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Non-Arbitrage In Financial Markets: A Bayesian Approach For Verification

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Abstract. The concept of non-arbitrage plays an essential role in finance theory. Under certain regularity conditions, the Fundamental Theorem of Asset Pricing states that, in non-arbitrage markets, prices of financial instruments are martingale processes. In this theoretical framework, the analysis of the statistical distributions of financial assets can assist in understanding how participants behave in the markets, and may or may not engender arbitrage conditions. Assuming an underlying Variance Gamma statistical model, this study aims to test, using the FBST - Full Bayesian Significance Test, if there is a relevant price difference between essentially the same financial asset traded at two distinct locations. Specifically, we investigate and compare the behavior of call options on the BOVESPA Index traded at (a) the Equities Segment and (b) the Derivatives Segment of BM&FBovespa. Our results seem to point out significant statistical differences. To what extent this evidence is actually the expression of perennial arbitrage opportunities is still an open question.

Keywords: Non-Arbitrage; Options; Variance Gamma; Full Bayesian Significance Test.
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INTRODUCTION

The concept of arbitrage (profit without risk), or rather its non-occurrence, plays a key role in finance theory. For non-arbitrage markets, under completeness and other standard regularity conditions, it is possible to prove the *Fundamental Theorem of Asset Pricing*. This theorem states the existence and uniqueness of a consistent pricing system for all the market's securities, when prices are expressed in a common *numeraire* asset (for example, nominal dollar prices discounted by a basic interest rate structure over time). Consistent current prices of an asset can then be computed as the expected value, with respect to an equivalent *risk neutral* or martingale probability measure, of the asset's *payoff structure*, that is, a full description of the asset's value in each of the possible states of nature in the future.

More synthetically, as pointed out in the preface of [10], the Fundamental Theorem of Asset Pricing states that, in a regular mathematical model for financial markets, the non-arbitrage principle holds if and only if there is an equivalent probability measure that makes prices martingale processes. For fundamental insights and interpretations of

[1] The ideas presented in this article express the authors' personal ideas, and not the view of any private or public institution.

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these results, see also [6], [9], [11], [15], and [16]. For a much wider perspective of equilibrium conditions in economical theory, see [17].

The objective of the study is to estimate and compare parameters of statistical distributions governing some specific assets. Using the FBST - the Full Bayesian Significance Test, we check if there is a significant difference on the parameters estimated for the same financial asset traded at two distinct markets. Detection of significant discrepancies may indicate distinct behaviors of the participating agents in these markets, a difference that, in turn, may generate arbitrage opportunities. Specifically, we will study contracts that are traded at two distinct markets, namely options on the Bovespa Index, Ibovespa, traded at (a) the Equities Segment and (b) the Derivatives Segment of BM&FBovespa.[2]

This paper is organized as follows: Section 2 introduces the Variance Gamma model, used as the basis of our statistical analyses; Section 3 describes the FBST methodology for testing the statistical significance of sharp hypotheses; Section 4 discusses the available empirical data bank; Section 5 covers empirical estimation procedures; Section 6 describes the implementation of MCMC algorithms used for numerical integration; and Section 7 gives our conclusions and final remarks.

THE VARIANCE GAMMA PRICE MODEL

Modern theory of option pricing started with the work of [3] and [26], assuming the hypothesis of normality for continuously compounded returns. Strong empirical evidence, like volatility smiles and smirks or sporadical market crashes, suggested the need to extend the theory to more general statistical models, exhibiting skewness, kurtosis and time-varying volatility structures. Some early examples of such extended models of particular historical importance were given by [30] and [22]; for general overviews, see [5] and [31].

The Variance Gamma (VG) model, initially presented in [24] and [23], and generalized in [22], has achieved in the past few years considerable popularity among financial market quantitative traders. According to its authors, VG models can accommodate very well the empirically observed volatility smiles as well as prize-for-asymmetry, by calibrating or estimating the model's parameters related to kurtosis and skewness.

