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FABIO STERPETTI*

FORMALIZING DARWINISM, NATURALIZING MATHEMATICS

1. Evolutionism and Mathematics

In the last decades two different and apparently unrelated lines of research have increasingly connected mathematics and evolutionism. Indeed, on the one hand, different attempts to formalize Darwinism have been made (see for a survey Barberousse and Samadi, 2015; Thompson, 2007), while, on the other hand, different attempts to naturalize logic and mathematics have been put forward (see for a survey Dutilh Novaes, 2012; Van Kerkhove, 2006).

The works of the scientists who have tried to shed light on the cognitive mechanisms underlying human reasoning and the evolutionary roots of our cognitive abilities (e.g. Dehaene *et al*. 2005), and the work of the mathematicians who have tried to construct more and more sophisticated mathematical models of the evolutionary processes (e.g. Batty *et al*. 2014), have stimulated interesting philosophical reflections in the fields of philosophy of mathematics and philosophy of biology.

These researches may appear either completely distinct or at least in some way convergent. They may in fact be seen as both supporting a naturalistic stance. Evolutionism is indeed crucial for a naturalistic perspective, and formalizing it seems to be a way to strengthen its scientificity and make biology more akin to the "hardest" sciences. This would allow evolutionism to better resist the attacks of the anti-naturalists. With regard to the researches devoted to showing how mathematics and logic are related to our biological features, given that mathematics and logic have been so long (and often are still) considered as unrelated to our human bodily constitution, and given their centrality for science, these attempts may even more easily be seen as connected to a naturalistic view.

On the contrary, in what follows we will underline some of the difficulties that such lines of research have to face. These difficulties lay in the possibility that the conception of knowledge on which they rest may be undermined by the consequences of accepting an evolutionary perspective.

* Dipartimento di Filosofia, Sapienza Università di Roma, fabio.sterpetti@uniroma1.it.
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1.2. Formalizing Darwinism

The numerous attempts to formalize Darwinism which have been put forward may be better understood considering the prominence of evolutionism in biology, and the paucity (if not the absence) of laws in biology, which has often been intended as a lack of rigorousness and reliability of such science.

Indeed, according «to most evolutionary biologists and philosophers of biology, the theory of evolution provides theoretical foundations, as well as conceptual unity, to all other domains of biology», but despite «their importance, the principles of evolutionary theory have seldom been explicitly formulated» (Barberousse and Samadi, 2015, pp. 229-230).

For example, Chaitin states that «it is a mathematical scandal that we do not have a proof that Darwinian evolution works» (Chaitin, 2012, p. 53).

Biology may in fact be defined an "inexact science" to the extent it is not "mathematical" or "mathematized". This is due to the fact that «since mathematics is often understood as (...) an exact science, the use of mathematics to specify or represent a scientific theory is one way to interpret a science as "exact". (…). Thus, non-mathematical (or as yet un-mathematized) biological sciences are inexact sciences» (Griesemer, 2013, p. 298).

The Darwinian core of evolutionism, i.e. the relevance and functioning of natural selection, was unsatisfactorily formulated and gave rise to many controversies. Thus, many have tried to formalize Darwinism «to provide a non-ambiguous non-tautologous translation of "survival of the fittest"» (Williams, 1973, p. 84).

Different authors try to prove the correctness of Darwinism in different ways: by giving a formalization of Darwin's arguments (Tennant, 2014); by constructing a mathematical argument aimed at showing that some core tenets of Darwinism may be formally proved (Chaitin, 2012); by mathematically representing the main evolutionary processes (Ao, 2005); or by showing that if biology is full of mathematical models which work and are built relying on Darwinism, this must mean that the distance of biology from physics with respect to the possibility of finding laws of nature is just «one of *degrees*, and not of *kind*» (Dorato, 2012, p. 109).

Formalizing Darwinism is aimed at strengthening it and at trying to make biology more akin to the "hardest" sciences. For example, Grafen states that:

> theories in physics are usually expressed in mathematics. Newton's mechanics and Einstein's theory of special relativity are essentially equations. Words are

needed to interpret the terms, but only for that. Darwin's theory of evolution by natural selection has obstinately remained in words since 1859. Of course, there are many mathematical models that show natural selection at work, but they are all examples. None claims to capture Darwin's central argument in its entirety (Grafen, 2007, p. 1243).

Similarly, Chaitin states that there «is empirical evidence in favor of Darwin's theory, but there is no mathematical proof. (…). If Darwin's theory is as fundamental as biologists think, then there ought to be a general, abstract mathematical theory of evolution that captures the essence of Darwin's theory and develops it mathematically» (Chaitin, 2012, p. 9).

1.3. The Formal Darwinism Project

To give just an example of one of the attempts to formalize Darwinism, we will briefly (and roughly) sketch the main lines of the Formal Darwinism Project (FDP) proposed and developed by Grafen (2014, 2007, 2002; Batty *et al*. 2014).

The Darwinian theory is considered to have «to explain two sets of phenomena: the organisms' adaptations and the transformations undergone by biological diversity over time» (Barberousse and Samadi, 2015, p. 232).

The first set of phenomena is usually modeled using optimality models, which indicate how fitness is maximized by an adaptive trait, while the second set of phenomena is modeled using population genetics models, which indicate how the gene frequencies change under the effect of natural selection. The problem is that such kinds of models differ on a crucial respect: population genetics models do not indicate that fitness is necessarily maximized (Grafen, 2014).

FDP is a debated (and rather controversial) attempt to construct a mathematical bridge between those two ways of studying natural selection.

Indeed, Grafen tries to prove that such approaches may be unified in order to secure what he consider to be the core of the Darwinian approach, i.e. the thesis that natural selection is the only relevant factor of evolution, and that this (roughly) means that fitness, on average, tends to increase.

For example, Grafen (2002) tries to show that for each population genetic model satisfying certain conditions, exists a corresponding optimality model of a specific kind. And the importance of the existence of this corresponding optimality model lies exactly in that «it provides an interpretation of some aspects of the population genetic model in optimality terms» (Grafen, 2002, p. 88).

One of the key assumptions of such a view is the claim that natural selection has a prominent role in evolution, i.e. adaptationism (Orzack and Forber, 2010). Adaptationism is a very debated view, and it has been fiercely criticized. But neither such well known criticisms, nor the mathematical details of FDP are relevant here. What is worth noticing is that Grafen searches for a confirmation of his view exclusively in some "formal" result: a «formal justification for» this kind of adaptationism «lies in the isomorphism between the dynamics of natural selection and an optimization program that captures the idea of the organism as striving to maximize her fitness» (Gardner, 2013, p. 791).

