General Relativity and the Probability Interpretation of Everett's Relative State Formulation

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One of the primary conceptual difficulties facing the multiple worlds interpretation (MWI) of quantum mechanics is the interpretation of the Born rule measure as a probability. Given that each world in the MWI is typically envisioned as being equally "real," a more natural rule would be to assign each of the N branches associated with a measurement the equivalent probability $1/N$, rather than the probability $|a|^2$ prescribed by the Born rule. This approach, the "alternate projection postulate" (APP), has been paid scant attention, however, since it leads to predictions that contradict those of standard quantum mechanics. In this paper, a further modification of the MWI is presented that not only incorporates the aesthetic advantages of the APP, but also is compatible with the predictions of quantum mechanics. This further modification involves an alternative method of enumerating branches that satisfies what is termed here the "Born identity," according to which there is not a single branch associated with a given experimental outcome, but rather more than one branch, with each branch being physically distinct and the number of branches being proportional to $|a|^2$. In place of the assumption of the Born identity, however, a feasibility argument for the derivation of the Born identity from more fundamental field-theoretic principles (such as those provided by general relativity) is sought. In this manner, it is proposed that quantum statistics may be derived from a purely classical (general relativistic) foundation without injecting the Born rule – either directly or in disguised form – as an independent postulate.

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I. INTRODUCTION

An essential feature of Everett's relative state formulation [6], a.k.a. the multiple worlds interpretation (MWI), is the application of the Born rule to assign a "probability measure" m to each branch associated with a measurement, with m being the squared norm of the complex coefficient a associated with the corresponding outcome, $m = |a|^2$, as calculated by the Schrödinger equation. Everett [6] and others (see, e.g., [2]) have attempted to demonstrate that this expression for m arises naturally from the essential makeup of the MWI. Recently, however, it has been argued [5] that other expressions for m can be conceived that are compatible with the basic structure of the MWI. In particular, an "alternate projection postulate" (APP) was proposed according to which each branch is assigned a probability measure equal to $1/N$, where N is the number of distinct possible experimental outcomes (also termed "branches" or "trajectories") associated with the measurement. Indeed, it was argued (correctly, in our view) that "the APP is in fact the most natural probability rule that goes with the Everett interpretation: on each 'branching' of an observer due to a measurement, all of its alternative 'worlds' receive an equal probability" [5]. The application of the APP to the MWI was then argued to give rise to a theory that is not only internally consistent, but also aesthetically superior to the standard MWI (SMWI), mired as it is by the interpretational difficulties of the Born rule. In addition, it was concluded (once again correctly in our view) that the application of the Born rule to the MWI should be seen as an assumption that stands independently from the basic structure of the MWI.

The difficulty with the APP, of course, is that it leads to predictions that disagree with those of standard quantum mechanics, a difficulty that was seen to "disqualify it as a possible candidate of a physical theory of our world" [5]. The goal of this paper is to explore how the MWI may be *further* modified to construct a formulation of quantum mechanics that reflects not only the interpretational advantages of the APP, but also the ability to make correct predictions. It will be shown that this is easily achieved through a reconceptualization of the number of branches associated with a given quantum mechanical measurement: rather than associating *one* branch with each experimental outcome, this alternate method, termed the "alternate enumeration postulate" (AEP), is allowed to associate *multiple physically distinct branches* with each individual outcome. Predictive accuracy is then restored by restricting the AEP to satisfy what is termed here the "Born identity," according to which the number of branches associated with an experimental outcome is proportional to $|a|^2$.

In place of the *assumption* of the Born identity, however, a method for the *derivation* of the Born identity from more fundamental principles of a field theory, such as general relativity, will be proposed. This method assumes a description of the observer, the system under observation, and indeed an entire "world" as geometric objects using the same mathematical language as that used for general relativity; in other words, it assumes the notion that "all is geometry" as envisioned (though not yet fully attained) by many, including Einstein, since the inception of general relativity. It will be shown that a small number of constraints (termed the "Born constraints") over this mathematical representation may be identified that give rise to predictions that are equivalent, at least in the approximation, to the Born identity. Although not achieved in this paper, it is envisioned that these contraints may be given the form of field equations and as such, could be derived from Einstein's equations themselves. In this manner,

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a general framework for the derivation of quantum statistics from general relativity is envisioned, one that does not require the Born rule as an independent postulate. The overall scheme presented in this paper, therefore, has two motivations: first, the aesthetic advantages of the APP as compared to the Born rule; second, the prospect (albeit not fully accomplished in this paper) of deriving quantum statistics from general relativity. This scheme will be referred to as the "modified MWI" (MMWI), which can be constructed conceptually via a series of modifications of the standard MWI (SMWI) as outlined in this paper.

Sec. II presents the argument for the aesthetic superiority of the APP compared to the Born rule. Sec. III presents in simple terms how the AEP and the Born identity confer the ability to make correct predictions to the MMWI. Special consideration will be given in Sec. IV to the issue of assigning "multiple physically distinct branches" to a *single* experimental outcome in a manner that is consistent with the essential structure of the Everett program. Sec. IV will also introduce the argument that general relativity can be incorporated naturally into the general scheme of the MMWI. In particular, the concept of a "multiplicity" of solutions to general relativistic systems that admit closed timelike curves is reviewed, with emphasis on the natural applicability of the APP to the probabilistic interpretation of "multiplicity." The notion that "all is geometry" is also discussed, whereby all material objects (particles) are represented as topologically nontrivial geometric objects referred to in the literature as "4-geons" [3] [8] using the mathematical language of general relativity. In Sec. V, a loose set of constraints (termed here the "Born constraints") over this geometry is proposed, and it is demonstrated explicitly that the Born identity is a feature of a simplified model that approximates the empiric predictions of the more complex model based on the Born constraints. This demonstration makes heavy use of the Feynman path integral (FPI) formulation of quantum mechanics. Since the Born constraints constitute essentially a description of paths through 4-space, it is envisioned (albeit not achieved in this paper) that the Born constraints could be expressed as a compact set of field equations and could be in turn derived from more generalized field equations, such as those of the theory of general relativity. It is conceivable that alternatives to the Born constraints may exist; therefore, the Born constraints are presented merely as part of an *existence proof* for the main thrust of this paper: that quantum statistics may be derived from general relativity. Sec. VI contains a summary discussion and presents areas of future work.

