An Adynamical, Graphical Approach to Quantum Gravity and Unification

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Abstract

We propose an adynamical, background independent approach to quantum gravity and unification whereby the fundamental elements of Nature are graphical units of space, time and sources (in parlance of quantum field theory). The transition amplitude for these elements of "spacetimesource" is computed using a path integral with discrete Gaussian graphical action. The unit of action for a spacetimesource element is constructed from a difference matrix K and source vector J on the graph, as in lattice gauge theory. K is constructed from graphical relations so that it contains a non-trivial null space (whence gauge invariance), and J is then restricted to the column space of K which ensures it is distributed in a divergence-free fashion over the spacetime defined by the element. This rule for the relational construct of K and J is our proposed fundamental axiom of physics and results in a self-consistency relationship between sources, the spacetime metric, and the stress-energy-momentum content of the element, rather than a dynamical law for time-evolved entities. In its most general form, the set of fundamental elements employed by lattice gauge theory contains scalar fields on nodes and links, and vector fields on nodes. To complete the fundamental set (unification in this view), we propose the addition of scalar fields on plaquettes (basis for graviton) and vector fields on links. We use this approach via modified Regge calculus to correct proper distance in the Einstein-deSitter cosmology model yielding a fit of the Union2 Compilation supernova data that matches ΛCDM without having to invoke accelerating expansion or dark energy.

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1. Introduction

1.1 Overview. In this paper, we introduce our adynamical, background independent approach to quantum gravity (QG) and the unification of physics. This approach is based in and motivated by our foundations-driven account of quantum physics called *Relational Blockworld*(1) (RBW) whereby the fundamental elements[1] of Nature are graphical units of space, time and sources[2]. Accordingly, the spacetime metric and source of each graphical element are co-determining, so there is no "background spacetime" connoting existence independent of matter-energy-momentum. To put it simply, these are elements *of* space, time and sources, not source elements *in* space and time. These graphical amalgams of "spacetimesource" are our beables. Are such beables local?

There has been a great deal of hand-wringing lately in the foundational literature on quantum gravity as to whether the most fundamental unifying theory from which spacetime emerges, must have local beables to be empirically coherent and make full correspondence with higher-level physical theories and the experienced world(2). Maudlin notes that(3) "local beables do not merely exist: they exist somewhere," or as Bell puts it(4), beables are "definitely associated with particular space-time regions." We share the consensus view that a successful theory of quantum gravity need not have local beables(5). Of course there is less consensus about the necessary and sufficient conditions for being a local beable, and that discussion is beyond the scope of this paper. To return to the main question about the status of spacetimesources, local beables are thought of as being separate from but located somewhere *in* spacetime, whereas, again, spacetimesources are *of* space, time and sources. That said, the various source values (observables) of our fundamental elements are certainly localized on the graphs. As will become clear, we recover a modified general relativity (and thus modified classical spacetime) in a way that makes clear why ordinary general relativity works as well as it does. Concerning the locality of beables, Einstein writes(6)

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[1] By "element" we mean "2D rectangle" or its 3D or 4D counterparts. We do not imply a meta-temporal process, nor do we conflate computational algorithms with the notion of an "evolving Now" in the blockworld.

[2] We use the word "source" in formal analogy to quantum field theory where it means "particle sources" or "particle sinks" (creation or annihilation events, respectively). When we want to specify "a source of particles" we will use "Source." Strictly speaking, our "source" is not so localized, but rather reflects a divergence-free property of the graphical element responsible for some property of a trans-temporal object.

> ..if one asks what is characteristic of the realm of physical ideas independently of the quantum theory, then above all the following attracts our attention: the concepts of physics refer to a real external world, i.e. ideas are posited of things that claim a `real existence' independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the other hand, brought into as secure a relationship as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived of as being arranged in a spacetime continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things ‘lie in different parts of space’. Without such an assumption of mutually independent existence (the ‘being-thus’) of spatially distant things, an assumption which originates in everyday thought, physical thought in the sense familiar to us would not be possible....

Einstein appears to conflate (or at least highlight) several different notions of “local” in this passage, including, (1) local as localized in spacetime, (2) local as possessing primitive thisness with intrinsic properties, (3) local as in no faster than light interactions and (4) local as in being otherwise independent (e.g., statistically) of entities at other points in spacetime. Our beables are local in the first and third sense.

The transition amplitude for these elements of “spacetimesource” is computed using a path integral with discrete Gaussian graphical action, i.e., a particle Source and sink are inseparably and relationally co-constructed. The action for a spacetimesource element is constructed from a difference matrix $\vec{\vec{K}}$ for field gradients on the graph and source vector $\vec{J}$ on the graph, as in lattice gauge theory (LGT). $\vec{\vec{K}}$ is constructed from graphical relations so that it contains a non-trivial null space (whence gauge invariance), and $\vec{J}$ is then restricted to the column space of $\vec{\vec{K}}$ which ensures it is distributed in a divergence-free fashion over the spacetime defined by the element. This rule for the relational construct of $\vec{\vec{K}}$ and $\vec{J}$ is our proposed fundamental axiom of physics and results in a self-consistency relationship between sources, the spacetime metric, and the stress-energy-momentum content of the element, so it is referred to as the “self-consistency criterion” (SCC).

Essentially, first, we’re assuming quantum field theory (QFT) is an approximation of LGT, which is the opposite of conventional thinking. Second, we’re underwriting its fundamental computational element, i.e., the free field transition amplitude, in a relational and adynamical fashion. More significantly, third, since LGT is the fundamental theory and it involves differences rather than derivatives, we are assuming that the size of spacetimesource elements can be as small or large as the situation requires. If not for these three differences, what we're proposing would be simplistic and naïve of course, since the free field amplitude is trivial. However, these changes provide obvious computational advantages without introducing empirical restrictions, since LGT works well using a cutoff length at the smallest experimental scales (those for QCD), and in the free field regime element size is irrelevant (except for short-lived particles). And, we find these changes discharge the technical and conceptual difficulties of QFT and quantum mechanics (QM) while leaving their computational structures and empirical successes intact, for all practical purposes. For example, the flexibility in element size provides an adynamical explanation of twin-slit interference (section 3.4) and a novel solution to the dark energy problem (section 5.2). We expect of course that this view of fundamental physics will suggest new experiments in other areas, as well. We will only briefly touch on such issues here, leaving that task for another venue, but most consequences will be obvious to the reader familiar with quantum physics. The focus of this paper will be on explaining how our proposed fundamental axiom of physics changes the approach and goals of unification and quantum gravity while vindicating the progress made to date on the Standard Model of particle physics[3].

In short, our fundamental axiom of theory X[4] does not involve a group structure that subsumes U(1) x SU(2) x SU(3) of the Standard Model, nor does it contain fundamental particles. Rather, it is a rule for the co-definition of sources and relations at the most fundamental level of Nature. This "self-consistency criterion" is to theory X as $F = ma$ is to Newtonian mechanics, it dictates the structure of a spacetimesource element. The spacetimesource elements of theory X then *underwrite* the action in the Standard Model and general relativity. Thus, according to theory X, the Standard Model represents

[3] Hereafter simply "the Standard Model."
[4] Here we follow the possibility articulated by Wallace (p 45) that, "QFTs as a whole are to be regarded only as approximate descriptions of some as-yet-unknown deeper theory," which he calls "theory X." Wallace, D.: In defence of naiveté: The conceptual status of Lagrangian quantum field theory. Synthese 151, 33-80 (2006). Our use of the term "theory X" herein refers exclusively to our particular version.

myriad and increasingly complex *applications* of the fundamental structure. That's why the Lagrangian density of the Standard Model is quite complex (Figure 14), in stark contrast to the set of fundamental elements per theory X (Figure 13). General relativity (GR) is viewed in analogous fashion in that the terms in the expansion of the Einstein-Hilbert action are not needed for every application, so it too deals with increasingly complex applications of the fundamental structure(7). Therefore, the major questions that need to be answered for unification per theory X, while related to those under investigation in current attempts, are certainly novel by comparison. They will become clear to the reader as the formalism is introduced and we will articulate a few in section 4.

Conceptually, the fundamental structure of theory X is responsible for the worldtubes of trans-temporal objects (TTOs). Accordingly, worldtubes are composed of spacetimesource elements (Figure 1), so that TTOs are understood as spatially distributed collections of sources $\vec{J}$ identified through time in Lorentz invariant fashion. Since these spactimesource elements account for the spatiotemporal distribution of $\vec{J}$, that two worldtubes have some spatial separation means that they must share elements, which entails that they exchange $\vec{J}$, i.e., they interact (Figure 2). Accordingly, LGT has been exploring the myriad forms of $\vec{J}$ needed in the construct of the fundamental elements of spacetimesource, and the manner by which these elements are to be assembled, in order to relationally construct the spatiotemporal distribution of worldtubes that model all observed phenomena. Obviously, LGT is physics that *builds on* theory X and needs to be done. In its most general form, the set of fundamental elements employed by LGT contains scalar fields on nodes and links, and vector fields on nodes. To complete the fundamental set (unification in this view), we propose the addition of scalar fields on plaquettes and vector fields on links. The vector fields on links are parallel transported (for computation of field gradients) via the scalar fields on plaquettes, thus underwriting quantum gravity (which is the standard view of particle physics). This means that Regge calculus (graphical form of general relativity) is understood as the curved assembly of graphical simplices (4D "tetrahedra") with M4 Newtonian gravity, as seen in the Einstein-deSitter (EdS) cosmology model. Each graphical simplex in this model harbors only a scalar field on plaquettes (Newtonian gravity), but a scalar field on links (photon field) is responsible for the distance modulus of supernovae. Correcting the proper distance in this cosmology model accordingly yields a fit[8] of the Union2 Compilation supernova data that matches ΛCDM without having to invoke accelerating expansion or dark energy[9].

1.2 Locality. The manner by which we correct EdS cosmology is a form of "disordered locality," i.e., spacetimesource elements can be arbitrarily large, similar to the situation in quantum graphity[10].Our physical model thus implements a suggestion made by Weinstein among others[11]:

> What I want to do here is raise the possibility that there is a more fundamental theory possessing nonlocal constraints that underlies our current theories. Such a theory might account for the mysterious nonlocal effects currently described, but not explained, by quantum mechanics, and might additionally reduce the extent to which cosmological models depend on finely tuned initial data to explain the large scale correlations we observe. The assumption that spatially separated physical systems are entirely uncorrelated is a parochial assumption borne of our experience with the everyday objects described by classical mechanics. Why not suppose that at certain scales or certain epochs, this independence emerges from what is otherwise a highly structured, nonlocally correlated microphysics?

As he says, every extant fundamental theory of physics assumes the non-existence of such non-local constraints[12]:

> Despite radical differences in their conceptions of space, time, and the nature of matter, all of the physical theories we presently use, non-relativistic and relativistic, classical and quantum, share one assumption: the features of the world at distinct points in space are understood to be independent. Particles may exist anywhere, independent of the location or velocity of other particles. Classical fields may take on any value at a given point, constrained only by local constraints like Gauss's law. Quantum field theories incorporate the same independence in their demand that field operators at distinct points in space commute with one another. The independence of physical properties at distinct points is a theoretical assumption, albeit one that is grounded in our everyday experience. We appear to be able to manipulate the contents of a given region of space unrestricted by the contents of other regions. We can arrange the desk in our office without concern for the location of the couch at home in our living room.

RBW provides an exact model (theory X) in which precisely this type of locality (type 2 and type 4 above) fails to obtain, thereby allowing us to explain a diverse range of phenomena from quantum entanglement to so-called dark energy. Furthermore, as will become clear shortly, the failure of locality in question, the way is it implemented at the bottom in theory X, is consistent with and driven by an appropriately modified GR. Bottom line, there are no space-like continuous worldlines in spacetime.

1.3 Adynamical Explanation. Our approach also differs from common practice (even quantum graphity) in that it is *adynamical*[13]. Carroll sums up nicely what we mean by a dynamical approach[14]:

> Let's talk about the actual way physics works, as we understand it. Ever since Newton, the paradigm for fundamental physics has been the same, and includes three pieces. First, there is the "space of states": basically, a list of all the possible configurations the universe could conceivably be in. Second, there is some particular state representing the universe at some time, typically taken to be the present. Third, there is some rule for saying how the universe evolves with time. You give me the universe now, the laws of physics say what it will become in the future. This way of thinking is just as true for quantum mechanics or general relativity or quantum field theory as it was for Newtonian mechanics or Maxwell's electrodynamics.

Carroll goes on to say that all extant formal models of QG, even those attempting to recover spacetime[15], are dynamical in this sense. While it is true that integral calculus and least action principles have been around for a long time, most assume these methods are formal tricks and not fundamental to dynamical equations. While our adynamical approach employs mathematical formalism akin to dynamical theories, e.g., LGT, we redefine what it means to "explain" something in physics. Rather than finding a rule for time-evolved entities per Carroll (e.g., causal dynamical triangulations[16]), our rule leads to the self-consistency of a graphical spacetime metric and its relationally defined sources. Again, while we do talk about "constructing" or "building" spatiotemporal objects in this paper, we are not implying any sort of "evolving blockworld" as in causet dynamics[17]. Our use of this terminology is merely in the context of a computational algorithm. So, one might ask for example, "Why does link X have metric G and stress-energy tensor T?" A dynamical answer might be, "Because link X-1 has metric G-1 and stress-energy tensor T-1 and the law of evolution thereby dictates that link X has metric G and stress-energy tensor T." Notice how this answer is independent of future boundary conditions; indeed, it's independent of conditions anywhere else on the graph other than those of the 3D hypersurface in the immediate past. Contrast this with an adynamical answer such as, "Because the values G and T on X satisfy the global self-consistency criterion for the graph as a whole." The changes we are proposing to the practice and understanding of quantum physics actually rest largely on our form of adynamical explanation couched in ontic structural realism.

2. Quantum Physics Reconceived: Ontic Structural Realism in a Blockworld

2.1 Dynamism Denied. Our account of spacetime and matter is very much in keeping with Rovelli's intuition that(18):

> General relativity (GR) altered the classical understanding of the concepts of space and time in a way which...is far from being fully understood yet. QM challenged the classical account of matter and causality, to a degree which is still the subject of controversies. After the discovery of GR we are no longer sure of what is space-time and after the discovery of QM we are no longer sure of what matter is. *The very distinction between space-time and matter is likely to be ill-founded*....I think it is fair to say that today we do not have a consistent picture of the physical world. [italics added]

We agree with Rovelli and believe a current obstacle to unification is the lack of a true marriage of spacetime with matter. That is, we believe one of the main obstacles to unification has been a form of 'spacetime-matter dualism' whereby the spacetime metric (or simply "metric") is subject to quantization distinct from the matter and gauge fields. This view is carried over from QFT and GR. In QFT, although matter-energy fields are imagined to pervade space, the metric is independent of the matter-energy content of spacetime. And, although Weyl characterized GR as providing *RaumZeitMaterie*(19), there are vacuum solutions in GR, i.e., spacetime regions where the stress-energy tensor is zero. Thus, neither QFT nor GR embody a true unity of "spacetimematter" and both employ a differentiable manifold structure for spacetime[5]. Herein we propose unification based on a true unity of space, time and sources, finishing Einstein's dream so to speak.

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[5] For an overview of problems associated with "the manifold conception of space and time" in quantum gravity see Butterfield, J., & Isham, C.J.: Spacetime and the Philosophical Challenge of Quantum Gravity (1999) http://arxiv.org/abs/gr-qc/9903072.

Fundamental theories of physics (e.g., M-theory, loop quantum gravity, causets) may deviate from the norm by employing radical new fundamental entities (e.g., branes, loops, ordered sets), but the game is always dynamical, broadly construed (e.g., vibrating branes, geometrodynamics, sequential growth process). As Healey puts it[20]:

> Physics proceeds by first analyzing the phenomena with which it deals into various kinds of systems, and then ascribing states to such systems. To classify an object as a certain kind of physical system is to ascribe certain, relatively stable, qualitative intrinsic properties; and to further specify the state of a physical system is to ascribe to it additional, more transitory [time dependent], qualitative intrinsic properties….A physical property of an object will then be both qualitative and intrinsic just in case its possession by that object is wholly determined by the underlying physical states and physical relations of all the basic systems that compose that object.

