## Leibniz's Tactile Binary Clock

Early in 2020, I began an extensive study of the manuscripts on binary arithmetic by the great German polymath Gottfried Wilhelm Leibniz (1646-1716). With the support of the Gerda Henkel Stiftung, this study yielded a number of articles and a book-Leibniz on Binary: The Invention of Computer Arithmetic (MIT Press, 2022, co-authored with Professor Harry Lewis of Harvard University) - containing English translations of Leibniz's 32 most important writings on the topic, many of which were previously unpublished. In addition to making these writings available, the aim of the book was to tell, for the first time, the true story of binary as it developed in Leibniz's hands, from first thoughts through to its dissemination, eventual publication, and then its posthumous influence, most notably on shaping our own computer age.

Trawling through Leibniz's unpublished manuscripts on binary yielded many surprises. Most notably, that Leibniz's invention of binary was not inspired by others, as has often been claimed, but occurred in response to his own work on three problems that exercised him in the mid-to-late 1670s, namely: devising methods and formulae to determine the divisibility of composite numbers, primality, and perfect numbers. Among other surprises were Leibniz's in depth work on binary fractions and binary expansions, along with his facility in developing techniques for handling and exploring them, and also his invention of base 16, or "hexadecimal" in modern parlance (Leibniz called it "sedecimal" or "sedenary"), another number system used in modern computing. Leibniz devised base 16-previously thought to have been invented in the nineteenth century - in 1679, and made occasional investigations of it afterwards, including on the back of an envelope!

But another surprise came after work on the book had been completed. By chance I came across a short manuscript in which Leibniz hit upon an innovative application of binary that has remained unknown till now. In this manuscript, which is reproduced and transcribed below, with an English translation available here, Leibniz sketches out a plan for a tactile clock based on binary reckoning that would enable one to tell the time at night. Leibniz's illustration shows a single hour hand that moves around the clock face. Between each numeral and the centre is a row of four points, each of which is either hollow (represented by an empty circle) or bumpy (represented by a filled circle). Hollow points denote 0 , and bumpy points denote 1 if they occur in the first position, 2 in the second, 4 in the third, and 8 in the fourth, with positions being counted from the centre outwards to the edge. The idea is that you feel for where the hand points, then trace your finger from the tip of the hour hand down towards the centre and then further down along the same line until you encounter a row of four points, which are then read by touch. Hence in the diagram, $a$ is $0, b$ is 0 , $c$ is 1 , and $d$ is 1 , and since $c$ and $d$ are in the third and fourth positions, they denote 4 and 8 respectively. Add them together, and one knows
 that the time is $12 o^{\circ}$ clock.

A casual glance at the clock face will reveal a mismatch between numerals and binary representations, as the row of holes and bumps leading from each numeral to the centre contains the binary representation not of that numeral, but of the one on the opposite side of the clock (e.g. the row between the numeral 7 and the centre is $000 \bullet$, which is the binary representation for 1 , and vice versa). The reason for this, of course, is that the clock face isn't
meant to be seen, only touched - the row of dots one reads is the one directly opposite where the hour hand happens to be pointing, not the one that would be underneath the hand (which wouldn't be accessible to touch anyway, because the hour hand would be over it).

Leibniz's tactile clock is as simple as it is ingenious, and the idea is all the more remarkable because it precedes the cognate inventions of night writing and Braille by more than a century. In the manuscript, Leibniz says nothing about the clock's mechanism, but this would have been easy enough for a horologer of the day to construct. The tactile clock design is certainly viable, although perhaps vulnerable to clumsy or heavy-handed touching of the hour hand. It is worth noting that the manuscript for the tactile clock was filed not among Leibniz's mathematical papers-hence why I missed it initially!-but among his correspondence with the engineer Gottfried Teuber (1656-1731), suggesting that his intention was for Teuber to build the clock. Yet Leibniz appears not have sent the design to Teuber, or approached anyone else to build the clock, despite its potential to benefit not just those in ill-lit rooms at night but also the blind and visually impaired. Accordingly, his tactile binary clock is much like the majority of his other ideas about binary and other nondecimal number systems, in that it was destined only for the admiration of future generations rather than the benefit of his own.

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Transcription of manuscript LBr. 916 Bl. $44^{\mathrm{r}}$ held by Gottfried Wilhelm Leibniz Bibliothek Niedersächsische Landesbibliothek, Hannover:

Der Gebrauch der Bimal rechnung kan unter andern dienen die Zahlen zu fühlen und zwar an der Uhr des nachts oder in der tasche wenn man wissen will worauff der zeiger stehet, und nicht zu sehen kan oder will.
a 0 ist 0 , und $b 0$ ist 0 und $c, 0$ ist 4 , und $d, 0$ ist 8
die hohle Puncte sollen bedeuten 0 , und die buckligen Puncte bedeuten Eins
wenn sie auff der ersten Stelle, 2 auff der andern, 4 auff der dritten, 8 auff der vierdten gesezt nun mann wolle fühlen worauff der Zeiger weise, und selbiger
weise fast auff 12. So fahre ich von der Spize derselben nach dem centro zu und weiter über das Centrum herab, und finde darunter die mit $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$
gezeichnete 4 Puncte, a o , so bedeuten 12 .
b o
c •
d •
Die erste beyde mit $a$ und $b$ auffm papier bezeichnete sind beyde 0 , weil sie hohl. Der dritte punct c ist bucklich gilt 4 weil er auff der dritten stelle, der vierdte d ist auch Bucklich gilt 8 weil er auff der vierdten stelle. Nun 4 und 8 macht 12.