II. MOTIVATION FOR THE APP: THE PROBABILITY CRITERION

As stated above, the primary motivation for the APP is that it provides the most "natural" probability rule that one may imagine upon first encountering the MWI. A simple example may be used to illustrate this argument. Consider an experiment in which M identically prepared spin-$1/2$ particles are prepared and that their spins are measured sequentially. Using the Born rule, each particle is predicted to be observed to be spin up or down with probabilities m_{up} or m_{down}, respectively. Since each individual spin measurement produces two separate branches (worlds), there will be a total of 2^M worlds at the end of the M measurements. In each of these worlds, upon the completion of the M measurements, the observer is imagined to calculate the frequency p_{up} with which the particles were observed to be in the spin up state. In other words, m_{up} is what the theory predicts, and p_{up} is what is actually observed (with m_{up} being calculated once prior to experiment, and p_{up} being calculated separately in each of the 2^M worlds, after completion of the experiment). From a practical perspective, the prediction is tested by comparing the prediction m_{up} with the observation p_{up} (and likewise for spin down), using as large a value for M as is practically feasible. In particular, it is hoped that as $M \to \infty$, the physical measure of the number of worlds in which the Born rule appears to be false – that is, in which p_{up} deviates from m_{up} by an arbitrarily chosen (small) number δ – approaches zero.

The notion of testing a "probability rule" by comparing the predicted frequency to the observed frequency may be stated more generally for any arbitrary quantum mechanical experiment, using the following definitions:

Definition 1. *Experiments M. M is the number of times that the experiment is run.*

Definition 2. *Outcomes N. N is the number of mutually exclusive possible outcomes for one experimental trial.*

Note the assumption that the spectrum of experimental outcomes is *discrete*. This assumption may be made without loss of generality, and is necessary for the purpose of defining a *measure* over the number of worlds in which a particular outcome is observed. A quantum mechanical experiment with a continuous spectrum of outcomes, such as a position measurement, should more properly be conceptualized as a theoretical limit as the number of discrete elements of the position measurement apparatus approaches infinity.

Definition 3. *Frequency p_n. For any individual world (of which there are N^M), $P_n \in [0, M]$ is the number times that the n^{th} outcome was observed, with $p_n = P_n/M \in [0,1]$ being the frequency of this outcome among the M measurements.*

Definition 4. *Probability measure m_n. The "probability measure" of the n^{th} outcome, $n \in [1, \dots, N]$, is defined as m_n, which is the predicted probability frequency associated with the n^{th} outcome.*

Note that p_n is an attribute of an individual world, whereas m_n is a predicted quantity that is independent of any individual experimental result.

Definition 5. *Error ϵ_n. The difference between the predicted probability measure m_n and the observed quantity p_n will be referred to as the "error" ϵ_n, that is, $\epsilon_n = |p_n - m_n|$.*

Definition 6. *Validity F_n. The proportion $F_n \in [0,1]$ of the N^M worlds in which the observer concludes that the probability measure is valid, as determined by the error being less than or equal to an arbitrarily chosen cutoff, $\epsilon_n \leq \delta$.*

Given these definitions, the probability criterion may be stated in the following manner:

Axiom 1. *Probability criterion. For any arbitrarily chosen δ,* $\lim_{M\to\infty} F_n = 1$.

The probability criterion is essentially a mathematical statement of the notion that "most" of the worlds will produce an observer who concludes that the theoretical prediction is correct, to within some arbitrarily chosen cutoff. In a sense, the probability criterion may be interpreted as a *definition* of the very notion of probability: it is a requirement that must be met by the probability measure m_n. It therefore may come as a surprise that the probability criterion is not generally met if m_n is calculated by the Born rule, $m_n = m_n^{Born} = |a|^2$. That is to say that in most of the N^M worlds, the observer will typically conclude that the Born rule is false. It is, however, met if m_n is calculated by the APP, $m_n = m_n^{APP} = 1/N$.

By way of illustration, suppose $m_{up} = 0.9$, $m_{down} = 0.1$, $\delta = 0.1$ and $M = 100$. These 100 measurements result in 2^{100} worlds, and it is asked: in what proportion of these worlds does the observer find that p_{up} falls within the interval $m_{up}^{Born} \pm \delta$? A quick calculation (using the formulae in the appendix to this paper) shows that the answer is a miniscule $5.58 * 10^{-8}$ percent. In contrast, the probability criterion *is* met if the probability measure is calculated using the APP, $m_{up}^{APP} = 1/N$. In this case, $m_{up}^{APP} = 1/2$, and the proportion of worlds in which the observer finds that p_{up} falls within the interval $m_{up}^{APP} \pm \delta$ is a much larger 96.4 percent. It is simple to see that for any arbitrary δ, as M approaches ∞, the former value approaches zero, and the latter approaches 100 percent.

It is not so difficult to see that the APP is *generally* immune to the above difficulty. Indeed, it is easily demonstrated (see the appendix) that the APP is a general solution to the probability criterion. The essential similarity between the APP and the probability criterion is that in both cases, each of the N possible "branches" associated with a single measurement – or equivalently, each of the N^M distinct worlds resulting from M measurements – is considered to be, ontologically speaking, on an "equal footing." The SMWI, on the other hand, does not seem to offer a clear ontological picture of the "reality" of alternate worlds. One is tempted to adopt a form of double-speak in the interpretation of the Born rule, whereby each world is "equally real," with some worlds being "more real" (more probable) than others. The fact that the APP, but not the Born rule, is a solution to the probability criterion is precisely what makes it the more natural choice for the probability measure.