The VG process is an extension of the standard Ito process, characterized by constant drift and diffusion, where time unfolds as a random variable following a stochastic Gamma distribution. The basic intuition of this model is that "time" accounts for relevant economic action, having as many random jumps as the market activity engenders. With a few small changes in the terminology used in [22],

$$X(t;\sigma,\nu,\mu) = \mu \cdot \gamma(t;1,\nu) + \sigma \cdot B(\gamma(t;1,\nu)), \tag{1}$$

in which $X(t;\sigma,\nu,\mu)$ is the VG process, μ is its drift, σ is its volatility; and *B(t)* is a standard Brownian Motion. Finally, the Gamma process with mean rate μ and variance

[2] BM&FBovespa, created in 2008, through the integration between the SÆo Paulo Stock Exchange (Bolsa de Valores de São Paulo) and the Brazilian Mercantile and Futures Exchange (Bolsa de Mercadorias e Futuros), is the 3rd largest financial exchange worldwide.

rate ν, $\gamma(t;\mu;\nu)$, has independent gamma increments over non-overlapping time intervals, $g(h) = \gamma(t+h;\mu;\nu) - \gamma(t;\mu;\nu)$, following a gamma density with mean μh and variance νh. This key compositional property in an expression of the reproductive property for the Gamma distribution, as discussed in [28], and can be generalized to other stochastic processes, see [33].

One appealing characteristic of the VG process is that it nests the Ito process (used in the Black and Scholes model) as a special case. Moreover, conditional on the realization of a random time change, *g(t)*, the process $X(t;\sigma,\nu,\mu)$ is normally distributed. Hence, the unconditional density for the *X* process can be obtained integrating on the gamma distributed increment,

$$f(X) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi g}} \exp\left(\frac{-(X-\mu g)^2}{2\sigma^2 g}\right) \frac{g^{\frac{t}{\nu}-1}\exp\left(\frac{-g}{\nu}\right)}{\nu^{\frac{t}{\nu}}\Gamma\left(\frac{t}{\nu}\right)} dg. \tag{2}$$

Using the VG process to replace the standard geometric Brownian motion, it is possible to greatly extend many standard models of dynamic evolution, and still obtain tractable analytical or semi-analytical solutions. In particular, [22] extend the classical Black-Scholes model expressing the evolution of a (fundamental) asset price as

$$S(t) = S(0) \cdot \exp(r \cdot t + X(t;\sigma,\nu,\mu) + \omega \cdot t), \tag{3}$$

in which the parameter $\omega = \nu^{-1} \cdot \ln(1 - \mu \cdot \nu - \sigma^2 \cdot \nu \cdot 2^{-1})$ is determined so that $E[S(t)] = S(0) \cdot \exp(r \cdot t)$, where r is the riskless interest rate.

In this setting, it can be shown that

$$[\ln(E(S(t))/S(0))] = r \cdot t \quad \text{or} \quad S(0) = E\left[S(t) \cdot \exp(-r \cdot t)\right] \ ,$$

implying a compatibility with the general framework given by Fundamental Theorem of Asset Pricing, as commented in the introduction.

In this framework, given a fundamental asset that follows a VG process, it is possible to compute the price of a call option with strike value of K and maturing at time t: Expressing this option price by the martingale condition, $c(S(0);K,t) = \exp(-r \cdot t) \cdot E[max[S(t) - K;0]]$. Furthermore, [22] demonstrates that this mathematical expectation can be analytically expressed as follows, where Ψ is a function defined in terms of Second Order Modified Bessel Functions and degenerate hypergeometric functions of two variables.

