# MARGINALIA IN WITTGENSTEIN'S COPY OF LAMB'S *HYDRODYNAMICS*


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As is well known Wittgenstein, before turning to philosophy, first studied mechanical engineering (Maschinen-Ingenieurwesen) at the Technische Hochschule (TH) in Berlin-Charlottenburg beginning in the Wintersemester 1906. His original intention had been to study physics under Boltzmann in Vienna. In his last year at the Realschule in Linz he gave "Physik" as his intended Beruf (career), whereas in the previous year he had hesitated between physics and engineering.[3] In an often cited list of influences on his own thought, which appears to follow a chronological order, he put Boltzmann's name first. The most obvious echoes of Boltzmann in his own writing are in the demonstration that most philosophical questions are meaningless, but there may, as we shall see shortly, have been a more concrete influence also. Boltzmann, whose sufferings were great, took his own life that year, which may have been the reason for the new choice of Berlin.

Certainly the choice of engineering was more in keeping with his father's expectations. When Ludwig on one (perhaps not unique) occasion absented himself from school, Karl, the father's, reaction was to say the boy might as well "faulenzen", loaf about, in Vienna. He would soon enough go into a workshop and there learn everything that he really needed. This expectation of the father's was probably in Ludwig's case (not in that of some of his brothers) an appropriate one. The boy had shown impressive technical ability already. At school he had shown particular interest in aeronautics. "Was ist aus der Flugmaschine geworden?" ("What became of the flying machine?") wrote one of his old schoolmasters when renewing contact in 1915. Needless to say the construction of such machines was not a normal university subject in those days, but it is a curious coincidence, if it is no more, that Boltzmann in an address of 1894 had predicted the superiority of the heavier than air machine over the dirigible.

It is likely that Wittgenstein in any case meant to follow his father's precept and example and stay at the TH only long enough to learn the things he required. At all events, something decided him to leave Berlin after three semesters and in the spring of 1908 to go to England. There he started work at a meteorological observatory and research station near Manchester[4]. In the autumn of 1908 he registered as a student doing research in the Engineering Laboratory of the University of Manchester. His work was to assist in the construction and flying of kites that were equipped with meteorological instruments, while he was allowed to use the facilities there for his own research on the design of kites. The experience with kites was important in two respects for aeronautical research: it could help in the design of flying machines and it would give a good understanding of the winds in the higher atmosphere. That Wittgenstein did intend to do aeronautical research when he came to England seems clear from the fact that he had brought with him from Berlin some prints of early balloons.

There are indications that Wittgenstein combined his experimental work on kites with some theoretical studies. His colleague W. Eccles recalled having found him in the living room

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[3] See Djavid Salehi, "Ludwig Wittgenstein als Schüler in Linz", *Wittgenstein Studies* 1997 disk 1.

[4] See e.g. B. F. McGuinness, *Wittgenstein: a life,* University of California Press, 1988.

of the inn in which they both lived at the time, surrounded by books and papers on the table and also on the floor. By October 1908 he had derived certain equations and had shown them to one of his professors, Horace Lamb[5]. Lamb told him that he wasn't sure whether those equations could be solved by the methods of the day. This must have frustrated Wittgenstein's theoretical studies in aeronautics, of which we hear nothing more. He also turned his attention from the experiments with kites to the development of a new engine, realising that unless some new kind of engine would be available the kite-flying experiments were not useful. But before much work was done on the engine that he designed and had built, his attention turned to the design of a propeller.

Unlike our knowledge of Wittgenstein's work on kites and combustion chambers, that of his attitude towards mathematics during his Manchester period and of his developing interest in philosophy is very limited. All we know is that he attended J. E. Littlewood's lectures on mathematical analysis, that (in the words of Wittgenstein's sister Hermine) "reflection on philosophical problems became such an obsession with him that he suffered terribly", and that by April 1909 he had corresponded with the mathematician P. E. B. Jourdain about Russell's paradox. It is not known for certain that Wittgenstein did visit Frege during his Manchester period but it seems likely. He continued to work in Manchester until 1911 when, though registered at Manchester for the new academic year, he went to Cambridge to study philosophy under Bertrand Russell.

Throughout his life Wittgenstein's struggles were with himself. We need not think that his inclinations made him a philosopher while duty to his father would have him be an engineer. Everything indicates that (like his sisters) he had internalised his father's values. Perhaps on another side (too little studied) his mother's also. Zwei Seelen wohnen ach in meiner Brust – Two spirits dwelt in his breast, perhaps even more than two. But certainly there was an opposition between an aptitude and inclination for the extremely practical and a craving for abstract thought (though in a form that often led to a self-denying hostility to abstraction). Frequently in his life he took refuge from his thoughts in a practical activity or at any rate devoted himself entirely to one. He was in a workshop for half his time in the army. He worked as a gardener, a joiner, an architect, a laboratory technician; he thought of becoming a doctor. His period as a schoolmaster should also be located here. So we should take seriously the inner conflict described by Hermine and the passion with which he put to Russell the choice he had to make between aviation and philosophy. The affect that he put into his thinking was clear to him when he told Russell (who cared for none of these things) that he was thinking about logic *and* his sins.

