SMOOTH QUANTUM MECHANICS

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ABSTRACT. I show that the contradiction between the unitary evolution and the condition to obtain, as a result of the measurement, an eigenstate of the observable, can be resolved without making use of discontinuities. The apparent state vector reduction can be replaced with a delayed initial condition, imposed to the unitary evolution of the observed system entangled with the measurement device used for the preparation. Since the quantum state of this device is not available entirely to the observer, its unknown degrees of freedom inject, by the means of entanglement, an apparent randomness in the observed system, leading to a probabilistic behavior. The condition imposed by the observable combined with the condition of minimal disturbance lead to the Born rule. Thus, we can construct a Smooth Quantum Mechanics, without the need of discontinuities in time evolution, and the probabilities appear from the lack of knowledge of the quantum states of all the systems involved.

As a consequence, the evolution is deterministic, but there is no way for an observer to make complete use of this determinism. For such an observer, and for an open quantum system, the evolution will still be indeterministic. The possibility to choose the initial conditions with a delay makes the determinism to be compatible with the free will at the same extent as the indeterministic version of Quantum Mechanics is. The apparent indeterminism at the observer's level also leaves room for a smooth version of the Many Worlds Interpretation.

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Contents

1. **Introduction**

The next section will provide some reasons to continue the pursuit of a description of the quantum world which is not based on discontinuities. Although the Quantum Mechanics is a great success, there are a lot of things about it that we may want to understand, and definitely this is one of them.

The main section of the article can be considered §3, which begins with some hints about the possibility of avoiding the discontinuities. The measurement apparatus used for the preparation of the quantum system is found to be in a deep connection with the result of the measurement, because of the entanglement. This feature allows the result of the observation to be correlated with the interaction between the preparation device and the observed system. This correlation may seem to defy the causality, but in fact it is nothing more than the usual entanglement combined with delayed initial conditions. Then, it is explained why, for some simple examples, the measurements not necessarily impose incompatible initial conditions. The entanglement with the preparation device, when this one is not entirely specified at the quantum level, provide enough degrees of freedom to the observed system, such that its state needs now a new measurement to specify its initial conditions. It is then showed how we can account for the Born rule and the Heisenberg's relations, even though the evolution is deterministic. The notions of consistent conditions and registries are introduced and employed to define the observer's level of reality, which indeed looks indeterministic.

Some problems of the Hilbert space, regarding the use of distribution that are not functions, and of wave functions that are not normalizable, are recalled in the section §4. The necessity of replacing it with a space of smooth and normalizable functions is presented. The idea of a rigged Hilbert space allows us to benefit from the standard development of Quantum Mechanics, keeping in the same time the physical reality in more reasonable parameters.

The section §5 recalls into discussion the suppositions I made in order to provide the smooth description of the measurement and the apparent state vector reduction. These suppositions provide some limits to the applicability of this scenario. It has to be like this, because, for example, at the macroscopic level we don't see that many quantum phenomena. There are some limits between the classical and the quantum levels, and we can make experiments to see whether these limits coincide with the theoretical limits.

The section §6 analyzes the implications of the Smooth Quantum Mechanics to other interpretations of QM. Various interpretations of the quantum phenomena are presented in a new light, showing that they may be more compatible that we thought.

Some implications on the apparent opposition between determinism and indeterminism are then explored, in section §7. It is found that the free will is not rejected more by the theory presented in this paper, than by the standard (indeterministic) Quantum Mechanics.

2. The problem of discontinuities in Quantum Mechanics

2.1. Why discontinuities seem to be required by Quantum Mechanics?

The successes of the Quantum Mechanics in predicting the physical behavior at small scales being so great, we have to accept this theory, despite some of its problems. One of these problems is represented by the discontinuities that occur under the name of *wave function collapse*, or *state vector reduction* [vN55].

When we deal with a quantum system that is isolated but subject to measurements, then the following problem occurs. On the one hand, the Schrödinger equation predicts a unitary time evolution for the state vector. On the other hand, the measurements expect to find the system in an eigenstate of the observable. The incompatibility between the unitary time evolution (which I will name the *wave-like behavior*) and the eigenstate condition (which I will name the *quantum behavior*) leads to the necessity of accepting that the state vector is projected under one of the eigenstates of the observable.

This projection is a discontinuity in the unitary time evolution. I will argue below why, despite the successes of the quantum theory, this discontinuity problem needs to be solved. I will also provide a solution that is very natural and restores the Schrödinger equation in its rights.

In the followings, I will consider the *state space* as being a complex vector space with a hermitian inner product $\langle|\rangle$, usually denoted by $\mathcal{S}$. I will try to avoid the assumption that the state space is complete (and thus a Hilbert space), for reasons that will be explained in §4. On the other hand, sometimes it will be convenient to work in the completion of $\mathcal{S}$, which of course is a Hilbert space, denoted by $\mathcal{H}$.

2.2. The discontinuity paradox

When we consider an isolated quantum system $|\psi\rangle$, the Schrödinger equation describes it very well. If the system is subject to incompatible quantum measurements, then the unitary evolution is broken and the discontinuities occur.

On the other hand, the measurement devices, say $|\mu_i\rangle$, $i \in I \subset \mathbb{Z}$, are themselves quantum systems. We can consider the measurement devices and the quantum system as being subsystems of a larger system $|\Psi\rangle$, which can be taken to be isolated. This implies that the Schrödinger equation should govern the evolution of $|\Psi\rangle$, and the discontinuities should not be needed, nor they should be allowed. The subsystem $|\psi\rangle$ should not collapse.

Both descriptions seem to be right, but one is definitely without discontinuities, while the other is discontinuous.

One possible solution to this problem can be suggested based on the Many World Interpretation. Here, the continuous evolution of $|\Psi\rangle$ implies that $|\psi\rangle$ also evolves continuously, but this continuous evolution can be viewed as a superposition of discontinuous evolutions. Each such possible discontinuous evolution will represent a different world, and the corresponding discontinuities take place only in that world. The following argument against the discontinuities is valid also in each of the worlds composing the total state, therefore the MWI doesn't cover it.

2.3. A mysterious cause for the discontinuities

The measurements seem to indicate that the state vector jumps discontinuously. On the other hand, there is no direct observation of such discontinuities, not there are any laws which can explain or allow them. When we assume that a wave function collapse took place, we don't know any possibility by which the laws of Physics allowed such a discontinuity. We may say that the observation caused the jump, but this is not an explanation. We need to know how the observation can disrupt the continuity in such a manner that it can not be observed.

If we believe that the observation, or something else, may cause a discontinuity, we have to explain how. We have to find the fundamental principle that allows the wave function to collapse. Then, we need to explain how this new law behaves such that it is usually not observed.

2.4. The "magical conservation principle"

In Quantum Mechanics, an observable that commutes with the Hamiltonian of the system $|\psi\rangle$ is conserved during the evolution of $|\psi\rangle$. But the conservation holds only as long as the system $|\psi\rangle$ evolves according to the Schrödinger equation. Since performing a measurement makes the system jump in a totally different state, it is expected that the conservation laws are broken.

For example, if we measure the momentum of the system $|\psi\rangle$, and then measure its position, then the initial momentum is lost. If we measure again the momentum, we should expect to obtain a totally different value than the first time. We can expect that, after several measurements, the conserved quantities of the system be totally blown up.

But the conservation laws don't break down as a result of measurements. Something happens always to restore them. We can assume that there is a nonlocal conservation law associated to these laws, such that the momentum is locally violated, but it is preserved globally. For example, the momentum disappears here and appears there. But, in this case, the relativity of simultaneity will allow us to observe that, in a different frame of reference, the momentum conservation is violated.

Even if we consider that sometimes $|\psi\rangle$ represents the state of the system, while some other times it represents just the probabilities for each possible outcome, this will not solve the problem. The truth is that the discontinuities are incompatible with the conservation laws. To make them compatible, we need to appeal to a "magical postulate":

During the state vector reductions, the conservation laws can no longer be deduced from the Hamiltonian, but they must be restored in some way or another.

The problem is that we don't know any cause for the conservation laws, other than the unitary time evolution.

2.5. **There must be an explanation**

If somebody presents to a physicist the blueprints of a device that produces more energy than it consumes, then the physicist become very skeptic. She can simply say that the energy is preserved in any condition, and dismiss the new invention. Or she can analyze the device, and find out where the flow is. This flow exists always. It may not be visible, being usually hidden in an unexpected place. But it has to be somewhere. There must be an explanation.

The quantum world is like a great illusionist, who has in his sleeves a lot of tricks that make us believing that the quantum system jumps discontinuously from time to time. But we have to remember that, at the end, there must be a logical explanation for the illusion number presented in the show. This paper is intended to show the strings.