Dynamism then encompasses three claims: (A) the world, just as appearances and the experience of time suggest, evolves or changes in time in some objective fashion, (B) the best explanation for A will be some dynamical law that "governs" the evolution of the system in question, and (C) the fundamental entities in a "theory of everything" will themselves be dynamical entities evolving in some space however abstract, e.g., Hilbert space. Our model rejects not only tenets A and B of dynamism, but also C. In our view *entities* or *things* are not fundamental and, in fact, it is in accord with ontic structural realism[21] (OSR):

> Ontic structural realists argue that what we have learned from contemporary physics is that the nature of space, time and matter are not compatible with standard metaphysical views about the ontological relationship between individuals, intrinsic properties and relations. On the broadest construal OSR is any form of structural realism based on an ontological or metaphysical thesis that inflates the ontological priority of structure and relations.

More specifically, our version of OSR (RBW[22]) agrees that[23] "The relata of a given relation always turn out to be relational structures themselves on further analysis." Note that OSR does not claim there are relations without relata, just that the relata are not individuals (e.g., things with primitive thisness and intrinsic properties), but always ultimately analyzable as relations as well (Figure 2). OSR already violates the dynamical bias by rejecting *things* with intrinsic properties and their dynamics as fundamental *building blocks* of reality – the world isn't fundamentally *compositional* – the deepest conception of reality is not one in which we decompose things into other things at ever smaller length and time scales.

A good deal of the literature on OSR is driven by philosophical concerns about scientific realism and intertheoretic relations, rather than motivated by physics itself(24). There has also been much debate in the philosophical literature as to whether OSR provides any real help in resolving foundational issues of physics such as interpreting quantum mechanics or in advancing physics itself. Consider the following claims for example:

> OSR is not an interpretation of QM in addition to many worlds-type interpretations, collapse-type interpretations, or hidden variable-type interpretations. As the discussion of the arguments for OSR from QM in section 2 above has shown, OSR is not in the position to provide on its own an ontology for QM, since it does not reply to the question of what implements the structures that it poses. In conclusion, after more than a decade of elaboration and debate on OSR about QM, it seems that the impact that OSR can have on providing an answer to the question of what the world is like, if QM is correct, is rather limited. From a scientific realist perspective, the crucial issue is the assessment of the pros and cons of the various detailed proposals for an ontology of QM, as it was before the appearance of OSR on the scene(25).
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> While the basic idea defended here (a fundamental ontology of brute relations) can be found elsewhere in the philosophical literature on 'structural realism', we have yet to see the idea used as an argument for advancing physics, nor have we seen a truly convincing argument, involving a real construction based in modern physics, that successfully evades the objection that there can be no relations without first (in logical order) having things so related(26).

As this paper will attest, theory X is a counter-example to Esfeld's claim and it provides exactly the physical model that Rickles is looking for. As Rickles says in the following passage, OSR has the potential to re-ground physics, dissolve current quagmires and lead to new physics(27):

> Viewing the world as structurally constituted by primitive relations has the potential to lead to new kinds of research in physics, and knowledge of a more stable sort. Indeed, in the past those theories that have adopted a broadly similar approach (along the lines of what Einstein labeled 'principle theories') have led to just the kinds of advances that this essay competition seeks to capture: areas "where thinkers were 'stuck' and had to let go of some cherished assumptions to make progress." Principle theory approaches often look to general 'structural

> aspects' of physical behaviour over 'thing aspects' (what Einstein labeled 'constructive'), promoting invariances of world-structure to general principles.

Rickles laments the fact that OSR has yet to be so motivated. He further anticipates theory X almost perfectly when he says[28]:

> The position I have described involves the idea that physical systems (which I take to be characterized by the values for their observables) are exhausted by extrinsic or relational properties: they have no intrinsic, local properties at all! This is a curious consequence of background independence coupled with gauge invariance and leads to a rather odd picture in which objects and [spacetime] structure are deeply entangled. Inasmuch as there are objects at all, any properties they possess are structurally conferred: they have no reality outside some correlation. What this means is that the objects don't *ground* structure, they are nothing independently of the structure, which takes the form of a (gauge invariant) correlation between (non-gauge invariant) field values. With this view one can both evade the standard 'no relations without relata' objection and the problem of accounting for the appearance of time (in a timeless structure) in the same way.

In this paper we provide physics that embodies Rickles' suggestion. Broadly speaking, we relate gauge invariance, gauge fixing, divergence-free sources, and relationally defined trans-temporal objects in an adynamic, graphical fashion. Specifically speaking, each row of our difference matrix $\bar{\bar{K}}$ for field gradients in the action for our spacetimesource element is a vector constructed relationally via the connectivity of some graphical element, i.e., nodes connected by links, links connected by plaquettes, or plaquettes connected by cubes. Thus, $\bar{\bar{K}}$ might rather be called the "relations matrix." Since each vector is relationally defined, its components sum to zero, which means [111…] is a null eigenvector of $\bar{\bar{K}}$. Our SCC then demands that the source vector $\vec{J}$ in the action for our spacetimesource element reside in the column space of $\bar{\bar{K}}$, so that it is orthogonal to [111…] which means its components sum to zero, i.e., it is divergence-free. A divergence-free source in each spacetimesource element then underwrites relationally defined, spatially distributed, trans-temporally identified properties, i.e., it provides the fundamental element for relationally defined trans-temporal objects per OSR. That $\bar{\bar{K}}$ possesses a non-trivial null space is the graphical equivalent of gauge invariance and restricting $\vec{J}$ to the column space of $\bar{\bar{K}}$ provides a natural gauge fixing, i.e., restricting the path integral of the transition amplitude to the column space of $\bar{\bar{K}}$. That $\bar{\bar{K}}$ possesses a non-trivial null space also means the determinant of $\bar{\bar{K}}$ is zero, so the set of vectors constituting the rows of $\bar{\bar{K}}$ is not linearly independent. That some subset of these vectors is determined by its complement follows from having the graphical set relationally constructed. Thus, divergence-free $\bar{J}$ follows from relationally defined $\bar{\bar{K}}$ as a consequence of our fundamental axiom of physics, i.e., the SCC.

2.2 Blockworld. As stated, we must further exacerbate this violation of dynamism by applying OSR to a blockworld. The blockworld perspective (the reality of all events past, present and future including the outcomes of quantum experiments) is suggested for example by the relativity of simultaneity in special relativity or, more generally, the lack of a preferred spatial foliation of spacetime in GR, and even by quantum entanglement according to some of us[29]. Geroch writes[30]:

> There is no dynamics within space-time itself: nothing ever moves therein; nothing happens; nothing changes. In particular, one does not think of particles as moving through space-time, or as following along their world-lines. Rather, particles are just in space-time, once and for all, and the world-line represents, all at once, the complete life history of the particle.

When Geroch says that "there is no dynamics within space-time itself," he is not denying that the mosaic of the blockworld possesses patterns that can be described with dynamical laws. Nor is he denying the predictive and explanatory value of such laws. Rather, given the reality of all events in a blockworld, dynamics are not "event factories" that bring heretofore non-existent events (such as measurement outcomes) into being; fundamental dynamical laws that are allegedly responsible for discharging fundamental "why" questions in physics are not brute unexplained explainers that "produce" events on our view. Geroch is advocating for what philosophers call Humeanism about laws. Namely, the claim is that relatively fundamental dynamical laws are *descriptions of regularities* and not the *brute explanation* for such regularities. His point is that in a blockworld, Humeanism about laws is an obvious position to take because everything is just "there" from a "God's eye" (Archimedean) point of view.

In addition there is the problem of time in canonical general relativity. That is, in a particular Hamiltonian formulation of GR the reparametrization of spacetime is a gauge symmetry. Therefore, all genuinely physical magnitudes are constants of motion, i.e., they don't change over time. In short, change is merely a redundancy of the representation.

Finally, the problem of frozen time in canonical QG is that if the canonical variables of the theory to be quantized transform as scalars under time reparametrizations, which is true in practice because they have a simple geometrical meaning, then[(31)] "the Hamiltonian is (weakly) zero for a generally covariant system." The result upon canonical quantization is the famous Wheeler-DeWitt equation, void of time evolution. While it is too strong to say a generally covariant theory must have $H = 0$, there is no well-developed theory of quantum gravity that has avoided it to date[(32)]. It is supremely ironic that the dynamism and unificationism historically driving physics led us directly to blockworld and frozen time.

Rickles notes that the problem of time can be solved by[(33)], "(1) global quantities defined over the whole spacetime and (2) 'relational' quantities built out of correlations between field values and/or invariants. There seems to be some consensus forming that the latter type are the way to go, and these will serve as the appropriate vehicle for defining time in an unchanging mathematical structure, as well as defining the structures themselves." Theory X, it will become clear, provides a solution precisely in terms of number 2.

We think therefore that both quantum mechanics, e.g., delayed-choice experiments, and relativity are telling us that it is a block universe, so it is time to promote this idea from mere metaphysics to physics. This is what RBW does.

2.3 OSR in a Blockworld. Putting it all together, reality is a blockworld best characterized as spacetimesource, as opposed to the "spacetime + sources" picture of current physics. In the foundations literature on the eternalism debate and the structural realism debate respectively, the biggest complaint is that the fate of these topics makes no real difference for physics itself, i.e., it does not lead to new models, new insights, or new predictions and it does not resolve conceptual problems. In short, the complaint is that such debates are nothing but pure metaphysics. We, however, actually do provide a new formal model for fundamental physics based on blockworld with relationally defined sources that has all the aforementioned virtues and the fundamental axiom for physics per this theory X is an adynamical self-consistency criterion.

2.4 Self-Consistency Criterion. Our use of a self-consistency criterion is not without precedent, as we already have an ideal example in Einstein's equations of GR

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Momentum, force and energy all depend on spatiotemporal measurements (tacit or explicit), so the stress-energy tensor cannot be constructed without tacit or explicit knowledge of the spacetime metric (technically, the stress-energy tensor can be written as the functional derivative of the matter-energy Lagrangian with respect to the metric). But, if one wants a "dynamic spacetime" in the parlance of GR, the spacetime metric must depend on the matter-energy distribution in spacetime. GR solves this dilemma by demanding the stress-energy tensor be "consistent" with the spacetime metric per Einstein's equations[6]. This self-consistency hinges on divergence-free sources, which finds a mathematical counterpart in $\partial\partial = 0$, i.e., the boundary of a boundary principle[(34)]. So, Einstein's equations of GR are a mathematical articulation of the boundary of a boundary principle at the classical level, i.e., they constitute a self-consistency criterion at the classical level. In fact, our SCC is based on the same topological maxim ($\partial\partial = 0$) for the same reason, as is the case with quantum and classical electromagnetism[(35)].

[6] Concerning the stress-energy tensor, Hamber and Williams write, "In general its covariant divergence is not zero, but consistency of the Einstein field equations demands $\nabla^{\alpha} T_{\alpha\beta} = 0$," Hamber, H.W., & Williams, R.: Nonlocal Effective Gravitational Field Equations and the Running of Newton's G (2005) http://arxiv.org/pdf/hep-th/0507017.pdf

3. Underwriting the Free Field Transition Amplitude

3.1 Boundary of a Boundary Principle. In Figure 3, the boundary of plaquette $\mathbf{p}_1$ is given by links $\mathbf{e}_4 + \mathbf{e}_5 - \mathbf{e}_2 - \mathbf{e}_1$, which also provides an orientation. The boundary of $\mathbf{e}_1$ is given by vertices $\mathbf{v}_2 - \mathbf{v}_1$, which likewise provides an orientation. Using these conventions for the orientations of links and plaquettes we have the following boundary operator for $C_2 \rightarrow C_1$, i.e., space of plaquettes mapped to space of links in the spacetime chain complex:

$$\partial_2 = \begin{bmatrix} -1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \quad (1)$$

The first column is simply the links for the boundary of $\mathbf{p}_1$ and the second column is simply the links for the boundary of $\mathbf{p}_2$. We have the following boundary operator for $C_1 \rightarrow C_0$, i.e., space of links mapped to space of vertices in the spacetime chain complex:

$$\partial_1 = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad (2)$$

which completes the spacetime chain complex, $C_0 \xleftarrow{\partial_1} C_1 \xleftarrow{\partial_2} C_2$. The columns are simply the vertices for the boundaries of the edges or conversely, each row shows which links leave (-1) or enter (1) each node. These boundary operators satisfy $\partial_1\partial_2 = 0$ as required by the boundary of a boundary principle.

3.2 Graphical Harmonic Oscillator and the SCC. The Lagrangian for the coupled masses of Figure 4 is

$$L = \frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}k(q_1 - q_2)^2 \tag{3}$$

so our transition amplitude is ($\hbar = 1$)

$$Z = \int Dq(t)\exp\left[i\int_0^T dt\left[\frac{1}{2}m\dot{q}_1^2 + \frac{1}{2}m\dot{q}_2^2 - \frac{1}{2}kq_1^2 - \frac{1}{2}kq_2^2 + kq_1q_2 + J_1q_1 + J_2q_2\right]\right] \tag{4}$$

giving

$$\vec{\vec{K}} = \begin{bmatrix} \left(\frac{m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k\Delta t & 0 & 0 \\ \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 & k\Delta t & 0 \\ 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} - k\Delta t\right) & 0 & 0 & k\Delta t \\ k\Delta t & 0 & 0 & \left(\frac{m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} & 0 \\ 0 & k\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{2m}{\Delta t} - k\Delta t\right) & \frac{-m}{\Delta t} \\ 0 & 0 & k\Delta t & 0 & \frac{-m}{\Delta t} & \left(\frac{m}{\Delta t} - k\Delta t\right) \end{bmatrix} \tag{5}$$

on the graph of Figure 3. The eigenvalues are $0, -2k\Delta t, \frac{m}{\Delta t}, 3\frac{m}{\Delta t}, \frac{m}{\Delta t} - 2k\Delta t, 3\frac{m}{\Delta t} - 2k\Delta t$ and the null space (space of eigenvalues 0) contains eigenvector [111111]. The space orthogonal to the null space of $\vec{\vec{K}}$ is called the column space of $\vec{\vec{K}}$. Therefore, any source vector $\vec{J}$ in the column space of $\vec{\vec{K}}$ has components which sum to zero and this is referred to in graphical approaches to physics as "divergence-free $\vec{J}$." If $\vec{J}$ is a force, this simply reflects Newton's third law. If $\vec{J}$ is energy, this simply reflects conservation of energy. We will use $\vec{J}$ on spacetimesource elements to underwrite conserved properties defining TTOs, so we require that $\vec{J}$ reside in the column space of $\vec{\vec{K}}$. Thus, $\vec{\vec{K}}$ must be constructed so as to possess a column space and non-trivial null space, which is the graphical equivalent of gauge invariance. As we shall see, this fundamental, adynamical rule results in a self-consistency relationship between sources, the spacetime metric, and dynamical properties such as mass, energy, and momentum, so we call it a "self-consistency criterion" (SCC). That explains the role $\vec{J}$ plays in the SCC, now we explain the graphical construction of $\vec{\vec{K}}$.

Giving weights to the links of Figure 3 to give Figure 5 we have the following boundary operator on Figure 5

$$\partial_1 = \begin{bmatrix} -\sqrt{\frac{m}{\Delta t}} & 0 & 0 & -\sqrt{-k\Delta t} & 0 & 0 & 0 \\ \sqrt{\frac{m}{\Delta t}} & -\sqrt{-k\Delta t} & -\sqrt{\frac{m}{\Delta t}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{m}{\Delta t}} & 0 & 0 & 0 & -\sqrt{-k\Delta t} \\ 0 & 0 & 0 & \sqrt{-k\Delta t} & -\sqrt{\frac{m}{\Delta t}} & 0 & 0 \\ 0 & \sqrt{-k\Delta t} & 0 & 0 & \sqrt{\frac{m}{\Delta t}} & -\sqrt{-k\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{-k\Delta t} & \sqrt{-k\Delta t} \end{bmatrix} \tag{6}$$

constructed analogously to Eq (2). One then finds *a la* Wise[36] that $\vec{\vec{K}} = \partial_1 \partial_1^T$. One can also read off the rows of $\vec{\vec{K}}$ by noting that row 1 says links of weight $\frac{m}{\Delta t}$ and $-k\Delta t$ are connecting nodes 1, 2 and 4, respectively. All other rows can be read off the same way. Either way, $\vec{\vec{K}}$ is understood to be constructed via graphical relations, so it might be called the "relations matrix."

