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Physical Models and Fundamental Laws: Using One Piece of the World to Tell About Another

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1. Introduction

In this paper, I will address a rather specific question that is seldom asked: "How are fundamental laws involved in the practice of scale modeling?" The reason this specific question deserves our interest, however, arises from what it has to do with the more general question of how laws and models can be related to each other: The answer to the specific question of how fundamental laws are involved in the practice of scale modeling is very different from answers usually given to the more general question.

The point of difference, we shall see, is this: the answer to the general question "How are fundamental laws involved in reasoning that uses models?" is often something to the effect that the only way for fundamental laws to apply to the real world in model-based reasoning is for them to be applied to --- or, sometimes, to be identified with features of --- abstract or idealized models. In contrast, this view does not cover scale models, which are concrete. Although the answer to the more general question is often regarded as comprehensive, i.e., meant to cover all model-based approaches, some of the authors who have provided proper foundations for these claims have realized that they must explicitly exclude scale models from their accounts of how models and laws are related. (e.g., Hesse, 1967; Boltzmann,1904)

Many different varieties of models are recognized in philosophy of science, but there is something they tend to have in common: on most views of scientific inference that involves models, the model is thought to act as some sort of intermediary between the actual world and something abstract, such as a principle, theory, or law. Sometimes the model itself is taken to be abstract. Here's an example of the sort typically given to illustrate the function of a model: suppose one has a theory or set of equations describing how electricity flows in a network. The theory may be a set of statements or it may be something even more abstract (e.g., on the semantic view, set-theoretic objects and relations between them). To apply a theory or a set of equations to an actual network, one might proceed by making an idealized model of a particular network. On the common view, after the idealized model of the network has been formulated, one then uses formal rules about how to draw inferences from the theory, equations, or statements describing the laws governing the behavior of electricity in networks to infer the behavior of the model network.

It is not going to be important to making my point whether the formal methods one uses in describing or making predictions about the model are geometrical, graphical, statistical, set theoretic, or logical. What is important to my point is that, on the common view, (i) the involvement of the formal methods is that they are used to infer features of, or the behavior of, the model network; and (ii) there is one further step: mapping the results drawn for the model onto some piece of the world: e.g., an expected observation, an action to be taken, and so on. The reason that I have not here made anything important depend upon whether one tends to use a "syntactic" or "semantic" view of theories is that, for the point I will be making in this paper regarding models, the difference does not matter: on either view, when it comes down to using the theory in explaining or predicting phenomena in our experience, the notion of model in play is that the model is abstract and needs to be mapped onto "the world" in some way.

That these features of models are common to so many different accounts of what a theory is reflects, I think, that these features of models are somewhat ingrained in pure science, philosophy of science, cognitive science, and artificial intelligence: so much so, that it doesn't seem as though this general notion of model could be excluding anything -- it seems to be comprehensive. However, it is not comprehensive. The alternative to it I discuss here is valuable in that it illustrates something that may not occur using the ingrained notion of model I've alluded to above. It illustrates how laws can be involved in model-based reasoning which does not treat models as mediators between the real world and our formal methods -- in which the models are <u>in</u> the world.

That such an alternative exists is significant to many views in philosophy of science. One example is the topic of scientific realism. For if it were true that laws can apply only via the use of models, and if, as many believe, the only way that laws can be involved in model-based reasoning is indirectly, by applying to a model, those who conclude that scientific laws can't be about the actual world we experience (but only about models) might have a point. I want to allow that their point might be well taken if it were true that fundamental scientific laws are only applicable to abstract models, but I want to show that in fact we <u>can</u> employ fundamental scientific laws in a way that is not restricted in application to idealized or abstract models. The methodology upon which scale modelling is based illustrates that the following is possible: a method that involves using basic fundamental laws along with models to predict behavior in a very specific, very complex, actual physical situation, even in cases in which the laws relied upon do not necessarily provide a way to describe behavior in either the model employed, or in the target situation.

To give a preview of the contrasts I want to illustrate:

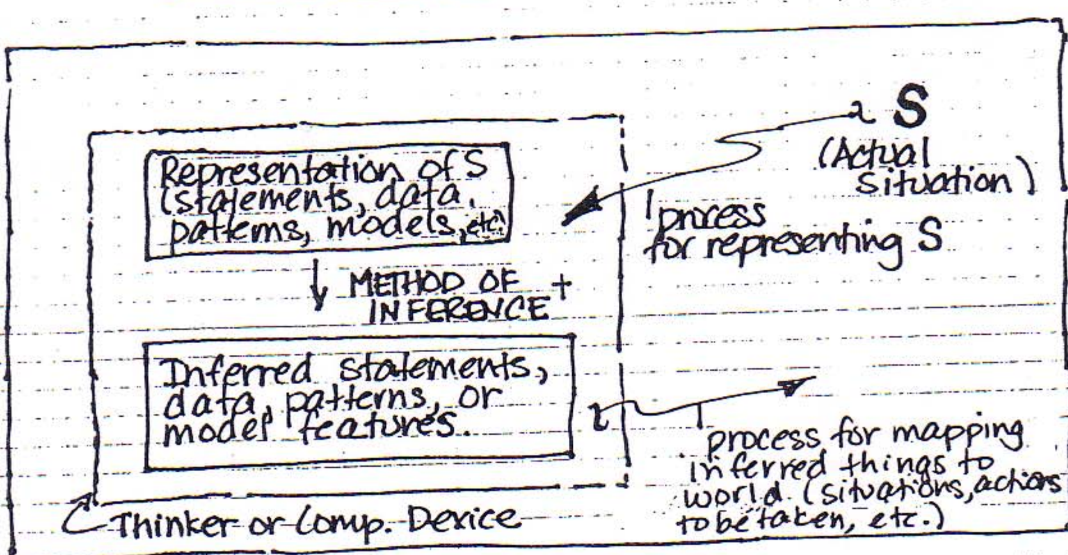
First, a contrast regarding the way models are used to draw inferences: on the standard view of models, inferences are drawn based upon models that function as intermediaries between something abstract & something concrete, whereas, on the method of physical similarity, inferences are drawn based upon models that are concrete, that is, the inference is drawn from a concrete situation (model) to a concrete situation (the thing modelled).

Second, a contrast regarding the way that models and fundamental laws are related: on the standard view of models, the laws describe and/or predict what happens in the model, whereas, on the method of physical similarity, the laws do not necessarily provide a means of predicting what happens in the model, but are used only to establish similarity of concrete situations.

2. Some Common Notions in Philosophy of Science

To see why these two contrasts -- one of which is associated with the ways models are employed when they are used in drawing inferences, the other with the way in which fundamental laws are involved in the use of models --- are of significance to questions in philosophy of science, it will be helpful if I identify what I consider basic to many different approaches to inference in philosophy of science. This is depicted in Figure I.

FIG. (I.) GENERAL SKETCH OF MOST VIEWS OF SCIENTIFIC INFERENCE:



+ METHOD OF INFERENCE need not be static; process can be iterative (e.g., neural nets constantly being updated) and/or adaptive.

Basically, the sketch shows scientific reasoning proceeding using representation (in the most general sense -- i.e., even neural networks with sensors take as their inputs a pattern; even statistical methods work on data). Models may be representations that have more structure than these other kinds of representations, but, on the view that models are abstract and that they represent, they fit this very general sketch.

My intent is to first present this picture, which I believe describes many accounts of scientific reasoning, including many accounts of model-based reasoning, in such a way that we can see why it seems plausible that it is a comprehensive account --- but then to describe a kind of model which cannot be accomodated by this picture. My purpose in presenting the method this way is to pinpoint the difference between the notion of models implicit in commonly-held views and the notion of models appropriate to describe the use of the methodology of applying the principles of physical similarity when using scale models to draw inferences.

Now, the principle of physical similarity is applied by building a model that is (by design) similar to another physical situation in certain respects, and inferring features about the latter physical situation, by using the principle of physical similarity to justify the conclusion that the two situations are similar in certain other respects. Thus it is a species of inference by analogy. In general, we might ask, how well do accounts of scientific analogical inference fit into the picture of scientific reasoning sketched in Figure 1?

A standard and general account of analogical reasoning goes as follows. The premises of the argument are that two objects (where object is to be construed in the most general way, to include situations, phenomena, processes, and so on) are compared. From the fact that object 1 (**S1**) and object 2 (**S2**) both have properties A, B, and C, and the fact that object 1 (**S1**) also has property D, it is inferred that object 2 (**S2**) also has property D. There are different accounts of what makes an analogical inference strong; in general, some account must be given to support the similarity of **S1** and **S2**. Generally the fact that **S1** and **S2** both have properties A, B, and C is not the only basis upon which the inference is based. Sometimes there is additional information supporting the claim that property D is causally related to properties A, B, and C. Sometimes we have some reason for thinking that **S1** and **S2** are structurally similar (e.g., model organisms and humans).

On this characterization of analogical inference, do analogical inferences fit into the general sketch of scientific inference given in Figure I, or do they present a problem for that sketch?