3. Quantum Mechanics without discontinuities

3.1. Can we eliminate the state vector reduction?

In this section I will construct a scenario of the measurement process, without the help of the state vector reduction. In doing this, I will make use of the entanglement [Sch35] between the measurement device used for preparation and the quantum system under observation. They become entangled as a result of their interaction, and because the initial conditions are not specified entirely, the observed system become undetermined. In this paper, the entanglement will refer to a situation when an initial condition imposed on a system implies an initial condition imposed on another system which previously interacted with this one.

The Schrödinger equation will be considered to be respected all the time, providing a deterministic evolution. Yet, a probabilistic behavior will emerge, but as a consequence of the observer's ignorance of the environment's quantum state[1].

When the initial conditions are not entirely specified, we need to use probability distributions. It is customary to represent sometimes these probability distributions by density operators, we will do the same, but with great care, because we don't want to mix the possible states. I will use the density operator only as a simple way to represent probability distributions for the initial data of a quantum system.

The incompatibility between the unitary time evolution and the eigenstate condition can be resolved without the help of the discontinuities entailed by the state vector reduction. There will be something in place of the reduction, but it will not be a projection, it will not bring in discontinuities. The role of the projectors describing the state vector reductions will be taken by a kind of "initial conditions", which will occur during the measurement, but without involving any discontinuity in the dynamics.

By considering the entanglement between the quantum system, the measurement device performing the preparation, and the measurement device, the reduction will no longer be needed.

3.2. Quantum measurement and entanglement

In this section I will argue that, after we perform an observation to a system, the system gets entangled with the measurement device. To check this, we need to perform a second measurement. As a matter of fact, we will consider this second measurement the main measurement, while the first one will be considered only as a preparation measurement.

For example, the two-slit experiment contains a preparation measurement: the wall containing the two slits measures the positions of the photons. The property measured is that the photon has passed through the slits. Then, the photons are detected by a screen. This is the main measurement.

As Einstein pointed out to Bohr, at the Fifth Solvay Conference (Brussels 1927), if we measure the recoil of the wall containing the two slits, when the light passes through

[1]The term *environment* will refer in this article to the systems – mostly measurement devices – interacting with the observed system.

it, we can deduce whether the photon passed through one slit or the other. As Bohr replied to him, then the interference pattern is destroyed. If the interference pattern is present, the measurement of the momentum did not detect a significant enough change.

Let's reverse a bit the reasoning, and apply it to the delayed-choice [Wei31, Sch49, Whe77, Whe78, WZ83] version of the two-slit experiment. We can decide after the photon has passed through the slit(s) whether to observe the "which way" or the "both ways" aspects. If we decide to observe the "which way" behavior, we cause the wall with the two slits to undergo a significant change of momentum (corresponding to the cases when the photon has passed through one slit or the other). If we choose to observe the interference, the change in momentum will be insignificant or, rather, undefined. The wall with the two slits will get in a superposition of eigenstates of momenta.

This analysis proves that indeed the wall with the two slits gets entangled with the photon, after they interacted.

A similar reasoning can be applied to other experiments, such as the Mach-Zehnder experiment (please refer to the figure 1). By attaching the two mirrors **A** and **B** (as in the figure 2), we can consider that the composed system undergoes a change in momentum, which can tell whether the photon has passed through one arm or the other of the interferometer. If we decide to remove the second beam splitter, then the change of momentum becomes significant. If we lave in place the second beam splitter, we cause the change in momentum to be undefined, to be a superposition. In this case too we can see that the measurement device gets entangled with the observed system.

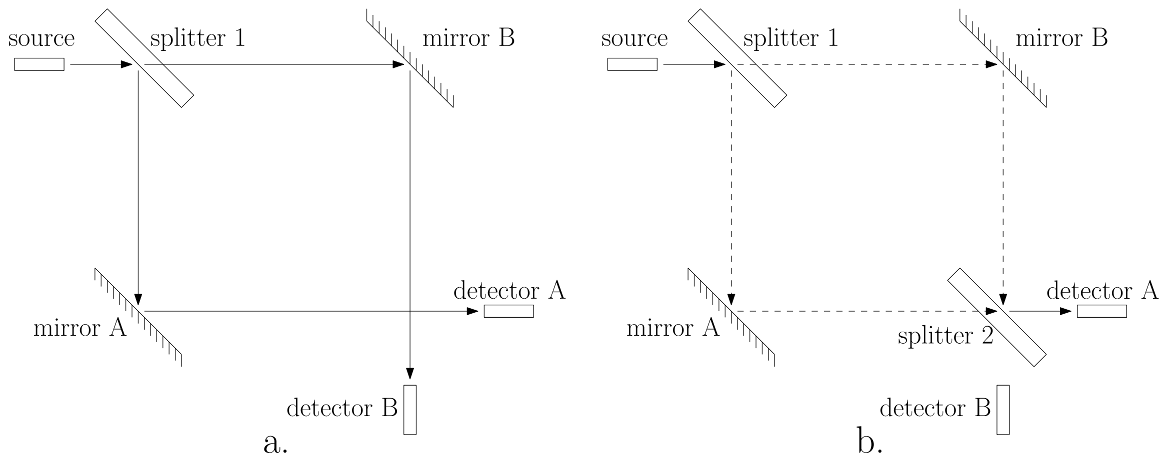

FIGURE 1. The first figure shows the *which way* type of observation. In this case, the electron is detected either by the detector **A**, or by **B**, and the conclusion is that the photon passes through one way or the other. In the second figure, the second beam splitter recomposes the photon and the interference allows only the detector **A** to report. The photon traveled *both ways.*

In the examples presented above, the entanglement between the measurement device and the observed system resolves the problem of the conservation of momentum. We will see that this entanglement can indeed explain the so called reduction of the state vector by unitary time evolution only, without making use of discontinuities.

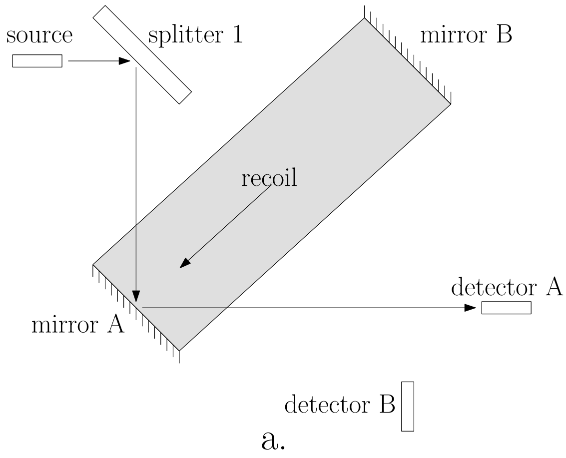

FIGURE 2. The figure **a** shows the recoil of the system composed by the two mirrors, if the the photon travels through the mirror **A**. The figure **b** shows the other case.

3.3. Smoothing out the discontinuities

The previous examples showed that the measurement device interacts with the observed system, and that this interaction depends upon what observable we choose to measure the next time.

If we consider the system being described by the Schrödinger equation corresponding to the Hamiltonian H, then successive measurements make the evolution look discontinuous. A measurement at t_0, performed with a measurement device $\mathcal{O}_0$, will find the system in the state $|\psi_0\rangle$, which is an eigenstate of the observable O_0. By evolving the solution to t_1, we obtain $|\psi(t_1)\rangle = U(t_1, t_0)|\psi(t_0)\rangle$, where $U(t_1, t_0)$ is the evolution operator associated to the Hamiltonian H. But a measurement of the observable O_1 at the time t_1 will find the system in an eigenstate of O_1. If $|\psi(t_1)\rangle$ is not an eigenstate of O_1, then it has to jump to such an eigenstate. This jump is usually interpreted as a projection, and it is considered to cause a discontinuity in the system's evolution.

We cannot say whether the jump takes place at t_1, at t_0 or some intermediate time. The unitarity of the time evolution guarantees that the probabilities are the same, so we cannot distinguish these cases [Sto08]. But the analysis of delayed-choice experiments seems to suggest that, if a projection happened, it took place in advance, at t_0 (figure 3).

When we analyzed what happened with a photon that traveled one way in the Mach-Zehnder experiment, we have seen that the mirror suffered a change in momentum. This implies that the mirror and the photon interacted. This interaction needs to be accounted in the Hamiltonian.