III. THE ALTERNATE ENUMERATION POSTULATE AND THE BORN IDENTITY

Heuristically and in simple terms, the representation of an individual experiment by the standard MWI involves two steps. First, one must enumerate each distinct possible outcome of the experiment, i.e. the number N of "branches" (variously referred to as "trajectories" or "worlds") associated with the measurement. Second, one must assign a probability measure m_n, with n an integer in $[1, N]$, to each distinct outcome using a probability rule such as the Born rule or the APP. As discussed above, the APP is presented as an aesthetically superior alternative to the Born rule in the second step. The straightforward way to incorporate the interpretational advantages of the APP with the ability to make correct experimental predictions, therefore, is to modify the first step of the SMWI, referred to here as the "standard enumeration postulate" (SEP).

The SEP is deceptively simple. As described in Everett's original paper [6], one begins (prior to measurement) with an observer in some definite physical state ψ^O, a system $\psi^S = \sum_n a_n \psi_n^S$ with eigenfunctions ψ_n^S, and a composite state ψ^{O+S}. The process of "observation" is defined as an interaction between the observer and the system that transforms the initial composite state ψ^{O+S} into the final state $\psi^{O+S'} = \sum a_n \psi_n^S \psi_n^O$. The notion of "branching" is introduced in the following manner: "[W]ith each succeeding observation (or interaction), the observer state 'branches' into a number of different states." In other words, the concept of "branching" is defined such that the number of branches is equal to the number of final observer-states ψ_n^O.

However, Everett also makes the statement that "each branch represents a different outcome of the measurement ..." [6]. Here, Everett seems to imply that the number of branches is equal to the number of final states of the *observed system* ψ_n^S – as opposed to the number of states of the *observer* ψ_n^O. Denoting the former by N_S and the latter by N_O, the tacit assumption that can be easily overlooked is that $N_O = N_S$. Furthermore, it is assumed that there is a one-to-one correspondence between the states ψ_n^O and the states ψ_n^S: "[T]he two have become correlated in a *one-one manner* ..." [6] (emphasis mine). This assumption of one-to-one correspondence will be referred to here as the "standard enumeration postulate" (SEP), and it is this assumption that will be replaced with an alternative way of counting observer states, which will be called an "alternate enumeration postulate" (AEP). In other words, the AEP is based upon the notion that N_O and N_S may be different. In keeping with the spirit of Everett's original program, the number of branches will be equated with N_O; this seems only reasonable, since it is the observer who (by definition) calculates the frequency p_n and compares it to m_n, as described in the previous section. In general, *any* alternative to the SEP will be referred to as an AEP – although of course, the primary objective will be to identify a *particular* AEP that restores quantum statistics.

To illustrate the difference between the SEP and the AEP, consider once again the observation of the spin state of a single spin-1/2 particle with $m_{up} = 0.9$. Prior to measurement, the system ψ^S can be expanded in terms of $N_S = 2$ eigenfunctions, $\psi^S = a_{up}\psi_{up}^S + a_{down}\psi_{down}^S$, with a_{up} equal to the complex square root of 0.9. According to the standard MWI, the process of measurement likewise transforms the observer ψ^O into $N_O = 2$ states, ψ_{up}^O and ψ_{down}^O. These two states are physically distinct by virtue of the fact that they are characterized by distinct internal physical recordings of the measurement result (for example, a bit in a computer's memory).

After the measurement, the observer will find himself in one of these two states with probabilities 0.9 and 0.1, respectively.

In the modified MWI, however, the APP is adopted, according to which each state is equally likely. In addition, consider a simple AEP for determining the number of observer states after the measurement: assume that the process of measurement transforms the observer, not into $N_O = 2$ states, but rather into $N_O = 10$ physically distinct states: $\psi^O_{up,j_{up}}$, $\psi^O_{down,j_{down}}$, with j_{up} an integer in $[1, 2, \ldots, 9]$ and $j_{down} = 10$. Since the number of distinct values of j_{up} is $J_{up} = 9$, and the number of distinct values of j_{down} is $J_{down} = 1$, then by the APP, it is seen that the probability that the observer will measure spin up (or down) is 0.9 (or 0.1), in agreement with the prediction made by the standard MWI.

The world-splitting diagrams of the SMWI and the MMWI are illustrated in Figure 1 A and B, respectively. Figure 1 B illustrates the generalized assumption that J_n is proportional to $|a_n|^2$ – that is, it is proportional to the probability m_n^{Born} associated with the n^{th} eigenfunction of the observed system, as prescribed by the Born rule. The relation $J_n \sim |a_n|^2$ may be thought of as a specific implementation of the AEP and is referred to here as the "Born identity." Just as the SMWI assumes quantum statistics by assuming the Born rule, the MMWI (in its simplest manifestation) assumes quantum statistics by assuming the Born identity.

The world-splitting diagram of Figure 1 B is the most simple one that one might conceive that gives rise to quantum statistics and fits the general scheme of the MMWI. However, other, more complex world-splitting diagrams could certainly be devised to meet these requirements. This is illustrated in Figure 1 C, in which the specific details of the world-splitting diagram are unspecified and are represented as a "black box." As will be discussed in the next section, the precise configuration of the world-splitting diagram inside this "black box" is, in principle, a consequence of the mathematical representation of ψ^O, ψ^S, and ψ^{O+S}, as well as the rules that govern their evolution.