1.4. The Epistemological Shortcomings of Mathematization

Any attempt to formalize some empirical domain may face different, and well known, epistemological risks.

One of such risks is that of mistaking a mathematical model for the reality itself. For example, Cellucci states that to «claim that the laws of nature or the mathematics used in formulating them exist in the world is to confuse the tools by which humans make the world comprehensible to themselves with the world itself» (Cellucci, forthcoming, p. 9).

Moreover, in searching for a model, we tend to oversimplify the phenomenon we are modeling in order to be able to manage it from a computational point of view. For example, Chaitin states that what he calls "metabiology" has to be «a highly simplified version of real biology», because otherwise he «wouldn't be able to prove any theorems» (Chaitin, 2012, p. 4).

Finally, focusing on mathematized models may lead us to neglect important aspects of the phenomenon we are investigating. For example, Schwartz states that «the mathematical-intellectual effort of applying» the formalized theory with which we deal, «fixes in us» that «particular point of view, making us blind to whatever appears neither as a dependent nor as an independent parameter in its mathematical formulation». Thus, «the mathematical formalism may be hiding as much as it reveals» (Schwartz, 2008, pp. 22-25).

Despite the relevance of these issues, in what follows we will instead focus only on the peculiar challenges which derive from the attempts to formalize Darwinism for a naturalistic perspective on knowledge.

2. *Formalizing and Naturalism*

The main idea behind the attempts to formalize Darwinism is that our knowledge is strengthened if it is presented in an axiomatic and deductive way, i.e. if we can isolate some few principles and deductively derive from them all our knowledge in some field. Moreover, there is the underlying conviction that a mathematical model may be isomorphic to the dynamics of the phenomenon it models, so that if we could establish a mathematical result relative to such model, this will increase our insight on the phenomenon we are investigating.

These issues put in relation the attempts to formalize Darwinism to the debate over naturalism. Indeed, evolution is central to naturalism. For example, Giere states that if «evolutionary naturalism is understood to be a general naturalism informed by the facts of evolution and by evolutionary theory, then no responsible contemporary naturalist could fail to be an evolutionary naturalist in this modest sense» (Giere, 2006, p. 53).

Thus, strengthening Darwinism seems, *prima facie*, a way to support a naturalistic stance. But if such strengthening is pursued through formalization, and if what we think formalization is is related to what we think mathematics is, then we have to confront with the issue of giving a naturalistic account of mathematics, in order to maintain a naturalistic view of knowledge.

Indeed, many authors think that «mathematics is an enormous Trojan Horse sitting firmly in the center of the citadel of naturalism», because even if «natural science is mathematical through and through», mathematics seems to «provide a counterexample both to methodological and to ontological naturalism». In fact, mathematics ultimately rests on axioms, which are «traditionally held to be known a priori, in some accounts by virtue of a form of intuitive awareness». The epistemic role of the axioms in mathematics seems «uncomfortably close to that played by the insights of a mystic. When we turn to ontology, matters are, if anything, worse: mathematical entities, as traditionally construed, do not even exist in time, never mind space» (Weir, 2005, pp. 461-462).

In order to avoid to be forced to draw anti-naturalist conclusions, the point is not to dismiss the attempts to formalize Darwinism, but rather to scrutinize the mainstream view on formalization, and try to rethink the traditional claims on mathematics and knowledge on which such view is based.

2.1. Formalizing and Platonism

It is thus relevant to analyse more precisely what does it mean "to formalize" and its relationship with mathematics.

According to Dutilh Novaes (2012) "to formalize" refers mainly to two related concepts: 1) de-semantification, and 2) being computable.

1) "De-semantification" refers to the idea of creating a set of symbols which conveys no meaning, performing an «abstraction from all meaning whatsoever» (Dutilh Novaes, 2012, p. 13).

2) "Being computable" refers to the idea that such symbols may be mechanically manipulated according to certain completely specified rules, so that starting from an initial state, after a finite number of discrete steps, a final state is reached.

There is a clear homogeneity in this view between logic, mathematics and formal languages: a «logic can be formulated in an axiomatic form with a very minimal set of rules of transformation (typically, only *modus ponens*), or these very axioms can be formulated as rules of transformation» (*Ibidem*, p. 59), while a formal language is normally conceived of as a mathematical object, whose properties can be mathematically investigated. Indeed, the concept of «formal language is a mathematically well-defined concept with precise borders. A structure is a formal language iff it is defined by a finite collection of symbols and rules of formation which determine exactly what is to count as a permissible combination of symbols» (*Ibidem*, p. 58).

Thus, the characteristic features of mathematics, logic, and formal languages are those of being axiomatic deductive closed symbolic systems. This view is at odds with a naturalist stance, because the way in which it defines mathematics and logic makes impossible to naturalistically account for our ability in producing and using such abstract devices (Cellucci, 2013a, 2013b). Indeed, Dutilh Novaes also considers formal languages a "technology", i.e. something which can be used to solve practical problems. More specifically, she thinks that formal languages may be considered as: 1) a tool to model empirical phenomena; 2) a de-biasing tool for improving our reasoning.

This approach to formalization raises at least two main questions.

1) With regard to the issue of modeling empirical phenomena, there is the problem of the applicability of such "formal devices" to the world, i.e. the problem of explaining their efficacy in solving practical problems if, as stated above, to be formal means to be deprived of "whatsoever meaning". This problem is equivalent to the very well known problem of explaining the applicability and the efficacy of mathematics to the world (for a survey

see Jacquette, 2012). In fact, it is precisely for their abstractness and computability that such "formal devices" may be considered as mathematical objects.

But if logic and formal languages are abstract in the same way in which mathematics is traditionally considered to be abstract, then we cannot adopt a realist stance towards them for the same reason we cannot easily defend a realist position in mathematics if we want to adopt a naturalistic stance. Indeed, Mathematical Platonism (MP), the most widespread realist position in philosophy of mathematics (Balaguer, 2009), is considered to be unable to naturalistically account for our reliability about mathematics (Steiner, 1998). MP is the claim that mind-independent mathematical abstract entities exist. MP's main argument, the Indispensability Argument (IA), claims that the indispensable role of mathematics in scientific theories justifies the realist's claim about the ontological status of mathematical entities.

The problem is that MP cannot account naturalistically for the capability of humans to grasp mathematics. For example, Brown admits that he has «no idea how the mind is able to "grasp" or "perceive" mathematical objects and mathematical facts», but that he nevertheless is certain that it «is not by means of some efficient cause» (Brown, 2012, p. 12).

