As this movement is a movement of returning, however, reflection also exhibits itself as a structure of stability. These two differing determinations are thus held simultaneously. Movement generates contradiction, but contradiction itself generates movement, as it forces the object to renounce its previous form and adopt one which resolves the tensions inherent within it. Ultimately, the very fact that the object of our inquiry has the structure of reflexivity means that it is impossible to resolve these contradictions. To do so would be to pay the price of banishing all movement from the world. As we saw in the second chapter, this was the approach taken by both Aristotle and Russell, who, by denying any self-referentiality to the object of our enquiry, were able to prevent us from needing to think contradiction. This was achieved, however, at the cost of both removing all movement from their systems, as well as removing the possibility of thinking the system (and by implication, the world) as a totality. Whilst those thinkers held to philosophies of identity, for Hegel, contradiction actually provides a proper understanding of this concept too, as, once we get beyond purely formal identity, a concept obtains real self-identity precisely when it "contains the determination of *distinction* essentially" (*EL*, §116).

The Speculative Proposition

We saw in the preceding chapters how Deleuze argues against the idea that the subject-predicate structure captures the total structure of the world. This criticism derives from Bergson's insight that propositional thinking essentially relies on a certain notion of space as a homogenous medium through which terms can be brought into relation. In conceiving of the world in this way, thought gains greatly in its ability to manipulate the world, as it allows the application to it of the categories of mechanism. In applying these categories, however, thought sacrifices its understanding of the true nature of relations as interpenetrative. As we have also seen in relation to Kant, Hegel is critical of the structure of judgement, seeing Kant's employment of it as leading to a schism between thought and its content. In attacking Kant's use of the judgement, however, Hegel is not opposing the use of language altogether.[26] That "we think in words"[27] is not something that can be overcome simply by showing the inadequacy of certain modes of linguistic thought, since if language is to be essential to thought, to do away with language is also to do away with thought. As Gadamer puts it, language "is not a self-effacing and temporary medium of thought or merely its casing"[28] but rather is that which gives determinacy to thinking. We can see furthermore that Hegel frequently uses linguistic terms to aid in the explication of dialectical transitions and concepts, such as the already noted examples of Hegel's plays on *Wesen* and *Erinnerung*, and revels in the "speculative spirit" (*SL*, 32) of the German language. These two aspects of Hegel's thought can be reconciled, however, provided we make a distinction between two different types of thought, and consequently, two different approaches to language. The ground for this distinction between two types of thought is to be found in the distinction between the finite and the infinite, and particularly in their reconciliation. In this regard, we will now return to the problem which occurred with Kant's philosophy in relation to the infinite, before moving on to the speculative proposition. This will furthermore give

us the understanding of the Hegelian infinite necessary to see in the next chapter how it relates to Deleuze's classification of Hegel as a thinker of infinite representation.

In discussing Kant, we noted how Hegel attacks the presupposition of the Kantian system that thought is finite. At the beginning of the *Logic*, we were not in a position to make any judgement about the actual nature of thought for two reasons, however. First, the nature of thought itself had not been elucidated, and second, and perhaps more importantly in this context, the meaning of the terms finite and infinite themselves had not been clarified. As such, Kant's classification of thought as finite rests for Hegel not just on his unwarranted assumption of this fact, but furthermore, on the unwarranted assumption that what it means to be finite is known in advance of the metaphysical deduction of the categories. As we shall see, once Hegel has made clear what he believes the infinite to be, he will agree with Kant that "finite thinking certainly has to do only with appearances" (*EL*, §60 Add. 1), whilst arguing that "this stage of appearance is not the end of it. There is a higher land, but for the Kantian philosophy it remains an inaccessible beyond" (*EL*, §60 Add. 1). The deduction of the category of infinity, and its correlate, the finite, takes place in the doctrine of being, but as we shall see, it mirrors the results which we have already found in regard to contradiction in the doctrine of essence. For this reason, we will only briefly sketch the deduction of the category of infinity before moving on to look at the implications of this new understanding of the category for Kant, and its relation to contradiction.[29]

The notion of the infinite develops for Hegel out of the idea of a determinate being, which is determined by having a certain quality. In being determined, however, we should note that the determination not only characterises what a thing is, but also acts as a limit, beyond which the thing would cease to be what it is. This limit, however, can also be seen as a negative limitation, and as such constrains the internal positivity of the being in question. As a result of this, being as positivity immanently generates an

'ought' to go beyond this negative limitation. The finite therefore becomes related to the beyond, which as opposed to the limited finitude of the determined being is seen as infinite. The infinite beyond, however, shows itself to also be finite, as it is an infinity which is also bounded by the limit which defines the finite being. As this being goes beyond itself, we have the beginnings of an infinite process of finite beings surpassing themselves. When we look more closely at this sequence, we see that the finite persists within the infinite, rather than dissolving into the infinite, as finite being only ceases to be on the condition that finite being comes to be. Thus we have a new form of infinite which is the infinite perpetuation of the finite. We can now see that not only is the infinite finite by its necessary contrast with finitude, but the finite also becomes infinite through its perpetuation. The infinite and the finite therefore show themselves to each contain the other, and, stepping back to see the process as a whole, we will realise that another form of infinity has emerged as the synthesis of the two which incorporates finitude into the whole. Thus we move from the bad infinite of infinite repetition to a new conception which is essentially the unity of two of terms. This is what Hegel calls in the *Jena Logic*[30] the infinitely simple. "Only the infinitely simple, or that unity-and-multiplicity, is one" (*JL*, 36).

It should be reasonably clear that the true infinite has an analogous structure to contradiction. "Genuine infinity … is not a series that always has its completion in some other yet always has this other outside itself. Rather, the other is in the determinate itself; it is a contradiction" (*JL*, 35). When we looked at contradiction on page 193, we saw that each of the terms both stood opposed to the other, but also necessarily united with it in that the two terms could only be made determinate when in relation with one another. Thus contradiction was an opposition which required no third to bring the terms into relation with one another. This same relation holds for the infinite: "The simple and infinity, or the absolute antithesis, make no antithesis save this very one that they are absolutely connected, and insofar as they are opposed, they are by the same token

absolutely one" (*JL*, 36). This process is what Hegel calls "the absolute contradiction of the infinite" (*JL*, 38). We saw that on Kant's reading, infinite thought was an impossibility, both in terms of the sublime, and in terms of the intellectual intuition of God, which was intuition precisely because of the impossibility of thought being infinite. That is, whilst we could think *of* the infinite, the finite structure of cognition meant that thought itself could not be infinite, as we shall see it is for Hegel once we properly understand the nature of the speculative proposition. For Kant, with the sublime, the infinite shows itself as infinite magnitude. As such, Kant's conception of the infinite is limited to the first concept explicated in the *Science of Logic*, what Hegel calls the bad infinite. According to Hegel's dialectic, this concept is inadequate, however. "The contradiction that bad infinity expresses, both that of infinite aggregate and that of infinite expansion, stays within the acknowledgement of itself; there is indeed a contradiction, but not *the* contradiction, that is, infinity itself. Both get as far as the requirement that the two alternating members [positing and surpassing the limit] be sublated, but the requirement is as far as they go" (*JL*, 33). We can see here that there are, therefore, two concepts of contradiction, just as there are two concepts of the infinite. The bad infinite, and 'bad contradiction' exclude the possibility of the unity of the two terms, whereas in the good infinite and 'good contradiction', this unity is incorporated, as each term "only is within its opposite" (*JL*, 35). Hegel wants to show, however, that this concept of the sublime, the bad infinite, overcomes itself into the concept of the true infinite. With the opposition between the finite and the infinite sublated, the possibility of a Kantian restriction of cognition to the finite collapses. The finite immanently overturns its own limitations, becoming infinite. Therefore, the infinite can be cognised by thought, and if this is the case, the Kantian restriction of the metaphysical deduction to a taxonomy of terms statically derived from the functions of judgement, as well as the possibility of a separation of thought and being collapses. Whilst Hegel therefore begins the *Logic* without assuming the incongruity of thought and being, the *Logic*, in its development, further shows the alternative position to arise

from a false problem. In this sense, Hegel is rebuking Kant not for relying on the finite understanding, but for not taking the understanding to its limit. "It is usually said also that the understanding should not go too far. This contains the valid point that the understanding cannot have the last word. On the contrary, it is finite, and, more precisely, it is such that when it is pushed to an extreme it overturns into its opposite" (*EL*, §80 Add. 1). Within this movement, however, we still preserve the distinctness of the two types of thought. "When we are discussing thinking we must distinguish *finite* thinking, the thinking of the mere understanding, from the *infinite* thinking of reason" (*EL*, §28 Add. 1). It is this distinction which will allow us to understand how the same structure of judgement can be used to express two different types of truths depending on whether it is meant to be taken up by the finite understanding or infinite reason.

We have already looked at some aspects of Hegel's misgivings over Kant's understanding of the judgement. It is in the *Phenomenology of Spirit* that he provides an exposition of the difference between the type of judgement found in Kant, which is to be associated with the understanding, and the speculative proposition, which relates to reason.[31] For Hegel, our ordinary understanding of the proposition relies on the presupposition of the subject of a proposition as an "objective, fixed self" (*PS*, 37). Once we have established a firm basis for the judgement, we can proceed by attaching predicates to the subject which provide us with knowledge about its nature. For the finite understanding, this process can be understood through what Hegel calls 'picture thinking' (*PS*, 37), a process whereby the connection is seen as taking place as if in a sensuous manifold, and thus in accordance with Kant's dictum that "*concepts of the understanding* must, as such, be demonstrable … i.e. it must always be possible for the object corresponding to such concepts to be given in intuition."[32] In this case, the connection between the subject and predicate, whilst informative, is not necessitated by anything other than the judging subject who associates the subject and predicate, since the "passive subject inertly supporting accidents" (*PS*, 37) cannot actively relate to the

predicates. The speculative proposition, which relates to infinite reason, differs from the empirical proposition in that it relates categories to categories, rather than subjects to predicates.[33] Whereas in a non-speculative proposition, we have a structure such as "the rose is red", Hegel gives us as an example of the speculative proposition "God is being" (*PS*, 38). Given that the second term is not a predicate, but is itself a subject, we cannot understand this proposition in the same way as a non-speculative proposition. This is because the second subject cannot be merely attached to the first, but instead determines the latter's essence. "The passive subject itself perishes" (*PS*, 37) in this movement, as the second term no longer has the difference from the subject that allowed the predicate to remain static in relation to the subject. What is lost, however, is not the subject itself, but rather its passivity, as the second term is meant to show us the essence of the first. Thus, we are "thrown back" (*PS*, 39) onto the original term as a subject, but as one which has been altered in the movement back and forth between the two terms. What is important about the speculative proposition is first that it allows terms to be identified which remain different from one another. "In the philosophical proposition the identification of subject and predicate is not meant to destroy the difference between them, which the form of the proposition expresses; their unity, rather, is meant to emerge as a harmony" (*PS*, 38). Second, the speculative proposition is inherently in motion, and in fact, "it is merely dialectical movement, this course that generates itself, going forth from, and returning to, itself" (*PS*, 40).

The speculative proposition embodies the structure of contradiction, and this contradiction can be interpreted in two ways. The finite understanding, in attempting to grasp the proposition "is checked in its progress, since that which has the form of a predicate in a proposition is a substance itself" (*PS*, 37). The contradiction is thus presented to finite understanding in the form which cannot be demonstrated in intuition, namely two subjects which both differ from one another, and yet are the same. There are two possible responses to this situation. The first is to attempt to maintain the

sovereignty of the finite understanding and complain of the "unintelligibility of philosophical writings" (*PS*, 39) by continuing to see the proposition as embodying either the subject-predicate structure, or the structure of a tautology. This arises "from the mistaken view that the inadequacy of the finite categories to express truth entails the impossibility of objective cognition" (*EL*, 2nd preface, 6). Alternatively, one may be forced to move beyond a finite understanding by giving up the attempt to understand the proposition through picture thinking. In this case, one continues to maintain that the proposition contains a contradiction, but no longer holds that a contradiction cannot be thought. By moving to an understanding grounded in infinite reason, the subject moves from a negative understanding of contradiction to a positive, contentful understanding. Once we have reached this point, movement has become an immanent part of thought, as the speculative proposition is a *transition* between the two terms. Without this mode of cognition, the *Science of Logic* as a whole is incomprehensible, beginning as it does with the identity and difference of being and nothing. In fact, the whole structure of the *Science of Logic* embodies this movement to speculative or infinite thought in its self-relation of form and content: "thinking is infinite because it is related in thinking to an ob-ject that is itself" (*EL*, § 28, Add.). One of the most important implications of this reading of the structure of the proposition is that it heads off any simple Bergsonian criticism of Hegel's use of language within his system of logic. This is the view that language is essentially spatial, and so the incorporation of language into metaphysics means that one has already debarred the possibility of understanding duration. Whilst this may be true of finite judgements, by moving to a conception of the proposition which by necessity cannot be captured in intuition, Hegel has kept open the possibility of a linguistic relation to the world which does not rely on a mechanistic, and atomistic, notion of space.

We need to look now at how Hegel's transformed conception of dialectic relates to the difficulties which we encountered in Aristotle's metaphysics. Whilst the question of the incorporation of movement into the dialectic is solved by the move to a philosophy which incorporates becoming into the structure of its logic itself through the inclusion of the infinite and contradiction, a second problem emerged concerning Aristotle's concept of focal meaning. This problem essentially affected Aristotle's philosophy at two levels, the large and the small. To begin with the large, Aristotle's problem was to show that the concept of being could be understood as a unified concept. This difficulty emerged because each concept was understood through a method of division, through which it was determined by the addition of a difference to a genus, as in the case of man as the rational animal, in which the difference, 'rational,' provides a further specification of the genus, 'animal.' When we proceeded to the top of this hierarchy, however, we reached a term which could not be specified in these terms. This highest term, being, could not be given a difference to define it, as this would presuppose a genus above it from which it was specified. This led to our understanding of the world being necessarily and structurally fragmented. To rule out this possibility, Aristotle moved to the position that each of the species of being was different from one another, but semantically related to each other. When we asked how we were to understand this idea of semantic relation, and in particular how to differentiate it from the view that the categories simply represent a homonymous 'heap' of unrelated terms, we were referred to the model of the individual, and in particular, how the semantic content of a term could 'drift' from its central meaning. This was in terms of the distinction between the essence of a thing and its accidents. The difficulty was how to determine which of the properties of the individual were to be taken as essential, and which as accidental. In understanding this distinction, however, we were referred back to the genera, since, "the proximate matter and the form are one and the same thing, the

one potentially, the other actually. Therefore to ask the cause of their being one is like asking the cause of unity in general; for each thing is a unity, and the potential and the actual are somehow one" (*MP*, 1045b). As the meaning of the general category of being or unity is what we were seeking, the relation of essence to accident now becomes problematic. The problem to be solved, therefore, was how one was to understand the movement between different semantic concepts, and particularly the concept of essence.

We should therefore note to begin with that the concept of essence in Aristotle was defined as "what it was to be that thing," thus carrying with it the idea of the pastness of the object. Likewise, essence for Hegel was "past – but timelessly past – being" (*SL*, 389). In the case of Aristotle, we saw on page 69 that the meaning of this reference to pastness was to be found in the stability of a retrospective analysis of an object. It was only once the object had ceased becoming (or at least, when the object was hypothetically considered under this aspect) that we could determine what that object essentially was. This meant, of course, that the essence of an object had to remain separate from the appearing of the object, since the appearing of the object, as movement, was antithetical to its fixed form. Essence, for Aristotle, therefore, in the end represented another way of banishing transition, an approach which further problematised our attempt to understand semantic drift, since semantic drift is precisely a movement of transition of meaning.

Whilst the Hegelian characterisation of essence is on the surface similar to that of Aristotle, as we have seen, the meaning of this characterisation is radically different. Whilst essence (*Wesen*) is still explicitly related to *gewesen*, the past participle of 'to be', the introduction of the idea of transition into the logic itself means that essence is now understood as the history of mediations which have allowed the object to appear in its immediacy. What this means is that the nature of essence is now conceived of along genetic lines: the essence of a particular term simply is the totality of mediations which

have generated that term. This means that the problem of explaining the issue of unity can be resolved. Instead of terms being externally connected together, leading to Aristotle's difficulty of explaining the nature of the connection which obtains between them, for Hegel, as the concept itself generates other terms, we no longer have the difficulty of explaining how these different terms are related to one another. "The sense of the definition, like its necessary proof, lies in its development alone." (*EL*, 2nd pref., 6) As we also saw above, since essence is the network of mediations (page 177) which underlie the apparent immediacy of the object, essence will itself emerge as not opposed to appearance, but rather as both different from and identical to it. The immediacy will only be a seeming immediacy, but the essence, likewise will only be understood as a result of its expression. Both terms will turn out to be unified in the same circuit of reflection. By moving to this understanding of essence, therefore, Hegel simultaneously solves the two problems of the Aristotelian system. Unity is given because the relations between categories can be understood simply by tracing back their genetic evolution, thereby showing their immanent relations, and essence is now seen as inseparable from the process of seeming, meaning that we no longer need to try to isolate a fixed element from the flux of appearing in order to understand the object.

Conclusion

In this chapter I hope to have shown that Hegel's ontology provides a coherent strategy for the resolution of the problems which plagued the Kantian and Aristotelian systems. This is achieved through the attempt to provide a purely immanent description of the development of thought, rather than relying on an arbitrary deduction of the categories. In adopting such an immanent strategy, thought initially brackets any assumptions which we could make about the subject matter. What we found, particularly in the case of the categories of the finite and the infinite is that, as the object of our

enquiry is thought itself, once the categories upon which any assumptions we could have made have been disclosed, we discover that their previous assumption would have been illegitimate. We can now see that Deleuze and Hegel provide two alternative accounts of how the difficulties in classical reasoning can be overcome. We have also seen that in their diagnoses, both systems agree on what is problematic in classical thought. In the final two chapters we will explore these two systems' relations to each other, and in particular whether difference itself is sustainable, or must always be pushed to its limit as contradiction.

Chapter Six – Hegel and Deleuze on Ontology and the Calculus

Introduction

The aim of this chapter will be to explore the differences and similarities between Deleuze and Hegel's ontologies through an analysis of their views on two fields, the differential calculus and the Kantian antinomies. In the field of mathematics, it is the calculus which most clearly expresses the Notion for Hegel. Indeed, whilst by Hegel's time, the foundations of the calculus had been frequently attacked, most notably by Berkeley, "it is the inability to justify the object [i.e. the differential coefficient *dy*/*dx*] as *Notion* which is mainly responsible for these attacks" (*SL*, 254). Thus for Hegel, if we are to understand the meaning of the contradictions which seem inherent in the foundations of the differential calculus, we need to move to a dialectical understanding of these foundations. Returning to Russell for a moment, we can see the implications of this relation between the antinomies of the calculus and dialectic in his early acceptance and later rejection of Hegelianism. Whilst the young Russell "found comfort for a time"[1] in a form of Hegelianism, this was premised on the fundamentally contradictory nature of mathematics, and in particular, the theory of infinite numbers,[2] and the foundations of the calculus. It was only after "Weierstrass, soon after the middle of the nineteenth century, showed how to establish the calculus without infinitesimals, and thus at last made it secure"[3] that Russell was able to apply the method of logical analysis to philosophy. Thus, for Russell, a dialectical understanding, not only of mathematics, but also of the world in general, was the only approach open in philosophy until the development of set-theoretical foundations of mathematics.[4] As we have dealt with some of the problems inherent in the set-theoretical perspective in chapter two, we will not return in detail to this interpretation here, but instead merely note that for Deleuze as well as Hegel,[5] what is important about the calculus is to be found in the "so-called barbaric or pre-scientific interpretations" (*DR*, 170). In fact, for Deleuze, the calculus is even more central to his

philosophy, since an interpretation of the calculus which moves away from the set-theoretic, and therefore spatial, understanding of mathematics opens the way for a philosophy of difference which truly understands difference apart from representation. Such an interpretation is essential for the Deleuzian project. The differences between these two approaches therefore provide archetypal exemplars of how Deleuze and Hegel approach the problems of representation. As we shall also see, Deleuze and Hegel's interpretations of the antinomies mirror those of the calculus, and show the applications of these principles outside of the mathematical realm. A return to Kant will once again show that Deleuze must be understood as a thinker in the post-Kantian tradition. We will now begin with a brief discussion of the workings of the calculus before moving on to look at Hegel's and then Deleuze's interpretations.

The Calculus

The basic approach of the differential calculus was first used in relation to motion by the 14th century mathematician Oresme,[6] and came with the recognition that the velocity of a system could be represented by a line on a graph. If we represent on a graph the relation between distance travelled and time taken to travel that distance by an object, it becomes possible to determine the velocity of the object by dividing the distance travelled by the time, hence velocity is measured in terms such as miles per hour, metres per second, etc. If the velocity is constant, the relation between distance and time will be proportional. This means that the line representing the moving object will be straight. If, therefore, we wish to determine the velocity of the object, we simply need to take a section of the line, and divide the distance travelled over that time, which will be represented by the change in the value on the distance axis over the length of the section, by the time, which will be represented by the change in the value of the time axis over the length of the section. As the two values are directly proportional, a section of any

arbitrary length will provide the same result. Moving on from Oresme, if we are dealing with an object moving at a velocity which is not constant, then this procedure cannot be used, as we were able to determine the velocity at any point using the previous method only because the velocity was the same at every point. Instead, however, we can measure the velocity at any point of a system with a changing velocity by drawing a graph of the function[7] of this change, that is, of the relation of distance to time, and drawing a line which runs parallel to a particular point on the curve.[8] This produces a vector of the velocity of the system at this particular moment. The difficulty with this approach is that it can only be approximate, as we are attempting to draw a line through a point, which in itself can seemingly have no direction. The alternative, to draw a line through two points of the curve, is equally flawed, as although it gives us an accurate line, we are dealing with a curve, and so the tangent we are now drawing will not represent the velocity at one particular moment. Leibniz's solution to this difficulty was to draw a line between the point whose velocity we wish to measure, and another arbitrary point on the curve, and then to imagine the distance between these two points decreasing towards zero. As we now have a straight line between these two points, we can treat the case in the same manner as the case of constant velocity described above, measuring the change in values of both axes along a length of the line. Thus mathematically, we end up with two lines, one representing the change over the section in terms of distance, and one in terms of time, neither of which on its own will have any determinate value, as the lines are infinitely short, but when divided, one by the other, will give a vector at the particular point. As the axes of the graph can represent more than simply time and distance, these values are referred to more generally as *dy* and *dx*. Further, using Descartes' insight that a function can be represented by a graph, and a graph by a function, we can define this procedure as a function of the original formula, dispensing with the graphical method altogether.

Hegel writes that "the whole method of the differential calculus is complete in the proposition that $dx^n = nx^{n-1}dx$, or $\frac{f(x+i)-fx}{i} = P$" (*SL*, 274). Whilst these statements look intimidating, they are in fact simply algebraic representations of the geometrical function. The second equation simply represents the gradient function of a curve. Gradient, as we saw in the case of straight lines, is defined in terms of the change in the value on the *y* axis divided by the corresponding change on the *x* axis, thereby giving a ratio. Now, if we take *i* to be a length along the *x* axis, assuming that $y = f(x)$, that is, that the value of *y* is dependent on the value of *x*, the top half of the equation will give us the difference between the values of *y* when $y = x$ and $y = x + i$. That is, the top half of the equation will give us the corresponding value of the rise on the *y* axis for a run of length, *i*, on the *x* axis, which begins at any arbitrary point on that axis. We thus have a function which will provide us with the value of the tangent at any point on the line. The first equation follows from this second equation through the substitution of an infinitesimal value for *i*. This substitution is the equivalent of making *dy* and *dx* infinitesimal values in Leibniz's work. Although showing that the first equation is true for all values of *n* involves some complex mathematics, it is an easy result to prove for any specific function. We will therefore focus on one particular function, that given the equation, $y = x^2$, then $\frac{dy}{dx} = 2x$.

The first step is to rewrite the equation, $\frac{f(x+i)-fx}{i} = P$, replacing the value *i* with *dx*, the Leibnizian infinitesimal, and replacing $f(x)$ with $(x)^2$, as in this case, $y = x^2$. We thus arrive at the function:

$$\frac{(x+dx)^2 - x^2}{dx} = \frac{dy}{dx}$$

We can now multiply out the brackets on the top half of the equation to give:

$$\frac{x^2 + 2xdx + dx^2 - x^2}{dx} = \frac{dy}{dx}$$

Which in turn reduces to:

$$\frac{2xdx + dx^2}{dx} = \frac{dy}{dx}$$

Cancelling out *dx* therefore gives us:

$$\frac{dy}{dx} = 2x + dx$$

Thus the differential of x^2 turns out to be $2x + dx$, where, given that the value *dx* is infinitesimal with respect to *x*, we can just write $\frac{dy}{dx} = 2x$ (*dx* retains a value in relation to *dy*, which is also infinitesimal, and so is retained on the left side of the equation). Thus, it is easy to show that for the value *n* = 2, Hegel's first equation follows from his second. For higher values, a similar result can be obtained with more time consuming applications of such elementary algebra. The procedure that we have just carried out is called differentiation. If we reverse this process, however, we have a procedure which allows us to find what is known as the primitive function of the derivative, i.e. the function from which the derivative is differentiated.[9] This process is known as integration. For modern set theory, the interpretation given above has given way to an interpretation in terms of the concept of limit, in order to avoid certain difficulties which emerge from the paradoxical use of the infinitesimal, which we will come back to below in our discussion of Hegel's theory of the calculus. In order to escape these difficulties, the modern set theoretic interpretation of the differential calculus largely ignores the individual values of *dy* and *dx*, instead giving a value simply to the ratio, *dy*/*dx*, as a whole, called a differential. This is accomplished by admitting the concept of a limit of an infinite series. An infinite series such as 1/2 + 1/4 +1/8 +1/16... may be said to approach the value, 1, and this allows us on the modern interpretation to equate the derivative, not with the final, infinitesimal value of the series, but instead with the limit of the series. We do not in the

case of the series need to worry about the fact that any finite summation will always be less than one. In a similar way, the differential is conceived of under the set-theoretic interpretation as precisely equalling the value of the gradient at any particular point. By defining the derivative to be the limit of the ratio, rather than the values of *dy* and *dx*, questions about what happens when or whether the ratio actually reaches this limit are put out of play. This is the method of Weierstrass, which allowed Russell to move away from an antinomic interpretation of mathematics, and therefore also from his early Hegelianism. We will now turn, however, to Hegel's theory of the calculus.