IV. GENERAL RELATIVITY AS A FOUNDATION FOR THE MMWI: CTC'S, MULTIPLICITY, 4-GEONS, AND THE NOTION THAT "ALL IS GEOMETRY"

The reader may be wondering at this point whether there really is, after all, any substantive difference between the SMWI and the MMWI, or whether the difference is nothing more than notational reshuffling. A careful consideration of the essential structure of Everett's original program indicates that there is, in fact, a substantive distinction to be made. The issue at hand is whether there is any substantive difference between the claim that the measurement transforms the observer (once again using the spin-1/2 experiment as an example) into one of a total of $N_O = 2$ states, as in the SMWI, or – alternatively – $N_O = 10$ states, as in the MMWI. The argument that this is a *substantive* distinction between the SMWI and the MMWI is that each of these N_O final states is properly conceptualized as being *physically distinct*. In the words of Everett's original paper [6], the final state is represented "in terms of a superposition, each element of which contains a *definite* observer state ..." (emphasis mine). Indeed, a compelling argument could be made that if the N_O states of the observer are not distinct *physically*, then it makes no sense to call them "distinct" at all. Therefore, it must be assumed that not only up and $down$, but also j_{up} and j_{down}, are *variables that represent some internal degree of freedom describing the physical state of the observer*. Thus, if the observer is a computer, then *up* and *down* represent bits in the computer's memory that record the result of the measurement, while j_{up} and j_{down} represent some other – as yet undescribed – internal physical degree of freedom of the computer. The essential makeup of the Everett program, therefore, requires the MMWI to demonstrate that the J_n states of the observer associated with the n^{th} outcome are, in fact, *physically distinct*.

A physical interpretation of the variables N and J will require, of course, some sort of mathematical representation of the physical state of the observer in its initial state ψ^O, as well as a set of rules that predict its evolution from a single definite state ψ^O into one of a *multiplicity* N_O of distinct, alternative, mutually exclusive physical states ψ^O_n. It is interesting to point out that classical general relativity appears to give rise under the right conditions to situations in which a system in a single definite state has multiple distinct solutions to its time dependent evolution. An example of such a system that has been studied by several groups (see, for example, [9]) is that of a billiard ball whose trajectory is analyzed in a spacetime model that is not simply connected – in particular, one that admits closed timelike curves (CTC's). These authors discovered that it is possible for there to be *more than one* distinct solution to the billiard ball's trajectory, each one of which is internally self-consistent. The number of distinct solutions to the billiard ball's trajectory, i.e. the "multiplicity," was found to differ for different initial conditions. This multiplicity of solutions exists despite the initial conditions being specified in full. In addition, the number of solutions was found to be not arbitrary, but rather determined by the particular initial configuration of the composite system.

In the realm of established classical physics, the existence of a multiplicity of solutions to the time-dependent evolution of a system appears to be unique to a model of GR that admits CTC's. By way of contrast, this sort of multiplicity is *not* characteristic, for example, of Newtonian mechanics. One might argue otherwise: for example, the Newtonian analysis of the trajectory of a pencil balanced on its tip might be expected to yield a multiplicity – in fact, an infinite number – of solutions to the direction of the pencil's fall. On further reflection, however, it may be noted that Newtonian mechanics would predict that a *perfectly* balanced pencil would not fall at all, but would *remain balanced*. If one were to inject random (say, quantum) fluctuations to this model so that the pencil would fall, then the multiplicity of solutions to the pencil's trajectory would be inherently attributable to quantum mechanics, not Newtonian mechanics.

It would be interesting, therefore, to attribute a probability interpretation to the existence of this multiplicity of solutions via application of the APP. In other words, ascribe to the observer ψ^O and system under observation ψ^S a general relativistic representation, one which admits CTC's, analogous to that given to the billiard ball and its enviroment discussed above. The "rules" that govern the evolution of these states are therefore nothing more than the laws of general relativity. The multiple distinct evolutions of the observer and system may be labelled $\psi^O_{n_o}, n_o \in [1, N_O]$ and $\psi^S_{n_s}, n_s \in [1, N_S]$, respectively. Via the probability criterion (the APP), each of these alternative N_O evolutions of the observer is attributed an equal probability $1/N_O$.

The composite observer and system state ψ^{O+S} will likewise have a multiplicity N_{O+S} of solutions to its time-dependent evolution. Each of these solutions is internally consistent, but inconsistent with any other solution; thus, there will be a separate 4-manifold for the representation of each solution. Any pair of evolutions $\psi^O_{n'_o}$ and $\psi^S_{n'_s}$ are deemed "compatible" if they coexist within any individual solution to the evolution of the composite observer-system state. Therefore, any individual of the N_S evolutions of the system must be compatible with at least one, but possibly more than one, of the N_O possible evolutions of the observer. This number is identified with the quantity J_{n_s} as discussed previously. (See Figure 1 B). The quantity J_{n_s} (as well as N_O, which is the sum of the individual J_{n_s}'s) is of course derivable, in principle, from the principles of general relativity, which govern this evolution.

Interestingly, an independent program for the "derivation" of quantum mechanics from general relativity has been proposed by Hadley [3] [8] that likewise relies in a fundamental sense on CTC's for its construction. In this program, particles are modelled as topologically nontrivial regions a of four-dimensional spacetime termed "4-geons," with particular topological features being associated with particular particle types. In this model, physical objects are not envisioned as living "on top of" spacetime, but rather are *built out of* spacetime itself. This is the essence of the notion that "all is geometry." A specific 4-geon model of the electron, for example, has been proposed, according to which the non-causal structure of the 4-geon spacetime is seen to underly the phenomena of spin-half [4] and charge [1].

