Kinds of Models

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Introduction

There could hardly be two more disparate uses of the word "model" than *animal model*

and *nonstandard model of arithmetic.* One refers to a biological organism, the other to

a mathematical structure. The term *animal model* is used to describe the employment

of an animal bred specifically for laboratory use to study some physiological process,

such as the employment of laboratory rats to study the effects of various toxins, drugs,

and therapies on humans. Here the interest is not in rat physiology per se, but in the

analogous human physiological processes about which the animal model can provide

insight. So referring to the rat as a model does not quite capture what is meant by ani-

mal model; what the rat is a model of, and in what way the rat models it, is involved in

the concept as well.[1] The term *nonstandard model of arithmetic* refers to a mathemati-

cal structure that happens to satisfy axioms originally formulated to axiomatize the ordi-

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[1] The point is explained in Rand (2004), which provides a general overview of the history and methodol-
ogy of animal models. [Rand, Michael S., DVM. "Selection of Animal Models", Lecture given September
27, 2004 at the University of Arizona-Tuscon.   Available online at
http://www.ahsc.arizona.edu/uac/notes/classes/animalmodels/animalmodels03.html Accessed June 10,
2005.]   The National Institutes of Health maintains a site devoted to model organisms for biomedical re-
search, at: http://www.nih.gov/science/models/

nary arithmetic[2] with which we are familiar, but is unlike (not isomorphic to) ordinary arithmetic. The interest in such models is often in the features of the nonstandard model of arithmetic that differ from those of ordinary arithmetic. Here again regarding the mathematical structure as a model does not quite capture what is meant by a nonstandard model, for it is important both that the structure satisfy the axioms for ordinary arithmetic, and also that it differ from ordinary arithmetic in a certain precise sense.

What makes these two kinds of models disparate is not the disparity that exists between the *things* that serve as a model --e.g., not the disparity between a living animal and an abstract mathematical structure -- but with the difference between these two models in terms of their relationships to what is modelled and in how they are used to model. To put it briefly, what makes these two kinds of models disparate are the things in virtue of which each is a model. It is probably fair to say that the latter notion -- a model is a model in virtue of satisfying some axioms or some other suitable (usually formal) specification -- is predominant in philosophy.

In this paper, I survey a broad variety of models with an eye to asking what kind of model each is in the following sense: in *virtue of what* is each of them regarded as a model? It will be seen that when we classify models according to the answer to this question, it comes to light that the notion of model predominant in philosophy of science covers only some of the kinds of models used in scientific contexts. The notion of a model predominant in philosophy of science requires that a model be related to some-

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[2] In the philosophical literature, "ordinary arithmetic" is generally further clarified as Peano arithmetic, after the mathematician Guiseppe Peano who developed a set of axioms (including an axiom schema for mathematical induction). These axioms can be found in numerous references; one monograph on the subject is Kaye (1991) [Richard Kaye, Models of Peano arithmetic. Oxford Logic Guides 15, OUP 1991, ISBN 0 19 853213 X, 292 pages.]

thing formal, such as equations or statements. Not all the examples provided in the brief survey in this paper fit that notion of a model. I identify another kind of model that ought to be taken more seriously in philosophical and foundational studies of scientific models, which I call a "piece of the world" kind of model, to contrast with a "realm of thought" kind of model.

Models and Reasoning

At this starting point in our survey, we want to cast our net widely and consider any kind of model that might be important in scientific endeavors. We are not interested in just any use of the word model, however; there are some uses of the word, such as "role model" or "showroom model" that do not denote the concept we are interested in. The concept of model arises in philosophy of science because models are employed in scientific contexts, not only in making predictions and applying science, but in inquiry as well. Let us say then that what we are interested in is models that are used in reasoning: models employed in making inferences or, even, in providing explanations or promoting understanding.

