Toward a Pragmatic/Contextual Philosophy of Mathematics: Recovering Dewey's *Psychology of Number*

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The philosophy of mathematics education is unnecessarily depriving itself of a potent source of material and inspiration. While John Dewey is regarded as a towering figure in the philosophy of education, he is conspicuously absent in contemporary conversations within the philosophy of mathematics education. This state of affairs is particularly interesting in that, early in his career, Dewey co-wrote a book in the area of mathematics education. *The Psychology of Number and its Applications to Methods of Teaching Arithmetic* (*PN*) was written by Dewey and James A. McLellan and first published in 1895.[1] *PN* receives little attention from Dewey scholars, philosophers of mathematics, and mathematics educators. For example, Paul Ernest's *The Philosophy of Mathematics Education* is regarded as a leading work in the field. It is telling that he refers to Dewey mostly in passing.[2] His references paint Dewey with the broad and stereotypical brush of the romantic progressive (in the most generic and inaccurate sense). This lack of regard is not uncommon.

The purpose of this essay is to present the idea that Dewey did, in fact, forward a clear, distinct and fundamentally original philosophy of mathematics education. *PN* will be considered in detail and other works by Dewey will also be cited in support of my thesis. Perhaps most importantly, I will conclude by suggesting that Dewey's unique contributions in this area, while under-appreciated, could be used as a much-needed mediating influence between firmly entrenched opposing schools of thought existent in the contemporary philosophy of mathematics education.

The Psychology of Number

In *PN*, consistent with his more general philosophy, Dewey situated mathematics within activity. Consequently, it is only fitting that these ideas were brought forth in the applied context of a guide for mathematics teachers. While *PN* is an interesting work on the methods of mathematics teaching in its own right, for the sake of this essay only the portions of the book immediately useful to Dewey's more theoretical views of mathematics will be considered. Insofar as some pedagogical techniques illuminate and compliment the underlying philosophy they are intended to elucidate, they will, to varying degree, be considered in this essay. This notion of the inseparability of action and thought is thoroughly Deweyan in spirit and plays a central role in his philosophy of mathematics.

I recently spoke with a fellow graduate student about Dewey's *Art as Experience*. He summarized Dewey's aesthetic project as attempting to put art back into the activity of life. He went on to say that, according to Dewey, we need merely to recognize that art is a part of our lives — art came out of our activities, but that it has been artificially separated from life for various reasons.

I had begun work on this philosophy of mathematics project prior to the conversation and I was having some difficulty putting Dewey's intentions into a concise mission statement. I have always been impressed by the consistency of Dewey's philosophy across disciplines and this Dewey-on-art conversation helped me see *PN* as an extension of this mission to the realm of philosophy of mathematics education. Thus, to the extent that it is possible to reduce Dewey's philosophy of mathematics to slogan length, it is as follows: mathematics is a product of human activity (in fact, it is indistinguishable from human activity) and when taught should be regarded as such.

Deweyan Use of "Psychology"

The first step toward a more complete understanding of how Dewey conceived of mathematics and the teaching of mathematics is to consider his use of the word psychology. Psychology has often been viewed largely as something to be overcome or ignored in philosophical work, as there is fear that mental processes can be a serious impediment to understanding how the world "really is." According to this traditional philosophic conception of psychology there is a sharp line between the mental and the physical. Dewey's philosophy works to mediate between those tendencies that focus disproportionately on either the mental or physical aspects of existence. To combat this artificial, static and, to Dewey, damaging polarization, Dewey cleverly employed "psychology" in an unorthodox manner.

