On the Sources and Implications of Carnap's *Der Raum*

Abraham D. Stone

July 7, 2008

Abstract

Der Raum marks a transitional stage in Carnap's thought, and therefore has both negative and positive implications for his further development. On the one hand, he is here largely a follower of Husserl, and a correct understanding of that background is important if one wants to understand what it is that he later rejects as "metaphysics." On the other hand, he has already broken with Husserl in certain ways, in part following other authors. His use of Hans Driesch's *Ordnungslehre*, in particular, foreshadows the theme of so-called "voluntarism" which will characterize his later thought.

Carnap's first publication, *Der Raum*,[1] remains relevant to controversies which continue today. Carnap's position is technically unpolished, but perhaps still to be taken seriously: in particular, as I will point out below, it resists classification according to the conventionalist/empiricist split which pervades discussions of general relativity. My own interest, however, is more in what this work reveals about Carnap's background and subsequent development. In this respect, *Der Raum* marks a transitional stage, and therefore has both negative and positive implications.

On the negative side, Carnap's later thought is based on the rejection of a traditional philosophical background, associated with "metaphysics." To understand him, then, we need a correct understanding of just what is being rejected, and *Der Raum*, coming before the anti-metaphysical turn, can supply exactly that. To this end I will argue that the transition begins with Husserlian phenomenology: Carnap here is a follower of Husserl.[2] It is Husserl's position, then, which he will soon reject as metaphysical. On the positive side, however,

---

[1] *Der Raum: Ein Beitrag zur Wissenschaftslehre*, Kant-Studien Ergänzungshefte 56 (Berlin: Reuther and Reichard, 1922).

[2] The importance of Husserl in *Der Raum* has been recognized, first, by Carnap himself (Reply to Grünbaum, in *The Philosophy of Rudolf Carnap*, ed. P.A. Schilpp [La Salle, IL:

Der Raum already exhibits deviations from Husserl, foreshadowing the bigger changes to come. To lend technical precision to Husserl's points and to explain his own departures, Carnap draws on other philosophers and mathematicians, such as Hilbert, Killing, Pasch, Poincaré, Russell, Weyl, and, above all, Driesch. Towards the end of the paper I will show how the use of Driesch, in particular, introduces the theme of so-called "voluntarism" which was both to turn Carnap against Husserl and to drive much of his later development.

1 Husserlian question, Husserlian answer

Der Raum aims to settle a question concerning "sources of knowledge" (*Erkenntnisquellen*) about the structure of space.[3] Given Kant's dominance over German-speaking philosophy at the time, the use of this Kantian terminology might seem to offer little clue as to the background of Carnap's inquiry.[4] But, in fact, the question is revealing.

For Kant, questions about *Erkenntnisquellen* arise because of his general theory that our knowledge has two such sources, intuition and thought.[5] But this view was rejected by many of his successors, including not only German Ide-

---

Open Court, 1963], 957), and then by M. Friedman ("Geometry, Convention, and the Relativized A Priori: Reichenbach, Schlick, and Carnap," in *Reconsidering Logical Positivism* [Cambridge: Cambridge University Press, 1999], 48; "Carnap and Weyl on the Foundations of Geometry and Relativity Theory," in idem, 51–8; *A Parting of the Ways: Carnap, Cassirer, and Heidegger* [Chicago and La Salle: Open Court, 2000], 67–8, 93 n. 128); S. Sarkar ("Husserl's Role in Carnap's *Der Raum*," in T. Bonk, ed., *Language, Truth and Knowledge: Contributions to the Philosophy of Rudolf Carnap* [Boston: Kluwer, 2003], 179–190); T. Mormann ("Geometrical Leitmotifs in Carnap's Early Philosophy," in M. Friedman and R. Creath, eds., *The Cambridge Companion to Carnap* [Cambridge: Cambridge University Press, 2007], 46–7); and T. Ryckman ("Carnap and Husserl," in idem, 103). See also Weyl's brief review of *Der Raum* (*Jahrbuch über die Fortschritte der Mathematik* 48 [1922]: 631–2), in which Husserl is the only other philosopher mentioned. Alan Richardson, in contrast, calls Carnap in *Der Raum* "an unabashed, if unorthodox, neo-Kantian" (*Carnap's Construction of the World: The* Aufbau *and the Emergence of Logical Empiricism* [Cambridge: Cambridge University Press, 1998], p. 139), while Yemima Ben-Menahem argues that *Der Raum* "echoes Poincaré in the problems it poses, the solution it reaches, and the character of its arguments" (*Conventionalism* [Cambridge: Cambridge University Press, 2006], p. 181). I will make some remarks about these competing interpretations below.

[3]*Der Raum*, Introduction, p. 5.

[4]See e.g. *Kritik der reinen Vernunft*, A260/B316. This work is no. 125 in the bibliography (Literatur-Verzeichnis) of *Der Raum*. Henceforth, I give this information in the form: RLV 125.

[5]See ibid., A294/B350, and see also A271/B327. (However, Kant sometimes gives other lists of *Erkenntnisquellen*: see A38/B55, A97, A299/B356.)

alists such as Hegel, but also the neo-Kantian schools of the late 19th and early 20th century. As the prominent Marburg neo-Kantian, Paul Natorp, explains:

> The subsequent philosophy which emanates from Kant, including the present, no less than "orthodox," neo-Kantian movement, has more and more taken offense at the dualism of pure intuition and pure thinking and finally broken with it decisively.... it was demanded by Kantian transcendental philosophy's own principle that one take together again in strict unity what in Kant is ... separated into two factors—pure intuition and pure thought—and seek to understand it as something unified.[6]

The B edition of the *Critique of Pure Reason* was thought to show traces of a move in this direction, since Kant says there that all synthesis has its origin in the understanding, and hence finds the understanding at work in the givenness of the manifold itself.[7]

But there were two interrelated groups among whom the question of *Erkenntnisquellen* was alive and well. First, a group of philosophically minded mathematicians and physicists, including Frege (whose life project was to answer this question with respect to arithmetic), as well as many others cited by Carnap—for example, Pasch, Killing, and Hausdorff. These generally dealt with a relatively unsophisticated, or anyway unreflective, version of the question. Typical is Frege's treatment in the *Grundlagen der Arithmetik*,[8] in which he barely pauses to say what he means by such terms as "analytic," "synthetic," "intuition," and "logic." Detailed epistemological discussions were found, instead, among the second group: the intellectual descendants of Brentano.[9]

---

[6] *Die logischen Grundlagen der Exakten Wissenschaften* (Leipzig and Berlin: Teubner, 1910) (RLV 179), 2. (Natorp refers here only to *pure* intuition; but, as for empirical intuition, he regards it as the infinite goal of *Erkenntnis*, rather than as a source of it: see pp. 273–4, 277.) Friedman, who is well aware of this feature of neo-Kantianism (see *Parting*, 28), therefore considers it "puzzling" that Carnap raises a traditional question about *Erkenntnisquellen* in *Der Raum* (66). He solves the puzzle, in fact, by invoking Husserl—but without, I think, appreciating the depth of Husserl's influence, and also based on an interpretation of Husserl which I would not accept.

[7] See Kant, *Kritik der reinen Vernunft* B129–30; Natorp, *Grundlagen*, 275–6.

[8] *Die Grundlagen der Arithmetik: Eine logisch-mathematische Untersuchung über den Begriff der Zahl* (Breslau: Koebner, 1884) (RLV 68).

[9] For discussion of this school, see Barry Smith, *Austrian Philosophy* (Chicago and La Salle: Open Court, 1994). Smith agrees in assigning Carnap to the "Austrian" tradition, while attaching the Marburg neo-Kantians to the "German" tradition which runs through Hegel (see especially p. 9 n. 3). Some of his other classifications (e.g. of Heidegger as "German") appear more questionable.

Questions about "the origin of our concepts" exercised both Brentano and the later members of the school.[10] The anti-Kantian Brentano traces such questions to Locke and Leibniz, but Husserl, who had, by the time of *Ideen* I, squarely identified himself as a Kantian, takes them as Kantian questions about *Erkenntnisquellen*. Even if Carnap never mentioned Husserl, then, and even if we did not know that, two years after publishing *Der Raum*, he was participating in Husserl's seminar in Freiburg,[11] the form of his question would give reason to suspect Husserlian influence.

