QUANTIFIER VARIANCE, VAGUE EXISTENCE, AND METAPHYSICAL VAGUENESS*

Ontological deflationism, in slogan form, is the view that "reality considered in itself is like some amorphous dough".[1] Metaphysical vagueness, also in slogan form, is the view that there is "vagueness in the world, vagueness in what there is as opposed to our descriptions or knowledge of what there is".[2] This paper asks whether one type of deflationism – quantifier variance – is committed to metaphysical vagueness.[3]

My answer is subtle. We need to distinguish between two elements of the slogan of metaphysical vagueness: a positive element which says where metaphysical vagueness is located ("in the world") and a negative element which says where metaphysical vagueness is not located ("in our descriptions or knowledge"). We can then treat the positive and negative elements as separate theses. *Positive metaphysical vagueness* is the claim that there is vagueness in the world. *Negative metaphysical vagueness* is the claim that vagueness is not due to our representations. And once separated, each thesis can be given their own careful regimentation. I'll present an argument for the following subtle answer:

Subtle Answer The quantifier variantist is committed to positive metaphysical vagueness – even if they aren't committed to negative metaphysical vagueness.

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[1] Matti Eklund, "The Picture of Reality as an Amorphous Lump," in Theodore Sider, John Hawthorne, and Dean W. Zimmerman, eds., *Contemporary Debates in Metaphysics* (Oxford: Blackwell Publishing, 2008), pp. 382–96, at p. 383.

[2] Elizabeth Barnes, "Ontic Vagueness: A Guide for the Perplexed," *Noûs*, XLIV, 4 (2010): 601–27, at p. 601.

[3] Quantifier variance is proposed and defended by Eli Hirsch, who draw inspiration from Carnap. See Eli Hirsch, *Quantifier Variance and Realism: Essays in Metaontology* (Oxford: Oxford University Press, 2011).

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That argument comes in two steps and goes via the theses of *restricted composition* and *vague existence.* In step 1, I'll argue that the variantist's commitment to restricted composition comes with a commitment to vague existence. In step 2, I'll present an argument that vague existence requires positive metaphysical vagueness. Afterwards, I'll ask whether the variantist is committed to negative metaphysical vagueness and consider how our answer to that question relates to my argument for positive metaphysical vagueness.

This two-step argument will have broader significance than its role in establishing the Subtle Answer. In the process of arguing for step 1, I will correct some misunderstandings in the literature surrounding vague existence. And the argument in step 2 will differ from extant arguments (e.g. by David Lewis and Ted Sider) linking restricted composition, vague existence, and metaphysical vagueness. It will thus be of interest to anyone concerned with those theses. Moreover, my argument for this second step will rest on purely logical considerations – none of the premises use non-logical vocabulary – and is a new instance of the recently-appreciated strategy of using logical theories to guide metaphysical inquiry.[4]

Before we begin the main argument, however, some background is required.

I. DIALECTICAL BACKGROUND

I.1. Assumptions of Vagueness. I will assume that we have primitive concepts of definiteness and indefiniteness that we apply when considering sorites series and borderline cases. We use those concepts when we say, for instance, 'for some people it's indefinite whether they are bald' or 'definitely John is bald'. As I intend to use it, our fluency with the concept of (in)definiteness leaves open the *nature* of the (in)definiteness; our use of the concept does not presuppose that the indefiniteness has a semantic, epistemic, or metaphysical source. This concept, I'll assume, has various theoretical roles associated with knowledge and assertion. Specifically, I'll assume that indefiniteness precludes knowledge and assertion:

IPKA If it's indefinite whether ϕ, then we do not know whether ϕ and should not assert that ϕ. Similarly: If for some x it's indefinite whether $\phi(x)$, then we do not know of x whether it is $\phi(x)$ and should not assert $\phi(x)$ of x.

[4] See, for instance, Timothy Williamson, *Modal Logic as Metaphysics* (Oxford: Oxford University Press, 2013).

In the first instance, the notions of definiteness and existence that I am interested in are expressed in English with expressions like 'it's definite that' and 'something'. But to help regiment our theorizing, we can work in a formal modal language that expresses the same concepts. In formalized English, we can express the concept of definiteness using the operator '$\triangle\phi$' and define the notion of indefiniteness – written as '$\triangledown\phi$' – in terms of it as '$(\neg\triangle\phi) \wedge (\neg\triangle\neg\phi)$'. We can also define the modality '$\Diamond\phi$' as '$\neg\triangle\neg\phi$' which we'll pronounce 'might ϕ'. And, formalized English can express the concept of existence using the existential quantifier '$\exists$'. So, formalized English includes sentence like '$\exists x \triangledown Bx$'.

I'll also make some logical assumptions. I'll assume that the logic of vagueness is a *normal modal logic* at least as strong as T. (That means, in part, that I'll assume excluded middle.) I'll only take the classical propositional inference rules (e.g. modus ponens) as primitive.[5] And I'll assume weak rules for the quantifier that are free-logic friendly. (I describe the logical system more fully in §?? and §?? of the appendix.) The weakness of those rules will help ensure that we don't beg any questions in our investigation.[6]

With these assumptions regarding vagueness in place, let's now introduce the position of quantifier variance.

I.2. Quantifier Variance. Consider the *Special Composition Question*: under what conditions does a class of objects compose a distinct object? There are lots of answers to this question which can be embedded in corresponding internally coherent theories of composition including the theories of mereological nihilism and mereological universalism. There are more moderate theories as well (even if no actual metaphysician defends them). For instance, (what we can call) the *n*-theory includes sentences like "a class composes a distinct thing when its members are stuck-together to degree greater than *n*".

[5] In particular, I won't assume ϕ entails $\triangle\phi$, which leads to violations of classical meta-inference rules like contraposition and reductio.

[6] Assuming stronger, classical rules would beg the question in two places. First, as I will explain in §??, the classical rules entail (with some minimal assumptions) there there is positive metaphysical vagueness. Second, as is well recognized from debates over contingentism, the classical rules entail (with some minimal assumptions) that everything definitely exists. I am interested in exploring the possibility of denying both of these claims.

Imagine a world with a number of isolated tribes.[7] The first tribe speaks Nihilese. Their linguistic behavior is the same as that of an English speaker except that in their ordinary interactions they speak as though the theory of mereological nihilism is unproblematically true – they are willing to assert the theory and its implications without much hemming and hawing.[8] Similarly with a second tribe that speaks Universalese and the theory of mereological universalism. Imagine further that for any *n*, there is a tribe that speaks *n*-ese, whose linguistic behavior is the same as that of an English speaker except that in their ordinary interactions, they speak as though the *n*-theory is unproblematically true.

Let 'unrestricted-quantifier-like expression' (UQL) refer to the counterparts of our quantifier in the languages spoken by these counterfactual tribes. According to a quantifier variantist:[9]

Variance For each of these mereological theories T, T is true[10] in the language spoken by the corresponding counterfactual tribe. This is because the UQLs in these languages express different meanings. Moreover, none of these UQLs are metaphysically distinguished.[11]

We must remember that the UQLs in other languages are merely unrestricted-quantifier-***like*** expressions. Only the UQL in our language

[7] See Cian Dorr, "What we Disagree about when we Disagree about Ontology," in Mark Eli Kalderon, ed., *Fictionalism in Metaphysics* (Oxford: Oxford University Press, 2005), pp. 234–86; Cian Dorr, "Quantifier Variance and the Collapse Theorems," *The Monist*, XCVII, 4 (2014): 503–70; and Rohan Sud and David Manley, "Quantifier Variance," in Ricki Bliss and J.T.M. Miller, eds., *The Routledge Handbook of Metametaphysics* (New York; Routledge, 2021), pp. 100–117.