Dewey's inclusive and activity-sensitive psychology is at the core of his more general pragmatic beliefs. In "The Postulate of Immediate Empiricism," Dewey presents an illustration that makes clear why, according to his pragmatic conception of how we know and what there is to know, philosophy and psychology are inextricably linked:

> I start and am flustered by a noise heard. Empirically, that noise is fearsome; it really is, not merely phenomenally or subjectively so. That is *what* it is experienced as being. But, when I experience the noise as a *known* thing, I find it to be innocent of harm. It is the tapping of a shade against the window, owing to movements of the wind. The experience has changed; that is, the thing experienced has changed not that an unreality has given place to a reality, nor that some transcendental (unexperienced) Reality has changed, not that truth has changed, but just and only the concrete reality experienced has changed. I now feel ashamed of my fright; and the noise as fearsome is changed to noise as a wind curtain fact, and hence practically indifferent to my welfare. This is a change of experienced existence effected through the medium of cognition.[3]

So, to Dewey, the world is as it is experienced and vice-versa. His immediate empiricism, or pragmatism, or whatever label we affix to his ideas gives philosophy the freedom to work without tethering itself to burdensome ontological/epistemological concerns. As a result, Deweyan philosophers are free (even obligated) to employ psychology in understanding experience. This is radical, as to many philosophers, psychology is a seen as a barrier to logic, obscuring the contents of the logical, *a priori* realm.

Dewey's reconception of logic and psychology in light of human activity posits psychology and logic as merely different modes by which we go about figuring our how to live our lives. Psychology is concerned with the mental processes by which we actually think (live), while logic is concerned with the formalization of such "psychological" thinking. Thus conceived, psychology and logic merely describe different ends of the spectrum of how we know.

10.47925/2003.436

This treatment of Dewey's use of the term psychology is warranted if the philosophical ideas in *PN* are to be understood. The very employment of the term psychology is, as I see it, a conscious effort on Dewey's part to minimize the ontological implications of the concept of number. Instead, Dewey forwarded more constructive and pragmatic notions of what numbers are and where they came from.

Philosophical Consideration of Number and Mathematics

Asserting that Dewey viewed number as a construct does not go far enough toward ensuring that the mistake is not made of thinking of the concept of number (or *a* number, such as 3) as an abstract entity. Dewey conceived of number as transactional in nature — it resides within the process of mathematical activity.

In a section of *PN* titled, "Number is a Rational Process, Not a Sense Fact," Dewey explained that we come to use number only after a great deal of rational, abstract thought. The raw sense data with which we work, while rich in gross information regarding the multiplicity of things in nature, does not offer any insight with regard to the notion of number: "There are hundreds of leaves on the tree in which the bird builds its nest, but it does not follow that the bird can count" (*PN*, 23).

Dewey considered the process by which number is produced and determined that discrimination and generalization are required. Discrimination involves the recognition that the objects in question consist of distinctly separate units. Consider the number three: a young child faced with three red blocks of identical size and shape first needs to be able to determine that the group of blocks are not one larger unit. Once individual units are discerned, the child next must be able to generalize.

Dewey divided generalization into two sub-processes, the first of which is abstraction. Abstraction requires that the child be able to consider only whatever qualities of an object necessitate its being considered as a part of one group or another. This abstraction of one or a set of qualities will also eventually include qualities that are not immediately observed by the senses: "But this very power to resist the stimulus of some sense qualities and to attend to others means also the power to group the different objects together on the basis of some principle not directly apprehended by the senses — some use of function which all the different objects have" (*PN*, 27).

Once a child can abstract single or groups of qualities from objects, the final process in the construction of the elementary notion of number is that of grouping. Grouping requires that the child gather the objects that are deemed similar according to the prior mental (and partially physical) activities of discrimination and abstraction. Only through this complex set of processes can crude sense data be considered in terms of number.

Dewey's philosophical explanation of the nature of number requires psychology because, to Dewey, number is necessarily psychological. Without the psychological processes outlined above, there would only be the ideas of'"much" and "many" but not the more refined notions of "how much?" and "how many?" From this psychological explanation of where basic notions of number come from, Dewey next explained how this simple sense of quantity came about as a result of the human need to measure in order to live more efficient and better lives: "Number arises in

10.47925/2003.436

the process of the exact measurement of a given quantity with a view to instituting a balance, the need of this balance, or accurate adjustment of means to end, being some limitation" (*PN*, 42).