Hegel and the Calculus

The basic function of the differential calculus is to find the gradient of a curve at a particular point. With a straight line, it is easy enough to find the gradient of a line, as the gradient will be constant along the length of the line. The gradient can be defined by the relation of the rate of change on the *y* axis to the rate of change on the *x* axis. Thus on a straight line, we simply need to take two arbitrary points on the line, (x_1, y_1) and (x_2, y_2), and find the rise or fall between them:

$$\frac{y_1 - y_2}{x_1 - x_2} = \text{gradient}$$

As the gradient is constant, we can find the rise or fall of the line by taking an arbitrary distance. The difficulty with a curve is that as the gradient varies, we need to find the gradient of a point. Gradient, however, seems to involve a difference. As we saw on page 209, for Leibniz, the solution was to resort to the notion of the infinitesimal, as an infinitely small difference between two points, *dy*. As this difference was infinitely small, it could be discounted for the purposes of calculation, but, as it retained a magnitude relative to *dx*, it could be used to form a ratio, *dy*/*dx* which had a determinate value. For

Deleuze, this approach forms the foundation for his interpretation of the differential calculus, although he takes advantage of later developments which finesse the paradoxes resulting from it. For Hegel, however, this Leibnizian approach must be rejected since the idea of neglecting infinitesimal values, whilst it may lead us to the correct results, does not give us the rigour essential to a proper mathematical proof. "Even if ordinary common sense in fairness allows such inexactitude, all geometricians reject this conception. It is quite obvious that in the science of mathematics there cannot be any question of such empirical accuracy; mathematical measuring by operations of the calculus or by geometrical constructions and proofs is altogether different from land-surveying, from the measuring of empirical lines, etc" (*SL*, 258). In speaking of Leibniz in particular, Hegel asserts that "it is chiefly this call to neglect which, along with a gain in facility, has given the calculus the appearance of inexactitude and express inexactness in its method of procedure" (*SL*, 258). The target of Hegel's criticisms is especially clear in the first postulate of the calculus given in L'hôpital's *Analyse* (1696), which provided the first general introduction to the method:

> *Postulate I.* Grant that two quantities, whose difference is an infinitely small quantity, may be taken (or used) indifferently for each other: or (which is the same thing) that a quantity, which is increased or decreased only by an infinitely smaller quantity, may be considered as remaining the same.[10]

It is this postulate which allows us to discount the effects of dx in the results of our derivation of the differential function. In rejecting the method of Leibniz, however, Hegel did not reject the calculus, but instead turned to the model provided by Newton.

Whilst Leibniz was the first to publish his theory of the calculus, Newton had independently developed a form of the calculus under the name of the method of fluxions.

This led to the dispute over priority which ultimately led to the stagnation of British mathematics for almost a hundred years as works written in the Leibnizian notation (which benefited greatly from Leibniz's parallel work on the possibility of a *characteristica generalis*), and dealing with the calculus from an algebraic perspective, surpassed what could be achieved with the more cumbersome notation of Newton with its geometrical emphasis.[11] Another important consequence of this dispute, however, was the work of Newton to try to differentiate his own method of fluxions from the calculus, which were practically equivalent, through his attempt to determine a more rigorous foundation. This was the method of prime and ultimate ratios which Newton first published in the *Principia* (1687). The key to this approach was to incorporate the idea of time into his demonstration. As he writes in the *Quadrature of Curves*:

> I don't here consider Mathematical Quantities as composed of parts extreamly small, but as *generated by a continual motion*. Lines are described, and by describing are generated, not by the apposition of Parts, but by a continual motion of points. Surfaces are generated by the motion of Lines, Solids by the motion of Surfaces, Angles by the Rotation of their Legs, Time by the continual flux, and so on in the rest. These *Geneses* are founded upon Nature, and are every Day seen in the motion of Bodies.[12]

Newton uses this dynamic conception of the generation of the curve itself to posit the idea that the curve could be understood as the movement of a point with a certain velocity. With this, it is a small step to see the gradient of the point on the curve as representing the instantaneous velocity of the point itself, which we find by noting that the time it takes the point to move can be reduced to zero. Under this interpretation, the differentials, called fluxions in Newton's system, do not have to be seen as being too small to affect the result of the calculation, but can actually be seen to vanish at the limit point, when $dt=0$:

> *Quantities, and the ratios of quantities, which may in any finite time converge continually to equality, and before the end of time approach nearer to each other than by any given difference, become ultimately equal.*
>
> If you deny it, suppose them to be ultimately unequal, and let *D* be their ultimate difference. Therefore they cannot approach nearer to equality than by that given difference *D;* which is contrary to the supposition.[13]

The ultimate ratio therefore functions much as the idea of a limit does in modern mathematics. If we take a normal ratio, such as 8/64, if we reduce the values of the denominator and numerator proportionately, for instance, to 4/32, or 2/16, we find that the ratio of terms itself remains unchanged. Newton uses the idea of an ultimate ratio to argue that at the limit case, where the numerator and denominator are reduced to 0/0 (where the lines become a point), we can still interpret the point as having a determinate ratio. It is this conception of the calculus, as grounded in the idea of an ultimate ratio, which Hegel takes to characterise it most adequately, albeit with the elimination of "those determinations which belong to the idea of motion and velocity (from which, mainly, he took the name, *fluxions*) because in them the thought does not appear in its proper abstraction but as concrete and mixed with non-essential forms" (*SL*, 255).[14] To see how Newton's understanding of the calculus is integrated into Hegel's dialectic, as well as why Hegel believed it to require a dialectical foundation, we will now turn to the idea of the ratio as it occurs in Hegel's remarks on the mathematical infinite in order to trace the genetic development of Hegel's own understanding of the ultimate ratio.

The Ratio in Hegel's discussion of Quantitative Infinity

In a similar way to what we saw on page 198 of the previous chapter in regard to the infinite, we need to distinguish when dealing with the quantitative infinite between a true infinite and a spurious infinite. As we shall see, in this, the true infinite will share many of the properties of the true qualitative infinite. Rather than beginning with the spurious infinite as we did in the case of quality, however, the explication of the mathematical infinite will be better explained by showing how two forms of infinite come into being together through the equation. As we saw with the qualitative infinite, the true infinite exhibited the negation and preservation of both the finite and the infinite within one whole. This overcame the spurious infinite, which was merely the perpetual repetition of the finite. Within the ratio, both these moments appear simultaneously. If we take Hegel's example of the fraction 2/7, we can represent it in the two following ways:

$$2/7 = 0.2857142\ldots$$

On the right, we have the decimal expansion of the fraction, which in this case forms an infinite series.[15] As we should recognise by now, this infinite series cannot for Hegel be construed as a true infinity, as it just represents a bare repetition of finite values. In this case, the *ought* of the series is patently clear. "What the series is meant to express remains an *ought-to-be* and what it does express is burdened with a beyond which does not vanish and *differs* from what was meant to be expressed" (*SL*, 248). When we assert an equality between 2/7 and its expansion, we have to note that while continuing the expansion of the series allows the value on the right of the equals sign to be made as accurate as required, it will always preserve a difference from the value on the left. This difference is the beyond which would have to be incorporated into the series to make it a representation of the true infinite, and therefore the source of the *ought*. For Hegel, the contradiction implied by the sign of equality combined with the necessary difference between the two

sides of the equation emerges from the fact that what we have on the left of the equation is no longer a quantitative notion, but has passed on into a qualitative determinateness, as the ratio is not tied to any specific value, but instead to their relation. This ratio cannot be captured by the expansion of the purely quantitative series of numbers on the right, no matter how closely it may approach the fraction. The fractional representation, as including that which is unobtainable from the quantitative determination (the moment of difference), has therefore become our first approximation to the qualitative mathematical infinite.

The fractional understanding of the calculus is ultimately not an adequate representation of true infinity, however. What is important about the fraction 2/7 is that it surpasses the purely quantitative determination of the series. As such, the two apparent quanta of the ratio show the property of variability, to the extent that the values of 2 and 7 can be replaced with other values without changing the overall value of the fraction. Thus, 2/7 is equivalent to 4/14 or 6/21. In all of these cases, however, we are still dealing with quanta for which the relationship is not essential. Thus, whilst the unity of the different terms is essential for the fraction itself, it is not essential for the terms which make up the fraction. Similarly, Hegel notes that moving to an algebraic description of the fraction does not overcome this limitation, as an algebraic formulation such as $y/x = a$ can equally be written in a form which does not contain a ratio, such as $y = ax$, the equation of a straight line. Variability, therefore is shown not to be the defining characteristic of the qualitative mathematical infinite, as even in the case of the algebraic variable, we still have a symbol standing in for an arbitrary quantum, that is, we have variability, but variability still conceived of as magnitude. It should be clear now why Hegel moves to the Newtonian interpretation of the calculus. In Newton's ultimate ratio, as we are dealing with the ratio of values at a point, neither of the terms in the ratio, dy/dx, can have any meaning outside of the ratio itself. "Apart from their relation they are pure nullities, but they are intended to be taken only as moments of the relation, as the

determinations of the differential co-efficient *dx/dy*" (*SL*, 253).[16] In the differential relation, we therefore have a situation whereby both the ratio itself as well as the terms can only be understood as a totality. It should be clear that this structure mirrors those of contradiction and the infinite as they were understood dialectically in the previous chapter. As we have seen that Hegel moves to a logic of contradiction in order to overcome the limitations of the finite understanding, however, we must now look at why Hegel believes that this understanding of the infinite cannot be arrived at purely through an analytic deduction.

Berkeley and the Foundations of the Calculus

Hegel writes that "the operations which [mathematics] allows itself to perform in the differential and integral calculus are in complete contradiction with the finite determinations and their relations and would therefore have to be justified solely by the *Notion*" (*SL*, 254). We have already looked at how the ideas of finitude and infinity have a different meaning when derived dialectically from those given to them by the understanding. Furthermore, we have seen that Hegel rejected Leibniz's interpretation as lacking rigour. Newton's interpretation of prime ratios in terms of instantaneous velocity also presents problems, however, as velocity is a rate of change. For Newton, these ratios were also described as nascent ratios, or the ratios at the point where the fluxions start to be. What is problematic in Newton's model, therefore, is whether such a thing as an ultimate ratio can actually be said to exist, since it also appears implicitly to rely on the dual properties of the ratio not yet having, but at the same time already having a determinate quantity. The problems of the foundations of the calculus, although known to Leibniz and Newton, were made notorious in Berkeley's treatise, *The Analyst; or, a Discourse Addressed to an Infidel Mathematician wherein it is examined whether the Object, Principles, and Inferences of the modern Analysis are more distinctly conceived,*

or more evidently deduced, than Religious Mysteries and Points of Faith. The essence of Berkeley's criticism lies in the statement that:

> If with a view to demonstrating any proposition a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterwards destroyed or rejected by a contrary supposition; in that case, all other points, attained thereby and consequent thereupon, must also be destroyed and rejected, so as from thence forward to be no more supposed or applied in the demonstration. This is so plain as to need no proof.[17]

Thus, Berkeley attacks Newton's notion of the ultimate ratio for both appearing to be unequal to zero (as the terms forming the ratio can be divided by one another), but also equal to zero (in order for the ratio to be applied to an instant). Going on to question the fluxions which make up the elements of both Newton's ultimate ratio as well as Hegel's mathematical infinite, Berkeley asks: "What are these fluxions? The velocities of evanescent increments? And what are these same evanescent increments? They are neither finite quantities, nor quantities infinitely small nor yet nothing. May we not call them the ghosts of departed quantities?"[18]

The origin of this critique therefore is that the idea of the fluxion is grounded in "direct impossibilities and contradictions"[19] which result from "the most incomprehensible metaphysics."[20] From a Hegelian perspective, however, Berkeley's criticisms can be seen as resting on a form of picture thinking which is an implication of his empiricist orientation and is based on taking a purely static view of the grounds of the calculus.[21] Hegel writes:

Although the mathematicians of the infinite maintained that these quantitative determinations are vanishing magnitudes, i.e. magnitudes which are not any particular quantum and yet are not nothing but are still a *determinateness* relative to an other, it seemed perfectly clear that such an *indeterminate state*, as it were called, between being and nothing does not exist (*SL*, 254).

The fact that there cannot be a state which contains both being and nothingness does not trouble Hegel, however. "The unity of being and nothing is, of course, not a *state*; … on the contrary, this mean and unity, the vanishing or equally the becoming is alone their truth" (*SL*, 254). Thus, the true foundation of the calculus, according to Hegel, is to be found in the results obtained in the dialectic of being and nothing which opens the *Science of Logic.* The differential calculus is therefore seen as being grounded in the fundamental dialectical moment of transition, within which the two moments of the fluxion, being and nothing, are to be taken as immanently related. We thus have a ratio of two terms, both of which are on the brink of vanishing, but which, when related to one another, give a determinate value. It is the two transitions of the fluxions, therefore, when related to one another through the form of the ratio, which generate the determinacy of the calculus. Berkeley's attempt to show the contradiction of the calculus reduces it to "a diseased condition externally induced through erroneous thinking" (*SL*, 254), essentially the thought of the finite understanding which eschews any consideration of movement. Berkeley's criticism shows that the calculus is the expression of a contradiction, but without a proper understanding of the nature of contradiction itself, this truth separates the differential calculus from speculative science, rather than being the principle of its unity with it. Ultimately, Hegel's characterisation of the differential calculus will therefore be premised on what is almost an axiom of dialectical thinking, that "*there is nothing which is not an intermediate state between being and nothing*" (*SL*, 105). When

these fluxions are incorporated into the ultimate ratio itself, we have a structure which is isomorphic with the structure of both contradiction and the infinite:

> The truth is rather that that which has being *solely* in the ratio is not a quantum; the nature of quantum is such that it is supposed to have a completely indifferent existence apart from its ratio, and its difference from another quantum is not supposed to concern its own determination; on the other hand, the qualitative is what it is only in its distinction from an other. The said infinite magnitudes, therefore, are not merely comparable, but they exist only as moments of comparison (*SL*, 255).

We thus have the unity of moments which can only exist in their difference through this unity. What this analysis has attempted to show is how, for Hegel, the differential calculus both requires a move to a dialectical understanding of mathematics, and also, in its dialectical development, comes to represent the structure of the system as a whole as it incorporates the movement from being to nothing. For Deleuze also, the calculus represents a founding metaphor. In the next section, therefore, we will move on to explore how Deleuze's understanding of the calculus differs from that of Hegel in order, through the large number of commonalities which they share in their interpretations, to highlight in precisely which ways they differ in their responses to the problems to be found in representation.

Deleuze and the Calculus

For Bergson, the transition to a method which takes movement as central presents us with "the most powerful method of investigation the human mind possesses" (*CM*, 25). Whereas before the differential calculus, there is a sharp distinction between the

straight line and the curve,[22] the differential calculus instead characterises the straight line as a special case of the curve, indeed the limit case. In doing so, mathematics mirrors Bergson's own relation of the two categories of space and duration. Furthermore, in making this assimilation, the calculus represents the static as a by-product of motion itself. Whilst mathematics may approach its object by this method, however, the same is not true of the sciences, even those which rely on the results of the differential calculus. Biology does not "get as close to its object as mathematics does to its own" (*CE*, 32). Whilst the differential calculus allows operations to be performed on quantities of change, it also allows the inverse procedure. As the method we have outlined shows, as the section of the curve by which we draw the tangent shrinks, the curve and the straight line become interchangeable. Instead of using this fact to open up a kind of thinking in duration, modern science for Bergson instead uses this fact to approximate the curve using a series of straight lines. Instead of, for example, using the idea of the curve to represent the relation between the biological as durational to matter as extensity, therefore, science attempts to reconstruct an approximation of the curve from a series of tangents, thus reducing duration to extensity (the tangent as the atomic element from which, for the scientist, the curve is constructed). This failure of the science of Bergson's time to adequately understand the role that motion plays also opens up the possibility of a new relation to science which Bergson recognises as a future possibility, and which Deleuze will try to actualise through his interpretation of the calculus.

Deleuze's exposition of the calculus begins with the pronouncement that "just as we oppose difference in itself to negativity, so we oppose *dx* to not-A, the symbol of difference to that of contradiction" (*DR*, 170). In spite of this, in his interpretation of the calculus, there are a great many similarities with the Hegelian model, to the extent that his analysis seems to show an engagement with Hegel's own text. One of Deleuze's main aims in giving his exposition of the calculus is to provide a metaphysic which will "take *dx* seriously" (*DR*, 170). For Deleuze, this approach can only be achieved by providing an

interpretation that will leave it "separated from its infinitesimal matrix" (*DR*, 170). What is driving Deleuze in this matter is, in parallel with Hegel, an attempt to provide an interpretation of the calculus which does not give the differentials the status of infinitely small quanta. Mirroring Hegel's argument about the representation of the infinite in the ratio, Deleuze writes, "quanta as objects of intuition always have particular values; and even when they are united in a fractional relation, each maintains a value independently of the relation" (*DR*, 171). For this reason, the Leibnizian interpretation of the calculus, which relies on infinitely small quantities must be rejected. Similarly, the calculus cannot be conceived of as operating with algebraic terms, since in this case "there must always be a particular value charged with representing the others and with standing in for them" (*DR*, 171). Thus, the equation of the circle, $x^2 + y^2 - R^2 = 0$, is opposed to its differential, $ydy + xdx = 0$, in which such a substitution of arbitrary values cannot be made. In this example, however, we can already see the difference between Hegel and Deleuze becoming clear. Whilst the differential of the circle can be represented as a ratio, Deleuze instead chooses the representation which maintains the separability of the differentials. His rhetorical device here signifies his intention to present an interpretation of the differential which falls outside of the positions of the infinitesimal and the ultimate ratio.

For Deleuze, "the symbol, dx appears as simultaneously undetermined, determinable, and determination" (*DR*, 171). To each of these three moments, we have a principle, which Deleuze claims together make up a principle of sufficient reason. The first stage, the undetermined as such, corresponds to the differentials themselves (dx, dy). Here, in what is his most important break with Hegel, Deleuze argues that whilst dx is strictly nothing in relation to x, this is not because the differential is not in a sense real, but rather because it cannot be captured by either (Kantian) intuition or the categories of quantity. This characteristic of falling outside of both quantity and intuition is what

Deleuze calls "continuousness" (*DR*, 171), presumably to highlight its difference from the discrete nature of quantitative determinations. In this sense, rather confusingly, continuousness is the "ideal cause of continuity" (*DR*, 171), continuity here understood as the kind of continuity discovered in the sensible realm. Just as with Hegel, Deleuze is opposed to thinking of the differential ratio in terms of variability, and continuousness instead represents the transcendental condition for variability, since it is through the reciprocal determination of differential elements, which themselves escape determination as quantities, that determinate quantities are generated by the differential function. In this sense, the differential, *dx*, as a symbol of difference, is "completely undetermined" (*DR*, 171), that is, as the representation of the "closest noumenon" (*DR*, 286), difference, it escapes the symbolic order. The symbols, *dy* and *dx*, and their values of 0 in respect to *y* and *x*, therefore represent the annihilation of the quantitative within them in favour of what Deleuze calls the sub-representational, or extra-propositional.

It should be clear from what we have said of Leibniz's interpretation of the calculus that once the differential is not seen as an infinitesimal quantity, we can no longer conceive of the process of differentiation as entailing the infinitesimal approximation of the tangent of a point. Instead, Deleuze introduces the notion of limit, but not the limit of the modern interpretation, whereby the differential is the value an infinite series converges on. Instead, "the limit must be conceived not as the limit of a function but as a genuine cut" (*DR*, 172). Deleuze's point is that whereas *dy* and *dx* are completely undetermined in relation to *x* and *y*, they are completely determinable in relation to one another. We can relate this back to our initial description of Leibniz's interpretation of the calculus by noting that Leibniz held differentials to be merely quantitatively distinct from the values of *x* and *y*, as infinitesimally small values. This led to the difficulty that on the one hand they relied on this quantitative moment to form a ratio and thereby determine the value of *x*, but on the other hand, this moment had to be ignored in order that we could determine the value of *x* itself, rather than $x + dx$. For

Deleuze, *dy* and *dx* are not to be characterised quantitatively, and so, when we determine the quantitative answer, they fall out of the equation, as they cannot be captured by the categories of quantity. That is to say, the differentials themselves escape representation, but, through their reciprocal interaction, are able to generate determinate representations. For Hegel, neither *dy* or *dx* could be understood outside of the ultimate ratio, precisely because each could only become determinate in relation to the other. For Deleuze, it is also true that *dy* and *dx* only become *determinate* in relation to one another, but this does not imply that the differentials are not *determinable* prior to their determination. They thus stand outside of the Hegelian notion of the ultimate ratio. This brings us on to the status of the reciprocally determinable, as *dy*/*dx*.

The differential calculus is at the root of Deleuze's notion of the Idea, and indeed, it relies on the results of differential geometry in order to escape from a reliance on sensuous intuition. As we saw on page 156 of chapter four, the Idea is set out by Deleuze using the Bergsonian analogy of light,[23] but the calculus is supposed to allow us to move to a more rigorous conception of it. In his discussion of reciprocal determination, Deleuze clarifies this connection to the calculus further by asserting that "the Idea has the differential relation as its object" (*DR*, 173). For Deleuze, as for Hegel, the differential relation is not to be understood in terms of quantities or variable magnitudes, but against Hegel, the value of the differential equation is to be understood according to its difference from the primitive function (the function which is differentiated). In the example which we used at the beginning of this chapter, the primitive function was the initial equation, $y = x^2$, which was differentiated to give, $dy/dx = 2x$. As we saw on page 211, differentiating a function gives us the tangent to that function, enabling us to determine the gradient at any point. To be more precise, however, we can say that the differential of a function is itself a function which will give us the tangent at any point on the primitive curve. The function, $dy/dx = 2x$, gives the gradient at any point on the curve, as we can insert any value of x into the equation. There is thus a change in quality between the differential

equation and the primitive function. The primitive function, which deals with the relations of actual magnitudes, is tied to representation, whereas the differential function, which specifies values in terms of *dy* and *dx* is instead tied to the Idea. The movement from the differential to the primitive function is therefore seen by Deleuze to be a movement of generation, akin to the movement from virtual to actual. There is also a third stage of Deleuze's analysis, complete determination, which corresponds to the actual values of *dy*/*dx*. The importance of this stage is that in specifying the tangent to various points on the curve, we can also determine the points, known as singularities, where the nature of the curve changes. These points, where *dy*/*dx* becomes null, infinite, or equal to zero, represent points of transition within the structure of the curve itself, as for instance, when the gradient of a tangent is equal to zero, we have a local maximum or minimum as the direction of the curve changes (as for instance, the fact that the gradient on the top of a hill is zero indicates that we can only go down from that point, although it may not be the tallest hill in the area[24]). Through repeated differentiation, we can further specify these points, determining their exact nature. The importance of these singular points is that they capture the specific nature of the curve, and correspond to the singularities of cycles and spirals which we saw on page 131 to be definitive of the geography of the phase portrait. This means, for Deleuze that the differential calculus allows us both to specify the general, in the form of the curve as a whole, as well as the singular, in the form of the series which emerges from the repeated differentiation of the area around a singularity. In this sense, it maps on to the example of the white light provided in chapter four. Reciprocal determination gives us the characteristic of generality, in the form of the line which all of the singularities, as partaking in it, together determine. It also gives us the singular, in the form of the local maxima and minima.

The Idea, which became so central in our exposition in chapter four, is to be understood as "a system of differential relations between reciprocally determined genetic elements" (*DR*, 173-4), and so is to be modelled on the calculus. In turn, Deleuze's

concept of dialectics, which as we saw on page 152 operated on problem fields to generate solutions, is to be understood in terms of the calculus. We further saw that the problem field must for Deleuze be both immanent and transcendent to the solution, as well as being different in kind. This difference on the one hand allowed for the genuine genesis of solutions, and on the other for the movement beyond the master-student characterisation of problematics, where the answer is known in advance, or in other words where the problematic is considered valid on the basis of the existence of a solution. In fact, "four terms are synonymous: actualise, differenciate, integrate and solve" (*DR*, 211). In this case, it is the last two of this list which are important, as if the problematic field is to be defined as a field of differential relations, its solution can be defined in terms of finding the primitive function which gives rise to those differential relations. Integration is the inverse operation of differentiation, and in this sense, the primitive function can be said to be contained within the differentiated function (i.e. immanent to it). At the same time, however, the problematic is different in kind from the solution, as the primitive function deals with magnitudes, whilst the problematic deals with continuousness as such. In this way, the problematic remains transcendent to the solution. Furthermore, whilst there is no mechanical way of moving from the derivative function to the primitive function, this does not mean that the truth of a problem is defined in terms of its solution. Rather, following the work of the mathematician Abel, the question of the nature of the solution is separated from the question of the existence of the solution. That is, we can mechanically determine where a particular formula is integrable prior to knowing what the result of the integration would be. This means that solvability does not rest upon an external criterion (the existence of a solution), but instead "solvability must follow from the form of the problem" (*DR*, 180) itself. The movement between the problem and the solution, as we saw in our previous discussion of it, is a fundamentally ontological movement. The way an organism is structured, for instance, is seen by Deleuze to be a solution to a problem. Because of this, the calculus is essential to Deleuze in allowing him to generate the kind of differential transcendental

philosophy which we saw was needed to overcome the limitations in the Kantian system thrown up by Sartre. We now need to look briefly at some of Deleuze's comments on the historical problematic of the metaphysics of the calculus to open the way for our discussion of the relation of Deleuze's philosophy to Hegel.

We have dealt with two conceptions of the foundations of the calculus apart from that of Deleuze. On the one hand, there is that of Hegel, which posits that the calculus is fundamentally dialectical, and as such can only be understood in terms of infinite thought. Central to this interpretation was the idea that the ultimate ratio, as a vanishing, fell outside of the procedures which a thinker of finite representation, such as Berkeley, was able to countenance. The differential, as a vanishing, could not be understood in the static terms today exemplified by set theory. The second approach was that of set theory itself, which proceeded by rejecting Berkeley's interpretation of the foundations of the calculus, and thereby seeking additional resources within finite representation itself with which to lend consistency to its operation. This led to the theory of limits. For Deleuze, "it is precisely this alternative between infinite and finite representation which is at issue when we speak of the 'metaphysics' of calculus" (*DR*, 176). We saw this question of representation played out in terms of the ontological status of the differential, with Hegel preserving it, and modern interpretations removing it. Deleuze attempts instead to finesse this problem by granting the differential a status outside of both forms of representation. In this sense, the differential is "neither real nor fictive, [instead] differentials express the nature of the problematic as such" (*DR*, 178). For this reason, we should not expect to find them in the realm of the solution, as the structure of the solution is defined by the structure of the proposition, which is representational in Deleuze's terms (whether finite or infinite). The question of the differential therefore bears on the question of the two forms of representation: "Perhaps the other alternative collapses as well, that between finite and infinite representation" (*DR*, 178), as "what is still missing is the extra-propositional or sub-representative element expressed in the Idea of the differential,

precisely in the form of a problem" (*DR*, 178). Such a sub-representational element would fall outside of the dichotomy of the finite/infinite.

We can now give an account of the relations between the differential, the phase portrait, and the Idea. As we saw in chapter four, Deleuze turns to Bergson to provide a model for a concept of difference which falls outside of the Aristotelian model. Bergson's concept of light differed from Aristotle's in that light was not defined through an abstraction from different shades of light, but instead though an interpenetration of these various shades which allowed each to maintain its specificity whilst participating in the concept of light as a whole. In order to move away from an understanding of the Idea built on analogy, Deleuze moved to the concept of phase space, which was essentially a space which allowed the total dynamic possibilities of the system to be specified simultaneously, whilst still affirming the reality of different possible trajectories. The phase space thus allowed us to see how the general structure of the system could be specified without the specific states being opposed to one another. As it relies on the differential calculus for its construction, if Deleuze was to be able to specify the phase space without falling back into representation, a theory of the calculus was needed which did not itself rely on either finite or infinite representation. This was provided through the notion of the differential as sub-representational, but determinable. With these structures now in place, we can go on to look at Deleuze's analysis of the Hegelian solution to the problem of representation, in relation to Deleuze's own differential solution.