A recent find in a London second-hand bookstore may throw some light on this at first sight contradictory (or at least divergent) movement in Wittgenstein's interests. On the one hand, he left engineering study for practical work on the construction of an aircraft engine; while on the other hand he concerned himself with the foundations of not just physics but mathematics itself. These were the tendencies that took him from Berlin to Manchester and then to Cambridge, where he arrived, divided in mind in the way indicated above.

The book that has been found is the authorised German translation of the third edition of *Hydrodynamics* by Horace Lamb (Cambridge University Press, 1906), published by B. G. Teubner in Leipzig and Berlin under the title *Lehrbuch der Hydrodynamik* in 1907. On the title page there is a signature in black ink "Ludwig Wittgenstein" in Wittgenstein's handwriting;

[5] Horace Lamb (1849-1934), see *Obituary Notices of Fellows of the Royal Society* **4**, (1935).

there are marginalia in pencil and in ink, in the same handwriting. As far as can be determined (some of the corrections of typographical errors are too insignificant to establish their authorship) no one else has written marginalia in the book. The main focus of our contribution is to give a summary and interpretation of the marginalia.

Before discussing the marginalia it is of interest to establish when Wittgenstein read this book. It was published during his stay in Berlin, though of course not necessarily used by him there or only there. It is a priori likely of course that the book was not bought in England, where the original English edition was readily available. There is actually evidence that the book was bought originally in Germany: on the inside of the front board is written in pencil "M20", with the 'M' usual in indications of currency, which is evidently the price at which the book was sold. In fact in other and contemporary volumes of the Teubner series to which it belongs this book is listed at the price of 20 Marks.

If bought in Berlin, this must have been during the period of Wittgenstein's study there (since there is no indication that he soon returned to Germany). The possibility that he needed it for his studies there must be examined. The curriculum of the TH Berlin for the relevant period contains only one course for which Lamb's book might have been used – the second general course on mechanics (Allgemeine Mechanik II) which was on the equilibrium and the motion of elastic solids and fluids. This does not seem likely however as the motion of elastic solids is not discussed in Lamb's book. More important, Lamb presupposed knowledge of partial differential equations, a subject taught only in an advanced course at the TH Berlin, making it unlikely that Lamb's book would have been used in the mechanics course, which could be attended by students as early as their second semester. It therefore seems likely that Wittgenstein bought the book either with the intention to use it in Manchester or for private study in Berlin, not related to any particular lecture course[6].

That Wittgenstein probably bought the book during his stay in Berlin somewhat modifies the current understanding of what actually drove him to Manchester instead of some German or Austrian university. The point usually made is that some role may have been played by Wittgenstein's father, who as a highly influential Austrian industrialist may have been involved in financing the large loan of radium that was made to the University of Manchester by the Austrian Academy of Sciences in the very year when Wittgenstein started his studies there. It is not impossible that this is a related fact, but we now have the additional information that at Manchester Wittgenstein would be studying in the very department where Horace Lamb, the author of the book he had just bought, was working. It is not the case, as might be thought that he had to move to Manchester in order to work at a meteorological station. Only 65 km from Berlin (in Lindenberg near Beeskow) similar experiments with kites were being carried out (the meteorological station officially started operations in 1905). It was from there in fact that the preface of Wittgenstein's Lamb volume was dated. But he may have been against that station for all too characteristic emotional reasons. In Berlin he lived with the family of Stanislaus Jolles, one of his professors, and clearly developed a relation of considerable intimacy with them, but this was followed by a painful break, which, to one with Wittgenstein's strong feelings, may have rendered a move away from Berlin desirable. This together with the presence at Manchester both of Lamb and of the meteorological station may have determined the choice of that city.[7]

[6] For information about courses at the Technische Hochschule we are indebted to Mr von Knobelsdorff of the library of the Technische Universität at Berlin, as the TH has now become, who provided it, and to Professor Ronald Jensen and Dr Margaret Möllmann of the Humboldt University, Berlin, who collected it. They had the onerous task of assisting in the attempt to prove a negative.

[7] There was, however, much correspondence in later years and in 1939 Wittgenstein (along with A. Einstein, a former colleague of Jolles) took some steps to help Jolles emigrate. These came to nothing and Jolles in fact died within the year. (Jolles's subject was projective geometry, a congenial subject for Wittgenstein.)

We shall now summarise and draw some conclusions from the marginalia. There are marginalia in the first four chapters (there are none actually in the second chapter but in a subsequent one there is a reference showing that that chapter had been read). The marginalia are most frequent at the beginning of the chapters. The book consists however of twelve chapters, the part that we know has been studied being no more than an introduction to the subject. This part consists of the equations of motion of a fluid, irrotational fluid motion and liquid motion in two dimensions. If this is all that Wittgenstein studied on the subject of fluid mechanics we must conclude that his knowledge was limited to general theorems and some elementary examples of calculations of idealised fluid flows. It is quite possible that this is indeed all he studied in hydrodynamics and that his encounter with Lamb discouraged him from any further studies in this area. The contribution of this knowledge to his work on developing a flying machine would have been very limited. Wittgenstein may well have concluded this himself and decided that further study of Lamb's book would not be useful either. We note here that this is not because of a possible difference between the subjects of hydrodynamics and aerodynamics (it is now well recognised that there is no fundamental difference): the part of Lamb's *Hydrodynamics* read by Wittgenstein is equally relevant to aerodynamics.