[8] The term 'unproblematically true' comes from Dorr, "Quantifier Variance," *op. cit.*. See Sud and Manley, "Quantifier Variance," pp. 100–117 for more discussion on how one might understand it.

[9] Technically, one might count as a 'quantifier variantist' while only accepting that some of these tribes speak truly or with respect to other ontological debates but not the debate over the Special Composition Question. However, I take such positions would be idiosyncratic and ill-motivated.

[10] Here and throughout, I will be using the bare term 'true' (and 'false') in its 'pleonastic' disquotational sense (Cf. Vann McGee and Brian McLaughlin, "Distinctions Without a Difference," *The Southern Journal of Philosophy*, XXXIII (1994): 203–51). I intend this as a stipulation: I don't mean to presuppose that this captures the meaning of 'true' as used by ordinary speakers or in other theoretical contexts.

[11] Different variantists give different glosses to the notion of metaphysical distinction as applied to UQLs. See Theodore Sider, "NeoFregeanism and Quantifier Variance," *Proceedings of the Aristotelian Society*, Supplementary Volume LXXXI (2007): 201–32; Theodore Sider. "Ontological Realism," in David Chalmers, David Manley, Ryan Wasserman, eds., *Metametaphysics* (Oxford: Oxford University Press, 2009), pp. 384–423; and Theodore Sider, *Writing the Book of the World* (Oxford: Oxford University Press, 2011) for one prominent development. But I'll stay neutral on how to understand the notion.

is the unrestricted quantifier, expressing the notion of existence as opposed to some existence-***like*** notion. And so, when we ask ontological questions about what exists, we must remember that we're asking that question in our language, using our notion of existence. Similarly, we must resist the temptation to treat the UQLs as quantifiers. The unrestricted quantifier (the UQL in our language) ranges over everything, so the UQLs in languages that are more "plenitudinous" than ours (i.e. languages whose speakers would describe our UQL as one that 'doesn't have everything in its domain') may not be quantifiers at all.

Quantifier variantists typically take their view to lead to a common sense theory of composition. According to commonsense, a class of objects composes just in case they satisfy some combination of intuitive desiderata such as *being stuck together*, *being adjacent*, *acting jointly*, etc. For simplicity, let's pretend that our judgments regarding composition correspond to just the single intuitive desideratum of *being stuck together* and say that the theory of commonsense mereology includes the claim that any class composes a distinct object just in case its members are stuck together. Let 'xCy' express the two place predicate 'x composes the distinct object y', '$C'x$' be short for '$\exists y(xCy)$' (that is, 'x composes something'), and 'Sx' express the predicate 'contains members that are stuck together'. Thus, according to commonsense mereology,

CM- $\forall x(C'x \leftrightarrow Sx)$

The path from variantism to CM- goes via the Charity Principle, according to which (without overriding meta-semantic pressure to the contrary) we are to interpret speakers in whatever way makes them most reasonable. Our community treats CM- as unproblematically true. Such treatment is most reasonable if that sentence is in fact true. So there is pressure to interpret CM- as true in our language, if there is such an interpretation. But, there is such an interpretation – indeed there are many! This follows *almost* immediately from the claim that each tribe speaks truly when asserting their respective n-mereological theories in n-ese. We need to only assume that if each tribe speaks truths when asserting their respective n-mereological theories in n-ese, a different tribe that is the same as the original tribe with the exception that they treat 'S' as short for 'contains members that are stuck together to degree n' also speaks truly. CM- is true in the mouths of each of these tribes. And, according to the variantist, there is no meta-semantic pressure that works against these interpretations. Thus, CM- is true in our mouths.[12]

[12] There is arguably a "quicker" path to CM-, which doesn't require positing tribal languages with alternative UQLs. Putnam has pointed out that, by the completeness

So (disquoting) $\forall x(C'x \leftrightarrow Sx)$. Moreover, variantists don't take this conclusion to be borderline or indefinite. They think we can know it (based on the reasoning we just gave) and are in a position to assert it (many variantists do). So, by IPKA, a variantist would accept its definitization:

Commonsense Mereology (CM) $\triangle\forall x(C'x \leftrightarrow Sx)$

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theorem, there will be several interpretations on which CM- (and the rest of consistent "total theory") is true. (See Hilary Putnam, "Models and Reality," *The Journal of Symbolic Logic*, XLV, 3 (September 1980): 464–82.) Some, if not all, of these interpretations will be massively gerrymandered. But unless there is some meta-semantic pressure to counteract charity, the availability of even such gerrymandered interpretations means that CM- will be true in our mouths.

Fans of reference magnetism will, of course, posit such countervailing meta-semantic pressure: ceteris paribus, we prefer interpretations that assign predicates to relatively *natural* extensions and no such interpretation is guaranteed by the completeness theorem. But Sider has argued that quantifier variantists are not well-positioned to accept the notion of naturalness used to block this path to CM-. According to Sider, once we accept that predicates have more-or-less natural semantic values, it's a short step to extend the notion to UQLs, and from there to accept that one UQL is particularly natural. But this conclusion plausibly contradicts the variantist's requirement that "none of the UQLs are metaphysically distinguished". (For more on reference magnetism, see David Lewis, "New Work for a Theory of Universals," *Australasian Journal of Philosophy*, LXI, 4 (December 1983): 343–77 and David Lewis, "Putnam's Paradox," *Australasian Journal of Philosophy*, LXII, 3 (September 1984): 221–36). For more on Sider's understanding of metaphysical distinction and his arguments that UQLs are so distinguished, see the references in fn. **??** above.

Should variantists endorse this second path to CM-? I doubt it: following Putnam's line-of-thought leaves the meanings of our subsentential expressions radically underdetermined, which is implausible. How can they block the path? One option is to posit some source of meta-semantic pressure, other than naturalness, which pushes against gerrymandered interpretations. Perhaps some causal constraint fits the bill. Or perhaps we can construe charity broadly so that interpretations on which we are talking about strange properties of strange objects (that e.g., are extremely far away from us) are simply less charitable, even if those interpretations have us speaking truly. Another option is to reject Sider's extension of the notion of naturalness to UQLs, even if we allow it for predicates. (See Eli Hirsch, "Language, Ontology, and Structure," *Noûs*, XLII, 3 (2008): 509–28, at pp. 522-23, and Eli Hirsch, "Comments on Theodore Sider's *Four Dimensionalism*," *Philosophy and Phenomenological Research*, LXVIII, 3 (May 2004): 658–64, at pp. 660-61.) Alternatively, a variantist might accept the extension and (i) claim that multiple UQLs are perfectly natural or (ii) deny that perfect naturalness of a unique UQL would violate the thesis of quantifier variantism. (See Hirsch, *Quantifier Variance*, *op. cit.*, p. xiii.) Whatever option variantists choose to block Putnam's argument, they will maintain that the argument in the main text for CM- goes through: a central tenant of variantism is that the availability of the various tribal languages combines with charity considerations to entail that common-sense ontological theses are true. So, the meta-semantic considerations that rule out Putnam-style interpretations of our language must not rule out the interpretations corresponding to the tribal languages. (Thanks to a referee for encouraging me to discuss this.)