Deleuze and Hegel

We can summarise the differences found in the interpretations of the differential calculus by noting that Hegel takes a broadly Newtonian line in his interpretation of the calculus, whereas Deleuze takes his inspiration from the work of Leibniz, albeit with a

non-intuitional interpretation of the differential. For Deleuze, the difficulty of differentials appearing in the resultant formulae is resolved through the belief that we are dealing with two different ontological planes, for Hegel once again through recognising that the status of the nascent ratio differs from that of normal numbers. Aside from this difference, we should note that what Hegel discovers in the calculus is "the infinity of 'relation'" (*DR*, 310, n. 9), meaning that what is important in the calculus is the relation between the two fluxions, to such a degree that this relation not only defines the determination of them, but also their existence. The questions of the determination and existence of the differential remain separate on Deleuze's interpretation, as the differential is given the characteristic of "determinability." The centrality of these differences is clear in that Hegel first refers to the differential calculus in his discussion of the notion of vanishing in the fourth remark to the dialectic of becoming, and it is this moment of vanishing which Deleuze uses to characterise Hegel's philosophy as a whole as one of infinite representation. Hegel adds movement to the dialectic by considering finite representation "not as having vanished and disappeared but as vanishing and on the point of disappearing" (*DR*, 43-4). That is to say, Hegel does not renounce finite representation, but instead calls for it to be reintegrated into the interiority of the infinite. "Opposition remains abstract so long as it does not extend to the infinite, and the infinite remains abstract every time it is posed outside of finite oppositions" (*DR*, 44). In this sense, for Deleuze, Hegel's criticism of those who went before him is that they had not taken difference to the level of absolute difference, the contradiction. In making this final move, Hegel goes from finite, organic representation to what Deleuze calls infinite, orgiastic representation. Organic representation is given content by participating in orgiastic representation, just as both being and the differential are maintained as being just on the point of vanishing.

Whereas Hegel focuses on the structural elements of the ratio dy/dx, Deleuze is instead interested in the difference in kind between the primitive function and the

differential, and the fact that by integration, the primitive function can be *generated* from the differential. In recognising the difference in kind between them, Deleuzian ontology returns to the Bergsonian idea that the absence of order is in fact the presence of a different kind of order, in Deleuze's terms a distinction which is played out between sedentary (representational) distributions and nomadic (differential) distributions. The differential provides the tools for this, presenting a logic which Deleuze believes is in itself pure affirmation, but which, through a process immanent to itself, develops a tendency which, at its limit, would contain both negation and opposition (we will return to the question of difference and affirmation in the last chapter). Everything, however, takes place in the middle for Deleuze, and negation and opposition appear in the world "only insofar as these are cut off from their virtuality which they actualise, and from the movement of their actualisation" (*DR*, 207). The fact that differential relations have a structure of their own, and, through Deleuze's analysis of the differential as escaping representation, a structure which itself escapes representation, means that Deleuze can attempt to provide an account of the genesis of determination which does not reduce the absence of determination to indifference. In doing so, Deleuze walks a tightrope by treating the virtual as both real and not actual. "Every object is double without it being the case that the two halves resemble one another, one being a virtual image and the other an actual image" (*DR*, 209).

The Kantian Antinomies

Whilst we have looked at the differences between Deleuze and Hegel so far in relation to mathematics, we will now briefly return to Kant in order to give another example of how these differences play out, this time in relation to Kant's discussion of the antinomies of pure reason (*CPR*, A405/B432), as it is here that all three figures see the construction of a problem for the subject which ultimately leads to a break with our

dogmatic interpretations of the world. Indeed, Kant went as far as to write that his system could have begun with the antinomies, as it is the inability of transcendental realism to resolve the paradoxes which emerge from our consideration of these cosmological concepts which lead, for Kant, to the recognition that Reason is subject to transcendental illusions which are endemic in both dogmatic and empiricist positions.[25] For Hegel, instead, the antinomies show the existence of objective contradiction in the world, which can be resolved only by reconceiving of Reason in such a way as to no longer define its mode of operation as "extend[ing] [a concept of the understanding] beyond the limits of the empirical" (*CPR*, A409/B435). For Deleuze, like Kant, the antinomies do not show the existence of objective contradiction, but instead create a rupture in the world of representation that can only be resolved through the incorporation of an element which escapes representation. The disagreement with Kant comes from Kant's inability to specify what this element is. As Kant claims that the antinomies provide "indirect proof of the transcendental ideality of appearances" (*CPR*, A506/B534), they have been the subject of some of the heaviest criticism to engage with the Kantian system. Since the aim of our discussion is to elucidate the relations between Hegel and Deleuze, we will keep discussion of this criticism to a minimum, outside of that employed by Hegel and Deleuze themselves.

The antinomy of reason deals with what Kant considers to be certain arguments which naturally develop from the use of reason to answer questions which go beyond direct experience.[26] Whereas the paralogisms deal with concepts which are outside of the remit of empirical knowledge, such as the soul, the antinomies deal instead with the attempt to extend the application of the categories beyond their proper use by tracing back the conditions of an object to the point at which we arrive at the unconditioned. In this regard, rather than dealing with concepts such as God or the soul, which relate to things outside of the world, the antinomies are cosmological, referring to the world in its totality. In order to accomplish this task, it is reason which takes up the categories of the

understanding, and attempts, by means of a regressive procedure which tries to move from the present, conditioned, to its conditions, to allow us to conceive of such a totality. As we shall see, it is the role of reason to attempt such a regression through the series of conditions which govern the object, but we fall into error when we confuse this general rule with a cosmological principle of "the absolute totality of the series of conditions, that is, viewed as actually present in the totality of the object (that is, in the appearances)" (*CPR*, A509/B537). To put this another way, the antinomies of reason arise, for Kant, when the regulative rule that we should look for the condition for the conditioned, is combined with the idea that that which is present to us is a thing in itself, rather than an appearance, so that the complete specification of the conditions will give us the unconditioned itself. This in turn gives rise to antinomy, as there are two separate ways of specifying the unconditioned through a regressive analysis, and these two procedures appear to produce equally valid, but mutually exclusive, interpretations.

What reason requires is to find the unconditioned from the conditioned. As such, it follows a regressive movement which traces back through the series of conditions to the unconditioned. In order for this movement of regression to function, we need to find the unconditioned of a series: "[The unconditioned] may be specified in either of two ways. It may be viewed as consisting of the entire series in which all members without exception are conditioned and only the totality of them is unconditioned. This regress is to be entitled infinite. Or alternatively, the absolutely unconditioned is only a part of the series – a part to which the other members are subordinated, and which does not in any way stand under any condition" (*CPR*, A417/B445). These two approaches correspond for Kant to empiricism and dogmatism, since on the one hand, the first, empiricism, stays entirely within the purview of the world of appearances, whilst the dogmatist approach, in seeking the unconditioned at the beginning of the series, is best characterised through "the presupposition of intelligible beginnings" (*CPR*, A466/B494). The debate between the two modes of explanation can therefore be understood to be whether the

unconditioned which provides the sense of the series is given as a first term, or as the totality itself. This, for Kant, is exemplified in "the contrast between the teaching of Epicurus and that of Plato" (*CPR*, A471/B499). Now we have some idea of the principles which govern the debate between dogmatism and empiricism, we can look at the debate itself. As cosmological ideas require the possibility of a regressive synthesis from the conditioned to the conditions, not all categories can furnish cosmological ideas (*CPR*, A409/B436), and in particular, it is the categories of quantity, reality, causality and necessity which lead to such ideas and their respective antinomies.[27] We shall consider just the first of these antinomies, which considers "the original *quanta* of all our intuition, *time* and *space*" (*CPR*, A411/B438). The first antinomy of reason therefore treats the limits of space and time, with the dogmatist thesis arguing that "the world has a beginning in time, and is also limited as regards space," and the empiricist antithesis holding that "the world has no beginning, and no limits in space; it is infinite as regards both time and space" (*CPR*, A427/B455). As there is a certain amount of repetition in the time and space arguments (and, as Hegel notes, the arguments in terms of space rely on those for time [*SL*, 234-5]), we will only consider them in terms of time here.

The argument for the thesis, that the world has a beginning in time, proceeds from the assumption of the opposite in order to generate a *reductio ad absurdum*. Given that the world has existed for an infinite amount of time, an infinite number of things must have happened. An infinite series is by definition, however, a series that can never be completed, and hence, the past (as an *infinite* series) cannot have passed away. This, therefore, necessitates that the world has a beginning. The empiricist antithesis follows a similar structure, first assuming that the world does have a beginning in time, meaning that there must have been time prior to that beginning. This time before the world must be an empty time, however, and in such a time, every moment must be identical to every other. In this case, it is impossible for anything to have come into being, meaning that there must be no beginning to time (*CPR*, A426-9/B454-7). Whilst there are objections to

both of these arguments,[28] Kant clearly wishes to maintain that the disagreements presented are real and serious, and such that they do not allow reason to "withdraw and treat the quarrel with indifference as a mere mock-fight" (*CPR*, A464/B492).

As Allison notes,[29] Kant's solution to the problem of the antinomies involves a two-fold process which involves on the one hand the therapeutic approach of recognising that the antinomies rest on the natural application of the faculty of reason combined with the extension of this application beyond its proper domain. As we saw, the antinomies rest on the desire of reason to relate the conditioned to its condition. As the relation between a condition and that which it conditions is analytic (*CPR*, A498/B526), and "human reason is by nature architectonic" (*CPR*, A474/B502), it is natural for reason to approach the conditioned in this way. The regression from conditioned to conditions is therefore "set as a task" (*CPR*, A498/B526) for reason, but the mistake reason makes is to presume that the conditions are already really given. This is the result of the understanding, which "represents things *as they are*, without considering whether and how we can obtain knowledge of them" (*CPR*, A498/B526). Now, given the transcendental realist interpretations of the object given by the empiricist and dogmatist positions, we can see, as in the case of the representationalist systems which we discussed in earlier chapters, the object of the understanding is considered to be the thing in itself. In this case, therefore, the conditions of the conditioned are likewise available to us, as both entities exist on the same ontological plain. Therefore, transcendental realism lacks the ability to differentiate between the claim that it is reason's task to find the conditions for the conditioned and the availability of these conditions, meaning that it is unable to find a solution to the antinomies. Transcendental realism suffers from the transcendental illusion that the totality of conditions can be given. Transcendental idealism, however, whilst unable to remove the transcendental illusion, which derives from the analytic presupposition that the conditioned has conditions, is able to mitigate this illusion by showing that once we recognise that the world cannot be conceived of as a thing in itself,

the arguments of both the dogmatists and empiricists can be sustained without leading to contradiction. This fact further leads to Kant's claim that the antinomies provide the grounds for an indirect proof of transcendental idealism.

Kant's solution to the antinomies is to play on the distinction which transcendental idealism generates between the thing-in-itself and appearance. For the transcendental realist, the availability of the totality of conditions means that we arrive at an unconditioned conclusion, leading to what Kant calls an analytical contradiction (*CPR*, A504/B532), but for the transcendental idealist, the unconditioned cannot be reached through the regressive application of the categories of the understanding, as these apply only to appearance, rather than to things-in-themselves. Whilst the categories can give us the empirical conditions for an event, as they represent the conditions for the possibility of experience, they cannot be applied to that which cannot be experienced, the in-itself, meaning that the kind of totality which the empiricist or dogmatist is looking for is simply unobtainable, as the sequence, no matter how far it is traced back, will remain conditioned. Once again therefore, the attempt to represent a totality has resulted in a contradiction. How does the introduction of the thing-in-itself allow us to resolve the antinomy? By recognising that between the two categorisations of the universe as either finite or infinite in magnitude, a third possibility exists. "If it be said that all bodies have either a good smell or a smell that is not good, a third case is possible, namely, that a body has no smell at all; and both the conflicting propositions may therefore be false" (*CPR*, A503/B531). Likewise, the noumenal, falling outside of the constraints of the categories of magnitude, is neither finite nor infinite. Finitude and infinitude are therefore considered by Kant as simple contraries, which means that no contradiction results in the denial of both. Kant in turn calls this kind of opposition dialectical. In this context, it is interesting to note that Kant will also take up the work of Zeno, arguing *contra* Hegel that Zeno's writings do not represent a sceptical form of dialectic, and *contra* Russell that he does not show the impossibility of movement, but instead that he prefigures Kant's own

work by showing that "if two opposed judgements presuppose an inadmissible condition, then in spite of their opposition, … both fall to the ground, inasmuch as the condition, under which alone either of them can be maintained, itself falls" (*CPR*, A503/B531).

For Deleuze, a parallel situation to that of Kant emerges. For Kant, the difficulty which transcendental realism suffers from is that it takes the totality of appearance to be totality in-itself, leading to the idea that the totality of appearances gives us the unconditioned, either immediately, or mediately through the first cause. This leads ultimately to an inability to understand how we are able to be acquainted with the world (the founding problem of the first critique). For Deleuze, the difficulty with representation is that it takes the totality of actual states of affairs to be totality in-itself. This leads ultimately to the inability to understand any concept of difference which is not purely actualised as negation (as actuality is equated with reality). "Forms of the negative do indeed appear in actual terms and real relations, but only in so far as they are cut off from the virtuality which they actualise, and from the movement of their actualisation" (*DR*, 207). The difference comes through Deleuze's claim that Kant has failed to follow his own distinction between analytical and dialectical forms of opposition in holding that, in not being determined according to the categories, the noumenal is undifferenciated. "However, to the extent that this pure [noumenal] thought remains undetermined – or it is not determined as differential – representation is not really overcome, any more than the propositions of consciousness which constitute the substance and the details of the antinomies" (*DR*, 178). Deleuze is not therefore so concerned with the specific application of Kant's procedure to the antinomies as with the fact that Kant introduces an element which is neither finite nor infinite in order to resolve the dilemmas. He disagrees with Kant's approach, however, in that Kant believes that this element, as being neither infinite nor finite, is not determinable. This parallels Deleuze's own introduction of a novel interpretation of the differential to attempt to undercut both sides in the debate between finite and infinite representation, and is the same approach which we saw him

take with the calculus. Before turning to Hegel's interpretation of the first antinomy, we should note that Kant's indirect proof of transcendental idealism is also susceptible to reformulation in transcendental empiricist terms. Kant writes that: "If the world is a whole existence in itself, it is either finite or infinite. But both alternatives are false (as shown in the proofs of the antithesis and thesis respectively). It is therefore also false that the world (the sum of all appearances) is a whole existing in itself. From this it then follows that appearances in general are nothing outside our representations – which is just what is meant by their transcendental ideality" (*CPR*, A506-7/B534-5). As Allison writes, the difficulty with Kant's argument is that it "rests ultimately upon the assumption that it provides the only possible basis for avoiding the antinomical conflict."[30] Whilst Allison attempts to shore up the argument against alternative explanations provided by such approaches as linguistic analysis, which attempt to maintain the equation of appearance with things in themselves, Deleuze's own system (where it is actuality rather than appearance at issue), in rejecting the idea of a totality being representational, is not susceptible to a similar approach. Instead, the debate between Kant and Deleuze take place on other grounds, such as those highlighted in earlier chapters.

Hegel also appreciates the value of the antinomies, which "more than anything else, brought about the downfall of previous metaphysics and can be regarded as the main transition into more recent philosophy since they, in particular, helped to produce the conviction of the nullity of the categories of finitude in regard to their content" (*SL*, 190). Already with this assertion, however, we can see a distance opening up between Hegel's interpretation of the antinomies and that of Kant. For Kant, the difficulty was not with the categories themselves, but rather with their extension beyond their proper field of application by reason. The target of Hegel's attack on Kant's reading of the antinomies will therefore be Kant's assertion that "reason does not really generate any concept. The most it can do is *free* a concept of *understanding* from the unavoidable limitations of possible experience" (*CPR*, A409/B435). For Kant, as we saw, the function of reason is

in this regard essentially negative, in that it generates antinomies through the use of the categories. Hegel instead wants to give a positive role to reason, and as we saw, for him, reason instead generates categories through the use of antinomies. The fact that contradiction is at the heart of the Hegelian method means that instead of the number of antinomies being limited by a subset of categories, "as many antinomies could be constructed as there are Notions" (*SL*, 191). Hegel's approach to the Kantian antinomies is therefore to show that they represent the emergence of infinite thought, as reason provides a genetic deduction of the categories through exposing the one-sidedness of them taken individually. Rather than seeing them as highlighting the need for in Kantian terms a noumenal, or in Deleuzian terms a sub-representational moment, Hegel will instead argue that it is only by removing the one-sidedness of these determinations that we can find any resolution to the antinomies. Hegel therefore berates Kant for an "excessive tenderness for the world" which is "never and nowhere without contradiction" (*SL*, 237-8). Hegel attempts to support this claim by showing how the first antinomy naturally dovetails into the dialectical movement itself, provided one does not, as Kant does, replace the final moment of reconciliation of the determinations with their suspension within subjective reason.

In order to show the truly dialectical nature of the antinomies, Hegel begins by showing that both arguments exhibit the fallacy of *petitio principii*, by affirming in the premises what they hope to prove in the conclusion. The thesis, which argues that the world has a beginning in time, actually assumes the concept of beginning, or rather of limit, in the form of the *now*. "Namely, there is assumed some or any *given point of time* up to which an eternity has elapsed" (*SL*, 235). This point cannot simply be any quantitative point, but must instead be the qualitative end, as it represents the end of an infinite synthesis of the past, and the beginning of the future. In it, therefore, "time is *broken off*" (*SL*, 235). Likewise, the antithesis, which asserts that there is no beginning to time, presupposes its content. In assuming the world comes to be in time, "this proof

presupposes that this existence *comes into being* and that the coming-to-be has an antecedent condition which is in time" (*SL*, 236). The idea that existence always requires such an antecedent condition is, however, the content of the antithesis. Taken together, the thesis and antithesis really assert two things: "that a *limit is*, and that the limit is equally only a *sublated* one" (*SL*, 237). The thesis contributes the notion of a limit, the antithesis, in the form of antecedent conditions, its overcoming. Whereas Kant keeps these determinations separate, for Hegel they are conjoined to form the immediate determination of the infinite, as "the beyond of the finite" (*SL*, 143). When these determinations are taken up positively by reason, the antinomy generates a new category, where both determinations are united through their self-relation. By recognising the place of contradiction within the world, therefore, and giving it a positive value, Hegel dissolves the negative implications which transcendental realism finds in the antinomy while staying within representation, albeit an infinitised form of representation. This union of the two determinations therefore also matches Hegel's approach to the calculus, which likewise takes place through the reconciliation in contradiction of the opposing determinations of being and nothingness.

Conclusion

In this section we have seen how the interpretations of the differential calculus prove to be of radical importance in understanding the ontologies of both Deleuze and Hegel. As we saw, the whole matter can be seen in terms of the question of the status of the differential. In this, finite representation, infinite representation and the philosophy of difference mark out the three alternatives. For finite representation, the differential vanishes from mathematics, truly becoming nothing as it is replaced in the foundations of the discipline by the notion of limit, which hopes to allow everything to be explained on the plane of representation. For infinite representation, the differential is preserved in its

vanishing, a point at which it falls outside of finite representation by no longer being in determinable as a state. Instead, in its vanishing, finite representation is put into motion, actively accepting the characterisation which finite representation gave it as contradictory, whilst forcing a positive reworking of our understanding of exactly what it is to be contradictory. For the philosopher of difference, the differential does indeed vanish, but only by falling outside of representation. It therefore becomes sub-representational, creating a fissure between the concepts of determination and structure. We have dealt sufficiently with finite representation, which is problematic for reasons outside of the calculus itself. In the final chapter, we will therefore focus on Hegel and Deleuze's alternatives as they play themselves out in a broader sphere of philosophy. The essential question has now revealed itself to be this: is it possible to have the undetermined (but determinable) without indifference? We will begin to answer this question by looking at Hegel's arguments against transcendental philosophy as they occur in respect to Kant, and their relation to Deleuze's own differential transcendental philosophy.

Chapter Seven – Force, Difference, and Opposition

Introduction

In the previous chapter, we explored the differences between the approaches of Deleuze and Hegel in detail, in relation to their interpretations of the differential calculus and the Kantian antinomies. These differences of approach were not related to each other directly, however, but were mediated through their work on Kant and the calculus. Whilst the logics of both Hegel and Deleuze relate to the same problematics, namely the antinomies and aporias of what Deleuze calls finite representation and Hegel, the viewpoint of the Understanding, they would seem to remain incongruent with each other. Deleuze writes, "the philosophy of difference must be wary of turning into the discourse of beautiful souls: differences, nothing but differences, in a peaceful coexistence" (*DR*, 207), and the danger of remaining a beautiful soul is clearly recognised also by Hegel, for whom it is a necessary, but partial moment of our relations to the world. In this chapter, therefore, we will try to see the relations these two different approaches have to each other directly, in order to determine what resources either position has to provide a critique of the other. This presents a difficulty in that Hegel for obvious reasons was not acquainted with the work of Deleuze, but this does not mean that the Hegelian system does not have resources to engage with transcendental empiricism. As we have already seen in chapter five, Hegel presents a dialectic of essence and appearance which tries to show that essence only appears to underlie appearance, as something akin to a transcendental illusion generated by the movement of Reason itself. Hegel takes this kind of approach in the *Phenomenology* in the section on force and the understanding, and presents a similar critique of Schelling's philosophy in the *Lectures on the History of Philosophy*. We will therefore try to see whether Hegel's approach in these cases can be used to provide a critique of Deleuze's dual ontology of virtual and actual, or if the novelties of Deleuze's approach lead to Hegel's critique

missing its mark. A further, and as will turn out, related difficulty which we will look at emerges from Deleuze's slogan that "just as we oppose difference in itself to negativity, so we oppose *dx* to not-A, the symbol of difference to that of contradiction" (*DR*, 170). This is once again the problem of the beautiful soul: "The beautiful soul is in effect the one who sees differences everywhere and appeals to them only as respectable, reconcilable or federative differences, while history continues to be made through bloody contradictions" (*DR*, 52). It is this position which Deleuze seeks to avoid. The question that therefore arises is, how are we to understand Deleuze's use of the concept of opposition here if difference is to be understood as purely affirmative, or in order words, non-negative? This problem, the relation of opposition and difference will therefore occupy us for the remainder of this chapter, in particular in regard to its expression in the problems of the one and the many, and of the super-sensible world. We will begin first with the problem of essence and appearance, and the super-sensible world, which we will explore through Hegel's dialectic of force and the understanding.

Force and the Understanding

As we saw on page 229, Deleuze characterises Hegel's philosophy as one of infinite, or orgiastic representation, in contrast to his own philosophy of difference which attempts to incorporate a sub-representational element. By this, he means that instead of attempting to resolve the difficulties of representation by moving to a transcendental position, Hegel uses these difficulties to move from the finite thought of representation to infinite thought through an acceptance of the contradictory nature of the world. We also saw on page 178 how this Hegelian dialectic led to a renunciation of the conception of essence as somehow providing the ground for appearance, since essence turns out to be, on a dialectical reading, "the seeming of itself in itself" (*SL*, 398), that is, the actual process of appearing. Such a result would clearly present

difficulties for a transcendental philosophy such as Deleuze's, where the object is conceived of as doubled, provided we could show that the Deleuzian concept of transcendental field can be captured within this structure of the dialectic. Hegel's dialectic of essence would allow the 'doubled' aspect of the object to be reincorporated into a single ontological plane, thus collapsing the distinction between virtual and actual. The approach which Hegel puts forward in the doctrine of essence does not only occur there, however. Hegel provides a summary of this method in his discussion of Schelling.[1] The point at issue is Schelling's efforts to derive the notion of an absolute which would overcome the duality of subject and object present in the Kantian system. Schelling here has "grasped the true as the concrete, as the unity of subjective and objective" (*LHP*, 3, 541), but has failed to truly overcome the dualism, as "the procedure is not immanent development from the speculative Idea, but it follows the mode of external reflection" (*LHP*, 3, 528). By providing a merely formal procedure of deduction, Schelling must presuppose the resolution of the dualism at the outset, as formal reasoning teaches us that there cannot be anything more in the conclusion than is contained in the premises. "The axiom assumes the main point in question, and all the rest follows as a matter of course" (*LHP*, 3, 528). Hegel claims to provide the correct method to overcome the difficulties which he sees in Schelling's approach to philosophy, a method which, as we have seen, involves allowing the movement of the concepts themselves to show the real status of the subjective and objective. "The true proof that this identity of subjective and objective is the truth could only be brought about by means of each of the two being investigated in its logical, i.e. essential determinations; and in regard to them, it must be shown that the subjective signifies the transformation of itself into the objective, and that the objective signifies its not remaining such, but making itself subjective" (*LHP*, 3, 526). In looking at the relation of virtual and actual for Deleuze, it will therefore be necessary to show that the virtual transforms itself into the actual and the actual into the virtual, in order for a dialectical critique to be effective. The section that we will look at to conduct this investigation is

'Force and the Understanding' in the *Phenomenology*, as here we find a dialectic which shows the dialectical process of overcoming the distinction between a field of differences (appearance) and a field of identity (the tautological law). Much as in the doctrine of essence, Hegel here argues that "the supersensible is the sensuous and perceived posited *in truth*; but the truth of the sensuous and perceived is to be *appearance*. The supersensible is therefore *appearance qua appearance*" (*PS*, 89). We will now explore how Hegel reaches this conclusion before looking at the Deleuzian response.

The dialectic of force emerges in the *Phenomenology* from the inability of consciousness to resolve adequately the dialectic of the one and the many at the level of pure perception. Having shown through the dialectic of sense-certainty that remaining with the pure particular is inadequate, Hegel goes on to show in the section on perception that an understanding in terms of the universal is equally inadequate. Consciousness first conceives of the object as a bundle of properties. If these properties are to be determinate, they must be seen as separate from one another, but this leads to the difficulty that the object dissolves into a simple plurality. If we posit the object over and above its properties, however, the object itself becomes a bare 'this' again which, having excluded all properties, itself becomes indeterminate. In order to escape from this dilemma, the understanding posits the notion of force. In the dialectic of perception, we finish with an unstable movement between two positions, that of unity, and that of difference. With the introduction of force, these two positions are reconciled. "The result has, implicitly, a positive significance: in it, the unity of 'being-for-self' and 'being-for-another' is posited; in other words, the absolute antithesis is posited as a self-identical essence" (*PS*, 80). This self-identical essence comes about through focussing on the moment of transition between the simple unity and the many. As we saw in Hegel's discussion of the infinitesimal calculus on page 219, however, the finite understanding is incapable of representing a transition *as a transition*, which in that case led to the

antinomy at the root of the calculus, the ambiguous status of the fluxion as both equal to zero, but still maintaining a determinate quantity. Whilst the conjunction of these two states led directly to contradiction in the case of the fluxion, in the case of force, the conjunction of the two states of force does not immediately seem to be contradictory. Instead, what is required is the separation of the two moments of force: "there emerges in [the object] the distinction of form and content; and in the shape of content the moments look like they did when they first presented themselves: on the one side, a universal medium of many subsistent matters, and on the other side, a One reflected into itself, in which their independence is extinguished" (*PS*, 81). This distinction of two sides cannot be at the expense of their unity, however, as what led to force resolving the dialectic of perception was the recognition of a necessary connection between the two moments in their vanishing into one another. Thus, in order to meet these two conditions of unity and difference, the understanding is compelled to resort to the concept of expression. Much as essence and seeming were related in the dialectic of essence, which we looked at in chapter five, force becomes distributed across two levels: force proper, which preserves the unity of force, and force as expressed manifoldly in appearance. This allows the contradiction to be dissolved, as we have two different aspects of force, allowing one of the contrary properties to subsist in each concept, so that they are not predicated of the same subject directly, but instead of two different moments of the same subject, force. "One of [force's] moments, the dispersal of independent 'matters' in their immediate being, is the *expression* of force; but force, taken as that in which they have disappeared, is force *proper*, force which has been driven back into itself from its expression" (*PS*, 81). Force therefore allows the understanding to reconcile the two aspects of the object, difference and identity, without having to bring in the notion of transition.