Dewey next worked to abolish the distinction that is typically made between counting and measuring. The traditional means of explaining the difference between counting and measuring focuses on how we count to determine how *many* of something there are and how we measure to determine how *much* of something there is. In a sense, this brings us back to Dewey's earlier discussion of whether some phenomena are a series of parts of one whole, or a related group made up of individual units. To Dewey, the answer was that they may be either, depending upon the context within which these phenomena dwell as well as the needs of the counter/measurer. To illustrate the sameness of counting and measuring Dewey offers a series of concrete examples:

> When we count up the number of particular books in a library, we measure the library — find out how much it amounts to as a library; when we count the days of the year, we measure the time value of the year; when we count the children in a class, we measure the class as a whole — it is a large class or a small class, etc. When we count the stamens or pistils, we measure the flower. In short, when we count we measure (*PN*, 48).[4]

In terms of the educational implications of this philosophy of mathematics, Dewey sought to establish the activity of measurement (as a part of genuine mathematical inquiry) as the primary means by which mathematical understanding is developed or created. Furthermore, he justified this course of action for mathematics education by explaining that since mathematics developed in this way over the course of human history, this is a natural way for children to learn the concept of number. In *John Dewey, E.H. Moore, and the Philosophy of Mathematics Education in the Twentieth Century*, Sidney Ratnor writes of this connection between the development of mathematics writ large and the learning of mathematics: "Dewey assumed an analogy between a child's progressive experience with elementary arithmetic and the development of these basic concepts in human history."[5]

Dewey next worked to develop his idea that measurement must be undertaken within a larger context. He offered a fairly complex example of how simply measuring a field (determining its area) will not yield a complete picture of its worth. We must know about what the field is capable of producing in order to truly measure it. In other words, we must know how it will affect our lives. Dewey explained that counting/measuring can help us with this problem, as well. The amount of corn, price per bushel, and cost of tilling the field, can all be addressed with measurement. In order to measure in any complex way, the wider context must be accounted for: "All numerical concepts and processes arise in the process of fitting together a number of minor acts in such a way as to constitute a complete and more comprehensive act" (*PN*, 57).

Deficiencies in Other Methods

The applied portion of *PN* also offers insight into Dewey's underlying philosophical beliefs about mathematics and mathematics education. While it is beyond the scope of this paper to explore Dewey's specific methodological recommendations, his account of the unacceptability of two rival methods serves to illuminate

10.47925/2003.436

Dewey's own ideas. *PN* refers to the two methods it opposes as the "symbols" method and the "things" method. The sketch of each method that follows is offered as a means of understanding Dewey by considering what he worked against.

The symbols method teaches number simply as part of a set of symbols. The concept of number that it forwards is essentially an abstract one. According to this method, mathematics primarily involves mastery of the mathematical rules that govern the manipulation of the symbols. Dewey questioned the meaning that children make of mathematics when the symbols method is implemented:"“It is little more than blind manipulation of number symbols. The child simply takes, for example, the figures three and twelve, and performs certain operations with them, which are dignified by the names addition, subtraction, multiplication; he knows very little of what the figures signify, and less of the meanings of the operations" (*PN*, 60).

The second method against which *PN* pits itself is the things method. According to the things method, mathematical meaning is derived from physical manipulation of things. Children learn mathematics through exposure to concrete objects. As Dewey explains the things method: "objects of various kinds"— beans, shoe-pegs, splints, chairs, blocks — are separated and combined in various ways, and true ideas of number and of numerical operations are supposed necessarily to arise" (*PN*, 60).

