Talk given at UNC-Chapel Hill on November 17, 2000

[editorial additions to the original version added in 2002 are indicated in brackets]

Physical Pictures:
Engineering Models circa 1914 and in Wittgenstein's Tractatus

by Susan G. Sterrett,
Department of Philosophy, Duke University

Today[1] I'm going to talk about an element in the milieu in which Wittgenstein conceived the *Tractatus Logico-Philosophicus* that has not been recognized to date: the formalization of the methodology of experimental scale models that occurred just about the time he was writing that early work.

Lest this seem too far-fetched or peripheral to warrant spending the good part of an afternoon on, let me provide some setting.

Wittgenstein remarked to a number of people that he got the main idea for the Tractatus Logico-Philosophicus upon reading an account of the use of a physical scale model with dolls and model automobiles and busses in a courtroom case about an traffic accident. He had, he said, been serving in the Austrian army at the Front in the Fall of 1914 at the beginning of World War I when he read a newspaper account of the court trial, and, upon reflecting upon the use of the dolls and model automobiles, suddenly realized that a proposition is a picture. After Wittgenstein's death, his friend von Wright wrote a memoir about him, in which he mentions the importance Wittgenstein put on reading about the use of a scale model. von Wright also suggests, in a footnote, that "it would be interesting to know whether Wittgenstein's conception of the proposition as a picture is connected in any way with the Introduction to Heinrich Hertz's *Die Prinzipien der Mechanik*", remarking that Wittgenstein held the book in high esteem.

1 [A shorter talk of the same title was given in Vienna on July 6, 2000 at HOPOS (History of Philosophy of Science Conference) 2000. The HOPOS 2000 version has since been published in History of Philosophy of Science: New Trends and Directions , Michael Heidelberger and Friedrich Stadler (Eds.) Dordrecht-Boston-London: Kluwer 2002 (Vienna Circle Institute Yearbook 9/2001), pp. 121-135.]

In an attempt to understand what Wittgenstein meant in saying a proposition is a picture, some people have gone off and running with von Wright's suggestion, which has led to some lopsided readings of the portions of the *Tractatus* that speak of a proposition as a picture or a model -- amazingly, the reading that often arises is that Wittgenstein meant here to be referring to abstract mathematical models, or to abstract mental models. Others, uninformed by the knowledge of what's involved in building a scale model, or of the fact that aeronautical engineering research at the time would have involved learning in practice how to construct and use scale models, have employed naive accounts of scale models in their attempts to render the remarks in the Tractatus intelligible.

Today, I'll explain the methodology of scale models, and explain why I find it helpful to keep in mind how this kind of model portrays when reading the Tractatus [abbreviated below as TLP] -- in particular, when reading the statements about pictures and models, such as:

- A picture is a fact [TLP 2.141],
- A picture is a model of reality [TLP 2.12],
- The "pictorial relationship" that makes a picture a picture is part of that picture [TLP 2.1.5.3] , and
- A picture must have its pictorial form in common with reality in order to able to depict it [TLP 2.17].

And, when reading the sections in the Tractatus about objects and states of affairs, or atomic facts, such as:

- The statement that the form of an object is the possibility of its occurring in states of affairs, or atomic facts [2.0141];
- that in a state of affairs, or atomic fact, objects fit into one another like links of a chain [2.03], and
- that if all the objects are given, then all possible states of affairs, or atomic facts, are also given [TLP 2.0124].

So, although I'll be telling a historical story today, the purpose is to get clear on the sometimes puzzling and obscure passages on pictures and models.

What I'll propose today is something that isn't found in the philosophical literature at all: that 1914 was a threshold year for the methodology of experimental scale models, in that a formal basis for the practice was announced and became well known in London just months before the crucial moment of insight Wittgenstein had in the Fall of 1914 when he read a newspaper account about the use of scale models in a Paris courtroom and suddenly thought that he had the key to the answer to all his problems about propositions. I'll explain the advance in the formal basis for the methodology of scale modelling, and then I'll show why I think the statements in the Tractatus about pictures, objects, atomic propositions, and states of affairs, as well as his unusual view that there are no logical constants in the general form of a proposition all fall into place once we understand the view that a proposition is a picture or a model along the lines of the mathematical basis for the construction and use of scale models.

[The timeline included as page 2 of the handout given out at the talk is shown on page 22 of this paper; it is a two-column chart entitled "Wittgenstein and Physical Similarity"]

Propositions as Pictures

To refresh your memory, the *Tractatus Logico-Philosophicus* was written early in Wittgenstein's career. He had studied engineering in Berlin, then gone to Manchester, England to do graduate research work in aeronautical engineering. While there, he studied mathematics as well, and, upon being told that Bertrand Russell was the person to study foundations of mathematics with, and/or after reading Russell's *Principles of Mathematics*, he left his position as a graduate research student in aeronautical engineering in Manchester to study philosophy with Bertrand Russell at the University of Cambridge. (It should be noted that both the legendary engineering professor at Manchester, Osborne Reynolds, and Horace Lamb, a mathematics professor at Manchester who wrote the classic engineering text, *Hydrodynamics*, as well as texts on mechanics and dynamics, both got their degrees in mathematics at

Cambridge. Thus there is not as great a gulf between engineering at Manchester and study in foundations of mathematics at Cambridge as one might suppose.) In various letters, Russell refers to Wittgenstein as a would-be “aeronautical engineer” -- it seems Wittgenstein still thought of himself as an engineer, and considered whether it was worth leaving that career path behind in order to study philosophy. Or, at least he sometimes gave Russell that impression. In 1913 - 1914, he spends time secluded in Norway, trying to solve the problems of logic. He produces what we now call “Notes on Logic” in 1913 and gets G. E. Moore to visit him there in early 1914, producing a manuscript we now refer to as “Notes dictated to G. E. Moore in April of 1914” during the visit. In the summer of 1914, he volunteers to serve in the Austrian army, and begins duty that Fall.