We therefore have two moments, force as the One, and force as its expression. When we look at each, however, we notice that each requires some other in order to

truly be what it is. Beginning with the One, force proper, we can note that the essence of force proper is to express itself. In order to do so, it is therefore necessary "that the said 'other' approaches *it* and solicits it" (*PS*, 83). If force is to be expressed, this expression must involve impressing itself on, or 'forcing' itself on, another. Force's expression is therefore called forth, or solicited, by another. However, once force has expressed itself, it exists as "a medium of unfolded 'matters'" (*PS*, 83). This is because in expressing itself, it is no longer force proper, as the One, but instead is the many, as expressed force, and is thus separated from its essence, One-ness, which exists outside of itself once again as something other. In order to allow force to return to itself, it requires some other to solicit its self-reflection. As Hyppolite notes, this other will in turn be force itself. As he writes, "to every attraction corresponds a repulsion; otherwise, the matter of the whole universe would coagulate at one point."[2] When we think back over the nature of force, this result should seem inevitable, to the extent that the concept of force was brought in precisely to allow two seemingly contrary ideas, identity and difference, to coexist with one another. From the start, it was therefore implicit in our understanding of force that it could only be force which was solicited by force, or in other words, that it is the nature of force itself which necessitates its expression.

In Hegel's dialectic, force therefore necessarily posits another, which also turns out to be force. The initial unity of force is therefore sundered, and we are left with two independent forces. This moment of independence quickly collapses, however, as each force has shown itself to be dependent on the other. They do not relate to each other as objects which stand beside one another, but are essentially constituted through their relations to each another. "These moments are not divided into two independent extremes offering each other only an opposite extreme: their essence rather consists simply and solely in this, that each *is* simply and solely through the other, and what each thus is it immediately no longer is, since it *is* the other" (*PS*, 86). Once this moment is reached, however, we are returned to a position which makes explicit the moment of

transition from which we began. That which is expressed is now recognised to be only a moment of its expression, as each moment of force is only formally distinct from the other. The moments now "collapse unresistingly into an undifferentiated unity" (*PS*, 86), as the purely formal determinations of the two aspects of force are not sufficient to allow them to be seen as genuinely distinct from one another. Force has now become, as Hyppolite puts it, "the thought of the phenomenal world which, as an interplay of forces, is now no more that the incessant exchange of determinations, a perpetual instability whose unity and consistency lie only in thought."[3]

Now that force is seen explicitly as this vanishing, understanding looks to this play of forces and recognises that this play for the senses is too immediate for it, lacking all determinacy. "The middle term which unites the two extremes, the Understanding and the inner world, is the developed *being* of force which, for the Understanding itself, is henceforth only a vanishing" (*PS*, 87). For the understanding, therefore, the play of forces is conceived of as a non-being, a surface show [*Schein*], the totality of which is appearance. Appearance, as a non-being of being, a being which vanishes, leads us to the notion of the being of being, "the *inner being* of things *qua* inner" (*PS*, 86) which is mediated by immediate appearing. In this movement, we therefore have a structure analogous to the relation of being to essence which we looked at in chapter five, where the immediate being led to the positing of its contrary, essence. Here, in contradistinction to the sensible world, a world of the supersensible is opened up, where the constant vanishing of the sensible is replaced with a "permanent *beyond*" (*PS*, 87). This beyond now becomes the object of consciousness, leading to the "first, and therefore imperfect, appearance of Reason" (*PS*, 88), but this in turn leads to further difficulties.

Whilst escaping from the focus on the vanishing of forces allows consciousness finally to resolve the difficulties which it found there in relating the individual and the

universal, the characterisation of the beyond now becomes a problem. The absolute universal of the inner truth of things appears as the rejection of appearance, as its pure negation. As such, the reason it was able to overcome the antinomies of perception and force is also its downfall. The supersensible resolves the difficulties in determining the sensible by precisely lacking all determinations. It is "positively the simple or unitary universal" (*PS*, 88). The beyond therefore turns out itself to be simply a void. The question of determination which Hegel is here raising is not, however, a problem of a lack of knowledge of the beyond, since at this stage in the dialectic, he is claiming that the limitation of consciousness in this respect is essentially structural, and therefore cannot on logical grounds be resolved by consciousness. For Hegel, consciousness has two choices in relating to this beyond. First, we can simply leave the inner world as it is, as an inaccessible beyond of consciousness. This approach, however, is problematic once again, as consciousness has characterised the beyond as the truth of appearance. Thus, to remain at the level of appearance, consciousness must resort to "perceiv[ing] something as true which we know to be untrue" (*PS*, 88). The only other option available to consciousness is instead to populate the void with determinations. Foreshadowing Nietzsche, Hegel proposes that this "holiest of holies" can indeed be determined, but in order to do so, consciousness must fill it with determinations which have their root in appearance. The reason for this is clear, in that if consciousness relates not to the beyond, but to appearances, then the only content it is able to apply to the beyond is that found in appearance itself. If consciousness were able to apply the determinations of the beyond to the beyond, then it would be clear that the beyond was not in fact a beyond, as it would be accessible to consciousness. What is important to note, however, is that the previous discussion does not imply that there is no supersensible world, but rather that the essence and 'filling' of the supersensible is appearance itself (*PS*, 89). This is because appearance cannot be treated as synonymous with the sensible world. Instead, "the truth of the sensuous is appearance" (*PS*, 89). This means that Hegel can assert that "the supersensible is therefore *appearance qua*

appearance" (*PS*, 89) without consequently having to reduce it to mere sensibility. Just as with the dialectic of seeming and essence, we are once again in a situation whereby what appears to be a beyond is in fact something akin to a transcendental illusion, generated by the movement of Reason itself. The dialectic, which begins with the play of forces, and ends with the equation of the supersensible with appearance, if it could be shown to apply to Deleuze's philosophy, would clearly offer strong grounds to reject his dual ontology of virtual and actual. This is particularly clear in relation to Deleuze's early work, *Nietzsche and Philosophy*, where the world of objects is conceived of as the expression of an underlying field of forces, necessarily inaccessible to consciousness. For the time being, however, we will return to the dialectic, in order to see how consciousness attempts to overcome these shortcomings through the concept of law.

Law comes about through the recognition that the truth of appearance is its own ceaseless movement. The pure beyond turned out to be a problematic concept, but the concept of law which replaces it has its grounds in appearance itself, and so presents the possibility of overcoming the difficulties which the dialectic of forces brought up. We begin with the absolute flux which remains following the collapse of the supersensible *qua* supersensible. The play of forces initially relied on the determination of force as two moments, the soliciting force, and the force solicited. As we saw on page 244, however, the moment of the supersensible, the pure universal, has now been recognised as appearance *qua* appearance, and with this recognition, the previous determinations of force now collapse into a flux. Within this flux, however, in which nothing is stable, we become aware of at least one permanence, the permanence of the flux itself. "What there is in this flux is only *difference* as a *universal* difference, or as a difference into which the many antitheses have been resolved. This difference, as a *universal* difference, is consequently the *simple element in the play of Force itself* and what is true in it. It is the *law of Force*" (*PS*, 90). Law therefore takes the element of universality from the beyond, but instead of making it a pure beyond, tranquilly subsists in the play of the flux itself.

Law thus attempts to resolve the difficulties of the beyond by relating itself directly to the sensible.

The notion of law, however, also turns out to be problematic, in that once again, it aims to combine two conflicting tendencies. On the one hand, the laws must capture the specificity of the play of forces in the world of appearance, that is, the moment of difference, and this moment requires that the laws capture a maximum of content, as not to do so would leave us with a moment which somehow escaped the laws. On the other hand, however, the laws are precisely the laws of appearance, and appearance represents the unity of force. Thus, the laws must express this unity in themselves. This leads to a dilemma. Either we maintain the diversity of laws, in which case, we end up with a diversity which cannot capture the unity of appearance, which is their purpose, or else, we abstract from these general laws in order to generate a unified theory, but in this case, the specificity of the law is lost, and we return to a beyond as, at the limit, an abstract universal.

Further, in developing our understanding of laws, we cannot grasp a key feature of them, namely their necessity. In moving from the phenomenal to the realm of laws, we cannot explain why the laws must function in the ways that they do. Although laws give definitions of forces which specify their properties necessarily, this necessity in turn "does not contain the *necessity of its existence*; it exists, either because we *find* it, i.e. its existence is not necessary at all, or else it exists through, or by means of, other Forces, i.e. its necessity is an external necessity" (*PS*, 93). The final problem is most serious for the notion of law, however. In attempting to explain phenomena according to laws, the understanding instead merely repeats the phenomena under law at the level of force. In order to introduce the necessity which Hegel argues is lacking from the notion of law, the understanding is forced to resort to explanation by tautology. This is achieved by creating an artificial distinction between two different moments, followed

by the cancellation of this difference. In the case of the law itself, this is achieved through taking a phenomenon as if it was structured from indifferent elements. Thus, for instance, "motion is not itself thought of as something *simple*, or as a pure essence, but as already divided; time and space are *in themselves* its *independent* parts or essences, or, distance and velocity are modes of being or ways of thinking, either of which can well be without the other" (*PS*, 94). This structure of the law is then repeated in that which is to explain it: "The single occurrence of lightning, e.g., is apprehended as a universal, and this universal is enunciated as the *law* of electricity; the explanation then condenses the *law* into *Force* as the essence of the law. This Force then, is *so constituted* that when it is expressed, opposite electricities appear, which disappear again into one another; that is, *Force is constituted exactly the same as law*; there is said to be no difference between them" (*PS*, 94-5). The movement of the law is therefore paralleled by a movement of force. It turns out that explanation in terms of law simply moves from the same to the same. The implication of this is that what is posited as a ground for appearance is in fact no ground at all, but is merely the same as appearance itself. If the understanding is to escape this dilemma, therefore, it must posit the inner world not simply as a tautologous replica of the world of appearance, but as different from it, and in fact as absolute difference.

The Inverted World

The structure of explanation proved to be tautologous, but this tautology opened up a moment of difference, as the $A = A$ of a tautology requires another moment in order to keep the two moments from collapsing into each other. Thus, as we saw in chapter five, identity requires a moment of difference. Likewise, the world of pure identity, law, opens up a world of pure difference, its absolute negation, the inverted world. "The *selfsame* really repels itself from itself, and what is not selfsame posits itself as

selfsame" (*PS*, 96). Whereas the supersensible world of laws captures that which is identical, or permanent within the perceived world, it was unable to capture the other aspect of the world, namely the movement inherent within it. The inverted world solves this problem for as the movement of the identical back into itself, it inherently contains this principle. Thus, "the first kingdom of laws lacked [the principle of change and alteration], but it obtains it as an inverted world" (*PS*, 97). The inverted world is not simply the world of change and alteration, however, but in its very determinateness seems to present itself as the opposite of the world of laws. "What tastes sweet is *really*, or *inwardly* in the thing sour; or what is north pole in the actual magnet in the world of appearance, would be south pole in the *inner* or *essential being*; what presents itself as oxygen pole in the phenomenon of electricity would be hydrogen pole in unmanifested electricity" (*PS*, 97-8). The inverted world appears to take us no further than the world of law therefore, as it simply posits the inner as being the exact opposite of the world. As such, it is just as inert as an explanatory principle. We should note, however, that this is only the case when the inverted world is "looked at superficially" (*PS*, 97), and indeed, just as difference and identity were reconciled in the *Science of Logic*, here too, we find that ultimately the separation of the inverted world and the world of laws cannot be maintained.

In our discussion of force and understanding, we have so far focussed on the development of the concept of force. It is important to note, however, that it is the interpretation of events from the point of view of the finite understanding which has been the main concern of Hegel in this dialectic. The viewpoint of the understanding maintains the idea developed from sensation of "fixing the differences in a different sustaining element" (*PS*, 99), meaning that difference cannot be understood in its own terms. Hegel makes clear this reliance on the spatial by describing the understanding's conception of difference: "Certainly, I put the 'opposite' here, and the 'other' of which it is the opposite, there; the 'opposite', then, is on one side, is in and for itself without the

'other'" (*PS*, 99). The understanding's notion of opposition therefore relies on a conception of space which underlies those differences, and thus allows the opposite and the other to subsist without one another. It allows the contraries to exist in different places, the 'here' and the 'there'. In this sense, the inverted world and the world of laws are seen by the understanding as being separate from one another in the same way in which two objects are spatially distinct from each other. Once we remove this spatialised conception of difference, and remove the 'sensuous idea' contained within it (a rejection of the discursivity thesis), we can move to a conception of difference which does not create a sharp divide between itself and that from which it differs: "But just because I have the 'opposite here in and for itself, it is the opposite of itself or it has, in fact, the 'other' immediately present in it" (*PS*, 99). Once this move has been made, Hegel's resolution of the problem of the inverted world is simple enough.

Once again, we find that this resolution of the dialectic involves the recognition of thought as infinite. Once we recognise that "the character of law is infinity" (*PS*, 99), then we can see that it is both self-same and different. That is to say, instead of seeing both moments as essentially indifferent to one another, they need to be seen as implicit moments of a single unity. "The two distinguished moments both subsist; they are *implicit* and are *opposites in themselves*, i.e. each is the opposite of itself; each has its other within it and they are only one unity" (*PS*, 100). The inverted world is now seen as containing the world which is its opposite, just as it itself was shown to be implicit in the world of law. Both moments subsist in a common unity. This movement is indeed clear in the inverted world provided we do not take the 'superficial' viewpoint of the understanding. In another example of the inversion, for instance, Hegel cites the case of revenge. In the immediate world, revenge requires me to "confront [my enemy] as himself a person who does not treat me as such, and in fact bids me destroy him as an individuality" (*PS*, 97), whereas the law of the inverted world instead proclaims that "the reinstatement of myself as a person through the destruction of the alien

individuality is turned into self destruction" (*PS*, 97). These two laws are opposites, and, taken as such, neither allows me to achieve personhood, as in the first case, I am not seen as a person by my enemy, and in the second, the destruction of my enemy is also the destruction of myself as a self. When understood according to infinite thought, however, we are able to achieve a resolution of this dichotomy. The difficulty emerges for the understanding because each subject in this case is seen as distinct, and self-subsisting, much in the same way that the immediate and inverted worlds were. From this perspective, it is likewise impossible to recognise myself in the other, and so the destruction of the alien individuality can only be understood as a destruction *tout court*. From the perspective of reason, however, both of these moments can be reconciled, as now destruction of the alien in the other amounts to my recognition of myself therein, and thus becomes a condition of my personhood, rather than of its destruction. Likewise, this equally opens the possibility of the other confronting me as a person in turn, thus generating the infinite movement of self consciousness which Hegel takes as determining the subject as a person. Thus, from the standpoint of reason, these opposites are reconciled precisely through an infinite movement which allows them both to subsist without a radical split into two worlds.

The dialectic of consciousness and self-consciousness is therefore implicit in the dialectic of the inverted world. As well as looking forward to later relations, we also find that moving to the position of infinite thought overcomes some of the problems we encountered in the structure of explanation itself. As we saw on page 250, explanation turns out to be tautologous because it relies on a difference being created which is in fact not a difference. Thus, for example, space and time are separated from one another in order to later reunite them in the laws of motion. The reason for the fall into tautology was essentially that identity was conceived of in such a way that it in itself excluded any possibility of maintaining concrete differences within it. It remained purely abstract. From the perspective of reason, however, the universal nature of law can be maintained

without having to abstract from all differences. Instead, we have a concrete universal which allows the differences between time and space to be preserved whilst at the same time maintaining the necessary connections between the parts. It is important to note in concluding our exegesis of Hegel's dialectic of force that this resolution of the inverted world is only available to us as readers of the *Phenomenology*. The consciousness which moves through the dialectic instead remains trapped in an understanding of the world as doubled: "it is true that infinity itself becomes the *object* of the understanding; but once again the understanding falls short of infinity as such, since it apportions to two worlds, or to two substantial elements, that which is a difference in itself – the self-repulsion of the selfsame and the self-attraction of the unlike" (*PS*, 102).

We can now ask to what extent this dialectic can be related to Deleuze's system of transcendental empiricism. As we can see from the structure of the dialectic, one of its targets is clearly the kind of transcendental philosophy proposed by Kant. Hegel's argument attempts to show how consciousness, in order to resolve certain antinomies which emerge in consideration of the object, is driven to posit appearance, and in positing appearance, to posit a beyond of appearance, much as we found Kant doing with the antinomies in the last chapter. Whilst consciousness may stop, as Kant does, with the notion of an empty thing-in-itself, the dialectic impels a move to the notion of law, which in turn, in order to capture the movement as well as the permanence of the phenomenal world, necessitates the positing of an inverted world. The inverted world is, however, the last trick of the understanding, and once we recognise that its positing of an indifferent relation between the immediate realm of laws and the inverted world is unsustainable, difference and identity are brought together in one unified world. The connection between infinite thought and contradiction is once again brought into play by Hegel. Infinite thought allows us to conceive of the coexistence of contrary, and hence, to the understanding, contradictory, determinations within the same world. It does this

by overcoming the understanding's desire to situate these determinations in a spatial *milieu* of the 'here' and the 'there'.

We shall begin by noting some similarities between Hegel's position and that of Deleuze. First, as Hyppolite notes, the concept of force which is at play in the dialectic is one that is derived from Aristotelian metaphysics, and, in the *Jena Logic*, is discussed in terms of possibility.[4] As we saw on page 245, there are two moments of force, force as expression, and force turned back on itself, force as that which is expressed. The difference between these two notions "exists only in thought" (*PS*, 82), however. In the *Jena Logic*, Hegel defines force as "the cause, as infinity, which itself is only in the form of possibility and has its actuality outside it" (*JL*, 47). This leads to the two moments of force being understood as, on the one hand, its possibility, and, on the other, its externalisation as its reality. Ultimately, this means that the concept of force simply relies on a doubling of the kind found in the parodies of Aristotelian physics, and this doubling leads to the inevitable tautologous nature of explanation. "To explain a rock's falling to the ground (that is, uniting with the ground) it is said that it unites with the ground not because it unites with the ground but because the force in the rock unites it, namely, the force uniting the rock with the ground" (*JL*, 62). These explanations come about because "the content of the appearances and of the force is the same; the totality of utterances is gathered together within the force. Internally sundered as the relation may be, it still counts as one in name, a simple togetherness; and the separating that is posited with respect to the relation is one that is alien to it, a separating of force as something possible from force as something actual; so the tautology of explanation remains the same" (*JL*, 63). What is therefore key to Hegel's rejection of the notion of force, and consequently the first notion of law, is that it provides a mere repetition of possibility in terms of actuality.[5] Within this repetition, nothing new is actually added to the explanation, since the division between the two is a division for the understanding and is not present in force itself. To this extent, Deleuze will follow Hegel's reading of the

situation, and indeed, Hegel's criticism of force as relying on an artificial modal distinction prefigures Deleuze's own critique of Kant. As we saw on page 28 of chapter one, Deleuze's chief concern with the Kantian schema was that, by providing conditions of possibility which simply mirrored the functions of judgement, Kant was only able to offer a theory of the conditioning of experience, rather than to actually explain the genesis of experience itself. In his discussion of Nietzsche, Deleuze claims that it is only by going beyond this mere moment of conditioning that we can complete the Kantian project.[6] Indeed, this motif finds its echoes in Deleuze's criticisms of the concept of possibility in *Difference and Repetition*, and perhaps here we can also find the influence of his teacher, Hyppolite himself. Here, Deleuze writes that "the only danger in all this is that the virtual could be confused with the possible" (*DR*, 211). By equating virtuality and possibility, we fall into a number of errors. First, given that possibility is opposed to reality, to equate virtuality with possibility is to exclude reality from it. Instead, "the virtual is opposed not to the real, but to the actual. *The virtual is fully real insofar as it is virtual*" (*DR*, 208). In elucidating the second error, Deleuze asks, "what difference can there be between the existent and the non-existent if the non-existent is already possible, already included in the concept and having all the characteristics that the concept confers upon it as a possibility? Existence is *the same as* but outside the concept" (*DR*, 211). In these comments, we can see that Deleuze is in agreement with the first movement of the dialectic of force and understanding. If force or law are posited in terms of possibility, then ultimately our explanations become empty. For both thinkers, therefore, the movement from possibility to actuality is the movement from the same to the same, and is only a movement in thought. This similarity leads to a further issue, namely, Deleuze's solution to this problem of identity. If we move from a view of the transcendental as the same to a view of it as different, do we not simply call for a further dialectical movement, that is, the reconciliation of these opposites in the infinite movement of reason?

Deleuze therefore requires a response to the immanent dialectic Hegel attributes to force. Given the failure of explanation by means of identity, consciousness is driven to recognise the moment of difference which is also at play, which, in order to overcome the limitations of tautologous explanations, becomes another world, the inverted world. As we saw on page 253, the inverted world completes appearance by providing the "principle of change and alteration" (*PS*, 97), which was lacking in the realm of law. Deleuze himself seems to take a similar approach to that of consciousness when it posits the inverted world. As others have noted,[7] claims by Deleuze that "all identities are only simulated, produced by an optical 'effect' of the more profound game of difference and repetition" (*DR*, xix) seem to open him up to the kind of criticisms Hegel is proposing. Likewise, the object itself must be seen as doubled – "as though the object had one part of itself in the virtual into which it plunged as though into an objective dimension" (*DR*, 209). In fact, he goes as far to claim that, "the whole of Phenomenology is an epiphenomenology" (*DR*, 52). What Hegel's response to this position would be is clear. Even whilst criticising philosophies of identity for creating an essentially spatial view of relations, Deleuze has himself fallen prey to a moment of spatialisation. Difference in itself is opposed to negativity (*DR*, 170), but in relying on opposition in this way, does Deleuze not simply fall back into the position of the finite understanding's relation to the inverted world, ignorant of infinite thought's reconciliation of these two moments? Thus, from a Hegelian point of view, in separating virtual and actual, Deleuze has spatialised their relation. In the remainder of the chapter, I wish to look at Deleuze's response to this dilemma, and in doing so, to bring out the reasons for Deleuze's own opposition to Hegel.

Deleuze and the Inverted World

To begin with, it is worth noting that Deleuze is not unaware of Hegel's work on the relations between appearance and the beyond. In the *Logic of Sense*, he points out that it was Hegel's 'genius' (*LS*, 259) to overturn the Aristotelian deployment of representation. In making this statement, Deleuze is attempting to highlight certain important differences between his understanding of the virtual and Hegel's understanding of essence.[8] In order to see how these changes affect the situation, we need to return to Deleuze's discussion of possibility. We have already seen two of Deleuze's criticisms of the notion of possibility, but he also makes a further statement: "every time we pose the question [of realisation or actualisation] in terms of possible and real, we are forced to conceive of existence as a brute eruption, a pure act or leap which always occurs behind our backs and is subject to a law of all or nothing" (*DR*, 211). Here, Deleuze is making two interrelated points. On the one hand, possibility creates a beyond, or rather, a behind, to appearance. On the other, possibility forces us to conceive of a sharp distinction between the realm of existence and non-existence. It is this second point which is key to Deleuze's rejection of Hegel's argument, as it is the rejection of this all or nothing dichotomy which allows the virtual to exist as real without having to be posited as separate from the actual. In order to understand Deleuze's position on this point, we need to recognise that the virtual and actual operate as tendencies within the real, rather than as states. In this sense, everything exists between the virtual and the actual, rather than as simultaneously in one or both states. Indeed, the pure virtual or pure actual are themselves abstractions created by taking tendencies to the limit. As Deleuze writes in *Negotiations*, "it's not beginnings and endings that count but middles. Things and thoughts advance and grow out of the middle, and that's where you have to get to work, that's where everything unfolds" (*N*, 161). In making this move, Deleuze is setting out a conception of the real as a process:

"the individual is not only the result, but the *milieu* of individuation" (*S*, 43). What Deleuze means by this is that individuation, or creation, cannot be understood as something which is indifferent to that which is produced, something which comes to an end with the coming into being of the created, but is rather continuous with the existence of the thing in question. It should be clear from the structure of Hegel's criticism that if Deleuze's conception of the transcendental field is such that ultimately it cannot be seen as indifferently related to the empirical, the further move, the unity of opposites, whereby each retains their distinctness likewise cannot be made. Without the difference between virtual and actual being pushed to the limit, there are no opposites to be reunited.

Of course, in taking this position, Deleuze raises as many difficulties as he removes. The first of these difficulties, one which we have already discussed in relation to Bergson is, given that nothing exists in a state of pure actuality, how are we to understand the emergence of the problems of representation which we discussed in chapter two? Deleuze explains this by arguing that it is only when the tendency towards actuality is conceived of as pure actuality itself that we develop a representational image of thought. In making this point, Deleuze is once again pointing to a transcendental illusion: as the virtual tends towards actualisation (this tendency is present in the real), we make the mistake of positing this tendency as itself a state, much as the task of reason is mistaken for the givenness of the materials to accomplish that task.[9] "Forms of the negative do appear in actual terms and real relations, but only in so far as these are cut off from the virtuality which they actualise. Then and only then, do finite affirmations appear limited in themselves, opposed to one another, suffering from lack or privation" (*DR*, 207). Although Deleuze's writings are weighted towards an exploration of virtuality, we should not on that basis ignore the fact that actuality is equally a facet of the real. To do so would be to fail to understand the significance of Deleuze's assertion that "pluralism = monism" (*B*, 29). This understanding of

representation allows us to answer the second question of the status of opposition in Deleuze's rejection of the symbol not-A, as here it is from the standpoint of representation that the opposition is being envisaged. There is therefore a tendency towards opposition in the Deleuzian conception of the real, but "opposition is a second order power, where it is as though things were spread out upon a flat surface, polarised in a single plain" (*DR*, 50). As opposed to this single plain of representation, or actuality, Deleuze puts forward the model of the stereoscopic image: "Stereoscopic images form no more than an even and flat opposition, but they depend upon something quite different: an arrangement of coexistent, tiered, mobile planes, a 'disparateness' within an original depth" (*DR*, 51). Here then, representation is seen as the result of a process, or rather as the result of creating an artificial difference between the process and the result.

In discussing Deleuze, we need to recognise that as well as presenting a philosophy of difference, identity still plays a key role in his system. Difference and identity are both tendencies at play in the real, and the transcendental field requires identity, or extensity, in order to express itself. Thus, here we find a Deleuzian parallel to the Hegelian maxim that "*there is nothing that is not an intermediate state between being and nothing*" (*SL*, 105). As we have seen, however, whereas for Hegel, the critique of essence and appearance aims to situate everything on a single plane, Deleuze instead argues that what is required is a field of depth to supplement this plane. This was particularly clear in his adoption of the concept of depth developed by Merleau-Ponty, who championed figures such as Klee and Cézanne against the perspectivalist tradition in order to try to show that depth could not be understood in the same way as the other dimensions. As should be clear by now, the Deleuzian concept of difference operates in a similar way, and thus must differ in kind from the plane of representation. The problem of difference is, therefore, not to be characterised in terms of the sheer empirical – "no two hands have the same distinctive points, no two typewriters have the

same strike, no two revolvers score their bullets in the same manner…" (*DR*, 26) – as to do so would be to understand difference once again on the same plane as identity. In claiming that it is the field of depth which engenders being and nothing, Deleuze requires a concept of difference which itself is not to be understood in terms of being and nothing. For Hegel, however, determinateness simply is "non-being … taken up into being in such a way that the concrete whole is in the form of being" (*SL*, 110). This means that the concept of a difference which is not understood in terms of negation is an impossibility. It is a concept which would simply collapse back into indeterminacy. Deleuze is therefore forced to walk a fine line between on the one hand not pushing difference as far as opposition, as this would allow the Hegelian dialectic of the inverted world to come into operation, but to give it content, in order to avoid it becoming a pure beyond which would leave it open to Hegel's claim against Schelling that we have an "Absolute as the night in which all cows are black" (*PS*, 9), where instead of inwardly saying "*Om*, *Om*, *Om*" (*SL*, 97), we are saying "*difference*, *difference*, *difference*." In fact, the Deleuzian concept of difference is supposed to show the possibility of a purely affirmative moment of determination, and as such, he opposes the beyond where "the man with sight sees as little in that pure light as in pure darkness, and just as much as the blind man, in the abundant wealth which lies before him" (*PS*, 88), where difference becomes indifferent through lack of determination, to his own analogy of light: "in a certain sense, all Ideas coexist, but they do so at points, on the edges, and under glimmerings which never have the uniformity of a natural light. On each occasion, zones of shadow correspond to their distinction. Ideas are distinguished from one another, but not at all in the same manner as forms and the terms in which these are incarnated" (*DR*, 186-7).