In the next section, a general relativistic description of matter in terms of 4-geons will be assumed with the goal of filling in the "black box" of Figure 1 C to produce the world-splitting diagram of Figure 1 E. This will involve the definition of a parameter χ as an internal physical characteristic of a 4-geon, with $(\chi)^2$ playing a role analogous to that of J in the above discussion. (Compare Figure 1 B and 1 E). Indeed, it will be shown that the simple MMWI model of Figure 1 B (the Born identity) makes the same empiric predictions as the more complex MMWI model of Figure 1 E (the Born constraints); therefore, the former may be interpreted as an *approximation* to the latter.

V. THE BORN CONSTRAINTS

The rules that govern the model of Figure 1 E will be referred to as the "Born constraints." This model assumes a general relativistic description in terms of 4-geons of the initial states of the observer ψ^O (say, a computer), the system under observation ψ^S (the particle), and the composite observer-system state ψ^{O+S}. As discussed above, there will be multiple 4-manifolds W to represent the multiplicity of evolutions of the composite observer-system state ψ^{O+S}; the ensemble of all such 4-manifolds, corresponding to a given experimental setup, will be labelled E.

In addition, assumptions will be presented concerning the actions of paths connecting two generic regions a and b of spacetime. These postulates will be presented in terms of generic paths λ connecting a and b; in this manner, they can be interpreted as generic mathematical constraints applied over the manifolds W. In the specific instance that a and b correspond to a 4-geon, however, the postulates form the basis for the construction of the physical descriptor χ, whose square will be shown to play a role analogous to that of J in the preceeding discussion. In this manner, the Born constraints serve as a *generalized field-theoretic set of mathematical constraints*, from which the model of the MMWI as depicted in Figure 1 E may be built. This is one reason that the 4-geon model is an attractive one to incorporate into the general scheme of the MMWI.

Consider a very general quantum mechanical experiment: given a particle that is observed within a spacetime region of 4-space $s(x, t)$ (where s indicates particle source), calculate the relative probability that it will be detected within the region $d_n(x, t)$ (where d indicates particle detector, with n representing the n^{th} detector element). This experiment can be used, for example, to represent the electron 2-slit experiment, which Feynman has argued contains the "essential mystery" of QM [7].

Following the general reasoning of Everett's original program [6], the observer is defined to be in a single, well-defined initial state, and to have already observed and recorded the full experimental setup: the presence of the particle at the source $s(x, t)$, a series of detectors at regions $d_n(x, t)$, a system of barriers between source and detector, and the system under observation ψ^S in its initial state. It should be emphasized that these internal recordings are *assumed* in Everett's original program to represent an *accurate* representation of external reality. Therefore, ψ^O may be interpreted as a boundary condition to be imposed upon a 4-manifold W, such that any W that meets the boundary condition defined by ψ^O contains one possible solution to the evolution of the composite state ψ^{O+S}. In other words, E is defined as the set of all 4-manifolds that "contain" ψ^O as a boundary condition; and it follows that each and every 4-manifold within E contains the full experimental setup. In contrast to the observer, the system under observation is not required to be in a *single* well defined state; rather, there may in fact be multiple initial states ψ^S, with different ψ^S's being represented by distinct W's.

Given a 4-geon corresponding to a region of spacetime a, the variable χ will be defined in terms of paths connecting the 4-geon a to other regions of spacetime. Within any $W \in E$, given any two regions in 4-space a and b (where s and d are specific instances of the more general variables a and b), consider the set of equivalence classes of all possible paths connecting a and b, where $\lambda^i_{a,b}$ denotes a class representative of the i^{th} equivalence class. (Given the assumption that spacetime is characterized by multiple-connectedness at the small scale via the 4-geon particle model, there will be a very large number of paths that cannot be deformed into one another, and hence a very large number of distinct equivalence classes.) There is further assumed a suitable definition for the general relativistic action S^i of path λ^i. Assume that one method of approximating this set of paths in the non-relativistic limit (using a flat background spacetime manifold) is to enumerate the set of "all possible" paths $\varphi^i_{a,b}$ from a to b by the technique of the Feynman path integral (FPI) [7]. Furthermore, assume that the action of λ^i is well-approximated in the non-relativistic limit by the action of the corresponding path φ^i as calculated by the FPI. Since the distibution of the amplitudes of the Feynman paths φ^i on the unit circle in the complex plane (see Figure 1 D) are completely determined by the experimental setup (according to the standard technique of the FPI [7]), the same must also be true of the amplitudes of the paths λ^i, given the postulated relationship between the φ^i and the λ^i.

Next, assume that given any set of paths λ^i between a and b, there is a natural method of placing them into disjoint subsets whose union equals the entire set of paths, and with each subset containing k paths, $k \in [1, 2, 3, \ldots]$. Thus, any individual path belongs to a single subset containing k paths, with k being called the "index" of the path; that is, the index is the number of paths with which any given path is coassociated (so that a path with index $k = 1$ is coassociated only with itself). The joining of these paths into subsets is assumed to depend upon the precise configuration of the region a. If a and b represent two different locations in spacetime of the same 4-geon, it is assumed that the placement of paths into subsets is preserved between a and b. Define $\chi^k_{a,b}$ as the total number of paths from a to b with index k. Replace a and b with s and d_n, respectively, and define χ^k_s as the sum of χ^k_{s,d_n} over all of the detector elements. Since s is interpreted as a 4-geon (corresponding to the state of the particle as it is emitted from the source), then χ^k_s is interpreted as a physical characteristic of this 4-geon.

Given any individual subset of k paths, assume that there is a natural cyclic ordering from 1 to k such that two adjacent paths λ^i and λ^{i+1} are $2\pi/k$ out of phase, that is, $S^i - S^{i+1} = 2\pi\hbar/k$. (See Figure 1, D, which illustrates the case for a subset with index $k = 5$. Recall that the amplitude for the i^{th} path whose action is S^i is given by the expression $\phi^i = e^{-iS^i/\hbar}$.)