At t_0, when the previous interaction took place, the Hamiltonian was changed for a small amount of time (figure 4). Macroscopically, this time seems to be too short, and can be considered 0. This would entail a discontinuity in the evolution of the measured system. But if we remember that a change of momentum cannot happen instantaneously, because this would imply an infinite force, we are forced to consider that the interaction really takes place during a nonzero time interval. By adding this

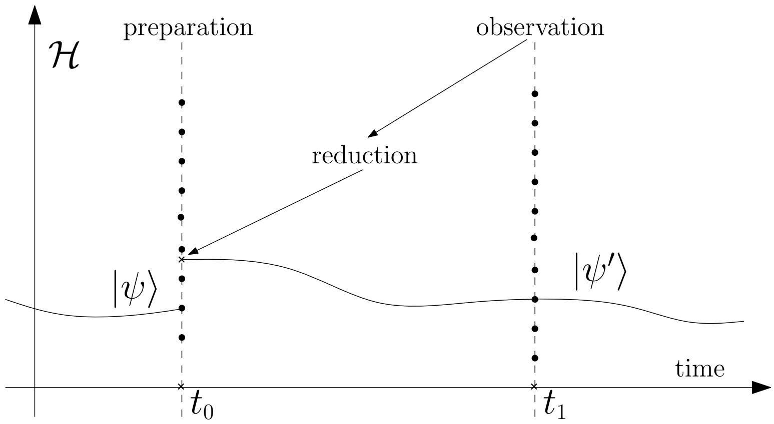

FIGURE 3. In a delayed-choice experiment, the reduction seems to "already happen" at the moment t_0. The reduction seems to take place in advance, anticipating the experimenter's choice of the observable O_1.

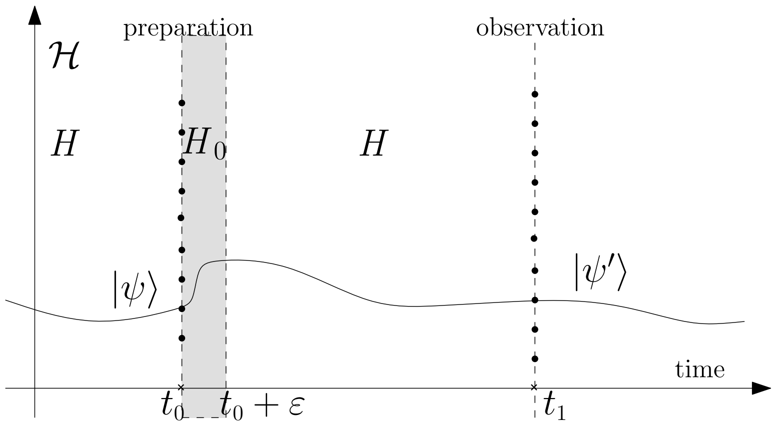

FIGURE 4. The disturbance in the evolution of the quantum system, introduced by the measurement device performing the preparation, needs to be taken into account by modifying the Hamiltonian to H_0 for the time interval $(t_0, t_0+\varepsilon)$. This will "repair" the discontinuity presented in the figure 3.

interaction to the Hamiltonian, for that small time interval $(t_0, t_0 + \varepsilon)$, we can obtain a continuous time evolution that looks like the discontinuous one outside the interval. Note that the unitary evolution operator will be continuous too, being obtained by integration from the Hamiltonian H, changed to H_0 during the interval $(t_0, t_0 + \varepsilon)$.

Two consecutive measurements impose two initial conditions to the quantum system, $|\psi(t_0)\rangle = |\psi_0\rangle$ and $|\psi(t_1)\rangle = |\psi_1\rangle$. The two conditions are incompatible when $U(t_1, t_0)|\psi_0\rangle \neq |\psi_1\rangle$. The discontinuities were introduced because the state predicted by the Schrödinger equation was not necessarily an eigenstate of the observable.

But, if we include in the Hamiltonian the interaction with the first measurement device, do we obtain $U(t_1, t_0)|\psi_0\rangle = |\psi_1\rangle$? By changing the Hamiltonian in this way, $U(t_1, t_0)$ changes accordingly, such that the equality really holds. There is a $U_\lambda(t_1, t_0)$ for each possible eigenvalue λ of the observable O_1. For example, in the Mach-Zehnder experiment, there is an U_A for the case when the photon hits the detector **A**, and an U_B for the case when it hits the detector **B**. This happens because the two interactions are different.

Needing more unitary evolution operators, one for each possible outcome, seems to violate the linearity. There must exist only one unitary evolution operator, for all the possible solutions of the Schrödinger equation. And this is indeed the case, globally. If we consider only a subsystem, and we observe it, we have to include the measurement devices. If we include them, then there is only one unitary evolution operator. But if we include them only partially, we ignore other parameters of the Hamiltonian. The operators $U_\lambda(t_1, t_0)$ are in fact instances of the same operator U, and λ is a parameter that holds the place of the parameters that were ignored. But what we ignored? The Hamiltonian is constant for a closed quantum system, we cannot change it. Only the state of the measurement device $\mathcal{O}_0$, used for the preparation, can contain ignored data. In general, this is a macroscopic system, and this means that we don't really know its quantum state in detail, but only the macroscopic result. It is the quantum state of $\mathcal{O}_0$ that we don't know completely. The fact that the state $\mathcal{O}_0$ is not completely specified provides us some degrees of freedom. Because after the interaction this device and the quantum system gets entangled, we can use the freedom in specifying $\mathcal{O}_0$ to make a choice of the state $|\psi\rangle$ of the quantum system under observation. The interaction between the two systems depends on their initial states, and that's why the interaction Hamiltonian, and consequently, the unitary evolution operator, eventually depends on the eigenvalue λ.

I want to mention that the measurement apparatus $\mathcal{O}_0$ can in fact be in much more unspecified states than those labeled by λ. We can choose, in principle, any observable O_1 for the second measurement, and for each eigenvalue there must be a set of initial conditions for $\mathcal{O}_0$ leading $|\psi\rangle$ to become a corresponding eigenstate. Moreover, finding the λ does not exhaust the freedom in the specification of the system $\mathcal{O}_0$.

There are several reasons why the interaction between the first measurement device and the observed system is usually ignored. Because ε is very small, this evolution looks like a discontinuity. It is not in the place where it is normal to look for it: we would expect that this interaction is between the measured system and the second measuring device, at t_1, but in fact happens at t_0, when the preparation measurement takes place. Moreover, the randomness of the result of the measurement O_1, and our possible choice of the observable O_1 itself, are hints that the interaction at t_0 is determined by something that happens later, at t_1. But this is just another face of the entanglement. It is strange, but it is nothing new to Quantum Mechanics. This seeming violation of the causality is an old feature of the Quantum Mechanics. It is due to the possibility to delay the choice of the initial conditions of the solution [Whe78, Sto08].

3.4. **Very simple examples**

The following examples will provide a grasp of the solution proposed in this paper to the problem of state vector reduction.

Imagine a quantum system whose state is $|\psi\rangle$, and let's consider that the first measurement ever performed to that system is made at the time t_0. The result of the measurement is one of the eigenvalues, say λ, of the observable considered. Accordingly, the quantum system is found to be in an eigenstate $|\lambda\rangle$ corresponding to that eigenvalue. $|\lambda\rangle$ is not uniquely determined, but since in general the observable is such that the eigenstates are nondegenerated, and since the state vectors are usually taken to be of norm 1, we can consider that for most cases $|\lambda\rangle$ is determined up to a phase factor.

Since this is the first measurement, the condition imposed by $|\psi(t_0)\rangle = |\lambda\rangle$ can be considered as the initial condition for the quantum system. This initial condition provides a unique solution to the Schrödinger equation, up to a phase factor, and the evolution of the system is completely determined.

The problem appears if we measure a new observable of $|\psi\rangle$, at the time t_1, which don't admit $U(t_1, t_0)|\psi(t_0)\rangle$ as eigenstate. In this case, the state vector is usually considered as being reduced. Before considering this case, it is useful to discuss a simpler situation.

Let's consider a system $|\Psi\rangle$ composed from m subsystems, each one represented on a state space $\mathcal{S}_i$. The state space of $|\Psi\rangle$ will be $\mathcal{S} = \mathcal{S}_1 \otimes \ldots \otimes \mathcal{S}_m$. $|\Psi\rangle$ can be written in general as

$$(1) \qquad |\Psi\rangle = \sum_{i_1,\ldots,i_m} C_{i_1,\ldots,i_m} |e_1^{i_1}\rangle \otimes \ldots \otimes |e_m^{i_m}\rangle,$$

where $(|e_k^j\rangle)_j$ form a basis for the state space $\mathcal{S}_k$, and $C_{i_1,\ldots,i_m}$ are complex coefficients.

We can perform m independent measurements to determine the initial state vector of each subsystem. Supposing that we obtain for each subsystem an eigenstate $|\psi_i\rangle \in \mathcal{S}_i$, then the total state is found to be

$$(2) \qquad |\Psi\rangle = |\psi_1\rangle \otimes \ldots \otimes |\psi_m\rangle.$$

This example shows a special situation, but it contains one of the key points of the solution proposed in this paper to the discontinuity problem. In this example, the state $|\Psi\rangle$ of the system is determined by performing measurements to the m subsystems composing it. The state vector reduction did not occurred. Instead, a specification of the initial conditions for all the subsystems also specified the solution of the Schrödinger equation for the whole system.