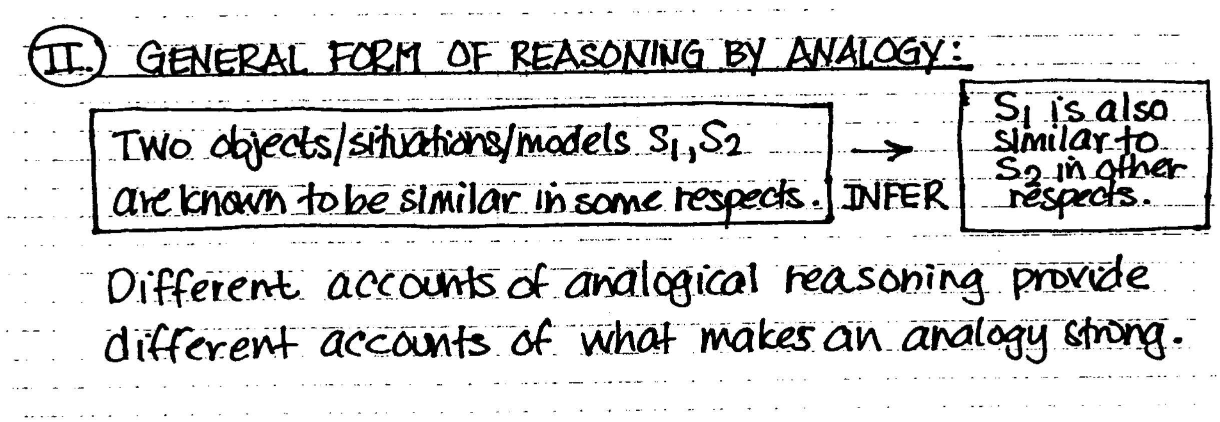


Figure 2

Actually, the account of analogical reasoning just given will fit nicely into the sketch in Figure 1, subject to some qualifications, namely: (i) that one can represent in some manner the premise that **S1** and **S2** are similar, and (ii) that the method of inferring the additional ways in which **S1** and **S2** are similar (i.e., in inferring that **S2** also has property D) can be accomplished by the thinker or computational device. So, whether or not a particular kind of analogical reasoning can be accomodated by the comprehensive sketch of scientific inference depicted in Figure 1 depends on whether or not these two conditions hold. Thus, we cannot make a blanket statement that all forms of analogical reasoning can be accomodated within the sketch in Figure 1. All we can say is that some kinds of analogical inference can be accomodated within the sketch in Figure 1; when it comes to a particular kind of analogical reasoning, however, we will need to examine whether these two conditions hold for the particular methodology under consideration.

Finally, let's examine how the general case of model-based reasoning might fit into the sketch in Figure 1, just as we did for analogical reasoning. In Figure 3, I've summarized what I take to be a fairly common presumption about models that covers many views. I will be challenging it later, but, for now, let's consider how the notion of models indicated in Figure 3 fits into the general sketch of scientific inference I've given in Figure 1.

FIG.

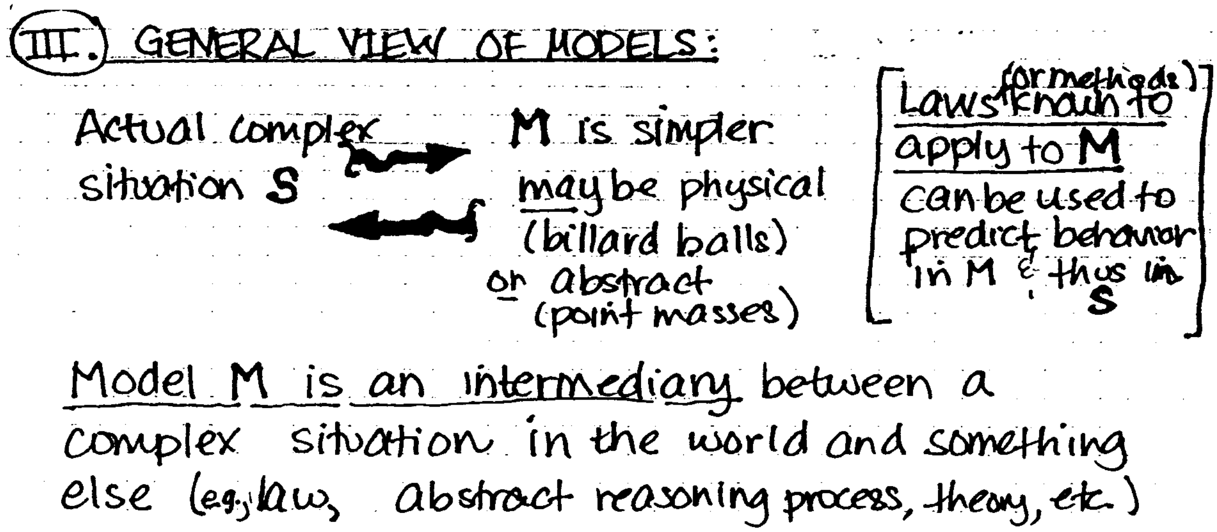


Figure 3

The idea behind the view I've shown in Figure 3 is basically that the model M is something that we can treat with the analytical tools we have at hand. These tools might be a theory, or some other method, such as a geometrical or computational algorithm, or a combination of them: the key thing is that we have a means of, a methodology for, making predictions about behavior in the model M. The sketch in Figure 3 is meant to be neutral with respect to syntactic or semantic views of theories and models.