Judging from the marginalia it seems that Wittgenstein was more concerned with grammar and with the importance of expressing in a sentence exactly what one means to say than with fluid dynamics. Many, indeed most, marginalia consist in some alteration of wording. There are several insertions that supplement the text. For example, at some point "a function" is introduced without immediate specification of what its arguments are; these have then been inserted by Wittgenstein. Also, when "the velocity" is discussed, there is an insertion asking what it is a velocity of. Then there are words that have been replaced by other words that seem more appropriate, for example in "Größe der Bewegung" Wittgenstein replaced "Bewegung" (=motion) by "Geschwindigkeit" (=velocity). Other changes are of a purely grammatical nature: the insertion of "bezw." (=respectively), the repositioning of a word or a part of a sentence. None of these changes in wording is required for an ordinary student to get a good understanding of the text. Wittgenstein has interpreted the text literally and has insisted that the wording should accurately reflect exactly what is meant by the text. To the average reader of texts on fluid dynamics this may seem to be the attitude of a purist. While it is often difficult to follow the arguments presented by an author (or translator), the degree of precision Wittgenstein insists on does not materially promote this aim. What his insistence does reflect is a strong interest in the use of language.

On a few occasions Wittgenstein has criticised the text. These criticisms often end with exclamation marks. It is a nice point whether this is due to an unusual degree of emotional involvement or simply an example of Wittgenstein's style, which later was always forceful with frequent use of emphasis. At one point here Wittgenstein argued that the result had been used in its own derivation (making the derivation a fraud). Lamb's presentation, although not a fraud, is indeed guided by the final result. A much simpler, less controversial derivation is readily available and could have been presented instead. More difficult to understand is a marginal note written at the point where Lamb has completed a derivation of an equation that describes the fluid flow generated by suddenly setting fluid in motion. Lamb indicates that the result could have been obtained from a previously derived equation. The new and old equations are indeed very similar, since they differ in only one term. That one term is, in the old equation, the extraneous force acting on the fluid (such as gravity). In the new equation it is the impulsive force that has set the fluid suddenly in motion, divided by its duration. It must of course be argued why these two are the same or, alternatively, one should consider the fact that they are equal as a separate result for the impulsive forces. But Lamb writes that we could have obtained the new equation from the old one by equating the two terms. Wittgenstein clearly objected emphatically, saying that the second set of terms are “defined here for the first time!” i.e. by questioning Lamb’s claim that both terms are equal. Of course Lamb knew that and why they were the same. It can be understood from the fact that the impulsive force (the new term) is nothing but the change in momentum of the fluid, which in turn must equal the extraneous forces that are acting on the fluid times the duration of the impulsive force (the old term). But from all Wittgenstein could know at this point the whole passage must have seemed a fraud.

At one point Wittgenstein seems to have been unable to envisage what Lamb was talking about. Lamb discusses there the motion of imaginary surfaces. Wittgenstein has written in the margin: was heißt: "sich mit der Flüssigkeit bewegen."?! (what is the meaning of: “move with the fluid”?!) One can easily think of the motion of the surface of the sea for instance, but what is an "imaginary surface" that "moves with the fluid"? Wittgenstein tried to think of such an imaginary surface in terms of -- for instance -- the surface of the sea. Such surfaces within a fluid have since been photographed and nowadays imaginary surfaces within the fluid are a common concept. The insistence on having a clear understanding of what is talked about is also reminiscent of the marginalia quoted above: marginalia that at first sight seem to be mere insertions of a purist may in fact rather be in the spirit of a Machian positivist.

It is only later on in the book that the marginalia tend to become less critical of the text. It is then also that marginalia are found in which whole sentences have been rewritten simply to get a better understanding of the text. For at those points one had to recall material discussed previously and Wittgenstein has included that information in his rephrased sentences.

Also later on in the book we can see Wittgenstein seeking for explanations. These marginalia are of great interest as they reveal what puzzled Wittgenstein when he read the text. Wittgenstein has drawn for instance a figure in which are indicated the positions of two points in physical space (used in the text to derive some theorem). Apparently Wittgenstein was puzzled as to why those particular points were chosen. A similar situation occurs further down on the same page. In the main text, several equations have just been presented. It is mentioned later on that terms in these equations can be divided into three groups of terms, each of which has a clear physical meaning. In this set of equations Wittgenstein has drawn further lines connecting the terms that make up one such group. He must have done this to find from the equations a geometrical explanation of why those terms were to be grouped together.

By looking into matters such as these Wittgenstein has really gone beyond what must have been Lamb’s intentions when he wrote his treatise. Clearly Wittgenstein did not simply try to follow the arguments presented in the text but searched – at least at the two occasions mentioned here – for explanations beyond the kind presented in the text.