In any case, Carnap answers in explicitly Husserlian terms. Parceling out knowledge of space between empirical, synthetic a priori, and analytic sources, he, like Husserl, expresses reservations about the terminology.[12] Husserl uses these terms only "in order to let historical parallels resonate" (loc. cit.). Carnap, in turn, overcomes his reservations only by making it clear whose usage he is following. Hence he explains that our knowledge is analytic insofar as it derives from "formal ontology in Husserl's sense," and distinguishes between a priori and a posteriori in terms of Husserl's characteristic distinction between essence, known through eidetic insight (*Wesenserchauung*), and "matters of fact."[13]

Others turn up in the same context, in particular Hans Driesch. Today mostly remembered, if at all, as a neo-vitalist, Driesch was known at the time for his work on general issues of logic, metaphysics, and epistemology. Carnap gives him prominent billing, both in *Der Raum* and in the *Aufbau*, with special attention to his logico-epistemological book, the *Ordnungslehre*.[14] In *Der Raum*, Carnap also gives Drieschian glosses to "analytic" and to "synthetic a priori."[15]

---

[10]As Smith notes: see *Austrian Philosophy*, 107.

[11]Although Carnap's dissertation was completed under Bruno Bauch at Jena, he had already moved to Buchenbach, near Freiburg, in 1919: see G. Gabriel, "Introduction: Carnap Brought Home," in S. Awodey and C. Klein, eds., *Carnap Brought Home: The View from Jena* (Chicago: Open Court, 2004), 18 n. 29. Carnap participated in Husserl's advanced seminars from the summer of 1924 to the summer of 1925: see K. Schumann, *Husserl Chronik* (The Hague: Nijhoff, 1977), 281 (by report of Ludwig Landgrebe).

[12]See, respectively, *Der Raum*, §V, p. 63; *Ideen* I, Introduction, p. 6. *Ideen* I = *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, erstes Buch: allgemeine Einführung in die reine Phänomenologie* (1st ed. 1913 [RLV 119]; reprinted as Husserliana [henceforth: Hua] 3, ed. K. Schuhmann, The Hague: Nijhoff, 1976). I cite this work by the first edition page numbers, which are printed in the margin of the Husserliana edition.

[13]*Der Raum*, Introduction, p. 6; §II, pp. 22–3; §IV, p. 61; §V, pp. 64–5. Cf. Husserl, *Ideen* I, Introduction, p. 3.

[14]*Ordnungslehre: Ein System des nicht-metaphysischen Teiles der Philosophie, mit besonderer Berücksichtigung der Lehre vom Werden*, 1st ed. (Jena: Diederichs, 1912) (RLV 50). The *Aufbau* cites the second, significantly revised edition (1923).

[15]*Der Raum*, §I, p. 8; §II, p. 22; §IV, p. 60.

But, while Driesch's influence is significant, as we will see, still, the main outline of Carnap's approach is Husserlian. To appreciate this, we need only compare Husserl's own answer to the same question about space.

The origin of our spatial concepts is an issue to which Husserl returned obsessively: new thoughts on the "origin of geometry" were to spur the developments of his final period. At the time of *Ideen* I, however, he had a fairly simple position. His answer, which occurs already in the *Logische Untersuchungen*, is that it depends what you mean by "space."[16] Space qua "world-space," "the well-known order-form of the world of appearance," is Euclidean: in this sense, "the talk of 'spaces' for which, e.g., the parallel axiom doesn't hold, is ... an absurdity" (loc. cit., 250–51). But the "categorial form" of this world-space can also be regarded on its own, as a form under which spatially related objects fall, leaving the objects themselves "fully indeterminate with respect to matter" (249). Space regarded as such a formal structure can indeed be taken, without absurdity, as one among many possible "spaces." This is the basis for "the generalizations which have grown out of geometrical theory," such as "the theory of n-dimensional manifolds, whether Euclidean or non-Euclidean" (250).

But if higher dimensional and/or non-Euclidean spaces are formally possible, why shouldn't the world-space actually instantiate one? Here we need to say more about the distinction, in Husserl's later terminology, between formal and material eidetic necessity. A material essence or "eidos" prescribes what is necessary to some genus or species of objects. Since every object belongs to a hierarchy of species and genera, each falls under a hierarchy of material eidetic laws, culminating in the laws of a "regional ontology," which apply to it by virtue of its highest genus.[17] But these highest laws encompass all the lower ones, insofar as they predelineate the essential possibilities for differentiation. So material eidetics and regional ontology are basically the same.

Formal eidetics, in contrast, is not about the objects of any region, but rather about the form of a region in general. Every region contains parts and wholes, objects and properties, classes and members, and so forth: the existence and nature of such structures are matters of formal eidetic truth. Husserl identifies the

---

[16]See *Logische Untersuchungen*, Prolegomena §70, vol. 1 (Halle: Niemeyer, 1900), pp. 248–52. This passage is specifically cited by Carnap (from the 2nd edition [1913] [RLV 118], which in this case is essentially identical): *Der Raum*, Literatur-Hinweise, p. 78 (note to p. 7).

[17]A "region," in Husserl's terminology, is a complex of highest genera which go together in a certain way (for example, the highest genera of an object and of one of its properties will always belong to the same region). See *Ideen* I, §16, pp. 30–31 for a more complete discussion. Examples of regions include (physical) nature, the psychological region, and the phenomenological region of pure consciousness.

discipline of formal ontology, which is concerned with such truths, with formal logic (in a broad sense which includes all of mathematics, set theory, mereology, and so on), and, as already noted, uses "analytic" as a synonym for "formal eidetic." So an analytic truth is one which concerns structures considered in abstraction from what they are structures *of*, ways in which things of any kind whatsoever might be related—or, as Husserl also says, "modifications [*Abwandlungen*] of the empty something" (§14, p. 28). The categorial form *Euclidean three-manifold*—defined in the *Logsiche Untersuchungen*, recall, as "fully indeterminate with respect to matter"—is formal in that sense. Our knowledge about it is thus formal eidetic, i.e., analytic.

If the idea of a non-Euclidean world-space is formally unobjectionable but nevertheless absurd, then the absurdity must involve violation of a regional, *material*-eidetic constraint. Such is the force of Husserl's remark, in *Ideen* I, that the material essence *space* is a "deformalization" of the formal essence, *Euclidean [three-]manifold*.[18] The point is that the world-space as matter-of-fact object (a high order, syntactically structured object belonging to the region of nature) is always, by virtue of its material essence, an instance of the formal category in question. The knowledge of this necessary truth about the natural world is synthetic a priori: based on material-eidetic insight. So while space, on one understanding, is a formal structure studied by mathematics, the knowledge of which is analytic, it is also, on a different understanding, an empirical, physical structure, about which, finally, we also know synthetic a priori (material-essential) truths.

Carnap in *Der Raum*, similarly, explains that space, in one sense, is a formal structure, by which objects of any kind might be ordered; assigns our knowledge about it, in that sense, to formal ontology; and, therefore, labels such knowledge analytic. He, too, thinks of physical space as a matter-of-fact structure of relations about which we know a posteriori. And he, too, finally, explains that space in another sense is "the space of intuition" (*Anschauungsraum*): an essentially necessary order to which anything intuitable, hence anything falling under physical spatial relations, must conform.

To understand this last point, one must keep in mind that the primary meaning of "intuition" (*Anschauung*), for most authors in this period, is ordinary sense perception (including, for some, also related acts such as imagination and memory). This results from a popular interpretation of Kant, which, while still popular today, is debatable: other interpretations (as in Natorp) yield other uses of the term. In no case, however, is *Anschauung* the name of a special

---

[18] *Ideen* I, §13, p. 27.

faculty for knowledge about essences or about mathematics.[19] Proponents of such special faculties do sometimes call them *Anschauung*, but only as a conscious broadening of the term, based on some *resemblance* between the faculties in question and ordinary sense perception.[20] Here is not the place to explore the untold confusion which has often resulted from this simple terminological point. The moral for our purposes is just that Carnap's term *Anschauungsraum* means: the space of all possible sense perception (and imagination), not: the space revealed by a special faculty of "intuition."[21]

Now, the three-way distinction between logical (formal) possibility, *anschauungsmäßig* possibility, and (physical) actuality is not unique to Husserl. Such classifications are widespread among the philosophically minded mathematicians mentioned above: found, for example, in Killing, Hausdorff, and Frege.[22]

---

[19] The closest I have found to this is an unusual usage in Weyl's *Philosophie der Mathematik und Naturwissenschaft* (Munich and Berlin: Oldenbourg, 1927), 93, where Weyl, explicitly following Fichte, distinguishes between sensation (*Empfindung*) and intuition (*Anschauung*), attributing our knowledge of spatial relations to the latter. But even there *Anschauung* refers to a non-qualitative component *of ordinary perception*. (Weyl's usage in *Raum-Zeit-Materie* is more standard.)