In chapters four and six, we saw how Deleuze attempts to introduce this concept of a difference which is not to be understood in terms of negation through the resort to new geometries, and in particular, geometries which are grounded in the differential

calculus. In doing so, he attempted to provide us with a new logic, one which operates outside of the categories of representation. In this chapter so far, I have tried to show how this concept allows Deleuze to escape from the dialectical reduction of the beyond to appearance. Deleuze tries to achieve this through defining a third way between the two options which Hegel puts forward. On the one hand, difference is not the same, as it is for immediate law, and so does not collapse into appearance immediately. On the other, it is not the difference of the inverted world, which is a difference taken to the level of opposition, and so is not open to reconciliation through contradiction. What defines this new approach, according to Deleuze, is that it proposes a new set of relations between the concepts of limitation, determination, and negation.

As we saw in the discussion of the infinite on page 196 of chapter five, the introduction of the notion of limit is a key moment in the *Science of Logic*. The limit determines the being of a something by showing that a something requires another in order to become determinate. That is, it is only in contrast to another something that a something can be truly determinate. In doing so, the limit once again repeats the structure which we found central to Hegel's characterisation of the differential calculus, as it both distinguishes elements from one another, as well as determining them as necessarily related to one another. "Limit is the mediation through which something and other each as well *is*, as *is not*" (*SL*, 127). The limit thus makes clear the idea that determination is inherently relational. The limit is essential to the determination of something as finite, as in being determined by something outside of itself, finite being can only become itself by ceasing to be. "[Things] *are*, but the truth of this being is their *end*" (*SL*, 129). As we saw, this notion of the finite limit brings the idea of the infinite into play, as in going beyond itself, each finite reaches its end, or ceases to be, but the overall movement of finitude persists, generating its contrary. We saw in the last chapter, however, that Deleuze provides an alternative account of the calculus which seeks to describe the relations of the two elements without recourse to negation. We can

therefore, by analogy, see how he believes it to be possible to bring in a similar conception here, in like manner to provide a purely affirmative notion of determination. In doing so, however, Deleuze is forced to move to a transcendental account in order to open up a space for difference which is not to be conceived of in spatio-temporal terms. There is thus a similarity between the approaches of Deleuze and Hegel to the problem of difference. Just as Hegel argues that it is only because we are dealing with a spatialised conception of difference that we cannot reunite the world of laws and the inverted world into one common world, Deleuze argues, against Hegel, that difference is understood in terms of negation only because we base our notion of difference (implicitly or not) on an empirical difference, the idea of something being determined by not-being something else. Deleuze brings out this distinction between two notions of limit with his discussion of two distributions, the sedentary and the nomadic. The sedentary "proceeds by fixed and proportional determinations which may be assimilated to 'properties' or limited territories within representation" (*DR*, 36). It is this type which Deleuze claims Hegel has infinitised in his theory of orgiastic representation. The second distribution, the nomadic, is "no longer a division of that which is distributed but rather a division among those who distribute *themselves* in an open space – a space which is unlimited, or at least without precise limits" (*DR*, 36). It is this second kind of distribution which Deleuze takes as characterising the transcendental field. In concluding our analysis of Deleuze's criticisms of Hegel, I want to look at the dialectic which gives rise to the question of the inverted world, the dialectic of perception, as it will allow us both to see why Hegel cannot think the Deleuzian concept of difference due to their differing approaches to the problem of the one and the many, and to focus attention on Deleuze's comment that "empiricism truly becomes transcendental … only when we apprehend directly in the sensible that which can only be sensed, the very being *of* the sensible" (*DR*, 56-7).

The One and the Many

The dialectic of force emerges from the dialectic of perception, in which consciousness struggles to reconcile two notions of the thing: on the one hand the notion of the thing as simply a collection of properties, and on the other, the thing as a unity. The first view emerges from our consideration of the 'also', and rests on viewing the object as a simple universal, a 'here', which results from the previous dialectic of sense-certainty. The object is conceived of as a collection of properties, and this collection is united in that it shares the same 'here'. "All these many properties are in a single simple 'Here', in which, therefore, they interpenetrate; none has a different Here from the others, but each is everywhere, in the same Here in which the others are" (*PS*, 68). This view of the object as a mere collection of properties proves, however, to be inadequate. This is because what holds the various properties together is merely an indifferent 'also'. "This salt is a simple Here, and at the same time manifold; it is white and *also* tart, *also* cubical in shape, of a specific gravity, etc." (*PS*, 68). This 'also' is an "abstract universal medium" (*PS*, 68), within which the various properties inhere. It functions much like the idea of space which we found in our chapter on Bergson, and this in turn is its problem. If the object is to be conceived of as a collection of properties, these properties cannot simply be related to one another indifferently through the medium. To be a collection requires a set of properties which are different from one another, and which must therefore differentiate themselves from one another. That is, the various properties must "*relate* themselves *to others* as to their opposites" (*PS*, 69). What Hegel seems to mean by this is that in conceiving of the object as a collection of many properties, we cannot simply see it as the indifferent unity of the 'also', as in this case, the very determinacy of the object itself would break down. A condition of the many is therefore that we have not just an indifferent connection between the elements, that it is a pure many, but instead that we view it as a One, as a unity of elements which relate to each other

directly by opposing one another. Once it is perceived as a One, however, we reach a new problem. Repeating the difficulties we encountered with the univocity of being in chapter two, the One cannot be conceived of in terms of any of the properties, as this would be to return to the problem of the many. The alternative, however, is to conceive of the One apart from these properties, but this leaves it, like Aristotle's concept of being, without determinacy, as there is nothing by which we can grasp it. For the rest of the dialectic, consciousness oscillates between these two conceptions, in turn treating one aspect as essential to the thing whilst rejecting the other as unessential, and provided by consciousness itself.[10] Finally, the failure to resolve this dichotomy leads consciousness to posit the notion of force, thus sacrificing the common sense notion of the object as a unity of properties for one which instead sees it as a relation of forces, whereby the two aspects of the many, as expressed force, and the one, as the unity of law, are given separate ontological realms. When this concept is pushed to the limit, both aspects are again reconciled.

This repetition of the problem of the univocity of being can therefore be seen in the problem of the one and the many on two levels. It seems like we have two problems. If we remain with the notion of the many, we lose the idea of essence, as there is nothing which unifies the various determinations. The individual therefore becomes merely a collection of accidental properties. If, on the other hand, we rely on the notion of a genus, at the level of the One, this genus itself becomes indeterminate (as we saw for Aristotle), and we risk losing the notion of the accident in the face of the abstract universality of the species. As we saw, this is what Deleuze calls the problem of the small and the large (*DR*, 30-42), and he writes that Hegel "tries to render [representation] infinite, to endow it with a valid claim to the unlimited, to make it conquer the infinitely great as well as the infinitely small, opening it up to Being beyond the highest genera and to the singular beneath the smallest species" (*LS*, 259). This is

also Robert Stern's evaluation of the Hegelian solution,[11] an interpretation he supports with the following passage:

> [E]ach human being though infinitely unique is so primarily because he is a *man*, and each individual animal is such an individual primarily because it is an animal: if this is true, it would be impossible to say what such an individual could still be if this foundation were removed, no matter how richly endowed the individual might be with other predicates, if, that is, this foundation can equally be called a predicate like the others. (*SL*, 36-7)

The problem of the one and the many is also a key problem for Deleuze, but here we can perhaps detect the influence of the assumption of the relation between determination and limitation in Hegel's dialectical movement, and we will now explore how, in moving to a different conception of limit, Deleuze is able to take the dialectic in an entirely different direction to Hegel, and in doing so, generate a concept of difference which cannot be captured by Hegelian dialectic. In doing so, Deleuze will rely on the idea of a continuous multiplicity which we discussed in chapter four, what he calls in *Difference and Repetition* a "geometry of sufficient reason, a Riemannian-type differential geometry which tends to give rise to discontinuity on the basis of continuity, or to ground solutions in the conditions of the problems" (*DR*, 162).

This geometry of sufficient reason, which corresponds to the Idea which we discussed in chapter four, directly represents Deleuze's alternative to the Hegelian solution of infinite thought. We saw on page 155 that Deleuze introduces the term dialectic to describe his method, a term which signifies an attempt to replace rather than reconcile himself with Hegel, and in discussing the Idea he likewise brings in the concept of a concrete universal. He writes that "Ideas are concrete universals in which

extension and comprehension go together – not because they include variety or multiplicity in themselves, but because they include singularity in all its varieties" (*DR*, 176), and whilst here the reference to the concrete universal, Hegel's solution to the problem of the large and small, must be taken as polemical, the term does highlight its analogous position in Deleuze's system, as the problem of the large and small is also the problem of extension and comprehension. The term comprehension defines the totality of properties which an object has, whilst extension defines the number of objects in a particular class (or, what objects the concept ranges over). Normally, these terms enter into a relationship of inverse proportionality, as the greater number of objects a term ranges over, the less it determines its object, and *vice versa*. Thus, being, a concept which is supposed to range over all objects, is essentially empty of content. For both Hegel and Deleuze, the aim therefore is to reconcile these two tendencies of the concept.

Deleuze takes as key to our notion of multiplicity that it is understood as a substantive: "multiplicity must not designate a combination of the many and the one, but rather an organisation belonging to the many as such, which has no need whatsoever of unity in order to form a system" (*DR*, 182). What Deleuze is hinting at here is the fact that the one and the many both take their roots in an adjectival mode of expression. That is, we talk of the one and the many in terms of the many *this-and-that*, and the one *this*. The one and the many therefore already point towards the idea of a set of discrete elements which essentially remain indifferent to one another. If the many is understood in these terms, the many, as adjectival, becomes something like the medium within which the elements reside, or the relations which hold between indifferent elements. It is that which allows the many indifferent elements to be brought together, through what is an extrinsic description. As we saw, Hegel takes up precisely this conception, and shows how the indifference of the elements proves to be insupportable, as their determination can only be understood mutually through their opposition to each other. Thus the concept of negation is introduced in order to both differentiate and relate the elements.

The Deleuzian conception of the multiplicity replaces the many not with a unity of the particular and the universal, but instead with the idea of a medium that is not indifferent, but contains difference within itself. Hence the Deleuzian example of the white light, within which all of the colours are united in such a way that they maintain their difference which we discussed on page 156. In choosing this example, which as we saw comes from Bergson, we move away from an objectival conception of elements. The elements do not now exclude one another, but interpenetrate. The example of light is supposed to show how the colours can coexist whilst still preserving their differences from one another. As such, it provides an idea of a concept of colour which, as it is not grounded in the idea of an object, does not require the idea of opposition or negation. In truth, however, we can see that light still requires a place within which it appears, and so is not free of the idea of a medium. This is why Deleuze moves to the conception of a differential geometry, as here the space itself is one in which the very fabric of its dimensions is no longer uniform, but contains deformations and perturbations. As such, differential geometry contains the possibility of capturing a field of differences without having to rely on objects which inhabit a medium. Here, the multiplicity becomes substantive in its own right. As we saw on page 136 of chapter four, such a geometry can be used to show all possible states of a system simultaneously, as any particular state represents one trajectory through its space. Thus, the space itself allows us to represent both the specificity of a particular state (comprehension), and the generality of the system as a whole (extension).

Instead of moving to the Hegelian concrete universal by rejecting the spatial notion of difference as relating *indifferent* elements, as we found Hegel doing, Deleuze instead rejects what he takes to be the essentially spatial idea of determination as involving *opposition* and *negation*. It is this difference in approach which leaves Hegel unable to think Deleuzian difference. As we saw, however, Deleuze is still able to understand the process of Hegelian dialectic, as for Deleuze, this involves making the

mistake of thinking the result of a process of generation apart from the process itself. For Deleuze therefore, Hegelian logic is in fact an epilogic, since the play of being and nothing relies on the exclusion of the generative field of depth. This does not of course prove Deleuze's point that Hegelian dialectic provides an inadequate tool for understanding the world, but does provide some circumstantial arguments for suggesting that Deleuze may have a more powerful alternative. Before concluding, I want to briefly look at some of the logical consequences Deleuze takes to follow from the structure of Hegelian dialectic.

The three critical statements about dialectic by Deleuze which I wish to discuss all follow from the move to a transcendental empiricist philosophy. The first of these concerns the status of the movement which Hegel creates with dialectic. As Deleuze writes, "[Hegel] creates movement, even the movement of the infinite, but because he creates it with words and representations, nothing follows" (*DR*, 52). Deleuze takes this criticism to derive from Kierkegaard and Nietzsche (*DR*, 8). In this sense, Deleuze is arguing that Hegel has misunderstood the cause of the movement of thought by continuing to represent it, rather than seeing this cause as sub-representational. The aspect of representation which Deleuze takes to be critical here is the universal. "'Everyone' recognises the universal because it is itself the universal, but the profound sensitive conscience which is nevertheless presumed to bear the cost, the singular, does not recognise it" (*DR*, 52). In this case, therefore, it is the singular, or singularity, which is neither particular nor universal, is excluded by beginning with a term whose universality instigates a set of related concepts which exclude the singular from consideration. The second criticism is that this movement is always around a particular point. This criticism is derived from Althusser's study of dialectic (*DR*, 186, 207), and argues that Hegel relies on a "monocentring of circles" (*DR*, 49) which Deleuze claims comes about through Hegel's adherence to the species-genus model. In doing so, all movement must be understood as being referred to a central point, just as the many must

be referred to the one in the dialectic of perception. The third point, which relates the previous two, is that the idea of opposition, which Hegel uses to unite the particular and universal, is too rough to provide an adequate description of the world. "Oppositions are roughly cut from a delicate milieu of overlapping perspectives, of communicating distances, divergences and disparities, of heterogeneous potentials and intensities" (*DR*, 50). That is, Deleuze asserts that simply relying on a reinvigorated understanding of the concrete universal will not provide the kinds of fine grained distinctions needed to adequately describe the world. "We can say 'the one is multiple, the multiple one' for ever: we speak like Plato's young men who did not even spare the farmyard" (*DR*, 182). The value of all three of these forms of criticism is tied, however, to Deleuze's understanding of difference. If we look at the first, the idea of false movement presupposes the discovery of a true movement which is responsible for the movement which Hegel posits. This is the movement provided by the difference between the transcendental and empirical fields. Without this difference, therefore, the criticism appears unfounded. Likewise, the third criticism that opposition is too rough cut a way of defining distinctions implies a knowledge of the true nature of the world, and hence another perspective. The second criticism, that Hegel relies on a monocentring of circles, is slightly different, in that what is at issue would not seem to be the assertion itself, but rather the normative value of it. That is, if Hegel completes the Aristotelian project, this is only a problem if the Aristotelian project is in itself misguided. But this claim in turn depends on the validity of the Deleuzian project. Where does this then leave the status of our enquiry? In showing that Deleuze is able to support a conception of difference which is unthinkable by Hegel, we have gone a long way towards showing that Deleuze offers a coherent alternative to Hegel's thought. In providing a differential ontology, Deleuze has avoided Derrida's famous challenge that to oppose Hegel is ultimately simply to reinstate the dialectic which reabsorbs the opposing position into the dialectic as a whole. Furthermore, now that we have more completely specified the point of disagreement between Deleuze and Hegel, we can resolve to move to another element

outside of the purely logical, in order to progress further in deciding between them. There is a sense in which situating a confrontation between Hegel and Deleuze in the grounds of pure logic is unfair to Deleuze, who after all, professes to be a transcendental empiricist. To explore these empirical possibilities here would be to go beyond the scope of the thesis, however.[12] To conclude, I want to now briefly survey the principle results of the thesis as a whole.

Conclusion

The aim of this thesis was to explore the relations between the responses of Hegel and Deleuze to the problems which arose from what Deleuze called the representationalist paradigm. As has often been noted, Hegel holds an anomalous position in the philosophy of Deleuze.[13] The philosophy of Deleuze is, to borrow a phrase of Salomon Maimon, a *Koalitionssystem*, and we have seen the importance of the work of Sartre, Bergson, and Kant in its construction. Each of these figures provides important elements which Deleuze takes up in his own synthetic philosophical system, and this list could be added to by a whole series of thinkers we have not considered.[14] Whilst Hegel remains largely absent from this list, Hegel develops much the same problematic as Deleuze, and Deleuze, in developing his own solution to what he terms representation, engages directly with structural elements in Hegel's own thought. The aim of this thesis was to explore these similarities in order to allow a comparison between Deleuze and Hegel, beginning by showing how both Hegel and Deleuze should be situated within the history of post-Kantian philosophy. We then saw how the problem of representation has specific consequences in terms of the large and the small: the problems of the univocity of being, and the specification of the individual. In particular, we discovered these problems both in the formulations of Aristotle, and of Russell. It is these problems which Deleuze claims Hegel attempts to solve by moving to a position of

infinite thought, where the genus and species are reconciled, what Deleuze calls infinite representation. Deleuze's response, instead, is to use the resources of differential geometry to expand certain Bergsonian insights into the dual nature of multiplicities. By providing a novel interpretation of the calculus, Deleuze aims to provide an equivalent of the concrete universal which does away with the generic concept of universality altogether. In order to resolve the problems of representation, as we saw in this chapter, both Hegel and Deleuze adopt a concept of difference which moves beyond that provided by empirical, spatial intuition. It is this move beyond spatial representation which means that Hegel's criticism of the notion of essence, here represented by the dialectic of the inverted world, no longer applies to the virtual. As Deleuze writes, somewhat polemically, "no doubt, if one insists, the word 'essence' might be preserved, but only on condition of saying that the essence is precisely the accident, the event, the sense; not simply the contrary of what is ordinarily called the essence but the contrary of the contrary: multiplicity is no more appearance than essence, no more multiple than one" (*DR*, 191). That is, Deleuze's contrary of the contrary involves neither term, rather than, for Hegel, both terms in an identity of identity and difference. In showing that Hegel is unable to think this concept of difference, I hope to have demonstrated the novelty of Deleuze's philosophy, which provides a viable alternative to Hegel. More than this, there is an asymmetry between their positions, as the Hegelian philosophy of negation *can* be understood from a Deleuzian perspective, albeit as a false movement. This fact does not on purely logical grounds entail a rejection of Hegelian dialectic, however, since the Hegelian can simply deny the actual existence of Deleuzian difference. In explicating these connections between Hegel and Deleuze, I hope I have opened up the possibility of a further test, however. In the *Phaedrus*, Plato writes that we should "cut up each kind according to its species along its natural joints, and not try to splinter any part, as a bad butcher might do."[15] This principle leads us back to the natural domain of logic, that is, the world itself. The ultimate test must therefore be the extent to which any logical theory captures the nature of the world, not simply by

ranging over all phenomena, but also by capturing their singularity, or used in the term`s loosest possible sense, their essence.

Notes to the Text

Notes to the Introduction

[1] For evaluations of and responses to Badiou's criticisms of Deleuze, see Ansell-Pearson, *Philosophy and the Adventure of the Virtual*, particularly chapter four, and Toscano, *The Theatre of Production*, chapter 6.5.
[2] I am here principally thinking of Hardt's *Gilles Deleuze: An Apprenticeship in Philosophy*, Widder's *Genealogies of Difference*, and Smith's *Deleuze, Hegel, and the Post-Kantian Tradition*. Hardt's work does not deal in any depth with Deleuze's relation to Hegel, his theme rather being the principal positive influences on Deleuze's development. Widder's book does relate Deleuze to Hegel, but as the title suggests, this work provides a *genealogical* account of this relation. The aim is therefore more to understand this relation historically rather than logically. Smith's article instead aims to show possible similarities between Deleuze and Hegel, but as an article is mainly suggestive.

Notes to Chapter One

[1] In this, we are following Allison's conception of Kant's transcendental idealism, which posits, "two radically distinct epistemic relations to objects, neither of which is *ontologically* distinct" (Allison, *Kant's Transcendental Idealism*, 47), as this provides the strong sense of the *radical break* with previous philosophies ("until the critical philosophy all philosophies are not distinguished in their essentials," Kant, cited in *Kant's Transcendental Idealism*, 16). This ontological identity leads to the classification of Allison's interpretation as a "one world view" of Kant's metaphysics, and as such is in opposition to the "two world views", put forward by thinkers such as Strawson, and Ameriks. This interpretation has the further advantage of mirroring Deleuze's own interpretation (and his own ontology to the extent that the virtual and the actual are differences in kind generated merely by differences in degree).
[2] Heidegger, Martin, *Phenomenological Interpretation of Kant's Critique of Pure Reason*, 133, "Pure thinking determines in advance the objectivity of the object, to such an extent that only on the basis of this determination can the object now be encountered as that to which ontic empirical knowledge fits. *The possibility of such an anticipation of determinations of objects prior to all experience, as well as the meaning and legitimacy of such an anticipation, are the basic problems of transcendental logic.*"
[3] Allison, *Kant's Transcendental Idealism*, provides a detailed discussion and deduction of these two characterisations of the world, albeit in defence of the synthetic nature of the Kantian system against what he calls the "theocentric" attitude of transcendental realism. As he writes, "what all forms of transcendental realism have in common may be negatively expressed as a failure to, or to put it less tendentiously, a refusal, to recognise that human cognition rests on *a priori* conditions of sensibility, which structure the way the mind receives its sensory data." (27), and further, "since cognition (of whatever sort) requires that its object somehow be 'given' to the mind, the denial of discursivity requires the assumption that the objects themselves (and as they are in-themselves), not merely the data for thinking them, be so given" (28). Here discursivity means that thought requires a synthesis of "both concepts and *sensible* intuition" (13). Allison is here arguing that if cognition is not seen (following Kant) as being grounded in the synthesis of concepts and the sensible, we are left with two options; the first, that of the empiricist tradition, holds the objects of thought not to be given, but instead merely to be inferred, thus leading to scepticism, such as that of Hume. Allison argues that in this case, scepticism emerges through the assumption that the only true mode of cognition of the object would be (infinite) intellectual intuition, against which the finite understanding of the mind is contrasted and judged inferior. Thus, whilst the empiricists hold to a limited epistemological anthropocentric model of cognition, this rests on an analytic ontological theocentric model of the structure of the world. Amongst the rationalists, this priority of intellectual intuition is even more obvious. Going beyond Allison's understanding of the distinction between transcendental idealism and transcendental realism, Deleuze is arguing that in both cases, the structure of the object under analysis will be that of an object holding certain properties. We will return to the exact nature of the Deleuzian alternative once we have an initial understanding of some of his basic concepts.

[4] Even in the case of thinkers such as Schopenhauer, and Nietzsche, the abyss as undifferentiated can only speak once it has been schematised, or is allowed to speak through Apollo.
[5] Judgements can of course also be formed of objects outside of the domain of the empirical (and also to the limits of the empirical, in terms of concepts such as world, which refer to the empirical as a totality). We will return to these applications of the functions of judgement in chapter six, where we will look at the transcendental illusions which can be generated by such applications. In this chapter, however, we will focus solely on their relation to the empirical world.
[6] By "objectival", I mean an entity which has a subject-property structure, or a proposition or logic which is isomorphic which this structure. Objectival therefore loosely means structured according to the classical conception of an object. A parallel meaning has been given to the term "subjectival". These neologisms have been coined to prevent confusion or conflation with the similar terms subjective and objective.
[7] Whilst there is clearly a distinction between the activities of synthesis and subsumption, at least as the former is pre-judgemental, whereas the latter is involved in empirical judgements themselves, the question as to whether the same operation is at play in both activities does not need to be answered for the purpose of the argument (although footnote 10 provides some textual evidence for their equation). Synthesis of intuitions is clearly non-hierarchical *contra* subsumption, which involves placing representations under other representations. Nevertheless, Heidegger, *Phenomenological Interpretation of Kant's Critique of Pure Reason,* for one, still equates subsumption with synthesis, stating that "judging, asserting, λέγειν means a gathering together, a grasping together, a *conjunction* or *synthesis*." (117) The weaker claim that "one and the same understanding, governed by the same set of rules or functions…is operative in both domains"(Allison, *Kant's Transcendental Idealism*, 152), or, more precisely, "the logical function and category express one and the same function of the understanding operating at two levels" (314) is all that is required.
[8] This conceptual reference, such as the reference to succession provided in the move from the hypothetical judgement to the causal judgement does not itself allow the application of the categories to intuition itself, as, as the aesthetic shows, intuition is *non-conceptual*. The actual application of the one to the other will be facilitated by the schemata, which are "intellectual on the one hand [and] sensible on the other" (*CPR*, A138/B177).
[9] "It is true that knowledge is expressible in propositions. But it does not follow that every form, or every formal feature, of propositions which the logician thinks it worthwhile to distinguish is absolutely necessary in the expression of knowledge. Given a certain indispensable minimum equipment of notions, the logician can, if he chooses, distinguish indefinitely many forms of proposition, all belonging to formal logic. If we allowed a category for each form, we would have indefinitely many categories" (Strawson, *The Bounds of Sense*, 79). Furthermore, "as far as logical forms are concerned, the logician's choice of primitives *is* a choice. For example, the idea of quantification in general may be said to be a primitive idea in logic in that it cannot, except for an antecedently limited language, be defined in terms of propositional constants, names, and predicates; but it is a matter of choice whether we introduce the existential quantifier without formal definition and define the universal quantifier in terms of it, or vice versa" (80).
[10] "The same function that gives unity to the different representations in a judgement also gives unity to the mere synthesis of different representations in an intuition, which, expressed generally, is called the pure concept of the understanding. The same understanding, therefore, and indeed by means of the very same actions through which it brings the logical form of a judgement into concepts by means of the analytical unity, also brings a transcendental content into its representations by means of the synthetic unity of the manifold in intuition in general, on account of which they are called the pure concepts of the understanding that pertain to objects *a priori*; this can never be accomplished by universal logic" (*CPR*, A79/B104-5).
[11] "In like manner, infinite judgements must, in transcendental logic, be distinguished from those that are *affirmative*, although in general logic they are rightly classed with them, and do not constitute a separate member of the division" (*CPR*, A72/B97). Here the table of judgements shows a difference between functions that simply cannot be discovered within judgement itself, but must be posited, due to its place within the transcendental logic.
[12] "But first I shall introduce a word of explanation in regard to the categories. They are concepts of an object in general, by means of which the intuition of an object is regarded as determined in respect of one of the logical functions of judgement" (*CPR*, B128).
[13] Allison, *Kant's Transcendental Idealism*, 164.
[14] *CPR*, B119. This is the kind of derivation of the categories that Kant claims Locke provides.