2) With regard to the de-biasing role of formal languages, there is the problem of linking the improvement of our cognitive abilities due to the use of formal languages with their evolutionary roots.

In fact, the mainstream (metaphysical) view in philosophy of science is Scientific Realism (SR), which claims that our best scientific theories are true, and that we can infer their truth from their empirical success (Psillos, 1999). The most shared view of truth among the realists is that of truth as correspondence[1]. For example, Sankey states that: «correspondence theories which treat truth as a relation between language and reality are the only theories of truth compatible with realism» (Sankey, 2008, p. 17). Moreover, many realists view themselves as naturalists, and thus should commit themselves with evolutionism.

If we take our best theories to be true, then it is obvious to measure the distance from the truth referring to such theory.

[1] The realism/anti-realism debate is so wide that it is impossible to account for all the realist positions which have been elaborated, so in what follows we will concentrate on what can be labeled a "standard" realist position. Analogously, many positions have been elaborated on the issue of truth (see for a survey Burgess and Burgess, 2011). Thus, even if truth as correspondence seems to be the most widespread view among the realists, not any realist adopts such view. Here we will focus on correspondence, but some of the objections we will deal with can be formulated even with respect to other conceptions of truth.

With regard to human reasoning, normally the experiments designed to measure our logical abilities have confronted the performances by some humans with the answers which were considered correct according to the theories elaborated by logicians, thus drawing the conclusion that our reasoning performances are deeply biased, and so that normally they are not reliable in attaining the truth (e.g. Nisbett and Ross, 1980).

Taking the very same theories we try to naturalize as true, because they are our best theories, i.e. the most successful, raises two main problems.

a) If our individual performances are so deeply biased, how can we come to elaborate a correct logical theory? In other words, given that the correct answers prescribed by the theory are considered to be true, because they are successful in dealing with the world, how can we reach knowledge of such truths, if, according to naturalism, our knowledge has to be put in relation to our evolved cognitive abilities, and scientific investigations tell us that such evolved cognitive abilities are unreliable?

The standard answer is: "culturally". For example, Dutilh Novaes argues that it is incorrect to try to find in our brains some encoded module for the elaboration of formal languages and deductive techniques, because such high level achievements may be seen as reachable only culturally. They are not "innate". They have to be in some way instructed by modifying through training those natural tendencies of thought which we can observe to be normally biased: «it is a mistake to look for deduction (…) in humans' spontaneous reasoning patterns, grounded in their biologically determined cognitive apparatus. Rather, I shall argue that deduction in fact corresponds to a method of reasoning and arguing developed specifically to be used in *scientific inquiry*; it is, in other words, a *cultural product*» (Dutilh Novaes, 2012, p. 148).

But in the meantime, these modifications have to be possible in relation to our biological constitution: «even if specific training is required, deductive reasoning is something that at least some of us humans *do* do, at least in some circumstances. So obviously, it must be grounded in cognitive possibilities that are available in humans from the start» (*Ibidem*, p. 148).

But how can we judge the correctness of the inferential steps and inferential rules that we perform when we are trained to do so? This is the problem of justifying the validity of logic. The problem of justifying our reliability about logic is analogous to that of justifying our reliability about mathematics (Schechter, 2010). How can we account for the reliability of mathematics and logic if we accept the idea that they are both produced by humans and humans are evolved organisms? There is no clear answer to this question (Pelletier, Elio and Hanson, 2008).

For example, Kyburg states that «our justification of deductive rules must ultimately rest, in part, on an element of deductive intuition: we see that MP is truth-preserving − this is simply the same as to reflect on it and fail to see how it can lead us astray» (Kyburg, 1965, p. 276). But there is no clear explanation of what such sort of "intuition" may be, and why it should be reliable.

On the other hand, Enoch and Schechter state that we are justified in employing the rules of inference and other belief-forming methods that we have to employ indispensably «for successfully engaging in some extremely important projects» (Enoch and Schechter, 2008, p. 553), i.e. they claim that the success justifies our inferences. But this argument does not show that our successful inferences "track the truth", unless we accept the assumption that "only the truth can lead to success".

In any case, relying on intuition and success seems the most widespread line of reasoning to justify the ability of our inferences to track the truth. Below we will highlight some of the difficulties that affect such approaches if we adopt an evolutionary perspective.

In fact, as we have seen, even if the trained performances cannot be reached instinctively, nevertheless the justification of the validity of any single inferential step we make in such performances rests, in ultimate analysis, on our cognitive structures and biological constitution.

Thus, to maintain a realist view is necessary to link evolution to the truth. For example, Wilkins and Griffiths state that to «defeat evolutionary skepticism, true belief must be linked to evolutionary success in such a way that selection will favour organisms which have true beliefs» (Wilkins and Griffiths, 2013, p. 134). The problem is exactly how to justify such a link.

In fact, even if we follow Dutilh Novaes in her distinction between an "immanent" and a "transcendent" approach to the justification of formal languages (Dutilh Novaes, 2012, § 7.1.3.), i.e. if we distinguish between the way of justifying "pure" and "applied" logic (for the former relying on some internal criterion, e.g. coherence; for the latter relying on some measure of the success of its application), we cannot in any case demonstrate to be able to justify our inferential moves independently of our constitution thanks to our trained ability.

On the one hand, we cannot immanently justify the choice of our axioms for immanent reasons:

> The mathematician gives a proof of a result by deriving it from other results already proved. They were proved by deriving them from other results already proved, and so on. Since mathematicians assume that this process must ultimately come to an end, the question then arises: How do mathematicians justify their

ultimate starting points? They cannot justify them by showing that they are true – in any sense of "true", including the weak sense of "being consistent" – because this is impossible by Gödel's second incompleteness theorem (Cellucci, forthcoming, pp. 14-15)[2].

On the other hand, if we refer to a transcendent approach, we can just refer to the empirical success of our formal languages:

> the most pressing question for an account of how formal, technical frameworks can be applied to investigate informal, nontechnical concepts and theories then becomes: how do the two realms "latch on" to each other? (…). But skepticism concerning the epistemic power of formal modeling is put under pressure by the sheer amount of successful cases of genuine novel knowledge (...) produced by means of formal modeling (Dutilh Novaes, 2012, p. 233).