According to the FPI, the *total* amplitude $\psi_{s,d_n} = a_n\phi_n$ for a particle emitted from the source s to be observed in the region d_n is calculated as the sum of the individual amplitudes ϕ^i_{s,d_n} over all paths from s to d_n. Given the assumptions above, performing the summation using the MMWI paths λ (in place of the Feynman paths φ) will yield the same result, since each path λ has the same action and hence the same amplitude as the corresponding path φ. Because of the cyclic ordering discussed above, it is plain to see that for $k > 1$, the amplitudes of the k paths within any individual subset will "cancel each other out," i.e. they will sum to zero. (See Figure 1, D.) Thus, the paths with index $k > 1$ may be eliminated from the calculation of ψ_{s,d_n}, so that ψ_{s,d_n} is the sum over all paths with index $k = 1$.

Consider next the details of the "black box" in Figure 1, C. This will be assumed to take the two-step structure illustrated in Figure 1, E. The first step involves the variable χ^1_s. Assume the tree diagram to be structured so that χ^1_s is minimized. (The minimization of χ^1_s implies the minimization of χ^1_{s,d_n} for each d_n.) That is, lower values of χ^1_s (and hence each χ^1_{s,d_n}) are *more probable* than higher values. The minimization of χ^1_{s,d_n} could equivalently be stated as grouping as many paths λ_{s,d_n} as possible into subsets of high (greater than one) index. As a result of the requirement that paths within any subset of $k > 1$ must be cyclically ordered (Figure 1 D), the minimum possible value of χ^1_{s,d_n}, which is denoted with a bar on top ($\bar{\chi}^1_{s,d_n}$), is of course restricted by the distribution of amplitudes on the unit circle. For example: suppose hypothetically that all of the paths from s to d_n have exactly the same phase. In this case, none of them can be placed into subsets of $k > 1$, so that each path must be "individually grouped" into subsets of $k = 1$. On the other hand, suppose hypothetically that half of the paths have phase π, and the other half have phase 0. In this case, it is possible to place each path with phase 0 into a subset of $k = 2$ containing one of the phase π paths, resulting in zero $k = 1$ paths. Each distinct possible way of dividing the set of paths into subsets will require a distinct global 4-manifold in E for its representation.

It can be noted that if all index $k = 1$ paths in a given $W \in E$ have the same phase, then there can be no *other* $W \in E$ in which these paths are placed into higher-index subsets. Therefore, if a 4-manifold is characterized by having all index $k = 1$ paths with the same phase, then χ^1_{s,d_n} takes its minimum value. A careful consideration of the technique of the FPI indicates that it is always possible to achieve a grouping of index $k = 1$ paths with the same phase. Therefore, the minimization of χ^1_{s,d_n} will produce a set of index $k = 1$ paths that all have the same phase. Since they all have the same phase, then the absolute value of the sum of their phases ($|\psi_{s,d_n}|$) is proportional to the number $\bar{\chi}^1_{s,d_n}$ of paths being summed over; that is, $\bar{\chi}^1_{s,d_n} \sim |\psi_{s,d_n}|$. Squaring this expression yields $(\bar{\chi}^1_{s,d_n})^2 \sim |\psi_{s,d_n}|^2 \sim |a_n|^2$.

Next, consider the set of all index $k = 1$ paths from s to any of the detector elements d_n. By experimental design, the particle traverses the source s as well as one and only one of the detector regions d_n. Assume that between any two regions a and b, both of which are traversed by a 4-geon, there exist *two* index $k = 1$ paths from a to b that can be uniquely associated with the 4-geon. Given $\bar{\chi}^1_{s,d_n}$ paths of index $k = 1$ from s to d_n, there are exactly $\frac{1}{2}(\bar{\chi}^1_{s,d_n}) * (\bar{\chi}^1_{s,d_n} - 1) \sim (\bar{\chi}^1_{s,d_n})^2$ unique ways to choose two of them. Assume that in the second phase of the trajectory diagram (Figure 1, E), the observation of the particle at detector element d_n is concomitant with identification of these two paths, and that there is a separate branch on the trajectory diagram associated with each possible pairwise combination. Thus, the number of branches associated with the n^{th} detector element is proportional to $(\bar{\chi}^1_{s,d_n})^2$. By the APP, the predicted probability P^{MMWI}_{s,d_n} of detection at d_n is proportional to this number of branches, so that $P^{MMWI}_{s,d_n} \sim (\bar{\chi}^1_{s,d_n})^2$. The FPI, of course, predicts that $P^{FPI}_{s,d_n} \sim |\psi_{s,d_n}|^2$. Since $(\bar{\chi}^1_{s,d_n})^2 = |\psi_{s,d_n}|^2$, it follows that $P^{MMWI}_{s,d_n} \sim P^{FPI}_{s,d_n}$; that is, the predictions of the MMWI are in agreement with the predictions of the FPI and hence of quantum mechanics in general. In other words, it is shown that the MMWI model of reality, using the world-splitting diagram of Figure 1 E, makes approximately the same predictions as the SMWI model of reality, using the world-splitting diagram of Figure 1 A. In this manner, the Born constraints are shown to be equivalent to the Born identity.