In general, we perform new measurements to the same system, and are forced to admit that the state vector is reduced, that it is projected on an eigenstate of the observable. We will see that such quantum systems, that seemingly undergoes discontinuities, can be embedded in a natural way in a simple system like the one above. All the observations that seems to cause jumps, will be explained naturally as independent observations. But how can this be possible, when we know that we performed repeated measurements of the same quantum system?

The solution is that each measurement is performed, actually, on a different system, which is an entanglement between our previous system and the measurement device that performed the previous measurement. The things go like this:

(1) We observe a property of the quantum system $\mathcal{Q}$.

(2) This is the first observation, and determines the system to be in a given state $|\psi_0\rangle$. Being the first observation, it is only an initial condition, and don't causes discontinuities.
(3) The system $\mathcal{Q}$ gets entangled with the measurement device $\mathcal{O}_0$ that performed the first observation. We assume that the state of $\mathcal{O}_0$ was not entirely determined at the quantum level. Then, the system $\mathcal{Q}$ being entangled with $\mathcal{O}_0$, it is no longer determined too, and we need to account all the possibilities. We will combine these possibilities in a probability distribution (due to the incomplete knowledge of the state $\mathcal{O}_0$). We can represent this probability distribution by a density operator ρ_0.
(4) The probability distribution may be chosen such that, in average, the possibilities give the state $|\psi_0\rangle$. As we shall see, this corresponds to the Born rule [Born26].
(5) If we perform a second observation of the system $\mathcal{Q}$, we need to consider not the unitary evolution of $|\psi_0\rangle$, but that of the density operator ρ_0. This entails not only a measurement of $\mathcal{Q}$, but also of the device $\mathcal{O}_0$, the two remaining entangled after their interaction.
(6) Let's say that, after the new measurement, the system is found in the eigenstate $|\psi_1\rangle$.
(7) A third observation will be performed on the system described by the new probability distribution (represented by another density operator ρ_1), obtained by the entanglement of $|\psi_1\rangle$ with the second measurement device $\mathcal{O}_1$, and so on.

We can describe in this natural way the observations of quantum systems, without requiring discontinuities.

3.5. Observation without state vector reduction

Let us consider a quantum system $\mathcal{Q}$ described by a time dependent state vector $|\psi\rangle$ valued in a state space $\mathcal{S}$. Let H be a time dependent Hermitian operator on $\mathcal{S}$ representing the Hamiltonian describing the time evolution of $|\psi\rangle$. We consider that the system undergoes two consecutive measurements, O_0 at the time t_0 and O_1 at the time t_1. The two measurement devices are considered to be quantum systems as well, and we denote them by $\mathcal{O}_0$, respectively $\mathcal{O}_1$. The first measurement device is described by a time dependent state vector $|\omega_0\rangle$, where $|\omega_0(t)\rangle$ is a vector from a state space $\mathcal{S}_0$, and the second by $|\omega_1\rangle$, valued in the state space $\mathcal{S}_1$.

The first measurement serves as preparation for the second one. If we consider that it is the first measurement in the entire history of the quantum system to be observed, then it serves as an initial condition. It selects a state vector $|\psi_0\rangle$ from the entire state space $\mathcal{S}$. The solution of the Schrödinger equation will be the one satisfying the initial condition $|\psi(t_0)\rangle = |\psi_0\rangle$. There is no discontinuity needed yet, because this is the first measurement performed to the system. It only serves as an initial condition for $|\psi\rangle$. We can express the possible states before the first measurement by a density operator which gives equal probability to all possible directions in the state space $\mathcal{S}$, being thus a multiple of the identity operator $1_{\mathcal{S}}$.

After the measurement, the quantum system $\mathcal{Q}$ becomes entangled with the measurement device. The system composed by $\mathcal{Q}$ and the first measurement device $\mathcal{O}_0$ is represented in the state space $\mathcal{S}\otimes\mathcal{S}_0$, and we will denote it by $\mathcal{T}_0$. This system can be represented by a density operator μ_0 acting on $\mathcal{S}\otimes\mathcal{S}_0$. The density operator will encode the missing information in the initial conditions of $|\omega_0\rangle$. Because of the interaction between the measurement apparatus and the observed system, the observed system is disturbed and its state become also incompletely specified. The density operator corresponding to the system $\mathcal{Q}$ is the partial trace:

$$\rho_0 = \mathrm{tr}_{\mathcal{S}_0}(\mu_0). \tag{3}$$

We will need now to make some considerations about the density operator ρ_0. First, we cannot say that the system $\mathcal{Q}$ jumped from the state $|\psi\rangle$ to the one represented by ρ_0. The density operator ρ_0 express only the lack of information about the measurement device $\mathcal{O}_0$. If we would know the state of $\mathcal{O}_0$ before and after the first measurement, then we would be able to predict what will happen to the system $\mathcal{Q}$, but since this information is not available, we can only consider a density operator representing a probability distribution of states.

It is known that a hermitian operator A on a state space $\mathcal{S}$ can be expressed as

$$A = \sum_i \lambda_i |e_i\rangle\langle e_i|, \tag{4}$$

where $(|e_i\rangle)_i$ is an orthonormal basis for $\mathcal{S}$. Here, and in other places in this paper, the sum sign $\sum$ is formal, sometimes representing integrals, or even sums of sums and integrals. Such a representation of A can be obtained by taking for $(|e_i\rangle)_i$ a complete set of eigenstates, and for λ_i the corresponding eigenvalues of A.

An operator A acting on a state space $\mathcal{S}$ determines a quadratic hermitian form by associating to each $|\varphi\rangle \in \mathcal{S}$ the complex number $\langle\varphi|A|\varphi\rangle$. If A is hermitian ($A^\dagger = A$), then $\langle\varphi|A|\varphi\rangle$ is always real. A density operators is a hermitian operator, which is also positive: $\lambda_i \geq 0$ for any i. Equivalently, the associated quadratic form is positive: $\langle\varphi|A|\varphi\rangle \geq 0$ for any $|\varphi\rangle \in \mathcal{S}$. The density operators satisfy also the condition $\sum_i \lambda_i = 1$.

The density operator ρ_0 after the first measurement can contain all the possible results of any possible measurement. It can, in principle, be a convex combination of any possible state vectors. Moreover, any form may have ρ_0, there is always possible to express it as in the equation (3), for some $\mathcal{S}_0$ and μ_0. But the second measurement rules out some of the possibilities, by the mean of the following principle:

The quantum principle. After a measurement, the quantum system is found in an eigenstate of the observable.

This principle requires that the density operator represents a mix of eigenstates of the observable. Remember that the density operator is obtained by partially tracing the density operator μ_0 corresponding to the entangled system $\mathcal{T}_0$, composed by the quantum system and the previous measurement device. Thus, this condition on ρ_0 translates as a condition on $\mathcal{T}_0$.

The *quantum principle* can be then expressed, in terms of the density operator ρ_0, in the following form:

The quantum principle applied to the density operator ρ_0. The density operator ρ_0 can be put in the form $\rho_0 = \sum_i \lambda_i |e_i\rangle\langle e_i|$, where $(|e_i\rangle)_i$ is a basis of eigenstates of the observable O_1.

In general, the eigenvalues λ_i can take any possible values, as long as they are positive and sum up to 1. These values result from the interaction between the measurement device $\mathcal{O}_0$ and the quantum system $\mathcal{Q}$. But we will want to characterize the situations when this interaction disturbs in a minimal way the system $\mathcal{Q}$. This condition can be expressed by saying that the state vector obtained by the previous observation O_0 is a sort of average of the possible states:

The minimal disturbance principle. The density operator ρ_0 can be expressed in the basis of eigenvectors of the observable O_1 in such a way that

$$\sum_i \sqrt{\lambda_i}|e_i\rangle = |\psi_0\rangle. \tag{5}$$

The observable O_1 fixes the eigenvalues and the eigenspaces, but the sum above is not uniquely defined, since the eigenvectors are not uniquely defined by the eigenvalues. This means that we will have more possible inequivalent values for the sum. One known problem of the density operator is that it obscures the phases of the pure states that compose it, and another is that it "forgets" the states contained in the probability distribution which it represents (see [Pen05]). This is why, in our case, it is important to remember that the density operator is just a convenient way to represent the probability distribution of the initial conditions.

The *quantum principle* and the *minimal disturbance principle* above define uniquely the density operator, and the corresponding probability distribution. By combining them, we obtain an expression for the eigenvalues λ_i of the density operator:

$$\lambda_i = |\langle e_i|U(t_1, t_0)|\psi_0\rangle|^2. \tag{6}$$

The *quantum principle* is an independent postulate. The *minimal disturbance principle* on the other hand, can be regarded as a definition of a "good measurement". Whenever it is broken, we can say that the interaction between the measurement device and the observed system was too disturbing.