But supporting the claim that if our reasoning is successful, our beliefs have to be true, amounts just to "assume" that "only the truth leads to success": it is not a justification neither of such assumption, nor of the inferences we perform in order to achieve success.

b) There is the epistemological difficulty of using the cultural sophisticated outputs of those cognitive abilities which should naturalize our cultural achievements in the very definition of the objects whose production we would like to explain. For example, Núñez states that:

> A major problem in most accounts of the concept of number is that scholars often introduce crucial elements of the *explanans* in the very *explanandum*. That is, they take number systems as pre-given and introduce them as a part of the explanatory proposal itself (…). Gallistel *et al*. (2006:247), for instance, speak of "mental magnitudes" referring to a "real number system in the brain", where the very *real numbers* are taken for granted, and put them "in the brain". But, we must not forget that the system of real numbers is an extremely sophisticated concept (...), shaped historically over centuries (...). How could such a number system be simply "in the brain"? (Núñez, 2006, p. 71).

This problem is analogous to that of using a logical theory in evaluating the individual reasoning performances while claiming that such a theory gives us the truths about such domain. There seems to be no independent-from-success way to define what the "truth" is, while the realists in

[2] Indeed, by Gödel's second incompleteness theorem «for any consistent, sufficiently strong deductive theory *T*, the sentence canonically expressing the consistency of *T*, Con(*T*), is undemonstrable in *T*» (Cellucci, 2013a, p. 4).

supporting their view need to justify the claim that only the truth can lead to success. We will now analyze such difficulty in relation to mathematics.

3. Naturalizing Mathematics

Is it possible to naturalize the human ability to grasp mathematics, in order to secure a realist view of science and mathematics and make it compatible with Naturalism? Some authors have tried to naturalize mathematics and explain naturalistically its applicability relying on evolutionism (De Cruz, 2007, 2006; Krebs, 2011).

The distance between an evolutionary account of mathematics and a platonist one is due to the radically different view of the nature of mathematics they adopt: «The most important corollary of adopting a universal selection theory to mathematics is that the history of mathematics is not the unfolding of eternal mathematical truths, but an evolutionary contingent process (…) a Darwinian approach may be able to dispel the essentialism that is still rampant in the philosophy of mathematics» (De Cruz, 2007, p. 281). In such a view mathematical objects cannot be thought as mind-independent objects as MP claims:

> If we take the idea that cultural objects, (...), are contingent products of evolution, adopting such an essentialist position for mathematics is both unjustifiable and untenable. The upshot of a Darwinian approach is that, if we take a modified version of Gould's (1989) thought experiment (…), namely rewinding the tape of the history of mathematics, we would quite likely end up with a mathematics that is different from the one we have today (De Cruz, 2007, p. 282).

So, a Darwinian approach to mathematics is thought to consent to the realist to block the IA while maintaining SR:

> An evolutionary approach makes the indispensability argument dispensable. Since basic mathematical constructions in the animal and human mind are the product of evolution by natural selection, they must somehow have promoted the survival and reproductive success of the ancestors of those organisms. This is only likely if there is some correspondence between those innate cognitive domains (...) and the physical world. Thus, mathematics can be considered a reliable tool to describe scientific phenomena – even if it has no existence outside the animal or human brain (De Cruz, 2004, pp. 80-81).

Combining selectionism and functionalism in order to naturalize realism may seem a coherent strategy: «the proper function of some mechanism,

trait, or process in evolved organisms is ultimately relative to fitness, and the brain has as proper function the production of beliefs that are fitness-enhancing» (De Cruz and De Smedt, 2012, p. 413). But this is not sufficient. Another step is necessary to achieve such goal: «the evolutionary argument (…) contends that natural selection will form animal brains that tend to produce true beliefs, because true beliefs are essential for adaptive decision making» (*Ibidem*, pp. 416-417). So, in order to naturalize realism, to produce beliefs that are "fitness-enhancing" has to mean to produce "true" beliefs.

There is a deep relationship between the evolved structure of the human brain and scientific knowledge. In fact, «from an evolutionary perspective, science is a recent development in our species. Thus, scientists have to draw on the same cognitive resources as other people, and they are subject to the same cognitive limitations» (De Cruz, 2011, p. 205). So, those cognitive resources have to be able to produce true beliefs in order to give us humans the ability to produce true scientific theories.

This view raises at least two main problems.

1) The first problem is that if the beliefs produced by humans have to be true in order to be fitness-enhancing, and mathematics is produced by humans because it has been fitness-enhancing, thus mathematics has to be true; and given that mathematics is fitness-enhancing, i.e. true, because «there is some correspondence between those innate cognitive domains (…) and the physical world», it seems to be fair to claim that De Cruz is adopting a correspondence theory of truth. In such a way, mathematics is applicable to the world because it corresponds to the world.

But how can we affirm a correspondence between mathematics and the world? This is a debated issue, because if we try to claim for a correspondence between mathematics and the world referring, as many realists do, to the Tarskian concept of truth, we have to face the well known objection that this kind of tool is adequate just to state the correspondence between a bit of language and a bit of metalanguage, and not between a bit of language and the world (e.g. Bunge, 2012).

Indeed, the correspondence relation may be formalized in Tarskian terms because it is possible to determine that an isomorphism obtains between two mathematical structures. If we try to adopt this line of reasoning we should admit that the world is a mathematical structure. In fact, an isomorphism can only obtain between two mathematical structures (Halvorson, 2012, p. 185), and if an isomorphism is claimed to obtain between mathematics and the world, the world has to be considered a mathematical structure. For example, Tegmark states that from «the definition of a mathematical structure (…), it follows that if there is an

isomorphism between a mathematical structure and another structure (…), then they are one and the same. If our external physical reality is isomorphic to a mathematical structure, it therefore fits the definition of being a mathematical structure» (Tegmark, 2008, p. 107). But this is obviously a sort of Platonism, more precisely of Pythagoreanism, which is at odds with the naturalistic stance De Cruz is arguing for (Steiner, 1998).

2) The second problem is the justification of the "realist core" of this view: the crucial premise of such adaptationist view is in fact that "only true beliefs can be useful". Stich has sketched this position as follows: «the argument seems to be that natural selection favors true beliefs, (…). So if an organism is the product of natural selection, we can safely assume that most of its beliefs will be true» (Stich, 2011, p. 83).

The problem is that if we commit ourselves to claim that human cognitive structures are true beliefs producers because they have been selected for, then we should be able to demonstrate that every cognitive adaptation is able to produce nothing else than (or at least mostly) true beliefs. But this is not an easy task, because, as Stich, among others, has stressed, «it is simply not the case that natural selection favors true beliefs over false ones. What natural selection does favor is beliefs which yield selective advantage. And there are many environmental circumstances in which false beliefs will be more useful than true ones» (*Ibidem*).