A Sampling of Models

Let us consider a few selected but very different examples of scientific reasoning employing models, each of which involves models in a slightly different way:

A. Mechanical Models of Electrodynamic Equations

An especially interesting example is a rather well-known example from history of science. In the late nineteenth century, Maxwell developed mathematical equations to de-

scribe electromagnetic phenomena.   Often, a mechanical model or fluid analogy was presented along with the equations as well to illustrate a certain phenomena implied by the equations. However, the model was not meant to be taken too literally.   Joseph Turner described Maxwell's use of physical analogy as a sort of golden mean: "The fluid was not offered as a physical hypothesis nor the theory developed in purely mathematical terms," citing Maxwell's remarks in his early "On Faraday's Lines of Force" that the fluid: ". . . is not even a hypothetical fluid which is introduced to explain actual phenomena. It is merely a collection of imaginary properties which may be employed for establishing certain theorems in pure mathematics in a way more intelligible to many minds and more applicable to physical problems than that in which algebraic symbols alone are used." [3]

There were other, more objective, reasons for exhibiting mechanical models of the electromagnetic phenomena described by Maxwell's equations: the existence of a mechanical model that would lead to the proposed equations was evidence for the consistency and plausibility of the equations, and agnosticism with respect to the actual mechanism responsible for the behavior described by the equations was often maintained even when a mechanism could be exhibited.

Likewise, the suggestion of alternative mechanisms did not call for arbitrating between them; the existence of several alternative possible mechanisms did not decrease the plausibility of the equations. Thus, the presentation of a model of the equations proposed was not a suggestion that the model was "true", but that the equations were con-

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[3] "On Faraday's Lines of Force," Papers, I, 160. Quoted in Turner, Joseph "Maxwell on the Method of Physical Analogy" *British Journal for the Philosophy of Science*, Vol. 6, No. 23 (Nov. 1955), 226-238.

sistent and compatible with the known laws of mechanics as well as with Maxwell's

equations.

B. Models of Axioms -- Arithmetic and Geometry

Another kind of model arises in logic and mathematics: a model *of* a set of statements,

regarded as axioms. The standard and nonstandard models of arithmetic described in

the introduction are examples of this kind of model. In this example, axioms are devel-

oped for something considered already familiar and known -- the natural numbers and

ordinary arithmetic. Then, it turns out that the axioms do not *characterize* the natural

numbers. That is, it turns out that there are things other than the natural numbers that

satisfy these axioms. So we refer to the natural numbers as the intended model, and

other models of the axioms as nonstandard models of arithmetic.

We find a slightly different example of this kind of model in the case of models of the

axioms of geometry.   It is possible to provide axioms of geometry that capture Euclid-

ean geometry. Hilbert formulated a set of axioms for Euclidean geometry such that

each axiom expressed some feature of or relation between the objects of geometry.

Besides asking whether there are other models of the axioms of Euclidean geometry,

we can ask whether there are models of some subset or revised version of these axi-

oms; the question can then be stated in terms of asking whether there are models of

geometries in which some of the features that the axioms aimed to capture are different

from Euclidean geometry. Non-Euclidean geometries were known at the time, and Hil-

bert showed that various non-Euclidean geometries were models of sets of axioms that

arose from replacing one of the axioms of Euclidean geometry with a slightly revised

axiom. The subject is very beautiful and its history fascinating, but that does not concern us here.

Despite the differences between these two examples --- models of arithmetic (both standard and nonstandard) and models of the axioms of geometry (both Euclidean and non-Euclidean) --- the kind of model at issue is an abstract model that is a model in virtue of *being a model of a set of statements*. Being *a model of a set of statements* means that the statements, suitably interpreted as statements about the abstract objects and relations between them, come out true under that interpretation for the objects and relations in the model.

Mathematical equations are a kind of statement, though. So one might wonder: does this kind of abstract model (example B.) differ from the kind of model involved in Maxwell's idea of a physical analogy (example A), since both are models in virtue of being models of a set of statements? The difference between the kinds of models in A. and the kinds of models in B. is just that the models Maxwell used were also supposed to be at least potentially physically realizable. In both abstract and physically realizable models, though, the fact that something is a model just means that it has the attributes needed for the statements to hold true for the model, not that those statements are true only for that particular model. With both kinds of models, there can be different models of the same set of statements. Thus the fact that the statements hold true for the model on a suitable interpretation does not say too much about whether a particular model is the model that actually gives rise to the behavior described by the equations. (In logical terminology, that the statements hold true in a model doesn't say whether the model is

the intended model; in physical terminology, that the statements hold true in the model doesn't say whether the model describes the mechanism responsible for the physical behavior described by the equations.)