What is important here about the *quantum principle* is that it does not require discontinuities. It simply refers to a new initial condition imposed to the quantum system, after the latter loses the specification of its initial data, as a result of the interaction with another quantum system whose state is incompletely specified.

We may think that there are situations when the density matrix ρ_0 cannot be expressed as a combination of projectors on eigenstates of the observable. This is indeed true, there are such situations, and indeed my reasoning does not apply to them. Their quantum state is too well specified to allow enough freedom for the density operator ρ_0 to be expressed in terms of eigenstates of the observable O_1. In this case $\mathcal{O}_0$ is not "macroscopic" and "irreversible" enough to impose the condition that the observed system is in an eigenstate of O_0. Thus, it cannot qualify as a measurement device, because it admits superpositions of eigenstates of O_0. This is good news, because not all interactions with other quantum systems are measurements. And it is not bad news for the scenario described here, because, in this case, the required degrees of freedom are provided by a measurement happening previous to the interaction with the system

$\mathcal{O}_0$. If there is no such prior interaction that can be considered measurement, then O_1 corresponds to the first observation. In both cases, $\mathcal{O}_0$ will be just a quantum device that interacted with our system. It can be integrated without problems in the evolution of the observed system.

3.6. The Born Rule

The density operator we employed simply represents the unknown from the larger system $\mathcal{S} \otimes \mathcal{S}_0$, subject to the quantum condition imposed by the observable O_1 and the one of minimal disturbance of the system $\mathcal{Q}$. Basically, the density operator ρ_0 represents the ignored parameters from the system $\mathcal{O}_0$. After the measurement, one specific state is determined for the system $\mathcal{Q}$, because of the *quantum principle*. The freedom in obtaining one or another result after the measurement is caused by our ignorance of some parameters. In this context, the probabilities in Quantum Mechanics are similar to those in Statistical Mechanics.

The eigenvalues of this density operator represent the probabilities to find the system, after the measurement, in the corresponding eigenstate, resulting in the Born rule [Born26]:

The Born rule. λ_i is the probability to find the system $\mathcal{Q}$ in the eigenstate $|e_i\rangle$ of the observable O_1.

In the theory presented in this paper, the Born rule results from the *quantum principle*, which need to be taken as axiom, at least for the moment, and the *minimal disturbance principle*, which is another axiom, or may simply be considered a characterization of a "good measurement". The Born rule describes now the lack of knowledge of the state of the observed system, in the same way in which a statistical law describes a classical ensemble.

3.7. Heisenberg's relations

We can derive the original Heisenberg relations [WZ83] from the relations $\Delta\omega\Delta t \geq 2\pi$ and $\Delta k_x \Delta x \geq 2\pi$, by multiplying with the reduced Plank constant $\hbar$. Here $\Delta\omega$ represents the size of the support of a function expressed in the frequency domain, in Fourier analysis. We can use other bases in the state space to obtain various such relations. These relations refer to how large can be the support of a state vector, when expressed in two different bases. For example, the relations $\Delta k_x \Delta x \geq 2\pi$ show that if the wave packet is too located in space, then in the momenta space it will be more spread. These relations, by themselves, have nothing probabilistic built in. We can obtain also the Heisenberg's relations from the commutation relations of the operators.

In general, it is used a version of Heisenberg's relations, $\sigma(p_x)\sigma(x) \geq \frac{1}{2}\hbar$, expressed in terms of the *standard deviation*, defined for an operator A by $\sigma(A) := \sqrt{\langle A^2\rangle - \langle A\rangle^2}$. Again, the probabilities have not yet entered into the play, because the standard deviations, in this case, refers to the components of the wave packet, expressed in two conjugate bases.

It is only when the Born's rule is applied to a state vector in relation to a basis of eigenstates of an observable, when Heisenberg's relations become the *uncertainty relations*[2]. Since the Born rule is a probabilistic description of a system when the environment parameters are not known, it follows that the probabilistic meaning of the Heisenberg's relations also reflects our ignorance of some parameters.

3.8. **Consistent conditions and registries**

Thinking in terms of projectors is thinking in terms of discontinuities. It is very useful for understanding important problems of the Quantum Mechanics, as the *consistent histories* approach [Gri84, Omn88, GH90a, GH90b, GH90c, Omn92, Omn94, Ish94] proved. But if we want to understand how the state evolution can be smooth, it would be more appropriate to think in terms of (more or less initial) *conditions*. In fact, there are no discontinuities, and the observations don't involve projections. The observations simply add new initial conditions to the Schrödinger equation. The initial conditions added by observations are, of course, not added at an initial time, but at the time when the measurement is performed. Such a condition is indeed equivalent with an appropriate condition at any other time, but if we want to relate the condition with the observation, we will consider it as being imposed at the time of observation. This is why we named such restrictions *delayed initial conditions*.

It is possible that two conditions imposed to the solution of the Schrödinger equation are incompatible. The usual approach is to consider that the solution must have a discontinuity between those conditions. But in the Smooth Quantum Mechanics, all the conditions imposed by the observations are compatible, and the incompatibility is only apparent. We can only have seemingly incompatible solutions, when the initial conditions of the measurement device used for preparation are not entirely known (when this one is macroscopic). In this case, the initial condition imposed by the observation is also an initial condition for the preparation device, because of the entanglement.

We will name a set of consistent conditions a *registry*. A registry is *complete* if it defines a unique solution for the Schrödinger equation (eventual up to an overall constant factor). We can think the registries as continuously growing islands of data about the quantum system. Two registries are said to be *compatible* if their union is also a registry. In this case, the set of solutions allowed by the total registry is the intersection of the set of solutions allowed by each one of the two registries. Two registries are said to be *equivalent* if they select the same set of solutions of the Schrödinger equation.

The observers, being subsystems of the Universe viewed as a closed quantum system, cannot know a complete registry. They can only work to collect data to enlarge their registry more and more. Since an incomplete registry admits an infinity of solutions of the Schrödinger equation, from the point of view of the observers the Universe evolves in an indetermined way. They may know that the evolution, as described by the Schrödinger equation, is deterministic, but they cannot use this fact because their registry is incomplete.

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[2] This remark applies also to the Direct Interpretation of Quantum Mechanics [Sto08].

Each new observation determines a new initial condition to be added to the registry. In the Quantum Mechanics, to *determine* should be understood by its both meanings: to find what the state is, but also to choose what the state is. Of course, the observer can only determine-choose the operator whose eigenstate is a particular subsystem at a particular time. And she can determine-find only the eigenvalue, hence the eigenstate.

3.9. The forever new quantum system

When we measure for the first time a quantum system whose state is unknown, we determine its state without adding discontinuities. Hence, any quantum system is entitled to a measurement, and this don't add discontinuities to its evolution.

Let's consider that a system $\mathcal{Q}$ is measured $n+1$ times, with the measurement devices $\mathcal{O}_i, 0 \leq i \leq n$. We will discuss the measurement number k, with $1 \leq k \leq n$. When we measure the system $\mathcal{Q}$, we measure in fact the system obtained by entangling $\mathcal{Q}$ and $\mathcal{O}_{k-1}$. Therefore, we measure a new system in fact, and not the original system $\mathcal{Q}$. Each measurement k results in the entanglement of the observed system and the measurement apparatus $\mathcal{O}_k$, forming a new system, and the next measurement is performed on that new system.

Even if we use the same measurement device more times, the measurements we perform don't determine entirely its state, because it is very large as compared to the quantum scale, and because it continuously interacts and changes also.

We conclude that, in general, sequences of measurements of what we consider the same quantum system, are in fact performed to new systems, obtained by successive entanglements. Each measurement is performed to a different system, which is not determined. Each undetermined system is entitled to a measurement, so it is possible that a sequence of measurements don't force the discontinuities to occur.

Each measurement is performed to a new quantum system, and each quantum system is entitled to a measurement. Like the mathematician Grigore Moisil used to say:

> Every human being is entitled to a glass of wine. But after a glass of wine, you are a new human being.

4. Hilbert spaces and continuity

4.1. Problems with the Hilbert space

So far, I showed that we can reconstruct the Quantum Mechanics without using discontinuities in the time evolution. But I haven't imposed restrictions to the state vectors themselves. They are in general considered to be vectors in a complex Hilbert space.

A Hilbert space is a vector space with an inner product (hermitian, in the complex case), which is complete (any Cauchy sequence of vectors converges in the norm defined by the inner product). A finite dimensional vector space with inner product is automatically Hilbert. But an infinite dimensional Hilbert space of functions, as the ones in Quantum Mechanics, may contain discontinuous functions, and distributions (like the Dirac distribution, which concentrates in a point an infinite value). This is because of the completeness property.