[15] I have here equated the transcendental unity of apperception with the transcendental ego. While there are some important differences between the two structures, I hope that the logic of the argument will justify the identification in this context at least.
[16] Although the first paralogism does indeed attempt to show that the ego cannot be seen as a substantial existent, it is still the case that for Kant the transcendental ego is a necessary presupposition of any form of consciousness capable of relating to objects. This would seem to contrast with Sartre's view, for whom it would certainly seem possible to conceive of a unified consciousness that could not (in fact) posit an "I" accompanying its representations (perhaps an animal consciousness). While Sartre is correct in asserting that the unity of consciousness is not a "real or metaphysical unity of the thing that thinks" (Allison, *Kant's Transcendental Idealism*, 343), from the implications he draws from his interpretation of this fact, it is clear that he is in fact also making the stronger (and false) claim that the unity of apperception is not a *logical* condition for Kant.
[17] "At sixteen, you see, I wanted to be a novelist. But I had to study philosophy in order to enrol in the Ècole Normale Supèrieure. My ambition was to become a professor of literature. Then I came across a book by Henri Bergson in which he describes in a concrete way how time is experienced in one's mind. I recognized the truth of this in myself" (Sartre, *Playboy Interview: Jean-Paul Sartre: Candid Conversation*, 69-77).
[18] Ricoeur, *Kant and Husserl*, 322-3. Ricoeur argues that there is also a genuinely phenomenological strain in Kant's thought which emerges when this epistemological preoccupation is not foregrounded.
[19] Sartre, *Intentionality: A Fundamental Idea of Husserl's Phenomenology*, 259.
[20] "So it is that all at once hatred, love, fear, sympathy – all these famous 'subjective' reactions which were floating in the malodorous brine of the mind – are pulled out. They are merely ways of discovering the world. It is things which abruptly unveil themselves to us as hateful, sympathetic, horrible, lovable. Being dreadful is a *property* of this Japanese mask, an inexhaustible and irreducible property which constitutes its very nature – and not the sum of our subjective reactions to a piece of sculptured wood" (Sartre, *Intentionality: A Fundamental Idea of Husserl's Phenomenology*, 259).
[21] "Kant says nothing about the actual existence of the *I Think*. On the contrary, he seems to have been perfectly aware that there are moments of consciousness without the *I*' (*TE*, 32). This stems directly from the fact that the existence of the transcendental ego is a question of validity, not one of fact. The transcendental is seen purely as providing necessary conditions for the possibility of experience. Questions about the existence in *fact*, or of the *nature* of the transcendental ego would thus stem from a confusion of matters of fact and matters of validity.
[22] These two positions, that on the one hand, consciousness is unified by the object, and on the other, that it is consciousness that unifies itself, seem to be reconciled in the belief that "consciousness refers perpetually to itself" (*TE*, 39), so that past consciousness is also an object for consciousness. In this way, it is always that related to which unites consciousness.
[23] By this, Sartre is presumably referring to the definition of substance as "that which is in itself and is conceived through itself; that is, that the conception of which does not require the conception of another thing from which it is formed." (Spinoza, *Ethics*, Def. 3). Of course, consciousness is always consciousness *of*, and so in this regard departs from Spinoza's definition, being reliant on the object of cognition, but Spinoza's definition still captures the sense in which consciousness, while dependent on the object, cannot be viewed as a product of the object itself (consciousness is contingent in relation to the object). The lack of interaction between consciousnesses can also be explained by Proposition 2 of the *Ethics*, which rests directly on this definition, and states that "two substances having different attributes have nothing in common." Here the attribute, or "that which the intellect perceives of substance as constituting its essence" (Def. 4) is different in the case of each individual consciousness, and therefore prevents their direct interaction. This lack of direct interaction will become the key to Sartre's critique of the master-slave dialectic, as it prevents the recognition of self in the other which is central to Hegel's resolution of the two consciousnesses relations.
[24] See Sartre, *Being and Nothingness*, chapter one.
[25] C.f. *NP*, 22, *Existence and Innocence*. Here Deleuze makes Nietzsche argue that the concept of responsibility emerges from the prioritisation of the actual, which makes forces appear as the result of the actions of bodies. In this case, the agent is responsible for actualising the force, or for holding back. This for Deleuze amounts to conflating the virtual and actual.

[26] "In fact this bestowal of sense [which corresponds to the notion of the virtual]... would correspond to the conditions posed by Sartre in his decisive article of 1937" (*LS*, 98). As late as 1990, the importance of these conditions is highlighted by Deleuze: "Transcendental Empiricism is meaningless indeed unless its conditions are specified. But transcendental 'field' must not be copied from the empirical, as in Kant. It must be explored on its own terms: 'experienced or 'attempted' (but it is a very particular type of experience). This is the type of experience that enables the discovery of multiplicities." (Deleuze, Gilles, *Letter-preface to Jean-Clet Martin*, in *Two Regimes of Madness*).

[27] Sartre, Jean-Paul, *Nausea*, which was written simultaneously with *The Transcendence of the Ego*, contains the phenomenological descriptions of both the chair and the tree which Sartre alludes to in the latter work. "I was saying to myself that the sea belonged to the class of green objects, or that green formed part of the sea's qualities...but all that happened on the surface." (182-3) "The root, the park gates, the sparse grass on the lawn, all that had vanished; the diversity of things, their individuality, was only an appearance, a veneer. This veneer had melted, leaving soft, monstrous masses in disorder – naked, with a frightening, obscene nakedness," (183). Sartre later would say of these descriptions that "I had to write in the style of a novel, as the idea was not yet clear enough for a philosophical book" (*Sartre par lui-meme*), and this would seem to be largely because both experiences seem to point away from the classical phenomenological method, and the Husserlian theory of essences, which defined his philosophical framework during his time in Berlin. The question of whether these analyses actually occur "clearly enough" within the framework of his later work, possibly through the ostensibly more Heideggerian writings of his mature philosophy, or indeed whether the early writing provide the grounds for a distanciation of Sartre's philosophy from the Heideggerian ontology to which it has so often been hastily assimilated, falls beyond the scope of this thesis.

[28] This result also has ties to that of gestalt psychology, possibly also mediated through Sartre or Merleau-Ponty, for example, Koffka's assertion that, "the relation between a set of conditions and the process which takes place under these conditions is not as a rule such that we can divide the total process into a finite number of part-processes each of which will depend upon a certain part of those conditions" (Koffka, *The Growth of the Mind*, 121).

[29] Sartre, *Being and Nothingness*, 255.

[30] "Formerly I believed that I could escape solipsism by refuting Husserl's concept of the existence of the transcendental 'Ego.' ... but actually although I am still persuaded that the hypothesis of a transcendental subject is useless and disastrous, abandoning it does not help one bit in solving the problem of the existence of others" (Sartre, *Being and Nothingess*, 235).

[31] Allison, *Kant's Transcendental Idealism*, 78 quoting Kant's *Lectures on Logic*: a concept is "a universal representation, or a representation of what is common to several objects, hence a representation, insofar as it can be contained in various ones."

[32] "In every judgement, then, there is a certain relation of different representations insofar as they belong to one cognition. E.g., I say that man is not immortal. In this cognition I think the concept of man, and it thereby happens that this cognition, which constitutes the unity of two different representations, becomes a judgement" (*Wiener Logic* 24: 928: 368, taken from Allison, *Kant's Transcendental Idealism*, 83).

[33] This is Deleuze's reading of *The Birth of Tragedy,* which "is only distinguishable from the dialectic by the *way* in which contradiction and its resolution are conceived" (*NP*, 11), clearly presuming a radical break between this early work and his later writings, whilst maintaining that *The Birth of Tragedy* contains the seeds of Nietzsche's later anti-dialectical approach. Other readings, such as Porter, *The Invention of Dionysus: An Essay on the Birth of Tragedy*, argue instead that the *Birth of Tragedy* already contains the critique of metaphysics which characterises Nietzsche's later work. Porter argues that *The Birth of Tragedy* provides an ironic discussion of the Greek concept of the will, and that Nietzsche's argument is already at this point that metaphysics is a necessary illusion (148, Chapter: *The Myth of Mythlessness*) in the face of the desolate picture of the world presented by atomism. This thesis is supported by an appeal to Nietzsche's earlier reading, and approval of the works of Lange, who held similar views, as well as through an analysis of Nietzsche's unpublished piece, *On Schopenhauer*, where Nietzsche argues that Schopenhauer's conception of the will is itself founded in representation ("On such a view, a phenomenal world is posited before the phenomenal world", *On Schopenhauer*, quoted from Porter, 60). The final, and most powerful piece of evidence, is the fact that Nietzsche begins his discussion with a reference to Lucretius' discussion of the gods, an atomist, and hence one opposed to metaphysics as propounded by those such as Schopenhauer, who holds that

"Dionysus and Apollo would be just one more appearance, one more complex bundle of atomic films drawn from the limitless fund of "images" (*phantasmata*) that abound in the universe and that the mind itself generates." (Porter, 37). Such a reading clearly (and explicitly – pp. 172 & 182) opposes that of Deleuze, but also throws into doubt whether such a Nietzsche could ever be conceived as moving beyond the problem of representation as put forward by Deleuze. Furthermore, Nietzsche's 1866 comment that "Schopenhauer still remains ours, in fact he becomes ours all the more" (Houlgate, *Hegel, Nietzsche, and the Criticism of Metaphysics*, 27), which followed an elaboration of the atomist, Lange's system, would seem to cast doubt on Porter's reading. The question of the veracity of Deleuze's interpretation of Nietzsche (which has strong textual support of its own) can be bracketed for the purposes of this thesis, however, as the central aim is to discover *Deleuze's* relation to Hegel. My discussions of Deleuze's treatment of Nietzsche and other thinkers will therefore be focussed on the degree to which they shed light on his own metaphysics, rather than on his justification for such interpretations.

[34] Kant's discussion of the *Ideal of Pure Reason* within the critique provides us with one formulation of this: *The Principle of Thoroughgoing Determination*, which holds that "Among all possible predicates of things, insofar as they are compared to their opposites, one must apply to it." (*CPR,* B579). "The task of defining an individual, which for Kant as for Leibniz is an infinite one, requires an exhaustive inventory or synthesis of these pairs [of predicates]" (Allison, *Kant's Transcendental Idealism*, 398).

Notes to Chapter Two

[1] Aristotle, *Physics*, 185a.

[2] For the place of formal reasoning in Plato, see Bochenski, *A History of Formal Logic*, 30.

[3] C.f. the opening to the *Topics*, "Our treatise proposes to find a line of inquiry whereby we shall be able to reason from reputable opinions about any subject presented to us, and so shall ourselves, when putting forward an argument, avoid saying anything contrary to it. First, then, we must say what deduction is, and what its varieties are, in order to grasp dialectical deduction; for it is the object of our search in the treatise before us."

[4] Aristotle, *Topics*, 102a.

[5] "Now one commonplace rule is to look and see if a man has ascribed as an accident what belongs in some other way. The mistake is most commonly made in regard to the genera of things, e.g. if one were to say that white happened to be a colour – for being a colour does not happen by accident to white, but colour is its genus." (Aristotle, *Topics*, 109a)

[6] This determination is not reversible, i.e. an animal is necessarily a substance, but a substance is not *necessarily* an animal.

[7] Aristotle, *Sophistical Refutations*, 165a, quoted from Bencivenga, *Hegel's Dialectical Logic*, 10.

[8] C.f. W. D. Ross, *Aristotle*, 24: "For logic is the study of thought, and that which the species contains over and above its specific nature is due to the particular matter in which it is embodied, and thus eludes thought. In so far as they can be known, the members of an *infima species* [species at the lowest level of the hierarchy] are identical, and it is only those properties of them which flow from their specific nature that can be grasped by science."

[9] Bencivenga, *Hegel's Dialectical Logic,* places great stress on the importance of these definitions. C.f. especially, chapter one. Whilst he recognises that these definitions are central to the problem of the definition of terms in Aristotle's logic, he does not extend his enquiry to the problems inherent in the structure of the logic itself which become clear especially in the scholastic idea of analogy. Bochenski, *History of Formal Logic*, notes these structural features of Aristotle's logic, and their relations to set theory, but fails to realise that these features may be problematic for an ontological enquiry such as Aristotle's. A thorough analysis of Aristotle's various uses of equivocity can be found in Owen, *The Doctrine of Being in Aristotelian Metaphysics*, 107-135.

[10] Aristotle, *Categories*, 1a.

[11] C.f., for example, Aristotle, *Topics*, 106a to 107b.

[12] "For things which differ in *genus* have no way to one another, but are far too distant and not comparable; and for things that differ in *species* the extremes from which generation takes place are the contraries; and the distance between extremes – and therefore that between the contraries – is the greatest" (*MP*, 1055a).

[13] A more rigorous dissection of this argument can be found in Bochenski, *History of Formal Logic*, 54.

[14] Aristotle, *Topics*, 102a.
[15] Given the conditional nature of this statement, another possibility opens itself up, that of rejecting the stability of the system, and maintaining the unity of being. This will of course be Hegel's position, whereby this inherent instability is, instead of being rejected, used as a motor to power the differentiation of being.
[16] Aristotle, *Categories*, 1b. For a discussion of the role of the categories in other works by Aristotle, and their development as well as significance, see Ross, *Aristotle*, 21.
[17] Bencivenga, *Hegel's Dialectical Logic*.
[18] "Even if one has several meanings, the other meanings will be related to the primary meaning" (*MP*, 1005a).
[19] C.f. Introduction to Aristotle, *Metaphysics*, Lawson-Tancred translation.
[20] Beistegui, *Truth and Genesis*, chapter one, particularly pp. 39-48.
[21] Beistegui, *Truth and Genesis*, 46.
[22] Ross, *Aristotle*, makes a fine-grained distinction in noting that the atemporal nature of form does not imply that essence is in fact eternal: "In generation form is not generated any more than matter… The most obvious interpretation of this passage [*MP*, 1033 a-b] would be that it teaches the eternity of form. Yet sometimes Aristotle speaks of form as coming into being and passing out of being instantaneously" (174). In this sense, the genesis of form still remains atemporal, hence the requirement of its instantaneous appearance.
[23] "Privation and possession are spoken of in connection with the same thing, for example sight and blindness in connection with the eye. To generalise, each of them is spoken of in connection with whatever the possession naturally occurs in. We say that anything capable of receiving a possession is deprived of it when it is entirely absent from that which naturally has it, at the time when it is natural for it to have it. For it is not what has not teeth that we call toothless, or what has not sight blind, but what has not got them at the time when it is natural for it to have them. For some things from birth have neither sight nor teeth yet are not called toothless or blind" (Aristotle, *Categories*, 12a).
[24] Bencivenga, *Hegel's Dialectical Logic*, 19, highlights the theoretical aspect of this problem.
[25] See Aristotle, *History of Animals*, 589b. Aristotle's definitions of land creatures as those that breathe air and sea creatures as those that breathe water break down in the discussion of the dolphin, which is a sea creature which breathes air, and the newt, which has gills, but which is "fitted for walking on dry land". In dealing with these cases, Aristotle proposes that we "supplement our definition of the term, 'aquatic'" in order to form a more precise definition.
[26] Ridley, *Evolution*, 47-49. We could also note that a ring species can be seen as a *transition* between two species. In this sense, it has been used to further justify the theory of evolution.
[27] This history of Porphyry the Phoenician is taken from the extended introduction to *ISA* by Edward W. Warren.
[28] See *ISA*, 39.
[29] Deleuze was not the first to raise this problem, and it can be seen in different forms in the works of, for example, Duns Scotus, Spinoza, and Heidegger. For an overview of the continuation of this strand of thought, see Beistegui, *Truth and Genesis*. Specific treatments can be found in *Difference and Repetition*, 34 (Duns Scotus), and *Expression in Philosophy: Spinoza*. The question of the sense of Being raised by Heidegger in *Being and Time* is also clearly dealing with the same problem.
[30] Aquinas, T., *Quaestiones Disputatae de Veritate*, 2, 11, c, taken from Bochenski *History of Formal Logic*, 178.
[31] *Ibid.*
[32] *Ibid.*
[33] The relation of the equivocity of being and language lends weight to Nietzsche's assertion that, "I am afraid we are not rid of God because we still have faith in grammar." Nietzsche, *Twilight of the Idols*, 483.
[34] Russell, *Principles of Mathematics*, 69.
[35] Suppose we take the proposition, "the barber shaves himself". It is clear from this that the further statement, "either the barber shaves himself or God does not exist", follows from this first statement, as the first part of the statement makes the whole true, regardless of the truth of the second part. Now, when we introduce the proposition, "the barber does not shave himself", and refer it to our second proposition, it becomes clear that the first part of the statement must be false, but, as we have asserted that the statement as a whole is true, this means that the second part, "God does not exist", must be the case. It is also clear that we could further prove the

contrary of this statement, or any other statement, by the same procedure, rendering all possible propositions both simultaneously true and false. Ross, *Aristotle*, notes that Aristotle holds the similar view that if an object contains contradictory predicates, it contains all predicates (160). although he provides different grounds for this belief.

[36] As Quine writes in relation to Russell's paradox in *Ways of Paradox*, 11, "The almost invariable way of specifying a class is by stating the necessary and sufficient condition of belonging to it. When we have stated such a condition, we feel we have 'given' the class and can scarcely make sense of there not being such a class. The class may be empty, yes; but how could there not be such a class at all? What substance can be asked for it that the membership condition does not provide?"

[37] Bochenski, *History of Formal Logic*, 389.

[38] Whitehead & Russell, *Principia Mathematica*, 37.

[39] Here I will focus on Russell's solution to the problem of self-referential paradoxes. It is important to note, however, that Tarski's own approach (Tarski, *The Semantic Conception of Truth*), which relies on a distinction between an object language and a meta-language solves the difficulty in much the same way, by removing all talk of the semantics of a given language to another, meta-language. The requirement that talk of the semantics of a language be conducted in a separate language removes the possibility of self reference (the language cannot generate propositions about the nature of truth in the language, for instance, which would themselves have to be evaluated in terms of this idea of the nature of truth). This requirement is explicitly aimed at excluding the possibility of avoiding paradoxes (65-6), and gives analogous results to those of the theory of types.

[40] Whitehead & Russell, *Principia Mathematica*, 60. We are here simplifying the situation somewhat by not considering the application of Russell's theory of types to his theory of relations. This complicates the situation as we require a type for relations between individuals, between classes of individuals, as well as relations of relations and relations of classes of relations, etc. This added complexity does not affect the fundamental approach taken by Russell, however.

[41] As Copi notes in *The Theory of Logical Types*, 17-18, we cannot simply reject contradictory classes (on an *ad hoc* basis) for three reasons. First, to do so would be to reject the fundamental rule of Boolean algebra that the negation of a well formed formula is also a well formed formula, as whilst the class of all classes which are members of themselves is well formed, the class of all classes which are not members of themselves would not be so. Second, it is possible to reconstruct the paradox using two classes, neither of which on its own is paradoxical. Given this, it would seem arbitrary which of the classes we rejected. Third, for any system powerful enough to include quantification (existential and universal operators, for instance), there is no decision procedure which will tell us that a class definition is not contradictory (although we can show that certain class definitions *are* contradictory by finding the contradiction).

[42] Russell gives a simpler example, based on the liar paradox which provides an illustration of this approach. The liar paradox dates back to Epimenides, who asserted, being a Cretan himself, that all Cretans were liars. This generates the paradoxical statement, 'all my assertions are false', which, if true, therefore is false, and false, therefore is true (leaving aside certain technical caveats). Russell reformulates this statement in the theory of types as, "all first-order propositions affirmed by me are false" (Bochenski, *History of Formal Logic*, 395). This statement does not lead to a paradox, as it is itself not the assertion of a first-order proposition, being itself of the second order.

[43] Quine in *A Logical Point of View*, 92, also recognises this point, albeit without relating it directly to the problem of univocity, writing, "not only are these cleavages and reduplications intuitively repugnant, but they call continually for more or less elaborate manoeuvres by way of restoring severed connections."

[44] Whitehead & Russell, *Principia Mathematica*: "Thus, for example,, the collection of *propositions* will be supposed to contain a proposition stating that 'all propositions are either true or false.' It would seem, however, that such a statement could not be legitimate unless 'all propositions' referred to some already definite collection, which it cannot do if new propositions are created by statements about 'all propositions.' We shall, therefore, have to say that statements about 'all propositions' are meaningless" (37). The notion of different orders of truth is brought in on page 42 of the same text.

[45] Whitehead & Russell, *Principia Mathematica*, 55.

[46] Whitehead & Russell, *Principia Mathematica*, 55.

[47] Russell, *Principles of Mathematics*, 347.
[48] Russell, *Principles of Mathematics*, 350.
[49] Russell, *Principles of Mathematics*, 350.
[50] Russell, *Principles of Mathematics*, 469.
[51] For an exhaustive study of the relation between Hegel and Aristotle see Ferrarin, *Hegel and Aristotle*. In particular, for Hegel's own opinions of Aristotle, chapter, 11: Pictures of Aristotle in Hegel's Formative Years, deals with this question, as well as Aristotle's eventual supplanting of Plato as Hegel's most influential thinker.
[52] Ferrarin, *Hegel and Aristotle*, 114.

Notes to Chapter Three

[1] This is not to say that other figures are not relevant to gaining a full understanding of Deleuze's philosophy. Hardt, *Gilles Deleuze: An Apprenticeship in Philosophy*, does a good job of highlighting the importance of other figures such as Hume, Spinoza, and Nietzsche.
[2] "If intuition must conform to the constitution of objects, I do not see how we could know anything of the latter *a priori*; but if the object (as object of the senses) must conform to the constitution of our faculty of intuition, I can see no difficulty in conceiving such a possibility" (*CPR*, Bxvii). The mechanics of Kant's solution were dealt with in detail in chapter one.
[3] Russell, *Russell's Logical Atomism*, 52.
[4] In the case of both of these propositions, different interpretations are possible. Whilst the purely psychological interpretation is put forward by Kemp-Smith, as Allison points out, this presents difficulties for the scope of Kant's argument. A purely logical interpretation would seem to go too far in the opposite direction, however, as it rules out the possibility of even conceiving of the possibility of things-in-themselves (and of intellectual intuition). Allison's own reading, which he takes to be "broadly psychological" is that "we cannot represent *outer appearances* without also representing them as in space" [italics mine].
[5] Čapek, *Bergson and Modern Physics*, 6.
[6] Čapek, *Bergson and Modern Physics*, 6.
[7] Čapek, *Bergson and Modern Physics*, 19.
[8] Čapek, *Bergson and Modern Physics*, 75.
[9] See also Descartes, *Rules for the Direction of the Mind*, particularly rules fourteen and fifteen. Here Descartes states that "the problem should be re-expressed in terms of the real extension of bodies and should be pictured in our imagination entirely by means of bare figures. Thus it will be perceived much more distinctly by our intellect."
[10] "By regarding intelligence as pre-eminently a faculty of establishing relations, Kant attributed an extra-intellectual origin to the terms between which relations are established" (*CE*, 358). Hegel makes a similar point about Kant's use of relation in the *Jena Logic*, writing that, "it matters not at all that what Hume calls things are [for Kant] sensations, perceptions, sense representations, or whatever else he likes-[for in any case] they are diverse, self-subsistent; the infinity of the relation, the necessity, is something separate from them. That being *per se* of the diverse in its objective aspect Kant calls a contingent togetherness; and the necessity remains something subjective… For in respect to them in themselves, what is connected is supposed to be outside of the connection – the sensations are self subsistent singulars; and likewise the infinity of the connection, the concept of the understanding in and for itself, is to be outside what is connected" (*JL*, 53-4).
[11] Russell, *History of Western Philosophy*, 788.
[12] Čapek, *Bergson and Modern Physics*, 224.
[13] Kant, *Prolegomena*, §13.
[14] Euclid enumerates the full list of axioms as follows:
Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and radius.
4. That all right angles are equal.
5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

Boyer, *A History of Mathematics*, 116.

[15] Cf Williams, *Descartes: The Project of Pure Enquiry*, 229, for a study of the relation of Descartes' conception of matter to Newton and geometry.
[16] Čapek, *Bergson and Modern Physics*, 75.
[17] Gleick, *Chaos*, 14.
[18] Boyer, *A History of Mathematics*, 60. What has been called the first quantitative law of acoustics concerns the ratio of the length of a string to tone produced by plucking it. First, it was noted that if a string of a certain length produces a certain note upon plucking, a string of twice the length will produce a note one octave below. Further, if string produces the tone C when plucked, other tones will be produced by strings in rational ratios of length with the first string, i.e. 16:9 for D, 3:2 for E, etc.
[19] Number mysticism, such as the Pythagorean idea of the counter-Earth posited to make the number of heavenly bodies equal to ten, while obscure is not unusual. Comparably, Galileo related the number of heavenly bodies to the number of Platonic solids.
[20] Boyer, *A History of the Calculus*, 19.
[21] Boyer, *A History of the Calculus*, 20.
[22] Boyer, *A History of Mathematics*, 305. The work of Chuquet (ca. 1500) first deals with the idea of a negative number in finding the solutions to cubic equations, although it was not until 1629, with the work of Girard that negative solutions to equations were used (pg. 336).
[23] Čapek, *Bergson and Modern Physics*, 58.
[24] Descartes writes, "I call extended only what is imaginable as having parts distinct from one another," cited in Merleau-Ponty, *The Incarnate Subject*, 50.
[25] Čapek, *Bergson and Modern Physics*, 90.
[26] Cohen and Stewart, *Collapse of Chaos*, 248.
[27] Cohen and Stewart, *Collapse of Chaos*, 249.
[28] For a more detailed summary of the topological approach to embryology, see Edelman, *Bright Air, Brilliant Fire*, 52-64.
[29] Morris, *Heideggerian Truth and Deleuzian Genesis*, 176.
[30] Delanda, *Intensive Science Virtual Philosophy*, 116.
[31] Bergson, *Matter and Memory*, 244.
[32] Bergson, *Matter and Memory*, 217.
[33] Čapek, *Bergson and Modern Physics*, 304. Whitehead's quotation relates to his parallel effort to develop a vibratory theory of matter in the light of some of the key results of quantum mechanics.

Notes to Chapter Four

[1] It needs to be noted here that the form of dialectic referred to here is not Hegel's.
[2] Kline, *Mathematics in Western Culture*, 478. The following exposition of non-Euclidean geometry is largely based on Kline's exposition in this work and *Mathematical Thought*.
[3] Technically, this is Playfair's axiom, but it is extensionally equivalent to Euclid's own, which is: "If two lines intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough."
[4] Kline, *Mathematical Thought*, 877.
[5] Kline, *Mathematics in Western Culture*, 496-7.
[6] Delanda, *Intensive Science and Virtual Philosophy*, 23. Delanda supports this claim with the following quote from Kline, *Mathematical Thought*, 1160: "What distinguishes space from a mere set of points is some concept which binds them together. Thus in Euclidean space the distance between points tells how close the points are to each other… As Frechet [a pioneer in the development of topology] pointed out, the binding property need not be the Euclidean distance function. In particular he generalised the notion of distance by introducing the class of metric spaces. In a metric space, which can be a two-dimensional Euclidean space, one speaks of the neighbourhood of a point and means all those points whose distance from the point is less than some quantity… However, it is also possible to suppose that neighbourhoods, certain subsets of a given set of points, are specified in some way, *even without the introduction of a metric*. Such spaces are said to have a neighbourhood topology."
[7] Gleick, *Chaos*, 46.
[8] Mainzer, *Thinking in Complexity*, 38.
[9] Cohen and Stewart, *The Collapse of Chaos*, 205.

