Probability without Tears
Julia Staffel

forthcoming in *Teaching Philosophy*
penultimate version; please cite final version when available

Abstract:
This paper is about teaching probability to students of philosophy who don't aim to do primarily formal work in their research. These students are unlikely to seek out classes about probability or formal epistemology for various reasons, for example because they don't realize that this knowledge would be useful for them or because they are intimidated by the material. However, most areas of philosophy now contain debates that incorporate probability, and basic knowledge of it is essential even for philosophers whose work isn't primarily formal. In this paper, I explain how to teach probability to students who are not already enthusiastic about formal philosophy, taking into account the common phenomena of math anxiety and the lack of reading skills for formal texts. I address course design, lesson design, and assignment design. Most of my recommendations also apply to teaching formal methods other than probability theory.

Introduction

Many areas of philosophy now make use of probability theory, but it is still not routinely taught at the undergraduate or graduate level, leaving many students without the necessary knowledge to understand philosophical debates that rely on probabilistic tools.[1] This paper is about how to teach probability to students who are not coming to philosophy with a prior interest in learning about it, but who would still greatly benefit from this knowledge. We might call these students *non-enthusiasts*, as opposed to probability *enthusiasts* who are intrinsically interested in studying probability and other formal methods, and who seek out opportunities to learn more about it. *Non-enthusiasts* have a neutral attitude towards probability, or even a negative attitude, or think of themselves as "not a math/logic person." In what follows, I will first discuss two factors that should inform how we approach teaching probability: math anxiety and students' lack of reading skills for formal material. I will then discuss how we can improve our course design, our lesson design and our assignment design in ways that are mindful of these factors. The goal is to impart knowledge of probability as a key formal tool in philosophy to a greater number of students in a way that is responsive to the reasons why students find this material difficult or scary. The teaching methods that can help us achieve this will contribute to a better learning environment for all students, including probability enthusiasts.

I am deliberately not setting a more ambitious goal, such as getting non-enthusiasts *excited* about learning probability. Setting such an overly ambitious goal strikes me as unrealistic, and possibly counterproductive to the more realistic goal of getting these students to be *comfortable* with learning probability. If we accidentally turn some of them into probability enthusiasts, so much the better.

1. Two Obstacles

---

[1] For a systematic study of how the use of probability theory in philosophy has increased between the late 2000s and the late 2010s, see Fletcher et al. (2021).

In order to effectively teach philosophy students about probability, we need to be aware of the prior knowledge and mindset with which they approach the subject matter. One important factor is that many students suffer from some degree of math anxiety. Estimates of the prevalence of math anxiety differ, but it seems that at least around 20% of students suffer from a high degree of math anxiety, while milder versions are even more common (see e.g. Ashcraft and Ridley 2005, Ramirez et. al. 2018). Symptoms of math anxiety include the following: feeling stressed out when thinking about math or trying to solve math problems, feeling helpless and paralyzed when trying to solve math problems, being afraid of failing, feeling like improvement is not possible, trying to avoid math, and feeling like the only one who "doesn't get it." High levels of math anxiety are correlated with decreased levels of achievement in math (Zhang et. al. 2019). The relationship between math anxiety and achievement is still being studied. Two plausible explanations of their relationship are that low performance on a test leads to increased fear of failing in the future, or that feeling anxious about math negatively affects the students' cognitive resources for solving math problems.

By the time we meet our students in college or graduate school, they have been studying math (or trying to avoid doing so) for many years, so we have little control over the relationship with math that our students bring into our classrooms. But even if we can't erase math anxiety, we can adapt our courses to try to eliminate or modify situations that trigger math anxiety as much as possible. These adaptations, which I will introduce below, in fact benefit all students, not just the ones who are anxious about math.

A second obstacle to be mindful of when teaching students about formal methods is their (in-)ability to read formal texts. Reading formal texts (i.e., texts containing formalisms, proofs, diagrams, etc.) requires a different approach than reading "informal" material that is written exclusively in prose. Math and logic textbooks are often written in a denser and more precise way than other textbooks, expecting readers to slow down, to work through examples, and to cross-reference graphs and earlier parts of the text as they read. Experts in formal methods know this, but students often expect reading formal material to work in roughly the same way as reading informal material. When this expectation is not met, they often give up. (Of course, this can also happen with difficult non-formal texts.) Even if they know in principle that they should be working through examples and reading slowly and carefully, they don't necessarily know how to detect errors in their own understanding, or what to do if they are confused (see e.g. Shepherd et al. 2012, Weinberg et al. 2012). This passage from a study of undergraduates' reading comprehension of mathematics textbooks illustrates this quite dramatically:

> Most of our students appeared to be strikingly unconcerned about their confusion or errors and did not seem to believe they could have independently done anything about them. Ten of the [eleven, J.S.] students stated at some point that they did not understand something, but made no attempt to understand whatever was causing confusion. Five students, three precalculus and two calculus, gave up at some point. They stated that they had no idea what to do, either while trying to work a task or when reading through a worked example. When questioned, one calculus student stated she would just move on, the other four stated they would quit and ask for help before continuing (Shepherd et al. 2012).

The cited study gives a lot more detail about the different ways in which the students struggled with their comprehension of the text. I recommend reading it in full for a vivid picture of how students engage with formal material. Interestingly, the students in the study had high ACT scores in math and reading comprehension. We can easily imagine how much more weaker students might struggle.