$$\begin{aligned} c(S(0);K,t) = \ & S(0) \cdot \Psi\left(d \cdot \sqrt{\frac{1-c_1}{\nu}}, (\alpha+s) \cdot \sqrt{\frac{\nu}{1-c_1}}, \frac{t}{\nu}\right) \\ & -K \cdot exp(-r \cdot t) \cdot \Psi\left(d \cdot \sqrt{\tfrac{1-c_2}{\nu}}, (\alpha \cdot s) \cdot \sqrt{\tfrac{\nu}{1-c_2}}, \tfrac{t}{\nu}\right), \end{aligned} \tag{4}$$

with

$$d = \frac{1}{s}\left[ln\left(\frac{S(0)}{K}\right) + r \cdot t + \frac{t}{\nu} \cdot ln\left(\frac{1-c_1}{1-c_2}\right)\right] \ , \tag{5}$$

$$\alpha = \zeta \cdot s\ ,\ \ \zeta = \frac{-\mu}{\sigma^2}\ ,\ \ s = \frac{\sigma}{\sqrt{1+\left(\frac{\mu}{\sigma}\right)^2 \cdot \frac{\nu}{2}}}, \tag{6}$$

$$c_1 = \frac{\nu \cdot (\alpha + s)^2}{2}\ ,\ \ c_2 = \frac{\nu \cdot \alpha^2}{2}. \tag{7}$$

SHARP HYPOTHESES AND THE FBST

As explained in the introduction, we intend to test the validity of the non-arbitrage condition in some Brazilian markets. In order to accomplish this task, we will analyze price time-series for option contracts on the BOVESPA Index traded at (a) the Equities Segment and (b) the Derivatives Segment of the BM&FBovespa Exchange. Specifically, we will estimate and compare parametric models for these two price series using the VG statistical model described in the last section. In this setting, our specific task is to test the significance of a statistical hypothesis stating a compatibility condition for the two price series under study.

If the two price series were unrelated, the corresponding and independent VG models would have six free parameters, namely, (σ_a, ν_a, μ_a) and (σ_b, ν_b, μ_b). However, as explained in the previous sections, under the non-arbitrage condition, the following hypothesis, H, expressed as a (vector) equality equation, must hold:

$$H:\ \ [\sigma_a, \nu_a, \mu_a] = [\sigma_b, \nu_b, \mu_b]\ .$$

This condition implies reducing by half the dimension (or degrees of freedom) of the parameter space under consideration. Hence, following this path, the abstract non-arbitrage condition is translated into a concrete sharp statistical hypothesis in our statistical model.

Testing the significance of sharp hypotheses poses several technical and epistemological difficulties for traditional Bayes Factors. The FBST was specially designed to give a measure of the *epistemic value* of a sharp statistical hypothesis H, given the observations, that is, to give a measure of the *value of evidence* in support of H given by the observations. This measure is given by the support function $\mathrm{ev}(H)$, the FBST *e-value*.

Let $\theta \in \Theta \subseteq R^p$ be a vector parameter of interest, and $p(x\,|\,\theta)$ be the likelihood associated to the observed data x, as in the standard statistical model. Under the Bayesian paradigm the posterior density, $p_n(\theta)$, is proportional to the product of the likelihood and a prior density,

$$p_n(\theta) \propto p(x\,|\,\theta)\, p_0(\theta).$$

The (null) hypothesis H states that the parameter lies in the null set, defined by inequality and equality constraints given by vector functions g and h in the parameter space.

$$\Theta_H = \{\theta \in \Theta \,|\, g(\theta) \leq \mathbf{0} \wedge h(\theta) = \mathbf{0}\}$$

From now on, we use a relaxed notation, writing H instead of Θ_H. We are particularly interested in sharp (precise) hypotheses, i.e., those in which there is at least one equality constraint and hence, $\dim(H) < \dim(\Theta)$.