C. Mathematical Models Used for Simulation

Mathematical models are often constructed for use in simulating some real or imagined situation. A familiar example is a flight training simulator; here, a complicated algorithm can be programmed and run on a personal computer, simulating what the plane's instruments will read and the pilot will see. The simulation produces responses such as instrument readings, depending upon how the values of various parameters such as weather, geography, and the actions of the person operating the simulator are varied. Mathematical simulations are used in a wide variety of disciplines. Simulations of how a building responds in an earthquake, the effect of fertilizer use on crop yield, the effect of a new policy restricting hunting on animal populations, how a rise in oil prices will affect the profits of corporations in various sectors of the economy, are just a few examples. In general, simulations are often called for when contemplating changes in policy. Sometimes mathematical models used in simulations take as input the values of parameters that are not generated by a mathematical function, but are empirical data that has been recorded, or are extrapolated from such empirical data. Even then, the mathematical model itself is an algorithm.

In mathematical simulations, the interest differs from that of examples of kind A. and B. in that the goal is not to obtain a model that satisfies a certain set of fundamental mathematical equations describing a physical phenomenon (such as Maxwell's equa-

tions), nor to obtain a model of a set of statements meant to capture mathematical structure (such as Peano's Axioms), but to obtain a model from which one can calculate the values of a set of parameters that agree with those observed in a real situation (or, counterfactually, would agree with in an imagined situation).   Here again, though, even though the goal is not conceived of as a matter of satisfying formal statements, it is possible to state the goal; the goal is that the values calculated by the formal or mathematical model agree with empirical observations or predictions based upon empirical observations.

D. Model Organisms in Biology

The use of rats to study physiological processes in humans is one example of an animal model, and an animal model is an example of the use of model organisms in biology. A model organism is an organism used to study either another specific organism (e.g., rats used instead of humans) or to study a specific biological phenomena that is common to many organisms (using fruit flies to illustrate inheritance of eye color).

Why use a model organism as an alternative or representative organism in lieu of the organisms about which one is making inferences? The restrictions against experimentation using humans explains why rats are used to study humans, but other advantages are that a model organism may be better suited to study a particular biological system, due to the simplicity of the system in that organism, or to the ease of observation or manipulation and control of the system in that organism. The fruit flies used for high school science education in genetics are chosen in part because they reproduce so quickly, allowing observations over several generations in a short period of time. Also,

as organisms are made the subject of laboratory studies, the organism used for such studies becomes better understood and more data is available about that specific organism. Standardized strains of the organism are produced and can be obtained for research activities.

Model organisms, then, involve some degree of idealization. What makes them models, however, is that the researcher has established some analogy between the model organism and the organisms about which conclusions are being made, and employs it to draw inferences about specific features of the organism of interest or the general biological systems or phenomena.

E. Experimental Scale Models

Experimental scale models are physical objects or systems used to test or predict the behavior of a machine or system. Like model organisms, scale models are models in virtue of the fact that the model is a physical thing that is used to investigate some specific behavior of another based on an analogy between them. They are usually mechanical systems, but the method is not restricted to mechanical systems, so electrical or chemical systems can be scale models, too. Generally, an experimental scale model is constructed so that there is a very specific analogy between the two physical systems, sometimes described as "physically similar systems". There are many analogies between any two given physical systems; the analogy that holds between the constructed model and the system of interest is usually specially chosen based on the physical phenomenon that governs the features of interest.

There is a formal, logico-mathematical aspect to this process, but it is not a set of statements that hold in the model. Rather, it is a statement that the value of a certain dimensionless ratio is the same in the model as it is in the system of interest. Thus, to state the criterion is to make a statement of the conditions for identity of the dimensionless ratios upon which physical similarity depends; to state that two systems are physically similar is to make a statement of identity between the relevant dimensionless ratios.