While these studies are not about formal writing in philosophy in particular, anecdotal evidence suggests that philosophy students experience similar difficulties when reading formal material. This should inform our lesson planning when we ask students to read texts that cover formal material. We need to manage their expectations about what their reading experience will be like and give them tools to get through it. In the section about lesson design below I will propose a variety of ways in which this can be accomplished.

What, then, should we aim for when teaching students who might be struggling with these two obstacles about formal methods in philosophy? Liam Kofi Bright offers this insightful answer: "We wish students to have the experience of seeing themselves as limited not by personal failings, but by something like real difficulty in the world." (Bright 2019) Philosophical questions are hard questions. Hence, it is appropriate for philosophers to experience confusion, lack of insight, and slow progress when dealing with them. If we are successful in teaching the use of formal philosophical methods, our students will experience all of these difficulties, but attribute them to the complex nature of the subject matter, rather than interpret them as a sign that they aren't suited to using formal methods.

2. Course Design

Students who aren't formal philosophy enthusiasts are unlikely to enroll in courses that teach formal methods such as probability, and that are non-mandatory. If a student doesn't particularly enjoy learning about formal material and doesn't realize that knowledge of it is essential to their area of study, it is entirely reasonable from their perspective to choose other courses instead. Yet, as I mentioned in the introduction, probability in particular is now a central tool in many debates in philosophy, and knowledge of it would benefit students in almost any area of philosophy. Learning this material via self-study is very difficult (c.f. the discussion in the previous section about reading formal material), so it is desirable that students learn about probability in their classes. The table below gives an overview over various debates that use probabilistic tools, but even undergraduate students who aren't going to take advanced philosophy classes can benefit from learning about probability. For example, in my critical thinking class, I teach a unit on how to use Bayes' theorem to interpret medical and other kinds of tests. I cover how hypothesis testing works in scientific research, how certain research and publication practices gave rise to the replication crisis, and how to use best practices in inductive reasoning to estimate probabilities of various kinds of events.[2]

A small selection of debates involving probability

---

[2] I have had great success with the following materials in my critical thinking class: an article by Gigerenzer et al. (2008) on statistical illiteracy in health contexts, the Radiolab podcast episode "Stereothreat" to illustrate the replication crisis (https://www.wnycstudios.org/podcasts/radiolab/articles/stereothreat)
and research by O'Connor and Weatherall on how the public can be misinformed through the deliberate spreading of a biased selection of research results (O'Connor and Weatherall 2019).

Philosophy of Language	semantics of epistemic modals, belief attributions, desire attributions
Philosophy of Mind	probabilistic perception, mental state contents, predictive processing, modeling reasoning and cognition, multisensory integration, reinforcement learning
Ethics	Risk and luck, normative uncertainty, population ethics, ethics of AI
Philosophy of Law	Evidence and standards of proof, algorithmic policing
Epistemology	All of formal epistemology, rationality, the nature of belief and knowledge, justification, judgment aggregation, reasoning
Metaphysics	The nature of chance, laws of nature
Philosophy of Science	Confirmation, philosophy of statistics, research ethics, laws of nature, scientific models, philosophy of physics, game-theoretic modeling in philosophy of biology
Philosophy of Religion	Fine Tuning arguments, Bayesian versions of the argument from evil, Pascal's wager

I propose two main strategies for increasing the number of non-enthusiasts who learn about probability as part of their philosophy coursework, either at the advanced undergraduate stage or at the graduate level. The first strategy is to redesign mandatory courses to cover probability. The most natural way to do this is to incorporate it into a required logic course. A standard way in which logic is taught at the advanced undergraduate and graduate levels is to cover predicate logic with identity, and to then move on to topics in metalogic, such as soundness and completeness proofs. While it is desirable to teach students about certain metalogical properties of the logical systems they employ, the usefulness of spending significant amounts of time on comprehending and reproducing the proofs of these properties is questionable. Assuming that most of our students are not aiming to become logicians, teaching them about formal methods that are widely used in many other areas of philosophy seems like a better use of their time. Hence, if we redesigned mandatory logic courses to cover predicate logic, basic probability, and perhaps some modal logic, we could ensure that even non-enthusiasts are equipped with some widely used fundamental formal tools.[3] Another possibility could be to add a formal methods course of the form just described to the curriculum in addition to a more traditional course in logic and metalogic, but allowing either one of these courses to satisfy the program's logic requirement. Of course, we can even incorporate this type of content into introductory critical thinking courses. As I briefly mentioned above, I teach basic probabilistic and statistical reasoning as part of such a course, emphasizing basic statistical literacy and understanding of the scientific method.

The second strategy for teaching probability to non-enthusiasts is to incorporate it into classes that aren't explicitly about probability or formal methods. Since most areas of philosophy contain debates that make use of probability as a tool, we can design our courses in these areas to incorporate these debates as well as some tutorials on probability and related material. For example, I teach a graduate seminar on

---

[3] See Bonino et al. (2021) for a study showing that the logical methods used in analytic philosophy are typically not very advanced.

Reasoning and Rationality that is usually listed either as a seminar in epistemology or in philosophy of mind. It covers questions such as: What is reasoning, and what distinguishes it from other mental activities? Does good reasoning have to follow the rules of logic and probability? What do we learn about human rationality from experiments about reasoning fallacies, such as the conjunction fallacy and the base rate fallacy? How should we model cognition? Are Bayesian models good models of cognition? How plausible are dual process models of reasoning?