It should perhaps go without saying that *Anschauung* also does not mean "intuition" in the contemporary Analytic sense (a kind of belief which one maintains even though one cannot give, or can't be bothered to give, any evidence for it). In the *Aufbau*, Carnap uses the Anglo-French loan word *Intuition* for something like that, although he goes on to demand that all such intuitions be rationally justifiable after the fact (see *Der logische Aufbau der Welt*, 4th ed. [Hamburg: Felix Meiner, 1974; reprinted 1998], §49, pp. 67–8; §54, p. 74; §100, p. 139; §143, pp. 191–2; §181, pp. 256–7). Carnap's usage reflects Russell's, which in turn reflects Bergson's (although Carnap also refers to Bergson directly): see Russell, *Our Knowledge of the External World* (New York: Routledge, 1993), 32–7. Husserl himself records the same distinction between *Anschauung* and *Intuition* (with disparaging remarks about the latter) in *Ideen* II: see *Ideen zu einer reinen Phänomenologie und phänomenologischen Philosophie, zweites Buch: Phänomenologische Untersuchungen zur Konstitution*, ed. M. Biemel, (The Hague: Nijhoff, 1952), §60d, Hua 4:273,33–274,2.

[20] See Husserl, *Ideen* I, §3, p. 11: "*Auch* Wesenserschauung ist eben Anschauung"; echoed by Carnap, *Der Raum*, §II, p. 23: "Im Allgemeinem mag aber der Ausdruck Anschauung *auch* die Wesenserschauung mit umfassen" (my emphasis in both cases).

[21] Note, however, that Husserl himself, in *Ideen* II, uses *Anschauungsraum* and "physical space" to mark a different contrast, namely between the subjective space of our perceptions and the constructed, mathematical, objective space of physics. Carnap was to adopt this terminology in the *Aufbau*.

[22] See W. Killing, *Die Nicht-Euklidischen Raumformen in analytischer Behandlung* (Leipzig: Teubner, 1885) (RLV 128a), iii; F. Hausdorff, "Das Raumproblem," *Annalen der Naturphilosophie* 3 (1904) (RLV 95): 3; Frege, *Grundlagen*, §14, pp. 20–21. Poincaré's concept of "representational space" is more distantly related (cf. Ben-Menahem, *Conventionalism*, 181 n. 5).

But these authors have no full-blown epistemological system, and Carnap never mentions them in general epistemological contexts. There is one point on which he does seem to follow Driesch, namely in emphasizing that the particular quality or "being-thus" (*Sosein*) of a given form of intuition is cannot be "conceptually defined," and is thus only communicable by "indicating contents of experience" (*auf Erlebnisinhalte hinweisen*).[23] But Driesch's complete view is utterly different from Carnap's and Husserl's. For Driesch, spatiality is one of the fundamental and irreducible order-constituents of consciousness (*Ursetzungen*).[24] From a Husserlian point of view, in other words, it impossibly combines aspects of the formal and the material eidetic (not to mention the noetic and the noematic, as well as the immanent and the transcendent). In short, Carnap's particular version of this classification is distinctively Husserlian. As I have already pointed out, Carnap himself was the first to recognize this. In *Der Raum*, he explains, where he allowed a synthetic a priori component of our geometrical knowledge, he was "following Kant and Husserl."[25]

2 Husserlian problems about general relativity, and Carnap's quasi-Husserlian solution

In fact, Carnap might not have broken with Husserl at all, at this stage, were it not for difficulties concerning general relativity. The form which these difficulties assume for him is in itself revealing. Euclidean geometry had served, historically, as a paradigm of necessary truth, so nearly everyone was troubled by claims of possible alternatives, let alone actual, physical ones. But the nature of the trouble varies. In a Marburg neo-Kantian system such as Natorp's, for example, the advance of logical/mathematical/scientific theory is understood as the discovery of more and more determinative conditions on the possibility of objective givenness.[26] The results of this process, as logical conditions for the very possibility of evidence, ought not to be subject to empirical challenge.[27] The homogeneity and isotropy of space were widely supposed to be among such conditions—where Natorp, at least, took this to exclude even spaces of constant

---

[23]See *Der Raum*, §II, pp. 22, 24; cf. *Ordungslehre*, 109: "ebensowenig wie von der als diese gesetzten Setzung 'grun' läßt sich aber mehr von ihnen angeben, als daß sie *da sind* in ihrem Sosein" (emphasis in the original).

[24]*Ordnungslehre*, 82.

[25]*Philosophy of Rudolf Carnap*, 957.

[26]For more discussion of this, see my "The Continental Origins of Verificationism: Natorp, Husserl and Carnap on the Object as Infinitely Determinable X," *Angelaki* 10 (2005): 129–43, and passages cited there, especially Natorp, *Grundlagen*, 354, 358–60, 366.

[27]See Natorp, *Grundlagen*, v.

(nonzero) curvature.[28] If general relativity is correct then, at a minimum, there is some error in the demonstration that just these conditions are necessary.

Husserl's problems are different, and may at first seem rather minor. It is surprising, of course, that theorems of Euclidean geometry, supposedly a matter of material-eidetic evidence, turn out to be false. But Husserl, unlike Natorp, never *argues* that they are (necessarily) true. Phenomenology explains our supposed knowledge of them only by referring us to acts of *Wesenserschauung* in which their necessity is posited—acts which, in a case like this (insight into the essence of an external object), are eminently fallible.[29] A deeper problem is raised, however, by the connection between eidetic possibility and imaginability. For it is supposed to be a matter of phenomenological necessity that certain types of transition are always possible between *Wesenserschauung* and ordinary intuition. A rational transition from intuition (perception) of a matter of fact to intuition of an essential truth which it instantiates must always be possible: we can always see (turn our attention to) the necessity of whatever is necessary in some actual state of affairs. But a transition in the other direction, in which an instance is produced for a rationally posited necessary law, is also always supposed to be possible—except that the instance in question may be only imaginary, rather than actually perceived.[30] This is what underwrites Husserl's so-called method of free variation: the limits of what is material-eidetically possible are also the limits of what can be, if not actually perceived, then at least imagined. But if, as was widely (though not universally) thought to be the case, non-Euclidean geometries are unimaginable, then, however formally consistent they may be, they would have to be ruled out as descriptions of actual spatial relations.

Carnap could have dealt with the first, superficial version of the problem

---

[28]See *Grundlagen*, 226–8, 307–9.

[29]*Ideen* I, §149, pp. 310–11; see also §60, pp. 113–15. This point, about the *inadequate* givenness of transcendent ("nonimmanent") essence, is also appealed to by Weyl, *Raum-Zeit-Materie: Vorlesungen über allgemeine Relativitätstheorie*, 4th ed. (Berlin: Springer, 1921), §18, pp. 133–4. (Note that this section does not yet appear in the third edition [Berlin: Springer, 1919] [RLV 264]. Both the second and the third editions appeared in 1919, so Carnap's bibliography entry is ambiguous. But the page reference at Literatur-Hinweise, p. 84 [note to pp. 41 and 57] shows that he is using the third edition.) Cf. Friedman's discussion of this passage, "Carnap and Weyl," 52–4. Friedman sees here a turn to the study of the constituting subject—in other words, to the study of *immanent* essence. Weyl would indeed be interested in such a procedure, but that does not seem to be his meaning in this passage. Unfortunately there is no room here to look into the true nature of the difference between Carnap and Weyl (or between either of them and Oskar Becker).

[30]See *Ideen* I, §§3–4, pp. 4–12.

simply by saying that eidetic insight does not fully settle the geometry of space (that it prescribes, say, only the property of being a Riemannian manifold), so that further details must be settled empirically and/or conventionally. The material essence of space would thus be the deformalization of a different, slightly more general formal category. But, although the thesis of *Der Raum* is sometimes summarized this way, its actual contents are more complicated. This is because Carnap has the second, deeper worry in mind. Certainly he subscribes to the relevant Husserlian doctrine, the connection between imaginability and material-eidetic possibility. Material-eidetic insight, as he explains, differs from formal-eidetic in that the latter takes place without (in abstraction from) any "intuitive significance."[31]   But the *anschauungsmäßig* character of material-eidetic insight does not mean that its deliverances are merely empirical rules gathered from repeated experience; rather, their hold over all *possible* experience can be established by free variation, based on only a single example—an example which may itself be merely imagined (§II, p. 22). So Carnap faces Husserl's problem here. Can non-Euclidean geometries be imagined? Can we, for example, imagine a two-sided polygon? And if not, why doesn't this limit to free imaginative variation found a material-eidetic insight that only Euclidean geometry is physically possible?

Carnap's answer is based on a substantial understanding both of differential geometry and of Husserl. He claims that eidetic insight reveals only the local character of the *Anschauungsraum*: its behavior in (arbitrarily) small regions. The mathematical motivation for this is that all Riemannian manifolds are "infinitesimally flat." In other words, roughly speaking, the geometry of a Riemannian manifold is always arbitrarily close to Euclidean in a sufficiently small region around any point. If eidetic insight prescribes only such infinitesimal Euclidean behavior, therefore, it will not rule out any geometry envisioned in general relativity.[32] Meanwhile, however, there are independent reasons for thinking that Husserl ought to have adopted this view in the first place.