Several times we have to observe that with more patience and longer study of the text Wittgenstein would have understood why things were presented in the way he objected to. Perhaps also, even when there is some basis for his criticisms, he would have habituated himself to the practice and way of speaking of practitioners of the subject. They would regard his difficulties as intellectual teething troubles. A failure to get over such difficulties, which are not surprising in a beginner, perhaps remained with Wittgenstein all his life. His later treatment of mathematics seems to proceed from a certain feeling for what is afoot in mathematics, but not enough acquaintance with the advances of the subject to inhibit his radical criticisms.[8] This is of a piece with what he reports about his reading of, say, Hume. He would very soon lay aside the volume and start thinking of the problems for himself. Of course by such a method he does not deal with Hume's problem, or Kant's, for example, but without it we should not have his own philosophy. There too he was always wrestling with himself.

To the question what Wittgenstein hoped to find in this theoretical treatise on hydrodynamics specifically relevant to his interest in developing a flying machine, we have to say that any such hopes will have been illusory at that date. Clearly the Wright brothers arrived at their design in a remarkable well-founded manner (even by modern standards)[9]. But they did not need much theoretical fluid dynamics to do that. Although fundamental work that *would* have been useful had already been done and presented in 1904 by Ludwig Prandtl, a professor at the University of Göttingen in Germany[10], it would take years before that work became widely accepted and recognised. Sydney Goldstein, a successor to Lamb in Manchester, wrote: "Lamb's textbook was the predominant "high-class" textbook for many years, certainly in England in the 1920's for students of applied mathematics, but we were not really happy with it. Rayleigh may have contrasted it with the "arid" textbooks of the 1860's [in a review of the 1916 edition of Lamb's book], but in the 1920's we were complaining that it was impossible to remember while reading Lamb's book that water is wet."[11]. The main difficulty with theoretical fluid dynamics at the beginning of the last century was the persistent difference between theoretical predictions and experimental observations of flow around objects. It was fundamental understanding of the cause of this difficulty (provided by the work of Prandtl) that would have been useful in developing a flying machine, but this became common knowledge only in the 1920's. Lamb did of course take account of this work in later editions of *Hydrodynamics* (the sixth edition – which is still available – appeared in 1932).

[8] This paragraph owes much to (though inevitably simplifies) the views of G.Kreisel. See *inter alia* his "Second Thoughts around some of Gödel's Writings", *Synthese* 114:119ff, 1998. The late George Temple, wryly accepted it when Wittgenstein's class for mathematicians in the 1930's never got beyond the number 2. But J.H.C.Whitehead, who, however, did not attend, was indignant, *"Why didn't he* do *some mathematics then?"*
[9] Cf. *The Wright Flyer*, by H. S. Wolko (ed.), Smithsonian Institution, Washington D.C., 1987.
[10] It may be of interest to note that Prandtl had been invited to come to Göttingen by the mathematician Felix Klein. It was at Klein's suggestion that Lamb's book was translated into German. Göttingen, not Berlin, dominated the theoretical and experimental work on fluid dynamics in Germany at the time.
[11] S. Goldstein, Fluid Mechanics in the first half of this century, *Annual Review of Fluid Mechanics* **1**, 1969, pp. 1-28.

# APPENDIX: THE MARGINALIA

Below follows a transcription of the marginalia[12]. The numbers refer to the section numbers in the book (these numbers differ only on a few places with the final, still available English edition). The passages of the German text to which they refer are given together with the marginal notes (in boldface), followed by the corresponding English text (in italic and taken from the third edition, Cambridge University Press, 1906). In the English text a translation of the marginalia is given (in italic boldface). Text between square brackets has been inserted by us. In most cases we have found it necessary to comment on the marginalia.[13]

§2(I). Wenn ~~die Spannung, die durch~~ **der Druck der auf** eine kleine Fläche im Innern der Flüssigkeit wirkt, gänzlich normal **auf diese** gerichtet ist, so ist ~~deren~~ **seine** Intensität, auf die Flächeneinheit bezogen, die gleiche für alle Richtungen der Fläche. *If the ~~stress~~* ***pressure*** *exerted across any small plane area situated at a point P of the fluid be wholly normal, its intensity (per unit area) is the same for all aspects of the plane.*

*Comment* W. here altered "stress" (Spannung) to "pressure" (Druck). At this point in the text it has not yet been explained that the pressure is actually the part of the stress that satisfies the condition stated in the quoted sentence (i.e. the pressure across any small plane area is wholly normal). Changing "stress" into "pressure" is therefore not correct, but W. could not have known this when he wrote the marginal note. Furthermore, although W. probably did not have the English text in front of him and so could not have been aware of it, there is a misunderstanding in the German translation at this point. The cause of this error is that the English text is ambiguous. "Any small plane area" could be taken to mean "any one", but it should be taken to be "every one". The translator picked the wrong option by translating it by "eine kleine Fläche". The theorem that is stated in the text to which the marginal notes refer, is therefore not proved in the subsequent text.