Lewis and Sider have presented an influential argument against CM.[13] Imagine a sorites series of classes, each of which contains a tabletop and a table base that are increasingly more stuck together. The members of the beginning classes are definitely not stuck together. The members at the end are definitely stuck together. And, for some middling classes, it's indefinite whether their members are stuck together. (For simplicity, suppose that the tops and bases are extended mereological simples and there are no other tabletops and bases outside the sorites.) Letting $\exists_A$ be an existential quantifier restricted to the members of this series, this last claim is: $\exists_A x \triangledown (Sx)$. With some uncontroversial assumptions, this, together with CM, entails:[14]

Indefinite Composition (IC) $\exists_A x \triangledown (C'x)$

But, Lewis and Sider object to IC with the following argument:

LS1. If it's indefinite whether some class composes (IC), then (at some possible world w) $\triangledown \phi^C$ where ϕ^C is the 'counting' sentence that says there are exactly two things: '$\exists x \exists y (x \neq y \wedge \forall z (z = x \vee z = y)$'.

LS2. If $\triangledown \phi^C$ then there are multiple "precisifying" interpretations of ϕ^C, where ϕ^C is true (at w) on one and false (at w) on another.

LS3. But there are no such interpretations.

LS4. So: It's not indefinite whether some class composes.

Here's the idea behind LS1. Imagine a world that contains no objects but the members of the indefinitely composing class (and any composite object they form), arranged as they are in the world with the sorites series. It's indefinite at this world whether there are exactly two things.

Lewis accepts LS2 because he rejects "metaphysical vagueness".[15] (Later on, we'll take a much closer look at what metaphysical vagueness amounts to.)

Sider argues for LS3 as follows.[16] If there are two precisifying interpretations of the counting sentence ϕ^C, then the two interpretations assign

[13] Note CM- is not straightforwardly rebutted by the Lewis/Sider argument. That argument purports to show that the connection between composition and the soritical common-sensical criteria cannot *definitely* hold, not that it cannot hold.

[14] See Proposition **??** and Proposition **??** in Appendix. The needed assumption is that all the classes in this series definitely exist (see claim (**??**) below).

[15] David Lewis, *On the Plurality of Worlds* (Malden: Blackwell Publishing, 1986), at p. 212.

[16] See Theodore Sider, *Four-Dimensionalism* (Oxford: Oxford University Press, 2001), Theodore Sider, "Against Vague Existence," *Philosophical Studies*, CXIV (2003): 135–46, Theodore Sider, "Against Vague and Unnatural Existence: Reply to Liebesman and Eklund," *Noûs*, XLIII (2009): 557–67.

different meanings – $\exists_1$ and $\exists_2$ – to the term '$\exists$'. A natural thought is to cash out these different interpretations in terms of varying domains:

Domains $\exists_1$ and $\exists_2$ are quantifiers ranging over different domains: there is something in the domain of $\exists_1$ that isn't in the domain of $\exists_2$.

But Sider argues that Domains cannot be definitely true.[17] If Domains is definitely true then, definitely there is something that isn't in the domain of $\exists_2$. So definitely $\exists_2$ has a different meaning from the unrestricted quantifier – it isn't a "precisifying" interpretation.

As one would expect, the quantifier variantist rejects LS3. Suppose that objects that are stuck-together to degree 0.435 are indefinitely stuck-together so that, according to CM, it's indefinite whether they compose a distinct object. Consider a world w that includes only a tabletop and table base (and any composite object they form) stuck-together to just that degree. Evaluated at w, the 0.43-ese sentence homophonic to our counting sentence ϕ^C is false and the 0.44-ese sentence that is homophonic to ϕ^C is true. So, there does appear to be two precisifying interpretations of our counting sentence ϕ^C and unrestricted quantifier '$\exists$' which differ with respect to truth-value at w – namely the interpretations of 0.43-ese and 0.44-ese and the UQLs in these respective languages.[18]

Indeed, Sider himself points out that the variantist will be unmoved by his argument.[19] That's because, for reasons independent of vagueness, variantists typically reject Domains: they do not, in general, treat the UQLs in the various tribal languages as quantifiers ranging over different domains. Remember: some of these UQLs are merely quantifier-*like* expressions!

Of course, an opponent can challenge the variantist to give an account of the meanings of these other UQLs which are supposed to serve as precisifying interpretations of our quantifier. The variantist has several responses to this challenge.[20] On one response, inspired by Sider, the variantist is happy to translate sentences between the various tribal languages, but simply declines to assign *in our language* semantic

[17] See Alessandro Torza, "Vague Existence," in Karen Bennett and Dean W. Zimmerman, eds., *Oxford Studies in Metaphysics, Volume 10* (Oxford: Oxford University Press, 2017), pp. 201–34 for one interesting way of retaining a sort of 'vague existence' while accepting Domains.

[18] Eli Hirsch, "Quantifier Variance and Realism," *Philosophical Issues*, XII (2002): 51–73 at p. 66; Eli Hirsch, "Ontological Arguments: Interpretive Charity and Quantifier Variance," in Theodore Sider, John Hawthorne, and Dean W. Zimmerman, eds., *Contemporary Debates in Metaphysics* (Oxford: Blackwell Publishing, 2008), pp. 367–81 at p. 376.

[19] Sider, "Against Vague and Unnatural Existence," *op. cit.*, p. 563.

[20] Thanks to an anonymous referee for encouraging me to discuss this.

values for the various UQLs. Rather, when they need to assign semantic values for the various UQLs, they move to the "bigger" language of Universalese and deliver their compositional semantic theory in this language. Assigning a compositional semantic value for each UQL is easy in Univeralese because in that language Domains *is* true, even if it's not true in our own language. (In Universalese, we can truly say "all of the UQLs in the tribal languages, including those that serve as precisifications of the English UQL, are quantifiers ranging over different domains".) As Sider notes, the inability to assign, in our own language, a semantic value for each UQL "wouldn't undermine quantifier variance" because "quantifier variantists can admit that bigger is better for certain purposes" including the purpose of assigning compositional semantic values for UQLs.[21]

A second responses follows Dorr in giving, in our language, non-set-theoretic semantic values for the various UQLs.[22] In particular, the UQLs express second-order properties. Our own UQL expresses the second-order property of *being a property that is instantiated.* So, '$\exists xFx$' expresses in our language that the property expressed by 'F' has the second-order property of *being instantiated.* Other UQLs express other second-order properties. Perhaps these second-order properties are inexpressible in our own language. Or (continuing to follow Dorr) perhaps we can express them using counter-possible or fictionalist operators. On this picture, the UQL in Universalese is the second-order property of *being a property that would be instantiated were mereological universalism the case* or *being a property that is instantiated according to the fiction of mereological universalism.* And there are other strategies for specifying the meanings of the UQLs besides these two.[23]

Summing up: it looks like the Lewis/Sider argument fails on the supposition of variantism. The relationship between quantifier variance and metaphysical vagueness will be a more subtle one. The first step in our investigation of these subtleties is to notice that variantists – and anyone that accepts CM – are committed to a particularly robust sort of *vague existence.*

[21] Sider, *Writing the Book of the World* op. cit., p. 182.

[22] Dorr, "What we Disagree About," *op. cit.*; Dorr, "Quantifier Variance" *op. cit.*.

[23] See Sud, Manley, "Quantifier Variance," *op. cit.* for an overview of the question of how to give a semantics for each UQL. For more see Arvind Båve, "How to Precisify Quantifiers," *Journal of Philosophical Logic*, XL (2011): 103–11; Sider, "NeoFregeanism," *op. cit.*; Sider, "Against Vague and Unnatural Existence," *op. cit.*; Eli Hirsch and Jared Warren, "Quantifier Variance and the Demand for a Semantics," *Philosophy and Phenomenological Research*, XCVIII, 3 (May 2019): 592–605; and the Dorr articles from the previous footnote.