The argument begins with the fact that we cannot perceive or imagine arbitrarily large regions of space. Husserl does not discuss this point in *Ideen* I, but he would acknowledge it. His first book, the *Philosophie der Arithmetik*,[33]

---

[31] *Der Raum*, §I, p. 7.

[32] Except that general relativity is not really a theory of space as a Riemannian three-manifold, but rather of space-time as a quasi-Riemannian four-manifold. Carnap, while aware of this complication, chooses to ignore it (see §III, pp. 40–41, 46, 57).

[33] *Philosophie der Arithmetik: Psychologische und logische Untersuchungen* (Halle: C.E.M. Pfeffer [Robert Stricker], 1891) (RLV 116), reprinted in Hua 12, ed. L. Eley (The Hague: Nijhoff, 1970).

is all about the contrast between the small finite realm (*Gebiet*) of intuition and the infinitely large structures of mathematics. The prime example there is the series of cardinalities which one obtains by successively adding members to a collectivum. But Husserl also mentions geometry: one example of something beyond the reach of our intuition is "the light-years of the astronomers" (214/Hua 12:192,3–4), and one example of an infinite set which can be symbolically, but not intuitively, represented is the set of points on a line (247/Hua 12:219,12–2). These two examples actually may hint at two different respects in which the infinitude of space escapes our imaginative capacities: infinite extension and infinite divisibility. In any case, by the time of the 1907 lectures later published as *Ding und Raum*, Husserl explicitly gives both these infinitudes (both familiar from Kant's Antinomies, and before that from Hume) as examples of the limits of intuition. Despite the double infinitude of space, he explains, "every possible presentation [*Darstellung*] must make do with limited means of presentation."[34] But then Carnap is right to point out, based on the connection between imaginability and essential possibility, that what material eidetic insight delivers directly can only be the *local* properties of space.

The question, then, is how to extend our eidetic knowledge about the finite realm accessible to intuition into a priori global knowledge about the surrounding infinite structure. Husserl is no stranger to this question. The whole plot of the *Philosophie der Arithmetik* turns around the way a limited *Gebiet* of intuitive, "proper" (*eigentlich*) representation—in this case, the representation, with respect to cardinality, of small collectiva—can be symbolically extended (*erweitert*) to infinity. Husserl's answer is that, although we can never have direct intuitive access to large cardinalities, we *do* have direct access to the clearly defined and infinitely iterable operation which, given any arbitrary (finite) collectivum, yields a new one whose cardinality is one greater. The "complete extension [*Erweiterung*] of which the concept of set or multitude is capable by symbolic means" is possible, in other words, because "the process of adding-on [*Hinzufügung*] of a unity to an arbitrary given number is an operation whose

---

[34] *Ding und Raum: Vorlesungen 1907*, ed. U. Claesges (The Hague: Nijhoff, 1973), Hua 16:122,13–14. I am not aware of any evidence that Carnap knew, in 1922, of the specific contents of these lectures. At a minimum, however, they show how accurate was Carnap's extrapolation from Husserl's published works. In fact, Husserl even mentions, in this context, that the considerations under discussion would apply in a space of any dimensionality and curvature (122,1–4). So he was already (in 1907!) in some ways quite close to Carnap's solution. Husserl does not allow for variable curvature, however (because it rules out rigid motion), and also adds that even constant-curvature non-Euclidean spaces are inconsistent with the essence of "our" spatial perception.

concept guarantees, a priori, that it leads to a new, determinate number."[35] In the same passage just cited, moreover, Husserl gives the operation of adding intermediate points between two points of a line as a further example of the same principle, and in *Ding und Raum* he says that the spatially large or small can only be presented "through the serial order form of the operations" by which the limited means of intuition "come into action again and again"[36]—a process which Husserl there, too, calls *Erweiterung* (e.g., 205,15; 209,16).

Here, again, Carnap follows Husserl. The *Erweiterung* of the *Anschauunsrgaum* beyond the limited *Gebiet* of our perceptual or imaginative field takes place, he explains, through a step-by-step extrapolation of the features of our limited perceptual space into an unlimited global structure, a *Gesamtgefüge*. We are justified in doing so, he says, because "if ... the species of a formation permits a second one of the same species to be added on to it in a determinate manner, we can demand [*fordern*] that this adding-on [*Anfügen*] should be further possible without end."[37] So Carnap and Husserl agree, not only on the initially local nature of a priori geometric knowledge, but also on the means by which such local knowledge can be extended. Why, then, does Husserl not agree with Carnap on the eidetic possibility of a non-Euclidean physical space?

The answer is that infinitesimal is not the same as local. Although every Riemannian manifold is everywhere infinitesimally Euclidean, only Euclidean space itself, and trivial variations thereon, are Euclidean in a small *finite* region around every point. Husserl is correct to argue, then, that local structure determines global structure. Since, as he puts it, the internal ordering (*Anordnung*) of the perceptual field prescribes a fixed order (*Ordnung*) to whatever falls within it, and since a continuous shifting of the field involves a continuous positing of unity in the continuously shifting images, "there arises the consciousness of a thing-manifold of fixed order [*von der fest geordneten Dingmanigfältigkeit*], and finally of the world."[38] The relation even of distant perceptible objects is held absolutely fixed by the order of all the intervening fields, between which in principle one might shift one's perception on the way from one to the other (218,9–12).

Carnap, conversely, must somehow argue that eidetic intuition reveals, not

---

[35]*Philosophie der Arithmetik*, 246–8/Hua 12:218,26–8, 220,11–14.

[36]Hua 16:122,14–18.

[37]*Der Raum*, §II, p. 23. For the terms *Erweiterung* and *Gebiet*, see Literatur-Hinweise, p. 81 (note to p. 26). This terminology is also found in other authors, however (including Killing and Pasch), so it is not unequivocal evidence of Husserl's influence. On *fordern* and *Gefüge*, see further below.

[38]*Ding und Raum*, Hua 16:217,30–35.

the finitely local, but only the infinitesimal structure of space. Unfortunately his presentation becomes vague at this crucial point, so that, although an argument can easily be reconstructed, it is difficult to be sure what he had in mind.[39] One way for him to go would be to claim that the realm revealed to intuition (i.e., to sense or imagination) is itself infinitesimal, rather than finitely small.[40] But that sounds implausible, if indeed it makes any sense at all. Carnap probably means, instead, to go in the opposite direction. For, as Husserl would notice only somewhat later, the fixed extension of local geometry into the arbitrarily large is blocked thanks to the other limit of perception, in the case of the arbitrarily small.[41] Perception gives only limited detail, or, in other words, is always only of relative straightness, equality, perpendicularity, etc.[42] And, although we can imagine indefinite improvements on this—can imagine, so to speak, focusing in more and more closely on a smaller and smaller region—an *infinitely* detailed view is literally unimaginable.[43] Even if imagined spatial relations are always perfectly Euclidean, then, that can only mean that they agree with Euclidean axioms up to the level of detail that is actually imagined. The corresponding eidetic knowledge is thus just what we need: namely, that space must be arbitrarily close to Euclidean in sufficiently small regions, but need not be perfectly Euclidean anywhere.[44] Although Carnap is not very clear about this, it does fit well with the way he talks: he constantly mentions

---

[39]For the view that Carnap was simply confused on this point, see Sarkar, "Husserl's Role," 188.

[40]This appears to be Friedman's interpretation: see "Carnap and Weyl," 48.

[41]See *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie*, ed. W. Biemel, Hua 6 (The Hague: Nijhoff, 1954), Abhandlung titled "Realitätswissenschaft und Idealisierung: Die Mathematisierung der Natur" (dated by Biemel to 1926–8), 290,21–6, and see also Beilage III (the 1936 text published by Fink as "Der Ursprung der Geometrie"), 384,10–45.

[42]Cf. Hume, *A Treatise of Human Nature* (Oxford: Clarendon, 1978), 1.2.4, pp. 49–50. Husserl gave a seminar on Part I of the *Treatise*, in conjunction with his lectures on the history of modern philosophy, in the summer of 1921, when Carnap was already living near Freiburg (Schuhmann, *Husserl Chronik*, 246). Interest in Hume was high in general among Brentano and his followers, and Husserl knew his Hume well. In *Erste Philosophie*, he goes so far as to call Hume's psychology "the first systematic attempt at a science of what is given in pure consciousness," i.e. the first attempt at transcendental phenomenology (*Erste Philosophie (1923/24), erster Teil: kritische Ideengeschichte*, ed. R. Boehm [The Hague: Nijhoff, 1956], Hua 7:156,39–157,2). I might add that, to the extent it makes sense to place Carnap in an empiricist tradition going back to Hume, the main connection is via Husserl and Brentano, not via the (anti-)neo-Hegelian Russell.

[43]See Husserl, *Krisis*, main text, §9a, p. 22,5–22.