It is easy to see from these questions that knowing basic logic and probability theory is essential to engaging with these questions. Logic and probability theory function in these debates both as potential normative standards for good reasoning and as parts of descriptive models of reasoning and cognition. As a result, this class brings up lots of opportunities for teaching students probability theory and then applying it to issues in epistemology and the philosophy of mind. However, the class is not perceived by students to be just about formal methods, but rather as a class that covers central themes in the philosophy of mind and epistemology. For this reason, it attracts students who are unlikely to enroll in a non-mandatory class that is explicitly about probability. This is just one example, but the reader can easily imagine how one might design topic-oriented courses in other areas of philosophy that lend themselves to incorporating lessons about probability. The same strategies can also be applied in upper-division undergraduate courses. For example, courses in epistemology, philosophy of mind or philosophy of science would be especially well-suited for this approach. In fact, I have taught a version of this class as a capstone course for graduating majors, with good success.

When integrating probability into a topic-oriented course, there are some pitfalls to avoid. The first pitfall concerns prerequisites. Students don't necessarily expect there to be any prerequisites for a course that is not advertised specifically as a formal methods course. However, for a course like the one I just described, knowledge of basic propositional logic and some high school algebra is essential. It is important to clarify this when advertising the course and at the beginning of the semester, so students can accurately gauge whether they are prepared to take it. Second, in designing the syllabus, it is important to actually leave enough time for students to learn and practice the formal material. It is tempting to allocate very little time to teaching probability, because we don't want to "sacrifice" too much substantive content. But if our goal is for the students to build enough of an understanding of probability that they can recall and deploy this knowledge on their own later, we need to allocate enough time for teaching and practicing it. This doesn't mean that several lessons in a row need to be devoted to just teaching probability. It's also possible to weave different aspects of probability into the lessons as they come up after an initial tutorial.

An additional pitfall is to not devote enough time to understanding the role probabilistic models play in the substantive theories under discussion. We can't expect that after teaching students the rules of probability, they will see on their own why probabilistic models are employed in a specific debate. We need to make the role and usefulness of the formal models an explicit topic of discussion. This involves slowing down enough to make sure students understand what the philosophical contribution of the formal models is supposed to be in a given debate, and allowing them to question whether the formal models are necessary and working as intended. In short: teaching formal material, including probability, to achieve understanding can't be rushed. There is little benefit in including this material in a course if we're not going to give students the time they need to master it.

3. Lesson Design
3.1 Readings

Let's suppose you are teaching a course that incorporates probability in some way. When designing the syllabus, an important component is selecting appropriate readings. The same material, such as rules of probability, arguments for probabilism, and applications to issues like scientific reasoning and confirmation, is presented very differently by different authors and the amount of formal knowledge and sophistication expected from the reader varies considerably. As we saw above, reading formal material is difficult for students even when their readings skills are generally good. Until very recently, it was difficult to find teaching materials covering probability that were written specifically with a philosophy audience in mind, but fortunately, things are looking much better now. The table in the appendix lists some recent texts and resources that I have either used to teach probability myself, or that have been recommended to me by other philosophers who have used them successfully. This list is by no means exhaustive, but it can serve as a good start, and it contains resources appropriate for both introductory and advanced classes. My current favorite for upper-level undergraduate and graduate courses is Titelbaum's forthcoming textbook *Fundamentals of Bayesian Epistemology*, which I consider the most accessible and comprehensive work to date that covers how probability theory is used in epistemology. For introductory critical thinking courses, I usually assign chapter 6 of Skyrms' *Choice and Chance*, which builds on their understanding of basic propositional logic and helps lay the foundation for the more applied readings mentioned in endnote 2.

Carefully selecting appropriate readings is a good first step towards helping students learn this material, but texts that contain formal content still present special challenges to students, even if they are written as accessibly as possible. To help students overcome these challenges, there are several things we can do in our teaching, most of which require little extra effort by the instructor.

One easy thing we can do is manage students' expectations about what it will be like to read these texts and equip them with strategies to succeed. Students often get discouraged because they expect to be able to read formal texts with the same amount of effort as informal texts, and when this turns out to be false, they give up. Hence, talking about how much time they should allocate and why reading this material takes extra skills and effort is a good start. This should also inform the instructor's decision on how much reading to assign. If students are usually asked to read 30-40 pages of informal reading per week, it is probably unrealistic to expect that they will be able to read as many pages of formal material. Further, we can also teach them about particular strategies for reading formal material. For this purpose, we can draw on existing resources. Many universities have put together handouts and worksheets for students with tips for reading their formal textbooks. These are usually intended for students in math and science classes, but the same advice applies for students reading about logic and probability in philosophical contexts. These worksheets advise students to work through examples on their own, consult graphs and diagrams, and go back and forth between trying to understand the big picture and the details, among other things. Googling "strategies for reading math textbooks" will bring a good number of results.[4]

---

[4] Here are some sample links with useful resources for students:
https://www.cuesta.edu/student/resources/ssc/study_guides/mathematics/214_math_text.html
https://learningcenter.unc.edu/tips-and-tools/readingmathtexts/
https://www.macalester.edu/max/wp-content/uploads/sites/120/2013/10/HowtoRead.pdf