II. PRELIMINARIES FOR FIRST STEP

In this section, I'll offer a regimentation of the claim of vague existence that has been mistakenly dismissed in the literature. To prepare ourselves, however, I'd like to discuss some subtleties regarding reasoning about vagueness.

II.1. The Barely Tall.* Consider a sorites series of people with height n inches for each positive real number n. For some range of people, it's indefinite whether they are tall. We can ask: are there any tall people that are merely indefinitely so? To put the question more succinctly, let's stipulatively use the term 'barely* F' for something that is F and indefinitely F. (The asterisk is to remind the reader that this is a defined technical term and should not be confused with the ordinary English term 'barely'.[24]) Are there any barely* tall members in the sorites? In our formal language, we're asking whether:

Barely* Tall $\exists x(Tx \wedge \nabla Tx)$

If we let '$\exists_T x$' abbreviate an existential quantifier semantically restricted to tall objects, we're asking whether $\exists_T x \nabla Tx$.

It's tempting to think that there are no barely* tall members of the sorites. Here's one line-of-thought that might lead us to this conclusion:

> In order for something to be barely* tall, it must be tall. But then how can it also be indefinitely tall? If it's tall, then it's not sort-of-tall and sort-of-not tall! That is, if it's tall then it's not indefinitely tall – it's definitely tall.

This line-of-thought rests on a mistake. Recall that we're assuming classical propositional logic. One consequence of this assumption is:

Lesson 1 We should not accept the schema $\phi(x) \rightarrow \neg\nabla\phi(x)$

As is widely acknowledged, accepting the problematic schema in Lesson 1, with classical propositional logic, would mean that there would be no vagueness. Substituting $\neg\psi$ for ϕ (and noting $\nabla\neg\psi$ is equivalent to $\nabla\psi$), the problematic schema also entails the following schema: $\neg\psi(x) \rightarrow \neg\nabla\psi(x)$. This entailed schema, together with the original schema and the law of excluded middle entail $\neg\nabla\psi(x)$ – i.e. that there are no borderline cases!

Here's a second line-of-thought that might lead one to the tempting claim that nothing is barely* tall:

[24] Thanks to an anonymous referee for pointing out that the two diverge.

> It would be wrong to assert of any particular object that it is barely* tall. To assert that it is barely* tall is to assert that it is tall and that it is indefinitely tall. But having committed oneself to the tallness of that object, it would somehow be wrong to draw back from the commitment and claim that it is indefinite whether it is tall. So, there are no barely* tall members.

This line-of-thought also rests on a mistake. The above reasoning is correct to conclude that we should not assert of any particular object that it is barely* tall. By IPKA, to assert of some x that it is barely* tall it must be definitely barely* tall. But (because the definiteness operator distributes over conjunctions) that means x must be definitely tall and definitely indefinitely tall. By the T-axiom, the second conjunct requires that x is indefinitely tall – but that contradicts the first conjunct. So, the above reasoning reveals that there is nothing that is definitely barely* tall: $\neg\exists x \triangle (Tx \wedge \triangledown Tx)$. But we cannot conclude, on this basis, that there are no barely* tall members, or even that it's not definitely the case that there are barely* tall members. Of course, we cannot say of any *particular* thing that *it* is barely* tall. But this does not entail that we cannot say that *something* is barely* tall. It is problematic to move from the claim that nothing is definitely barely* tall to the claim that definitely nothing is barely* tall, or even the weaker claim that nothing is barely* tall:

Lesson 2 If we cannot point to any *particular* object and say of *it* that it is barely* tall, that does <u>not</u> imply that *nothing* is barely* tall. We should not, in general, accept the schema $(\neg\exists x \triangle \phi(x)) \rightarrow (\neg\exists x \phi(x))$.

Indeed, the distinction between these two claims plays a crucial role in classical resolutions of the sorites paradox. In a sorites over F we can say that definitely there is a last F, while respecting the intuitions of vagueness by saying that nothing is definitely the last F.

Both attempts to show that there are no barely* tall members were mistaken. More generally, there is nothing logically inconsistent with the claim that something is barely* tall or even the claim that *definitely* something is barely* tall.[25] With this ground cleared, I'd like to argue that, at least in a sorites series with enough steps, there *are* some barely* tall members.

[25] See Proposition ?? of the Appendix.

II.2. The Collapse Argument. Here's my first argument for this claim. Note that if no one is barely* tall, the tall and the definitely tall "collapse", in the sense that something is tall iff it is definitely tall. That's because, if no one is barely* tall, then everyone that is indefinitely tall is not tall. That is, if no one is barely* tall, everyone that is tall is definitely tall. And, of course, (by the T-axiom) everyone that is definitely tall is tall. So, if no one is barely* tall, then everyone is tall iff they are definitely tall.

But we can reject the consequent: it's not the case that the definitely tall and the tall collapse. Such a collapse does not reflect the meaning of 'definitely': the extension of 'definitely tall' is smaller than the extension of 'tall'. One way to see this is to reflect on how we *use* the terms 'tall' and 'definitely tall'. I claim that we are more stringent in our application of 'definitely tall' than our application of 'tall'. In other words (holding context fixed) we are disposed to assert that something is tall in a strictly wider range of cases than those in which we assert that something is definitely tall.

How would we verify this claim? We might try asking typical English speakers to (i) pick the smallest height-in-cm h that makes someone tall and (ii) to pick the smallest height-in-cm h^* that makes someone definitely tall, and then compare the extensions. Of course, we have to be delicate in how the questions are presented. For instance, we would expect participants to be reluctant to give an answer to either question, given that no answer is definitely correct. And because English speakers may confuse 'ϕ' and 'definitely ϕ', we should make sure the questions are clearly distinguished for the respondent. Finally, because these terms are highly context sensitive it's important that respondents are asked for the heights in the same context (which might shift depending on which question we ask). We can circumvent these three challenges by demanding a numerical answer (and so not allowing answers like 'I don't know') and asking the same respondent both questions at the same time (in order to distinguish the questions and hold context fixed). So, imagine we present an English speaker with both of the following questions at the same time: *what is the smallest height-in-cm h that makes someone tall?* and *what is the smallest height-in-cm* h^* *that makes someone definitely tall?* (and we only allow numerical answers). I am claiming that h will come back less than h^*.

While I know of no study that has carried out this particular experiment,[26] my own linguistic competence does verify this claim. I would

[26] Although Phil Serchuk, Ian Hargreaves, and Richard Zach, "Vagueness, Logic, and Use: Four Experimental Studies on Vagueness," *Mind and Language*, XXVI, 5 (November 2011): 540–73 have conducted related experiments, aimed at testing the so-called "Confusion Hypothesis". Unfortunately, I do not think their results settle the present question.

reply (after a bit of hemming and hawing!) with an h less than h^*. So, I conclude that typical English speakers treat 'being definitely tall' as a more stringent condition than merely 'being tall'. Given that there is nothing logically inconsistent with this use, and absent some sophisticated error-theoretic story, charity considerations weigh in favor of accepting that there is a gap between the tall and the definitely tall. That is, we should conclude that the tall and the definitely tall do not collapse. By *modus tollens*, then, we can conclude that (at least in a sorites series with enough steps) something is barely* tall.