[10] Gleick, *Chaos*, 45-53.
[11] Mainzer, *Thinking in Complexity*, 43.
[12] Cohen and Stewart, *The Collapse of Chaos*, 207.
[13] Mainzer, *Thinking in Complexity*, 44.
[14] Morris, *Heideggerian Truth and Deleuzian Genesis*, 171.
[15] Gleik, *Chaos*, 15.
[16] Mainzer, *Thinking in Complexity*, 44.
[17] Gleick, *Chaos*, 26.
[18] Mainzer, *Thinking in Complexity*, 67.
[19] Lewin, *Complexity*, 51.
[20] Gleick, *Chaos*, 298. The classic example of linear systems becoming synchronised with each other is two pendulum clocks hung on the same wall. After a certain amount of time, minor vibrations travelling through the wall will lead to each pendulum swinging not just at the same rate but also beginning each swing at the same moment, even if they are initially set up to start their swings at different times.
[21] Delanda, *Intensive Science and Virtual Philosophy*, 26.
[22] C.f. Kline, *Mathematics in Western Culture*, "For several reasons, the problem of depicting the real world led the Renaissance painters to mathematics. The first reason was one which could be operative in any age in which the artist seeks to operate realistically. Stripped of colour and substance the objects that painters put on canvas are geometrical bodies located in space. The language for dealing with these idealised objects, the properties they possess as idealisations, and the exact relationships that describe their relative locations in space are all incorporated in Euclidean geometry. The artists need only avail themselves of it" (150).
[23] Merleau-Ponty seems here to be referring to Descartes' brief discussion of brass etchings in discourse four of the *Optics*.
[24] This follows from Da Vinci's discovery of perspective projection distortion, i.e., the impossibility of portraying a field of depth on a two dimensional plane without distortion. The first of the three main distortions are the use of horizontal parallel lines in perspectival painting. Whilst in the painting, these lines do not converge, in natural perspective, these lines do appear to meet. The second is that in perspectival painting, parallel lines are portrayed as straight, whereas they appear curved as they approach the horizon in natural perception. Third, artificial perspective does not take account of the fact that the surface of the retina itself is curved (Kline, *Mathematics in Western Culture*, 164).
[25] "Depth and intensity are the same at the level of being, but the same in so far as this is said of difference" (*DR*, 231).
[26] Lewin, *Complexity*, 40.
[27] The particular trajectory which is actualised is determined by a process of counter-actualisation, which reverses the direction of actualisation. In *Nietzsche and Philosophy*, Deleuze uses the analogy of a dicethrow which takes place on two tables: "the earth where the dice are thrown and the sky where the dice fall back" (*NP*, 25). These two tables correlate with the actual and the virtual, and the dicethrow to the actualisation of a particular trajectory. For Deleuze, this dicethrow reconciles two different concepts. On the one hand, "the dice which are thrown once are the affirmation of *chance*" (26), and on the other, "the combination which they form on falling is the affirmation of *necessity*" (26). The word 'chance' here is used to describe the fact that the virtual (from which the dice are thrown) capture the total possibilities of the system, whereas only one trajectory is *actualised*. What is important for Deleuze is the way this relation of chance and necessity is conceived. Understanding the relation in terms of possibilities, or rather probabilities, relies on "a great number of throws" (26) for the player to achieve the correct result. This method of understanding the relation of potentiality to actuality is for Deleuze illegitimate, as it understands the two tables in the same terms. What are affirmed are possibilities as "fragments of chance" (28) which are structured themselves like the throw of the dice. An example of this kind of affirmation would be to roll the dice having calculated the possibility of a double six in advance. Here, the possibility is structured isomorphically with the result. Deleuze instead claims that he wishes to affirm the whole of chance: "There are many numbers with increasing or decreasing probabilities, but only one number of chance as such, one fatal number which reunites all the fragments of chance" (26). This way of affirming chance is to understand chance in terms of the virtual Idea, where the potentialities are not understood as separable, but must be affirmed all at once as reciprocally determining. These two elements, the virtual sky and the actual earth are further characterised in terms of two elements. Deleuze writes that "fire is the

element which plays, what has no opposite" (29). This is the element of the virtual, and is characterised by a non-oppositional logic. The logic of actuality, which is used by the bad player in stead has its "root in reason" (27), which is the terrain of "the spirit of revenge, the spider" (28). What is important to note, however, is that as well as the dice being thrown, they also "fall back" onto the table of the sky. Deleuze writes that "bringing the dicethrow back the number puts chance back into the fire, it maintains the fire which reheats chance" (29). Whilst the presentation is made somewhat obscure by the language, it is clear that Deleuze sees the movement between the two planes as being bi-directional. That is, not only is the virtual actualised, but the particular actualisation in turn determines the virtual. In a similar manner, the actual state of affairs determines the particular trajectory of the virtual to be actualised.

[28] In this case, interpenetration emerges as the dimensions of the system together capture its dynamic properties. Just as we saw with the sugar-water example, to remove one of the dimensions would be to change the dynamics of the event, thus changing the event itself. In this sense, the dimensions cannot be understood analytically.

[29] "'*Perplication*' is what we called this state of Problems-Ideas, with their multiplicities and co-existent varieties, their determination of elements, their distribution of mobile singularities and their formation of ideal series around these singularities" (*DR*, 280).

Notes to Chapter Five

[1] For discussions of the strength of Hegel's other criticisms of Kant, see in particular, Ameriks, *Hegel's critique of Kant's Theoretical Philosophy*, Smith, *Hegel's Critique of Kant*, and Houlgate, *Hegel, Kant, and the Formal Distinctions of Reflective Understanding*.

[2] Allison, *Kant's Transcendental Idealism*, 77.

[3] Kant, *Critique of Judgment*, §59.

[4] C.f. *CPR*, A311/B368.

[5] Kant, *Critique of Judgment*, §29.

[6] Hegel comments on Kant's claim that the sublime leads to the subject withdrawing into his invisible ego and opposing "the absolute freedom of his will as a pure ego to all the terrors of fate and tyranny" (*SL*, 230) by arguing that because the pure ego is abstract and empty, it in fact reinstates the idea of a beyond through its own lack of determination in the face of the determinate world it finds before it. In this sense, Kant's analysis of the sublime proves inadequate for Hegel.

[7] Concepts, as universals, of course contain within themselves a relation to what they subsume under themselves, but are not related to other concepts as universals.

[8] In actual fact, it is not exactly the relation to the sensible manifold which is problematic, as abstract determinations "certainly do not stand in the sensible mutual externality of space" (*EL*, §20), Hegel goes on to say that "nevertheless, these determinations, which are in themselves spiritual, stand at the same time in isolation upon the broad field of inner abstract universality of representation in general." (*ibid.*) It is clear from what Hegel says later that this is to be understood as a spatial, if not sensible, way of representing concepts: "Representation leaves [its determinations] *side by side*, in its undetermined space, linked only by the simple 'and.'" (*ibid.*).

[9] This is not to say, of course, that in following the movement of the logic, we can presume in advance that new content will emerge, as this would be to presuppose the success of the procedure in advance. Rather, it is only in analysing the movement of the dialectic itself that this will become clear.

[10] Rosen, *Hegel's Dialectic*, raises this issue of the counter intuitive nature of Hegel's concept of determinate negation (30), arguing that determinate negation ultimately rests on the claim that immanent critique leads to positive results (33). On the reading given here, this second claim is indeed controversial, as dialectic is not to be seen as inherently teleological. If dialectic can show the immanent development of the object, however, this will in turn lead to the justification of his concept of determinate negation.

[11] Rosen, *Hegel's Dialectic*, 26.

[12] For the relation of scepticism to dogmatism, see also *EL*, §32.

[13] Here I am following Maker, *Philosophy without Foundations*, 99, Rosen, *Hegel's Dialectic*, 26, and Houlgate, *The Opening of Hegel's Logic*, 32. Althusser puts forward this view in attempting to create a radical break between Hegel and Marx, writing that, "it is inconceivable that the essence of the dialectic in Hegel's work should not be contaminated by Hegelian ideology, or, since such a 'contamination' presupposes the fiction of a pure pre-'contamination'

dialectic, *that the Hegelian dialectic could cease to be Hegelian and become Marxist by a simple, miraculous 'extraction'*" (Althusser, "Contradiction and Overdetermination" in *For Marx*, 91). Given Deleuze's affirmation of Althusser's interpretation of the relation of Hegelian and Marxist dialectics (for example, *DR*, 186 and 207), it would be a reasonable assumption that Deleuze also holds to this view of Hegelian dialectic.

[14] Strictly speaking, these three moments cannot be understood separately from each other. "All of them can be put together under the first moment, that *of the understanding;* and in this way they can be kept separate from each other, but then they are not considered in their truth" (*EL*, §79).

[15] Taylor, *Hegel*, 226.

[16] For clarification of this point, see *EL*, §24 Add. 1.

[17] This argument is fully outlined in Maker, *Philosophy without Foundations* (chapter 3), as well as Hyppolite, *Logic and Existence*, part 2, chapter 1.

[18] Maker, *Philosophy without Foundations*, chapter 3.

[19] Hyppolite, *Logic and Existence*, 57-70.

[20] I am here indebted to Stephen Houlgate's analyses of the doctrine of being, both in *The Opening of Hegel's Logic*, and *Hegel's Critique of Foundationalism* for the basic structure of this introduction.

[21] We will return to the paradoxical nature of the difference and similarity of being and nothing when we discuss the implications of the structure of contradiction for infinite thought.

[22] Burbidge, *On Hegel's Logic*, 63.

[23] Burbidge, *The Logic of Hegel's Logic*, 56.

[24] "The various forms of idealism, Leibnizian, Kantian, Fichtean, and others, have not advanced beyond this being as determinateness, have not advanced beyond this immediacy, any more than scepticism did." *SL*, 396. Cf. , Michael Baur, *Sublating Kant and the Old Metaphysics*.

[25] Giovanni, *Reflection and Contradiction*, 156.

[26] Suchting, in his introduction to the translation of the *Encyclopaedia Logic* (xv), refers to a section of Rosenkranz's *Das Leben Hegels* where he quotes Hegel as follows: "Properly speaking, it belongs to the highest cultural development of the people *to say everything in their own language*. The concepts that we mark with foreign words seem to us to be themselves something *foreign* and not to belong to us immediately as our very own. The elements of things appear to us not to be the *present* concepts with which we are environed and have to deal with all the time and in which the most ordinary man expresses himself." He goes on to say that "our thought is still not properly at home in our language, it does not dominate the language, as it should, and we cherish in this regard, a blind reverence for what is brought from abroad." This last sentence shows that Hegel's reliance on language should not uncritically be seen as leading him simply to a derivation of the categories of ontology from the categories of language.

[27] Hegel quoted from Houlgate, *Hegel Nietzsche and the Criticism of Metaphysics*, 142.

[28] Gadamer, *Hegel's Dialectic*, 94.

[29] For a fuller analysis of this dialectic, c.f. Burbidge, *On Hegel's Logic*, and Houlgate, *The Opening of Hegel's Logic*.

[30] The *Jena Logic* predates the '*Greater Logic*', and lacks some of its sophistication, such as the separation of the doctrine of being from the doctrine of essence. Whilst this elides what is a fundamental difference between the infinite and contradiction, it also makes clear the structural parallels which occur between the two categories. Whilst the *Jena Logic* can therefore give insight into the origin of these parallels, and provide circumstantial evidence for their existence in the *Greater Logic*, we must rely on the *Greater Logic* itself to determine if these parallels still hold.

[31] The importance of the speculative proposition is noted especially by Houlgate (*The Opening of Hegel's Logic*, and *Hegel, Nietzsche and the Criticism of Metaphysics*), and Hyppolite, *Logic and Existence*, 129-148.

[32] Kant, *Critique of Judgment*, § 57, Comment I.

[33] The main analysis of the speculative proposition occurs in the *Phenomenology of Spirit*. It is also prefigured in Hegel's analysis of the Kantian question, "How are synthetic judgements *a priori* possible?" in *Faith and Knowledge*. Whilst he reproaches Kant for stopping at the "*subjective* and external meaning of this question" (69) as he "believed he had established rational cognition was impossible," (ibid.) Hegel argues that what makes these judgements possible is that "these heterogeneous elements, the subject which is particular and in the form of being, and the predicate, which is the universal and in the form of thought, are at the same time

absolutely identical." (ibid.) Whereas the understanding and intuition are connected for Kant by the discursivity of the understanding, for Hegel, it is instead "the productive imagination which must be recognised as what is primary and original, as that out of which subjective Ego and objective world first sunder themselves into the necessarily bipartite appearance and product, and as the sole In-itself." (73). By deriving both faculties from the imagination, the discursivity thesis is made to collapse, as the imagination now relates to itself in synthetic *a priori* propositions. When we ask further what the imagination is for Hegel, we are told that it is "nothing but Reason itself" (73). Thus, the synthetic *a priori* judgement is reason in a relation of difference to itself. Hegel complains, however, that Kant only understood the imagination to be the appearance of Reason in empirical consciousness (73), thus failing to move beyond an understanding of the categories in terms of the intellect.

Notes to Chapter Six

[1] Monk, *Was Russell an Analytic Philosopher?*, 40, quoting Russell, *Portraits from Memory*, (London, Allen and Unwin, 1956), 21.

[2] Russell's worries concerning infinite numbers are best summed up in Leibniz's paradox, which relies on the fact that we can associate an even number with every number, like so:

1	2
2	4
3	6
4	8
…	

From this we can infer that the cardinality (total number) of even numbers is equal to the cardinality of numbers as a whole. This seems absurd, however, as on the other hand there must be more numbers than even numbers, since the set of numbers includes the set of even numbers but also contains all of the odd numbers. This paradox was 'solved' by Cantor's theory of transfinite numbers, which proposed that arithmetical operators operated differently on infinite (in Cantor's theory, transfinite) numbers than on finite numbers. One of the axioms Cantor's theory of transfinite numbers is that $m+m=m$, where m is a transfinite number. Thus, the number of even numbers added to the number of odd numbers *is* equal to the number of even numbers. Whilst this result is paradoxical from the viewpoint of finite arithmetic, the axiom set of transfinite number theory is entirely consistent. For more details, see Copi, *The Theory of Logical Types*, 2-5. This move actually bears some similarity to Hegel's argument that infinite numbers are qualitatively distinct from finite numbers.

[3] Russell, *History of Western Philosophy*, 783.

[4] For more on this point, and Russell's early Hegelianism, see Monk, *Was Russell an Analytic Philosopher?*

[5] This interpretation of the role of the differential calculus in the work of Hegel and Deleuze is opposed to that of Simon Duffy in *The Logic of Expression: Quality, Quantity and Intensity in Spinoza, Hegel and Deleuze*. Leaving aside minor technical problems in his interpretation of Hegel, the main difficulty is his assertion that "Deleuze … establishes a historical continuity between Leibniz's differential point of view of the infinitesimal calculus and the differential calculus of contemporary mathematics thanks to the axioms of non-standard analysis which allow the inclusion of the infinitesimal in its arithmetisation; a continuity which effectively bypasses the methods of the differential calculus which Hegel uses in the *Science of Logic* to support the development of the dialectical logic." (74-5) The structure of Duffy's argument is that whereas Hegel asserts that the differential calculus is inherently dialectical (*SL*, 254: "The operations which [mathematics] allows itself to perform in the differential and integral calculus are in complete contradiction with the nature of merely finite determinations and their relations and would therefore have to be justified solely by the notion"), Deleuze is able to avoid the threat of dialectic by showing the possibility of a non-contradictory understanding of the calculus. He achieves this by following the interpretation of the infinitesimal provided by Abraham Robinson in his *Non-Standard Analysis*. Whereas prior to Robinson, the only rigorous foundation to the calculus was provided by an interpretation in terms of limit, as championed by Weierstrass, which operated by removing all reference to infinitesimal quantities, Robinson's work meant that "the infinitesimal was given a rigorous foundation, and a formal theory of the infinitesimal was constructed, thus allowing Leibniz's ideas to be 'fully vindicated.'" (Duffy, 56). I have two principle objections to the incorporation of the work of Robinson into Deleuze's theory. First, on

a historical note, Robinson's *Non-Standard Analysis* first appeared in 1966, merely two years before the publication of *Difference and Repetition*, and four years after Deleuze's publication of *Nietzsche and Philosophy* in 1962, which contains his first foray into differential metaphysics. This makes it unlikely that Deleuze would have had time to assimilate and incorporate the novel approach of Robinson into the system of *Difference and Repetition*. Second, and most importantly, the work of Robinson runs precisely counter to Deleuze's own approach. The difficulty with the infinitesimal is that it must have a value in order to form a ratio of values at a particular point on the curve, but must also strictly be equal to zero in order to provide a tangent to a point (rather than between two points) on the line. Robinson's solution is to posit a new class of numbers, called hyperreal numbers, one of which is defined as infinitesimally close to each real number. By defining the infinitesimal as a separate class of number, Robinson is able to avoid the problems which emerge when *dy* is considered to have the same properties as *y*. The end result of this is that Robinson allows the incorporation of infinitesimals into set theory, paralleling the result of Weierstrass for the limit.

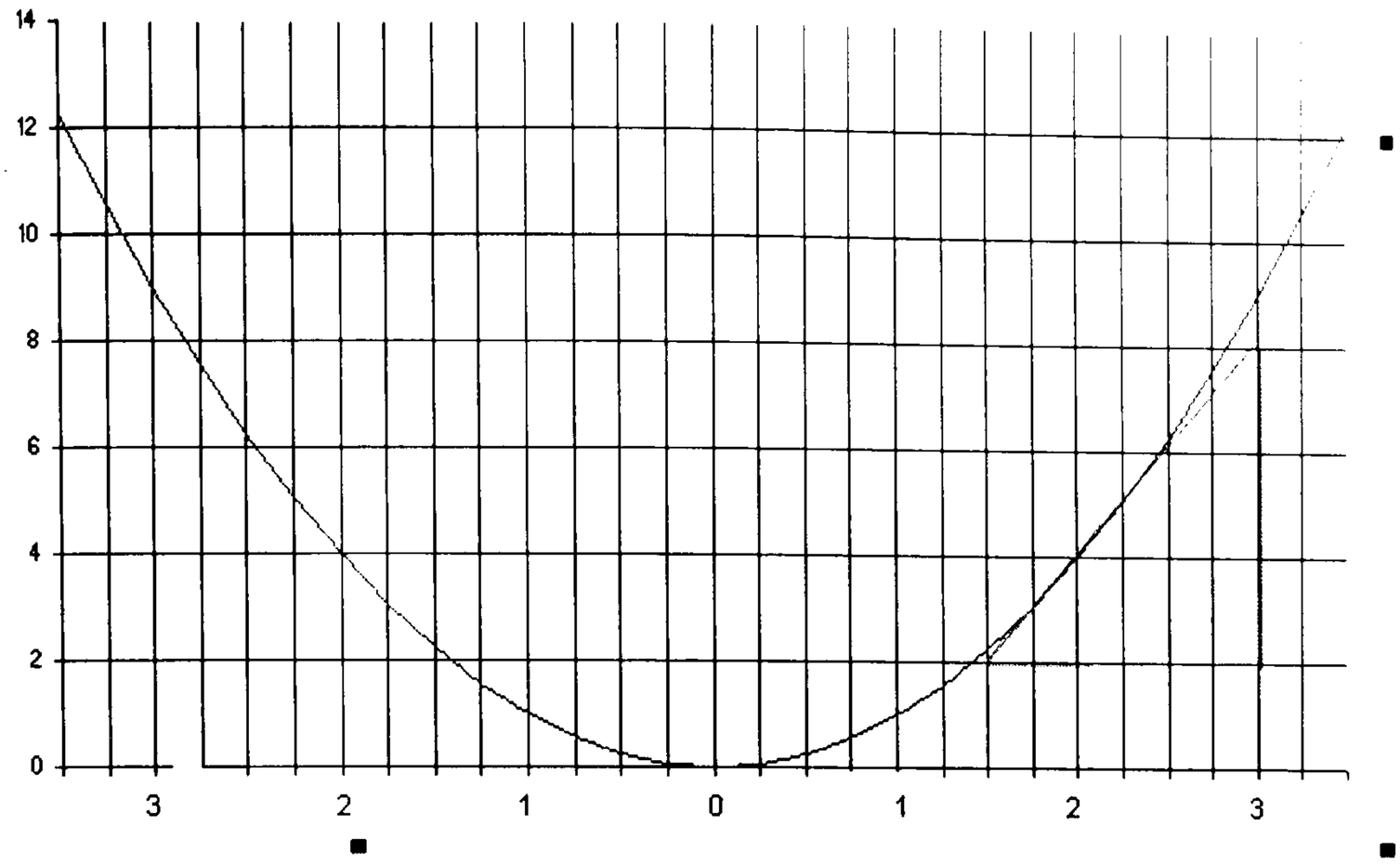
When we look at the reason for Deleuze's move to a logic of the differential, however, we see that this was not made principally in order to avoid an engagement with dialectic, but rather to allow the possibility of a thought which goes beyond representation. Thus what he criticises about the Weierstrassian interpretation is not that it relies on a Newtonian model of limit, but rather that it grounds the calculus in set theory, which, as we saw in chapter two, it the very model of a representational logic. "What is still missing [in the modern mathematical interpretation] is the extra-propositional or sub-representative element expressed in the Idea by the differential." (*DR*, 178). As non-standard analysis is simply a method for incorporating infinitesimals within set theory, it is clear that Deleuze cannot accept this approach, as it does to the infinitesimal what set theory had done to the limit in "[losing] its phoronomic character and [involving] only static considerations." (*DR*, 176) Finally, Deleuze's comment that "the principle of a general differential philosophy must be the object of a rigorous exposition, and in no way depend on the infinitely small" (*DR*, 171), (as this would be to spatialise the virtual) clearly excludes the infinitesimal from consideration. In fact, Deleuze explicitly excludes the notion of an "intercalary series" on the basis that it relies on characteristics "borrowed from sensible or … geometric intuition." (*DR*, 171)

Therefore, both Deleuze and Hegel must be seen as relying on what Deleuze calls "so-called barbaric or pre-scientific interpretations of the differential calculus" (DR 170). In this sense, when Deleuze writes that "just as we oppose difference in itself to negativity, so we oppose *dx* to not-A, the symbol of difference to that of contradiction" (DR 170), he sees both difference philosophies and philosophies of contradiction as being separate from, and indeed attempting to explain the possibility of, a philosophy of identity such as that of set theory.

[6] Boyer, *The History of the Calculus*, 82, although it was Archimedes who provides the first recognised anticipation of the calculus in the method of exhaustion used to find the area of a circle. Archimedes' use was concerned with static figures, rather than rates of change however.

[7] A function consists in a domain (or range) of numbers, and a rule which associates each of these numbers with a value of another variable. Thus, $f(x) = x^2$ associates any real number, positive or negative, with a value equal to that number squared.

[8]



Given the curve, $y = x^2$ (the curve), we can determine the velocity, or gradient of the curve at any point by drawing a tangential line (the triangle), and dividing the value on the y axis by that on the x axis. In this case the resultant vector has a value of 2.

[9] The two operations are not strictly inverse, as differentiation removes any constant values from the initial function, so, for instance, if $f(x)$ is $y = x^2 + 1$, the first stage of the differentiation will give us: $\frac{(x+dx)^2 + 1 - (x^2+1)}{dx} = \frac{dy}{dx}$, which is equivalent to the function derived from $y = x^2$. In integration, the constant values are therefore normally represented by the value c. The value of c can be determined if any value of the function is known, however.

[10] OU, *Origins and Development of the Calculus,* IV.10. Quoting L'hôpital (1696) *Analyse des Infinements Petits*, Chapter I, from *A Sourcebook in Mathematics*, ed. Struik, J. (Harvard University Press, 1969).

[11] C.f. Eves, *Introduction to the History of Mathematics*, 398.

[12] OU, *Origins and Development of the Calculus,* III.26. Quoting *The Mathematical Works of Isaac Newton* (assembled Whiteside, D. T.), Johnson Reprint, New York, 1964, I, p. 141

[13] OU, *Origins and Development of the Calculus*, III.26.

[14] See also Eves, *Introduction to the History of Mathematics*, 400 on the redundancy of time in Newton's method.

[15] Hegel notes (*SL*, 247) that fractions such as 1/10, which do not normally lead to an infinite sequence can be made to do so by interpreting them in another number base (i.e. instead of base 10). Thus, for instance, 1/10, when interpreted in the octal system (base 8), also leads to an infinite sequence of 0.0631463…

[16] Hegel renders the differential relation as *dx/dy*, which would give the line perpendicular to the tangent at the point. Whilst this does not in any way affect the logic of Hegel's argument, I will continue to render the relation as *dy/dx*, as this is the more common rendition of the differential relation. It is also worth noting that Hegel uses Leibniz's notation instead of Newton's. Once again, this does not materially affect the exposition, as both procedures are mathematically equivalent.

[17] Berkeley, *The Analyst*, paragraph 12.

[18] Berkeley, *The Analyst*, paragraph 35.

[19] Berkeley, *The Analyst*, paragraph 8.

[20] Berkeley, *The Analyst*, paragraph 48.

[21] C.f. Berkeley, *The Analyst*, paragraph 4: "Now as our sense is strained and puzzled with the perception of objects extremely minute, even so the imagination, which faculty derives from sense, is very much strained and puzzled to frame clear ideas of the least particles of time, or the least increments generated therein: and much more so to comprehend the moments, or those increments of the flowing quantities in *statu nascenti*, in their very first origin or beginning to exist, before they become finite particles."

[22] Millet, *Bergsonian Epistemology*, 32.

[23] C.f. Chapter four, *Bergson on Ravaisson.*

[24] In fact, it is also important that we have a positive gradient before the summit, followed by a negative gradient after the summit, since a dip will also have a gradient of zero, but will be preceded by a negative gradient and succeeded by a positive one.
[25]"Yet I have a plan in mind according to which even *popularity* might be gained for this study, a plan that could not be carried out initially, however, for the foundations needed cleaning up, particularly because the whole system of this sort of knowledge had to be exhibited in all its articulation. Otherwise I would have started with what I have entitled the 'Antinomy of Pure Reason,' which could have been done in colourful essays and would have given the reader a desire to get at the sources of this controversy" (Kant, *Correspondence*, To Marcus Herz, after May 11, 1781).
[26] The term antinomy is used in two senses by Kant. First to refer, in the singular, to the root cause of all four contradictions, i.e., the totalising approach of reason, and the absence of such a totality, and second to refer to the expressions of this antinomy, in the specific antinomies which are the expression of the generic antinomy.
[27] Allison, *Kant's Transcendental Idealism*, 362.
[28] Allison, *Kant's Transcendental Idealism*, 366-376, does perhaps as good a job as can be done of defending Kant's arguments against the objections of Russell, Strawson, Bennett, and Kemp-Smith.
[29] Allison, *Kant's Transcendental Idealism*, 384.
[30] Allison, *Kant's Transcendental Idealism*, 389.

Notes to Chapter Seven

[1] For a discussion of the adequacy or lack thereof of Hegel's account of Schelling, see Bowie, *Schelling and Modern European Philosophy*, and Houlgate, *Schelling's Critique of Hegel's Science of Logic*. The disagreement centres on Schelling's claim that Reason is essentially discursive, meaning that following Kant it can only provide the conditions of possibility of knowledge by explicating the nature of a particular being in question. The being of the being in question falls outside of Reason's remit, requiring non-rational mode of access.
[2] Hyppolite, *Genesis and Structure of Hegel's Phenomenology*, 123.
[3] Hyppolite, *Genesis and Structure of Hegel's Phenomenology*, 123.
[4] Hyppolite, *Genesis and Structure of Hegel's Phenomenology*, 122.
[5] As we have seen in the thesis as a whole, Hegel provides a genetic theory of the relation of the categories, and in this sense, the form of possibility discussed here does not represent Hegel's definitive understanding of the concept. The *Science of Logic* traces the development of this concept, which begins as abstract possibility, but develops into the concept of contingency. The abstract possibility described here, essentially, the possibility which precedes actualisation, is shown by Hegel to also include its negation, as precisely in order to allow something to be conceived of as possible, we need to also recognise the fact that the alternative state of affairs may obtain. Thus, in order that it be possible that it is day, it must also be possible that it is night. The possibility of A therefore has the structure, A or not-A. This structure is not simply that of an abstract possibility, however, as it presents an actual logical truth, that is, it is actually the case that only one of the disjuncts can be actualised. Possibility therefore becomes actual possibility. The result of this is the introduction of contingency into actuality, as actuality now contains a moment of possibility, or arbitrariness, within it. We will not be analysing this dialectic here, as we will instead focus on the issues of the inverted world, and the one and the many, but we should note that here Hegel takes a similar approach to that of the dialectic of the inverted world by taking two opposed moments which exist at different conceptual levels, and reincorporating them into one actuality. As such, a similar analysis to that given here of the inverted world could be provided. We should note here that whereas contingency for Hegel is understood as A or not-A, that is, in terms of negation and disjunction, virtuality for Deleuze is understood as A and B and C, that is, in terms of affirmation and conjunction. This comparison is of course a massive oversimplification of both philosophers' understanding of the terms (in particular, it risks implying a reading of both in terms of discrete, spatial multiplicities, and of Deleuze as a philosopher of contradiction), but may provide a rough sketch of their differences when taken in conjunction with the more fine-grained analyses presented in the thesis as a whole. For a more detailed account of Hegel's concept of contingency, see Houlgate, *Necessity and Contingency in Hegel's Science of Logic*.