The completeness condition is redundant in the finite dimensional case, and in the infinite dimensional case allows discontinuities. Does this mean that we should eliminate it?

The most fundamental observables in Quantum Mechanics are the positions and the momenta. The positions operators admits, as eigenstates, Dirac distributions. The momenta admit as eigenstates functions that have infinite norms. It is natural to admit that the state space satisfies the completeness condition, being so a Hilbert space. But each of these two basic observables come with peculiar behavior: they both bring in infinities. Does the physical world really contain such strange wave functions?

Let's take first the positions. Whenever we measure the position of a particle, we make use of other particles. In fact, in general we use atoms or more complex systems that can detect the particle whose position we want to measure. In the optimal case, a photon is absorbed by an atom, and thus being detected. We then deduce the position of the photon from the position of the atom. But the atom is not a point, it is larger. Moreover, the atom, even when it is bound with others atoms, is in continuous thermal motion. We can conclude that there is no way to detect the position of a particle with infinite accuracy. The state vectors of a position operator are not realistic. The Dirac distributions are not observed in reality. The eigenstates of momenta have a problem too: they extend infinitely in space (and time). It is hard to imagine how we can detect them by any possible experiment.

We conclude that in the real world we can detect not the position, but states which are close to eigenstates of the position, and not momenta, but states which are closed to momenta eigenstates. The wave functions encountered in the real world are continuous, and without infinite norms.

There is yet another reason for eliminating the discontinuities of the state vectors. If we will consider the Special Relativity, a state vector which evolves continuously in time, but it is discontinuous in space, it will appear in some reference frames to be discontinuous in time. And this will bring back again the discontinuities in the time evolution.

4.2. **The rigged Hilbert space**

We can use as a state space a space $\mathcal{S}$ of continuous or even smooth functions. As a matter of fact, because the Schrödinger equation is differential, we will need the wave functions to be differentiable. The functions will be considered to be defined on the Euclidean space $\mathbb{R}^3$ and valued in a complex vector space with hermitian inner product $(,)$. We define, as in the case of the Hilbert space, the inner product:

$$\langle\psi|\varphi\rangle = \int_{\mathbb{R}^3} (\psi(t), \varphi(t))\mathrm{d}x. \tag{7}$$

Further, we will require that any vector $|\psi\rangle \in \mathcal{S}$ have a finite norm.

The continuity will rule out the Dirac functions from $\mathcal{S}$, therefore the position operators will also be ruled out. The true position operators will be in fact only approximations of the ideal position operators on the Hilbert space $\mathcal{H}$. The condition of finiteness of the norm will eliminate the momenta operators.

By completing the state space $\mathcal{S}$ we will obtain a Hilbert space $\mathcal{H}$. $\mathcal{S}$ is then dense in $\mathcal{H}$. Or, more generally, we can consider a rigged Hilbert space $\mathcal{S} \subset \mathcal{H} \cong \mathcal{H}^* \subseteq \mathcal{S}^*$. The state vectors will be then elements of $\mathcal{S}$, but the (ideal) eigenstates of various operators will belong to $\mathcal{S}^*$.

I want to recall that not all the Hilbert spaces contain discontinuous functions or distributions. For example, the complex holomorphic functions forms a Hilbert space. If $\mathcal{S}$ is of this type, it is already Hilbert, and if we want to include distributions we will need to extend it.

Nevertheless, the Hilbert space formalism is very useful. This is why we can continue to use it as an approximation, where the observables can be idealized. This will simplify our concepts when we discuss measurements. On the other hand, it is important to remember that the real world which is represented by these idealizations may not support discontinuities and infinite norms. From physical point of view, the condition of completeness of the state space is a postulate with no background in experience.

The necessity to use a Hilbert space of distributions suggests that the observables of Quantum Mechanics, this including the operators for positions and momenta, are idealizations, and they may not maintain the central role they have today in the theory. But, so far, they proved to be useful, at least as idealizations of the true type of wave functions admitted in the physical world.

5. Making testable predictions

5.1. The importance of making testable prediction

We can consider this article as a proof that Quantum Mechanics does not require discontinuous state vector reductions. Thus, if we have to choose between the Smooth Quantum Mechanics and the standard version, we may say that it is the charge of the standard Quantum Mechanics to prove the existence of the wave function collapse. But, as Bohm and Hiley [BH93] observed, because the standard interpretation was the first one, it is in the charge of the new theory, even if it is simpler, to bring the proofs. On the other hand, the quantum paradigm is very counterintuitive, and its smooth version doesn't seem to be much easier to understand, so I think that it is justified to search for tests that can differentiate the two theories.

Because the Smooth Quantum Mechanics provides a mechanism for what appears to be the state vector reduction, we have to explore the limits of applicability of this mechanism. The limits of applicability can be identified by analyzing the assumptions required by this mechanism. After these limits will be established, experiments can be carried out to test them.

But in order to identify these limits, a better understanding of the measurement devices and the measurement processes is needed, and it seems that this understanding is not yet completely available.

5.2. Assumptions made in the description of the measurement

In this paper I showed how the Schrödinger equation, by the means of the entanglement, allows something that looks like a state vector reduction, but is not discontinuous, moreover, it is deterministic. By accounting the measurement device used for preparation, to which the quantum system is entangled since the previous measurement, the state space is enlarged enough to allow the observable's condition of finding the system in an eigenstate to become simply a delayed initial condition. In accomplishing this, I made use of the entanglement and the possibility to specify the initial conditions with a delay. In addition, I made three suppositions about the measurement devices, suppositions which may need to be justified better.

The first assumption was that the measurement device will impose an initial condition to the system, forcing it to be in an eigenstate of the observable. But the Schrödinger equation and the superposition admits solutions in which the measurement results in a superposition of eigenstates. The main purpose of this paper was to show that the system can be found in an eigenstate without the need of discontinuities, and this possibility indeed exists, under some general assumptions. But an important question is "why don't we observe superpositions of eigenstates when we perform measurements?". More general the question is "why don't we observe superpositions of classical objects?". Presently, the favorite approach in the quest for answering these questions is given by the decoherence [Omn88, Omn92, Omn94, Zeh96, Zur98, Zur02, Zur03a, Zur03b, Zur04].

The second assumption was that a "good measurement", that disturbs less the observed system, is one that respects the condition named here the *minimal disturbance* *principle*. This is obvious only in the particular case when the state $|\psi\rangle$ obtained after the preparation measurement i is an eigenstate of the observable of the measurement $i+1$. For the general case, the condition will become justified for a large number of measurements. Basically, it saids that a kind of average of the results for a large number of measurements performed in identical settings is $|\psi\rangle$. By "identical" we mean that the observed system is prepared in the same state, and the preparation device is considered to be the same at macroscopic level only, its real quantum state being not known.

A third assumption made about a measurement device is that, when entangled with the observed system, it has enough freedom to allow the density matrix to be the one imposed by the next measurement's observable, combined with the *minimal disturbance principle*. The freedom refers to the parameters describing the measurement device that are not yet fixed.

5.3. **Limits of validity**

To find the limits of validity of the mechanism described here, it is important first to understand how complex quantum systems may behave in a way which can be described very well classically. This is the problem of the emergence of the classical level from the quantum level. There must be a mechanism explaining how the quantum systems become well localized in space, and also their momenta become well localized. Also, the superposition seems to become inappropriate at this level. For example, a gulf ball cannot be in two places simultaneously. A cat cannot be both dead and alive. At least, we have to show that these situations happen very seldom.

After this, we need to find a good enough definition of a measurement device. According to the mechanism presented in this paper for determining the quantum system to be in an eigenstate of the observable, the measurement devices and the measurements need to respect the assumptions presented in the previous section. A better understanding of the measurement process will allow us to find more precisely the conditions under which the solution presented here to the problem of discontinuities will apply.

Perhaps the most important role in the elucidation of these problems will be played by the quantum thermodynamics, entanglement, and the better understanding of decoherence.

Once the limits established, they can be tested experimentally. It would be interesting to find the critical threshold between the usual quantum interactions, and those that can be considered measurements, as well as the limits between the quantum and classical. Then, tests will be designed and performed, and used to the confirmation, rejection or improvement of this solution.

6. Implications to other interpretations

6.1. Schrödinger's view

When Schrödinger discovered his famous equation, he was very happy that this one described not only the de Broglie waves, but also the spectral lines for hydrogen. Initially, he believed that this equation provides the complete description of Quantum Mechanics. In order to prove this, Schrödinger tried to interpret the wave function's amplitude as a charge density. Soon, Max Born proposed his probabilistic interpretation, and Bohr tried to explain to Schrödinger that the collapse should be added to the theory. Despite the success of the Copenhagen interpretation, Schrödinger was never glad about the wave function collapse. We can see now that he had a point. The collapse is just an illusion which can be explained by another phenomenon discovered by Schrödinger, the entanglement.