The SCC is our proposed fundamental axiom of physics, as its status in theory X is akin to Newton's laws of motion or Einstein's equations of GR. Just as Newton's second law co-defines force and mass, and Einstein's equations co-define the spacetime metic and stress-energy tensor, the SCC co-defines relations and sources at the most fundamental level of Nature. We will provide examples in this section for the Schrödinger, Klein-Gordon, Dirac, Maxwell, and Einstein-Hilbert actions.

Now that we have explained our SCC, our choice of gauge fixing is obvious. The discrete, graphical counterpart to Eq (4) is

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} dQ_1 \ldots dQ_N \exp\left[i\frac{1}{2}\vec{Q}\cdot\vec{\vec{K}}\cdot\vec{Q} + i\vec{J}\cdot\vec{Q}\right] \tag{7}$$

with solution

$$Z = \left(\frac{(2\pi i)^N}{\det(K)}\right)^{1/2} \exp\left[-i\frac{1}{2}\vec{J}\cdot\vec{\vec{K}}^{-1}\cdot\vec{J}\right] \tag{8}$$

However, $\vec{\vec{K}}^{-1}$ does not exist because $\vec{\vec{K}}$ has a non-trivial null space. This is the graphical characterization of the effect of gauge invariance on the computation of *Z*. Because we require that $\vec{J}$ reside in the column space of $\vec{\vec{K}}$, the graphical counterpart to Fadeev-Popov gauge fixing is obvious, i.e., we simply restrict our path integral to the column space of $\vec{\vec{K}}$. Nothing of physical interest lies elsewhere, so this is a natural choice. In the eigenbasis of $\vec{\vec{K}}$ with our gauge fixing Eq (7) becomes

$$Z = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} d\widetilde{Q}_2 \ldots d\widetilde{Q}_N \exp\left[\sum_{n=2}^{N}\left(i\frac{1}{2}\widetilde{Q}_n^2 a_n + i\widetilde{J}_n\widetilde{Q}_n\right)\right] \tag{9}$$

where $\widetilde{Q}_n$ are the coordinates associated with the eigenbasis of $\vec{\vec{K}}$ and $\widetilde{Q}_1$ is associated with eigenvalue zero, a_n is the eigenvalue of $\vec{\vec{K}}$ corresponding to $\widetilde{Q}_n$, and $\widetilde{J}_n$ are the components of $\vec{J}$ in the eigenbasis of $\vec{\vec{K}}$. Our gauge independent approach revises Eq. (8) to give

$$Z = \left(\frac{(2\pi i)^{N-1}}{\prod_{n=2}^{N} a_n}\right)^{1/2} \prod_{n=2}^{N} \exp\left[-i\frac{\widetilde{J}_n^2}{2a_n\hbar}\right] \tag{10}$$

Thus, we find that the self-consistent co-construction of space, time and divergence-free sources entails gauge invariance and gauge fixing. After quickly checking the general structure for unweighted scalar fields on the hypercube, we will apply this idea to the Schrödinger, Klein-Gordon, Dirac, Maxwell, and Einstein-Hilbert actions.

3.3 Unweighted Scalar Fields on the Hypercube. We now provide $\vec{\vec{K}}1 = \partial_1\partial_1^T$, $\vec{\vec{K}}2 = \partial_2\partial_2^T$, $\vec{\vec{K}}3 = \partial_3\partial_3^T$, the eigenvalues for each $\vec{\vec{K}}$, and the structure of the column space for each $\vec{\vec{K}}$ on the hypercube (Figure 10) with unweighted links, plaquettes and cubes. These boundary operators satisfy $\partial_n\partial_{n+1} = 0$. We have for $\vec{\vec{K}}1 = \partial_1\partial_1^T$ (note that there are 16 nodes, the nodal numbering system does not use 9 and 10 for obvious reasons, as you can see in Figure 10):

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{18}
v_1	4	−1	−1	0	−1	0	0	0	−1	0	0	0	0	0	0	0
v_2	−1	4	0	−1	0	−1	0	0	0	−1	0	0	0	0	0	0
v_3	−1	0	4	−1	0	0	−1	0	0	0	−1	0	0	0	0	0
v_4	0	−1	−1	4	0	0	0	−1	0	0	0	−1	0	0	0	0
v_5	−1	0	0	0	4	−1	−1	0	0	0	0	0	−1	0	0	0
v_6	0	−1	0	0	−1	4	0	−1	0	0	0	0	0	−1	0	0
v_7	0	0	−1	0	−1	0	4	−1	0	0	0	0	0	0	−1	0
v_8	0	0	0	−1	0	−1	−1	4	0	0	0	0	0	0	0	−1
v_{11}	−1	0	0	0	0	0	0	0	4	−1	−1	0	−1	0	0	0
v_{12}	0	−1	0	0	0	0	0	0	−1	4	0	−1	0	−1	0	0
v_{13}	0	0	−1	0	0	0	0	0	−1	0	4	−1	0	0	−1	0
v_{14}	0	0	0	−1	0	0	0	0	0	−1	−1	4	0	0	0	−1
v_{15}	0	0	0	0	−1	0	0	0	−1	0	0	0	4	−1	−1	0
v_{16}	0	0	0	0	0	−1	0	0	0	−1	0	0	−1	4	0	−1
v_{17}	0	0	0	0	0	0	−1	0	0	0	−1	0	−1	0	4	−1
v_{18}	0	0	0	0	0	0	0	−1	0	0	0	−1	0	−1	−1	4

The eigenvalues are {8,6,6,6,6,4,4,4,4,4,4,2,2,2,2,0} and the null space is span{[111…]}, which we know from the fact that the rows of $\vec{\vec{K}}$ sum to zero. The SCC then means $\vec{J}$ sums to zero globally (all 16 nodes).

We have for $\bar{\bar{K}}2 = \partial_2 \partial_2^T$:

3	-1	-1	-1	1	1	1	-1	0	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
-1	3	-1	-1	-1	0	0	1	1	1	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
-1	-1	3	-1	0	-1	0	0	-1	0	0	0	1	1	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
-1	-1	-1	3	0	0	-1	0	0	-1	0	0	0	0	-1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
1	-1	0	0	3	-1	-1	-1	0	0	1	1	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
1	0	-1	0	-1	3	-1	0	0	0	-1	0	-1	0	0	1	1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
1	0	0	-1	-1	-1	3	0	0	0	0	-1	0	0	0	0	-1	0	0	0	-1	0	0	1	1	0	0	0	0	0	0	0
-1	1	0	0	-1	0	0	3	-1	-1	1	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	1	-1	0	0	0	0	-1	3	-1	-1	0	0	-1	0	0	0	1	1	0	0	0	0	0	0	0	-1	0	0	0	0	0
0	1	0	-1	0	0	0	-1	-1	3	0	-1	0	0	0	0	0	0	-1	0	0	-1	0	0	0	1	1	0	0	0	0	0
0	0	0	0	1	-1	0	1	-1	0	3	-1	0	0	0	-1	0	-1	0	1	0	0	0	0	0	0	0	-1	0	0	0	0
0	0	0	0	1	0	-1	1	0	-1	-1	3	0	0	0	0	0	0	0	-1	0	0	0	-1	0	-1	0	1	0	0	0	0
-1	0	1	0	0	-1	0	0	0	0	0	0	3	-1	-1	1	1	-1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
0	-1	1	0	0	0	0	0	-1	0	0	0	-1	3	-1	-1	0	1	1	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	1	-1	0	0	0	0	0	0	0	0	-1	-1	3	0	-1	0	-1	0	0	0	-1	0	0	0	0	0	1	1	0	0
0	0	0	0	-1	1	0	0	0	0	-1	0	1	-1	0	3	-1	-1	0	1	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	1	-1	0	0	0	0	0	1	0	-1	-1	3	0	0	-1	0	0	0	0	-1	0	0	0	-1	0	1	0
0	0	0	0	0	0	0	-1	1	0	-1	0	-1	1	0	-1	0	3	-1	1	0	0	0	0	0	0	0	0	0	0	0	-1
0	0	0	0	0	0	0	0	1	-1	0	0	0	1	-1	0	0	-1	3	-1	0	0	0	0	0	0	-1	0	0	-1	0	1
0	0	0	0	0	0	0	0	0	0	1	-1	0	0	0	1	-1	1	-1	3	0	0	0	0	0	0	0	-1	0	0	-1	-1
-1	0	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	3	-1	-1	1	1	-1	0	0	-1	0	0	0
0	-1	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1	3	-1	-1	0	1	1	0	0	-1	0	0
0	0	-1	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	-1	-1	3	0	-1	0	-1	0	1	1	0	0
0	0	0	0	-1	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	-1	0	3	-1	-1	0	1	0	0	-1	0
0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	-1	-1	3	0	0	-1	-1	0	1	0
0	0	0	0	0	0	0	-1	0	1	0	-1	0	0	0	0	0	0	0	0	-1	1	0	-1	0	3	-1	1	0	0	0	-1
0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	0	-1	0	0	1	-1	0	0	-1	3	-1	0	-1	0	1
0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	0	0	0	0	-1	0	0	0	1	-1	1	-1	3	0	0	-1	-1
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	0	-1	0	0	0	-1	0	1	0	-1	0	0	0	3	-1	1	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	-1	0	0	-1	1	0	0	0	-1	0	-1	3	-1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	-1	0	0	0	-1	1	0	0	-1	1	-1	3	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	0	0	0	0	0	-1	1	-1	-1	1	-1	3

The eigenvalues are {8,8,8,6,6,6,6,6,6,6,6,4,4,4,4,4,4,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}. The null space contains [111…], since the rows of $\bar{\bar{K}}$ sum to zero. The SCC then means $\bar{J}$ sums to zero globally (all 32 links). The null space contains vectors which correspond to $\bar{J}$ conserved on the links at each node (Figure 11). Thus, we can understand the 17 dimensions of the column space as resulting from degrees of freedom. If we specify $\bar{J}$ on all 12 links of the "inner" cube, all the time-like links connecting the "inner" cube to the "outer" cube are determined by local conservation. Then if you specify the 4 link values on one face of the "outer" cube, local conservation leaves only one free link to specify on the opposite face, 12 + 4 + 1 = 17.

We have for $\vec{\vec{K}}3 = \partial_3 \partial_3^T$:

2	-1	-1	1	1	0	-1	-1	0	1	1	0	0	-1	0	0	0	0	-1	0	0	0	0	0
-1	2	-1	-1	0	1	1	0	-1	-1	0	0	0	1	1	0	0	0	0	-1	0	0	0	0
-1	-1	2	0	-1	-1	0	1	1	0	-1	0	0	0	-1	0	0	0	1	1	0	0	0	0
1	-1	0	2	-1	1	-1	0	0	1	0	-1	0	-1	0	1	0	0	0	0	-1	0	0	0
1	0	-1	-1	2	-1	0	-1	0	0	1	1	0	0	0	-1	0	0	-1	0	1	0	0	0
0	1	-1	1	-1	2	0	0	-1	0	0	-1	0	0	1	1	0	0	0	-1	-1	0	0	0
-1	1	0	-1	0	0	2	-1	1	-1	0	0	-1	1	0	0	1	0	0	0	0	-1	0	0
-1	0	1	0	-1	0	-1	2	-1	0	-1	0	1	0	0	0	-1	0	1	0	0	1	0	0
0	-1	1	0	0	-1	1	-1	2	0	0	0	-1	0	-1	0	1	0	0	1	0	-1	0	0
1	-1	0	1	0	0	-1	0	0	2	-1	1	-1	-1	0	0	0	1	0	0	0	0	-1	0
1	0	-1	0	1	0	0	-1	0	-1	2	-1	1	0	0	0	0	-1	-1	0	0	0	1	0
0	0	0	-1	1	-1	0	0	0	1	-1	2	-1	0	0	-1	0	1	0	0	1	0	-1	0
0	0	0	0	0	0	-1	1	-1	-1	1	-1	2	0	0	0	-1	-1	0	0	0	1	1	0
-1	1	0	-1	0	0	1	0	0	-1	0	0	0	2	-1	1	-1	1	0	0	0	0	0	-1
0	1	-1	0	0	1	0	0	-1	0	0	0	0	-1	2	-1	1	-1	0	-1	0	0	0	1
0	0	0	1	-1	1	0	0	0	0	0	-1	0	1	-1	2	-1	1	0	0	-1	0	0	-1
0	0	0	0	0	0	1	-1	1	0	0	0	-1	-1	1	-1	2	-1	0	0	0	-1	0	1
0	0	0	0	0	0	0	0	0	1	-1	1	-1	1	-1	1	-1	2	0	0	0	0	-1	-1
-1	0	1	0	-1	0	0	1	0	0	-1	0	0	0	0	0	0	0	2	-1	1	-1	1	-1
0	-1	1	0	0	-1	0	0	1	0	0	0	0	0	-1	0	0	0	-1	2	-1	1	-1	1
0	0	0	-1	1	-1	0	0	0	0	0	1	0	0	0	-1	0	0	1	-1	2	-1	1	-1
0	0	0	0	0	0	-1	1	-1	0	0	0	1	0	0	0	-1	0	-1	1	-1	2	-1	1
0	0	0	0	0	0	0	0	0	-1	1	-1	1	0	0	0	0	-1	1	-1	1	-1	2	-1
0	0	0	0	0	0	0	0	0	0	0	0	0	-1	1	-1	1	-1	-1	1	-1	1	-1	2

The eigenvalues are {8,8,8,6,6,6,6,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}. The null space contains [111…], since the rows of $\vec{\vec{K}}$ sum to zero. The SCC then means $\vec{J}$ sums to zero globally (all 24 plaquettes). There are vectors in the null space which correspond to $\vec{J}$ conserved on the plaquettes at each link (Figure 12). Thus, we can understand the 7 dimensions of the column space as follows. Start by specifying $\vec{J}$ on the six plaquettes of the "inner" cube. Then local conservation dictates the value of $\vec{J}$ on all plaquettes connecting the "inner" cube to the "outer" cube. That means we need only specify the value of $\vec{J}$ on one plaquette of the "outer" cube and local conservation will dictate the rest of its values on the "outer" cube. We next apply this approach to the free-particle Schrödinger action.

3.4 Non-relativistic Scalar Field on Nodes. The non-relativistic limit of the Klein-Gordon (KG) equation gives the free-particle Schrödinger equation (SE) by factoring out the rest mass contribution to the energy E, assuming the Newtonian form for kinetic energy, and discarding the second-order time derivative[37]. To illustrate the first two steps, plug $\varphi = Ae^{i(px-Et)/\hbar}$ into the KG equation and obtain $\left(-E^2 + p^2c^2 + m^2c^4\right) = 0$, which tells us E is the total relativistic energy. Now plug $\psi = Ae^{i(px-Et)/\hbar}$ into the free-particle SE and obtain $\frac{p^2}{2m} = E$, which tells us E is only the Newtonian kinetic energy. Thus, we must factor out the rest energy of the particle, i.e., $\psi = e^{imc^2t/\hbar}\varphi$, assume the low-velocity limit of the relativistic kinetic energy, and discard the relevant term from our Lagrangian density (leading to the second-order time derivative) in going from φ of the KG equation to ψ of the free-particle SE. We will make these changes to Z for the KG equation and obtain ψ(x,t), which we will then compare to ψ(x,t) from QM to obtain a self-consistency relationship between source and space *a la* Einstein's equations of GR. We will also contrast QM's "mediated" account of twin-slit interference with the adynamical spacetimesource account of our theory X.