I think you can see that, in this sketch, the role of the model is like the role of a representation: the model is easier to handle than what it represents and it is representative of something. As I explained above, my purpose in laying out this sketch is to clarify the common presumptions of most views of model-based reasoning, for I will be presenting a model-based methodology that I believe is not easily accomodated by such a sketch: the method of scale models, which are a special case of physical models.

3. The Method of Physical Models

The key idea in making scale models was illustrated in the text of a glossy, multi-page automobile advertisement that appeared in magazines a few years ago. There was very little text; just this:

> Whispers in the wind tunnel . . . screams on the autobahn. (Audi advertisement)

The first part of the text appeared on a page showing a car body inside a wind tunnel, as the air streamed smoothly over the body of the automobile. The second part of the text showed the actual automobile being driven on the highway.

This expresses the method of scientific inference used in scale modeling. Unlike the common view on which laws or other analytical methods apply to or within a model, what the method of physical similarity, which underpins the use of scale models, tells us is this:

> "**If** it whispers in the wind tunnel,
> **then** it will scream (go fast) on the autobahn (a highway with no speed limits)"

The inference is from physical situation to physical situation.

I don't have sufficient space here to fully present the method, or the arguments for its validity, but I will attempt to describe the methodology sufficiently to clarify why it is significantly different from the standard view of models. There are of course some rather straightforward uses of scale modelling, such as the use of geometrical scale models in the architectural layout process, where the only purpose of the model is to represent spatial relationships. These are such special cases of scale modelling that they hide the extremely interesting and sophisticated methods involved in more general cases of scale modelling, in which fluid phenomena, mechanical phenomena that depend upon stress-strain or other material properties, and heat transfer phenomena are modelled by small objects whose behavior can be used to predict the behavior of larger machines and situations. The basic ideas behind the method are not totally absent from mainstream general science curricula, but they are seldom emphasized if indeed they are pointed out at all. A high school physics text may mention in passing Galileo's point that the partial knowledge that the period of a pendulum is independent of the mass but varies as the square root of the length can be used to infer the period of a very large pendulum merely by making measurements on a scale model and multiplying the result by the square root of the size ratio between the model pendulum and the full-size pendulum, but the more general point that there is a methodology of physical similarity to be appreciated and formalized is generally not made. The focus still tends to be on the specific equations or conservation laws which can be used to compute quantities such as the period from values of other quantities, and not on how one could answer questions about one pendulum by building another that is a scale model of it.

Scale modeling that is based on the principle of physical similarity is a method for predicting the behavior of certain quantities in a specific, complex, situation **S1**. Different foundations for the method have been developed and refined over the past half-century. Here I ignore subtleties arising from those differences, and describe the method in terms of the most common foundation given, which is based on dimensional analysis, i.e., on an analysis of the dimensions of the quantities upon which the phenomenon of interest depends.

Using consistency of dimensions and the knowledge of the dimensions of the quantities that are relevant, for all the quantities that are (i.e., one must know which quantities the phenomenon of interest depends upon), one can:

--- Build (i.e., construct) a physical situation **S2** that is similar to **S1** in the relevant respects,

--- Develop the rules for transforming the values of quantities in **S2** to values of quantities in **S1**.

Once the model (**S2**) is constructed, one measures the quantities in/observes the behavior of the physical situation **S2**: **S2** is the scale model, or experimental model.

Then, from one's observations or measurements on **S2** (the scale model), one infers the quantities of interest in **S1** (the situation modelled).

Now, let's reflect on the method:

First, is this method of reasoning analogical? It is, and in fact it is quite naturally accomodated by the account of reasoning by analogy given above and illustrated in Figure 2: the model and the thing modelled can be considered two objects or situations **S1** and **S2** known to be similar in some respects (the respects in which you kept **S2** the same as **S1** when you built **S2** in such a way that it would be (in a sense that can be made rigorous) *physically similar* to **S1**) are taken to be similar in other respects (the feature of **S1** which you inferred based upon your observations of **S2** (the scale model)). To avoid any misunderstanding about the inferred feature: we do need to take care in how we specify the feature of **S1** to be inferred, so that the description of that feature (e.g., period of oscillation) allows one to infer the <u>corresponding</u> feature of **S1**; the inference is from the value of a quantity in **S2** (the scale model) to the corresponding, or transformed, value of the quantity in **S1**. However, this subtlety occurs in many kinds of analogical reasoning and is not special to scale modelling. When the analogy involves a mapping from two very different domains this point is obvious. I mention it here only for clarification: as the kind of quantity is the same in both **S1** and **S2**, we need to keep in mind that we must apply a multiplicative factor (which can be different for different quantities) to the value of a quantity in **S2** in order to obtain the value of the corresponding quantity in **S1**.