Beyond giving this type of general advice, there are also strategies that target individual readings and need to be implemented specifically for every new reading. For example, we can devote 5-10 minutes of class to giving students a preview of an upcoming reading. By letting students know what the main point of the reading will be and perhaps telling them what they should focus on and what they can ignore, students know what to expect, and they have at least some prior knowledge that will help them orient themselves in the text. For especially difficult texts, or texts that contain a lot of new formal content, it might also be useful to provide the students with some annotations to the reading. This kind of "cheat sheet" might contain explanations of unusual notation, fill in background information or explain a difficult concept, or explain in plain English what some of the formulas say or how to interpret a diagram. Either way, it's important to remember that students are easily discouraged by unfamiliar notation, even if a text is otherwise accessibly written. Hence, at a minimum, we should familiarize the students with the notation they need to know to understand the reading.[5]

A more interactive strategy is to set students up to help each other with the reading. Many universities provide instructors with access to software that lets students annotate a text collaboratively, such as *Perusall*. It allows the instructor to upload a reading, and then the students can post questions and comments in the margins for other students to see and interact with. This can be set up either for small groups or for the whole class. Instructors can require students to provide some number of annotations, and they can also respond to queries or add explanations themselves. This method works similarly to the instructor-provided cheat sheet, but it gets the students involved in helping each other. However, it is slightly more labor-intensive for the instructor, because they would have to check for unanswered questions and incorrect explanations. Discussion boards or similar types of platforms could be used for the same purpose.

Another good method for helping students understand formal material is to provide them with reading questions to answer before class. If the reading questions are designed to promote understanding, they can help students identify what is important and guide their attention to particular aspects of the text. Examples of such questions are: What is the main question the author is trying to answer? What function does the example on p. X serve? What does part Y of the formalism represent? Why is the author modifying the model on p. Z? What philosophical assumption is underlying this part of the formal model? These reading questions can either be formulated as short essay questions, or students can be given a multiple-choice quiz. On online platforms, students can be given multiple attempts to complete a quiz to encourage them to think about the reading carefully and figure out the correct answers on their own.

The approach I have used in my own classes with good results combines these methods as follows: before students do the reading, I spend some time in class explaining the main points and possible sticking points of a reading, usually with the help of a short handout. I illustrate new probabilistic concepts or

---

[5] Much of the literature on pre-reading activities focuses on teaching K-12 mathematics, but a recent article by Butler has some useful examples of pre-reading activities for college classes. In section 3, she lays out various ways of teaching new formal vocabulary to students prior to reading, and she also suggests various strategies for drawing connections to students existing knowledge prior to reading (Butler 2019). A method that could be adapted very well to teaching probability is to have students draw tables in which they record the following for new notation: a symbol and its meaning in words, an application of that symbol in context and a written explanation of its meaning in context, and alternative notations capturing the same concept. Butler illustrates a few variations on this basic method.

applications with very basic examples, which build a foundation for understanding more complex applications in the reading.[6] Students are then asked to write down and send me questions on the reading as well as critical comments, which form the basis of discussion for the following class meeting. The primer I provide makes the reading less intimidating to the students, and by checking their reading notes, I get a very good sense of what they already understand and what needs further clarification.

3.2 Presenting Formal Material in Class

When presenting formal material in class, our teaching methods should be informed by the fact that many students find engaging with formal material inherently stressful and that they might lack confidence in their ability understand and master it. A good strategy (not just in light of this, but more generally) is to start with the simplest possible examples of the general phenomenon we're trying to illustrate, and to allow the students to become familiar and engage with those examples. For example, when learning about Dutch book arguments, students can experiment with setting up simple Dutch books and explore which combinations of buying and selling bets will lead to guaranteed losses. Research on how to reduce math anxiety recommends providing many simple examples (O'Leary et al. 2017), as well as making time for hands-on manipulation of examples (Iossi 2007).[7] Once students have really grasped the relevant concept, such as setting up a betting arrangement with a guaranteed loss, in a simple setting, they are more prepared to understand it in a more abstract, generalized setting. Starting with simple examples also has the benefit of allowing students to at least partially grasp the material. If a student understands how the examples work, but perhaps doesn't follow entirely how a more general, abstract theorem is derived, the student still benefits from having this more concrete understanding of the matter.

If possible, students should be given the opportunity to work through examples *in class*, not just as part of a homework assignment. Since students tend to have trouble persevering on their own and find it difficult to correct their mistakes and misunderstandings, especially when encountering new material, it is suboptimal to delegate work on examples exclusively to homework assignments. By generating and manipulating examples in class, students can work collaboratively and get help from the instructor when they get stuck, which then prepares them to do further work on their own. To enable students to take full advantage of these lessons, instructors should make sure that students can access class materials later. For example, instructors can make handouts, specifically set aside time for students to take notes, allow them to photograph the board, record the lecture, etc. Trying to understand new material while also taking accurate notes in real time can be overwhelming, so easing the students' cognitive load by allowing for

---

[6] For example, students tend to initially have trouble understanding why conditional probabilities are not the same when we switch the two propositions, i.e., why $P(A|B)$ can differ wildly from $P(B|A)$. I use intuitive examples to get the students to work out why they must often be different. If I assign A = 'a person gets a speeding ticket', and B = 'a person is speeding', students can figure out that $P(A|B)$ is very low, because most of the time we don't get a ticket for speeding, but $P(B|A)$ is high, because given that someone gets a speeding ticket, they were probably speeding. Another example that works well is A = 'Peter has a cockatoo' and B = 'Peter has a pet'. By using these examples as easy to remember anchor points, students are no longer tempted to confuse $P(B|A)$ with $P(A|B)$ later on.