We have the Notebooks he kept in his rucksack, in which he wrote almost daily. The incident that Wittgenstein said prompted him to think about a proposition as a picture occurred during the first few months of military service, in the fall of 1914, and is recorded in a diary entry. He wrote:

> "In a proposition a world is as it were put together experimentally. (As when in the law-court in Paris a motor-car accident is represented by means of dolls, etc.)" [Wittgenstein 1979, p. 7e]

That insight, that in a proposition a world is put together experimentally, occurs in the context of considering the most general concept of the proposition, and of co-ordination between proposition and situation; an earlier entry on the very same day reads thus:

> "The general concept of the proposition carries with it a quite general concept of the co-ordination of proposition and situation: The solution to all my questions must be extremely simple." [Wittgenstein 1979, p. 7e]

His diary indicates that he thinks his insight about a world being put together experimentally does hold the solution to all his questions, for he then notes:

> "This must yield the nature of truth straight away (if I were not blind)."
> [Wittgenstein 1979, p. 7e]

The comment to friends that reading the newspaper account about how dolls were used to represent a motor-car accident in a law-court was pivotal in coming to think that propositions represent by being pictures really does appear accurate: with one exception, there is no mention of pictures in previous entries in the Notebooks, in any of the manuscripts entitled *Notes on Logic* (1913) nor in the “Notes dictated to Moore in Norway” in April of 1914. But, there is an abundance of entries on the proposition as picture in writings after that. And, the question of how a picture portrays is salient throughout his subsequent writings, although that question undergoes shifts in emphasis as time goes on.

Wittgenstein's biographer, Brian McGuinness, has remarked that this particular development in Wittgenstein's thinking (i.e., that a proposition is a picture or model) is probably not as crucial as it is often taken to be. [McGuinness, 1974, p. 57]. I think he has a point, in that, although this entry in September of 1914 marks the advent of the discussion of pictures, some of the features of the account in the Tractatus of what picturing is already appeared in earlier notes, though they are not explicitly formulated in terms of picturing. For example, in the *Notes on Logic*: "Propositions . . . are themselves facts: that this inkpot is on this table may express that I sit on this chair," and (Summary) "that a certain thing is the case in the symbol says that a certain thing is the case in the world." That these ideas appear in his writings over a year before he made the notebook entry about the dolls in the law court indicates that the idea of a proposition as a picture may be a way to put certain thoughts he had already been formulating, rather than marking a totally unprecedented turn of thought.

Mental Models and Experimental Models

That Wittgenstein specifically mentions "experimentally" ("probeweise") in the key diary entry (“ that in a proposition a world is put together experimentally”) is more significant than might at first be obvious. For, when we look at what Boltzmann had to say about experimental models, we see Wittgenstein has put a surprising twist on things. Recall that Boltzmann appears first on the list of those whose work Wittgenstein said: "I have only seized on it immediately with a passionate urge for the work of clarification." [McGuinness 1988, p. 84] . In an essay on models, Boltzmann explicitly described experimental models as of a different sort than the kind with which he was comparing mental models. Boltzmann explains why they must be distinguished:

> A distinction must be observed between the models which have been described and those experimental models which present on a small scale a machine that is subsequently to be completed on a larger, so as to afford a trial of its capabilities. Here it must be noted that a mere alteration in dimensions is often sufficient to cause a material alteration in the action, since the various capabilities depend in various ways on the linear dimensions. Thus the weight varies as the cube of the linear dimensions, the surface of any single part and the phenomena that depend on such surfaces are proportionate to the square, while other effects --- such as friction, expansion and condition of heat, &c., vary according to other laws. Hence a flying-machine, which when made on a small scale is able to support its own weight, loses its power when its dimensions are increased. The theory, initiated by Sir Isaac Newton, of the dependence of various effects on the linear dimensions, is treated in the article UNITS, DIMENSIONS OF." [Boltzmann 1974, p. 220]

Thus, for Boltzmann's account of mental models, the experimental models represent a challenge to the account: the relationship between model and what is modelled is *not* like the relationship between a mental model and what is modelled by it. This puzzled me for awhile, for it's clear that Boltzmann realizes that there is a reliable methodology involved in experimental models. Why does he leave the topic in such an unresolved state, then? I found an answer to this in his correspondence; it turns out that the piece really was unfinished: when the *Encyclopedia Brittanica* article was commissioned, Boltzmann tried to beg off on it, asking that it be assigned to someone else if possible. He protested that "what I write about technical and machine models will not be complete and will have to be enlarged by an English technician." [Letter #3, January 7, 1900 in Blackmore 1975, p. 57 - 58.] When transmitting it, he says that he is sending it only because he has promised to do so, but that he is not at all pleased with it. [Letter #5 in Blackmore 1975, p. 58] It is unlikely that Wittgenstein, who is reported to have made scale models of a sewing machine [McGuinness 1988] and airplanes [Engelmann 1968] in his youth, would not have noticed the glaring difference between mental models and experimental scale models to which Boltzmann is drawing attention. Thus, what was striking about the use of physical models in a courtroom does not fit tidily with the notion of model found in Boltzmann.