§2(II). Durch *P* ziehen wir drei gerade Linien *PA*, *PB*, *PC*, die senkrecht zueinander stehen, und zeichnen eine Ebene in unendlicher Nähe von *P*, die diese Linien in *A*, *B* und *C* trifft, und deren Richtungskosinusse gegen dieselben *l*, *m*, *n* sind. Es seien $p$, $p_1$, $p_2$, $p_3$ die ~~Intensitäten der Spannungen~~ **Drücke** ~~**pro Flachen**~~**einheit** auf die Flächen *ABC*, *PBC*, *PCA*, *PAB* des Tetraeders *PABC*. *Through the point P draw three straight lines PA, PB, PC mutually at right angles, and let a plane whose direction-cosines relatively to these lines are l, m, n, passing infinitely close to P, meet them in A, B, C. Let* $p$, $p_1$, $p_2$, $p_3$ *denote the ~~intensities of the stresses~~* ***pressures*** ~~***per***~~ ***unit*** ~~***area***~~ *across the faces ABC, PBC, PCA, PAB, respectively, of the tetrahedron PABC.*

*Comment* This marginal note refers to the beginning of the proof of the theorem that was stated under §2(I) above. Being consequent, W. changed again "Spannung" (=stress) into "Drücke" (=pressure). Because it is not clear what could be meant by "Intensitäten der Spannungen/Drücke" (=intensities, i.e. magnitude, of the stresses or pressure), W. just used "Drücke". The result reads then "Drücke pro Flacheneinheit" (=pressure per unit area).

[12] The authors would like to thank the Wittgenstein Trustees, who hold the copyright on the marginalia, for their kind permission to reproduce the marginalia here.

[13] The authors would like to thank Dr. C. J. Lawrence of the Chemical Engineering Department, Imperial College, London, and Prof. L. van Wijngaarden of the Departments of Applied Physics and Mechanical Engineering, University of Twente, the Netherlands, for useful suggestions on the interpretation of the marginalia.

However, pressure is by definition force per unit of surface area, so the "pro Flacheneinheit" (=per unit area) is incorrect, hence it has been crossed out.

§2(III). Wenn wir also die Bewegungsgleichung des Tetraeders parallel zu *PA* bilden, so haben wir …*Hence if we form the equation of motion of the tetrahedron parallel to PA we have …*

*Comment* The word "Bewegungsgleichung" (=equation of motion) has been underlined with a wiggly line. The reader cannot know at this point what is meant by "equation of motion of the tetrahedron". A much clearer phrase would be "force balance".

§4(I). Es seien $u,v,w$ die Komponente der Geschwindigkeit **wessen?** parallel zu den Achsen im Punkte $(x, y, z)$ zur Zeit $t$. *Let u,v,w be the components, parallel to the co-ordinate axes, of the velocity* ***of what?*** *at the point (x, y, z) at the time t.*

*Comment* One of the main concepts in fluid dynamics is that of a velocity field. This is an abstraction: with "the velocity" is meant "the velocity of the fluid at a specific point in space and in time". Lamb did in fact try to explain this in the preceding paragraph. W. asks now what (wessen) that is the velocity of. This is probably related to the next marginal note, §4(II), in which W. challenges (or is confused by) Lamb's viewpoint that not only sensorily distinct things can have a velocity, but also a continuum like a fluid.

§4(II). Bei einer so definierten stetigen Bewegung ist die Relativgeschwindigkeit zweier nachbarter Teilchen *P* und *P*' immer unendlich klein, so daß die ~~Linie~~ **Strecke** *PP'* immer dieselbe Größenordnung behält. Wenn wir uns eine kleine geschlossene Fläche um *P* gelegt denken und diese sich mit der Flüssigkeit bewegen lassen **Was heißt: "sich mit der Flussigkeit bewegen."?!**, so wird sie folglich immer dieselbe Substanz einschließen. Und jede beliebige Fläche, welche mit der Flüssigkeit bewegt, trennt vollständig und dauernd die Substanz auf ihren beiden Seiten. *In continuous motion, as thus defined, the relative velocity of any two neighbouring particles P, P' will always be infinitely small, so that the ~~line~~* ***distance*** *PP' will always remain of the same order of magnitude. It follows that if we imagine a small closed surface to be drawn, surrounding P, and suppose it to move with the fluid* ***What is the meaning of: "move with the fluid."?!****, it will always enclose the same matter. And any surface whatever, which moves with the fluid, completely and permanently separates the matter on the two sides of it.*

*Comment* (i). In "the line … will always be of the same order" (die Linie behält dieselbe Größenordnung) not the line itself, but its length is meant. Possibly Wittgenstein objected to this use of "Linie". (ii). At the point of the second marginal note Lamb is talking about imaginary surfaces within fluid. If the fluid moves, those surfaces "move with the fluid". Such a surface cannot be thought of as resembling for instance the surface of the sea, i.e., it is not a sensorily distinct thing that is said to be "moving with the fluid". Such surfaces within a fluid have since been photographed and nowadays "imaginary surfaces" is a common concept.