Secondly, suppose we consider this method of reasoning "model-based", inasmuch as it employs **S2** as a model (albeit a concrete one) to draw inferences about **S1**. Is the notion of model involved in this method accomodated by the account of models given above and summarized in Figure 3?

I think the answer to this is "No". For, the way in which scientific laws are involved in scale modeling is not as described in Figure 3. Why not? Well, first of all, because, even on a very permissive notion of law, the laws and equations involved are <u>not</u> used to describe or predict behavior of the model. Nor can the method of scale modelling be reformulated so as to be redescribed or reduced to another of the form shown in Figure 3, for this reason: in the general case, the laws can't be used to predict what happens in the model M because the method only requires partial knowledge to supplement the use of the model. By partial knowledge, I mean that the information used in conjunction with the model is not enough information to determine the value of the quantity inferred without the use of the model.

The method requires a lot less than that required by any method that fits Figure 3. In Figure 3, we establish that the model behaves like the thing to be modelled on the basis of some governing equation that could, at least in principle, be used to determine what happens in the object or situation modelled. Let me emphasize this point about the method of physical similarity: we need not be in possession of laws or equations that, even in principle, would be sufficient to predict what happens in either the model M (**S2**) or the object modelled (**S1**). This is why I say that the notion of scale models is significantly different from the notion of model illustrated in Figure 3, and that the methodology of physical similarity is not well-accomodated within the sketch of scientific inference illustrated in Figure 1.

4. Physical Similarity Versus Geometrical Similarity

This point – that the use of scale models (sometimes called analogue models) is qualitatively different from the use of abstract models – is often presented as a negative sort of point.

For instance, people are often concerned to point out that one can't make certain kinds of inferences based on scale models --- for example, if "scale model" is taken to mean simply built out of the same materials but to a different geometrical scale, people point out that there's no guarantee that the behavior of the model will be similar to the behavior of the full-size geometrically similar version. But there are two sides to this: we can become interested in how scale modelling works, when it does work, or we can set scale models aside as misfits that do not belong under the notion of model we want to use in philosophy of science. The point that scale models use a methodology that is significantly different from most other kinds of model-based reasoning, put in this negative way, as a warning, has been made on occasion by a number of philosophers writing about models. The response has been to set scale models off to the side. These philosophers, although correct in the negative aspect of the point (be wary of thinking scale models can be treated according to whatever account is being proffered), have seldom even noticed that there's a positive aspect to the point as well --- a very fruitful one, in fact!

The positive aspect of the point is this: the methodology of scale modeling can be used to draw inferences from scale models, though 'scale model' does not mean simply built to the same geometrical proportions but on a different size scale. Though a number of philosophers do respect that there is something substantially different going on in scale modeling, few take the time to give an account of <u>how</u> – or <u>why</u> ---- it works!

That such a methodology existed was mentioned early on by those writing about mental models, a topic of great interest around the turn of the century. Here's a remark from Ludwig Boltzmann's entry for "Model" in the <u>Encyclopedia Britannica</u>. (Boltzmann (1904)) First, he describes models used "in the mathematical, physical, and mechanical sciences," especially the view on which "thoughts stand to things in the same relation as models to the things they represent." Then, he remarks:

> A distinction must be observed between the models which have been described and those experimental models which present on a small scale a machine that is subsequently to be completed on a larger, so as to afford a trial of its capabilities. (Boltzmann (1904), p. 218)

Now, the view that Boltzmann had just presented in the discussion he gave preceding this remark included mechanisms, both naturally occurring and constructed by man, of which his account was:

> . . it is perfectly clear that these models of wood, metal, and cardboard are really a continuation and integration of our process of thought; for, according to the view in question, physical theory is merely a mental construction of mechanical models, the working of which we make plain to ourselves by the analogy of mechanisms we hold in our hands, and which have so much in common with natural phenomena as to help our comprehension of the latter. (Boltzmann, 1904 , p. 218)

That is the account of models he gives in his essay, but which, he says, does not fit scale models of machines. Why not?