An opponent might worry, however, about the probative value of the elicited linguistic intuitions.[27] For instance, they might point out that the ordinary English term 'definitely' need not correspond to the rather technical usage that we're using the term for in the study of vagueness, in which case our linguistic behavior with respect to the non-technical term 'definitely' shouldn't guide the semantics for the technical term. Or they might worry that the juxtaposition of the two questions suggests to respondents that the answers should be different. Or they might simply not share my own linguistic intuitions regarding h and h^*.

Some of these worries, I think, can be eased. (For instance, with respect to the first complaint: we introduced the technical concept by pointing to a familiar theoretical role that was described in ordinary English. So, we can replace the English term 'definitely' with that theoretical role and rerun our experiment. For instance, we can ask participants to (i) pick the smallest height-in-cm h that counts as tall and (ii) to pick the smallest height-in-cm h^* where we can *know* that the person is tall. Again, relying on my own linguistic competence, I predict h will come back less than h^*.) But instead of trying to address every potential objection, let me bolster my case by offering a different argument in favor of the barely* tall, which, unlike the present argument, relies only on our linguistic behavior with respect to definiteness-free claims.

II.3. The Seamlessness Argument. This argument takes as a starting point a particular sort of seamlessness associated with vague predicates. Suppose we run someone through a forced march sorites, beginning with someone who is 4'0" and ending with someone who is 7'0". For each member of the sorites we require the respondent to classify the member as tall or as not tall. The subject will begin by classifying the members as not tall and will, at some point, switch to classifying them as tall. Moreover, their "willingness" to give an answer changes as the march proceeds. By this I mean that their earliest and latest verdict will be given with the features that are characteristic of judgments

[27] Thanks to an anonymous referee for raising these concerns.

of definiteness: the respondent will give their answer willingly without hemming and hawing, their confidence that their answer is correct will be high, they will resist changing their mind, and so forth. And, for some in-between answers, the respondent will give their verdicts with the features characteristic of judgements of indefiniteness: they will hem and haw and be unwilling to give an answer unless one is demanded, they will have little confidence that their answer is correct, and so forth.

Note two characteristics of how a subject's willingness to answer changes throughout the forced march. First: the willingness changes seamlessly, rather than in fits and starts. Imagine that someone is very willing to declare everything up to a particular point as "not tall" and then suddenly becomes dramatically less willing. Or imagine someone hems and haws when asked about a particular member, and then suddenly becomes dramatically more willing to classify the very next member. Such a person is not using a vague term like 'tall' competently. Rather, a competent speaker's willingness to answer will drop off slowly and continuously during the early stages of the march and will increase slowly and continuously at the latter stages. In other words, their willingness would change in accordance with Figure ?? rather than, say, Figure ??. This phenomenon is familiar from the literature on higher-order vagueness. Indeed, it is a key motivation for thinking that tallness is vague at higher-orders.

Second: a competent speaker's willingness to respond will bottom out in the cases around the point at which they switch from classifying member as not tall to classifying them as tall. In other words, a competent speaker's willingness to respond will seamlessly decrease as they move across the march until they switch from classifying member as "not tall" to classifying them as "tall", at which point their willingness will seamlessly increase. No competent speaker is *more* willing to declare someone who is 5'7" not-tall than someone who is 5'6" and they are not *less* willing to declare someone who is 5'10" tall than someone who is 5'9". In other words, their willingness would change in accordance with Figure ?? rather than, say, Figure ??.

However, these two features of linguistic behavior – (i) a seamless decrease and increase in willingness, that (ii) bottoms out at the point where classifications switch – supports the claim that competent users of vague terms implicitly accept that there are barely* tall individuals. There are a couple ways to see this.

First: We know that some of the reactions in the march fall within a range of willingness that makes for judgments of indefiniteness (that is, we at least implicitly judge that some cases are borderline and not all cases are definite). I'm not sure what range of willingness this is, and

Figure 1. Normal Case

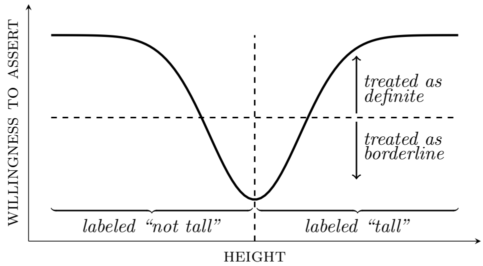


Figure 2. A Deviant Case

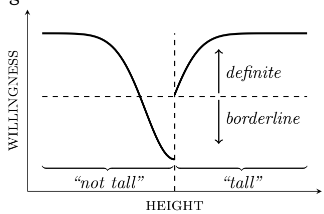
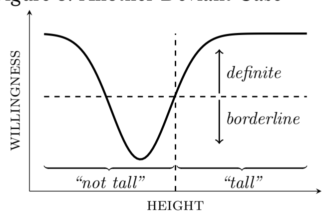


Figure 3. Another Deviant Case



surely it's a vague matter just how low our willingness must be to constitute judgements of indefiniteness. But *any* such level of willingness – anywhere we choose to draw the horizontal line in Figure 1, demarcating judgments of indefiniteness – will include *both* judgments of not-tallness and judgments of tallness.[28] So, we implicitly accept that there are tall objects among the borderline ones – i.e. that there are barely* tall individuals. If we didn't implicitly accept this claim, linguistic behavior reflected by Figures 2 and 3 would be entirely competent. Again, absent some sophisticated error-theoretic story, we should vindicate this judgment by accepting the barely* tall.

Second: What Figure 1 shows us is that (in a sufficiently fine-grained sorites) once we've declared a member *F* – however hesitantly – we don't immediately declare the next member not-*F* ***with the high level of***

[28] **Objection**: What if the level of willingness that makes for judgments of indefiniteness only includes the *very* minimum of the graph? What if only our single *least* willing judgement counts as a judgment of indefiniteness? In that case, it's possible for our willingness to change seamlessly and bottom out around the point where we switch verdicts, without us judging anyone tall with the low willingness that makes for judgements of indefiniteness. **Response**: We can rule out such a case. In such a case, the vagueness of our terms wouldn't be modally robust: if we were ever-so-slightly more willing to issue our verdicts in the forced march sorites, our term 'tall' would not be vague (because there would be no judgements of indefiniteness). But that's absurd: 'tall' would remain vague in such a case.

willingness characteristic of definiteness. Either we also declare the next member F (perhaps with less willingness than the previous judgement) or we declare the next member not-F with the unwillingness, hesitancy, and hemming and hawing characteristic of judgements of indefiniteness. In other words, our linguistic behavior implicitly commits us to the principle that, if some member in the sorites is F then the next one *might* be F. That is, we implicitly accept the principle that if something is F the next one isn't definitely not-F.

Seamlessness $F(x) \rightarrow \neg\triangle\neg F(x')$

Seamlessness entails that some member of the sorites is barely* F. We can see this as follows. Imagine a sorites of increasing height. By classical logic, there is bound to be a last not-tall member in the sorites. By Seamlessness, the next member immediately following the last not-tall member might be not-tall. We also know, of course, that this member is tall (after all, this member follows the *last* not-tall member, so must be tall). So, this member is tall but might be not-tall. That is to say, it is tall but not definitely so – it is barely* tall.

II.4. Epistemicism and Supervaluationism on the Barely Tall. The preceding arguments focused on the linguistic behavior of competent users of vague terms – behavior that any theory of vagueness should vindicate. Here I want to point out that this conclusion – that in a sorites that is fine-grained enough, we will find barely* tall individuals – follows very naturally from the most popular theories of vagueness, and so should be welcomed by their proponents.