[44]Assuming that imagined objects have no intrinsic scale. See Husserl, *Ding und Raum*, Hua 16:121,19–33, and cf. Hume, *Treatise*, 1.2.1, p. 28.

small regions of space, without saying exactly how small, and never mentions infinitesimal "regions" at all. It is also suggested by his technical sources, the most important in this respect being Killing and Pasch.[45]

In mathematical approach, Carnap is closest to Killing, who begins with a subset of what he takes to be Euclid's assumptions, leaving out the non-local parallel axiom and the assumption that the straight line is infinite. On this basis, Killing attempts to show, first, that all of Euclid's theorems hold in a "limited *Gebiet*." This turns out to mean that they hold arbitrarily well in every sufficiently small neighborhood—or, as Killing more typically expresses things, that they hold perfectly in every infinitely small neighborhood.[46] Four global possibilities—Euclidean, hyperbolic, and single or double elliptic geometry—are then shown to follow from different ways of extending local (infinitesimal) segments into geodesics. Carnap updates this approach mostly by substituting Hilbert's axioms for Euclid's. He also claims to include the general case of variable curvature, though without enough mathematical detail to determine how or whether that is possible.[47]

Epistemologically, however, Carnap is closer to Pasch, who explicitly claims to derive all his axioms from our finite sense perception.[48] Unlike Killing, Pasch never speaks of infinitely small regions in which the Euclidean axioms hold

---

[45]See W. Killing, *Nicht-Euklidischen Raumformen*, 1–17, and M. Pasch, *Vorlesungen über neuere Geometrie*, 2d ed. (Leipzig and Berlin: Teubner, 1912) (RLV 190), 4–20, both cited by Carnap, *Der Raum*, Literatur-Hinweise, p. 81 (notes to pp. 26 and 27).

[46]Killing alternates between these two types of formulation. See e.g. *Nicht-Euklidischen Raumformen*, 3, where the main theorem, "In every triangle whose sides are, all taken together, infinitely small, the sum of the angles equals two right angles," is followed by a long paraphrase in terms of allowing a given finite triangle to shrink, etc.

[47]Killing himself says that his original intention had been to take on the general case, but decided against it as being too large a project (*Nicht-Euklidischen Raumformen*, vi). The technical result Carnap probably would have needed to back up his claims was published by Weyl in the same year as *Der Raum*, too late for Carnap to have seen it (although he does cite some of Weyl's earlier works, and recommends Weyl as the primary source to consult on general relativity). See H. Weyl, "Zur Infinitesimalgeometrie: Einordnung der projektiven und der konformen Auffassung," *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* (1921): 99–112, and see the discussion in Torretti, *Geometry and Relativity* (New York: Dover, 1996), 191–4. Carnap also allows another generalization, to the *n*-dimensional case (see *Der Raum*, §II, p. 30). He spends little time discussing this point in *Der Raum*, since he knows of no empirical evidence best accommodated via more than three spatial dimensions. But he was open to the possibility, as can be seen from his reference to Kaluza in a slightly later paper: "Dreidimensionalität des Raumes und Kausalität: Eine Untersuchung über den logischen Zusammenhang zweier Fiktionen," *Annalen der Philosophie und philosphischen Kritik*, 4 (1924):117 n. 1.

[48]*Vorlesungen*, 14, 17.

perfectly. And whereas Killing, in explaining why we cannot determine the precise geometry of physical space, appeals to the limits of our physical measurement devices,[49] Pasch sees a limitation intrinsic to our form of intuition. This presumably is why Carnap criticizes Killing on this issue, but not Pasch.[50] The absence of a determinate metric and affine structure cannot be supplied, as Killing implies, even by infinitely improved physical measurement (as if an infinitely straight and regularly marked ruler could be used to establish the very definitions of "straight" and "regularly marked"). On the other hand, if *imagination* were infinitely precise then, on the Husserlian view Carnap takes, the affine and metric structures would be determined in advance of all physical measurement. This is because imagined pictures include an irreducible phenomenological character of, for example, straightness: one need not make any measurement to determine whether one is imagining a straight line. The key point, then, is that the straightness of such a line, like every other feature of the imagined situation, is never given with infinite precision.

In any case, whatever the details of the argument, Carnap's conclusion is that material eidetic insight founded on our actual imaginative capabilities is *not* sufficient to fix a global geometry. It is important to understand the depth of the problem, however. If eidetic insight fails to prescribe a determinate global geometry, then it fails to fix any relation at all between distant objects, and hence leaves us with no global intuitive space whatsoever. We get no a priori reason to think, for example, that a thing outside the limits of our perception must be *somewhere* (must lie in some determinate spatial relation to the perceived objects). Yet mathematical physics, general relativity included, makes constant use of the assumption that some determinate geometry applies to arbitrarily large regions, or even, in cosmology, to space as a whole.[51] If eidetic insight does not justify the assumption that physical space, though known to us only through imprecise perception, nevertheless instantiates some determinate geometrical structure—then what does? Husserl himself ultimately concluded that this assumption, along with others which go into Galileo's "mathematization of nature," is indeed unjustifiable.[52] But Carnap instead, fatefully, appeals

---

[49] *Nicht-Euklidischen Raumformen*, 13.

[50] See *Der Raum*, Literatur-Hinweise, p. 83 (note to p. 54).

[51] Actually, GR as ordinarily understood assigns to space(-time), not a geometry, but an equivalence class of geometries. Again, Carnap ignores this complication.

[52] See *Krisis*, §9h, Hua 6:48–54. Cf. Husserl's treatment of the same achievement of Galileo at *Ideen* II, §16, Hua 4:49,8–17 (a text dating from 1915); §18d, Hua 4:76,19–77,21 (a text of unknown date, but before 1917, although with a slight change by Husserl post-1925—see critical apparatus, Hua 4:406–7 for details); and see also *Ideen* I, §9, p. 20.

to the notion of a *Forderung*—a "demand" or "postulate."[53] Thus he transforms the question from: to what extent does the local nature of eidetic insight leave geometry undetermined? to: what minimal a priori conditions must we demand of possible experience, *beyond* those derivable from eidetic insight, in order to guarantee that it conform to *some* global spatial order?

3 Carnap's use of Driesch and Poincaré

This turn of thought also has Husserlian precedent. Even the term *Forderung*, in fact, is found in relevant contexts in the *Philosophie der Arithmetik*.[54] Still, there can be little doubt that Carnap's main source in this respect is Driesch, whose system in the *Ordnungslehre* is built around the twin concepts of *Forderung* and *Gefüge*. The full system is complicated and not entirely coherent, but the relevant points can be summarized as follows.

All philosophy, according to Driesch, begins with phenomenological reflection upon the stream of experience. Nothing much could be said about this, however, if there it were not for certain "order constituents" in the stream of experience thus revealed.[55] Such an order constituent is a sign of "final conclusiveness" (*Endgültigkeit*) (5). It establishes that what has been experienced has been experienced as such-and-such, and that this "as" *ought* to hold for all further experience, as well. The second part of philosophy, *Ordnungslehre*, thus arises as "the theory of final conclusiveness on the grounds of a methodological solipsism" (8).

Because thought, in this sense, is what first establishes or reveals an ought, Driesch says that its "primordial achievement" (*Urleistung*) is a demand, a *Forderung*, of order (7). Every order constituent presents itself as the fulfillment of this primordial demand (18). Ordering thought thus fulfills its own demands in the act of making them: it cannot be constrained by any order, because it first produces and recognizes order. In this sense, all order is its "free achievement" (34). Nevertheless, according to Driesch, there is a kind of *Forderung* which reflects back on the ordering ego itself, an "ought to posit positings which ought to hold" (6). This is what eventually allows thought to posit objects which make

---

[53]*Forderung*, in a mathematical context, might naturally be translated as "postulate" (equivalent to Euclid's term αἴτημα). This usage occurs in Carnap's mathematical sources. But, as will become clear, Carnap is interested also in the term's non-technical force, as "demand."

[54]See, e.g., p. 222/Hua 12:199,9–11: "doch steht nichts im Wege, dem Begriffe des Processes die Forderung beizufügen, daß er alle erdenklichen Glieder in sich aufnehme."

[55]*Ordnungslehre*, 6.

demands on *it*, and about which it can therefore be incorrect. Such objects, which first occur in the realm of natural actuality, are then "as if" independent of the ego (161–2).