Well, he says – and rightly so --- that, with experimental models:

> A mere alteration in dimensions is often sufficient to cause a material alteration in the action, since the various capabilities depend in various ways on the linear dimensions. (Boltzmann, 1904, p. 220)

He recognizes, however, the very significant point that such variation reflects physical laws:

> Thus the weight varies as the cube of the linear dimensions, the surface of any single part and the phenomena that depend on such surfaces are proportionate to the square, while other effects --- such as friction, expansion, and condition of heat, etc., vary according to other laws. Hence a flying machine, which when made on a small scale is able to support its own weight, loses its power when its dimensions are increased. (Boltzmann, 1904, p. 220)

But he discusses only the negative point that the ratios of volume to length are not invariant under geometrically similar transformations of a model, and does not address what would lead to the positive aspect of the point, namely the question: "How do scale models inform us?"

More recently, other writers such as Mary Hesse, Patrick Suppes, and Rom Harre, to name a few, have repeated this recognition of the negative point, while neglecting to develop the positive point. For instance, in her survey article in The Encyclopedia of Philosophy, Mary Hesse remarks on the distinction between scale models (also known as 'analogue machines') and logical models of formal theories; she warns that analogue machines are constructed in just those cases where "the explicit drawing of deductive consequences is impossible or impracticable," and that, in such complex cases, "it is dangerous to attempt to apply to scientific models those arguments that are valid in connection with logical models of formal systems," but she does not go on to give a positive account of how one would build an analogue machine for such a case. She doesn't discuss, so far as I can see, any other role for a formal theory in her account of physical models, than that of defining a formal relation that a model can be said to satisfy, or fail to satisfy. Examining the positive point in the practice of scale modeling will show us that there is another role that formal methods can play in reasoning that uses models.

We actually got a hint of the positive point in Boltzmann's explanation of why scale models weren't like the other models he discussed: there, he referred to the relation of the ratios of volume to length: it follows a rule; volume grows by the third power of the linear dimensions.

This is significant, for we can use this to draw inferences from small scale models to larger objects. To take an example of a rather straightforward case where the phenomenon I care about is simply a matter of volume – for instance, suppose that I want to know the volume of material (wax, or plaster, or bronze, or whatever) that will be required to fill a certain mold. I can figure this out by using a geometrically similar scale model – i.e., a smaller size object in which all the ratios of the linear dimensions in the small object are the same as the ratios in the large object about which I want to make inferences – IF I account for the fact that the volume increases or decreases as the cube of the linear dimension.

So, I can do the following: I make a scale model of a mold that is one tenth the size of the mold of some object X – if it's some complicated shape, such as a dolphin, or the shape of someone's head, I can get an artist to fashion a small version having the same proportions. Then, I see (experimentally determine) how much fluid it takes to fill the scale model, and multiply that volume by the cube of the ratio I used in making the scale model. For instance, if I used a scale of 1:10 in building the model, then I will multiply the amount of fluid it takes to fill the scale model by 10x10x10, or 1000, to infer the amount of fluid it will take to fill the mold for a (full-size) object X. Notice that I can do this even if I have no formula at all by which to determine the volume or shape of the mold.

You can see that this would be an effective method for determining the volume of a large statue without doing any volume calculations or estimations at all, so long as an artist could use his or her sense of proportion to fashion a smaller version. Nor is there idealization involved: we are not approximating the complicated shape of the statue by some more geometrically tractable shapes. That is, this process is not like the process of approximating a circle by a polygon so that you can handle the circular shape using methods you have developed that apply only to figures with sides that are straight lines. The example of determining the volume of a statue is a very simple example, in that the only scaling involved was scaling of lengths. Nevertheless, the example illustrates the basic steps of the process.

The key things required to use the method are just this:

(i) Keep the relevant dimensionless ratios the same in the model as in the thing modeled. In our example, the relevant ratios were ratios of linear dimensions (i.e., lengths); for other phenomena, such as heat transfer, different quantities would be important, and the relevant ratios we'd need to keep the same between the model and the object of interest would be different ones.

(ii) Know how the quantity of interest – in this case, volume – varies due to changes in linear dimension. Boltzmann assumes volume varies as the cube of the linear dimension. What knowledge is that based upon? Only that the space is taken to be Euclidean. The point here is that the kind of information is very fundamental, not that it is universally applicable or exceptionless.

Of course there are some phenomena for which we would not want to regard Euclidean geometry as fundamental, but there are plenty where we do. The important point here is that we can use such *very fundamental* assumptions to achieve *so much* in situations of partial knowledge. The kind of information used has to do with the nature of space, and not with specifics about the situation being modelled.