§5. Um die Änderung einer Funktion $F(x,y,z,t)$ für ein bewegtes Teilchen zu bestimmen, beachten wir, daß zur Zeit $t+\delta t$ das Teilchen, welches sich ursprünglich in dem Punkte $(x, y, z)$ befand, im Punkte $(x+u\cdot\delta t,\ y+v\cdot\delta t,\ z+w\cdot\delta t)$ ist, sodaß …. *To calculate the rate at which any function* $F(x,y,z,t)$ *varies for a moving particle, we remark that at the time* $t+\delta t$ *the particle which was originally in the position* $(x, y, z)$ *is in the position* $(x+u\cdot\delta t,\ y+v\cdot\delta t,\ z+w\cdot\delta t)$*, so that …*

*Description* W. moved the part "Zeit $t+\delta t$" (intending no doubt "zur Zeit $t+\delta t$") to before "im Punkte". This change is not actually reproduced here.

§6(I). Wir nehmen ein Element, das als Zentrum den Punkt ($x, y, z$) und die Kanten $\delta x$, $\delta y$, $\delta z$ parallel zu den Achsen des rechtwinkligen Koordinatensystems hat. Der Betrag, um den die $x$-Komponente des Momentes **in der Zeiteinheit** dieses Teilchens wächst, ist …. *Let us take an element having its centre at (x, y, z) and its edges δx, δy, δz parallel to the rectangular co-ordinate axes. The rate at which the x-component of the momentum of this element is increasing is…*

*Comment* The translation is poor at this point: "the rate" at which something "is increasing" had been translated as "der Betrag" (=amount), rather than Änderung, so that the insertion "in der Zeiteinheit" (=per unit time) is called for.

§6(II). Der Druck auf die $yz$-Ebene, die nächst dem Koordinatenursprung liegt, ist schließlich: … **Hier wird schon das Resultat vorausgesetzt** *The pressure on the yz-face which is nearest the origin will be ultimately …**Here the final result is being presupposed***

*Comment* In this section a force balance is derived for a small element of (frictionless) fluid. The only non-trivial part of the final result of the derivation is the contribution referred to in the marginal note. The way in which Lamb has expressed this contribution mathematically is indeed, as indicated by Wittgenstein, guided by the final result of the derivation. It is possible to write this contribution in a much easier and less controversial way.

§6(III). An "x" is quite correctly changed into a "y" in one of a set of three similar formulas.

§10. Um die Größe der ~~Bewegung~~ **Geschwindigkeit** $v$ der Oberfläche $F(x,y,z,t)=0$ in senkrechter Richtung zu sich selbst zu finden, schreiben wir: … *To find the rate of ~~motion~~* ***velocity*** *(v) of the surface F(x,y,z,t)=0, normal to itself, we write …*

*Comment* "Größe der Bewegung" is not a good translation of "rate of motion". "Motion" (=Bewegung) cannot have a magnitude (=Größe), but velocity (=Geschwindigkeit) can.

§11. Wenn wir dies mit $\delta x\delta y\delta z$ multiplizieren und über irgend einen Bereich **der Flüssigkeit** integrieren, finden wir… *If we multiply this by δxδyδz, and integrate over any region* ***in the fluid****, we find …*

§12. Diese Gleichungen können auch von Gleichung (2) des §6 abgeleitet werden, indem man die letztere mit $\delta t$ multipliziert und für die Grenzen 0 und $\tau$ integriert, wobei man $X'=\int X\,dt$, $Y'=\int Y\,dt$, $Z'=\int Z\,dt$ setzt … **Diese Gleichungen definieren überhaupt erst *X´ Y´* etc !** *These equations might also have been deduced from (2) of Art. 6 by multiplying the latter by δt, integrating between the limits 0 and τ, putting …**These equations are absolutely the first definition of X' Y' etc!***

*Comment* In this section the motion of fluid that is suddenly set into motion by application of an impulsive force is described. The components of the impulsive forces (per unit mass) are denoted by $X'$, $Y'$ and $Z'$. The point of the passage to which the marginal note refers is to show that it is possible to derive the result just presented by using a previously derived result. But in that previous result some variables $X$, $Y$ and $Z$ (the components of the external forces -

e.g., gravity - acting on the fluid per unit mass) were employed, rather than *X´*, *Y´* and *Z´* (the components of the impulsive forces per unit mass). The equations to which the marginal note refers give the relation between both sets of variables. The way it is presented in the main text, it seems as if this relation has been obtained by comparison of both results (alternatively one might consider these relations as a new result for the impulsive forces). W. may have objected to this unconvincing strategy. Of course Lamb knew that this relation was correct. It follows namely from the fact that the impulsive force (with components *X´*, *Y´* and *Z´*) is nothing but the change in momentum of the fluid, which in turn must equal the forces that are acting on the fluid (with components *X*, *Y* and *Z*) times the duration of the impulsive forces, hence the relation between *X* and *X´* etc.