[7] O'Leary et al.'s study aims to identify factors that are related to increased and decreased math anxiety. They report that "there was a significant decrease in MA when participants reported that their teachers provided plenty of examples and practice items, and this remained after controlling for general and test anxiety." (O'Leary et al. 2017, p.9) In a study by Harper and Daane (1998), providing hands-on active learning experiences significantly decreased math anxiety in a group of elementary school teachers in training.

various ways of accessing class content later is a good teaching strategy. This takes some of the stress out of trying to "keep up" during class.

How students perceive their progress and chance of succeeding partly depends on what they think is required to master the material we want them to learn. As research by Leslie et al. (2015) has shown, both philosophy and mathematics are fields in which the belief is common that one must have a special innate talent to be successful. The authors argue that the prevalence of such "genius beliefs" in a field is highly correlated with whether women and African Americans are underrepresented in it, as these groups are stereotypically not viewed as having an exceptional amount of innate talent in those areas. This research is relevant for our purposes, because we don't want to reinforce the belief in our students that they can't master formal methods in philosophy (or more generally be a good philosopher) unless they possess some sort of innate talent. Countering these stereotypes not only benefits students who are members of marginalized groups, but also any students with math anxiety. Students with math anxiety often display a fixed mindset, which means that they perceive their ability to learn formal material as limited and unchangeable (see Dweck 2016 for research on fixed vs. growth mindsets). To reach these students and inspire confidence in them that they can learn probability theory and other formal methods, it is especially important to avoid reinforcing any sort of "genius" beliefs about succeeding as a philosopher.

Many readers probably think now that they are already doing pretty well in this respect. After all, who tells their students things like "You better be a mathematical genius, or else there is no point for you in studying formal philosophy?" Yet, we may be less aware of the many subtle ways in which we emphasize talent and achievement rather than growth and progress. For example, we might praise certain philosophers for their great minds and insights, without ever talking about the struggles and wrong turns they took in developing their views, thus inadvertently creating the impression that these philosophers followed a straight and easy path to success. This notion is easily reinforced by the articles our students read, and the talks they attend, in which formal philosophy is presented. Students only see the final result, because all the errors and unsuccessful attempts to solve a problem are usually omitted from the final product. Again, this inadvertently creates the impression in an inexperienced audience that there simply were no mistakes or clunky first attempts. Hence, unless we make a point of emphasizing to our students that all the polished and elegant results they encounter went through a messy and laborious creation process, students have little reason to believe that they did not spring fully formed out of the mind of a genius. If we keep this in mind, we can easily incorporate ways of presenting formal material that don't elide the creation process. Even sharing short anecdotes or stories about the struggles that were involved in generating ideas can make a difference.[8] Further, if we arrange for students to meet with philosophers, for example as part of a colloquium talk or if they visit a seminar, we can ask them to talk to the students about the genesis of their work, thus helping the students see the messy business of "how the sausage is made."

---

[8] In one study with 400 9th and 10th grades students, the hypothesis that hearing about the ways in which researchers struggled would positively impact student performance was confirmed. Students (see e.g. Ashcraft and Ridley 2005, Ramirez et. al. 2018) either read stories about scientists' great achievements, or stories about how scientists struggled in their life or work. Reading achievement stories had a slightly negative effect on subsequent grades, but reading struggle stories had a positive effect. This was especially true for low achieving students (Lin Siegler et al. 2016).

A related issue in teaching formal material concerns students' willingness to ask questions and get help. Students with math anxiety might worry that asking a question might make them look stupid, or that they are the only one not understanding. Unsurprisingly, the usual "does anyone have any questions?" is often followed by silence, although it is completely obvious to the instructor that many students have not fully grasped the material. A way around this problem is to get every student to ask a question, for example by using a "muddiest point" assignment. Students are asked to write a brief note during or at the end of class where they describe what they found most confusing or difficult. The instructor collects them and decides on their basis which points need reinforcement. Depending on how the instructor plans on responding, the questions can, but don't need to be, submitted anonymously. Asking students to submit a question as part of a reading notes assignment is another good option, which I frequently use in my classes.

Two further points are worth keeping in mind. First, students should be allowed to question the usefulness of applying formal models to specific philosophical problems. If they are putting in the effort to study these models, the question of whether doing so is worth the payoff is entirely legitimate, and there should be room for discussing it explicitly. Second, instructors might be worried that spending time on exploring simple examples and going through formal material slowly might leave their best students bored. This might occasionally be the case, and we can often keep these students interested by giving them additional readings or questions to think about. But generally, our aim should not be to tailor our classes to the needs of our highest achieving students. They'll be fine.

4. Assignment Design

Students, especially students with math anxiety, tend to experience assignments about formal material as being far more stressful than other assignments, such as short essay exams or a research papers. It is up to the instructor to choose assignment formats that are less stressful for students. One way to achieve this is to design exams as take-home exams rather than timed in-class tests. A possible concern with this is that it might increase academic dishonesty, since cheating can be difficult to detect for assignments that have clear right and wrong answers, as logic and probability exercises often do. This problem can be circumvented to some extent by asking students to add explanations to their calculations, and by asking short-essay comprehension questions about the material. This is feasible at least in small classes, although it could lead to a significantly increased grading load in larger classes (compared to, e.g., in-class multiple choice tests).