Even Schrödinger's idea of interpreting $\langle\psi|\psi\rangle$ as the electric charge density can now be revived. We have to remember to distinguish between the wave function which is the result of a measurement, and the density matrix before the measurement, which represents the quantum system entangled with the previous measurement device. It is the density matrix who describes the unknown, hence the probabilities. The state vector obtained by a measurement is just the state of the system. It is physical. In the case of the electron it can be used to calculate the charge current.

As we can see now, the Quantum Mechanics can be thought without the help of the state vector reduction, and we can say that Einstein, Schrödinger and de Broglie were right not to accept this compromise. The world is still deterministic, as they believed all the time.

6.2. Bohr's view

But in the same time, there is no way to collect all the initial data for determining the evolution of the Universe. This limitation make impossible to benefit of this deterministic feature. Even more important is the fact that we can still choose the past, as long as we don't contradict what is already established, what is already recorded in the "registry". Wheeler explained this very nice in [Whe78].

Therefore, we can see that Niels Bohr, together with Heisenberg and Born, were right as well. Not because, as they may thought, the world is not deterministic, but because there is no way to accede to the deterministic level. The determinism is hidden, and it is also eluded by the capacity of choosing the past, not as much to violate the causality and to change the history, but enough to allow the keeping of randomness. And Bohr was right to say that "no phenomenon is a phenomenon until it is an observed phenomenon" [Bohr28, WZ83].

This brings us back to Einstein, who wanted to have a real world, existing independent of the observer. The world is deterministic, because there is no collapse. It is causal, and local, because each possible solution of the Schrödinger equation is causal and local. The non-locality refers only to the choice of the solution, by imposing "initial conditions" to the system, when we observe it. Indeed, when we choose a solution of the Schrödinger equation, we make our choice by imposing some conditions to the solution. The choice is then valid everywhere and anytime. In this sense, the choice is non-local and even acausal. We make a non-local and acausal choice to select a local and causal solution. Therefore, we can understand precisely the level where the world is local and causal, and the level where it is not so. The solution is local and causal, the choice of the solution among the infinity possible is nonlocal and acausal. Two different categories, two different levels of existence.

6.3. **Reality and Many Registries**

If the solution "already" exists somewhere when we choose it, then we can say that the world is *real*, and has some independence of the observer. It is out there, among the other possible solutions, and the observer only chooses in what solution to live. The initial conditions imposed by the observer are then conditions by which the observer makes a choice. This vision automatically leads to a version of the Many World Interpretation. All the possible worlds exist; by "possible" we understand that they are solutions of the Schrödinger equation. The initial conditions we gather by observations only serve as determining in what version of the world we live.

If we perform a delayed-choice experiment, we can choose, by choosing the observable to measure, one set of solutions of the Schrödinger equation, or a totally different one. One of the solutions results to be the real state, but this is because we decided to measure one particular observable. Had we decided to measure another observable, another solution would have been observed. If our decision is not predetermined, it follows that there must exist more solutions, so that we can make our choice. We choose not between solutions, but between observables, but each observable comes with a distinct set of possible solutions. For example, by choosing the direction to measure the spin, we force the Nature to respond with a spin oriented along that particular direction. So there must exist distinct solutions for each possible direction we could select. But this holds only if we really are free to choose the observable.

In the context of the Smooth Quantum Mechanics, the reality and independence of the world goes hand in hand with a version of the Many World Interpretation. All possible solutions not only have a mathematical existence, but a physical one. The relative states of Hugh Everett III [Eve57, Eve73], metamorphosed in the many worlds of Bryce DeWitt [dW71, dWG73], in the many minds, and in David Deutsch's multiverse [Deu85, Deu91], are several versions of this interpretation.

To emphasize better this point, let's remember the EPR experiment [EPR35], in Bohm's version [Bohm51]. Two observers measure the two electrons, when they are separated by a long enough spatial interval. Are their results correlated? Of course. Had they measure the spin in the same direction, if the first electron is found in the state $|\uparrow\rangle$, then the second one must be with certainty in the state $|\downarrow\rangle$. But, assuming that the results are not correlated (which is, of course, impossible), what happens? Let's say that one of the observes finds the first electron in the state $|\uparrow\rangle$, and the other finds the second electron in the same state $|\uparrow\rangle$, and then they exchange information about the results of the experiments, or even meet each other. They will belong to different solutions of the Schrödinger equation, and no matter how hard they will look for each other, they cannot exchange any information, nor they can meet. Each one of them will meet a double of the other one, whose result of the measurement is precisely $|\downarrow\rangle$.

I will call the smooth version of the MWI the *Many Registries Interpretation.* The main difference is that each world is a smooth solution of the Schrödinger equation. We cannot move from a world to another, and the world cannot split, its evolution being deterministic. But we can have the illusion of splitting, because, from the point of view of an open quantum systems, each observation has a random output. If two solutions of the Schrödinger equation are equal at a time t_0, they are equal at any other time. But if two registries of consistent conditions are equal at a time t_0, the next measurement can provide different outcomes, and the two registries can extend differently. This is like a split of the world. And this is all that matters, because the observer will never know the whole solution of the Schrödinger equation, she will only know a registry of initial conditions. A pair of distinct worlds deserves the name *parallel worlds*, because they are distinct in each of their points (although they may be defined by registries that are not disjoint).

We can see that we can save almost all what Einstein cherished, by allowing the Many Registry Interpretation. The world will be independent of the observer, as long as we accept that all possible worlds exist. The physical laws are deterministic, causal and local, since they all are derived from the Schrödinger equation, without any discontinuity. The only, and central, non-classical effect is that of the "delayed initial conditions" - the possibility to decide later what already happened, as long as it was not recorded into the registry.

As a counterargument against the Many Worlds Interpretation, was raised the "Occam's razor argument": we should choose the simplest, or the most economical solution. This counterargument can be raised also against Many Registries Interpretation, but the truth is that this interpretation is simpler. The simplest explanation is that the Schrödinger equation is everything. To obtain a particular world, we have to add extra conditions. In the standard interpretation, or in the Consistent Histories Interpretation, the extra conditions are projectors. In the Smooth Quantum Mechanics, they are a consistent set of initial conditions - a registry. It may be simpler to assume that there are no such extra conditions, but each possible solution perceives, from within, such conditions defining it.

It is simpler to have only the vector space of the solutions of the Schrödinger equation, and not the more complicated vector space of functions that are only piecewise such solutions. If we admit discontinuous solutions, we need to add the magical conditions that take care of the conservation laws.

6.4. Ignored variables

One idea of Louis de Broglie was to complete the Quantum Mechanics with extra parameters that "repair" the determinism. His idea of pilot wave, and of hidden thermodynamics, together with J.P. Vigier's stochastic interpretation of Quantum Mechanics, culminating with David Bohm's causal or ontological interpretation of Quantum Mechanics [Bohm52, BH93], are theories whose purpose is to restore the determinism, the causality, and the reality and independence of the world. The price, as we now know, was the locality [Bel64, CHSH69, CS78, ADR82, Asp99].

It proved to be very difficult to add hidden variables that complete the Quantum Mechanics to a deterministic theory, which in the same time is able to give the same predictions. It also proved to be very difficult to fulfill this idea by adding simple principles.

The Smooth Quantum Mechanics is naturally deterministic. The indeterminism occurs only when the system is open. In this case, the quantum system is embedded in a larger system which obeys the Schrödinger equation. This larger system is of course deterministic. But the open system is not, and they both behave as predicted by the Quantum Mechanics.

We can complete the quantum system which is open, by adding all the parameters in the larger system. These parameters, which are ignored from the open subsystem, can be thought as hidden variables. But they are not hidden in a subquantum level. They are just "hidden" in the environment. This is a new sense of Bohm's idea of *wholeness*. What type of hidden variables can be simpler than plain quantum parameters of the environment? And what can be a better expression of the wholeness, than the idea that everything that was considered hidden, can be retrieved from the environment?

There are several important differences between the idea of hidden variables, and that of ignored variables in Smooth Quantum Mechanics. The Smooth Quantum Mechanics manifests the seemingly acausal behavior of the Standard Quantum Mechanics - the possibility to fix the parameters of what happened already, if they were not fixed. The alternative of Bohmian Mechanics is to allow non-local action in the true sense, to allow faster than light (instantaneous) communication. Even if they take place at a subquantum level, since that level is part of reality, these "spooky actions" are annoying. The Smooth Quantum Mechanics solves the problem without such interactions, but the price is given by the "delayed initial conditions". This price is payed by other interpretations too, exception making the Bohmian Mechanics, which replaced it with the non-local interactions at subquantum level.