The problem of connecting selection and truth is analogous to that of linking the truth of a theory to its empirical success. As the success cannot guarantee the inference to the truth of its (hypothesized) cause, so the survival is not able to discriminate among its possible causes. So both success and survival are not reliable indicators of truth (Wray, 2013). The realists face a dilemma: either they reduce their explanation to a vacuous tautology, or accept that success is caused not only by true beliefs, but this would amount to dismiss realism.

The only way to maintain an evolutionary realist perspective, and secure that success is implied only by true beliefs, is to presuppose such a realist connection (Sage, 2004). For example, Millikan states:

> "Assuming that" the capacity to form and to use beliefs has survival value mainly in so far as the beliefs formed are true (…), and "assuming that" humans currently have this capacity in part because, historically, having it had survival value, the mechanisms in us that produce beliefs, (…) all have in common at least one proper function: helping to produce true beliefs (Millikan, 1984, p. 317, emphasis mine).

3.1. The Self-Defeating Objection

It has been objected that a position like that of Stich (call it: the Debunking Position, DP) is self-defeating, in the sense that DP denies that natural selection favors true beliefs, thus that the correspondence view of truth can be justified through evolution, and so that science can be justified, while DP itself adopts that very same concept of truth in scientifically stating that in some evolutionary contexts "false" beliefs are more useful than "true" ones. For example, De Cruz states that:

> This line of reasoning is self-defeating. Evolutionary accounts according to which human cognitive capacities are so deeply biased and defective that knowledge is ruled out are self-undermining. There would be no good reason to assume that scientific theories are justified, or that philosophical reflection and argumentation (…) provides us with sound conceptual knowledge (De Cruz *et al*. 2011, p. 525).

But this objection is inadequate. In fact, it does not give a real defence of the realist position (call it: the Evolutionary Position, EP). But such objection helps us to underline the difficulties that affect both DP and EP, difficulties which derive from the conception of knowledge that both DP and EP adopt.

Indeed, DP *per se* does not deny the correspondence view of truth, nor that science aims at or obtains truth, it just says that natural selection cannot account for the truth of the beliefs that humans produce. In fact, DP can be stated starting from a realist point of view: scientific theories are true, so it is possible to discriminate between true and false beliefs, and science says that false beliefs are in some evolutionary contexts more useful than true ones. Thus DP just affirms that it is true that science says that natural selection cannot account for the truth of human beliefs, and just denies the assumption that "only true beliefs can be adaptive". So, since DP does not deny the possibility of discriminating true beliefs from false beliefs, and its conclusions do not contradict any of its premises, DP is not self-defeating, and the above objection is inadequate.

DP would be self-defeating only if the evolutionary strategy adopted by EP would be the only acceptable means to establish a criterion of truth, i.e. a tool to tell the true from the false (Cellucci, 2014). If such premise is assumed, than denying the possibility that selection gives us a true beliefs producer system would amount to deny that it is possible to have a criterion of truth, and so that it is not possible to discriminate between true beliefs and false beliefs. In such case DP would be self-defeating.

But there is no evident reason why such premise should be accepted by those who advocate for DP. In fact, it is not clear how could a realist demonstrate that the evolutionary strategy is the only acceptable one. In giving such a demonstration she should either use 1) an evolutionarily established criterion of truth, or 2) a non-evolutionarily established criterion of truth. Both cases are problematic. In fact, if the realist uses 1), she adopts as established a criterion which is still under examination, given that the evolutionary strategy to establish a criterion of truth is exactly what is contested, so this option would amount to a sort of *petitio principii*; if the realist uses 2) this would lead to a self-defeating argument, because if the demonstration of the uniqueness of the evolutionary strategy to establish a criterion of truth rests on a non-evolutionarily established criterion of truth, this very fact undermines the claim of uniqueness that should be demonstrated.

What can be conceded is just that DP shares the same problem of its opponent EP, i.e. that of giving a criterion of truth. But, given that this is a problem for both the positions at issue, it seems unfair for EP to move a criticism to DP based on such problem.

In fact, even EP should give a criterion of truth independent from the survival in order to justify the claim that natural selection favours true beliefs over false ones without reducing such claim to a tautology (Downes, 2000). In other words, EP needs a criterion of truth other than the survival in order to identify which beliefs are true among the set of beliefs made up by natural selection. If instead the beliefs that are true are defined as those beliefs which have been selected, then we have just a tautology, and we cannot really claim that natural selection has favoured true beliefs over false ones, because this conclusion would just be implicit in our definition of "true beliefs" in terms of "selected beliefs"[3].

Finally, if it would be demonstrated that the survival is the only available evolutionarily established criterion of truth, the fact that EP needs a criterion of truth other than the survival would imply that EP cannot be but self-defeating (see point 2) above).

So, EP cannot fairly accuse DP of being self-defeating, given that EP is in a similar (if not worst) situation: to secure its criterion should rely on some other criterion. So, if it is not possible to rely on some criterion different from the evolutionary one, than both EP and DP fall, given that

[3] If EP adopts such "tautological strategy", it can affirm to have not to give a criterion of truth, and that the selected beliefs are true. But this amounts to claim that false beliefs cannot exist. If this would be the case the problem arises of how to account for: 1) our misbeliefs; 2) the fact that the history of science shows us that the scientifically produced beliefs change over time.

the latter would be self-defeating, while the former would be impossible even to be substantially formulated. If instead it is assumed that it is possible to rely on some different criterion, then DP is not unfairly formulated, nor self-defeating, while EP risks to be self-defeating.

There is at least an impasse here. Analyzing the inadequacy of the objection of being self-defeating moved to DP shows that the conception of knowledge on which both EP and DP rests is unsatisfying, because it is exactly that conception which leads to the impasse described above.

Indeed, it is important to remind that they may be both considered two realist positions. In fact, the problem in both cases rests on the realist's side: either she cannot accuse DP of being self-defeating, and thus cannot defend her own attempt to naturalize truth, or she falls together with DP. On the contrary, discarding DP as self-defeating is not a problem for an anti-realist position which denies that knowledge is related to the concept of truth, such that, e.g., proposed by Cellucci (forthcoming, 2014, 2013a).

4. Naturalizing Knowledge

Despite the difficulties it implies, the traditional account of knowledge as "justified true belief" seems to resist in wanting of a better proposal, which seems not easy to be found (Lycan, 2006). For example, Nozick states that, if asked what knowledge is, a «reasonable philosopher today might say that, in view of the difficulties thus far encountered, he just does not know» (Nozick, 1995, p.146). Notwithstanding, many realists continue to subscribe to (some formulation of) the traditional correspondence view of truth and to the traditional view of knowledge (Sankey, 2008).