Another good strategy, which actually reduces the instructor's workload, is to offer completion credit on homework assignments instead of assigning grades. The idea here is that students get credit for working through a problem set, independently of whether they get the correct answers. The instructor can then release an answer key that students can use to check their own work and to ask for feedback when needed. Another option is to go over the problems in class to make sure everyone knows the correct solutions. By offering completion credit only if a student attempts to answer each question, students are encouraged to at least give each problem a try. This addresses the problem that students with math anxiety sometimes get low scores because they don't even try to answer questions that look unfamiliar or difficult, although they could have solved them with a little effort. In my logic and probability classes, the weekly assignments usually have completion credit, and later there is a take-home assignment that is graded. This

gives students opportunities to practice the material in a low-stakes context before their performance is graded.[9]

For small classes, the following strategies might also be feasible, though they would be difficult to scale up: Students must set up a mandatory office hour appointment before or while they are working on an assignment, so the instructor can help each student with aspects of the material they don't understand before the student submits their work. Another possibility is to allow students to retake exams until they get a satisfactory score. This method is called "mastery grading," and it emphasizes reaching a certain level of competence with the material instead of measuring a student's performance by giving them a grade for a specific assignment or exam. Depending on the task, students can either keep working on improving a specific assignment, for example rewriting an essay, or they can take additional versions of an exam to demonstrate that they have mastered a particular skill, such as using Bayes' theorem or working with stochastic truth-tables (see, e.g., Cilli-Turner et al. 2020 for specific ideas of how to implement mastery grading). However, this technique can be difficult to implement in larger classes, because it potentially creates an unmanageable amount of grading for the instructor. Intermediate solutions are also possible, in which students get limited opportunities to retake assignments for a better grade.

When designing exams and assignments, instructors should include some easier and some harder questions. Bloom's taxonomy provides a great guide for classifying how difficult a question is (see Anderson et al. 2001, overviews are easy to find online). More difficult questions require students to apply their knowledge to new types of cases and to think independently, whereas easier questions ask for factual knowledge or for familiar ways of applying knowledge. By varying the difficulty of the questions included in an assignment, students who have not fully mastered the material are given the chance to complete at least part of the assignment and earn a passing grade. By contrast, an assignment that can only be understood and completed by someone who has fully mastered the material does a poor job of measuring the degree of the student's competence. This not only defies the purpose of exams and assignments, it is also demoralizing to students who have worked hard to attain at least a partial understanding of the topic.[10]

---

[9] I have found in my formal epistemology seminar that giving assignments with completion credit really encourages students to try working through the material. My most remarkable experience was with a student, a philosophy major, who had been homeschooled and who had learned very little math. Experimenting with the probability exercises showed him that he was a lot more capable than he had feared, and he went on to finally catch up on the math education he had previously missed.

[10] Here are some sample question types for each of Bloom's levels:

Knowledge: Stating definitions and rules of probability, other concepts that can be memorized.

Comprehension: Identifying what falls and doesn't fall under a definition or concept, applying one rule to a predefined case or exercise. This could involve determining which probability rules should be used in particular cases, judging the applicability of different interpretations of probability to specific cases (logical, frequentist, subjective, …) etc.

Application: Solving problems that require applying more than one rule or concept, but in a way that follows a familiar, previously practiced pattern. This could involve, for example, applying Bayes' theorem to a word problem in a way the students have practiced before.

Analysis: Solving problems that require students to figure out the appropriate rules to apply, and to explain their answers. This requires a higher level of skill than application tasks, since the students have to flexibly apply their knowledge to cases that don't exactly follow a known pattern.

Synthesis: Similar to analysis questions, but students are asked to develop more general rules or methods to solve certain types of problem. This could also involve asking students to classify problems. For example, if devising a particular Dutch book could be a question at the application or analysis level, showing why any violation of a specific probability rule would generally lead to a Dutch book is a synthesis-level question.

Further, as mentioned before, instructors should take care to stay consistent with formal notation that is used in the class and on exams, and they should help students understand alternative notation when it comes up.

Conclusion

Knowledge of probability is essential for philosophers in many different subfields, even if they don't primarily conceive of themselves as doing formal philosophy. Yet, students who aren't formal philosophy enthusiasts are unlikely to seek out courses about probability, because they often don't realize how many current debates in philosophy rely on probabilistic models in some way. I have offered two strategies for teaching probability to those students: we can either modify required (logic) courses to incorporate probability theory, or we can design courses in particular areas of philosophy to incorporate material from debates that use probabilistic models.

In teaching probability and other formal material to students who are not formal philosophy enthusiasts, we should be mindful of two phenomena that can make it difficult for these students to learn: math anxiety and difficulties with reading formal texts. There is good reason to think that they affect many of our students, and that they can prevent our students from reaching their full potential. I have offered strategies for course design, reading selection, lesson planning, and assignment design that are intended to help *all of our students* master formal methods in philosophy, even if they initially arrive in our classroom being anxious about math, or without knowing how to read formal material.

Appendix

This table contains a non-exhaustive list of resources for teaching probability to undergraduates and graduate students. All entries are also listed in the bibliography, and links are provided there. For forthcoming pieces that are not linked in the bibliography, contact the authors directly.

Text/Resource	Description	Level
Bradley (2015): *A Critical Introduction to Formal Epistemology*	Textbook that covers probability and rational belief in relation to various classic problems in epistemology	Undergraduate

---

Evaluation: Questions that require a high level of judgment regarding which information needs to be used and how the rules should be applied to it. This could involve analyzing common epistemological problems, for example about updating on testimony, judgment aggregation, learning or forgetting in probabilistic terms. The questions commonly discussed in formal epistemology papers are on this level.