The special type of *Forderung* in question is "the self-demand of *parsimony of positings*" (110).[56] Driesch first discusses it at length in his section on space—the very section which Carnap cites in *Der Raum*. But the full importance of parsimony comes out only in the realm of natural actuality, where thought achieves its goal by establishing a quasi-logical necessity, according to which later experiences are "as if co-posited" with earlier ones (146). This can be accomplished in a certain selected piece, a certain *Ausschnitt*, of experience (132), within which experiences are interpreted as the appearance of physically actual objects at different points in their process of becoming (with the remaining experiences then dismissed as dreams, hallucinations, illusions, etc.). Now a "system," or *Gefüge*, for Driesch, is an ordering-type in which each element implies all the others, so that the generic character of the *Gefüge* already predetermines all the different ways it can be instantiated (93, 121).[57] Nature, therefore, from the point of view of *Ordnungslehre*, is just a selection of experiences in which "a particular *Gefüge* of ordering thought-demands is found to be *fulfilled*: a *Gefüge* of demands with respect to *becoming*."[58]

This selection of experience is in turn possible, however, only thanks to more fundamental types of *Gefüge*. Suppose, to take Driesch's example, that I experience three houses. This will count as a *correct* experience, an actual perception, only if it does not contradict the unified positing of nature. That a contradiction could arise here at all is due, first, to the status of the natural world as *Gefüge*: the positing of three houses runs the risk of being inconsistent with my other experiences only because something else (e.g., four houses, or none) may be "as if co-posited" with them. But the possibility of contradiction depends, also, on the unified order of space and time. Three houses don't per se contradict anything: there can be three houses on this street and none on the next. It is only once we add a time and location that a contradiction becomes

---

[56]I have changed all of Driesch's small capitals to italics.

[57]Driesch indicates several times in the text (pp. 87, 93, 138) and also in the index (344) that *Gefüge* is supposed to be the equivalent of, or a replacement for, the term *System*. This is part of his general policy of using German, rather than Greek or Latinate, terms (see p. 10). Cf. T. Mormann, "Synthetic Geometry and the *Aufbau*," in Bonk, ed., *Language, Truth and Knowledge*, 47. Mormann is puzzled by Carnap's use of the term *Ordnungsgefüge*: he notes that it is "not a terminus technicus in mathematics," but doesn't suggest any alternative origin.

[58]*Ordnungslehre*, 132.

possible. To say that an experience is correct, in other words, is to say that "*here* and *now*, that is, in these points of the space of nature ... and the time of nature ..., *this such* exists with natural actuality" (160). But the unified structure of space is itself a *Gefüge*. For example: there is no contradiction in the idea of a closed two-sided polygon; only, in Euclidean space, the third side is as if co-posited with the other two. This spatial *Gefüge* must therefore be the result of an earlier and more fundamental demand for parsimony.

It is in these terms that Carnap justifies his own global *Anschauungsraum*, as the *Gefüge* answering to the a priori *Forderung* of global spatial structure. Moreover, both he and Driesch understand this as a demand that the extension of the finite realm of intuition via serial "adding-on" should be completable in a determinate way.[59] Both hold that, while the primitive phenomenological character or *Sosein* of spatiality directly yields a list of local characteristics or axioms, a *Forderung* of thought is needed to back up the judgment that "segments, planes, and parts of space can be increased by a *so much* through adding-on others of the same kind."[60] Carnap simply gives this a more precise technical content, based on Killing and Pasch. Finally, Carnap and Driesch even agree that there is more than one way we could impose the demand: as Driesch puts it, "a genuine choice between several final-conclusivenesses which present themselves as possible to adhere to" (109–10). They disagree only about how, and at what stage, we are to choose.

Driesch thinks further demands of parsimony require space to have one specific type of order: in particular, to satisfy the parallel postulate (111–16). Carnap agrees that the "spatial system" (*Raumgefüge*) is ultimately determined by such further demands. The choice between different geometries remains in principle open; only from an "end-positing" standpoint which invokes the "teleological and methodological principle" of simplicity does one choice emerge as the sole permissible one.[61] Carnap maintains, however, that these further demands must not be added to our positing of the *Anschauungsraum*, i.e. to the structure of space as it is posited in advance of all experience. The positing of the *Anschauungsraum*, in other words, should include only such *Forderungen* as are required to insure *some* global (Riemannian) structure, but not any

[59]See the important note already cited several times above: *Der Raum*, Literatur-Hinweise, p. 81 (note to p. 26), where Carnap lists sources to consult on "the *Erweiterung* of the spatial *Gebiet*." Besides Killing, Pasch, and Driesch, the list includes only Benno Kerry's vaguely relevant *System einer Theorie der Grenzbegriffe: ein Beitrag zur Erkenntnisstheorie* (Leipzig and Vienna: Deuticke, 1890) (RLV 127).

[60]Driesch, *Ordnungslehre*, loc. cit.

[61]*Der Raum*, §III, p. 56.

further ones which would serve to specify that structure more precisely. The reason is not a lack of theoretical justification: since the demand for any global structure whatsoever, according to Carnap, is not justified by eidetic insight, we would be no *less* justified in adding a further *Forderung* of Euclideanness (§II, p. 28). Rather, the problem is practical. The spatial *Gefüge* is not posited for its own sake, but, as Driesch would agree, for the sake of what comes next, the construction of a "contradiction-free *Gefüge*" of physical actuality (§IV, p. 61). What the overall demand of parsimony requires here, then, is a simplicity, not in geometrical determinations per se, but only in "the structure which follows on the ground of those determinations" (§III, p. 56). If we were free to impose simplicity at the purely geometrical stage, then, Carnap agrees, we would presumably always demand Euclidean order. But this demand would interfere with the ultimate goal of ordering natural actuality, and is therefore *practically* ruled out. In other words, what is true of Carnap's flat-earth metric (§III, p. 52) is true also, in a subtler way, of the a priori choice of a Euclidean metric: we cannot seriously (*ernstlich*) choose it.

This brings us to Carnap's conventionalism, hence to his appropriation of Poincaré. He and Poincaré are working towards different goals, based on different assumptions.[62] Poincaré's idea of the synthetic a priori, for example, has not much to do with the Husserlian concept used by Carnap. Moreover, Carnap takes a far more radical position about space, claiming that not only our particular geometry, but our notion of global spatiality as such, rests on a choice we have made in response to the demand of parsimony—on a convention, if you like. Such differences are beside the point, however, because what Carnap wants from Poincaré are the philosophical and technical means to fill in a very specific place in his own project. Having argued, against Husserl, that the limits of imaginability do not establish a full global structure for space, and having argued, against Driesch, that the demand for geometrical parsimony must lose in any conflict with the demand for physical parsimony, he must now demonstrate that such a conflict can actually arise, and explain how, in such cases, the overriding demand of physical parsimony can be sufficient to impose a full affine and metric structure. In §III of *Der Raum*, Carnap turns to Poincaré for help in carrying out those limited tasks.

Poincaré might seem an unlikely ally, since his ultimate conclusion appears to agree with Driesch's: that no experiment will ever get us to choose a non-Euclidean geometry. But, as Carnap understands him, Poincaré bases this argu-

---

[62]See Friedman, "Poincaré's Conventionalism and the Logical Positivists," in *Reconsidering Logical Positivism*, 71–86.

ment on an assumption about the contingent, empirical behavior of our world. Given other circumstances, which Poincaré thought counterfactual, the same considerations may well lead to a different choice (§III, p. 56). Whether or not this view is correctly imputed to Poincaré, moreover,[63] he does provide examples which make the point vivid. To allude to the most famous one: hyperbolic geometry seems simpler than Euclidean geometry plus specially rigged thermal gradients and laws of refraction.[64] What Carnap takes from these examples is that the means for making geometrical measurements, i.e. for establishing geometrical matters of fact, are always physical objects, so that the results of such measurements are always subject to reinterpretation under a change of view about the physical laws which apply to the measuring instruments. Carnap thus takes his cue from Poincaré in pointing out how a change in view about law of thermal expansion could change the result of measuring length with a rod, and how a change in view about the laws of refraction could change the result of establishing straightness by line of sight—i.e., how metric and affine *Tatsachen* would have to change to accommodate different physical theories.[65] If there are such interchanges between the physical and the geometrical, then an intrinsically more parsimonious geometry might be purchasable only at the cost of a less parsimonious total theory, so that Driesch is, at least in principle, incorrect. Which global geometry *ought* to be demanded will depend on empirical matters of fact.

Thus understood, Carnap's position fits oddly into the debate between conventionalist interpretations of general relativity and those which take it to show that geometry is physical.[66] Whatever may be the case for others, Carnap re-

---

[63]Poincaré appears to make a stronger, psychological prediction that *we*, given our upbringing, will always continue to use Euclidean geometry, even if we turn out to live in a world where some other geometry would be more convenient. (See *Wissenschaft und Hypothese*, tr. F. and L. Lindemann [Leipzig: Teubner, 1906] [RLV 204], 52, 73, 74–5. The other source Carnap cites, "Raum und Zeit," in *Letzte Gedanken*, tr. K. Lichtenbecker [Leipzig: Akademische Verlagsgesellschaft, 1913] [RLV 207], 54–5, is only slightly more conciliatory.) Note that this reading is at odds, not only with Carnap, but also with Ben-Menahem, who sees Poincaré as making the "recommendation" that we choose Euclidean geometry (*Conventionalism*, 65).