The FBST defines $\mathrm{ev}(H)$, the e-value supporting (in favor of) the hypothesis H, and $\overline{\mathrm{ev}}(H)$, the e-value against H, as

$$s(\theta) = \frac{p_n(\theta)}{r(\theta)} \ , \ \ s^* = s(\theta^*) = \sup_{\theta \in H} s(\theta) \ , \ \ \widehat{s} = s(\widehat{\theta}) = \sup_{\theta \in \Theta} s(\theta) \ ,$$

$$T(v) = \{ \theta \in \Theta \,|\, s(\theta) \leq v \} \ , \ \ W(v) = \int_{T(v)} p_n(\theta) \, d\theta \ , \ \ \mathrm{ev}(H) = W(s^*) \ ,$$

$$\overline{T}(v) = \Theta - T(v) \ , \ \ \overline{W}(v) = 1 - W(v) \ , \ \ \overline{\mathrm{ev}}(H) = \overline{W}(s^*) = 1 - \mathrm{ev}(H) \ .$$

The function $s(\theta)$ is known as the posterior surprise relative to a given reference density, $r(\theta)$. Its role in the FBST is to make $\mathrm{ev}(H)$ explicitly invariant under suitable transformations on the coordinate system of the parameter space. The truth function, $W(v)$, is the cumulative surprise distribution.

The tangential (to the hypothesis) set $\overline{T} = \overline{T}(s^*)$, is a Highest Relative Surprise Set (HRSS). It contains the points of the parameter space with higher surprise, relative to the reference density, than any point in the null set H. When $r(\theta) \propto 1$, the possibly improper uniform density, $\overline{T}$ is the Posterior's Highest Density Probability Set (HDPS) tangential to the null set H. Small values of $\overline{\mathrm{ev}}(H)$ indicate that the hypothesis traverses high density regions, favoring the hypothesis.

Notice that, in the FBST definition, there is an optimization step and an integration step. The optimization step follows a typical *maximum probability* argument, according to which, "a system is best represented by its highest probability realization". The integration step extracts information from the system as a probability weighted average. Many inference procedures of classical statistics rely basically on maximization operations, while many inference procedures of Bayesian statistics rely on integration (or marginalization) operations. In order to achieve all its desired properies, the FBST procedure has to use both, having a simple and intuitive geometric characterization.

In general, the optimization and integration steps are performed numerically. The optimization step can be implemented using general porpose numerical optimization algorithms, like [1] and [2]. The integration step, is often tailor coded for each specific application using standard computational tools and techniques of Bayesian statistics like Monte Carlo and Markov Chain Monte Carlo procedures, see [13] for a general overview.

MARKET DATA

This study covers two options on Ibovespa (BOVESPA Index) spot, that are traded, respectively, at the Equities Segment of the BM&FBovespa Exchange and at the Ibovespa Futures at Derivatives Segment operated in the same institution. Regardless of some fine contractual distinctions concerning the definition of the underlying asset for each of these options, the definition of their liquidation process at maturity depends on the same economic variable, namely, the value of Ibovespa Settlement.[3]

The options at the Equities Segment are liquidated at maturity, according to the difference between the Ibovespa Settlement and the strike of the option. Meanwhile, on the expiration date, the parties in an option on Ibovespa Futures traded at the Derivatives Segment take positions in Future Ibovespa contracts. However, according to the rules of the Ibovespa Futures market, the underlying value that is traded in these contracts is the same Ibovespa Settlement. Hence, on the expiration date, these Futures are also liquidated accordingly to the Ibovespa Settlement. Thus, exercising the option on the expiration date, the parties in this contract on Ibovespa Futures are also (implicitly) assuming a liquidation according the difference between the Ibovespa Settlement and the strike of the option. Besides this subtle difference, all other relevant aspects of the options contracts are the same for both segments.

This study is based on market data for the aforementioned european call options on Ibovespa within the two months period starting on 15/12/2012 and ending on 14/02/2012. This data was captured for the Equities Segment and Derivatives Segment for strikes ranging from 54,000 to 72,000, with bands at 1,000 points. Because trading in Ibovespa Future option at Derivatives Segment only becomes bulky two months before its maturity, we considered only the contracts expiring in Feb./2012. In order to avoid non-synchronization effects, we used data captured in trades close to the marking to market call. The Ibovespa spot values were obtained near the time of each trade. In total 543 observations are available for analysis.