It does, however, exactly fit the notion of experimental scale model. The striking thing about it was that it suggested that the notion of an experimental model might explain how a proposition can portray; on this notion of model, if a proposition is a model of reality it does reach right out to reality.

That is, the model does *not* exist in a mental or nonphysical realm somewhere, and the representing relation is no unexplained bridge between a mental or nonphysical realm and a physical realm. The model is, as propositional signs are, physical, and it is connected with what it represents in virtue of being in the same law-governed physical world. The methodology of experimental models explains, in a straightforward way based upon the extremely general logical principle that the units in a complete equation must be homogeneous, how we can read what's happening in one piece of the world off another. One can use the experimental model to predict the value of a quantity in one situation (reality) from an experimental model constructed in such a way as to have the same dimensionless ratios as that situation, but often what experimental models are used for is to determine whether some qualitative behavior obtains or not. One example of this is one of the most famous examples of the use of the principle of similarity: the work on turbulence done at Manchester by Osborne Reynolds in 1883 (this would have been a few years before Wittgenstein was born).

I've given you a handout pertaining to this, from Reynolds's scientific paper [p. 3 of the handout is an excerpt from Reynolds 1883 reproduced in Osborne Reynolds

Centenary Symposium (1968), p. 84]: See the sketch of a man standing behind an experimental apparatus? Here's what's going on: the problem of when flow would become turbulent, rather than flow in smooth layers, was an important and mathematically intractable problem. What Reynolds showed was that the phenomenon of turbulent flow depended upon a dimensionless ratio, rather than on the value of some dimensioned quantity, such as velocity. What the picture shows is an apparatus wherein we can see the onset of turbulence: he has a bit of red dye put in as the flow enters the pipe, and if the flow is in smooth layers, or laminar flow, then we see a horizontal red stripe or stripes. Once the flow becomes turbulent, we see the red stripes becomes curls that turn upon themselves and the red dye gets distributed throughout the flow in an unpredictable chaotic pattern. Now look at what Reynolds says in the paper: "The results of this investigation have both a practical and a philosophical aspect." (at top of box (a)); then at bottom of (a): "The results as viewed in their philosophical aspect were the primary object of the investigation." Here's what Stokes said of the work in the speech he gave on the occasion of Reynolds's being awarded the Royal Medal in 1888 (less than a year before Wittgenstein was born):

> In an important paper published in the *Philosophical Transactions* for 1883, he has given an account of an investigation, both theoretical and experimental, of the circumstances which determine whether the motion of water shall be direct or sinuous, or, in other words, regular and stable, or else eddying and unstable. The dimensions of the terms in the equations of motion of a fluid when viscosity is taken into account involve, as had been pointed out, the conditions of dynamical similarity in geometrically similar systems in which the motion is regular; but when the motion becomes eddying it seemed no longer to be amenable to mathematical treatment. But Professor Reynolds has shown that the same conditions of similarity hold good, as to the average effect, even when the motion is of the eddying kind; and moreover that if in one system the motion is on the border between steady and eddying, in another system it will also be on the border, provided the system satisfies the above conditions of dynamical as well as geometrical similarity. This is a matter of great practical importance, because the resistance to the flow of water in channels and conduits usually depends mainly on the formation of eddies; and though we

> cannot determine mathematically the actual resistance, yet the application of the above proposition leads to a formula for the flow, in which there is a most material reduction in the number of constants for the determination o f which we are obliged to have recourse to experiment." [p. 80 in Jack Allen's paper in Osborne Reynolds Centenary Symposium (1968)]

I think the emphasis others have put on Boltzmann's and Hertz's discussions of models [e.g., Griffin 1964; Barker 1980]   is not well placed -- a more fitting place to look is towards such experimental scale models. I actually have a section examining the burgeoning literature claiming the so-called "picture theory" in the Tractatus is a development of a one-page section on dynamical models in Hertz's *Principles of Mechanics*. [Hertz 1956 (1895)] Basically, I show that the idea they find in Hertz -- dynamical similarity -- is an important one, but it is a ubiquitous idea found in any eighteenth century mechanics text of the time, that every single point they try to draw out of it can be found in any basic text on mechanics, and that physical similarity is a more general case of dynamical similarity. I've left that section out so as not to overrun the time in this talk, but I can explain this afterwards should anyone be interested. I'm more interested today in explaining the strong reasons for looking to engineering models than in explaining how weak in comparison the reasons to look to Hertz's page-long discussion of dynamical models are.   Once one takes a look instead at what's happening with engineering experimental models, what one finds is that not only is this kind of model more fitting, but that it is just at this point in time when Wittgenstein has the thought to think of propositions as models in which a world is put together experimentally that there is a major event in the formalization of the methodology.