We will briefly analyze two realist attempts to naturalizing knowledge made by Kornblith (2002) and Plotkin (1997), to show that these attempts completely rest on such traditional view, and that, for this reason, they are unsatisfying for a naturalist.

Referring to Nozick's work, it will be argued that the confidence in the fact that only the truth leads to success rests on a sort of "intuition", which cannot be naturalistically justified, but which can instead be evolutionarily explained.

4.1. Kornblith and Plotkin

Kornblith tries to naturalize knowledge by claiming that knowledge is a natural kind. He takes «natural kinds to be homeostatically clustered

properties, properties that are mutually supporting and reinforcing in the face of external change» (Kornblith, 2002, p. 61).

The problem of this position lays in the notion of truth on which it relies to define what a natural kind is. In fact, Kornblith tries to connect truth and success relying on Darwinism:

> If we wish to explain why it is that members of a species have survived, we need to appeal to the causal role of the animals' knowledge of their environment in producing behavior which allows them to succeed in fulfilling their biological needs. (…). The knowledge that members of a species embody is the locus of a homeostatic cluster of properties: true beliefs that are reliably produced, that are instrumental in the production of behavior successful in meeting biological needs and thereby implicated in the (...) selective retention of traits (*Ibidem*, p. 62).

This amounts to say that knowledge is useful in achieving successful behaviors because it is a natural kind, and natural kinds are such that consent to fulfill animal's biological goals because in some way they "carve nature at its joints".

Thus, the concept of knowledge maintained by Kornblith is equivalent to the classical one: knowledge is reliable produced true beliefs. But how can we support the idea that science can know the natural kinds?

The only way to support such claim is to accept the traditional view of truth. In defining "natural kinds" in the way Kornblith does, the idea of a necessary correspondence between our knowledge and the world is just reaffirmed, but not demonstrated to be necessary.

Kornblith tries to support his view in the same way the realists support SR, i.e. abductively: «we are committed to the existence of knowledge because (...) knowledge plays an explanatory role in our theories. We are thus committed to the existence of knowledge, on this view, for the same kinds of reasons that we are committed to the existence of quarks and quasars (…). Knowledge is a feature of the world» (*Ibidem*, p. 159).

The problem is that even if we concede that we should commit ourselves to the existence of knowledge, given the role it indispensably plays in our explanations, this does not by itself guarantees that the features we traditionally attribute to knowledge, i.e. the way we say that knowledge is, correctly corresponds to what knowledge really is.

We have here the same problem which is at the core of the debate over SR: are we justified in claiming that the world is exactly the way we say it is, because those concepts we use to describe the way the world is figure in our best scientific explanation?

We could be justified in making such claim only if we could justify the claim that only the truth (as correspondence) implies success. But realism merely assumes such claim, it does not justify it.

Plotkin states that «what one knows corresponds to something happening in the world» (Plotkin, 1997, p. 7). Thus, he seems to adopt the traditional correspondence view to account for what knowledge is. Plotkin naturalizes this view of knowledge relying on an adaptationist view of evolutionism. In fact, he says that evolutionary theory is a theory of knowledge because evolution explains adaptations, knowledge «is a special kind of adaptation. And all adaptations are knowledge» (*Ibidem*, p. 228). In fact, since evolution «leads (…) to the thoughts and ideas that we have in our heads», and knowledge may be thought of as a «relationship between the organization of the brain and specific features of order in the world outside» (*Ibidem*, pp. 20-21), we should infer that those adaptations which consent our knowledge give us the ability to mirror the way the world is.

Thus Plotkin claims that since adaptations and knowledge are successful they have to be true. This view is obviously based on the assumptions that "only the truth can lead to success", that truth is correspondence, and that truth is necessary to have knowledge. But there is nothing in Plotkin's argument which justifies such assumptions, except the asserted impossibility of conceiving otherwise the explanation of our success.

Plotkin also acknowledges that there is a difficulty in justifying knowledge relying on evolutionism. In fact, organisms are not made only by adaptations, they are «bundles of organized features», only some of which are «good adaptations» (*Ibidem*, p. 228). What is possible to assess is just the whole organism's success in surviving. Fitness is a way of measuring such success. But fitness is a "statistical abstraction", in the sense that if an organism survives, it is not possible to assess whether its survival is due to the fact that a specific adaptation correctly corresponds to the world or not. The situation is analogous to that we have in the debate on the confirmation of scientific theories. We cannot judge of any individual claim in a theory, because confirmation holistically supports the whole theory, so we cannot know if any claim of the theory is true, even if we think that confirmation is a guide to truth: one «may not always be certain as to whether in any one case survival is contributed to by knowledge or whether we are just fooled into thinking that this particular organism has good knowledge because it survives» (*Ibidem*, p. 232).

Thus, «having "incorrect" or false knowledge, does not mean that the individual will fail to survive and reproduce» (*Ibidem*, p. 233). Moreover, the difficulty of linking truth and survival is not a problem of degree, because even «if it were only very rarely untrue, but the holders of the

untruths survived and reproduced, that would be enough to nullify any foolish claim by evolutionary epistemology to overcoming the justification problem», i.e. to support the claim that success is a good indicator of truth. In fact, only «if survival and reproduction are absolutely correlated with knowledge could they be an infallible guide to true belief», but Plotkin maintains that «this is not the case» (*Ibidem*, p. 234). Thus we cannot safely rely on the success of some organism to assess the truth of such organism's knowledge.

Plotkin tries to overcome such problem through an abductive defence of SR. Put it briefly, his argument runs like follows: knowledge is necessary to survive; knowledge is «correspondence to the things-in-themselves» (*Ibidem*, p. 240); if we could not have knowledge of the way the world is, we could not have survived; we survived; thus we have knowledge of the way the world is. He thinks that the fact «that we survive at all is proof that knowledge is possible» (*Ibidem*, p. 240), and that this means that "Kantianism" has to be incorrect: «if Kant were correct, if living things could never know the things-in-themselves, then life would never survive (…). Since we and the myriad other forms of life do survive, Kant must be wrong» (*Ibidem*, p. 241).

But the fact that knowledge is possible defeats "Kantianism" only if we accept that knowledge cannot be anything but "correspondence to things-in-themselves". But why should we accept such premise? We should do that only if we accept the idea that only the truth as correspondence can lead to success. But how could we support such a claim? If we rely on survival we have to face the fact that Plotkin himself has underlined: survival cannot justify our claim that only the truth can lead to success, because not only the truth leads to survival.