The basis for the inferences we can make is the principle of physical similarity. Now, similarity of physical situations is not an absolute matter; similarity will be with respect to a certain phenomenon or kind of phenomenon, and will be determined in the more general case by dimensionless ratios of various sorts. For the example above, the dimensionless ratio that was relevant was a ratio of lengths. For some other kinds of phenomena, the relevant dimensionless ratio for determining similarity is Mach number, which is the ratio of the velocity of an object to the velocity of mechanical waves in the medium with respect to which it is moving (here the velocity of mechanical waves depends upon the temperature and pressure of the medium). Another famous one is Reynolds number, which is a product of density, velocity, and length, divided by viscosity. The details of the ratio are not important for us right now; what is important here is that having the same Reynolds number indicates similarity of flow regime and many other important hydrodynamical behaviors. If one wants to know whether an incoming flow (say an underground stream flowing into a lake) is going to mix with the fluid in the larger body, or will instead remain separated and essentially flow as a stream within the lake, the relevant dimensionless ratio is Richardson number. There are dozens of such important dimensionless ratios in hydro- and aerodynamics.

The strategy used in building aerodynamic and hydraulic models is to build a model with the same important (i.e., important for the phenomena being investigated) dimensionless ratios; this results in models that are similar to the situation of interest with respect to the particular kinds of aerodynamic and hydraulic phenomena being investigated.

This is in fact the basis upon which scale models are based, and the methodology is used for constructing things for which being right matters very much, i.e., on which human lives and large investments depend: the operation of airplanes and ships, and the stability of buildings, bridges, and machinery. As I mentioned above, space does not permit a proper presentation and justification of the methodology, which by now has been provided with a very formal justification using the methods of abstract algebra. However, I want to emphasize two things about the process that bear on the philosophy of science:

First, the method does <u>not</u> require that you have equations that determine the behavior of either the model or the situation being modeled.

Secondly, the method <u>does</u> employ fundamental scientific laws. Examples of the kind of thing I mean by "fundamental" scientific laws are very general principles, such as conservation laws and the second law of thermodynamics.

One of the earlier uses of engineering scale models was that of modeling a ship travelling in a canal. As Boltzmann correctly pointed out, a geometrically similar model may behave very differently from its full scale counterpart. In the case of a model of a canal that is geometrically similar, the effect of the surface tension of the water in the scale model becomes disproportionately salient. So, researchers realized, one can't simply scale the linear dimensions of the canal by a fixed ratio, keeping the geometrical ratios the same. Yet, this didn't mean that the method of scale models had to be abandoned. It did, however, have to be modified.

Rather, the same methodology is applied, with some modification, as follows: use the method of physical similarity to determine the set of relevant dimensionless parameters for the phenomenon of interest – here, the phenomenon of interest is wave formation in canals. All that is required to do so is the knowledge (sometimes, a well-founded guess) of which quantities the phenomenon depended upon; exact equations are not required. Then, one constructs a scale model subject to the constraint that one strives to keep the relevant dimensionless ratios as close as possible between the scale model and the thing modeled. What one finds is that the resulting scale model of the ship and canal should be built using one scale for the horizontal linear dimensions, and another scale for the vertical linear dimensions. This is in fact how research on ships travelling in canals was carried out. This involves a compromise; giving up total geometrical similarity in order to keep other important dimensionless ratios the same between the model and the canal being modelled. Hence some practical judgement is involved in deciding what tradeoffs to make.

But, this kind of tradeoff is not an approximation of law to reality, nor of law to an idealized model. The laws are used to help determine what conditions yield physically similar situations (with respect to a certain phenomenon). How are the laws used to do this? Well, only very general pieces of information are obtained from them. Here, they are not used to predict anything directly, but, rather, to figure out what quantities a certain phenomenon depends upon (i.e, viscosity, density, velocity, temperature, length, and so on).

5. Something From Practically Nothing?

But if it isn't necessary to possess an equation or method that enables one to compute the quantity of interest, or to predict model behavior in a specific case, one may well wonder how so much can be inferred from so little. The most common foundation given for the method is dimensional analysis. (There have been further developments in the foundations of the method, but for the sake of simplicity I will not discuss such subtleties here.) The author of one of the main texts on the subject describes it this way:

> Dimensional analysis is a method by which we deduce information about a phenomenon from the single premise that the phenomenon can be described by a dimensionally correct equation among certain variables." (Langhaar, 1951, p. 1)

The philosopher Brian Ellis addresses the puzzlement many feel over dimensional analysis:

> The theory of dimensions has always been an enigma to the physical scientist. For it appears that here is a genuinely a priori method of obtaining knowledge. (Ellis, 1966)

In response to such puzzlement, Ellis makes two points:

(i) The information as to which quantities are related, and the information that those are the only ones involved, is actually a great deal of information, and

(ii) The significance of dimensionless ratios in science arises from the demand that numerical laws be (measurement) scale-invariant; i.e., that that expression of the law be the same no matter what measuring scales are used.