The generalization and formalization of the methodology of experimental scale modelling I am referring to was presented in London in 1914 just several months before the incident that Wittgenstein said prompted him to think of a proposition as a picture, but he would certainly have encountered specific applications of the methodology in the experimental work he had done prior to studying logic with Bertrand Russell.

This formalization of the method of experimental scale modelling, which is based upon a dimensional analysis of the quantities involved in the equations, showed how the most general form of a physical equation arises, not from the algebraic form of the equation, but from the characters (that is, the kinds) of the physical quantities involved in the equation. Let me repeat that, since it's crucial: ***the most general form of a physical equation arises, not from its algebraic form, but from the kinds, or types, of the physical quantities involved in the equation***. It's the kinds, as well as the number, of the physical quantities involved in the equation -- by physical quantities I mean quantities such as velocity, density, temperature, viscosity, surface area; actually, there is no restriction on the kind of quantities that can be considered, that's what makes the method completely general --- it's the kinds, as well as the number, of physical quantities involved in the equation that determine how these quantities can be combined to form different dimensionless parameters that together determine the behavior of the physical situation. Perhaps some examples will help: On your handout, I have shown two examples of dimensionless ratios: [Mach number, which is the velocity divided by the celerity, and which has dimensions of $L^{1}T^{-1}L^{-1}T^{1}$ = [1]. Reynolds number, which is (density)(velocity)(length)/(viscosity) and which has dimensions of $M^{1}L^{-3}L^{1}T^{-1}L^{1}M^{-1}L^{1}T^{1}$ = [1] where M stands for the dimension of mass, L for the dimension of length, and T for the dimension of time.]

Once the values of all the dimensionless parameters are determined, so is the behavior of the physical system (behavior of course is behavior with respect to some phenomenon of interest). Often it is possible to build a scale model of a physical system, such that the two systems have the same values of the dimensionless parameters that characterize a certain kind of behavior of the system. Then, it is possible to set up correspondences between the quantities in the two systems, so that you can read what would happen in the system being modelled off the scale model, using what are called modelling laws: the modelling laws tell you how to transform the quantity in the scale model into the value in the system being modelled. The scale model by itself really tells you nothing, without knowledge of these modelling laws. These tell you the corresponding time in the target situation corresponding to the time intervals in the situation you are regarding as a model: so, for instance, a time period of a week in a riverbed model might correspond to three months in the actual riverbed it was designed to be a model of.

The generalization of the methodology presented in 1914 proceeded by way of considering "The Most General Form of Physical Equations" and showed that, if you know all the quantities that are involved in the physical equation, and you know the kind of quantity each is, that's all you need to know in order to do all of the following:

> (i) figure out the minimum number of dimensionless parameters that characterize the behavior of the system,
>
> (ii) actually construct the combinations of these quantities into the required mutually independent dimensionless parameters. [e.g., Mach number, Reynolds number, and Froude number. As I've shown on the handout, you can see the quantities fit together to form a quantity in which all the dimensions "cancel" so to speak; this is indicated by a dimension of “1”, which is what you get for any quantity raised to the exponent of zero.]
>
> (iii) figure out the correspondences between quantities in physically similar systems, that is, get the "modelling laws" (“modelling laws” is the technical term, but it is not the best term -- in the Tractatus, I think these show up in the representing relation that makes a picture a picture) used to co-ordinate values of quantities between the two systems. An example here would be the corresponding elapsed times in the thing modelled and the model, or corresponding velocities.

Now, if this seems pretty astounding . . . well, it is.

Buckingham showed how to do all these things in the paper in which he presented what he called "the pi-theorem" in April of 1914 in London. He was systematizing things that had already been proven, putting things that were already in use in special cases into a more general form, and trying to explain their significance. He remarked:

"While this theorem appears rather noncommittal, it is in fact a powerful tool and comparable . . . to the methods of thermodynamics or Lagrange's method of generalized coordinates."

He presented the "pi-theorem" the following year in the United States in a paper to an Engineering Society, under the title: “Model Experiments and the Form of Empirical Equations” one of the commentators remarked:

> . . . the pi-theorem is closely analogous to thermodynamics and the phase rule. Thermodynamics affords certain rigid connecting links between seemingly isolated experimental results, while the phase rule tells us the number of degrees of freedom of a chemical system. The pi-theorem likewise affords rigid connecting links . . . just as the phase rule tells us [the number of degrees of freedom in a chemical system] , so also the pi-theorem tells us [the number of degrees of freedom in a physical system]
> . . The kernel of the paper is a theorem which is merely a restatement of the requirements of dimensional homogeneity, announced by Fourier nearly a hundred years ago, and extensively used by Rayleigh and others. But [. . .] Gibbs' phase rule, too was new only in form, not in substance, yet it served as the crystallizing influence which caused an immense number of latent ideas to fall into line, and we may expect the pi-theorem to play a similar role.
>
> This inevitable development of technical physics into a unified branch of science . . . , can be facilitated if writers on the problems of hydro- and aerodynamics, heat transmission and the like will be as introspective as possible, explicitly calling attention not only to their results, but to their methods of reasoning as well. For in every successful artifice of reasoning , there must be some element which is universal and capable of being generalized . . .
> [Buckingham 1915, p. 292]