For the KG equation we have

$$Z = \int D\varphi \exp\left[i\int d^4x\left[\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}\overline{m}^2\varphi^2 + J\varphi\right]\right] \tag{11}$$

which in (1+1)D is

$$Z = \int D\varphi \exp\left[i\int dxdt\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{c^2}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2 - \frac{1}{2}\overline{m}^2\varphi^2 + J\varphi\right]\right] \tag{12}$$

($\hbar = 1$ and $\overline{m} \equiv \frac{mc^2}{\hbar}$). Making the changes described above with $\psi = e^{i\overline{m}t}\sqrt{2\overline{m}}\varphi$, Eq (12) gives the non-relativistic KG transition amplitude corresponding to the free-particle SE[38]

$$Z = \int D\varphi \exp\left[i\int dxdt\left[i\psi^*\left(\frac{\partial\psi}{\partial t}\right) - \frac{c^2}{2\overline{m}}\left(\frac{\partial\psi}{\partial x}\right)^2 + J\psi\right]\right] \tag{13}$$

In order to obtain the spacetimesource graphical element for Eq (13) we assume a simple four-node graph (Figure 6) so that

$$Z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dQ_2 dQ_3 dQ_4 \exp\left[i\frac{1}{2}\vec{Q}\cdot\vec{\vec{K}}\cdot\vec{Q} + i\vec{J}\cdot\vec{Q}\right] \tag{14}$$

with

$$\vec{\vec{K}} = \begin{bmatrix} \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & im & \frac{\hbar\Delta t}{m\Delta x} & 0 \\ im & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & 0 & \frac{\hbar\Delta t}{m\Delta x} \\ \frac{\hbar\Delta t}{m\Delta x} & 0 & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) & im \\ 0 & \frac{\hbar\Delta t}{m\Delta x} & im & \left(-im - \frac{\hbar\Delta t}{m\Delta x}\right) \end{bmatrix} \tag{15}$$

The eigenvalues of $\vec{\vec{K}}$ are $a_1 = 0$, $a_2 = -\frac{2\hbar\Delta t}{m\Delta x}$, $a_3 = -2im$, and $a_4 = a_2 + a_3$ with eigenvectors $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\1\\-1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}$, respectively. The eigenvectors form the unnormalized H_4 Hadamard matrix[7] and the eigenvalues are consistent with this fact, i.e., 0 and -2 times the off diagonal entries of $\vec{\vec{K}}$. We choose $\vec{J}$ proportional to the unit eigenvector associated with a_2 (since it will give real J^2), which is in keeping with the SCC. Computing Z per Eq (10) and using this as a propagator with a delta function Source we have

$$\psi(x,t) \propto \exp\left[\frac{iJ_o^2}{2\frac{2\hbar t}{mx}\hbar}\right] \tag{16}$$

where J_o is the magnitude of $\vec{J}$ ($\Delta t \to t$ and $\Delta x \to x$ for notational simplicity).

---

[7] All of our Hadamard matrices are unnormalized.

The corresponding QM propagator is obtained via the path integral with action

$$S = \int \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 dt \tag{17}$$

which gives[39]

$$\psi(x,t) = A\sqrt{\frac{m}{2\pi\hbar it}} \exp\left[\frac{imx^2}{2\hbar t}\right] \propto \exp\left[\frac{imx^2}{2\hbar t}\right] \tag{18}$$

with delta function Source $\psi(x,0) = A\delta(x)$. In this view, a particle of mass m is moving through space from Source to detector, so we call this a "mediated" view (as with standard field theoretic accounts).

Comparing the exponents of Eq (16) and Eq (18) we have $J_o^2 = 2\hbar x$. Thus, in GR-like fashion, we obtain a self-consistency relationship between source and space resulting from our fundamental axiom of physics.

Eq (16) is an oscillatory solution like that of Eq (18), so it is easy to see how both results lead to twin-slit interference. However, the results are quite different conceptually. Eq (16) was obtained in spatiotemporally holistic fashion, as we described in section 1 (and Figure 2), and the view of how its amplitudes are combined is shown in Figure 7. By contrast, QM's Eq (18) was obtained dynamically and the view of how its amplitudes are combined is shown in Figure 8. This illustrates nicely that per theory X the interference pattern of the twin-slit experiment does not entail "quantum entities" moving through space as a function of time to "cause" detector events. Rather, interference is understood adynamically via 'competition' between fundamental elements of spacetimesource.

Again, in our view, physics is concerned with explaining the relative spatiotemporal locations of TTOs and physics currently says TTOs are composed of smaller TTOs, i.e., smaller subsets of trans-temporally identified properties (fundamental particles). We propose a more fundamental decomposition of TTOs in terms of spacetimesource elements. Accordingly, quantum physics is telling us something very important about the composition of TTOs, i.e., their properties combine via interference at the level of spacetimesouce elements. We next study the Klein-Gordon action and compare it to the Schrödinger result.

3.5 Scalar Field on Nodes. We now consider Eq (12). The 4-node graph of Figure 9 depicts our spacetimesource element for this case and gives

$$\bar{\bar{K}}=\begin{bmatrix} \left(\frac{\Delta x}{\Delta t}-\frac{c^2\Delta t}{\Delta x}-\overline{m}^2\Delta t\Delta x\right) & -\frac{\Delta x}{\Delta t} & \frac{c^2\Delta t}{\Delta x}+\overline{m}^2\Delta t\Delta x & 0 \\ -\frac{\Delta x}{\Delta t} & \left(\frac{\Delta x}{\Delta t}-\frac{c^2\Delta t}{\Delta x}-\overline{m}^2\Delta t\Delta x\right) & 0 & \frac{c^2\Delta t}{\Delta x}+\overline{m}^2\Delta t\Delta x \\ \frac{c^2\Delta t}{\Delta x}+\overline{m}^2\Delta t\Delta x & 0 & \left(\frac{\Delta x}{\Delta t}-\frac{c^2\Delta t}{\Delta x}-\overline{m}^2\Delta t\Delta x\right) & -\frac{\Delta x}{\Delta t} \\ 0 & \frac{c^2\Delta t}{\Delta x}+\overline{m}^2\Delta t\Delta x & -\frac{\Delta x}{\Delta t} & \left(\frac{\Delta x}{\Delta t}-\frac{c^2\Delta t}{\Delta x}-\overline{m}^2\Delta t\Delta x\right) \end{bmatrix} \tag{19}$$

The eigenvalues of $\bar{\bar{K}}$ are $a_1 = 0$, $a_2 = -2\left(\frac{c^2\Delta t}{\Delta x}+\overline{m}^2\Delta t\Delta x\right)$, $a_3 = \frac{2\Delta x}{\Delta t}$, and $a_4 = a_2 + a_3$ with the same eigenvectors $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\-1\\1\\1\end{bmatrix}$, $\begin{bmatrix}-1\\1\\-1\\1\end{bmatrix}$, and $\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}$, respectively, that we found for the non-relativistic case. Thus, again we have the H_4 Hadamard matrix with eigenvalues of 0 and -2 times the off-diagonal entries of $\bar{\bar{K}}$. As with the non-relativistic case, we choose $\vec{J}$ proportional to the unit eigenvector associated with a_2. In this case, our Z gives (dropping Δ)

$$\varphi(x,t)\propto \exp\left[\frac{iJ_o^2}{4\left(\frac{c^2t}{x}+\overline{m}^2tx\right)\hbar}\right] \tag{20}$$

Again, we wish to compare with the mediated counterpart, so we compare with the two-point correlation function for the free scalar field[40]

$$G(x,t) \propto e^{i\frac{px}{\hbar} - i\frac{Et}{\hbar}} \tag{21}$$

Comparing Eq (20) with Eq (21) we obtain $J_o^2 = 4\left(\frac{c^2 t}{x} + \overline{m}^2 tx\right)(px - Et)$. Here the SCC leads to the self-consistent relationship between source, time, space, mass, momentum, and energy. To see how this reduces to our non-relativistic result, we first reintroduce the scaling factor $\sqrt{\overline{m}}$ so that $a_2 = -2\left(\frac{c^2\Delta t}{\Delta x} + \overline{m}^2 \Delta t \Delta x\right) \to -2\left(\frac{c^2 \Delta t}{\overline{m}\Delta x} + \overline{m}\Delta t \Delta x\right) = -2\left(\frac{\hbar \Delta t}{m \Delta x} + \frac{mc^2 \Delta t \Delta x}{\hbar}\right)$. Then our non-relativistic result follows from $\frac{\hbar t}{mx} + \overline{m}tx \to \frac{\hbar t}{mx}$, $p = m\frac{x}{t}$ and $E = \frac{1}{2}m\left(\frac{x}{t}\right)^2$, as we would expect. We next study the Dirac action and find that it extends the Hadamard structure of the Schrödinger and KG results.

3.6 Vector Field on Nodes. We apply this approach to vector fields on nodes and note that the KG operator for scalar fields is the square of the Dirac operator for vector fields, i.e., $(-i\gamma^\mu \partial_\mu - m)(i\gamma^\mu \partial_\mu - m) = (\partial^2 + m^2)$. In order to construct $\vec{\vec{K}}$ for the Dirac operator on the hypercube of Figure 10 we have the following link weights on *t*, *x*, *y*, and *z* links respectively:

$$T = \begin{bmatrix} \frac{i}{t} - m & 0 & 0 & 0 \\ 0 & \frac{i}{t} & 0 & 0 \\ 0 & 0 & \frac{i}{t} & 0 \\ 0 & 0 & 0 & \frac{i}{t} \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 0 & 0 & \frac{i}{x} \\ 0 & -m & \frac{i}{x} & 0 \\ 0 & \frac{i}{x} & 0 & 0 \\ \frac{i}{x} & 0 & 0 & 0 \end{bmatrix} \tag{22}$$

$$Y=\begin{bmatrix} 0 & 0 & 0 & \frac{i}{y} \\ 0 & 0 & -\frac{i}{y} & 0 \\ 0 & \frac{i}{y} & m & 0 \\ -\frac{i}{y} & 0 & 0 & 0 \end{bmatrix} \qquad Z=\begin{bmatrix} 0 & 0 & \frac{i}{z} & 0 \\ 0 & 0 & 0 & -\frac{i}{z} \\ \frac{i}{z} & 0 & 0 & 0 \\ 0 & -\frac{i}{z} & 0 & m \end{bmatrix}$$

Then the 64 x 64 matrix $\bar{\bar{K}}$ is simply given by:

$$\bar{\bar{K}}=\begin{bmatrix} (-T-X-Y-Z) & Z & Y & 0 & X & \dots \\ \vdots & & & & & \end{bmatrix} \tag{23}$$

This has the same form as $\bar{\bar{K}}$ for the Schrödinger (Eq (15)) and KG (Eq (19) actions. That is, reading across the rows for each node one simply has a collection of the link weights relating the nodes which are connected. Thus, as claimed in section 2, we can understand how $\bar{\bar{K}}$ instantiates graphical relationalism and divergence-free $\vec{J}$ per the SCC as follows.

Each row of $\bar{\bar{K}}$ is a vector constructed relationally via the connectivity of some graphical element, i.e., nodes connected by links, links connected by plaquettes, or plaquettes connected by cubes. Since each vector is relationally defined, its elements sum to zero, which means [111…] is a null eigenvector of $\bar{\bar{K}}$. Thus, the determinant of $\bar{\bar{K}}$ is zero, so the set of row vectors is not linearly independent. That some subset of the vectors is determined by its complement follows from having the graphical set relationally defined. This allows for divergence-free $\vec{J}$ as we showed with the hypercube in section 3.3. Therefore, divergence-free $\vec{J}$ follows from relationally defined $\bar{\bar{K}}$ as a consequence of our SCC.

To study the eigenstructure, we point out that $\bar{\bar{K}}$ is in nested form. $\bar{\bar{K}}_{block}=\begin{bmatrix} A & TI \\ TI & A \end{bmatrix}$

where *TI* is the 8x8 identity matrix *I* times *T* and *A* is the 8x8 matrix $A=\begin{bmatrix} B & XI \\ XI & B \end{bmatrix}$.

Continuing the nesting we have $B=\begin{bmatrix} C & YI \\ YI & C \end{bmatrix}$ where $C=\begin{bmatrix} D & ZI \\ ZI & D \end{bmatrix}$ and

$D=\left[-T-X-Y-Z\right]$. The eigenvalue problem for $\bar{\bar{K}}$ then takes a nested form in terms of Hadamard matrices H_1, H_2, H_4, H_8, and H_{16} as follows. $DH_1 = H_1\left[-T-X-Y-Z\right]$ where $H_1 = [1]$. $C\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -T-X-Y & 0 \\ 0 & -T-X-Y-2Z \end{bmatrix}$ where $H_2=\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$BH_4 = H_4 diag\left[-T-X, -T-X-2Z, -T-X-2Y, -T-X-2Y-2Z\right]$ where

$$H_4=\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

$$AH_8 = H_8 diag\left[-T, -T-2Z, -T-2Y-2Z, -T-2X, -T-2X-2Z, -T-2X-2Y, -T-2X-2Y-2Z\right]$$

where $H_8=\begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix}=H_2\otimes H_4$. Thus, $\bar{\bar{K}}_{block}H_{16} = H_{16} diag\left[vector\right]$ where

$H_{16} = H_2 \otimes H_8$ and

$$vector=-2\begin{bmatrix} 0 \\ Z \\ Y \\ Y+Z \\ X \\ X+Z \\ X+Y \\ X+Y+Z \\ T \\ T+Z \\ T+Y \\ T+Y+Z \\ T+X \\ T+X+Z, \\ T+X+Y \\ T+X+Y+Z \end{bmatrix}$$

Finally, the eigenvalue problems for each of the 4x4 matrices in *vector* are solved and the eigenvectors are located in a 64x64 matrix built from H_{16}. So, for example, the first column of H_{16} is [111…] and the four-dimensional null space is be spanned by [1,0,0,0], [0,1,0,0], [0,0,1,0] and [0,0,0,1], so the first four columns of the eigenbasis matrix for $\bar{\bar{K}}$ are (column entries top to bottom read left to right here):

```
{{0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,
  0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,
  0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0},
 {0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,
  0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0}, {1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,
  0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0}}
```

$\bar{J}$ being orthogonal to each of these vectors simply means that the global sum over each spacetime component of $\bar{J}$ at each node gives zero, as required for vector addition over all 16 nodes. We next study the Maxwell action.

3.7 Scalar Field on Links. We now apply this approach to gauge fields for the exchange of energy via photons. In order to model the construct of action for the exchange of energy via photons, we use the Maxwell Lagrangian density L for free electromagnetic radiation

$$L = -\frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta} \tag{24}$$

with the field strength tensor given by

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha = \left[\frac{\left(A_\beta(n+\hat{\alpha}) - A_\beta(n)\right)}{\ell_\alpha} - \frac{\left(A_\alpha(n+\hat{\beta}) - A_\alpha(n)\right)}{\ell_\beta} \right] \tag{25}$$

on the graph[41] where n is the node number, ℓ_i the lattice spacing in the ith direction, and $\hat{\alpha}$ and $\hat{\beta}$ are displacements to adjoining nodes in those directions. Applying this to the (1+1)D case $\bar{\bar{K}}$ is Eq (26)

$$
\begin{matrix}
\frac{1}{x^2} & -\frac{1}{x^2} & -\frac{1}{tx} & \frac{1}{tx} \\
-\frac{1}{x^2} & \frac{1}{x^2} & \frac{1}{tx} & -\frac{1}{tx} \\
-\frac{1}{tx} & \frac{1}{tx} & \frac{1}{t^2} & -\frac{1}{t^2} \\
\frac{1}{tx} & -\frac{1}{tx} & -\frac{1}{t^2} & \frac{1}{t^2}
\end{matrix}
$$

where we have ignored overall factors $\frac{-1}{4\mu_o}$ and the volume of the element, and $c = 1$. The eigenvalues are 0, 0, 0, $2\left(\frac{1}{x^2}+\frac{1}{t^2}\right)$. The dimensionality of the column space represents the degrees of freedom available with local conservation of $\vec{J}$, as explained in section 3.3. That is, specifying $\vec{J}$ on just one link dictates the other three values per conservation of $\vec{J}$ on the links at each node.