Dewey's primary objection was that each method acknowledges only part of mathematics. The symbols method deals only with operations in the mind and the things method deals only with concrete objects. Keeping in mind that, to Dewey, mathematics is the activity by which the mind deals with objects, it becomes easy to see that the two methods combined offer mathematics as mind and mathematics as objects, but the critical component of activity is neglected. Dewey acknowledged that although the symbols method is almost wholly abstract and one-sided, the things method is somewhat less so, as children must engage in some thought as they group and separate various objects. The problem with the things method is that this thoughtful activity is incidental and subordinated to a preoccupation with the things themselves. Dewey offers an outright dismissal of the symbols method, claiming that "no educationist defends it" (*PN*, 61). The things method, however, has many adherents. Dewey was trying to make clear to admirers of this system that it is as incomplete as the symbols method, but that its incompleteness is of a different kind: "it is not the mere perception of the things which gives us the idea, but "*the employing of the things in a constructive way*" (*PN*, 61).

The Two Methods and Rationalism and Empiricism

In his more general philosophy, Dewey frequently depicts his brand of pragmatism as a way to mediate between the equally dogmatic theories of knowledge, rationalism and empiricism. His conception of thought as mental *activity* is based upon his experimental conception of truth, knowledge and how we understand. In *The Quest for Certainty*, Dewey addresses how neither rationalism nor empiricism captures the dynamic nature of human existence: "Inquiry proceeds by reflection, by thinking; but "*not*, most decidedly, by thinking as conceived in the old tradition, as something cooped up within the 'mind.' For experimental inquiry or thinking

10.47925/2003.436

signifies *directed activity*, doing something which varies the conditions under which objects are observed and directly had and by instituting new arrangements among them."[6]

The elements present in the empiricism/rationalism battle, to Dewey, are also hard at work in the philosophy of mathematics education. Without explicitly referring to rationalism or empiricism, Dewey makes it clear that the symbols method is either analogous to, heavily influenced by or actually is a branch of rationalism and that the things method possesses a similar relationship to empiricism. Dewey uses familiar language to describe the two methods: "The method of symbols supposes that number arises wholly as a matter of abstract reasoning; the method of objects (things) supposes that it arises from mere observation by the senses — that it is a property of things, an external energy just waiting for a chance to seize upon consciousness" (*PN*, 62).

Once Dewey set up the two methods as analogs to rationalism/empiricism it becomes clear that his philosophy of mathematics is not expressly against everything contained within the two methods. Instead, Dewey wanted to salvage what he could from the two traditions in an effort to forward a philosophy of mathematics and mathematics education that focused on how mathematics is a mental activity involving objects with which we engage to help us improve our lives. The symbols method can contribute the mental and abstract ingredients, while the things method can help ensure that our mental activities stay tethered to real occurrences in our actual lives.

Dewey's Philosophy of Mathematics and Philosophy of Mathematics Education

Dewey's philosophy of mathematics is simply an extension of his more general pragmatic philosophy. Mathematics is defined by (in fact it actually is) its use. Brute sense data is no more mathematical than random symbol manipulating. The concept of the number three does not reside somewhere within a collection of three apples or beans any more than it does in the symbol "3." The concept of three is part of the activities in which we engage that require quantification (measuring) as a means to some end.

Dewey used this philosophy of mathematics to develop a philosophy of mathematics education centered on measurement (in its broad Deweyan conception — remember, all counting is measuring and all measuring is counting). Making measurement the vehicle for mathematical explorations ensured, according to Dewey, that number symbols will always be linked to concrete units. Additionally, mathematics through measuring encourages an active conception of the discipline.

Philosophy of Mathematics in the Twenty-First Century

In these final sections, I will consider Dewey's philosophy of mathematics and philosophy of mathematics education as they pertain to contemporary issues. I hope to establish, as I suggested in this paper's introduction, that Dewey's pragmatic mathematics could be a useful way to reconceive the fields. As was the case in Dewey's day (think of the things versus symbols debate), the philosophy of mathematics currently has two deeply entrenched positions — formalism and

10.47925/2003.436

Platonism.[7] I acknowledge that there are other philosophies of mathematics, it is just that the two I am presenting tend to be the most widespread.