The actual statement of the "pi-theorem" in the paper presented in London in early 1914 is as follows :

> . . . any equation which describes completely a relation subsisting among a number of physical quantities of an equal or smaller number of different kinds, is reducible to the form
>
> $\psi(\pi 1, \pi 2, \pi 3, \pi 4, \text{ etc. }) = 0$
>
> in which the π's are all the independent dimensionless products of the form Q1x, Q2y, . . ., etc. that can be made by using the symbols of all the quantities Q." [Buckingham 1914, p. 376]

Notice that in this "most general form of a physical equation", there are no algebraic constants, i.e., no plus or minus signs, no function signs, or signs for derivatives, integrals, or any functional operators --there are no signs for any mathematical relations. (As Buckingham explains, the exponents of the Q's are just shorthand and they indicate repeated operations, i.e., el superscript 2 just means you perform the operation of taking some length or other twice. It doesn't indicate the square of a quantity, or repeated multiplication or even iteration of the same operation.)

[[See §3 "Illustration" of Buckingham 1914 (p. 348-350) (provided on talk handout)]

Would Wittgenstein have known of this specific paper? It's hard to say what he would have heard of between the time he spent isolated in Norway in 1913 and entering the military in 1914. Rayleigh had been writing about a less formalized version of the method of physical similarity for a over a decade; he wrote about Manchester engineering professor Osborne Reynolds' remarkable successes using the principle of "dynamic similarity" in hydrodynamics and he explained the method of dimensional analysis used to establish it.

But, the main ideas behind the methods of physical similarity, even if they were taking a generalized form in England only that year, may have already been around in Germany during the time Wittgenstein studied there. For one thing, one of Helmholtz's scientific papers over a decade before Reynolds's striking work with turbulence, though ostensibly a practical paper about similar motions and the steering of air balloons, basically provides all the mathematical basis for the principles of dynamical similarity required for aerodynamics. Helmholtz says, basically, well, we've got the governing differential equations, but we know we can't solve them. Yet, he says, there is a way we can use them to tell us how to use observations from one case to inform us about an unobserved case. He proceeds to derive the dimensionless parameters that must be kept the same between the observed and the unobserved case in order that the two cases will have "similar motions"; and the reasoning he uses is dimensional analysis, that is, he looks only at the dimensions of the quantities involved. Then he says, if you construct a situation you can observe in such a way that these dimensionless parameters hold, you will get motions that are similar to those of the situation you are interested in. This paper by Helmholtz on steering air balloons was published in the 1873 in Berlin.

Another reason I say that the basic ideas may have been known in Germany at the time Wittgenstein took an engineering course of study there is that another commentator at Buckingham's 1915 presentation in the United States remarked that Buckingham " has struck the keynote of a new development of technical physics. . . The importance of technical physics, as a branch of subject matter, is already so clearly recognized in Germany that laboratories are being established devoted exclusively to this field." [Buckingham 1915]

Thus, although it is very hard to say just how much of this, and in what form, was around in Wittgenstein's milieu at various stages in his career, I think it is fair -- actually quite a modest claim -- to say that the practice of dimensional analysis, and of efforts to formalize it and generalize it, were part of the milieu of anyone studying aeronautics in the years just prior to 1914. And that is what Wittgenstein was doing then. Manchester was famous for Osborne Reynolds's work in lending sophistication and respectability to the practice of scale modelling, and his striking and famous work on the onset of turbulence was done at Manchester. One of the earliest and most famous applications of physical similarity and scale modelling was the screw propeller, and Wittgenstein was working on propellers. Another is the flow of jets through orifices, which Wittgenstein was working on in a laboratory.

That he was reading Russell's *Principles of Mathematics* at the same time might have made him especially sensitive to the fact that the basis for similarity to be found in dimensional methods was not yet completely formalized, and heightened his interest in the consequences that could be drawn paying attention only to the kind, or type of physical quantities involved in an empirical equation. Here is an excerpt from a letter Wittgenstein wrote to Russell in early 1913:

> "What I am most certain of is . . . the fact that all theory of types must be done away with by a theory of symbolism showing that what seem to be *different kinds of thing*s are symbolized by different kinds of symbols which cannot possibly be substituted in one another's places. . . . Propositions which I formerly wrote e2(a, R, b) I now write R(a, b) . . ." [Letter to Russell #9 dated January 1913 in Wittgenstein (1974)]

And we know that it was subsequent to Wittgenstein's conviction quoted above (that Russell's approach to a theory of types was not the way to go, that he was looking for "a theory of symbolism showing that what seem to be different kinds of things are symbolized by different kinds of symbols which cannot possibly be substituted in one another's places" so that he could dispense with symbols for relations) -- it was soon after this that Buckingham's formalization is presented in London, which emphasized how the whole basis for scale modelling really follows from the general principle of dimensional homogeneity, that is, from looking at the most general form of an equation and paying attention only to the various possible combinations of different types of magnitudes that cannot be substituted in one another's places. And that paper did make an impact. The next year Lord Rayleigh published an essay devoted to the subject, entitle "Physical Similarity". Rayleigh had been a proponent of the practice of physical similarity, often urging that its power was underappreciated, and he often lauded Reynolds's work in essays on various subjects, but 1915 was the first time he devoted a whole essay to the topic. Then there is an explosion of literature on the methods of physical similarity and of dimensional analysis in the next few years.