Consider the most popular form of epistemicism, as developed by Timothy Williamson.[29] According to the epistemicist, the extensions of vague terms are classical, and vagueness results from the fact that these classical extensions are highly sensitive to global patterns of use. There are counterfactual worlds where, compared to the actual world, our terms are used every-so-slightly differently and this slight difference in global patterns of use generates slight differences in the extensions of the homophonous words used in those worlds. Indeed, the difference in use – and the corresponding difference in extensions – is so slight that actual individual speakers' dispositions to apply the terms are not sufficiently sensitive to these differences. Call such worlds *semantically indiscriminable* from the actual world. According to the epistemicist, x is definitely tall iff in any semantically indiscriminable world, the term 'tall' as used in that world includes x in its extension.[30]

[29] Timothy Williamson, *Vagueness* (London: Routledge, 1994).

[30] At least to a first approximation. See Michael Caie, "Vagueness and Semantic Indiscriminability," *Philosophical Studies*, CLX, 3 (September 2012): 365–77 on difficulties for an epistemicist analysis of the notion of indefiniteness.

The term 'tall' is vague because there are semantically indiscriminable worlds where the term has a different cut-off than it actually does. But surely some of these semantically indiscriminable worlds are ones where the term 'tall' has a higher cut-off than it actually does, as a result of its being used ever-so-slightly more stringently. (After all, if this isn't the case – if all the semantically indiscriminable worlds are ones where the term has a lower cut-off than it actually does – our ability to detect small shifts is *perfectly* fine-tuned to detect slightly more stringent usages despite lacking that ability with respect to slightly less stringent usages. That's implausible.) But that entails that (in a sufficiently fine-grained sorites) something is barely* tall. Here's why. If there is a semantically indiscriminable world where the cut-off for "tall" is higher than it actually is, then anything in the gap between the actual cut-off for "tall" and this higher cut-off will be tall (because it meets the actual cut-off) but not definitely so (because it fails to meet the cut-off at all semantically indiscriminable worlds). Epistemicists should welcome this result, especially given our earlier arguments that our linguistic behavior commits us to the barely* tall.

The supervaluationist should also embrace this result. In order to account for higher-order vagueness, Williamson argues that the supervaluationist should introduce the notion of one precisification p accessing another precisification q.[31] We can then say that 'Definitely ϕ' is true on a precisification p iff 'ϕ' is true on all precisifications that are accessible from p. Whether or not q is accessible from p will depend on whether the classical extensions q assigns are similar enough to p's own assignments. (In Williamson's gloss: "an interpretation might admit just those interpretations that are reasonable by its lights, because they do not differ from it by too much.").[32] This naturally motivates various structural features on the accessibility relation and corresponding results for the logic of the definitely operator. For instance, because p is most similar to itself, it's natural to require that p deem itself reasonable. Thus the supervaluationist can explain the T schema: $\triangle\phi \rightarrow \phi$. And because being similar enough is non-transitive, it's natural to deny schema 4: $\triangle\phi \rightarrow \triangle\triangle\phi$. And, because the notion of being similar enough may be treated as vague, different precisifications can be more or less demanding in their applications of the notion. So it's natural for the supervaluationist to allow p to treat q as similar enough without requiring that q treat p as similar enough. This means the supervaluationist need not accept the B schema ($\phi \rightarrow \triangle\Diamond\phi$).

[31] *Ibid.*, 156–162.
[32] *Ibid.*, 159.

What other structural features are natural for the supervaluationist to adopt? Suppose p assigns cutoff h^p to "tall" and deems some other precisification q similar enough, where q assigns cutoff h^q to 'tall' and $h^q < h^p$. That is, p treats a slightly lower cut-off for 'tall' as similar enough to the cut-off p assigns. It's natural for the supervaluationist to require that p also treat some slightly higher cut-off as similar enough to the cut-off that p assigns: if some slight *decrease* to p's cut-off is similar enough, then surely so is some slight *increase*. But this structural requirement guarantees the existence of the barely* tall. We can see this as follows. Pick an arbitrary precisification p that conforms to that structural requirement. Suppose the claim that there is a borderline tall member ($\exists x \triangledown Tx$) is true on that precisification. Then the precisification must deem some other precisification of 'tall' as reasonable. By the structural requirement, it must deem a precisification that assigns a *higher* cut-off to 'tall' as reasonable. But then, according to p, anything with a height that falls in-between its own cut-off and the reasonable higher cut-off is an instance of the barely* tall: the claim '$\exists x Tx \wedge \neg \triangledown Tx$' is also true on p. So, the conditional "if something is borderline tall then something is barely* tall" is true on p. But p was arbitrary, so the conditional is true simpliciter. And of course the antecedent is true: something is borderline tall. So, by modus ponens, we can conclude that something is barely* tall.

Admittedly, the notion of similarity is unclear enough that a skeptical supervaluationist could reject the proposed structural constraint: they might claim that lower cut-offs for 'tall' are similar enough but no higher cut-off is. But *prima facie* that is a strange position to take. And, more importantly, given my previous arguments that our linguistic behavior commits us to the barely* tall, it's an ill-motivated one too: the supervaluationist should follow the epistemicist and welcome the barely* tall.

Summing up: I first rebutted arguments that purported to show that it's logically inconsistent to accept the existence of the barely* tall: we learned that these seemingly-plausible arguments rested on mistakes. Having established its consistency, I then argued that, for vague predicates F, there will be barely* F members of a fine-grained sorites. Let's turn now to trying to understand the claim that existence is vague.

II.5. The Thesis of Vague Existence. The term 'vague existence' has been used to refer to various different theses. But, I'd like to focus on a particular thesis that I'll call Bare Existence (BE):

BE $\exists x \triangledown \exists y(y = x)$

In order to illustrate the parallel between BE and claims like Barely Tall, we can define an existence predicate Ex as short for $\exists y(y = x)$ and let

'$\exists_E x$' abbreviate an existential quantifier trivially restricted to existing objects. Then, BE is equivalent to:

- $\exists x(Ex \wedge \triangledown Ex)$
- $\exists_E x \triangledown (Ex)$

In other words, the claim that there is a borderline case of existence is the same as the claim that something barely* exists.

Several authors have thought that BE is plainly absurd. But, upon careful reflection, we can see that this resistance deploys lines of thought paralleling the mistaken ones that purported to show the absurdity of the barely tall.[33]

Consider first David Lewis's reactions to BE:

> There is such a thing as the sum, or there isn't. It cannot be said that…there sort of is and sort of isn't. What is this thing such that it sort of is so, and sort of isn't, that there is any such thing?[34]

One may feel tempted to read the following argument into Lewis's rhetorical question:

> In order for something to barely* exist – to be a borderline case of existence – it must exist. But then how can it also indefinitely exist? If it exists, then it's not sort-of-existing and sort-of-not-existing! That is, if it exists then it doesn't indefinitely exist.

Such reasoning ignores Lesson 1 by relying on the problematic schema $\phi(x) \rightarrow \neg\triangledown\phi(x)$ as applied to the existence predicate. Absent some special and non-question-begging reason to think this schema is unproblematic in the case of existence, this strawman can be dismissed for the same reason we dismissed the parallel argument against Barely Tall.