Formalism, as a philosophical movement, is steeped in the abstract, symbols-oriented conception of mathematics. One who subscribes to a Deweyan philosophy of mathematics might find it difficult to understand the formalist perspective and so a word on its origins is in order. As mathematics developed and pure mathematics became less immediately and obviously tied to the natural world (think of set theory or non-Euclidean geometry), it became harder to make sense of or justify the foundations of mathematics. What resulted was a reconception of mathematics in the early part of the twentieth century that sought to explain mathematics in terms of following rules or axioms. In *Mathematics: The Loss of Certainty*, Morris Kline explained the origins of the formalist movement. Mathematician David Hilbert developed the idea that mathematics could avoid the ambiguities and confusions that some branches of purely abstract mathematics introduced by conceiving of mathematics in purely formal or symbolic (as opposed to intuitively meaningful) terms. Kline viewed this move as an effort to hold on to the old conception of mathematics as perfect and certain:

> Whether or not the symbols represent intuitively meaningful objects, all signs and symbols of concepts and operations are freed of meaning (according to formalism). For the purpose of foundations the elements of mathematical thought are the symbols and the propositions, which are combinations or strings of symbols. Thus, the formalists sought to buy certainty at a price, the price of dealing with meaningless symbols.[8]

While symbolic formalism has been incredibly useful in the areas of computer programming and artificial intelligence, its function in a few areas does not qualify it as *the* correct way of thinking about the nature and origin of mathematics. Dewey would, I am sure, applaud its use as a means of improving our lives, but probably caution dyed-in-the-wool formalists not to confuse function and ontology (I will develop this assertion in the following section).[9]

Klenk explains Platonist philosophy of mathematics as: "the belief that mathematical theories have as their subject matter sets of abstract mathematical objects....The objects exist independently of the human mind or mathematical system, and it is the job of the mathematician to discover or uncover their properties, just as it is the job of the physical scientist to discover facts about the empirical world."[10]

Platonism, and its consequent view that regards mathematical activity as the discovery of already existent mathematical realities would, it seems, be unacceptable to Dewey. There is no acknowledgement of the role of human activity in the actual construction or development of mathematics. According to mathematical Platonism, the focus of mathematics is on abstract entities.

If one considers Dewey's philosophy of mathematics as taking what is useful (and rejecting what is not) from both the formalist and Platonist perspectives, it is easy to imagine a pragmatic philosophy of mathematics that bridges the philosophical gap between the two intractable positions. In this paper's final section I will sketch such a mediation within the more specific area of the philosophy of mathematics education.

10.47925/2003.436

Toward a Pragmatic Philosophy of Mathematics Education

While formalism led to the failed experiment that was the "new math" (recalling Dewey's dismissal of the symbols method, it seems that had *PN* been consulted, the whole new math debacle could have been avoided!), vestiges of it remain in today's mathematics classrooms. It is my sense that the accountability movement of recent years is encouraging the development of a particularly troublesome strain of formalism whereby teachers teach children abstract rules to follow as a means of ensuring that students pass high stakes tests.

Likewise, Platonism is also embodied in a variety of ways in contemporary mathematics classrooms. It seems that Platonism undergirds the trend toward "conceptual understanding," insofar as teachers rely on manipulatives to convey the essence of mathematics to their students. Plato's *Meno* offers an early version of this technique, as Socrates used diagrams to help the slave boy uncover or discover pre-existent mathematical knowledge. While I am not stating an opposition to the use of manipulatives in the mathematics classroom (they can serve as an invaluable aid in developing conceptual understanding of mathematical concepts), like Dewey, I am concerned that an *ove*r-reliance on their use can lead teachers to fail to embark upon the most crucial facet of mathematics education, namely the cultivation of classrooms that encourage *meaningful* mathematics explorations.