Let me go back to what Buckingham actually said; I'm just going to repeat it now, so you can notice that the emphasis is on SYMBOLS:

> ". . . any equation which describes completely a relation subsisting among a number of physical quantities of an equal or smaller number of different kinds, is reducible to the form
>
> $$\psi(\pi1, \pi2, \pi3, \pi4, \text{ etc. }) = 0$$
>
> in which all the ''s are all the independent dimensionless products of the form Q1x, Q2y, . . ., etc. that can be made by using the symbols of all the quantities Q. [Buckingham 1914, p. 376]

We don't know details about the specific scientific papers Wittgenstein read, but we do know that this was a major advance -- to this day it is Buckingham's Summer of 1914 paper that is cited in English-language texts as the landmark paper that provides the formal mathematical basis for the method of scale models -- that according to Wittgenstein's own account, it was subsequently reading about the use of an experimental scale model in the Fall of 1914 -- though in a courtroom!, rather than in a laboratory -- that stimulated him to think of a proposition as a picture.

All right. That's the historical story. What difference does it make to the philosophy?

I promised that everything about objects, atomic facts, and propositions would fall into place once we understood this notion of model. That's all I'm going to claim: that a coherent reading falls out of this understanding, not that it's a good account of language. But I'll also hint at why I think it helps us see a thread between this early work and Wittgenstein's later work.

Let me return to the points in the Tractatus I mentioned early in the paper, and let's look at them in the light of this view of model or picture as experimental model:

- that a picture is a fact [TLP 2.141]

[That’s easy: a model is a fact in the world, in that it is that certain proportions or dimensionless parameters hold in a physical situation that makes it a model of something else]

> **- that a picture is a model of reality** [TLP 2.12]**,**

> **- that the "pictorial relationship" that makes a picture a picture is part of that picture** [TLP 2.1513]

This remark makes perfect sense: A model isn’t a model all by itself, only insofar as we regard it as a model of some other situation, and in scale modelling, this means using what are called “modelling laws”.   A nice example of this is in model trains: time runs faster in a model than in the real situation. This means frequency will be higher. So the train whistle in the model will be high -- this can be disconcerting, until you have developed the experience such that you “hear” the corresponding note instead of the actual one. That a model is a model of some other situation is perfectly clear to fluid modellers: a lab setup using oil on a small-diameter pipe is used to model a whole family of situations: monographs are provided showing how to apply the results to situations involving different fluids flowing in various sizes of pipes.

> **- that a picture must have its pictorial form in common with reality in order to able to depict it** [TLP 2.17]**,**

I think it’s clear that the reason it works is because it is also a piece of the world. The facts are the dimensionless ratios that are the same between the model and the situation being modelled, so depicting is having the same dimensionless ratios.   It seems to me that the pictorial form would be having the same objects, i.e., the same quantities. So, in the screw propellor problem, it doesn’t matter that we are using water in the model to picture a situation involving air, what does matter is that the relevant quantities are viscosity, temperature, velocity, and so on; something like that.

And, when reading the sections in the Tractatus about objects and states of affairs, or atomic facts, such as:

- the statement that **the form of an object is the possibility of its occurring in states of affairs, or atomic facts**

If we see physical quantities as objects, and dimensionless parameters as atomic facts, Wittgenstein's statement here seems to express exactly that the kind of quantity an object is [length] is determined by the way it can be combined with other quantities to yield dimensionless quantities. This makes a lot of sense -- it's not claiming that there are certain kinds of quantities that are basic (such as mass, length, time, etc.) but that the what's important about the kind of quantity is how it combines with others to form dimensionless parameters. In the degenerate case, if an object combines with some other object to form a dimensionless parameter, then they are of the same kind. He stays neutral as to which ones are basic or simple. This is consistent with a later entry in his notebook, in the context of puzzling over which objects are simple and which are not: he suddenly realizes this is not a question of which objects are absolutely simple: he exclaims: "The object is simple -- for <u>me</u>!"

- **that in a state of affairs, or atomic fact, objects fit into one another like links of a chain, and**
- **that if all the objects are given, then all possible states of affairs, or atomic facts, are also given.**

There's a huge literature on what people call "the picture theory" in the Tractatus -- I mention here only one problem that some people think the most serious obstacle to making sense of it, and show it is no problem at all, but the most natural thing you'd want to say about picturing using models.

OK. Here's the supposed obstacle: If A can represent B and B can represent A, this is thought to be a problem for an interpretation of how A is a picture of B, for, the objection goes, how do you know which is reality and which is the picture? For people who think of Wittgenstein as proposing the picture theory to bridge a divide between two realms, one of which is reality and the other of which is some sort of representation of reality, this is supposed to be a knock-down argument for the dismissal of the whole lot of attempts to make sense of how propositions could be pictures.

Now let's think of this along lines of a scale model. Both are *in the world*. What happens is that either the scale model does represent the situation or it doesn't. Now how do we tell? Well, we can ask whether or not the model behaves like the situation represented. Both A and B are *real;* the question is *whether A pictures B*, and the picture A includes the representing relationship. In the courtroom, the model made a claim: the situation was like this. Then we use the model to investigate the consequences of the claim that the situation was like the model or picture: for instance, in the picture, could the car have stopped at the stoplight and still have run into the person? Was the improperly parked bus blocking the view of the pedestrian? And so on.