F. Re-enactments of Events

When forensic scientists want to investigate whether a certain piece of evidence -- an injury, a blood spatter pattern, the path of a bullet, the skid marks left by a car -- could have been left by a proposed event, they sometimes re-enact the event. Of course the exact conditions of the universe cannot be created; some tiny subset of the conditions that existed in the original event are selected as the significant ones and used to stage a re-creation. So the re-enactment can be thought of as a model, a model of the hypothesized original event being investigated. The goal here is to produce the phenomena that would have been produced by the hypothesized event, and the phenomena produced may or may not agree with the evidence.

It could be argued that, in re-enactments, there is some idealization of the original event involved in the sense that, in choosing which features to include in constructing the re-enactment, the original event is characterized in terms of only some of its features, others being neglected. However, the re-enacted event is just as concrete as the original event, so the model is not abstract. The re-enacted event is a model in virtue of its simi-

larity to the hypothesized original event with respect to the causes of the phenomenon being investigated (e.g.,, blood spatters, damage to structures, injuries, fingerprints, etc.). As with experimental scale models, the conditions for similarity between model and thing modelled are stated in terms of items that must be identical between them, and a statement that they are similar is a statement of identity of the features relevant to producing the evidence of interest.

Classification of Models into Kinds

Reflecting upon the similarities and differences of these six examples, there is a natural clustering of the first three into one kind of model, and the last three into another. For the first three kinds of model:

A.   Mechanical Models of Electrodynamic Equations

B.   Models of Axioms -- Arithmetic and Geometry

C.   Mathematical Models Used for Simulation

the model is a model in virtue of its relation to some equations or formal statements. The models are abstract in that they are mathematical structures, algorithms, or descriptions of mechanisms. They are something grasped in thought (as Frege might put it), rather than something located in time and space.

In contrast, in the last three examples of models surveyed above:

D.   Model Organisms in Biology

E.   Experimental Scale Models

F.   Re-enactments of Events

the model is a model in virtue of a similarity or analogical relation it bears to some other

physical object. In earlier papers about the methodology of experimental scale models,

I have referred to this as "using one piece of the world to tell about another"; the phrase

is apt for model organisms in biology and re-enactments of events as well. The model

is not abstract in any of these last three examples, for it is a concrete piece of the actual

world.

 For ease of discussion, let me refer to the kind of model associated with the first cluster

of examples as a "realm of thought" kind of a model. (I do not mean that the phrase

"realm of thought" captures or defines the kind of model; I am only using it here in this

paper as shorthand to refer to the kind of model that is associated with the cluster of the

first three examples of kinds models (A. B. and C. above)) By way of contrast, let me

refer to the alternative kind of model associated with the second cluster (D., E., and F.

above) as the kind of model that is a "piece of the world".

Towards a More Comprehensive Notion of Models

In philosophy of science, the notion of model that is dominant comes from the first clus-

ter of examples of models, the "realm of thought" kind. In all of the examples given

above of this kind, the model is a model in virtue of being a model *of* some equations or

formal statements. As a result, the notion of model that is dominant in philosophy is that

a model is a model of some equations or formal statements.

The "piece of the world" kind of model is not totally ignored in philosophy of science --

after all, the idea is ubiquitious in our everyday life; a map, for instance, is a scale

model, and a measuring instrument is one piece of the world often used to tell about

another piece -- but the problem is that in philosophy it is soon put off to the side as not

really having the kind of formal structure or generality that lends itself to philosophical

reflection. Constructing scale models or setting up re-enactments or choosing animal

models is often seen as an applied art, a skill not expressible as a formal method. In

other papers I brought to the attention of philosophers the point that, actually, there are

quite formalized methodologies for inferences based upon experimental scale models

(Sterrett (2002), Sterrett (2005/2006))! The point holds more generally, for other "piece

of the world" kinds of models. The "piece of the world" models are concrete, but often

there are formal methods of showing that one concrete thing models another.

Another significant difference between the more mainstream "realm of thought" kind of

models and the "piece of the world" models appears upon stepping back and looking at

the overall picture of how model and theory are interrelated in each kind of model.