§13. Diese Gleichungen enthalten die Ableitungen nach *x, y, z,* während unsere unabhängigen Variabeln *a, b, c, t* sind. Um diese Differentialquotienten zu eliminieren, multiplizieren wir die obigen Gleichungen **bezw.** mit $\partial x/\partial a$, $\partial y/\partial a$, $\partial z/\partial a$, und addieren sie; … *These equations contain differential coefficients with respect to x, y, z, whereas our independent variables are a, b, c, t. To eliminate these differential coefficients, we multiply the above equations by* $\partial x/\partial a$, $\partial y/\partial a$, $\partial z/\partial a$, *respectively, and add; …*

§15. Wenn wir ~~darum~~ **ferner** ~~$\chi$=~~ … [a lengthy mathematical expression follows] **=$\chi$** ~~schreiben~~ **setzen**, so finden wir … *Hence if we ~~write~~* ***put*** *~~$\chi$=~~…=$\chi$, we find …*

*Comment* To simplify the notation in the text that follows it was stated that from then on a certain lengthy mathematical expression is simply replaced by "$\chi$". The point of this marginal note is that one should write this as "from now on, let us call this lengthy expression $\chi$", rather than (as was done in the text) "let us define a variable called $\chi$ and its value is the lengthy expression". The variant suggested by Wittgenstein does make it clear that we are merely dealing with a matter of notation, not with a new variable that has physical significance.

§30(I). Wittgenstein added a diagram/figure, representing three dimensions, with three axes (labelled x,y and z) in which the position of two points is indicated. These points (at (x,y,z) and ($x+\delta x$, $y+\delta y$, $z+\delta z$)), which are close together, are considered in the text: the difference between the fluid velocity at these points is split up into three elementary types of motion (translation, rotation and compression). The drawing suggests that W. was puzzled by the choice of these points – at first sight their positions seem to have been chosen arbitrarily.

§30(II). The differences in (three) velocity components at the two points mentioned above have been approximated in the text. They are approximately linear functions of $\delta x$,$\delta y$ and $\delta z$. Pencil lines have been added connecting some of the coefficients (that is, the factors that multiply $\delta x$, $\delta y$ and $\delta z$ in the expressions for the difference in velocity components) in these equations. One set of connected coefficients represents one set of variables (*a*,*b*, and *c*) defined later on in the text; two other lines connect terms that make up two more variables. It seems that W. has tried to find some geometrical explanation why these coefficients were to be grouped together. This would have been useful in order to come to an improved notation; Lamb's notation is elaborate (he did not use the then still uncommon vector notation).

§30(III). Deshalb kann man sich ~~die Bewegung~~ **Den ~~Uberschuß~~ Zuwachs der Geschwindigkeit aus zwei Teilen …** eines kleinen Elementes, dessen Zentrum im Punkte (*x,y,z*) liegt, aus drei Teilen zusammengestetzt denken. *Hence ~~the motion~~* ***The relative motion out of two parts …*** *of a small element having the point (x,y,z) for its centre may be conceived as made up of three parts.*

*Comment* The discussion so far in the text has been on the fluid velocity at a point relative to the velocity at another point. This relative velocity (the Uberschuß or Zuwachs in the marginal note) consists of two parts, as stated in the note. But in the text there is a sudden departure from the consideration of this relative velocity; it is the motion of a fluid element that is considered henceforth (this consists of the same two parts as the relative velocity, plus the translation). Apparently W. had a clear picture of the text just presented, and tried to interpret the text (to which the marginal note refers) within that picture. The marginal note is therefore a translation of what Lamb states from his new viewpoint into the viewpoint that he had previously used (and to which W. prefers to adhere). The capital letter at the beginning of Wittgenstein's note is of course illogical.

§31. In a figure in which a periodic pattern of arrows had been drawn, additional arrows are drawn in pencil, continuing the periodic pattern, as if to verify that the pattern is indeed periodical. Also, a typographical error is corrected.

§36. In a mathematical expression a term $\partial\phi/\partial n$ occurs. In the margin a note reads: **$\partial\phi/\partial n$ Geschwindigkeitscomponente in Richtung von $\delta n$ (§19)**, that is: ***$\partial\phi/\partial n$ velocity component in the direction of $\delta n$ (§19)***. A minor error in this note is that there should be a minus sign in front: $-\partial\phi/\partial n$ is the velocity component (Geschwindigkeits-component) in the direction of $\delta n$. §19 is in Chapter 2 and there are no marginalia in that Chapter so this is the only indication that that Chapter had been studied. The spelling Component(e) was more usual in Austria than in Germany.