[6] "If, on the contrary, the will to power is a good principle, if it reconciles empiricism with principles, if it constitutes a superior empiricism, this is because it is an essentially *plastic* principle that is no wider than what it conditions, that changes itself with the conditioned and determines itself in each case along with what it determines" (*NP*, 50). "Nietzsche seems to have sought (and to have found in the 'eternal return' and the 'will to power') a radical transformation of Kantianism, a re-invention of the critique which Kant betrayed at the same time as he conceived it, a resumption of the critical project on a new basis and with new concepts" (*NP*, 52).
[7] Houlgate, *Hegel's Critique of Foundationalism*, 31.
[8] Deleuze makes this clear in the subchapter on sense in *Nietzsche and Philosophy*. He writes that "Nietzsche substitutes the correlation of sense and phenomenon for the metaphysical duality of appearance and essence and for the scientific relation of cause and effect" (*NP*, 3). Deleuze will take a similar approach in his own philosophy, sense being roughly equated with virtuality.
[9] C.f. Chapter Six, *The Kantian Antinomies*.
[10] For a more detailed discussion of this dialectic, see Stern, *Hegel and the Phenonemology of Spirit*.
[11] Stern, *Hegel and the Phenomenology of Spirit*, 66.
[12] For studies of Deleuze's relation to biology, see Ansell Pearson, *Viroid Life* and *Germinal Life*. De Landa also puts forward an interpretation of Deleuze based heavily on empirical research. One obvious place to begin to evaluate the relation of Hegel to Deleuze would be the theory of evolution, as here the value of the concept of the species has been heavily questioned. For Hegel, "the single man is what he is in the particular, only insofar as he is, first of all, human as such, and within the universal; and this universal is not just something over and above other abstract qualities or mere determinations of reflection, but it is rather what permeates all the particulars and embraces them within itself" (*EL*, §176, Add. 1). In making this move, he is highlighting the parallels between his solution to the problem of the one and the many, and of the relation of the individual to the species. The issue is not whether Hegel has a theory of evolution, which seems unlikely (C.f. Houlgate, *An Introduction to Hegel*, 173), but rather whether the notion of species can have any greater meaning than the characterisation of certain statistical regularities within a population. Deleuze's notion of an a-centred multiplicity, not governed by the One, may provide a better model of an evolutionary theory governed by accidents rather than essences.
[13] C.f. Malabou, *Who's Afraid of Hegelian Wolves*.
[14] The figures dealt with in this thesis in no way provide a comprehensive list of thinkers who Deleuze incorporates into his system. Other major thinkers used by Deleuze include Spinoza, Leibniz, Nietzsche, Proust, the Stoics, Masoch, and Duns Scotus.
[15] Plato, *Phaedrus*, 265e.

Bibliography

Alexander, J. Davidson, "Kant, Hegel and the Problem of Grounds," *Kant-Studien,* 1979 70:451-470.

Alliez, Eric, *The Signature of the World, or, What Is Deleuze and Guattari's Philosophy?* trans. Albert, E. R. and Toscano, A. (London: Continuum, 2004).

Allison, Henry E., *Kant's Transcendental Idealism*. (New Haven: Yale University Press, 2004).

Althusser, Louis, *For Marx*, trans. Brewster, Ben (London: Verso, 1990).

Ameriks, Karl, "Hegel's Critique of Kant's Theoretical Philosophy," *Philosophy and Phenomenological Research,* 1985 46:1-36.

Ansell-Pearson, Keith, *Germinal Life: The Difference and Repetition of Deleuze* (London: Routledge, 1999).
———, *Philosophy and the Adventure of the Virtual: Bergson and the Time of Life* (London: Routledge, 2002).
———, *Viroid Life: Perspectives on Nietzsche and the Transhuman Condition* (London: Routledge, 1997).

Aristotle, *Metaphysics*: *Penguin Classics*, trans. and ed. Lawson-Tancred, H. C. (London: Penguin Books, 1998).

Badiou, Alain, *Deleuze: The Clamor of Being*, trans. Burchill, L. (London: University of Minnesota Press, 2000).

Barnes, Jonathan, *Aristotle* (Oxford: Oxford University Press, 1983).

Baugh, Bruce, *French Hegel: From Surrealism to Postmodernism* (London: Routledge, 2003).

Baur, Michael, "Sublating Kant and the Old Metaphysics: A Reading of the Transition from Being to Essence in Hegel's Logic," *The Owl of Minerva,* 1998 29: 139-64.

Beistegui, Miguel de, "Response to David Morris," *Pli,* 2006 17: 184-95.
———, *Truth and Genesis: Philosophy as Differential Ontology*: *Studies in Continental Thought* (Bloomington, Ind.: Indiana University Press, 2004).

Bencivenga, Ermanno, *Hegel's Dialectical Logic* (Oxford: Oxford University Press, 2000).

Bergson, Henri, *Creative Evolution*, trans. Mitchell, Arthur (Mineola, N.Y.: Dover Publications, 1998).
———, *The Creative Mind: An Introduction to Metaphysics*, trans. Andison, M. L. (New York: Wisdom Library, 1946).
———, *Matter and Memory*, trans. Paul, Nancy Margaret, and Palmer, W. Scott (New York: Zone Books, 1988).

Berkeley, George, *De Motu and the Analyst*, trans. Jesseph, Douglas M. (Dordrecht: Kluwer, 1992).

Bochenski, Joseph M., *A History of Formal Logic*, trans. and ed. Thomas, Ivo (New York: Chelsea Pub. Co., 1970).

Bos, H. J. M., "Newton, Leibniz, and the Leibnizian Tradition," in Ivor Grattan-Guinness, ed., *From the Calculus to Set Theory, 1630-1910*, 49-93.

Bowie, Andrew, "The Actuality of Schelling's Hegel-Critique," *Bulletin of the Hegel Society of Great Britain,* 1990: 19-30.
———, *Schelling and Modern European Philosophy: An Introduction* (London: Routledge, 1993).

Boyer, Carl B., *The History of the Calculus and Its Conceptual Development* (New York :, London: Dover, Constable, 1959).
———, *A History of Mathematics*. 1st Princeton paperback printing. (Princeton, N. J.: Princeton University Press, 1985).

Burbidge, John W., *On Hegel's Logic: Fragments of a Commentary* (Atlantic Highlands, N.J.: Humanities Press, 1981).

Caygill, Howard, *A Kant Dictionary*: *Blackwell Philosopher Dictionaries* (Oxford: Blackwell, 1995).

Čapek, Milic, *Bergson and Modern Physics: A Reinterpretation and Re-Evaluation*: *Synthese Library* (Dordrecht: Reidel, 1971).

Cohen, Jack S., and Stewart, Ian, *Collapse of Chaos: Discovering Simplicity in a Complex World* (London: Penguin, 2000).

Copi, Irving Marmer, *Theory of Logical Types* (London: Routledge, 1971).

De Landa, Manuel, *Intensive Science and Virtual Philosophy* (London ; New York: Continuum, 2002).

Deleuze, Gilles, *Bergsonism*, trans. Tomlinson, H. and Habberjam, B. (New York: London : Zone, 1988).
———, *Desert Islands and Other Texts 1953-1974*, ed. Lapoujade, David and trans. Taormina, Mike (Los Angeles, Calif.: Semiotext(e), 2004).
———, *Difference and Repetition.*, trans. Patton, P. (New York: Columbia University Press, 1994).
———, *Différence et Répétition* (Paris: PUF, 1997).
———, *Essays Critical and Clinical*, trans. Smith, Daniel W., and Greco, Michael A. (London: Verso, 1998).
———, *Expression in Philosophy : Spinoza*, trans. Joughin, M. (New York: Zone Books, 1990).
———, *Francis Bacon : The Logic of Sensation*, trans. Smith, D. (Minneapolis: University of Minnesota Press, 2003).
———, *Kant's Critical Philosophy: The Doctrine of the Faculties*, trans. Tomlinson, H. (London: Athlone Press, 1984).
———, *The Logic of Sense*, trans. Boundas, Constantin V. (London: Continuum, 2001).
———, *Negotiations, 1972-1990*, trans. Joughin, Martin (New York; Chichester: Columbia University Press, 1995).

———, *Nietzsche and Philosophy*, trans. Tomlinson, Hugh (London: Athlone Press, 1983).
———, *Proust and Signs*, trans. Howard, R. (London: Athlone, 2000).
———, *Pure Immanence: Essays on a Life*, trans. Boyman, Anne (London : Zone Books, 2001).
———, "Review of Simondon (1966)," *Pli - The Warwick Journal of Philosophy,* 2001 12: 43-49. Translated by Ivan Ramirez.
———, *Spinoza, Practical Philosophy*, trans. Hurley, R. (San Francisco: City Lights Books, 1988).
———, *Two Regimes of Madness: Texts and Interviews, 1975-1995*, ed. Lapoujade, David and trans. Hodges, Ames and Taormina, Mike (London : Semiotext(e), 2006).

Deleuze, Gilles, and Guattari, Felix, *What Is Philosophy?* trans. Birchill, Graham, and Tomlinson, Hugh (London: Verso, 1994).

Deleuze, Gilles, and Parnet, Claire, *Dialogues II*, trans. Tomlinson, J., Habberjam, B. and Albert, E. (London: Continuum, 2002).

Deligiorgi, Katerina, "Kant, Hegel, and the Bounds of Thought," *Bulletin of the Hegel Society of Great Britain,* 2002: 56-71.

Descartes, René, "Rules for the Direction of the Mind," in *The Philosophical Writings of Descartes, Vol. 1.* Trans. Cottingham, John, Stoothoff, Robert, and Murdoch, Dugald (Cambridge: Cambridge University Press, 1985).

Di Giovanni, George, *Essays on Hegel's Logic* (Albany: State University of New York Press, 1990).
———, "Reflection and Contradiction: A Commentary on Some Passages of Hegel's Science of Logic," *Hegel Studien,* 1973 8.

Dillon, M. C., *Merleau-Ponty's Ontology* (Bloomington: Indiana University Press, 1988).

Duffy, Simon, *The Logic of Expression: Quality, Quantity, and Intensity in Spinoza, Hegel and Deleuze* (Aldershot: Ashgate, 2006).
———, *Virtual Mathematics: The Logic of Difference* (Manchester: Clinamen Press, 2006).

Duquette, David A., "Kant, Hegel and the Possibility of a Speculative Logic," *Essays on Hegel's Logic,* 1990.

Edelman, Gerald M., *Bright Air, Brilliant Fire: On the Matter of the Mind* (London: Penguin, 1994).

Edwards, C. H., *Historical Development of the Calculus* (New York; London: Springer, 1979).

Eves, Howard Whitley, *An Introduction to the History of Mathematics* (Philadelphia: Saunders College Pub., 1990).

Ferrarin, Alfredo, *Hegel and Aristotle* (Cambridge: Cambridge University Press, 2001).

Fleischhacker, L., "Hegel on Mathematics and Experimental Science," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht ; Boston: Kluver Academic Publishers, 1993), 209-28.

Gadamer, Hans Georg, *Hegel's Dialectic*, trans. Smith, P. C. (London: Yale University Press, 1982).

Gardner, Sebastian, *Kant and the Critique of Pure Reason* (New York: Routledge, 1999).

Gioli, Giovanna, "What Is Transcendental Empiricism? Deleuze and Sartre on Bergson," *Pli - The Warwick Journal of Philosophy,* 2007 18: 182-203.

Gleick, James, *Chaos : Making a New Science* (London: Heinemann, 1988).

Goldstine, Herman Heine, *A History of the Calculus of Variations from the 17th through the 19th Century* (New York: Springer-Verlag, 1980).

Grattan-Guinness, Ivor, "The Emergence of Mathematical Analysis and Its Foundational Progress, 1780-1880," in *From the Calculus to Set Theory, 1630-1910 : An Introductory History*, ed. Grattan-Guiness, Ivor (London: Duckworth, 1980), 94-148.
———, "Hegel's Heritage in Applied Mathematics: A Plurality of Traditions," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht ; Boston: Kluver Academic Publishers, 1993), 201-08.

Gray, Jeremy, *Ideas of Space: Euclidean, Non-Euclidean, and Relativistic. 2nd ed* (Oxford: Clarendon, 1989).

Guicciardini, N., "Newton and British Newtonians on the Foundations of the Calculus," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht ; Boston: Kluver Academic Publishers, 1993), 167-78.

Hallward, Peter, *Out of This World: Deleuze and the Philosophy of Creation* (London: Verso, 2006).

Hardt, Michael, *Gilles Deleuze: An Apprenticeship in Philosophy* (London: Routledge, 1993).

Hegel, Georg Wilhelm Friedrich, *The Encyclopaedia Logic, with the Zusätze*, trans. Geraets, Theodore F., Suchting, Wallis Arthur, and Harris, Henry Silton (Indianapolis: Hackett, 1991).
———, *Faith & Knowledge*, trans. Cerf, Walter, and Harris, H. S. (Albany: State University of New York Press, 1977).
———, *Hegel's Science of Logic*, trans. Miller, Arnold V. (Atlantic Highlands, NJ: Humanities Press International, 1989).
———, *The Jena System, 1804-5: Logic and Metaphysics*, trans. Harris, H. S., Burbidge, John W., and Di Giovanni, George (Kingston: McGill-Queen's University Press, 1986).
———, *Lectures on the History of Philosophy*. 3 vols., trans. Haldane, Elizabeth Sanderson, and Simson, Frances H. (Lincoln: University of Nebraska Press, 1995).
———, "On the Relation of Skepticism to Philosophy," *Between Kant and Hegel : Texts in the Development of Post-Kantian Idealism*, eds. Di Giovanni, George, and Harris, H. S., (Indianapolis: Hackett Pub. Co, 2000).
———, *Phänomenologie des Geistes: Bd 3*, ed. Moldenhauer, E. (Frankfurt am Main: Suhrkamp, 1969).
———, *Phenomenology of Spirit*, trans. Findlay, J. N., and Miller, Arnold V. (Oxford: Clarendon Press, 1977).
———, *The Philosophy of History*, trans. Sibree, J. (New York: Dover, 1956).

———, *Wissenschaft der Logik I. Die objektive Logik.: Bd 5*, ed. Moldenhauer, E. (Frankfurt am Main: Suhrkamp, 1969).
———, *Wissenschaft der Logik II. Die subjektive Logik.: Bd 6*, ed. Moldenhauer, E. (Frankfurt am Main: Suhrkamp, 1969).

Heidegger, Martin, *Being and Time*, trans. Macquarrie, John (Oxford: Blackwell, 1973).
———, *Hegel's Phenomenology of Spirit*, trans. Martin, Emad, Parvis, and Maly, Kenneth (Bloomington, Ind.: Indiana University Press, 1988).
———, *Phenomenological Interpretation of Kant's Critique of Pure Reason*, trans. by Emad, Pervis and Maly, Kenneth (Bloomington: Indiana University Press, 1997).

Houlgate, Stephen, "Hegel's Critique of Foundationalism in The "Doctrine of Essence"," *Bulletin of the Hegel Society of Great Britain*, 1999: 18-34.
———, "Hegel's Critique of the Triumph of Verstand in Modernity," *Bulletin of the Hegel Society of Great Britain,* 1997: 54-70.
———, "Hegel, Kant, and the Formal Distinctions of Reflective Understanding," in *Hegel on the Modern World*, ed. Collins, Ardis B. (Albany, N.Y.: State University of New York Press, 1995).
———, *Hegel, Nietzsche and the Criticism of Metaphysics* (Cambridge: Cambridge University Press, 1986).
———, *An Introduction to Hegel: Freedom, Truth and History*. (Malden, Mass. ; Oxford: Blackwell, 2005).
———, "Necessity and Contingency in Hegel's *Science of Logic*," *The Owl of Minerva,* 1995 27: 37-49.
———, *The Opening of Hegel's Logic* (Indiana, Ind.: Purdue University Press, 2006).
———, "Schelling's Critique of Hegel's *Science of Logic*," *Review of Metaphysics,* 1999 LIII: 99-128.

Hyppolite, Jean, *Genesis and Structure of Hegel's Phenomenology of Spirit*, trans. Heckman, John and Cherniak, Samuel (Evanston: Northwestern University Press, 1974).
———, *Logic and Existence*, trans. Lawlor, Leonard and Sen, Amit (Albany: State University of New York Press, 1997).

Inwood, M. J., *A Hegel Dictionary*: *Blackwell Philosopher Dictionaries* (Oxford: Blackwell Reference, 1992).

Jaeger, Werner Wilhelm, *Aristotle: Fundamentals of the History of His Development* (Oxford: Clarendon Press, 1948).

Kainz, Howard P., *Hegel's 'Phenomenology'* (Alabama: University of Alabama Press, 1979).

Kant, Immanuel, *Correspondence*, trans. And ed. Zweig, Arnulf (Cambridge: Cambridge University Press, 1999).
———, *Critique of Pure Reason*, trans. Smith, Norman Kemp (London: New York : Macmillan ; St. Martin's Press, 1929).

Khalfa, Jean, *Introduction to the Philosophy of Gilles Deleuze* (London: Continuum, 2002).

Kline, Morris, *Mathematical Thought from Ancient to Modern Times* (New York: OUP, 1972).
———, *Mathematics in Western Culture*: *Pelican Books* (Harmondsworth: Penguin, 1972).

Koffka, Kurt, *The Growth of the Mind: An Introduction to Child-Psychology*, trans. Ogden, Robert Morris (London: K. Paul, Trench, Trubner, 1928).

Lambert, Gregg, *The Non-Philosophy of Gilles Deleuze* (New York; London: Continuum, 2002).

Lewin, Roger, *Complexity: Life at the Edge of Chaos* (London: Dent, 1993).

Llewelyn, John, "Kantian Antinomy and Hegelian Dialectic," in, *Hegel's Critique of Kant*, ed. Priest, Stephen (Oxford: Clarendon Press, 1987).

Löwith, Karl, *From Hegel to Nietzsche : The Revolution in Nineteenth-Century Thought*, trans. Green, David E. (New York: Columbia University Press, 1991).

Lovejoy, Arthur O., *The Reason, the Understanding, and Time* (Baltimore: Johns Hopkins Press, 1961).

Mainzer, Klaus, *Thinking in Complexity: The Complex Dynamics of Matter, Mind, and Mankind* (Berlin; London: Springer, 1997).

Maker, William, *Philosophy without Foundations: Rethinking Hegel* (Albany: State University of New York Press, 1994).

Malabou, Catherine, "Who's Afraid of Hegelian Wolves?" in *Deleuze: A Critical Reader*, ed. Patton, Paul, (Oxford: Blackwell, 1996), 114-38.

Merleau-Ponty, Maurice, "Eye and Mind," in *The Primacy of Perception*, ed. Edie, James M. (Evanston: Northwestern University Press, 1964).
———, *The Incarnate Subject : Malebranche, Biran, and Bergson on the Union of Body and Soul*, trans. Bjelland, Andrew G., and Burke, Patrick (Amherst, N.Y.: Humanity Books, 2001).
———, *The Merleau-Ponty Aesthetics Reader: Philosophy and Painting*, trans. Johnson, Galen A.(Evanston, Ill: Northwestern University Press, 1993).
———, *Phenomenology of Perception*, trans. Smith, Colin (London: Routledge, 2002).
———, *The Visible and the Invisible*, trans. Lingis, Alphonso (Evanston: Northwestern University Press, 1968).

Millet, Jean, "Bergsonian Epistemology and Its Origins in Mathematical Thought," in, *Bergson and Modern Thought: Towards a Unified Science: Models of Scientific Thought*, eds. Papanicolaou, Andrew C. and Gunter, P. A. Y. (New York: Harwood Academic Publishers, 1987), 29-37.

Monk, Ray, "Was Russell an Analytic Philosopher?" in *The Rise of Analytic Philosophy*, ed. Glock, Hans-Johann, (Oxford: Blackwell, 1997), 35-50.

Moore, A. W., "The Method of Exhaustion as a Model for the Calculus," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht; Boston: Kluver Academic Publishers, 1993), 139-48.

Moretto, A., "Hegel on Greek Mathematics and Modern Calculus," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht; Boston: Kluver Academic Publishers, 1993), 149-66.

Morris, David, "Heideggerian Truth and Deleuzian Genesis as Differential 'Grounds' of Philosophy: Miguel De Beistegui's *Truth and Genesis: Philosophy as Differential Ontology*," *Pli – The Warwick journal of Philosophy,* 2006 17: 166-83.

Nancy, Jean-Luc, *The Speculative Remark: One of Hegel's Bons Mots*, trans. Surprenant, Celine (Stanford, Calif.: Stanford University Press, 2001).

Nietzsche, Friedrich, *The Gay Science*, trans. Kaufmann, Walter Arnold (New York: Vintage Books, 1974).
———, "Twilight of the Idols," in *Portable Nietzsche*: *The Viking Portable Library*, ed. And trans. Kaufmann, Walter (Harmondsworth: Penguin, 1976).

OU. History of Mathematics Course, Team, *History of Mathematics*. (Milton Keynes: Open University Press, 1974).

Owens, Joseph, *Doctrine of Being in the Aristotelian Metaphysics: A Study in the Greek Background of Mediaeval Thought*. (Toronto: Pontifical Institute of Mediaeval Studies, 1963).

Pears, David, *Bertrand Russell and the British Tradition in Philosophy*: *The Fontana Library*: Collins, 1967).

Pedersen, K. M., "Techniques of the Calculus, 1630-1660," in *From the Calculus to Set Theory, 1630-1910 : An Introductory History*, ed. Grattan-Guiness, Ivor (London: Duckworth, 1980), 10-48.

Pinkard, Terry, "Hegel's Philosophy of Mathematics," *Philosophy and Phenomenological Research,* 1981 41: 452-64.

Plato, "Phaedrus," in *Plato: Complete Works*, ed. Cooper, John M., and Hutchinson, D. S. (Indianapolis ; Cambridge: Hackett, 1997), 506-56.

Porphyry, *Isagoge*, trans. Warren, Edward W. (Toronto: Pontifical Institute of Mediaeval Studies, 1975).

Porter, James I., *The Invention of Dionysus : An Essay on the Birth of Tragedy* (Stanford, Calif.: Stanford University Press, 2000).

Priest, Stephen, *The Subject in Question: Sartre's Critique of Husserl in the Transcendence of the Ego* (London: Routledge, 2000).

Quine, Willard Van Orman, *From a Logical Point of View: Logico-Philosophical Essays*. (Cambridge, Mass.: Harvard University Press, 1961).
———, *Ways of Paradox, and Other Essays* (Cambridge, Mass.; London: Harvard University Press, 1976).

Ridley, Mark, *Evolution*. (Oxford: Blackwell Science, 1996).

Rosen, M., *Hegel's Dialectic and Its Criticism* (Cambridge: Cambridge University Press, 1982).

Rosen, Stanley, *G.W.F. Hegel: An Introduction to the Science of Wisdom* (New Haven ; London: Yale University Press, 1974).

Ross, William David Sir, *Aristotle* (London: Methuen, 1964).

Russell, Bertrand, *A History of Western Philosophy: And Its Connection with Political and Social Circumstances from the Earliest Times to the Present Day* (London: G.Allen and Unwin, 1946).
———, *The Principles of Mathematics* (London: Allen and Unwin, 1937.
———, *Russell's Logical Atomism*, ed. Pears, David Francis (London: Fontana, 1972).

Sainsbury, R. M., *Paradoxes*. (Cambridge: Cambridge University Press, 1995).

Sartre, Jean Paul, *Being and Nothingness: Essay on Phenomenological Ontology*, trans. Barnes, Hazel E. (London: Routledge, 1989).
———, "Intentionality: A Fundamental Idea of Husserl's Phenomenology," in *Edmund Husserl: Critical Assessments of Leading Philosophers*, eds. Bernet, Rudolf, Welton, Donn, and Zavota, Gina (UK: Routledge, 2005), 257-260.
———, *Nausea*, trans. Baldick, R. (Harmondsworth: Penguin, 2000).
———, "Playboy Interview: Jean-Paul Sartre: Candid Conversation," *Playboy: Entertainment for Men,* 1965 12: 69-77.
———, *The Transcendence of the Ego: An Existentialist Theory of Consciousness*, trans. Williams, Forrest (New York: Noonday P, 1972).
———, *Truth and Existence*, ed. Aronson, Ronald and trans. van der Hoven, Adrian (Chicago: University of Chicago Press, 1992).

Smith, Daniel W., "Deleuze, Hegel, and the Post-Kantian Tradition," *Philosophy Today,* 2000 44 (Supplement): 119-31.

Smith, John E., "Hegel's Critique of Kant," *The Review of Metaphysics,* 1973 XXVI.

Spinoza, Benedictus de, *The Ethics: Treatise on the Emendation of the Intellect; Selected Letters*, trans. Shirley, Samuel, and Feldman, Seymour (Indianapolis: Hackett Pub. Co, 1992).

Stern, Robert, *Hegel, Kant and the Structure of the Object* (London; New York: Routledge, 1990).
———, *Routledge Philosophy Guidebook to Hegel and the Phenomenology of Spirit* (London: Routledge, 2002).

Stewart, Jon, *The Phenomenology of Spirit Reader: Critical and Interpretive Essays* (Albany, NY: State University of New York Press, 1998).

Strawson, P. F., *The Bounds of Sense: An Essay on Kant's Critique of Pure Reason* (London: Routledge, 2005).

Tarski, A., "The Semantic Conception of Truth and the Foundations of Semantics," in *Philosophy of Language*, ed. Martinich, A. P. (New York ; Oxford: Oxford University Press, 1996), 61-84.

Taylor, Charles, *Hegel* (Cambridge: Cambridge University Press, 1975).

Toscano, Alberto, *The Theatre of Production: Philosophy and Individuation between Kant and Deleuze* (Basingstoke: Palgrave Macmillan, 2006).

Toth, I., "The Dialectical Structure of Zeno's Arguments," in *Hegel and Newtonianism*, ed. Petry, Michael John (Dordrecht; Boston: Kluver Academic Publishers, 1993), 179-200.

Westphal, Kenneth R., *Hegel's Epistemology: A Philosophical Introduction to the Phenomenology of Spirit* (Indianapolis, IN: Hackett Pub., 2003).

Westphal, Merold, *History and Truth in Hegel's "Phenomenology"* (Hassocks: Harvester Press, 1982).

Whitehead, Alfred North, and Russell, Bertrand, *Principia Mathematica.* (Cambridge: CUP, 1927).

Widder, Nathan, *Genealogies of Difference* (Urbana: University of Illinois Press, 2002).

Williams, Bernard Arthur Owen, *Descartes : The Project of Pure Enquiry* (Hassocks: Harvester Press, 1978).

Williams, James, *Gilles Deleuze's Difference and Repetition: A Critical Introduction and Guide* (Edinburgh: Edinburgh University Press, 2003).

Žižek, Slavoj, *Organs without Bodies: Deleuze and Consequences* (New York: Routledge, 2003).