A second line-of-thought that might lead someone to find BE nonsensical is discussed by Katherine Hawley:

> The thought is that to posit an object, to quantify over it or refer to it is already to be committed to its existence. Having committed oneself to the existence of an object…it would somehow be wrong to draw back from the commitment and claim that it is indeterminate whether the object exists.[35]

[33] In his talk ("Vague Existence") at the New York Institute of Philosophy conference on Metaphysics in Higher Order Languages (where a version of this paper was also given), Jeff Russell discussed related confusions around vague existence. Thanks to Jeff and other attendees of the conference for helpful discussion.

[34] Lewis, *Plurality*, *op. cit.*, 212–213.

[35] Katherine Hawley, "Vagueness and Existence," *Proceedings of the Aristotelian Society*, CII, 1 (June 2002): 125–40 at pp. 134–55.

This reaction, however, ignores Lesson 2. Just as we were right to conclude that we cannot say of anyone that they are barely* tall, Hawley is right that it would be wrong to say of a particular thing that *it* barely* exists. To be in a position to assert of some x that it indefinitely exists, x must definitely indefinitely exist, which is absurd. So, nothing definitely barely* exists. Recall, however, that we should not move from this claim to the claim that nothing barely* exists. Indeed, I think it's the failure to appreciate this distinction that has lead others, such as Elizabeth Barnes, to worry that BE is "hard, if not impossible, to make sense of".[36] As we've just seen, it would be wrong to conclude, on the basis that nothing definitely indefinitely exists, that nothing indefinitely exists. In response, it's tempting to concede this point, but retain the slightly weaker conclusion that it's never *definite* that something indefinitely exists. After all, one might reason, if it's definite that some x barely* exists (i.e. is a borderline case of existence), then x would definitely barely* exist, which is nonsense. But, recall from above, that even this weaker conclusion rests on a logical mistake: we cannot move from the claim that it's definite that some x is barely* F to the logically inconsistent claim that some x is such that *it* definitely is barely* F. The upshot here is that, for all logic tells us, we may be in a position to assert that something indefinitely exists, so long as we aren't saying of any particular thing that it indefinitely exists.[37]

At long last, we have all the ingredients to take the first step in our two-step argument: I'll argue that the variantist is committed to BE.

III. FIRST STEP: VARIANTISM REQUIRES VAGUE EXISTENCE

Recall that, according to the variantist:

CM $\triangle\forall x(C'x \leftrightarrow Sx)$

Consider again the sorites series of classes with members that contain (mereologically atomic) table tops and table bases that are progressively more stuck-together. Presumably, all of the classes in the sorites definitely exist (otherwise, we'd have established BE immediately!). Letting $\forall_A$ be a universal quantifier restricted to the members of this series, we have:

(1) $\forall_A x\triangle(Ex)$

[36] Elizabeth Barnes, "Arguments Against Metaphysical Indeterminacy and Vagueness," *Philosophy Compass*, v, 11 (2010): 953–64 at p. 960.

[37] See Proposition ?? of the Appendix.

Together with CM, (??) entails:[38]

(2) $\forall_A x \triangle (Sx \leftrightarrow C'x)$

As we argued above, in a sorites series for F with enough steps, something will be barely* F. In the present sorites, some class contains members that are stuck together, but only barely*:

(3) $\exists_A x(Sx \wedge \triangledown Sx)$

Together with (??) this entails that some class barely* composes:[39]

(4) $\exists_A x(C'x \wedge \triangledown C'x)$

With one more assumption (??) entails BE. The needed assumption is that the tables in this sorites don't have vague parts. More carefully: if a class in the sorites composes a table then, definitely, if that table exists it is composed of that class rather than some class of other parts:

(5) $\forall_A x \forall y(xCy \rightarrow \triangle(Ey \rightarrow xCy))$

At first blush (??) might seem controversial. After all, it might seem odd to ignore the possibility of vague parthood, especially in the context of vague composition. However, when we reflect on what (??) is and is not claiming, we can see that it is surely true. Imagine a class in the sorites composes a table. (??) denies that it's vague whether *that very table* has some *other* class as its parts. One way to make the plausibility of the claim more apparent is by adding a few details to the imagined sorites. Imagine (i) that the world doesn't contain any concrete objects except for the table tops and table bases (and anything they compose); and (ii) that the table top-base pair in one class in the sorites is far (say 1 mile) away from the table top-base pair in the subsequent class in the sorites. The following principle is plausible (especially when it comes to tables and their parts): for any two objects, if one might have the other as a part, then the two objects must be in regions that are near or overlapping one another. This principle doesn't preclude the possibility of objects with vague parts: for all I've said, Kilimanjaro is such that it's indefinite whether it has a particular rock as a part. The principle merely requires that its vague parts are in Tanzania instead of, say, New York City. Now, for any table in the sorites, definitely, if it exists, it will be composed of some table top-base pair and will be in the

[38] See Proposition ?? in the Appendix.
[39] See Proposition ?? in the Appendix.

region containing that pair – it won't be near any other such pair. Thus it cannot be vague whether any other table top-base pair composes it. That is, it won't be vague what parts it has – whatever parts it has will definitely be its parts so long as it exists.

Together with (??), (??) entails BE. Roughly: (??) says that if a class composes a table then, definitely, that table cannot exist if that class fails to compose it. But (??) says that some class composes some table, but might have failed to compose anything at all. So, the composed table doesn't definitely exist.[40] Of course, we cannot point to any particular table and say that it indefinitely exists. But we can be confident that (at least in a sorites with enough steps) there is such an indefinitely existing table.

This concludes step 1 of the main argument: the variantist is committed to BE. For the rest of the paper, I'll explore BE's relationship to metaphysical vagueness.

IV. PRELIMINARIES FOR SECOND STEP: WHAT IS METAPHYSICAL VAGUENESS?

Recall the guiding slogan of metaphysical vagueness:

Slogan There is "vagueness in the world, vagueness in what there is as opposed to our descriptions or knowledge of what there is".[41]

As I mentioned in the introduction, Slogan has two elements – a positive element saying where metaphysical vagueness *is* ('in the world itself') and a negative element saying where metaphysical vagueness *is not* ('in our descriptions or knowledge of what there is'). Different authors' regimentations of metaphysical vagueness have focused on one or other of these two elements. And, there are various ways we might choose to cash out each. For instance, there are various ways to give voice to the positive element, that vagueness is 'in the world':

- There is a state of affairs that indefinitely obtains[42]
- There is an indeterminate or incomplete state of affairs that (definitely) obtains[43]

[40] See Proposition ?? in the Appendix.

[41] Barnes, "Ontic Vagueness", *op. cit.*, p. 601.

[42] Timothy Williamson. "Vagueness in Reality," in *The Oxford Handbook of Metaphysics* (Oxford: Oxford University Press, 2003), pp. 690–716. See also Barnes, "Ontic Vagueness," *op. cit.*, at p. 611 and Nathan Salmon. "Vagaries about Vagueness," in Richard Dietz and Sebastiano Moruzzi, eds., *Cuts and Clouds* (Oxford: Oxford University Press, 2010), pp. 131–48 at pp. 133–34.

[43] Jessica Wilson, "A Determinable-based Account of Metaphysical Indeterminacy," *Inquiry*, LVI, 4 (2013): 359–85; Jessica Wilson, "Are There Indeterminate States of Affairs?