As a proxy of the Reference Interest Rate of the economy, we used the value of the fixed rate implicit in DI Futures contracts traded at BM&FBovespa. When maturity dates of the DI Futures differed from maturities dates of the options, the interest rates for the options were estimated using an exponential interpolation based on the standard 252 business days convention.[4]

EMPIRICAL LIKELIHOOD AND ESTIMATION PROCEDURES

One of the standard approaches to perform a Bayesian analysis in finance econometrics, is to formulate an empirical observation error model, combine it with the basic stochastic models driving the price evolution of financial assets, and derive a joint empirical likelihood function, see [7], [12], [20], [18] and [19].

This approach is used in [22] to estimate the parameters of the VG model. The authors formulate a simple observation error model for observed prices, w_i, relative to theoretical model prices, $\hat{w}_i$, using an exponential multiplicative structure, $w_i = \hat{w}_i \exp(\eta \varepsilon_i - \eta^2/2)$, where ε_i stands for the standard white noise, that is, a zero-mean unit-variance Gaussian process. According to [22], this formulation is well suited to deal with heteroskedasticity in option prices for different strikes. The combined model

[3] The Ibovespa Settlement is defined as the arithmetic mean of the BOVESPA Index in the three last hours of trading of the last trading day, including the end of the closing call.
[4] The source of the data on the options, DI Futures and Ibovespa spot was BM&FBovespa.

TABLE 1. Median ML-Estimators for the VG-Model, 12/2011-02/2012.

	Equities Segment	**Derivatives Segment**
σ (Volatility)	0.4092	0.2519
ν (Kurtosis)	0.0001	0.0001
μ (Skewness)	0.1411	0.0234
Total Number of Observations	481	62

renders the following empirical log-likelihood function,

$$\ln \ell = -\frac{1}{2}\sum_{i=1}^{M}\left(\frac{ln(w_i)-ln(\hat{w}_i)}{\eta}+\frac{\eta}{2}\right)^2-\frac{M\cdot ln(2\pi)}{2}-M\cdot ln(\eta)-\sum_{i=1}^{M} ln(w_i), \qquad (8)$$

where

$$\eta = \sqrt{2\cdot(\sqrt{1+k^2}-1)}\ , \quad k^2=\frac{1}{M}\cdot\sum_{i=1}^{M}(ln(w_i)-ln(\hat{w}_i))^2\ , \qquad (9)$$

M is the number of observations, w_i stands for the i-th observation of the option's price in the market, and $\hat{w}_i$ is the corresponding value calculated from the pricing model presented in Equation 4.

The numerical maximization of Equation 8 would render, as optimal arguments, the Maximum Likelihood or ML-estimators for this combined empirical model. However, in this exploratory work, we use the easier to compute and asymptotically equivalent non-linear least squares estimator that miminizes k, the sum-of-squares term in Equation 9.

Following this approach, we estimated approximate optimal parameters for options traded at the Equities and Derivatives Segments, considering the databank described in the last section, from 12/15/2011 to 14/02/2012, divided into ten blocks of weekly data. The median of these ten block estimates are displayed in Table 1, apparently showing a sensible dissimilarity between the two market segments. For example, considering the parameters in Table 1, an annual interest of 10.34%, strike at 66,000 and the BOVESPA Index trading at 61,820 pts, an option with 19 days to expiration would be priced at approximately \$189.00 on the Equities Segment and \$193.00 on the Derivatives Segment.

In Section 7 we will investigate the statistical significance of this apparent divergence between the VG-model estimated parameters for the aforementioned trading segments. The next section explains some technical details that are necessary for an efficient numerical implementation of the FBST integration step.