6.5. What about point-like particles?

Sometimes it can be an advantage to consider the particles as being point-like. This may help us to reuse some intuitions from the Classical Mechanics. Bohmian Mechanics managed to keep the particles point-like, with definite positions and momenta.

The Feynman's *path integral formulation* is another way to express the wave behavior of quantum systems in terms of positions and momenta. In fact, it can be used for other bases of the state space too. But it helps the intuition more to think in terms of spacetime trajectories, and is very powerful at calculations.

Another way to keep the classical positions and momenta, for restoring to the particles the point-likeness, is to change the logic [BvN36, Lup47]. If, by discussing about position and momenta simultaneously, we obtain contradictions, then we can modify the logic so that the opposition is no longer a contradiction.

It is very difficult to keep the classical representations when passing to the counterintuitive quantum phenomena. In the Smooth Quantum Mechanics, this becomes simpler.

The waves are classical enough to help the intuition. Expressing the phenomena in terms of point-like particles having simultaneously definite positions and momenta entails severe complications. It is much simpler to think in waves. Everything is a wave. Each particle is a wave, part of the larger wave. The waves describing particles, or more complex quantum systems, have the purely quantum property of being indivisible, in the sense that any measurement will find them in a definite eigenstate. There is no way to find the particle in two different eigenstates of the observable simultaneously. This is an extra principle that is added to the plain Schrödinger equation.

But, of course, we can always think in terms of point-like particles, or we can apply the techniques developed from this way of thinking, since they are well tested and effective.

6.6. **"Contraria Sunt Complementa"**

By browsing superficially through the views of the great physicists who contributed to the Quantum Mechanics, we find opinions that seem in profound contradiction one another. But we can see that, in the Smooth Quantum Mechanics, many of them have found their place. Ideas apparently irreconcilable complete one another without contradiction in a vision of the quantum world. The idea of a world of waves and no collapses promoted by Schrödinger coexists with the Copenhagen view of random outputs of the measurement. Einstein's, de Broglies and Bohm's deterministic views coexists peacefully with Born's probabilistic amplitudes and Heisenberg's uncertainty. And this coexistence of such different views is logical and natural.

They all were right, they only spoked about different aspects of the quantum world.

7. Determinism

7.1. Determinism vs. indeterminism

We have seen that the Universe evolves, according to the Schrödinger equation, deterministically. On the other hand, the observers, as subsystems of the Universe, can't determine the future. They are limited to an incomplete registry. From the point of view of that registry, the evolution is indeterministic.

When a quantum system is observed from the outside, it is open. The observations disturbs it, and the system seems to evolve indeterministically. A Laplacian *intelligence* can still predict the evolution of the quantum system, by taking into account all the environment parameters. From this point of view, the *ignored parameters* can play the role of *hidden variables*.

The Smooth Quantum Mechanics allows a world where the determinism and the indeterminism coexist in a compatible way, at different levels, of course. Like Forrest Gump[3] saids:

> I don't know if Momma was right or if, if it's Lieutenant Dan. I don't know if we each have a destiny, or if we're all just floating around accidental-like on a breeze, but I, I think maybe it's both. Maybe both is happening at the same time.

7.2. Hidden causality and determinism

The causality, the determinism, the reality, are partially hidden (ignored), they belonging to the solution of the Schrödinger equation, but to the observer being accessible only an incomplete registry of initial conditions to determine that solution. For each solution, the causality and the determinism are respected. For each incomplete registry, they are not.

For an incomplete registry, the reality is not totally independent. By making the choice of the observable to be measured, the observer selects a subset of possible solutions, the ones that are, on the measured quantum subsystem, eigenstates of that observable.

Einstein is quoted to say the famous words "God doesn't play dice". At a level, this is true, since the evolution is deterministic. At the level of the registry on the other hand, it seems not to be true. God does play dices, but the dices are loaded[4].

7.3. Determinism and free will

One of the reasons why people accepted the discontinuities in Quantum Mechanics was the belief that a probabilistic evolution may allow the free will. It is usually thought that the free will is incompatible with a deterministic universe. Of course, some philosophers named *compatibilists* (Thomas Hobbes, David Hume, and the ancient Greek Stoics) employ a weaker definition of *freedom*, such that the free will is compatible with the

[3] http://www.imdb.com/title/tt0109830/

[4] Joseph Ford is credited for this paraphrase, in the context of Chaos Theory.

determinism. But the free will in the strong sense is considered to be incompatible with the determinism.

I don't know how we can define the free will in a consistent way. In physics, the world is described as being material, and any possible influence of the behavior of a system can come from the past of that system, from outside the system, if this one is open, and from indeterminacy, in the standard Quantum Mechanics. For the point of view of an open system, any external influence can be perceived as an indeterminism in its evolution.

Note that the indeterminism only does not guarantee the free will. It may even jeopardize the free will, because it is important, for a system to control in a rational way another system, that the latter to behave in a predictable way. For example, if the player cannot control a pair of (true random) dices, she cannot impose her will to the dices, despite the randomness of those dices. A car that is functioning in an arbitrary way cannot be driven. Analogous, if the human body is a physical system that is based on random input (such as the state vector reduction), the only way a "mind" which is not part of that system can control it is by the mean of that random input. Thus, the randomness would be only an expression of an influence from outside the physical system. The physical system (in our case the human body) should be an open system, in order to be influenced, and at least a part of what we think to be the quantum randomness should represent an input from the outer system controlling the body.

Conway and Kochen proved a "free will theorem" [CK06], which shows that if the human performing a quantum experiment is endowed with free will, so are the particles in the experiment. This theorem, as its stronger version [CK08], applies to a very wide range of quantum theories. Their results also applies to the Smooth Quantum Mechanics.

But we cannot draw from their theorem the conclusion that the free will is incompatible with the determinism. The Smooth Quantum Mechanics is an example of theory where the determinism takes place at one level, but the level perceived by the human beings is indeterministic. The human beings can decide what to observe, and by this they decide the possible outcomes of the experiment. Their decision influences the system, which, by entanglement, chooses some parameters of the past history, that were not chosen until that moment. The compatibility between the determinism and the free will is obtained by the possibility to delay the decisions concerning the initial conditions.

Of course, this does not guarantee in any way the existence of the free will. Even if the observer can choose the observable, there is no evidence that she can influence the specific result among the eigenstates of that observable. Moreover, it remains open the possibility that even the choice of the observable is predetermined by the past of the observer.

The conclusion is that, although we cannot guarantee the existence of the free will, the Smooth Quantum Mechanics is compatible with it, although it is deterministic at a fundamental level.

8. **Conclusions**

The standard Quantum Mechanics seems to be affected by discontinuities, both in time evolution and in the position representation of the state vector as a wave function. But the conservation laws and the observations suggest that, if the discontinuities exists, they should be masked, that the state vector behaves as if there are no discontinuities. All experiments failed to provide direct evidence of the state vector reduction.

This article suggests a simple way to give the same output as the standard Quantum Mechanics, by avoiding to appeal to discontinuities. By using only the Schrödinger equation and an appropriate choice of the initial conditions, the observed quantum system may behave “as if it collapses”. The initial conditions needed for specifying the solution of the Schrödinger equation are usually restrained by the *quantum condition* of finding the system in an eigenstate of the observable. After each observation, the quantum system gets entangled with the measurement device. Thus, even if the system is found in a precise state by the measurement, the entanglement with the measurement device makes its state to be again undetermined. The next measurement selects again an initial condition, to specify the state of the observed system. But now the system gets entangled with the measurement apparatus used for the last observation, and the cycle continues.

This scenario shows that we can indeed conceive a Smooth Quantum Mechanics. There is no need to bring into discussion discontinuities in time evolution. Moreover, the observations don’t provide evidence for quantum systems that are eigenstates of the position or momentum operators. Thus, there is no need to assume that the wave function can be a distribution which is not a function, nor that it can have an infinite norm. A rigged Hilbert space can save us from this kind of problems. It follows that indeed we can limit ourselves to smooth solutions of the Schrödinger equation.

The assumptions made in providing this description have some limits of validity. Most of the assumptions regard the measurement device and the process of measurement. If we can develop the theory of measurements in a more detail, we can predict better these limits, and then we can devise experiments to confirm or to infirm them.

The description of the means by which the Nature creates the illusion of the state vector reduction reveals that there are two levels: an indeterministic level, and a deterministic one. The deterministic level is the solution of the Schrödinger equation. But when the system is observed, the observer can access only a limited amount of information. It is at this level where the indeterminism seems to govern. We can see thus that the quantum world can be, in the same time, but at different levels, realistic and non-realistic, deterministic and indeterministic, local and non-local, and that the state vector is a complete description of reality. Moreover, if the indeterministic version of Quantum Mechanics allowed the existence of the free will, the smooth version also allows it, by the means of the delayed initial conditions.

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