§37. Die Funktion $\phi$ kann in einem Punkte innerhalb der Flüssigkeit kein Maximum oder Minimum haben; denn wenn dies der Fall wäre, so müßten wir auf einer kleinen geschlossenen Fläche, die diesen Punkt einschließt, $\partial\phi/\partial n$ überall positiv oder überall negativ haben. Jeder dieser Fälle ist mit Gleichung (2) nicht verträglich**. In diesem Falle würde in den, von der geschlossenen Fläche eingeschlossenen Raum von allen Seiten Flüssigkeit stromen oder nach allen Seiten solche abströmen müssen was ohne Quellen oder Senken unmoglich ist.** *The function $\phi$ cannot be a maximum or minimum at a point in the interior of the fluid; for, if it were, we should have $\partial\phi/\partial n$ everywhere positive, or everywhere negative, over a small closed surface surrounding the point in question. Either of these suppositions is inconsistent with (2).* ***In that case, fluid would flow from all sides into the volume enclosed by the closed surface or fluid would flow out through all sides, which is impossible in the absence of any sources or sinks.***

*Comment* The marginal note differs from the text (apart from the wording) in two respects: "$\partial\phi/\partial n$" has been replaced by its physical meaning (a particular velocity component) and instead of "Either of these suppositions is inconsistent with (2)" the reasoning in that statement is given directly in the note, thereby avoiding mention of equation (2) and its notation. The rephrasing is correct. Wittgenstein occasionally overlooks an Umlaut, but usually not, as here, three times in a short space.

§39. Wenn jedoch das ursprüngliche Gebiet, in dem die wirbellose Bewegung stattfindet, nach außen hin unbegrenzt ist, und wenn die ersten (also auch alle höheren) Ableitungen von $\phi$ im Unendlichen verschwinden **d.h. Die Flüssigkeit im unendlichen ruht da $\partial\phi/\partial x$ etc = u etc = 0 , ...**, ... *If however the original region throughout which the irrotational motion holds be unlimited externally, and if the first derivative (and therefore all* *the higher derivatives) of $\phi$ vanish at infinity* ***i.e. The fluid at infinity is quiescent because $\partial\phi/\partial x$ etc = u etc = 0 , …***, *...*

*Comment* In this marginal note a statement in mathematical terms is rephrased in physical terms. W. has made the same (minor) sign error as in §39 by equating $\partial\phi/\partial x$ to the component of the velocity in the x-direction (=u).

§40. Wenn $\partial\phi/\partial n$ in jedem Punkte der Oberfläche des oben beschriebenen Gebietes 0 ist, so ist $\phi$ im Inneren überall konstant. Denn die Bedingung $\partial\phi/\partial n=0$ drückt aus, daß keine Stromlinien in das Gebiet eintreten **also daß die Stromung nur entlang solchen Linien $\partial\phi/\partial n=0$ vorsichgeht** oder es verlassen, sondern daß sie alle in ihm eingeschlossen sind. *Again, if $\partial\phi/\partial n$ be zero at every point of the boundary of such a region as is above described, $\phi$ will be constant throughout the interior. For the condition $\partial\phi/\partial n=0$ expresses that no lines of motion enter* ***so that the flow proceeds only along such lines $\partial\phi/\partial n=0$*** *or leave the region, but that they are all contained within it.*

*Comment* Wittgenstein offers an explanation in this marginal note that is different from the one presented in the main text. In the main text it is stated that no so-called "lines of motion" (imaginary lines in the fluid along which the fluid flows, nowadays usually referred to as streamlines) can exist under the given circumstances. This is then proved by first noting that the given circumstances imply that if there are any such streamlines, they have to be lines that close upon themselves. The closed streamlines are subsequently eliminated by referring to a previously derived result. The approach in the marginal note seems however to be to consider (hypothetically) a streamline and then to show that it cannot exist. The note suggests that if there were flow through the boundary, it would have $\partial\phi/\partial n=0$ along its line of motion, but that is as much as to say that there is no motion (along the line of motion), hence the theorem has been proved. The word "nur" (=only) has been inserted in the marginal note, possibly referring to the fact that this would only be true along such a line, not in the entire volume enclosed by the boundary.

§42. Es seien *U, V, W* drei beliebige Funktionen **von x, y, z**, welche .... *Let U, V, W be any three functions* ***of x, y, z,*** *which ...*

*Comment* Notation for a certain function was introduced without mentioning what variables it is a function of; Wittgenstein has inserted these arguments.

§47. Wenn zwei Wege oder zwei geschlossene Kurven ineinander überführbar sind, so muß es möglich sein, sie durch eine stetige Fläche zu verbinden, welche gänzlich in dem Raume liegt, und deren vollständige Begrenzung **Alle Stadien bei der Überführung** sie bilden, und umgekehrt. *If two paths or circuits be reconcilable, it must be possible to connect them by a continuous surface, which lies wholly within the region, and of which they form the complete boundary* ***All stages of the transformation****; and conversely.*

*Comment* Two identically shaped (equal-sized) lines are considered whose locations (and orientations) are different. By moving one line towards the other, and showing all the intermediate stages, one obtains the "continuous surface" mentioned in the main text. Lamb does not mention that the continuous surface shows all the intermediate stages of the transformation, but it is explained in the marginal note. The marginal note therefore makes it clear how the continuous surface spoken of is formed.

§59. Darum können wir ψ ~~als~~ bis auf eine additive Konstante **als** bestimmt betrachten. …; *so that we may, if we please, regard ψ as indeterminate to the extent of an additive constant.*

§71. A typographical error in a mathematical formula is corrected.