On the cube $\bar{\bar{K}}$ is Eq (27)

$$
\begin{matrix}
-\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t^2} & -\frac{1}{tx} & \frac{1}{tx} & \frac{1}{y^2} & 0 & 0 & 0 & \frac{1}{xy} \\
\frac{1}{t^2} & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{tx} & -\frac{1}{tx} & 0 & \frac{1}{y^2} & 0 & 0 & 0 \\
-\frac{1}{tx} & \frac{1}{tx} & -\frac{1}{x^2}-\frac{1}{y^2} & \frac{1}{x^2} & 0 & 0 & \frac{1}{y^2} & 0 & -\frac{1}{ty} \\
\frac{1}{tx} & -\frac{1}{tx} & \frac{1}{x^2} & -\frac{1}{x^2}-\frac{1}{y^2} & 0 & 0 & 0 & \frac{1}{y^2} & 0 \\
\frac{1}{y^2} & 0 & 0 & 0 & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{t^2} & -\frac{1}{tx} & \frac{1}{tx} & -\frac{1}{xy} \\
0 & \frac{1}{y^2} & 0 & 0 & \frac{1}{t^2} & -\frac{1}{t^2}-\frac{1}{y^2} & \frac{1}{tx} & -\frac{1}{tx} & 0 \\
0 & 0 & \frac{1}{y^2} & 0 & -\frac{1}{tx} & \frac{1}{tx} & -\frac{1}{x^2}-\frac{1}{y^2} & \frac{1}{x^2} & \frac{1}{ty} \\
0 & 0 & 0 & \frac{1}{y^2} & \frac{1}{tx} & -\frac{1}{tx} & \frac{1}{x^2} & -\frac{1}{x^2}-\frac{1}{y^2} & 0 \\
\frac{1}{xy} & 0 & -\frac{1}{ty} & 0 & -\frac{1}{xy} & 0 & \frac{1}{ty} & 0 & -\frac{1}{t^2} \\
0 & \frac{1}{xy} & \frac{1}{ty} & 0 & 0 & -\frac{1}{xy} & -\frac{1}{ty} & 0 & \frac{1}{t^2} \\
-\frac{1}{xy} & 0 & 0 & -\frac{1}{ty} & \frac{1}{xy} & 0 & 0 & \frac{1}{ty} & \frac{1}{x^2} \\
0 & -\frac{1}{xy} & 0 & \frac{1}{ty} & 0 & \frac{1}{xy} & 0 & -\frac{1}{ty} & 0
\end{matrix}
$$

with eigenvalues

$$\{0,0,0,0,0,0,0,-\frac{2(t^2+x^2)}{t^2x^2},-\frac{2(t^2+y^2)}{t^2y^2},-\frac{2(x^2+y^2)}{x^2y^2},-\frac{2(t^2x^2+t^2y^2+x^2y^2)}{t^2x^2y^2},-\frac{2(t^2x^2+t^2y^2+x^2y^2)}{t^2x^2y^2}\}$$

of a combinatorial nature analogous to (1+1)D. Again, the dimensionality of the column space (five) represents the degrees of freedom available with local conservation of $\vec{J}$. That is, specifying $\vec{J}$ on the four links of one face (front, say) gives $\vec{J}$ on the links connecting the front face to the back face by local conservation. Then specifying $\vec{J}$ on just one link of the back face specifies the remaining links by local conservation.

$\bar{\bar{K}}$ for the hypercube is too large to display here, but its eigenvalues are

$$\Big\{\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\ \Big\{-\frac{2}{t^2}-\frac{2}{x^2}\Big\},\ \Big\{-\frac{2}{t^2}-\frac{2}{y^2}\Big\},\ \Big\{\frac{2}{x^2}+\frac{2}{y^2}\Big\},\ \Big\{\frac{2(t^2-x^2)}{t^2x^2}+\frac{2}{y^2}\Big\},\ \Big\{-\frac{2(t^2+x^2)}{t^2x^2}-\frac{2}{y^2}\Big\},$$
$$\Big\{-\frac{2}{t^2}-\frac{2}{z^2}\Big\},\ \Big\{\frac{2}{x^2}+\frac{2}{z^2}\Big\},\ \Big\{\frac{2(t^2-x^2)}{t^2x^2}+\frac{2}{z^2}\Big\},\ \Big\{-\frac{2(t^2+x^2)}{t^2x^2}-\frac{2}{z^2}\Big\},\ \Big\{\frac{2}{y^2}+\frac{2}{z^2}\Big\},$$
$$\Big\{\frac{2(t^2-y^2)}{t^2y^2}+\frac{2}{z^2}\Big\},\ \Big\{-\frac{2(t^2+y^2)}{t^2y^2}-\frac{2}{z^2}\Big\},\ \Big\{\frac{2(x^2+y^2)}{x^2y^2}+\frac{2}{z^2},\ \frac{2(x^2+y^2)}{x^2y^2}+\frac{2}{z^2}\Big\},$$
$$\Big\{\frac{2(t^2x^2+t^2y^2-x^2y^2)}{t^2x^2y^2}+\frac{2}{z^2},\ \frac{2(t^2x^2+t^2y^2-x^2y^2)}{t^2x^2y^2}+\frac{2}{z^2}\Big\},\ \Big\{-\frac{2(t^2x^2+t^2y^2+x^2y^2)}{t^2x^2y^2}-\frac{2}{z^2}\Big\}\Big\}$$

of a combinatorial nature akin to the lower-dimensional versions. Again, the dimensionality of the column space (17) represents the degrees of freedom available with local conservation of $\vec{J}$, as explained in section 3.3 for links of the hypercube. We next study the Einstein-Hilbert action.

3.8 Scalar Field on Plaquettes. This is linearized GR, i.e., the harmonic terms only. We have for the Einstein-Hilbert Lagrangian density[42]

$$L=-\partial_\lambda h_{\alpha\beta}\partial^\lambda h^{\alpha\beta}+2\partial_\lambda h_{\alpha\beta}\partial^\beta h^{\alpha\lambda} \qquad (28)$$

omitting trace terms not relevant to the lattice. To discretize this on the hypercube (Figure 10) we first label our scalar field on each plaquette according to its span. For example, the front face of the "inner" cube is spanned by x and z, so it's labeled h_{13}. Of course, there are three other such plaquettes, one displaced from the front towards the back (in y) of the "inner" cube, one displaced in t to the front of the "outer" cube, and one displaced in t and y to the back of the "outer" cube. There are six fields (h_{01}, h_{02}, h_{03}, h_{12}, h_{13}, h_{23}) which generate such a quadruple, accounting for all 24 plaquettes of the hypercube. Likewise, for the cube we have (h_{01}, h_{02}, h_{12}) and their pairing partners giving us the six plaquettes.

We see that the first term of S is just the sum of the squares of the gradients formed in each set of $h_{\alpha\beta}$ values, e.g.,

$$\left(\frac{h_{13}(back-in)}{y}-\frac{h_{13}(front-in)}{y}\right)^2+\left(\frac{h_{13}(back-out)}{y}-\frac{h_{13}(front-out)}{y}\right)^2+$$
$$\left(\frac{h_{13}(back-out)}{ct}-\frac{h_{13}(back-in)}{ct}\right)^2+\left(\frac{h_{13}(front-out)}{ct}-\frac{h_{13}(front-in)}{ct}\right)^2$$

for h_{13} where "in" stands for "inner" cube and "out" stands for "outer" cube. The second term of S is formed by mixing gradients, just as with the photon field in section 3.7. For example, we would have terms like $(\partial_0 h_{12})(\partial_2 h_{10})$ which on the lattice would have forms such as

$$\left(\frac{h_{12}(bottom-out)}{t}-\frac{h_{12}(bottom-in)}{t}\right)\left(\frac{h_{10}(back-in)}{y}-\frac{h_{10}(front-in)}{y}\right)$$

Using these conventions on the cube (again, ignoring overall scaling factors and letting $c = 1$), $\vec{\vec{K}}$ is Eq (29)

$$\begin{matrix}
\frac{1}{t^2} & -\frac{1}{t^2} & -\frac{1}{ty} & \frac{1}{ty} & -\frac{1}{tx} & \frac{1}{tx}\\
-\frac{1}{t^2} & \frac{1}{t^2} & \frac{1}{ty} & -\frac{1}{ty} & \frac{1}{tx} & -\frac{1}{tx}\\
-\frac{1}{ty} & \frac{1}{ty} & \frac{1}{x^2} & -\frac{1}{x^2} & -\frac{1}{xy} & \frac{1}{xy}\\
\frac{1}{ty} & -\frac{1}{ty} & -\frac{1}{x^2} & \frac{1}{x^2} & \frac{1}{xy} & -\frac{1}{xy}\\
-\frac{1}{tx} & \frac{1}{tx} & -\frac{1}{xy} & \frac{1}{xy} & \frac{1}{y^2} & -\frac{1}{y^2}\\
\frac{1}{tx} & -\frac{1}{tx} & \frac{1}{xy} & -\frac{1}{xy} & -\frac{1}{y^2} & \frac{1}{y^2}
\end{matrix}$$

which looks much like Eq (26) for the scalar field on links. The eigenvalues of $\bar{\bar{K}}$ are

$$\{0,0,0,2(\frac{1}{x^2}+\frac{1}{y^2}),\frac{xy-\sqrt{x^2y^2+4t^2(x^2+y^2)}}{t^2xy},\frac{xy+\sqrt{x^2y^2+4t^2(x^2+y^2)}}{t^2xy}\}$$

and a basis for the null space is

$$\{\{0,0,0,0,1,1\},\{0,0,1,1,0,0\},\{1,1,0,0,0,0\}\}$$

which represents conservation of $\vec{J}$ among each pair of plaquettes associated with (h_{01}, h_{02}, h_{12}). [Of course, the rows of $\bar{\bar{K}}$ sum to zero so, as always, [111…] is a null eigenvector meaning we have global conservation of $\vec{J}$.]

$\bar{\bar{K}}$ for the hypercube is too large to display here, but one null eigenbasis is

$$\{\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1\},\{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0\},$$
$$\{0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0\},\{0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0\},$$
$$\{0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\},\{1,1,1,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0\}\}$$

$\vec{J}$ orthogonal to each of these null eigenvectors means it is conserved across each set of four plaquettes associated with (h_{01}, h_{02}, h_{03}, h_{12}, h_{13}, h_{23}). We point out that this null space structure exists for the gauge equivalent form[(43)] $L=-\partial_\lambda h_{\alpha\beta}\partial^\lambda h^{\alpha\beta}$ on the lattice[8], but we included the mixing terms for comparison with the particular gauge choice in the photon case. We next interpret the Standard Model per theory X.

4. Unification and Quantum Gravity

4.1 The Standard Model. Strictly speaking, when finding the gradient of a vector field on the graph as we did with the Dirac operator, we need to specify a means of parallel transport. So, in our view and that of LGT, local gauge invariance is seen as a modification to the matter field gradient on the graph required by parallel transport per U_μ, i.e.,

$$\gamma^\mu D_\mu\psi=\gamma^0\left(\frac{U_0\tilde{\psi}_0-\psi}{ct}\right)+\gamma^1\left(\frac{U_1\tilde{\psi}_1-\psi}{x}\right)+\ldots \tag{30}$$

[8] The second term $2\partial_\lambda h^{\lambda\mu}\partial_\alpha h_\mu^\alpha$ of the gauge equivalent form would be used for juxtaposed graphical elements, which represents a more complex arrangement.

where $\widetilde{\psi}_i$ is the vector field on the node adjacent to ψ in the positive i^{th} direction. The Lagrangian density $L = \frac{1}{2}\overline{\psi}\left(\gamma^{\mu} D_{\mu} - m\right)\psi - \frac{1}{4\mu_o} F^{\alpha\beta} F_{\alpha\beta}$ is therefore seen as the addition of parallel transport U_μ and a curvature term $A^{\dagger}\left(\partial_2{}^{\dagger}\partial_2\right)A$, where A generates U_μ, to $L = \frac{1}{2}\overline{\psi}\left(\sqrt{\partial_1{}^{\dagger}\partial_1}\right)\psi$ to produce a well-defined field gradient between $\widetilde{\psi}_i$ and ψ. Thus, the action of the Standard Model results from the self-consistent co-construction of space, time and sources via field gradients on the graph as ultimately underwritten by the SCC.

If one introduces two vectors at each node, this same standard requires

$$\gamma^{\mu} D_{\mu}\psi = \gamma^{0}\left(\frac{\begin{bmatrix} C_{011} & C_{012} \\ C_{021} & C_{022} \end{bmatrix}\begin{bmatrix} \widetilde{\psi}_0^1 \\ \widetilde{\psi}_0^2 \end{bmatrix} - \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}}{ct}\right) + \gamma^{1}\left(\frac{\begin{bmatrix} C_{111} & C_{112} \\ C_{121} & C_{122} \end{bmatrix}\begin{bmatrix} \widetilde{\psi}_1^1 \\ \widetilde{\psi}_1^2 \end{bmatrix} - \begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}}{x}\right) + \ldots \qquad (31)$$

where the matrix $C_{\mu ab}$ is an element of SU(2) associated with the link in the positive μ^{th} direction from $\begin{bmatrix} \psi^1 \\ \psi^2 \end{bmatrix}$. Again, we have the same form for our field gradients, i.e., the nodal field gradients parallel transported by the link field, which still contributes a gradient to the action $-\frac{1}{4g^2}\left(F^{a\alpha\beta} F^{a}_{\alpha\beta}\right)$ (sum over a) where g is the coupling constant, $F^{a}_{\alpha\beta} = \partial_{\alpha} A^{a}_{\beta} - \partial_{\beta} A^{a}_{\alpha} + f^{abc} A^{b}_{\alpha} A^{c}_{\beta}$ (sum over b and c) and f^{abc} are the structure constants of SU(2). The pattern is extended to SU(3) for three vectors at each node and all possible mixing between U(1), SU(2) and SU(3) forms the Standard Model.

With this understanding of the Standard Model, we see that the next logical addition to our collection of fundamental spacetimesource elements would be those constructed from the gradient of vector fields on links. The scalar field on plaquettes (basis for quantum gravity) would define parallel transport for this field gradient in the manner scalar fields on links defines parallel transport for the vector fields on nodes. Thus, underwriting

TTOs via spacetimesource elements leads to a relatively simple picture of unification (Figure 13) compared to that based on fundamental particles (Figure 14). However, while we do not view particle physics as the study of what is ultimately fundamental in Nature, it has been essential to understanding how the fundamental elements of spacetimesource are to be combined, and what properties are represented by $\vec{J}$.

The major questions to be answered in this view of unification are clear. Is there a limit to the number of vectors that can be (or need be) introduced on nodes and links? If so, does it have to do with information density? Is it related to quark confinement? Or, is there a purely mathematical fact that underwrites it? Why is there no physical counterpart to a scalar field on cubes? Is this because it requires (4+1)D to close graphically and satisfy the boundary of a boundary principle for all graphical entities? What physical objects correspond to vector fields on links? Are they just quarks and leptons interacting gravitationally? Or, will this generate new fermions that only interact gravitationally, e.g., dark matter? How many terms in the lattice Einstein-Hilbert action are truly needed to account for all observed phenomena, i.e., how much of GR will remain? Will we need sources that are functions of $h_{\alpha\beta}$? Obviously, the program of unification changes non-trivially in this approach. We next explain particle physics per theory X.

4.2 Particle Physics. In our approach, the role of the field is very different than in QFT where it pervades otherwise empty, continuous space to mediate the exchange of matter-energy between sources. Per theory X (and that of LGT), a field is simply a map of scalars and vectors to the graph. One obtains QFT results from LGT by letting the lattice spacing go to zero. In fact, one can understand QFT renormalization through this process of lattice regularization[(44)]. As it turns out, however, this limit does not always exist, so calculated values are necessarily obtained from small, but non-zero, lattice spacing[(45)]. With this picture in mind, we can say simply what we are proposing: The lattice is fundamental, not its continuum limit. Once one accepts this premise, it's merely a matter of degree to have large spacetimesource elements, which is the basis for our explanation of the twin-slit experiment (section 3.4 above) and dark energy (section 5.2 below). In this approach, *there is no graphical counterpart to "quantum systems" traveling through* *space as a function of time from Source to sink to "cause" detector clicks.* This implies the empirical goal at the fundamental level is to tell a unified story about detector events to include individual clicks – how they are distributed in space (e.g., interference patterns, interferometer outcomes, spin measurements), how they are distributed in time (e.g., click rates, coincidence counts), how they are distributed in space and time (e.g., particle trajectories), and how they generate more complex phenomena (e.g., photoelectric effect, superconductivity). Thus in theory X, particle physics per QFT is in the business of characterizing large sets of detector data, i.e., all the individual clicks.