MCMC IMPLEMENTATION

As explained in Section 3, the FBST methodology requires the computation of definite integrals with the probability measure given by the posterior density for the statistical model in study, $p_n(\sigma,\nu,\mu \mid W)$, given the observed data, W. This section describes an efficient implementation of a Markov Chain Monte Carlo used to compute the required integrals, see [14] for general surveys and overviews of MCMC algorithms and their application in Bayesian Statistics.

The implementation of our MCMC is based in a hierarchical combination of the Gibbs and Metropolis-Hastings algorithms, used to generate a random sample, $(\sigma, \nu, \mu)^{(k)}$, $k = 1 \ldots m$, according to the desired posterior density, see [8].

At the higher level, the MCMC uses a Gibbs sampling procedure following a cyclic chain along conditional posterior densities, that is,

$$\sigma^{(k+1)} \sim p_n(\sigma \mid W, \nu^{(k)}, \mu^{(k)}) \; , \quad (10)$$

$$\nu^{(k+1)} \sim p_n(\nu \mid W, \sigma^{(k+1)}, \mu^{(k)}) \; , \quad (11)$$

$$\mu^{(k+1)} \sim p_n(\mu \mid W, \sigma^{(k+1)}, \nu^{(k+1)}) \; . \quad (12)$$

At each step of the Gibbs chain, one still has to sample from non-standard, although univariate, statistical distributions. This task is performed by a lower level sampling method based on the Metropolis-Hastings algorithm. This Metropolis-Hastings procedure uses a dynamically adjusted Gaussian kernel, $N(0, \xi)$, for generating random proposals, followed by the standard acceptance/rejection phase.

The numerical simulations in this article had, at the higher hierarchical level, the burn-in period, spacing among realizations and sample size, pre-set at, respectively, 200, 10 and 500. Fine tuning of these parameters for the very best performance in FBST applications, can be accomplished using the error control method presented in [21].

CONCLUSIONS AND FINAL REMARKS

In the preceding sections we defined a coherent framework for testing the non-arbitrage condition in some financial markets based on:

1. the Fundamental Theorem of Asset Pricing;
2. the Variance Gamma stochastic model for price evolution of a fundamental asset and the associated formula for pricing european options;
3. a carefully defined empirical likelihood function well suited for data analysis in financial econometrics;
4. the Full Bayesian significance test methodology;
5. the efficient implementation of computational algorithms; and
6. a carefully assembled data bank with price series of options on the BOVESPA Index traded at (a) the Equities segment and (b) the Derivatives Segment of the BM&FBovespa Exchange.

A direct translation of the non-arbitrage hypothesis presented in section 3 requires the optimization of a six-parameter model under a three dimensional vector constraint, followed by a six-dimensional integration operation. In this exploratory work, we will test a pair of much simpler hypotheses, namely, the point hypothesis stating that: H_{ab}: the "true" parameters for the price model in segment (a) are equal to the maximum likelihood estimates obtained in segment (b), and vice-versa, namely, the hypothesis

TABLE 2. *e*-values supporting complentary point hypotheses of non-arbitrage for two distinct Ibovespa market segments.

	H_{ab}	H_{ba}
e-value of H	0.242	0.066

H_{ba} stating the converse situation. Table 2 presents the FBST *e*-value supporting these two hypotheses.

The computed *e*-values do not support either of the two complementary point hypotheses. The following empirical observation seems to corroborate this conclusion, pointing to divergent behavioral patterns for the agents participating in the two market segments: While trade at Equities Segment seems to be more evenly distributed over time, at the Derivatives segment it peaks around one month before expiration.

To what extent the statistical evidence obtained in this study actually expresses a perennial arbitrage opportunity between the two market segments is still an open question. However, our study seems to indicate that, at some moments, traders in the two segments appear to price the same future payoff structure differently, suggesting that, at least at these specific moments, real arbitrage opportunities could be found.

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