Ellis is mostly concerned with using dimensional analysis to derive formulae, such as the pendulum equation. I am instead concerned with how dimensional analysis can be used to tell us how to build a model and use it to draw inferences about a situation that may be inaccessible to us by any other means. That is, with how it permits us to *"use one piece of the world to tell about another"*. I agree with both of Ellis's points, and would add to them another, which is that what makes the fruitfulness of the method of scale modeling less surprising is that there is an actual slice of the world involved as well: the concrete scale model itself is an essential part of the bases upon which the inference is drawn.

6. Conclusion

Let's return to the general sketch of models I opened my talk with. My claim is that the way fundamental laws are used in the methodology of scale modelling is not as depicted in the sketch. In the sketch given in Figure 1, fundamental laws are employed to tell us what will happen in a model; in the methodology of scale modelling, they are instead used to tell us: "here's how to tell". They tell us what structural features of real situations to look for to identify situations that are physically similar with respect to some phenomenon, so that we can examine or build one within our practical reach, in order to determine what will happen in another situation that is not in our reach. Some examples of objects or situations not in our current observational reach include vehicles, machines and large projects such as dams, lakes, or canals that are being designed but do not yet exist, as well as events that have occurred in the past (e.g., unrecorded geological events) or will occur in the future (meteorological predictions).

Thus models employed in scale modelling are not well-described as mediating between a world of abstraction or theory and a world of phenomena: instead, the models are pieces of the world, and the methodology allows us to remain in the world of phenomena, so to speak, but gives us insight into phenomena so that we can tell what will happen in one situation we cannot observe based upon another situation we can observe.

It seems to me that this kind of model also provides an idea for what a basis for case-based approaches to problem-solving might look like. To investigate this suggestion, how would it work for some specific disciplines?

In Economics, it is quite natural to want to apply what can be learned about one specific, unique, economic situation to another. That is, despite the existence of analytic approaches that attempt to find principles and equations from which predictions in a specific situation could be made, it is also of interest to try to learn lessons from one unique economic situation or economy and apply them to another economic situation or economy. There are parameters thought to reflect overall structural features of an economy, for instance, velocity of money. So, we might ask whether we could apply the method of physical similarity, or at least dimensional analysis, to determine the relevant dimensionless ratios for certain kinds of economic phenomena. While the approach of building a scale model here would be impracticable, not to mention ethically questionable, there are some features of the methodology we might employ by using the method in reverse. For instance, even if we cannot build a scale model of an economy or economic situation in order to answer our questions about the consequences a certain policy (such as a tax break, an interest cut, or a tuition voucher program), we might instead work from the concrete cases about which we do have information, and see what kinds of things these concrete cases could be models of. If dimensional analysis could give us some insight into the dimensionless ratios important to labor or unemployment phenomena, we might attempt to see how they are related to critical economic phenomena. We could investigate whether there are critical values of these economic dimensionless ratios, just as a Mach number below 1.0 means there will not be shock waves, and just as Reynolds numbers below a certain number mean the flow will be laminar and that there will not be turbulent eddies. This is just a suggestion; I don't know the answer, nor whether there are analytical economic methods in use that might be described this way.

Sociology presents one of the most well known examples of a scale effect, i.e., of the phenomenon that an increase in (group) size may result in qualitatively different (group) behavior. It's well known that the kind of interaction that occurs in small groups does not occur when the group is larger. Just as the effects of surface tension become more salient in geometrically similar but smaller-sized models of canals, so there do seem to be some effects of social interaction that are proportionately more significant in smaller groups. Now, we might speculate: could we follow the path here that we took in response to Boltzmann's remarks? That is, instead of giving up on modelling social groups by smaller pilot programs, could we instead get more sophisticated about what a model is, and ask: well, if simple scaling by group size isn't the appropriate way to obtain a model that behaves as the thing we want to investigate, what kind of scaling might be helpful? That is, how might a smaller pilot program be designed so as to give us insight into the larger situation we want to investigate? We might explore whether there are dimensionless parameters associated with social interactions in groups rather than just paying attention to the number and proportions of types of people in the group: does the way the discussion is structured, the power relationships between the group members, the length of the average verbal contribution, the barriers that prevent interruption, how homogenous or disparate the members of the group are, matter to the nature of the group interaction? If so, we might ask: could we figure out the relevant dimensionless ratios for some kinds of social phenomena?

These last suggestions are very speculative, and I leave them so. Due to space limitations here, I have not been able to fully explain the methodology of scale models, nor to address the interesting topic of how this notion of models complements and illuminates work by other philosophers on the relationship of models and laws and on the use of models in scientific discovery. I hope, however, that I have been able to convey why I feel it is a rich source of ideas about model-based reasoning.

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