As was eminently apparent from our examples in section 3, it is practically impossible to compute *Z* in theory X for all possible spatiotemporally relative click locations in a particle physics "event," which contains "approximately 100,000 individual measurements of either energy or spatial information[(46)]." However, we know from theory[(47)] and experiment that, with overwhelming probability, detector clicks will trace classical paths[9], so it makes sense to partition large click distributions into individual trajectories and treat these as the fundamental constituents of high energy physics experiments[10]. This is exactly what QFT does for particle physics according to our interpretation. Since the individual trajectories are themselves continuous, QFT uses propagators in continuous spacetime which entails an indenumerably infinite number of locations for both clicks and interaction vertices. *Thus, issues of regularization and renormalization are simply consequences of the continuum approximation necessary to*

[9] Individual detector clicks (called "hits in the tracking chamber") are first localized spatially (called "preprocessing"), then associated with a particular track (called "pattern recognition"). The tracks must then be parameterized to obtain dynamical characteristics (called "geometrical fitting"). See Fernow, R.C.: Introduction to experimental particle physics. Cambridge University Press, Cambridge (1986), sections 1.7.1, 1.7.2 & 1.7.3, respectively.

[10] Some assumptions are required, e.g., "Sometimes it is necessary to know the identity (i.e., the mass) of at least some of the particles resulting from an interaction" (Fernow, 1986, p 17), "Within the errors [for track measurements], tracks may appear to come from more than one vertex. Thus, the physics questions under study may influence how the tracks are assigned to vertices" (Fernow, 1986, p 25), and "Now there must be some minimum requirements for what constitutes a track. Chambers may have spurious noise hits, while the chambers closest to the target may have many closely spaced hits. The position of each hit is only known to the accuracy of the chamber resolution. This makes it difficult to determine whether possible short track combinations are really tracks" (Fernow, 1986, p 22). Despite these assumptions, no one disputes the inference. While we do not subscribe to the existence of "click-causing entities," we agree that clicks trace classical paths. Indeed, this is the basis for our approach and consequently, the results and analysis of particle physics experiments are very important.

deal with very large click distributions, having decided to parse the click distributions into continuum trajectories.

Essentially, we're saying a particle physics detector event is one giant interference pattern and the way to understand a particular pattern involving thousands of clicks can only realistically be accomplished by parsing an event into smaller subsets, and the choice of subsets is empirically obvious, i.e., spacetime trajectories. These trajectories are then characterized by mass, spin, and charge. The colliding beams in the accelerator 'create' a spatiotemporally small but complex configuration of spacetimesource elements linking the accelerator beam collision event to the surrounding detectors. The possible field configurations on the graph are used to compute Z with anharmonicity terms in the action used to offset disordered locality and deviations from regular lattice spacing. In standard LGT → QFT the calculated outcomes are found by taking the limit as the lattice spacing goes to zero via renormalization, but we needn't assume the spacing goes to zero, only that it's 'small' as defined by the experimental uncertainties. Likewise, assuming the accelerator and detectors are sufficiently isolated during the brief period of data collection, the graph size is not infinite as in QFT.

This severely undermines the dynamical picture of perturbations moving through a continuum medium (naïve field) from source to source, i.e., it undermines the naïve notion of a particle. In fact, the typical notion of a particle is associated with the global particle state of n-particle Fock space and "the notion of global particle state is ambiguous, ill-defined, or completely impossible to define(48)." What we mean by "particle" is a collection of detector hits forming a spacetime trajectory and doesn't entail the existence of an object with intrinsic properties, such as mass and charge, moving through the detector to cause the hits.

Our view of particles agrees with Colosi & Rovelli(49) on two important counts. First, that particles are best modeled by local particle states rather than Fock n-particle states computed over infinite regions, squaring with the fact that particle detectors are finite in size. The advantage to this approach is that one can unambiguously define the notion of particles in curved spacetime as excitations in a local M4 region, which makes it amenable to the graphical form of GR, Regge calculus (introduced below). Second, this theory of particles is much more compatible with the quantum notion of complementary observables in that every detector has its own Hamiltonian (different sized graph), and therefore its own particle basis (unlike the unique basis of Fock space). Per Colosi & Rovelli, "In other words, we are in a genuine quantum mechanical situation in which distinct particle numbers are complementary observables. Different bases that diagonalize different H_R [Hamiltonian] operators have equal footing. Whether a particle exists or not depends on what I decide to measure." Thus, in our view, particles simply describe how detectors and Sources are relationally co-defined. There are no unique "fundamental particles" understood as the "elements of matter." Rather, spacetime trajectories of identified properties, i.e., particles, are constructed from fundamental elements of spacetimesource.

That the spacetimesource elements of theory X can be large suggests a modification of the graphical approach to GR. We next explain how such a modification to graphical GR, i.e., Regge calculus, can be used to eliminate the need for dark energy.

5. Implications for Astrophysics and Cosmology

5.1 Regge Calculus. In Regge calculus, the spacetime manifold is replaced by a lattice geometry where each 4D cell (simplex) is Minkowskian (flat). Curvature is represented by "deficit angles" (Figure 15) about any plane orthogonal to a "hinge" (triangular side to a tetrahedron, which is a 3D side of a 4D simplex). The Hilbert action for a 4D vacuum lattice is $I_R = \frac{1}{8\pi}\sum_{\sigma_i \in L}\varepsilon_i A_i$ where σ_i is a triangular hinge in the lattice L, A_i is the area of σ_i and ε_i is the deficit angle associated with σ_i. The counterpart to Einstein's equations is then obtained by demanding $\frac{\delta I_R}{\delta \ell_j^2} = 0$, where ℓ_j^2 is the squared length of the j^{th} lattice edge , i.e., the metric. To obtain equations in the presence of matter-energy, one simply adds the appropriate term $I_{M\text{-}E}$ to I_R and carries out the variation as before to obtain

$\frac{\delta I_R}{\delta \ell_j^2} = -\frac{\delta I_{M-E}}{\delta \ell_j^2}$. One finds the stress-energy tensor is associated with lattice edges, just as the metric, and Regge's equations are to be satisfied for any particular choice of the two tensors on the lattice.

5.2 Dark Energy and Other Astrophysical Implications. Since one recovers GR from Regge calculus by making the simplices small (as in LGT → QFT), it seems that empirical evidence of the deviation from GR phenomena posed by large spacetimesource elements, i.e., modified Regge calculus (MORC), might be found in the exchange of photons on cosmological scales. Therefore, we modified the Regge calculus approach to Einstein-deSitter cosmology (EdS)[(50)] and compared this MORC model, EdS, and the concordance model ΛCDM (EdS plus a cosmological constant Λ to account for dark energy) using the data from the Union2 Compilation, i.e., distance moduli and redshifts for type Ia supernovae[(51)] (Figure 16). We found that a best fit line through log(D_L/Gpc) versus log(z) gives a correlation of 0.9955 and a sum of squares error (SSE) of 1.95. By comparison, the best fit ΛCDM gives SSE = 1.79 using a Hubble constant of Ho = 69.2 km/s/Mpc, Ω_M = 0.29 and Ω_Λ = 0.71. The parameters for ΛCDM yielding the most robust fit to[(52)] "the Wilkinson Microwave Anisotropy Probe data with the latest distance measurements from the Baryon Acoustic Oscillations in the distribution of galaxies and the Hubble constant measurement" are Ho = 70.3 km/s/Mpc, Ω_M = 0.27 and Ω_Λ = 0.73, which are consistent with the parameters we find for its Union2 Compilation fit. The best fit EdS gives SSE = 2.68 using Ho = 60.9 km/s/Mpc. The best fit MORC gives SSE = 1.77 and Ho = 73.9 km/s/Mpc with the EdS proper distance D_p corrected by a factor of $\sqrt{1+\frac{D_p}{A}}$ where A = 8.38 Gcy. A current "best estimate" for the Hubble constant is Ho = (73.8 ± 2.4*)* km/s/Mpc[(53)]. Thus, MORC improves EdS as much as ΛCDM in accounting for distance moduli and redshifts for type Ia supernovae even though the MORC universe contains no dark energy is therefore always decelerating. So, per theory X, it is quite possible that this data does not constitute "the discovery of the accelerating expansion of the Universe," (Nobel citation, 2011), i.e., there is no accelerating expansion, so there is no need of a cosmological constant or dark energy in any form[(54)].

Theory X has other possible implications for astrophysics and cosmology as well. Perhaps MORC's version of the Schwarzschild solution will negate the need for dark matter as its counterpart to Einstein-deSitter cosmology did with dark energy. What will MORC have to say about the event horizon and singularity in the Schwarzschild solution, i.e., black holes? Perhaps, the singularity will be avoided as in Regge calculus cosmology where backwards time evolution "stops" at a time determined by the choice of lattice spacing[11]. And, with an adynamical approach, cosmological explanation takes on an entirely new form. No longer is one seeking explanation in the form of a time-evolved spatial hypersurface of homogeneity – an explanation that cannot be satisfied with the Big Bang or even a non-singular "stop point." Thus, such dynamical explanation results in contentious, misleading or unverifiable notions about[(55)] "creation from nothing," the multiverse, etc. Rather, explanation via adynamical self-consistency writ large doesn't rest ultimately on the Big Bang or any other region of the graph. The reason the fields on node X and link Y have the values they do is required by the solution for the entire graph, i.e., it is required by the values of the fields on all the other nodes and links. As we pointed out in section 1 when we contrasted dynamical explanation with our adynamical/self-consistency explanation, no region of the graph is distinguished over any other in this explanatory scheme.

6. Summary

We proposed a graphical, adynamical version of theory X underwriting QFT based on our OSR interpretation of quantum physics called Relational Blockworld. Theory X results in a novel approach to unification and quantum gravity whereby temporally identified spatial distributions of properties, i.e., trans-temporal objects (TTOs), are ultimately decomposed into simple units of space, time and source. These fundamental spacetimesource elements are not themselves TTOs, so this differs from the current view

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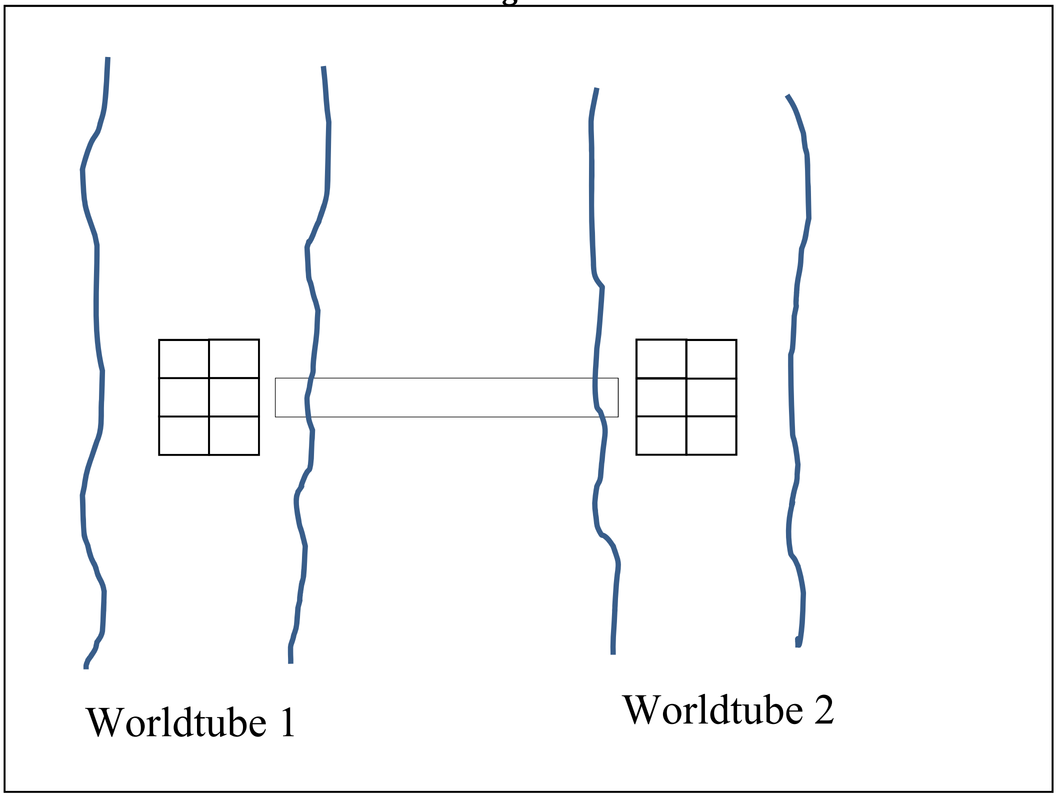
[11] This is the "stop point problem" of Regge calculus cosmology. Of course it's not a "problem" for our approach, since Regge calculus is fundamental to GR, not the converse, one does not require Regge calculus reproduce the initial singularity of GR cosmology.

that TTOs are ultimately decomposed into fundamental particles, which are themselves TTOs. Since the fundamental spacetimesource elements are not TTOs, their construction is not dynamical, but as these elements must account for TTOs, the rule for their construct must underwrite dynamism. The rule we proposed is the graphical counterpart to gauge invariance and divergence-free sources. That is, the difference matrix $\bar{\bar{K}}$ for field gradients of the discrete graphical action is constructed with a non-trivial null space from graphical relations, and the source vector $\vec{J}$ resides in the column space of $\bar{\bar{K}}$. This fundamental axiom of physics leads to the self-consistent relationship of sources, space, time, and stress-energy-momentum of the graph, so it is called the "self-consistency criterion" (SCC). Therefore, the SCC is perfectly consistent with the notion that symmetry is the key to fundamental physics. However, we did introduce a major formal deviation from current practice by allowing spacetimesource elements to be large and irregularly shaped. Thus, anharmonicity terms in the actions of QFT and GR are required to account for the disordered locality and the irregular structure of the spacetimesource elements of theory X. Correcting for disordered locality in the Einstein-deSitter cosmology solution leads to a fit of the Union2 Compilation supernova data that matches ΛCDM without accelerating expansion, a cosmological constant or dark energy. Other possible astrophysical implications were noted.

Theory X is certainly not complete, as indicated by some major outstanding questions we presented in section 4. And, until all such questions are answered, we cannot say exactly what a unified picture will contain. But, in the taxonomy of quantum approaches versus GR approaches to unification and quantum gravity, we are clearly in the quantum camp. Exactly how much GR will be modified remains to be seen, but it will be have to be modified as indicated by our approach to dark energy, for example. In contrast, the Standard Model of particle physics with its focus on gauge symmetry is viewed as a direct application/extension of theory X to aggregates of spacetimesource elements. Thus, theory X doesn't suggest any sweeping change to the formalism of particle physics, but rather it vindicates the formalism by providing rationale for some of its questionable techniques, e.g., UV and IR cutoffs in regularization. But, theory X does move the focus of unification away from fundamental particles and dynamical explanation as a whole.