[64]See Poincaré, *Wissenschaft und Hypothese*, 66–70. Not that I know of a rigorous definition of relative simplicity from which this would follow.

[65]See §III, pp. 34–6, 52. The fact that these examples work under strange idealizing assumptions (e.g. that thermal expansion and contraction are instantaneous, and need not be accompanied by other thermal effects, such as black-body radiation) only serves to strengthen the impression that Carnap is following Poincaré, rather than coming up with examples on his own. In the flat-earth example, which really is Carnap's, he makes no attempt to tie the required law of length change to a familiar physical mechanism.

[66]See M. Friedman, *Foundations of Space-Time Theories* (Princeton: Princeton University

gards these two as compatible and even identical. The conventionality of geometry, in the sense that there really are choices to be made about it, is precisely a consequence of the fact that geometry is physical, i.e. that the methodologically correct choice of physical theory involves inter alia choices about the structure of space. If geometry were self-enclosed, then, although it would still depend on a kind of choice, the correct choice would be completely determined by considerations internal to geometry itself: the best theory would be the simplest theory *qua pure geometry*. For Carnap, on the other hand, it is considerations external to geometry proper—*physical* considerations—which determine the best choice. Hence for him the choice of metric is logically (*denkmäßig*) free and independent of experiences, but is "*scientifically* determined or, better, to be determined":[67] the methodological principles which demand a certain choice, on the basis of the empirical matters of fact, are part and parcel of what is meant by "science." Studying the physical world means accepting such principles, because our positing of a coherent and independent physical world, a *Gefüge* of natural actuality, is itself only a response to the overriding practical demand of parsimony.[68]

4   Implications for Carnap's later thought

As I indicated to begin with, all this detail about *Der Raum* interests me primarily because of the insight, negative and positive, which it yields into Carnap's further development.

On the negative side is Carnap's rejection, in the *Aufbau*, of the synthetic a priori, leaving all truths as either logical or empirical—in other words, the beginning of "logical empiricism."[69] We can now see how this relates to the *Aufbau*'s other rejection, namely of "metaphysics" as a body of unverifiable, and therefore meaningless, statements.[70] If material-eidetic knowledge derives,

---

Press, 1983), especially chapters 1 and 7; R. Torretti, *Geometry and Relativity*, 230–47; Ben-Menahem, *Conventionalism*, ch. 3; and the further sources they cite.

[67] *Der Raum*, §III, p. 56; my emphasis.

[68] Note that this is not so different from the view taken by Friedman in *Foundations* (see especially 338–9). But cf. Friedman, "Carnap and Weyl," 58, and see also Ben-Menahem, *Conventionalism*, 125.

[69] *Aufbau*, §106L, p. 148; §179, p. 253.

[70] Friedman has recently argued, mostly on the basis of a single statement in Carnap's "Intellectual Autobiography," that the rejection of metaphysics in the *Aufbau* has nothing to do with unverifiability (see "The *Aufbau* and the Rejection of Metaphysics," in Friedman and Creath, eds., *Cambridge Companion*, 147). But what Carnap actually says was new after the *Aufbau* is not the characterization of metaphysical statements as unverifiable, but the consequences attached to that characterization: lack of all cognitive meaning, as opposed to mere exclusion from science. And even this is an exaggeration: see *Aufbau*, §179, pp. 252–3;

as Carnap maintains in *Der Raum*, from a pure phenomenological *Sosein*—a purely subjective character which cannot be described or communicated—then it is unsurprising that metaphysics in the *Aufbau* consists mostly of attempts to solve various "problems of essence."[71] Also related is the *Aufbau*'s "main thesis" (*Hauptthese*), that "there is only one realm of objects and therefore only one science."[72] If there is only one region, then mere formal ontology suffices to characterize all possible types of object. This then plays into Carnap's assertion that only purely formal descriptions are communicable.[73]

On the positive side, note first that, because eidetic insight is analogous to sense perception, Carnap in *Der Raum* is already a kind of empiricist.[74] Carnap will soon turn against eidetic "perception," for the reasons just discussed. He always remains relaxed, however, on the question of what is to count as empirical data, and eventually even concludes that this is a matter of convention.[75] The point is that there be *something* to which our speech and thought is publicly responsible, against which it can be publicly checked. Eidetic insight was rejected because Carnap found it unsuitable for that role, rather than, say, because of a naturalistic worry about causal interactions with essences. In *Der Raum*, in any case, Carnap still treats eidetic insight as an admissible source of quasi-empirical data. And, if "empiricism" is understood in that broad sense, then there are striking similarities between the form it takes here and the forms it takes much later in Carnap's development.

One theme is that the realm of scientific knowledge vastly exceeds, and yet continues to be justified by, its empirical basis. In the *Aufbau*, with its strict reductionism and verificationism, this theme is muted, but still present in several

---

§180, p. 254. The *Aufbau* is milder only in rhetoric.

[71] See ibid., §20, pp. 25–6; §161, pp. 222–3.

[72] *Aufbau*, §4, p. 4.

[73] See *Aufbau*, §§14–19, and see also, applying this back to the definition of space, §125, p. 166. A view like this (but without the anti-metaphysical conclusion) is found in Husserl's posthumously published *Ideen* II. It quite possible that Carnap was familiar with the contents of that work, since, as Richardson points out, Landgrebe was engaged in editing it during the time when Carnap was both participating in Husserl's seminar and working on the *Aufbau* (see "The Geometry of Knowledge: Becker, Carnap, and Lewis and the Formalization of Philosophy in the 1920s," *Studies in History and Philosophy of Science* 34 [2003]: 175; cf. the Biemel's description of this stage of editing, Hua 4:xviii, 400). That the *Aufbau*'s empiricism is essentially a rejection of the private or absolutely subjective, and the connection between this fact and the unity of the object domain, have been noted by Ryckman: see "Designation and Convention: A Chapter in the Early History of Logical Empiricism," *PSA* 1990: 152, 154.

[74] As, indeed, Husserl claims to be in *Ideen* I: see §20, p. 38.

[75] See "Über Protokollsätze," *Erkenntnis* 3 (1932): 215–28.

ways. First, empirical statements in the *Aufbau* system are in general reduced to statements about the *entire* stream of the subject's experiences. But, as Carnap himself emphasizes, the idea that I might have simultaneous access to all that is a wild fiction[76]—no better than the fiction that I might intuitively grasp a huge collectivity, or the whole of space, or a four-dimensional shape. All such statements, then, go far beyond the actual data which back them up. Secondly, Carnap says in the *Aufbau* that a constitutional system with a different basis— for example, a physical basis—would be equally as acceptable, meaning equally as *empirical*, as the methodologically solipsistic system actually adopted there.[77] Carnap does not explain how the concepts in such a system could be tested for empirical validity, but it could not be by reducing them to the given. Finally, Carnap at one point defends the status of pure mathematics not, as one might expect, on the basis that mathematical language is tautologous, but rather on the basis that there is *some* "relation of dependence," in one direction or the other, between mathematical and empirical statements.[78]

Parallel ideas gain prominence in Carnap's later thought. When he begins to say that universal statements can never be completely verified by any (finite) amount of data,[79] he is only countenancing a slight (or slightly transfinite) expansion of the idealized empirical subject of the *Aufbau*. When he says that even particular statements about the physical basis can in general not be reduced to protocol sentences but are instead justified by the fact that protocol sentences can be deduced from *them*,[80] he is in a way just fleshing out a possibility he had already raised in the *Aufbau*. And when, finally, he says that the theoretical language is connected to the observation language at only a few points,[81] he simply gives to the whole of physical science the status that pure

---

[76]See *Aufbau*, §101, pp. 139–41.

[77]For the possibility of a system with physical basis, see is §59, pp. 80–81. That such a system would draw the same boundary between empirical and metaphysical is asserted (without proof) at §176, p. 247.

[78]Because, supposedly, mathematical statements can stand in contradiction to empirical ones. See §181, pp. 257–8.

[79]First in "Die physikalische Sprache als Universalsprache der Wissenschaft," *Erkenntnis* 2 (1932):440. At this stage, however, it is only strictly *verificationist* reduction of universal statements that is rejected: Carnap gives up on translational reductionism in general, even to a physical basis, only in "Testability and Meaning," *Philosophy of Science* 3 (1936): 467–8 (cf. "Die physikalische Sprache," 440, 448; *Logische Syntax der Sprache* [Vienna: Springer, 1934], §82, p. 248).

[80]Also first in the "Physikalische Sprache," loc. cit.