Now, for the model to represent the situation, it has to match it in terms of the consequences. It's a picture if it does. The question of which one is "correct" is a question one can ask only in the context of the claim that one of the situations is being claimed to be like the other. They are both real, neither of them is outside the realm of reality or violates natural laws. The question is whether or not the one you are using as a proposition matches reality or not. The model reaches right out to reality; you can ask: is the value of the quantity, such as elapsed time, that the picture projects (i.e., you have to apply the “modelling laws” that tell you, for instance, that one second in the model is ten seconds in the real traffic situation) the same, are the geometric traces of the projectile the same, are the flowlines the same, and so on. For a model to be a picture of something, one thing is relevant: is the model the same with respect to some phenomenon or other as the thing the model is of? Yes or No? Yes for True, No for False.

~~~

That’s the philosophical significance of the historical story. I have two speculations about the human side of things to add in closing:
~~~

First, given that I am emphasizing how ubiquitous scale models were in laboratories, then why is it the use of a scale model in the courtroom, rather than a laboratory, that kicks off the thought that a proposition portrays by being a picture. Here's my speculation: in a courtroom, there are "statements of fact" and "statements of law". "Statements of fact" would include statements about how things happened, and what the consequences of certain actual and counterfactual events were or would be. "Statements of law" would include statements about responsibility and blame. One can use the scale model only to establish statements of the first sort. In fact, once all the questions of the sort that could be settled by a scale model are settled, questions about responsibility, blame, punishment, and regret, are still untouched. In such a context, anyone who thought that empirical propositions might have anything to say about such questions should -- and often would --- be brought to realize that they don't. In a law-court, someone following such a line of thought might be silenced by being told that the question they are attempting to provide evidence for to the jury is a question of law, not of fact.

Wittgenstein did say that the most important point of the Tractatus was an ethical one, and was made by what he *didn't* say. Perhaps the point about ethics arose out of, or resonated when, reflecting on the limits to what a scale model could portray about a situation in the context of a courtroom, rather than a laboratory. He ended his preface to the Tractatus by saying that, if he was correct in believing that he had found the solution of the problems of logic, that this shows "how little is achieved when these problems are solved." So perhaps it was the interest in exploring limits to what can be portrayed that set him to work on working out the idea that a proposition is a picture.

Second, I think the fact that he framed the question in terms of how a model is used in a foreign court of law, where there are rules about what counts as evidence, and conventions of courtroom procedure established by social institutions, is a thread between his earlier and later work. Once he makes the notebook entry about the dolls in law-court, he never stops thinking about a proposition being a picture. It is in part in working out that idea all over again that the later work arises. In his later work, social and cultural aspects of language become more important than they seem to be in the Tractatus, but I suggest this: That he was first stirred to think about propositions this way by a context that involves social institutions, responsibility, blame and punishment shows us that his personal interest in logic was never something apart from such concerns. I can hardly think of a more culturally embedded, emotionally charged setting in which to think about "the general concept of the proposition" and "the co-ordination of proposition and situation" than a dispute about who was in the right legally in a traffic accident!

Wittgenstein & Physical Similarity

ENGLAND

Note: This chart does not show the contributions from Russia. Moscow had wind tunnels in 1891, and Buckingham's now-landmark 1914 paper is based on the work of a Russian mathematician.

1883
Osborne Reynolds, Engineering Prof. at U. of Manchester, England, shows that onset of turbulence depends on a dimensionless number, not on velocity.

1884
Reynolds paper on "conditions of similarity under any geometrically similar circumstances"

1899
Lord Rayleigh's paper in which he states that "the principle of dynamical similarity" is based on dimensional analysis. Cites Reynolds's work.

1908 (Spring)
Wittgenstein arrives at Kite Flying Upper Atmosphere station Spring 1908.

1908 (Autumn)
Wittgenstein at University of Manchester, studies aeronautical engineering. Discussions with Lamb.

1909
Lord Rayleigh is made president of the 1st British Council of Aeronautics. Rayleigh popularizer of principles of similarity.

1911
Wittgenstein goes to Cambridge to study logic and foundations of mathematics with Bertrand Russell.

1913 Autumn -1914 Spring
Wittgenstein in Norway. "Notes on Logic" (1913)
"Notes Dictated to G. E. Moore" (1914)

1914 Summer
Lord Buckingham presents "Physically Similar Systems: Illustrations of the Use of Dimensional Equations"

1915
Lord Rayleigh "Principle of Similarity" (London)
Lord Buckingham "Model Experiments and the Forms of Empirical Equations (presented in America)

EUROPE

1873
Helmholtz's "On Geometrically Similar Motions of Fluid Bodies with an application to the steering of air balloons" uses dimensional analysis to establish the dimensionless ratios required to achieve physical similarity between two situations. (Berlin)

1889-1906
Wittgenstein born in Vienna. Educated at home until age 14. Then at Linz, Upper Austria.