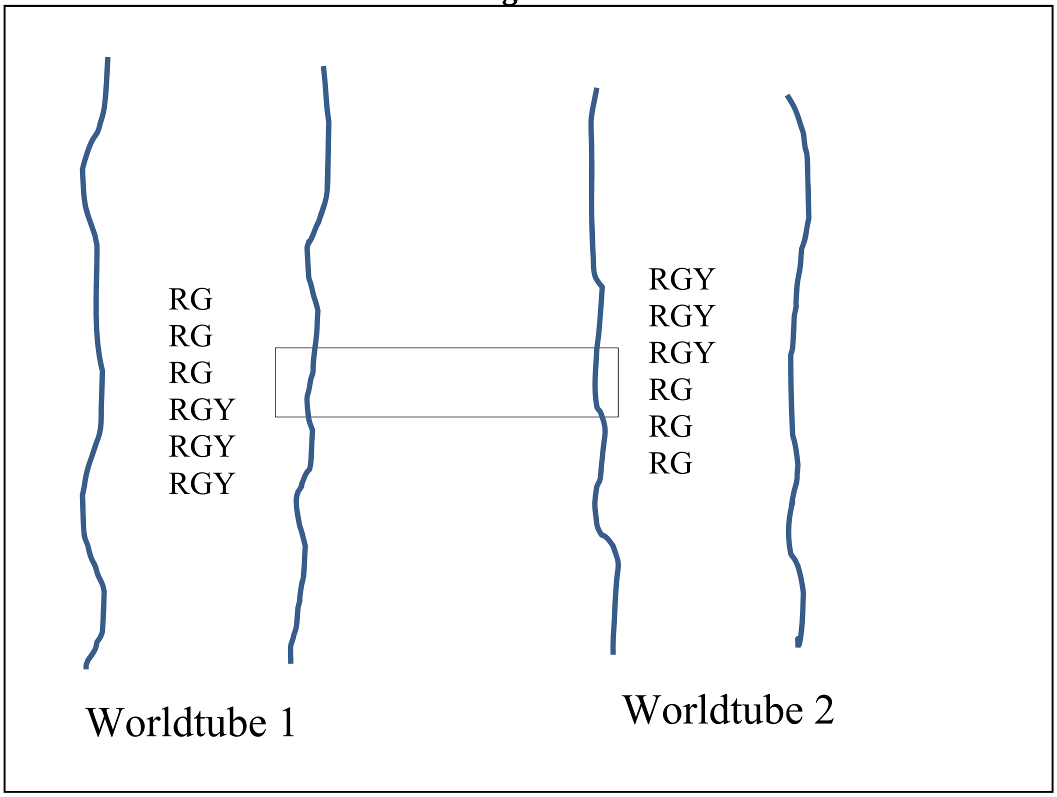
So, particle physics does not escape unscathed, at least conceptually, in our view. Given the incipient nature of theory X, we won’t speculate further.

Figure 1



Composition of Trans-Temporal Objects (TTOs) – Six elements of spacetimesource are shown in each TTO's worldtube. A TTO is simply a compilation of such elements, as they account for the spatial extent of the TTO and the time-identified properties $\vec{J}$ that define the TTO. That the TTOs are themselves spatially separated means they must share elements of spacetimesource, so they must exchange $\vec{J}$ (interact). One such element is shown in this figure.

Figure 2



Analogy – The property Y is associated with the source $\vec{J}$ on the spacetimesource element shared by the worldtubes. As a result, property Y disappears from worldtube 1 (Y Source) and reappears later at worldtube 2 (Y detector). While these properties are depicted as residing in the worldtubes, they don't represent something truly intrinsic to the worldtubes, but are ultimately contextual/relational, i.e., being a Y Source only makes sense in the context of/in relation to a Y detector, and vice-versa.

Figure 3

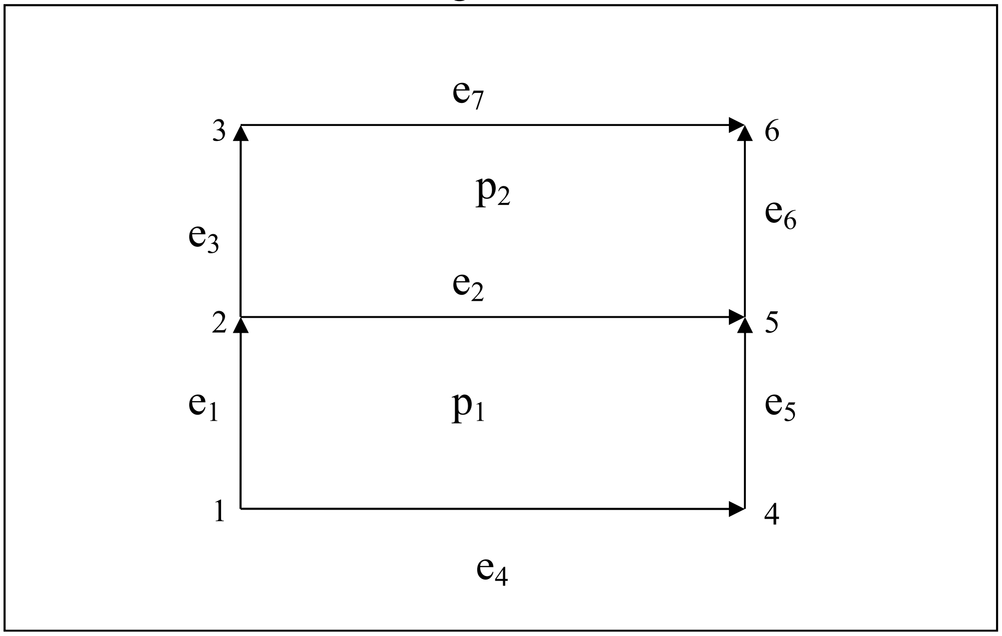


Figure 4

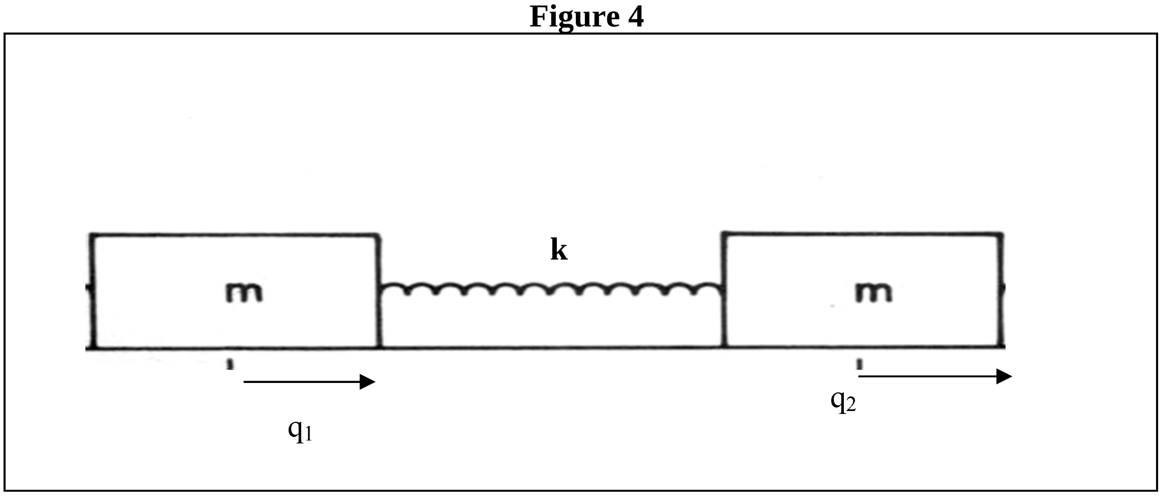


Figure 5

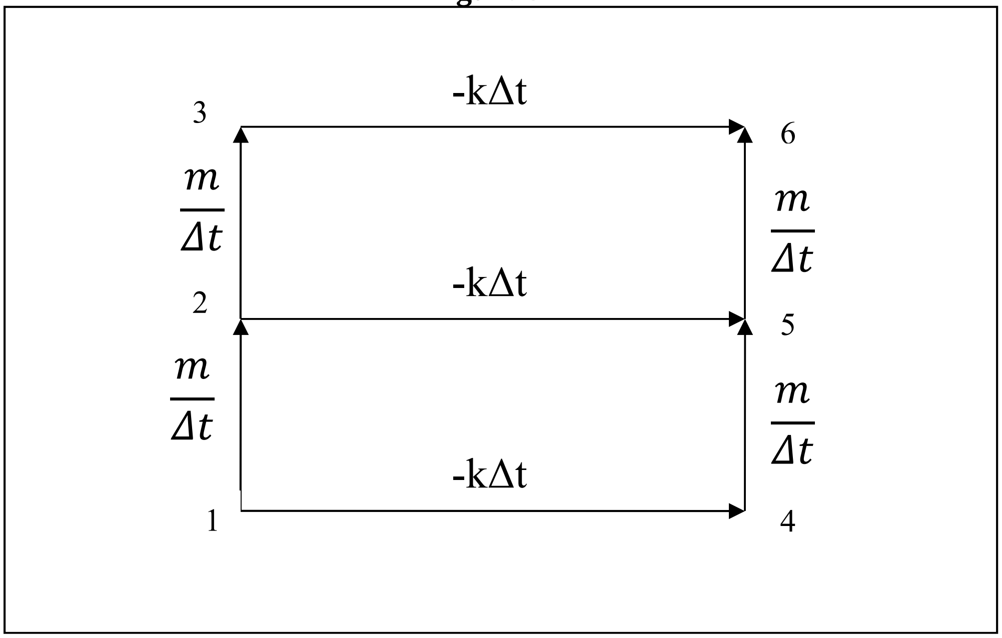


Figure 6

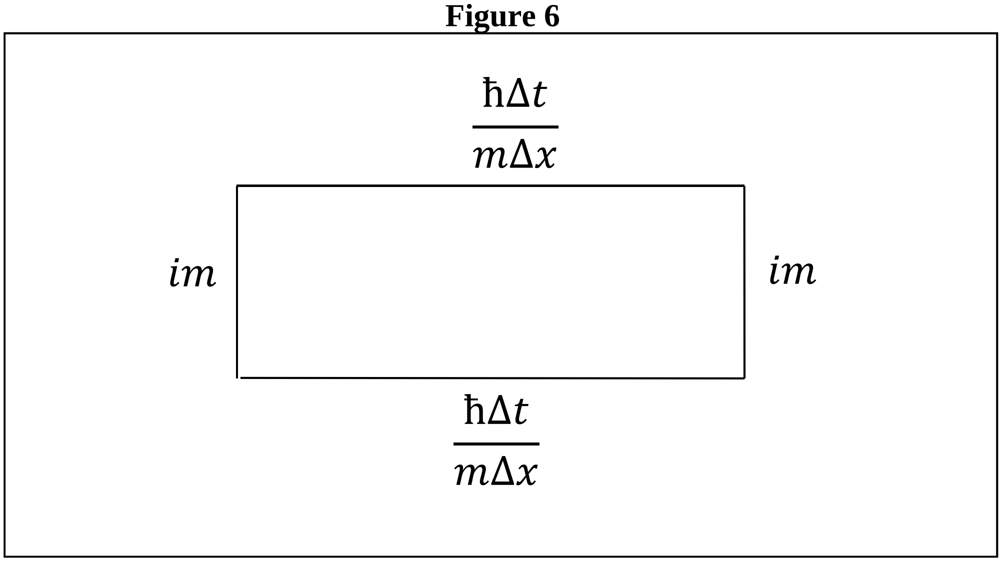


Figure 7

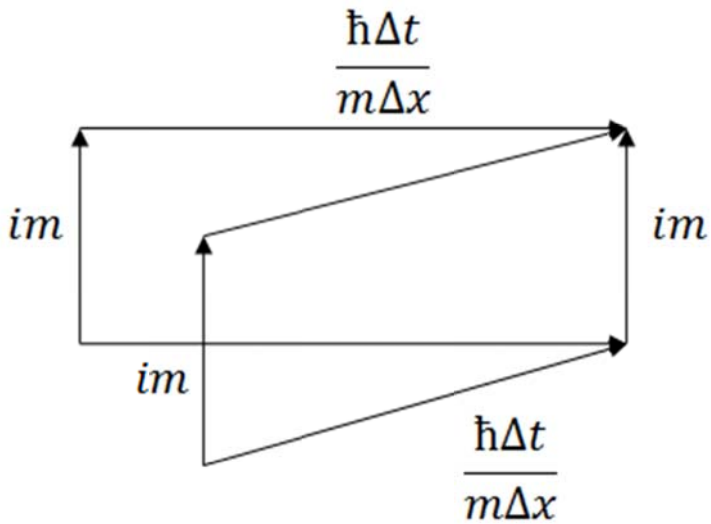


Figure 8

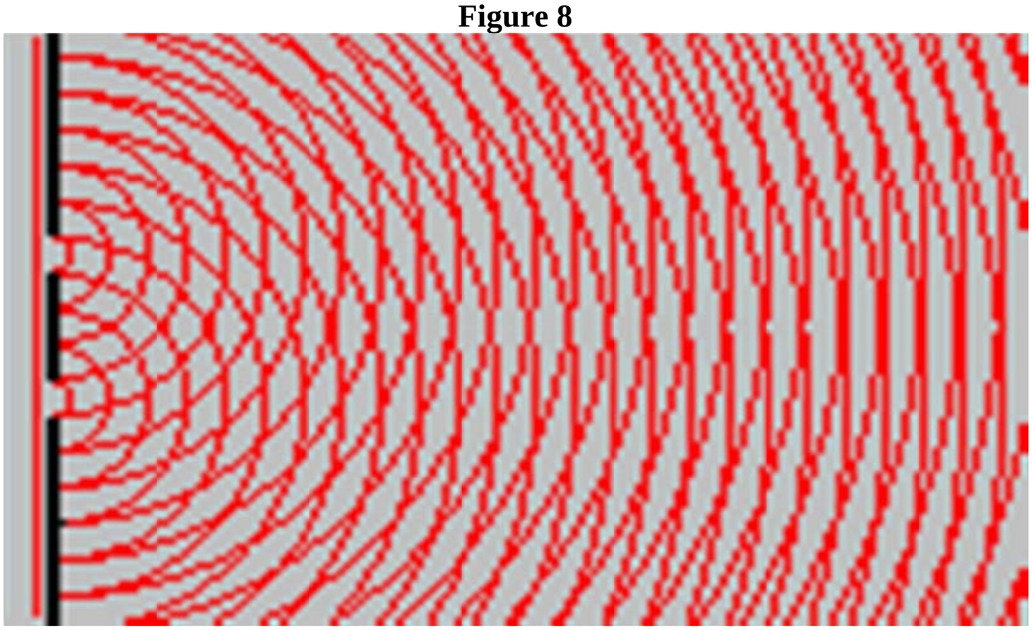


Figure 9

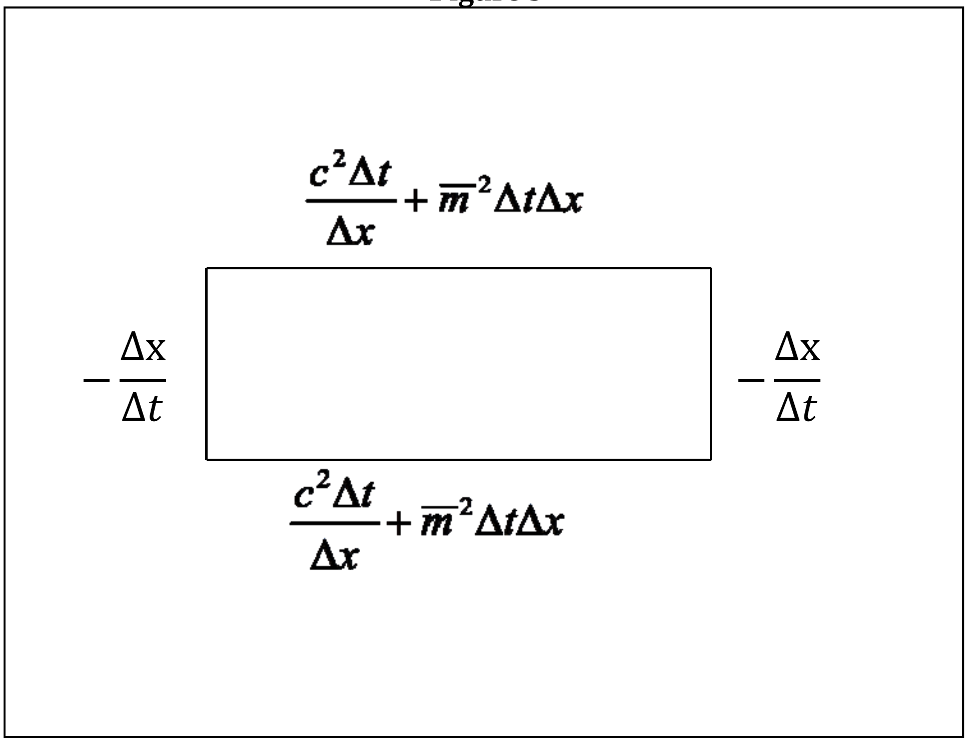

$\frac{c^2\Delta t}{\Delta x} + \overline{m}^2\Delta t\Delta x$
$-\frac{\Delta x}{\Delta t}$
$-\frac{\Delta x}{\Delta t}$
$\frac{c^2\Delta t}{\Delta x} + \overline{m}^2\Delta t\Delta x$