This is a helpful table for interpreting Bloom's taxonomy in the context of mathematics: http://www.math.toronto.edu/writing/BloomsForMath.html

Christensen (2004): *Putting Logic in its Place*	Research monograph that covers rationality constraints on belief based on logic and probability theory. Written at a very accessible level	Advanced undergraduate and graduate
Easwaran, Kenny (forthcoming): *An Opinionated Introduction to the Philosophical Foundations of Bayesianism*	Textbook that covers Bayesianism, in particular the philosophical justification for using Bayesian models of belief in epistemology	Advanced undergraduate and graduate
Fitelson, Branden: PrSAT (decision procedure and Mathematica plugin). Described in "A Decision Procedure for Probability Calculus with Applications" (forthcoming)	PrSAT is a decision procedure for the classical probability calculus. It has a Mathematica implementation, i.e., there is a plugin for this popular software that can solve various probability problems.	Advanced undergraduate and graduate
Hacking (2001): *An Introduction to Formal Epistemology and Logic*	Textbook that covers probability and its interpretations, some decision theory, the problem of induction	Undergraduate
Hacking (2006): *The Emergence of Probability*	Research monograph that covers the history of probability theory	All levels
Hájek & Hitchcock (2016): *The Oxford Handbook of Probability and Philosophy*	Comprehensive handbook that covers probability theory and its applications in philosophy. Has some entries on historical issues and alternatives to standard probability	Advanced undergraduate and graduate
Halpern (2017): *Reasoning about Uncertainty*	Reference book that covers different formal ways of representing uncertainty, not just standard probability theory	Advanced undergraduate and graduate
Howson & Urbach (2006): *Scientific Reasoning*	Textbook that covers probability theory in the context of scientific and statistical reasoning	Advanced undergraduate and graduate
Knauff & Spohn (2021): *Handbook of Rationality*	Handbook that covers formal theories of rationality as well as applications of formal methods in cognitive science	Advanced undergraduate and graduate
Lindley (2014): *Understanding Uncertainty*	Textbook that provides a very detailed introduction to probability theory, some decision theory and scientific reasoning	Advanced undergraduate and graduate
Pettigrew & Weisberg (2019): *The Open Handbook of Formal Epistemology.*	Online handbook with detailed entries on various topics in probability theory, doxastic logic, conditionals, and alternatives to probability theory	Advanced undergraduate and graduate

Pettigrew (2020): *Dutch Book Arguments*	Short introductory text focused on Dutch book arguments	Advanced undergraduate and graduate
Rowbottom (2015): *Probability*	Textbook that covers probability theory and its different interpretations	Beginning undergraduate
Skyrms (2000): *Choice and Chance*	Textbook that covers basic logic, probability, and various issues related to induction	Undergraduate
Sprenger & Hartmann (2019): *Bayesian Philosophy of Science*	Research monograph that covers basic probability and applies it to a wide variety of debates in current philosophy of science	Advanced undergraduate and graduate
Strevens (2017): *Notes on Bayesian Confirmation Theory*	Lecture notes that cover probability theory and its application to confirmation	Advanced undergraduate and graduate
Sturgeon (2020): *The Rational Mind*	Primarily a research monograph about the nature of belief, but written for readers who aren't necessarily familiar with formal epistemology	Graduate
Titelbaum (forthcoming): *Fundamentals of Bayesian Epistemology*	Textbook that covers probability theory and its main applications in formal epistemology, decision theory, arguments for probabilism	Advanced undergraduate and graduate
Weatherson (2015): *Lecture Notes on Decision Theory*	Lecture notes that cover probability theory, decision theory, and some game theory	Advanced undergraduate and graduate
Weirich (2021): *Rational Choice Using Imprecise Probabilities*	Short introductory text on using imprecise probability in decision theory	Advanced undergraduate and graduate

Bibliography

Anderson, Lorin W. and Krathwohl, David R. (eds). 2001. *A Taxonomy for Learning, Teaching and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. New York: Longman.

Ashcraft, Mark H. and Ridley, Kelly S. 2005. Math Anxiety and its Cognitive Consequences: A Tutorial Review. In: Jamie I. D. Campbell (ed.), *Handbook of Mathematical Cognition*. Ney York: Psychology Press, 315-330.

Bonino, Guido; Maffezioli, Paolo and Tripodi, Paolo. 2021. Logic in Analytic Philosophy: A Quantitative Analysis. *Synthese* 198, 10991-11028.

Bradley, Darren. 2015. *A Critical Introduction to Formal Epistemology*. London/New York: Bloomsbury.

Bright, Liam Kofi. 2019. Just a Humble Philosopher. Blog post available at: https://sootyempiric.blogspot.com/2019/11/just-humble-philosopher.html

Butler, Melanie. 2019. Preparing our Students to Read and Understand Mathematics. *Journal of Humanistic Mathematics* 9 (1), 158-177. https://scholarship.claremont.edu/jhm/vol9/iss1/8

Christensen, David. 2004. *Putting Logic in its Place*. Oxford: Oxford University Press.

Cilli-Turner, Emily; Dunmyre, Justin; Mahoney, Thomas and Wiley, Chad. 2020. Mastery Grading: Build-A-Syllabus Workshop. *PRIMUS* 30, 8-10, 952-978.

Dweck, Carol S. 2016. *Mindset: The New Psychology of Success*. New York: Ballantine.

Easwaran, Kenny. Forthcoming. *An Opinionated Introduction to the Philosophical Foundations of Bayesianism*. London/New York: Routledge.

Fitelson, Branden. Forthcoming. A Decision Procedure for Probability Calculus with Applications. *Review of Symbolic Logic*.