[81]"The Methodological Character of Theoretical Concepts," in H. Feigl and M. Scriven, eds., *Minnesota Studies in the Philosophy of Science*, vol. 1 (1956), 47. The C rules "connect sentences of L_O with certain sentences of L_T, for instance, by making a derivation in the one

mathematics has in the *Aufbau*.

These huge extra-empirical structures are *freely* constructed, in that the empirical basis does not determine them. From *Der Raum* and the conventionalist papers which followed it, through the *Aufbau*, down to Carnap's late writings, he always maintains that. *Der Raum* itself, moreover, already hints at two aspects of this freedom which were to remain constant. First, that it is a freedom to choose between forms of language—although actually *Der Raum*, following Poincaré, merely *compares* the choice of geometries to a choice of language.[82] Second, Carnap already admits that we in fact find ourselves having already chosen (a geometry, a language, a system of scientific and everyday concepts). What philosophy does here is to awaken us to the possibility of alternatives, so that, even if we choose to go on as before, we can now do so responsibly and freely.[83] The freedom from empirical constraint, however, makes the question pressing: what justifies science in using such extra-empirical structures at all? What, in other words, marks science off from metaphysics—which Carnap never gave up on attacking for its lack of empirical content?

This brings us to the topic of what Richard Jeffrey has called Carnap's "voluntarism."[84] The term is extremely misleading, because, as Jeffrey himself immediately points out, Carnap always holds, as he does in *Der Raum*, and as Driesch does in the *Ordnungslehre*, that this freedom is not the freedom of indifference. A free choice is not an arbitrary (*willkürlich*) choice, but rather one in accordance with practical principles (*Grundsätze*).[85] The oft-cited dictum of the *Logical Syntax*, that "in logic, there are no morals," is sometimes read as a retreat from this. That it is no such thing is shown even by the immediate continuation:

---

or the other direction possible."

[82] See *Der Raum*, §V, pp. 64–5.

[83] See *Der Raum*, §III, p. 35, and cf. *Aufbau*, §179, pp. 252–3; "Empiricism, Semantics and Ontology," reprinted in *Meaning and Necessity: A Study in the Semantics of Modal Logic*, 2nd ed. (Chicago: University of Chicago Press, 1956), 207; *Logical Foundations of Probability*, 2nd ed. (Chicago: University of Chicago Press, 1962), Preface to the 2nd ed., xv; *Aufbau*, Foreword to the second edition (1961), p. xvii. This aspect of Carnap's views (and its continuity throughout his thought) is also noted by T.E. Uebel, "Anti-Foundationalism and the Vienna Circle's Revolution in Philosophy," *British Journal for the Philosophy of Science* 47 (1996):431.

[84] See *Probability and the Art of Judgment*, 2nd ed. (Cambridge: Cambridge University Press, 1991), 28 and (in an identical passage) "Carnap's Voluntarism," in *Logic, Methodology and Philosophy of Science IX*, ed. D. Prawitz, B. Skyrms, and D. Westerståhl (Amsterdam: Elsevier, 1994), 847.

[85] *Der Raum*, §III, p. 36; cf. *Aufbau* §§103–5, pp. 143–6; "Empiricism, Semantics and Ontology," 220–21.

> Everyone may construct his logic, i.e. form of language, as he wishes.
> Only, *if he wishes to discuss it with us, he must clearly report* how
> he intends to do so, and give syntactical determinations instead of
> philosophical arguments [*Erörterungen*].[86]

The "must" records a linguistic imperative. The aim of the book, in fact, is the same as the *Aufbau*'s: "to provide a structure of concepts [*Begriffsgebäude*], a language" (Foreword, p. iii) which eliminates unclarity and inexactness and thus exposes pseudo-questions (v).[87]

This leaves open, however, the nature of the imperative in question. The extremely popular characterization of Carnap as a pragmatist, if taken seriously, amounts to the claim that that all such imperatives are, for him, hypothetical, rather than categorical.[88] The use of the Kantian term "principle," in a context where the issue is the nature of freedom, suggests, however, that the *supreme* practical principles ought to be ethical in nature, and that ethical imperatives should be understood in a Kantian way, as categorical.

I will not try to decide this issue definitively here: there is not enough evidence either way in *Der Raum* itself. But it is worth pointing out the following.

Carnap's dissertation advisor, Bruno Bauch, published in 1904 an essay on ethics in which the main opposition is between utilitarianism—described as a moral theory on which all individuality is sacrificed for the good of the whole—and Nietzschean "amoral individualism."[89] Bauch offers his own solution to the dilemma in his third section, on "critical ethics." Although the relationship between Carnap and Bauch was not at all close, still Carnap may have been familiar with this particular piece, if only because it is highly recommended by Driesch.[90] Driesch himself, seemingly unimpressed with Bauch's solution, leaves it an open question what sort of wished-for human "totality" (*Ganzheit*) is correctly demanded: whether an order in which all humans are ultimately interchangeable, or one in which each individual must find his or her special role.[91] Carnap, for his part, addresses exactly the same issue in the Foreword to the *Aufbau*, and claims—or, to speak more cautiously, expresses the faith—

---

[86] *Logische Syntax*, §17, p. 45 (my emphasis).

[87] Cf. *Aufbau*, Foreword to the Second Edition, xvii.

[88] See, most explicitly and carefully, Richardson, "Carnapian Pragmatism," in *Cambridge Companion*, 295–315 (and see especially 298 n. 3, 302 n. 7). But less careful versions are too common to list. Jeffrey himself is ambiguous on this point.

[89] "Ethik," in W. Windelband, ed., *Die Philosophie im Beginn des zwanzigsten Jahrhunderts: Festschrift für Kuno Fischer*, vol. 1 (Heidelberg: Carl Winter, 1904), 54–103.

[90] *Ordnungslehre*, 262 n. 1.

[91] *Ordnungslehre*, 275–6.

that the new attitude, the *neue Sachlichkeit*, of which scientific philosophy is a limited but essential part, will resolve it.

It is not immediately clear how technical issues about the epistemological status of space could be relevant to this problem, and I don't wish to claim with any certainty that Carnap made such a connection in 1922. What is certain is that, in the *Aufbau*, Carnap claims that his solution is connected to the unity of science. Roughly speaking this is because, first, the scientific attitude is one of "responsibility"—i.e. of making one's own individual claims in a forum where one is answerable to the audience of ones colleagues—and, second, the unity of science makes this forum into a universal one ("the forum of the understanding," as Carnap calls it there): i.e., one's audience consists of every (possible) earnest investigator. And we do know that Carnap, by 1929, describes the unity of science as a further development of the unity of space, and moreover connects both to the ethical project of liberating humanity from the grip of potentially oppressive claims to supersensible knowledge. The ancient "discovery" of "the one, all-inclusive space," he says, allowed for the refutation of mythology (because one could always demand to know *where* a given mythical being was supposed to be found); the modern understanding, building on this, establishes also the unity of *logical* space, which will allow the elimination of metaphysics (because one can always ask for the observational criteria of any concept).[92]

These "discoveries" can hardly be understood as empirical (as if someone checked for disconnected bits of physical or logical space and failed to find them). Nor, in 1929, can Carnap mean that they are discovered by eidetic insight. They are, rather, not theoretical discoveries at all; what is discovered is the *practical* necessity of certain demands (the demands, as Carnap puts it there, that there be both a physical/geometrical path from here to any body,[93] and a logical/ epistemic "path" of definition from "here" to any object whatsoever). The data do not directly require any particular choice in this respect, but we must make the correct choice if we want to maximize our responsibility to them, i.e. to

---

[92] "Von Gott und Seele: Scheinfragen in Metaphysik und Theologie," printed in T. Mormann, ed., *Scheinprobleme in der Philosophie und andere metaphysikkritische Schriften* (Hamburg: Meiner, 2004), 49–62; see especially pp. 54–5 and 57–8.

[93] As Carnap explains: "'path' not in the sense of a blazed trail, but rather in the sense of a line drawn in thought [*einer gedachten Linie*], a possibility of motion" (54). From this it is still not clear that there must be geodesics from here to everywhere, but Carnap adds on the next page that the path can be specified by giving direction and distance (55). And this is crucial to the epistemological point: if someone claims that a certain mythical thing exists, I can challenge her to supply a *local* characterization of the path from here to that thing.

maximize the ways they can refute or justify our judgments. This, recall, is the whole point of positing an external reality: in Drieschian terms, of the ego's "free achievement" of something binding on itself, of an "ought to posit positings which ought to hold." And the demand might well be regarded as categorical, at least in the sense that we can't wait to derive it, or demand that others derive it, from any theoretically describable (i.e., empirical) state of affairs—for example, from facts about human nature, or about what is convenient for us, or about what we personally prefer. For to begin such a theoretical description is to begin trying to communicate, and hence already to have accepted the demand in question. The imperative is then binding on whoever "wants to discuss" anything with anyone—which is to say, on any citizen of the kingdom of ends, the community of all rational beings.