VI. DISCUSSION

Conceptually, the modified MWI (MMWI) may be constructed from the standard MWI (SMWI) through a series of stepwise modifications. First, the aesthetic advantages of the probability criterion are incorporated by replacing the Born rule with the APP. Next, the ability to make empirically correct predictions is restored by replacing the standard enumeration postulate (SEP) with an alternate enumeration postulate (AEP) that obeys the Born identity. Finally, more complex versions of the MMWI may be constructed using more complicated world-splitting diagrams. Careful consideration has been paid to the construction of the MMWI in a manner that is consistent with the original intent of Everett's original program. In particular, the MMWI, perhaps more than the SMWI, respects the original ontology of Everett's program, according to which each world (or equivalently, each branch) is attributed an equal degree of "reality." In addition, the MMWI makes sure to distinguish different branches of the world-splitting diagram by virtue of a physical distinction between different observer-states – once again, in a manner that respects the ontology of Everett's original program. A distinction should perhaps be made, at this point, between the terms "branch" and "world," with the term "world" being used to correspond to an element (a 4-manifold) of the ensemble E. A discussion of the relationship between the MMWI and other ensemble formulations of quantum mechanics, such as Einstein's so-called "statistical formulation," will be the subject of a separate paper.

Although the initial motivation for the MMWI is the aesthetic appeal of the APP over the Born rule, a second and perhaps more powerful motivation is the prospect of deriving quantum statistics from field-theoretic constraints, such as Einstein's equation of general relativity. This derivation is envisioned to proceed as follows: general field-theoretic constraints → Born constraints → Born identity → quantum statistics. The last step – quantum statistics from the Born identity – is a straightforward application of the APP. Indeed, it should be noted that the goal of deriving quantum statistics from relativity is worthwhile only if one accepts the APP as being sufficiently natural as to require no further justification. In other words, it would be pointless to attempt a derivation of quantum statistics from GR in the context of the SMWI, since the SMWI assumes the Born rule, and as such, it assumes quantum statistics already. Note that at no point in the above derivation does the MMWI outright *assume* quantum statistics. Indeed, the Born rule is entirely *bypassed* in the MMWI; in effect, the Born rule has been sidestepped by the Born constraints. One way to view this situation would be to note that the Born rule gives rise to correct predictions only if worlds (branches) are counted using the standard enumeration postulate (SEP), as in the SMWI. When worlds are counted according to the AEP, as in the MMWI, it is the APP rather than the Born rule that leads to correct experimental predictions. This is an either-or choice: we may adopt the SMWI and the Born rule, *or* we may adopt the MMWI along with the APP, AEP, and Born constraints. Either model makes the same predictions.

This paper is concerned primarily with the second step: the derivation of the Born identity from the Born constraints. These constraints are offered merely as an "existence proof" for the feasibility of the overall scheme of this paper; alternative sets of constraints may exist that serve the same purpose, and that fill in the "black box' of Figure 1 in a different manner. Assuming the 4-geon particle model and the notion that "all is geometry," the Born constraints may be summarized loosely as a set of relations applicable to the paths λ connecting any two regions of spacetime a and b within a 4-manifold M. These include variables such as the placement of paths into groups of k elements and the cyclic ordering of their actions S; the assumption that certain properties such as the action S can be well approximated in a flat manifold; and assumptions governing the makeup of the world-splitting diagram of Figure 1 E. Given these assumptions, it is demonstrated that the predictions of the MMWI are equivalent (in the approximation) to the predictions of standard quantum mechanics, with the Feynman path integral formulation being used to represent standard quantum mechanics.

The largest gap in the above proposed derivation is in the first step: the derivation of the Born constraints from a more general expression of field-theoretic constraints, such as Einstein's equation. One of the arguments that is often put forward for the incompatibility of GR and QM is that QM is fundamentally a theory of probability, whereas GR is a deterministic theory that has no inherent concept of probability. This argument is challenged, however, by the appearance of "multiplicity" within GR, provided that one accepts the APP as a natural probability interpretation of multiplicity. The primary accomplishment of this paper is that quantum statistics has essentially been reformulated in terms of the Born constraints, which take the form of mathematical statements that may be applied to the 4-manifolds used for the expression of general relativity. Given this common mathematical language, it becomes at the least *feasible* to envision that the Born constraints may be derived from more fundamental general relativistic relations (that is, the Einstein equation). As discussed above, in the absence of an inherent notion of probability within GR, it is difficult to imagine – in fact, it may be pointless to attempt – such a derivation.

There are many mathematical languages for the expression of general relativity (such as differential geometry, differential forms, or geometric algebra), and it is unclear to the author which might be the most appropriate in a search for the derivation of the Born constraints from Einstein's equation. In the language of differential forms, for example, one might begin with the definition of a 1-form w over a 4-manifold that can be expressed as the pull-back of the standard 1-form on the unit circle, so that the integral of w along any closed loop C will be integer-valued. This is a field-theoretic statement, and coupled with a suitable conception of a 4-geon, might form the basis for the characterization of the paths λ that play a major role in the statement of the Born constraints. Further work will be necessary to determine whether this notion of w is implied by a field-theoretic description of general relativity; if so, then a link between GR and the Born constraints might result. On the other hand, alternatives to the Born constraints may be conceived that are implied by GR and that in turn imply quantum statistics. It should also be noted that the arguments put forth in this paper do not point to any particular field theory. Other theories, such as the Evans unified field theory or perhaps some version of string theory, may serve as an alternative to GR. The reasons that GR are singled out in this paper are essentially that GR is the most obvious place to look first, and that GR exhibits multiplicity.

The vision offered by the present work for the derivation of quantum statistics from GR is in broad outline only, and is admittedly incomplete. To our knowledge, however, it is unique as a prospect for the explicit *derivation* of QM from GR. Other programs have been devised for the demonstration of the *compatibility* between QM and GR. Hadley's program [3] [8], for example, is one such program. Indeed, Hadley's program and the present work may be viewed as complementary in many ways, with the notion of 4-geons playing a central role in both. Further development of the general scheme proposed in this work will undoubtedly require much mathematics. However, it will hopefully not require any new physics.

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APPENDIX A: RELATIONSHIP BETWEEN THE PROBABILITY CRITERION AND THE APP

Assume the definitions of Sec. II. It is further assumed that m_n can be expressed as a function of N. (Compare this to Everett's assumption [6], discussed in [5], that m_n can be expressed as a function of a_n.) The goal is a solution for m_n that satisfies the probability criterion.