Moreover, Plotkin's argument seems to be based on a sort of ambiguity. He often says that organisms have to successfully relate to their environment in order to survive. But saying that knowledge is "knowledge of things-in-themselves" and that "truth is correspondence" is far stronger than merely claiming that living beings have to "successfully" relate to their environment, or "match" with it. If we do not qualify "match" as "correspondence up to isomorphism", then such a claim is inadequate to define a scientific realist position, because it could be compatible with different, and broadly speaking anti-realist, positions (e.g. instrumentalism, pragmatism, skepticism, etc.) which deny that our knowledge corresponds to the way the world is independently of us (Unger, 1971; Cellucci, 2014;

Stanford, 2006)[4]. But if we try to defend the claim that knowledge is "correspondence up to isomorphism", we have to face the well known difficulty of showing how it is possible to assess if the correspondence between our knowledge and the world actually occurs. It seems that we cannot do that but indirectly relying on success. But in this way we come again to the problem that success is not a good indicator of truth.

Kornblith's and Plotkin's way of naturalizing knowledge seems more a way of securing the realist conception of knowledge, than a way to propose a truly new way of conceiving of knowledge reflecting on the way in which science proceeds.

4.2. Nozick and Evolution

Nozick (2001, 1993) takes into account evolution to assess the traditional claims on what knowledge is. His work allows us to assess the two different but related aspects of the scientific theorizing which underpin the traditional views of knowledge and truth that we have already seen above in relation to the attempt to justifying our inferences: 1) some statements appear to us as self-evident; 2) the theory derived from such statements are successful. The combination of these two aspects seems to support the claim that scientific theories are able at attaining the truth.

Think about mathematics: we have no other way to justify our confidence in the truth of the axioms of our theories than relying on the fact that they seem to be evident to us (or to the community of mathematicians). A sort of confirmation of our ability in "seeing" their being self-evidently true is claimed to derive from the empirical success of our scientific theories which indispensably rely on mathematics, which at its turn rests on such self-evident axioms. Thus, such axioms have to be true.

But if we accept a naturalist stance, then we have to accept to conceive of humans and human understanding as natural facts, and so that the justifications of our beliefs have to be evaluated in the same way in which we evaluate the behaviour of all other living beings, i.e. referring to evolution.

[4] This is the problem of a sort of "double standard" that many realists seem to adopt: when they refer to scientific theories and mathematical models they define "truth" in the rigorous Tarskian terms of "correspondence up to isomorphism", while when they try to root human epistemic ability in attaining the truth in evolution, they seem to content themselves with a definition of "truth" in terms of "sufficiently good to survival". But the two conceptions of truth are clearly not equivalent (Downes, 2000). Thus, the realists cannot naturalize and justify the former relying on the latter.

So, if we analyse the first aspect mentioned above ("some statements appear to us as self-evident") from an evolutionary stance, what at most we can conclude is that the ability in judging something as self-evident may have been selected for because of its usefulness in some relevant contexts of our evolutionary past.

But this leave us with two problems: a) the problem of induction: even if in the past what we now perceive as self-evident has been useful, this does not mean that it will continue to be so; b) the fact that in the past what we now perceive as self-evident has been useful does not guarantee that it ever was, strictly, true.

These are the key points of Nozick's rebuttal of the idea that there are metaphysical necessities: if we accept evolutionism we cannot state neither if what appears to us as self-evident and necessary is instead contingent, nor if it is even true. This position clearly debunks some traditional claims on what knowledge and truth are.

Nozick gives as an example of this second problem ("that something has been useful in the past does not guarantee that it ever was, strictly, true") the Euclidean geometry intended as a theory of the physical space: indeed, even if since antiquity to the 19th century no one conceived the possibility that the physical space could not be Euclidean (Kragh, 2012), today we think that Euclidean geometry «is true enough for almost all practical purposes; (…) but, strictly, it is not true» (Nozick, 1993, p. 109). This fact could be due to the selective advantage we had in perceiving the physical space as Euclidean, and this very fact could explain our tendency to perceive the Euclidean geometry as self-evidently true.

It is important to clarify that, from a scientific realist point of view, the only acceptable use of "true" is Nozick's "strictly true", i.e. "true because it corresponds up to isomorphism to the mind-independent world", while Nozick's "true enough" is clearly so vague, contextual, and pragmatically defined that could not fit with the non-epistemic view of truth the majority of the realists advocates (Sankey, 2008).

Euclidean geometry is used as an example of a useful-but-false belief that the cognitive structures instilled in us by the evolution could let us think to be true also by Peirce[5]. But if this were the case, why we should think that our present theory could not be in the same situation in which the

[5] Cf., e.g., Peirce (CP 6.29): «It is true that according to the axioms of geometry the sum of the three sides of a triangle are precisely 180°; (…). They are expressions of our inborn conception of space, and as such are entitled to credit, so far as their truth could have influenced the formation of the mind. But that affords not the slightest reason for supposing them exact». Peirce is considered to be the first philosopher to claim that Euclidean geometry "cannot" be the geometry of the physical space (Dipert, 1977; Kragh, 2012).

Euclidean geometry was? The fact that Euclidean geometry is "false" may in fact be asserted from the point of view we reached after the elaboration of non-Euclidean geometries. This development in mathematics has affected both the above mentioned aspects we are dealing with: 1) the very fact that finally alternatives geometries were conceived and shown to be coherent and that physical theories based on such alternative geometries have also been conceived, speaks against the claim that what we take to be unconceivable at some time will be such forever. Thus, we should refrain from claiming that our present theory is true because we cannot conceive (or find) an alternative to it; 2) we claim that Euclidean geometry is not really true as a theory of the physical space not only because a different geometry may be coherently elaborated, but because such new geometry has been used in some fundamental physical theory, which is empirically more successful than the physical theory based on the Euclidean geometry.

Thus, the fact that some statements appear to us as self-evidently true is not by itself a guarantee of their truth, if our ability in evaluating the self-evident truth of a statement is an evolved capacity. Our "sense" of the self-evident may be not only oriented towards contingent connections which were useful in the past and that do not reflect necessary and eternal truths, but given that we are not able to demonstrate that only the truth leads to success, we cannot eliminate the possibility that an ability in perceiving as self-evident some falsities has been selected because of the usefulness deriving from perceiving such falsities as self-evident truths.

Now we come to the second aspect mentioned above ("the theory derived from such statements are successful"). Here it seems that there could be a line of defence for the traditional view: in fact, the argument goes, "how could be possible" that empirically successful theories derive from false self-evident statements? The combination of the two aspects seems to be the key: some false-but-useful statements may be mistakenly taken for true ones, but the fact that the theories elaborated starting from some statements bring us at attaining strong empirical success must mean that we reached the truth.