Formalism focuses on the teaching and learning of algorithms. Platonism encourages "conceptual understanding" through the use of physical objects in an effort gain access to the realm of abstract but existent mathematical objects. I submit that the algorithmic and conceptual approaches are both impoverished forms of mathematics education. I am dubbing Dewey's pragmatic mathematics education the contextual approach, because it seeks to place mathematics within the context of genuine activity and it fosters recognition of the links between mathematics and its historical development. It is my belief that this contextual approach can, in a Deweyan spirit, save what is salvageable from each of the two methods through emphasizing activity — thus enlivening mathematics instruction, creating fertile opportunities for the construction of mathematical concepts.

A Dewey biography explains how his dynamic and non-absolutist ideas acknowledge a world that benefits from the plural and often uncertain conditions within which we live:

> "Opposing narrow-minded positions that would accord full ontological status only to certain, typically the most stable or reliable, aspects of experience, Dewey argues for a position that recognizes the real significance of the multifarious richness of human experience."[11] Likewise, it is my hope that applying Dewey's pragmatic philosophy to mathematics education will allow students of mathematics to recognize "the multifarious richness" of the mathematical experience.

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1. James A. McLellan and John Dewey, *The Psychology of Number and its Applications to Methods of Teaching Arithmetic* (New York: D. Appleton and Company, 1895). For all subsequent references this text will be cited as *PN*. Although Dewey and McLellan co-authored the book, I will refer only to Dewey throughout the essay, as the philosophy of mathematics I am attempting to recover is Dewey's.

2. Paul Ernest, *Social Constructivism as a Philosophy of Mathematics* (Albany: State University of New York Press, 1998), 183-84.

10.47925/2003.436

3. John Dewey, "The Postulate of Immediate Empiricism," in *The Influence of Darwin on Philosophy and Other Essays* (New York: Henry Holt, 1910), 230.

4. This argument seems somewhat reductive, as it suggests that gross quantitative measurement can detail the quality of what is being measured (measuring a library solely by number of books, for example). Dewey deals admirably with this in *PN*, 42-44.

5. Sidney Ratnor, "John Dewey, E.H. Moore, and the Philosophy of Mathematics Education in the Twentieth Century," *Journal of Mathematical Behavior* 11, (1992): 105-16. This rationale smacks of the post-Darwinian belief that "ontogeny recapitulates phylogeny." While today, this is not a very attractive rationale, it probably held some sway with Dewey, an avowed Darwinian. Interestingly, Stephen J. Gould discusses how school curricula in the late nineteenth century were reshaped to facilitate that belief. See Gould's *The Mismeasure of Man* (New York: W.W. Norton and Company, 1996), 143-44.

6. Dewey, *The Quest for Certainty* (New York: Minton, Balch, 1929), 123.

7. Reuben Hersh, *What is Mathematics Really?* (New York: Oxford University Press, 1997).

8. Morris Kline, *Mathematics: The Loss of Certainty* (New York: Oxford University Press, 1980), 248.

9. Larry Hickman's invited essay at the 2001 Philosophy of Education Annual Meeting, "Philosophical Tools for a Technical Culture," dealt with the neo-Deweyan theme of function trumping ontology. Larry Hickman, "Philosophical Tools for a Technical Culture, in *Philosophy of Education Society 2001*, ed. Suzanne Rice (Urbana, Ill.: Philosophy of Education Society, 2002), 25-35.

10. V.H. Klenk, *Wittgenstein's Philosophy of Mathematics* (The Hague: Martinus Nijoff, 1976), 8.

11. Richard Field, "John Dewey: Metaphysics," from *Internet Encyclopedia of Philosophy*, available at: <http://www.utm.edu/research/iep/d/dewey.htm> 2001.

10.47925/2003.436