Figure 10

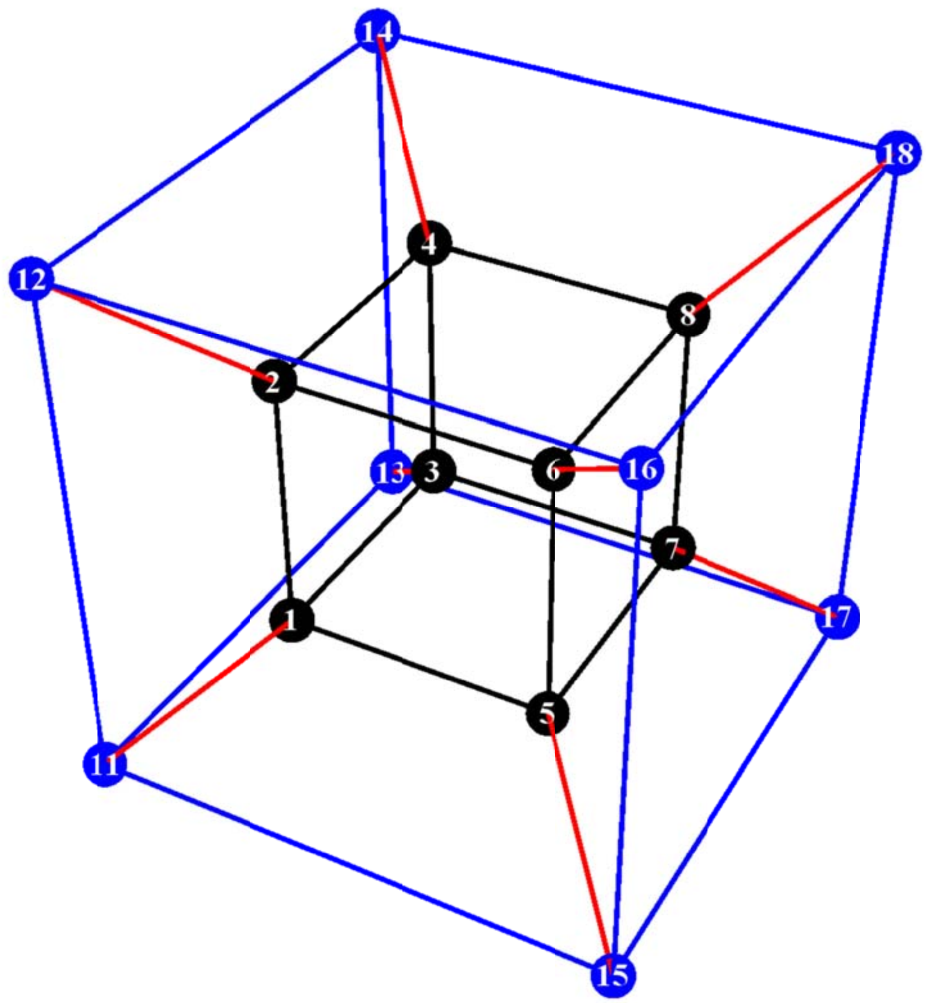


Figure 11

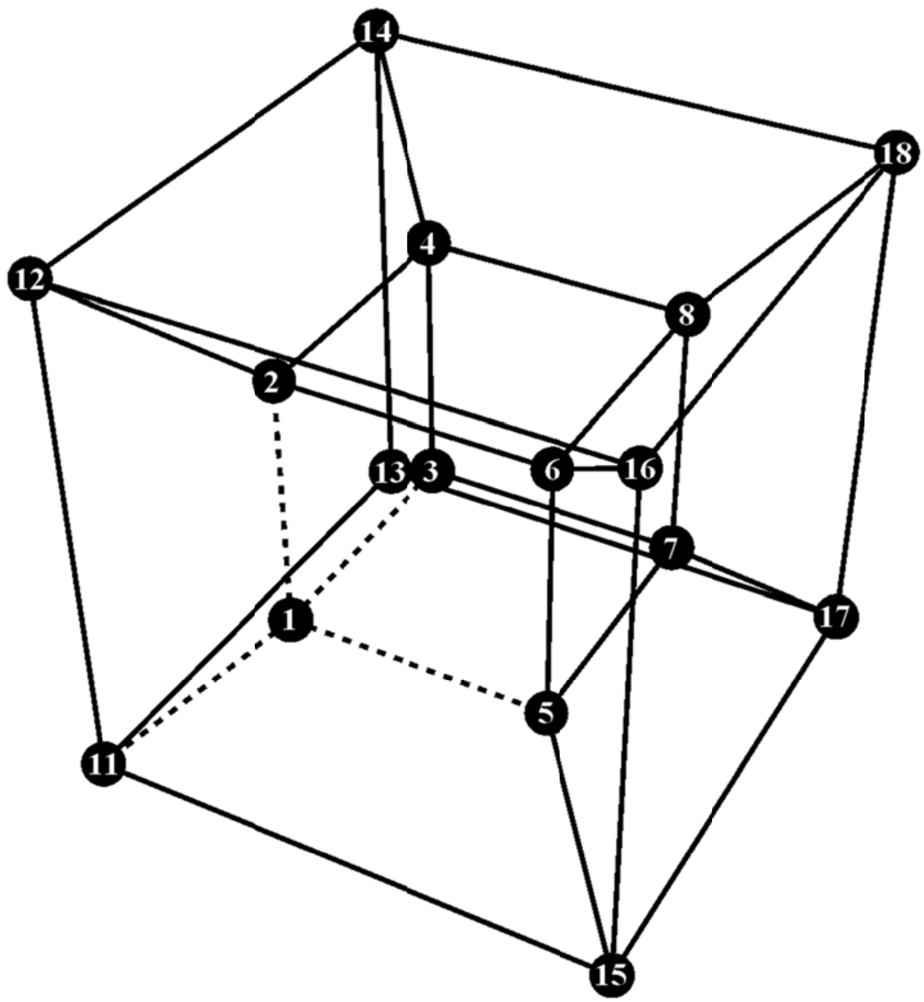


Figure 12

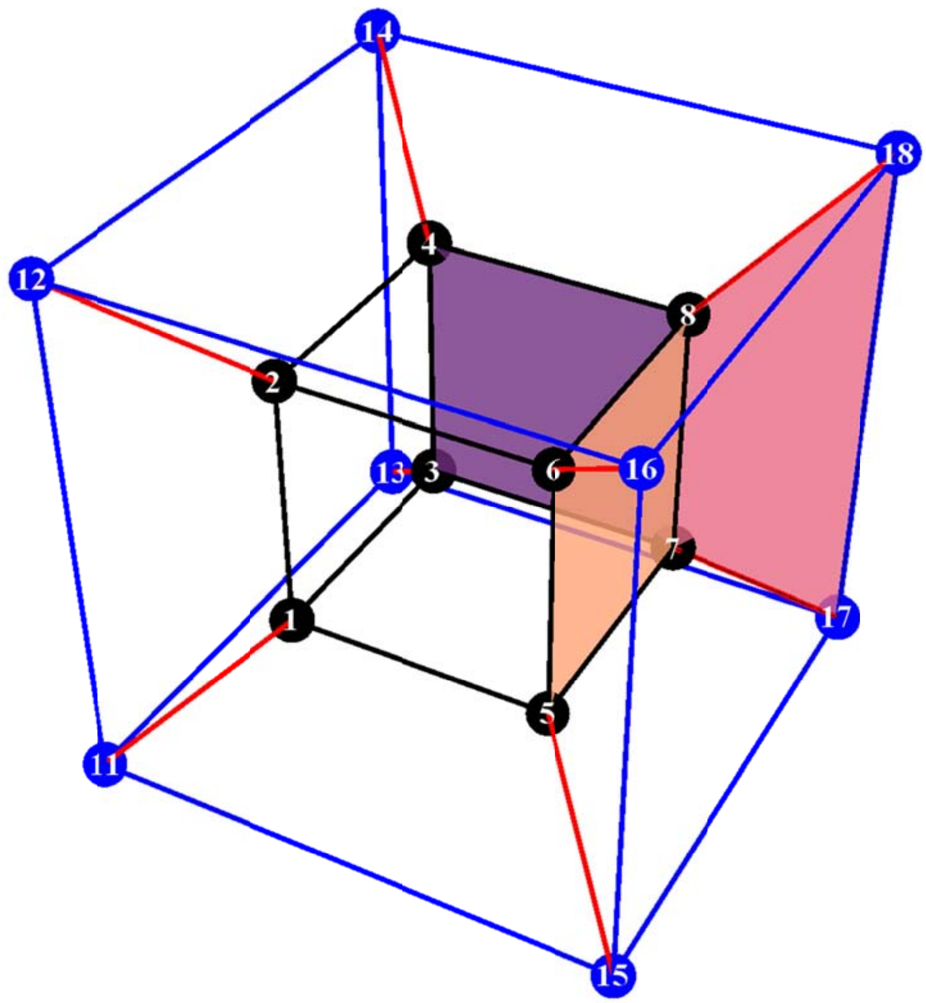


Figure 13

Scalar field on nodes	One vector each node	One vector each link
Scalar field on links	Two vectors each node	Two vectors each link
Scalar field on plaquettes	Three vectors each node	Three vectors each link

Fundamental spacetimesource elements for unification via theory X

Figure 14

The Standard Model Lagrangian Density. Credit: T.D. Gutierrez

http://nuclear.ucdavis.edu/~tgutierr/files/sml.pdf

$$
\begin{gathered}
-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\frac{1}{2}ig_s^2(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu\partial_\mu A_\nu - \frac{1}{2}\partial_\mu H\partial_\mu H - \\
\frac{1}{2}m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
\frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\frac{1}{8}g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - \\
gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \\
\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial+m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + \\
m_d^\lambda)d^\lambda_j + igs_wA_\mu[-(\bar{e}^\lambda\gamma e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma d^\lambda_j)] + \frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+ \\
\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2-1-\gamma^5)u^\lambda_j) + \\
(\bar{d}^\lambda_j\gamma^\mu(1-\frac{8}{3}s_w^2-\gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(1+ \\
\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)] + \\
\frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + \\
i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+ \\
\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] - \\
\frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \\
\bar{X}^+(\partial^2-M^2)X^+ + \bar{X}^-(\partial^2-M^2)X^- + \bar{X}^0(\partial^2-\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + \\
igc_wW^+_\mu(\partial_\mu\bar{X}^0X^- - \partial_\mu\bar{X}^+X^0) + igs_wW^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+Y) + \\
igc_wW^-_\mu(\partial_\mu\bar{X}^-X^0 - \partial_\mu\bar{X}^0X^+) + igs_wW^-_\mu(\partial_\mu\bar{X}^-Y - \partial_\mu\bar{Y}X^+) + \\
igc_wZ^0_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) + igs_wA_\mu(\partial_\mu\bar{X}^+X^+ - \partial_\mu\bar{X}^-X^-) - \\
\frac{1}{2}gM[\bar{X}^+X^+H + \bar{X}^-X^-H + \frac{1}{c_w^2}\bar{X}^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar{X}^+X^0\phi^+ - \\
\bar{X}^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-] + igMs_w[\bar{X}^0X^-\phi^+ - \\
\bar{X}^0X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0]
\end{gathered}
$$

Figure 15

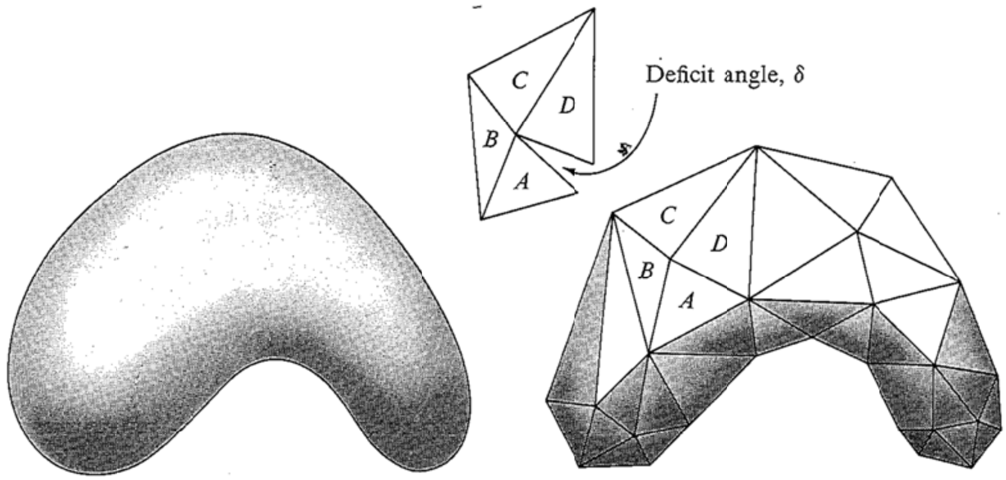
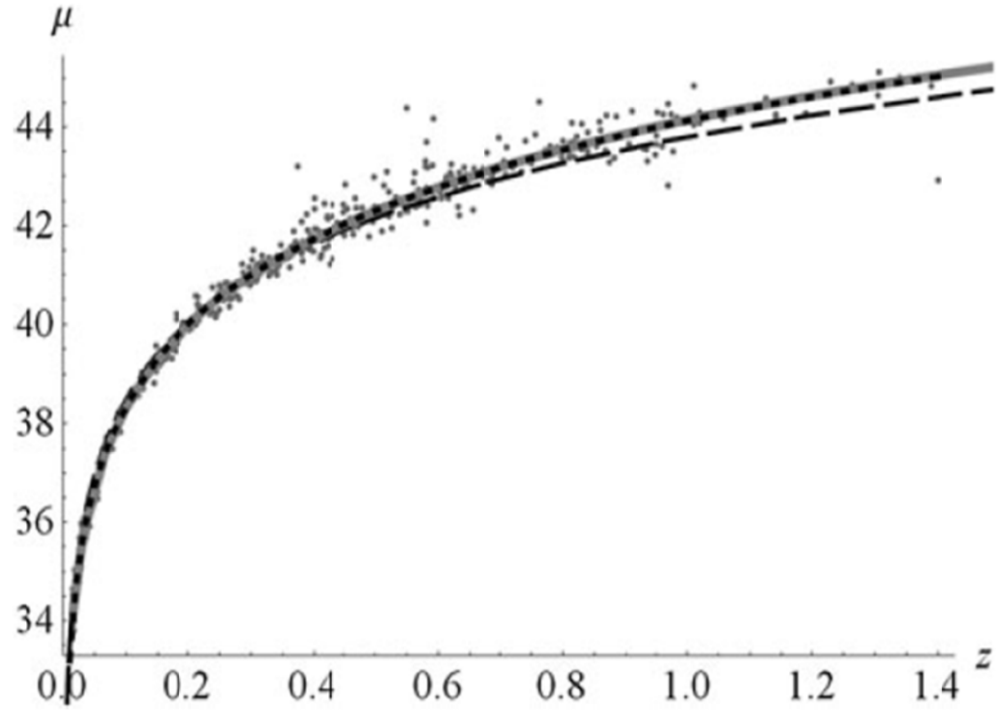


Figure 42.1.
A 2-geometry with continuously varying curvature can be approximated arbitrarily closely by a polyhedron built of triangles, provided only that the number of triangles is made sufficiently great and the size of each sufficiently small. The geometry in each triangle is Euclidean. The curvature of the surface shows up in the amount of deficit angle at each vertex (portion *ABCD* of polyhedron laid out above on a flat surface).

Reproduced from Misner, C.W., Thorne, K.S., Wheeler, J.A.: Gravitation. W.H. Freeman, San Francisco (1973), p. 1168.

Figure 16



Plot of Union2 data along with the best fits for EdS (*dashed*), ΛCDM (*gray*), and MORC (*dotted*). The MORC curve is terminated at z = 1.4 in this figure so that the ΛCDM curve is visible underneath**.**

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