1906
Wittgenstein wants to go to Vienna to study physics with Ludwig Boltzmann. Boltzmann dies.

1906 -1908
Wittgenstein attends Technical Hochschule to study engineering. (Berlin -Charlottenberg).
[? Highly probable: Wittgenstein purchases in Germany a German translation of Hydrodynamics, by Horace Lamb, who is a Professor of Mathematics at U. of Manchester.] At end of program, decides to go to England for graduate research in aeronautical engineering.

1910
Prandtl's paper on importance of model experiments.

1910 +
Propeller Turbine being developed in Munich.

1914
Prandtl explains that the drag coefficient of a body is a function of the dimensionless "Reynolds number".

1914 Summer
Wittgenstein volunteers for Austrian Army, WWI.

1914 Autumn
Wittgenstein reads about use of model in a Paris law-court, gets insight that "in a proposition a world is put together experimentally" and that a proposition is a picture

[**References**

Barker, Peter (1980) “Hertz and Wittgenstein” , *Studies in History and Philosophy of Science* 11: pp. 243-256

Blackmore, John (1995) *Ludwig Boltzmann, His Later Life and Philosophy, 1900-1906*. Dordrecht: Kluwer Academic.

Boltzmann, Ludwig (1974) *Theoretical physics and philosophical problems : selected writings.* Edited by Brian McGuinness with a foreword by S. R. de Groot ; translations from the German by Paul Foulkes. Dordrecht ; Boston : Reidel Pub. Co.

Buckingham, Edgar (1914). “On Physically Similar Systems: Illustrations of the Use of Dimensional Equations” Physical Review 4: 345 -376.

Buckingham, Edgar (1915). “Model Experiments and the Forms of Empirical Equations”, in *Transactions of the American Society of Mechanical Engineers* 37, pp. 292.

Engelmann, Paul. (1968) *Letters from Ludwig Wittgenstein, with a memoir.* Translated by L. Furtmüller. Edited by B. F. McGuinness. New York: Horizon Press.

Griffin, James (1964) *Wittgenstein's Logical Atomism*. Oxford: Clarendon Press.

Helmholtz, Hermann (1873) “Ueber ein Theorem, geometrisch ahnliche Bewegungen flussiger Korper betreffend, nebst Anwendung auf das Problem, Luftballons zu lenken” delivered to the Berlin Society, in *Monatsberichte der Konigl. Akademie der Wissenschaften zu Berlin, vom 26 Juni 1873*. S 501-504. Reprinted in Helmholtz’s Wissenschaften Abhandlungen, Vol. 1, p. 158ff.

Hertz, Heinrich (1956) *The Principles of Mechanics*. New introduction by Robert S. Cohen, preface by Hermann von Helmholtz. New York: Dover Publications. [Translation of Hertz 1910, first published in 1895.]

Lamb, Horace (1906) *Hydrodynamics*, 3rd edition. (First edition published as "Treatise on the mathematical theory of the motion of fluids", in 1879. The second edition, largely remodeled and extended, appeared under the title *Hydrodynamics* in 1895.) Cambridge: Cambridge University Press.

McGuinness, Brian. (1974) "The *Grundgedanke* of the *Tractatus*" in Vesey (1974) pgs. 49 - 61.

McGuinness, Brian (1988) *Wittgenstein, A Life: Young Ludwig 1889 - 1921*. Berkeley, CA: University of California Press.

Osborne Reynolds Centenary Symposium (1968) *Osborne Reynolds and engineering science today*; papers presented at the Osborne Reynolds Centenary Symposium, University of Manchester, September 1968, by Jack Allen [and others] Edited by D. M. McDowell and J. D. Jackson. Manchester, England: Manchester University Press.

Reynolds, Osborne (1883) "An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous and of the law of resistance in parallel channels." *Royal Society, Phil. Trans*.

[Sterrett, Susan G. (2002). "Physical Pictures: Engineering Models circa 1914 and in Wittgenstein's Tractatus" (text of talk presented July 2000 at HOPOS 2000), which appeared in *History of Philosophy of Science - New Trends and Perspectives* , Michael Heidelberger and Friedrich Stadler (Eds.) Dordrecht-Boston-London: Kluwer 2002 (Vienna Circle Institute Yearbook 9/2001) pp. 121-135]

Vesey, Godfrey (1974) *Understanding Wittgenstein*. First published 1974 by Macmillan Press Ltd. (c. The Royal Institute of Philosophy 1974); London and Basingstoke: Macmillan.

Wittgenstein, Ludwig (1922) *Tractatus Logico-Philosophicus.* Translated from the German by C. K. Ogden, with an introduction by Bertrand Russell. London and New

York: Routledge & kegan paul Ltd., 1922. Another translation used here is: Ludwig Wittgenstein, Tractatus Logico-Philosphicus. Translated by D. F. Pears and B. F. McGuinness, with the Introduction by Bertrand Russell. London and Henley: Rougledge & Kegan Paul, 1974 (revisedversion of edition first published in 1961). The numbers used to cite portions of the text refer to the statement numbers of the original German text.

Wittgenstein, Ludwig (1974) *Letters to Russell, Keynes, and Moore*. Oxford: Basil Blackwell.

Wittgenstein, Ludwig (1979) *Notebooks 1914-1916.* Edited by G. H. von Wright and G. E. M. Anscombe. With an English translation by G. E. M. Anscombe. Chicago: University of Chicago Press.]