On the mainstream view of the "realm of thought" kind of models, the way models help

is that the model is an intermediary of some sort between the theory (conceived of as a

set of equations or a set of formal statements) and the actual world. Often, it is re-

marked, the theory applies only to some idealized or abstract representation of the

world. So, to apply the theory to the world, e.g., to make predictions or retrodictions,

one has to first construct a model, and then apply the theory to the model. Even when

the model is considered physically realizable, as in mechanical models of Maxwell's

equations, the complaint has been made that the model is in some sense "abstract" in that it is not a specific object in the world, but is imagined or visualized, a mental construction. The criticism based upon this view of how scientific equations or theories apply to the world goes something like this: if theories apply to the world only in virtue of applying to such abstract objects, then theories aren't really about the actual world, are they? They're just about abstract models!

Whether or not you think such a charge fair, it is interesting that this criticism can't even be formulated for the "piece of the world" kind of models. The criticism just melts away, because the models are not, as with the first cluster of examples, models in virtue of being models of a set of equations or a set of statements.

Yet, there is still formal methodology involved in reasoning employing the "piece of the world" kind of models. Generally formal methods or scientific laws come into play in establishing criteria for similarity or identity of two situations (always with respect to some phenomenon or feature) and in showing that these criteria are satisfied. In the case of experimental scale models, the formal methodology is the methodology of establishing physical similarity by dimensionless parameters. For model organisms, certainly sophisticated knowledge of biological systems and processes is involved in selecting an appropriate model organism as a model for a particular biological phenomenon of other organisms, and these are not a matter merely of a individual's skill even in cases where that skill has led to the choice of model organism, for the analogy between the model and target organism can be formalized and rationalized. The reasoning might employ laws of biology to argue for the generalization of a process from a model organism to

other organisms like it in the relevant ways, and to specify what these relevant features are. Likewise, in re-enactments of events for forensic purposes, the argument that the re-enactment is a good model of the hypothesized original event is based upon establishing similarity between the two situations. This involves formal scientific knowledge about the kinds of factors that need to be kept the same for the phenomena to be the same between the two situations: which masses, velocities, densities, viscosities (of blood) and so on causally determine the important features of the forensic evidence one is investigating. The researcher doesn't necessarily need to calculate or even know the values of the important parameters, so long as he or she can establish that they are the same in the re-enactment as in the hypothesized original event.

Thus, even those who are interested only in formal methods in philosophy of science will gain from the more comprehensive notion of models I am advocating here. I have clustered the kinds of models surveyed into "realm of thought" and "piece of the world" only for the purpose of illuminating the differences between the kinds of models generally included in analytic philosophy of science under the rubric of "model" and the kinds that it has neglected. The "piece of the world" kind of model is actually used a lot in scientific endeavors. In fact, I believe it is common to use both kinds of models to investigate and solve a given practical problem. A number of philosophers have argued for the recognition of more pluralism in scientific methods. My hope in this paper is to advance the acceptance of the neglected kinds of model I have loosely referred to as "a piece of the world" kind of model by pointing out that the methodology of such modelling can be embraced by analytic philosophers of science without giving up the rigor that has characterized the discipline.

References

Kaye, Richard. (1991) *Models of Peano Arithmetic.* Oxford Logic Guides 15, Oxford University Press.

Rand, Michael S. (2004) "Selection of Animal Models", Lecture given September 27, 2004 at the University of Arizona-Tuscon.   Available online at
http://www.ahsc.arizona.edu/uac/notes/classes/animalmodels/animalmodels03.html Accessed June 10, 2005

Sterrett, Susan G. (2002) "Physical Models and Fundamental Laws: Using One Piece of the World to Tell About Another."   *Mind and Society*   5, 2002, Vol. 3: 51-66. (Preprint available on the Philosophy of Science Archives at http://philsci-archive.pitt.edu/archive/00000720

Sterrett, Susan G. (2005/2006) "Models of Machines and Models of Phenomena." Presented at Workshop "Applying Science", PSA 2004 (Philosophy of Science Biennual Meeting 2004) Austin, Texas, November 18, 2004. *International Studies in the Philosophy of Science* (forthcoming) Preprint available at http://philsci-archive.pitt.edu/archive/00002088

Turner, Joseph (1955) "Maxwell on the Method of Physical Analogy." *The British Journal for the Philosophy of Science*, Vol. 6, No. 23 (Nov. 1955), 226-238.