But let's analyse this second aspect having in mind the evolutionary perspective we illustrated above: what we can safely say is just that we have successful theories, or that a theory is more successful than some previous or alternative one. This is not at odds with the fact that the statements from which we derive the theories may not be true. In fact, we have not been able to establish that only true statements may lead to successful theories. The fact that a theory is successful should imply that its fundamental "axioms" are true only if it is the case that only the truth leads to success. But, as already noted above, there is not such a direct link

between success and truth (or at least it is not easy to demonstrate that this link obtains relying on evolutionism).

Obviously our ability in coping with the world and in elaborating theories which are more successful than the previous ones is something that we have to try to explain. The point is assessing if the traditional proposed solution is satisfying. Indeed, traditionally, what has been proposed as an explanation of our success is that our theories are true, and that truth is correspondence. But from an evolutionary perspective this hypothesis is equivalent to one of the statements that we judge as self-evident. How could we evaluate such hypothesis? If we rely on our "sense for self-evident truths", we will not be able to say if our intuition of what truth is is really true according to our view of truth or if it just belongs to that kind of statements which has been useful to judge as self-evident even if they are "strictly" false. On the other hand, if we rely on the fact that our theories are successful to support the claim that they have to be true, we have to face the fact that if we try to propose the truth as an explanation for the success of our theories we should support and explain what truth is independently of success. Indeed, we cannot presuppose exactly that link between success and truth that we should instead support.

Most of the time, in supporting their claims, realists appeal to the fact that "it would be impossible to conceive" a different way to account for the success of our theories other than referring to the truth (Kitcher, 1978). But how could we support the idea that our ability in conceiving alternative explanations and in judging what is impossible to conceive is reliable? If we want to take naturalism seriously, we have to refer to evolutionism. Thus, we come again to the objection that we have seen above relative to the reliability of our innate ability in perceiving what is necessary or possible.

Thus, we have to face the possibility that accepting an evolutionary stance would mean not only that even if a concept of truth seems self-evident to us it may nevertheless be false, but also that the fact that it seems impossible to us to conceive an alternative to a concept may not be a reliable way to assess the truth of the impossibility to conceive an alternative to such concept, nor a reliable way to confirm the truth of such concept. So, neither the appealing to our "intuition", nor the appealing to the "empirical success", seems able to justify the traditional realist conception of knowledge.

The fact is that we cannot reach an Archimedean point of view and assess if our hypothesis that "only truth can explain success" is true. We can only rely on success. And we can safely comparatively measure success. But does this make the hypothesis (i.e. the truth of our theories)

that we proposed to explain the success of our theories more confirmed? The answer is in the negative. The point is that it is easy to mistake an increase in the success for an increase in the confirmation of the hypothesis that it is the truth which explains our success, exactly because we tend to implicitly assume that it is only the truth that leads to success. But as many other implicit assumptions we make, and notwithstanding how much this assumption about the truth seems evident to us, such assumption may be unreliable.

4.3. The Inadequacy of the Traditional View of Knowledge

What we have said makes explicit the need to elaborate and adopt a different view of knowledge, in order to support a naturalistic stance. For example, Cellucci (forthcoming, 2014, 2013a), following Kant, argues for the inadequacy of the traditional view of knowledge for a naturalist. In fact, if knowledge is justified true belief, and truth is correspondence, then it is not possible to assess if knowledge has been reached, because it is not possible to confront our knowledge to the things-in-themselves, thus knowledge is impossible for us, it is beyond human reach; but given that knowledge is necessary to survive, and that we indeed survived, knowledge must be possible; thus we have to conclude that knowledge is not correspondence.

What has to be underlined here is that the realists and the skeptics share the same conception of knowledge. They just draw different conclusions, because they adopt different secondary premises. Indeed the realist argues that: knowledge is justified true beliefs, thus truth is necessary for knowledge; we do have knowledge; this means that we can attain the truth. On the contrary the skeptic argues that: knowledge is justified true beliefs, thus truth is necessary for knowledge; we cannot attain the truth; this means that we cannot have knowledge (Unger, 1971).

A naturalist position as the one advocated here shares with the realist the conviction that we do have knowledge and that knowledge is necessary to survive, while shares with the skeptic the conviction that we cannot reach knowledge if knowledge is justified true beliefs. But from such premises draws the conclusion that it is incorrect to define knowledge in the traditional way, and that the intuition that knowledge can be nothing but justified true beliefs is ungrounded.

Another option that shares the realist's and the skeptic's main premise is anti-naturalism (Plantinga, 2006). The anti-naturalist argues that: knowledge is justified true beliefs, thus truth is necessary for knowledge; if

we accept naturalism we have to deny that it is possible to reach the truth; but given that we do have knowledge, this means that we have to reject naturalism. Moreover, naturalism would be self-defeating: how could it justify its own claim to be the true view if it leads to the conclusion that it is impossible to attain the truth? Anti-naturalism is obviously entirely based on the assumption that knowledge has to be related to the truth. In this case Naturalism would be self-defeating. But it is not clear at all why we should accept such premise on what knowledge is.

The realist attempts to naturalize the traditional view seem not compelling. Indeed we have no satisfying definition of the traditional concept of knowledge (Lycan, 2006), nor a really satisfying definition of the concept of truth and a related definition of a workable criterion of truth (Cellucci, 2014). Thus it is not even clear "what" we should accept and "why".

5. Conclusions

It has been argued for the necessity of rethinking the concept of knowledge and decoupling it from the concept of truth in order to elaborate and support a naturalistic stance, which could take Darwin, and science more generally, seriously.

Briefly, we have tried to show that if mathematics does not depend on our biological make-up, mathematics may be considered not to be contingent, but then the problem arises of how we can attain mathematical truths. With regard to the formalization of Darwinism, this means that we could safely assume that the formalization leads to a more secure foundation of Darwinism, but we would not be able to naturalistically account for this very fact.

If, on the contrary, mathematics depends on our biological make-up, then we cannot take mathematical knowledge to be certain, in the sense that it may be contingent, but we can try to give a naturalistic account of our ability in producing mathematics. With regard to the naturalization of mathematics, this means that even the very theory we take for granted in order to naturalize mathematics, i.e. Darwinism, cannot be taken for certain, in the sense that it could not be considered securely founded even if a complete formalization of it could be given.

We cannot maintain both certainty and naturalism. But this does not mean that we cannot have knowledge, and in fact we do have knowledge. This just means that we should accept to modify our view of what knowledge is.

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