The number of ways to make P_n observations of the n^{th} outcome is calculated to be $\frac{(N-1)^{(M-P_n)}M!}{P_n!(M-P_n)!}$, as follows. There is only one way to get the n^{th} result on P_n out of P_n trials. Next, the number of ways to get anything other than the n^{th} result on $M-P_n$ out of $M-P_n$ trials is $(N-1)^{M-P_n}$. Next, the number of ways to mix an ordered sequence with P_n elements and an ordered sequence of $M-P_n$ elements, i.e. the number of ways of distributing P_n elements over M elements is M choose P_n, i.e. $\frac{M!}{P_n!(M-P_n)!}$. Multiplying these expressions yields: $\frac{(N-1)^{(M-P_n)}M!}{P_n!(M-P_n)!}$. Dividing this expression by the total number of worlds N^M yields the proportion $f(p_n) \in [0,1]$ of such worlds: $f(p_n) = \frac{(N-1)^{(M-P_n)}M!}{P_n!(M-P_n)!N^M}$. The integrated proportion $F(m_n,\delta) \in [0,1]$ of worlds in which the observed frequency p_n is close to the predicted frequency m_n, i.e. falls anywhere within the closed interval $m_n \pm \delta$, i.e. falls within $[m_n-\delta, m_n+\delta]$, is calculated as the sum: $F(m_n,\delta) = \sum_{p_n=m_n-\delta}^{p_n=m_n+\delta} f(p_n)$. The probability criterion states that for any δ, $\lim_{M\to\infty} F(m_n,\delta) = 1$. It is readily seen that the APP, $m_n = 1/N$, is a solution to this equation. Computerized numeric calculations confirms this solution. Therefore, it can be concluded that the APP is a solution to the probability criterion.

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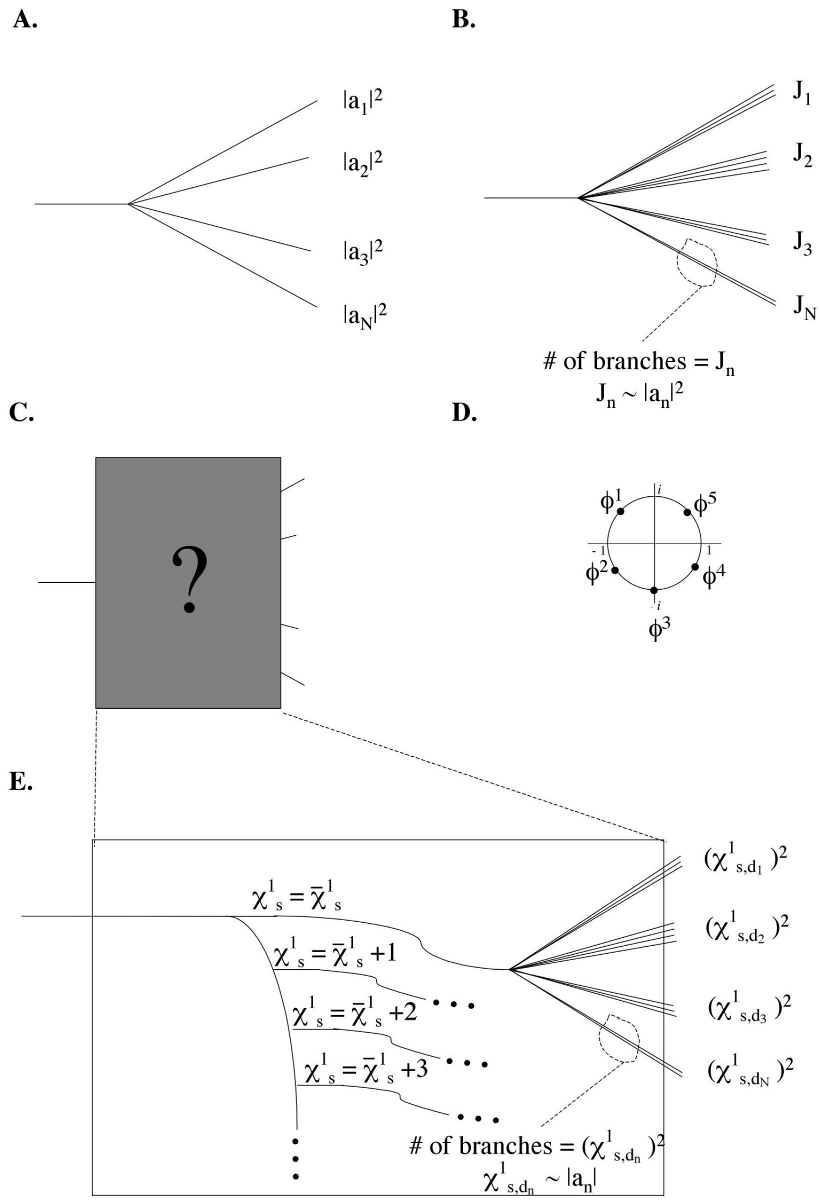

FIG. 1: **Diagrammatic representation of world-splitting. A.** Standard MWI (SMWI). Each branch is realized with probability $|a|^2$. **B.** The modified MWI (MMWI). Each branch is equally likely, and the number J_n of branches associated with the n^{th} outcome is assumed (by the Born identity) to be equal to $|a|^2$. **C.** More complicated versions of the MMWI that reproduce quantum statistics may be envisioned if the Born identity is replaced with more complex world-splitting diagrams. **D.** Cyclic ordering of the amplitudes of the k paths of a subset with index $k = 5$, illustrating that for any $k > 1$, the amplitudes will "cancel each other out" when summed together. **E.** The Born constraints.