Fitelson, Branden. 2021. PrSAT plugin for Mathematica. http://fitelson.org/PrSAT/

Fletcher, Samuel C.; Knobe, Joshua; Wheeler, Gregory and Woodcock, Brian A. 2021. Changing Use of Formal Methods in Philosophy: Late 2000s vs. Late 2010s. forthcoming in *Synthese*, available at: http://philsci-archive.pitt.edu/19575/

Gigerenzer, Gerd; Gaissmaier, Wolfgang; Kurz-Milcke, Elke; Schwarz, Lisa M and Woloshin, Steven. 2008. Helping Doctors and Patients Make Sense of Health Statistics. *Psychological Science in the Public Interest* 8 (2), 53-96.

Hacking, Ian. 2001. *An Introduction to Probability and Inductive Logic*. Cambridge: Cambridge University Press.

Hacking. Ian. 2006. *The Emergence of Probability: A Philosophical Study of Early Ideas About Probability Induction and Statistical Inference (2nd edition)*. Cambridge: Cambridge University Press.

Hájek, Alan and Hitchcock, Christopher. 2016. *The Oxford Handbook of Probability and Philosophy*. Oxford: Oxford University Press.

Halpern, Joseph. 2017. *Reasoning about Uncertainty (2nd edition)*. Cambridge: MIT Press.

Harper, Norma Wynn and Daane, C.J. 1998. Causes and Reduction of Math Anxiety in Preservice Elementary Teachers. *Action in Teacher Education* 19 (4), 29-38.

Howson, Colin and Urbach, Peter. 2006. *Scientific Reasoning: The Bayesian Approach (3rd edition)*. Chicago: Open Court.

Iossi, Laura. 2007. Strategies for Reducing Math Anxiety in Post-Secondary Students. In: S. M. Nielsen and M. S. Plakhotnik (eds.), *Proceedings of the Sixth Annual College of Education Research Conference: Urban and International Education Section*. Miami: Florida International University, 30-35. http://coeweb.fiu.edu/research_conference/

Knauff, Markus and Spohn, Wolfgang. 2021. *The Handbook of Rationality*. Cambridge: MIT Press.

Leslie, Sarah-Jane; Cimpian, Andrei ; Meyer, Meredith and Freeland, Edward. 2015. Expectations of Brilliance Underlie Gender Distributions Across Academic Disciplines. *Science* 347 (6219), 262-265.

Lindley, Dennis V. 2014. *Understanding Uncertainty (2nd edition)*. Hoboken: Wiley.

Lin-Siegler, Xiaodong; Ahn, Janet; Chen, Jondou; Fang, Fun-Fen Anny and Luna-Lucero, Myra. 2016. Even Einstein Struggled: Effects of Learning about Great Scientists' Struggles on High School Students' Motivation to Learn Science. *Journal of Educational Psychology* 108 (3), 314-328.

O'Connor, Cailing and Weatherall, James. 2019. *The Misinformation Age: How False Beliefs Spread*. New Haven: Yale University Press.

O'Leary, Krystle; Fitzpatrick, Cheryll L. and Hallett, Darcy. 2017. Math Anxiety is Related to Some, but Not All, Experiences with Math. *Frontiers in Psychology* 8, Article 2067.

Pettigrew, Richard and Weisberg, Jonathan. 2019. *The Open Handbook of Formal Epistemology*. PhilPapers Foundation, Open Access.

Pettigrew, Richard. 2020. *Dutch Book Arguments (Cambridge Elements in Decision Theory and Philosophy)*. Cambridge: Cambridge University Press.

Ramirez. Gerardo; Shaw, Stacy T. and Maloney, Erin A. 2018. Math Anxiety: Past Research, Promising Interventions, and a New Interpretation Framework. *Educational Psychologist* 53 (3), 145-164.

Rowbottom, Darrell P. 2015. *Probability*. Cambridge: Polity.

Shepherd, Mary D.; Selden, Annie and Selden, John. 2012. University Students' Reading of Their First-Year Mathematics Textbooks. *Mathematical Thinking and Learning* 14, 226-256.

Skyrms, Brian. 2000. *Choice and Chance (4ᵗʰ edition)*. Belmont: Wadsworth/Cengage.

Sprenger, Jan and Hartmann, Stephan. 2019. *Bayesian Philosophy of Science.* Oxford: Oxford University Press.

Strevens, Michael. 2017. *Notes on Bayesian Confirmation Theory.* Manuscript, available at: http://www.strevens.org/bct/BCT.pdf

Sturgeon, Scott. 2020. *The Rational Mind*. Oxford: Oxford University Press.

Titelbaum, Michael. Forthcoming. *Fundamentals of Bayesian Epistemology*. Oxford: Oxford University Press.

Weatherson, Brian. 2015. *Lecture Notes on Decision Theory*. Manuscript, available at: http://brian.weatherson.org/DTBook-15.pdf

Weinberg, Aaron; Wiesner, Emilie; Benesh, Bret and Boester, Timothy. 2012. Undergraduate Students' Self-Reported Use of Mathematics Textbooks. *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies* 22 (2), 152-175.

Weirich, Paul. 2021. *Rational Choice Using Imprecise Probabilities (Cambridge Elements in Decision Theory and Philosophy)*. Cambridge: Cambridge University Press.

Zhang, Jing; Zhao, Nan and Kong, Qi Ping. 2019. The Relationship Between Math Anxiety and Math Performance: A Meta-Analytic Investigation. *Frontiers in Psychology* 10, Article 1613.