- The indefiniteness operator, or facts involving it, are fundamental or ungrounded.[44]

- There are ungrounded or fundamental facts that are indefinite.[45]

Other theses give voice to the Slogan's negative element, that vagueness is "not in our descriptions or knowledge":

- Vagueness would remain in a perfectly precise language: A sentence S in a perfectly precise language is still vague.[46]

- Facts involving the indefiniteness operator are not explained by facts about language or mental states.[47]

I won't be asking which of these many theses is the "correct" way to understand our Slogan, or even one particular element of that Slogan. I take a more ecumenical stance. Each thesis can be regimented and investigated on its own terms. And each provides us with some way of giving voice to elements of our guiding slogan. Nor should we demand that the theses stand and fall together: we might have "metaphysical vagueness" of some flavors but not others.[48] Indeed, I'll argue that this is the case for the quantifier variantist. In the rest of this section, I'll argue that the variantist's commitment to vague existence comes with

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Yes," in Elizabeth Barnes, ed., *Current Controversies in Metaphysics* (New York: Routledge, 2017), pp. 105–19.

[44] Sider, *Writing the Book*, *op. cit.*, p. 137. Elizabeth Barnes and J.R.G. Williams, "A Theory of Metaphysical Indeterminacy," in Karen Bennett and Dean W. Zimmerman, eds., *Oxford Studies in Metaphysics, Volume 6* (Oxford: Oxford University Press, 2011), pp. 103–48 at pp. 106–7.

[45] Cf. Elizabeth Barnes, "Fundamental Indeterminacy," *Analytic Philosophy*, LV, 4 (December 2014): 339–62.

[46] Cf. Barnes, "Ontic Vagueness," *op. cit.*, at pp. 603–605.

[47] *Semantic* or *epistemic* accounts of vagueness are often described as *explaining* facts involving indefiniteness in terms of language or mental states and are juxtaposed with *metaphysical* accounts. Compare: Lewis, *Plurality*, *op. cit.*, p. 212 uses the locution "the reason it's vague" and Barnes, "Ontic Vagueness," *op. cit.*, at pp. 603 uses the locution "sources of vagueness".

[48] For instance, in Rohan Sud, "Vague Naturalness as Ersatz Metaphysical Vagueness," in Karen Bennett and Dean W. Zimmerman, eds., *Oxford Studies in Metaphysics, Volume 11* (Oxford: Oxford University Press, 2018), pp. 243–277, I argue that the thesis that there are fundamental facts that are indefinite does not entail any of the other theses associated with "metaphysical vagueness". And Bradford Skow, "Deep Metaphysical Indeterminacy," *The Philosophical Quarterly*, LX, 241 (October 2010): 851–58 has argued that certain interpretations of quantum mechanics use the notion of "metaphysical indeterminacy" in a way that cannot be accommodated by the semantics for metaphysical vagueness presented in the works of Barnes and Williams. On one view, the upshot of Skow's discussion is that the notion of metaphysical vagueness appealed to in quantum mechanics differs from the one Barnes and Williams are trying to regiment.

a commitment to one particular thesis that gives voice to the positive element of our Slogan. And later, I'll argue that they are not committed to various theses giving voice to the negative element of our Slogan.

IV.1. Positive Metaphysical Vagueness. According to the positive element of our slogan, metaphysical vagueness is vagueness "in the world itself" or "in reality". As Williamson and others argue, one way of regimenting the claim that reality is vague is as the claim that there are vaguely obtaining states-of-affairs:[49]

> Reality is vague if and only if at least one state-of-affairs is borderline. For if reality is vague, it is vague how things are, so for some way it is vague whether things are that way; thus, for some state-of-affairs *S*, it is vague whether *S* obtains. Conversely, if for some state-of-affairs *S* it is vague whether *S* obtains, for some way it is vague whether things are that way, so it is vague how things are; thus reality is vague.[50]

Compare metaphysical possibility. One way to cash out the claim that the 'world is infused with possibility' – that modality is a worldly phenomenon – it to say that it's contingent what states-of-affairs obtain. What is a state-of-affairs? For any object and any property we can talk about the state-of-affairs of that particular object instantiating that property (e.g. the state-of-affairs *Williamson's being a shepherd*). These state-of-affairs we can call *non-qualitative.* Other states-of-affairs don't involve a particular object (e.g. *there being a philosopher*). We can call these *qualitative* states-of-affairs. Some states-of-affairs obtain while others do not. And, some obtain contingently.

If contingently obtaining states-of-affairs is a way the world itself is 'infused with possibility', this suggests that *indefinitely* obtaining states-of-affairs represents a way the world itself would be infused with vagueness. So, on the Williamsonian proposal, metaphysical vagueness is understood as the claim that there is a state-of-affairs such that it's indefinite whether that state-of-affairs obtains. That's one way (albeit not the only way) to give clear sense to the positive element of Slogan. I'll focus on this particular regimentation in what follows, and so will refer to this thesis as 'positive metaphysical vagueness'.

IV.1.1. Restricting Existential Generalization. One might worry that this characterization of vagueness in reality follows trivially from the banal claim that it's indefinite whether Harry is bald. After all, if it's indefinite whether Harry is bald, then it's indefinite whether the state-of-affairs of

[49] I focus on Williamson because he offers the most formally rigorous development of this strategy and does so using higher-order logic (which will be relevant below).

[50] Williamson, "Vagueness in Reality" , *op. cit.*, p. 701.

Harry's being bald obtains. So, isn't there a state-of-affairs – *Harry's being bald* – that indefinitely obtains?

As Williamson points out, this suggestion depends on existentially generalizing across the indefiniteness operator, which is suspect. We are happy to restrict existential generalization in the case of possibility: it's contingent whether the number of planets is odd but there is no number such that it's contingent whether it's odd. We can retain Williamson's understanding of vagueness in reality by making similar restrictions in the case of indefiniteness. This restriction is independently motivated by cases of (i) vague identity statements and (ii) the problem of the many.

Consider vague identity statements. Following Gareth Evans, many philosophers deny de re vague identity ($\exists x \exists y \triangledown (x = y)$).[51] But, it seems obvious that there are cases of indefinite identity (e.g. it's indefinite whether the tallest short man is Harry). As many have pointed out, we can accept such vague identity statements while rejecting de re vague identity so long as we restrict existential generalization.[52]

A similar conclusion follows from problem of the many cases. For various reasons, philosophers have been hesitant to accept the claim that there are 'vague objects' – objects with vague parts. That is, they accept $\neg(\exists x \exists y \triangledown (yPx))$ (where 'xPy' says x is part of y). But they also accept that it's indefinite whether a particular rock x is part of Kilimanjaro: $\exists x \triangledown (xPk)$. Once again, these two conclusions can be jointly held if we restrict existential generalization.[53]

Indeed, we can bring the lessons of vague identity and the problem of the many cases together to explain away the appearance of "vague objects". Suppose again that it's indefinite whether a particular rock x is part of Kilimanjaro. Assume there are a plenitude of distinct (albeit massively overlapping) "precise" giant hunks of rock $r_1, r_2, \ldots$ in the vicinity of Kilimanjaro. If there are no vague objects, then it's not as if Kilimanjaro is some distinct "vague object" over-and-above these precise hunks of rock. Rather, there are *only* these precise hunks of rock, and it's simply indefinite which one is Kilimanjaro.

On the same basis, we can accept that it's indefinite whether *Harry's being bald* obtains without accepting vagueness in reality – without accepting vaguely obtaining states-of-affairs. There are a plentitude of

[51] Gareth Evans, "Can There be Vague Objects?" *Analysis*, XXXVIII, 4 (1978): 208.

[52] Richmond H. Thomason, "Identity and Vagueness," *Philosophical Studies*, XLII (1982): 329–32; David Lewis, "Evans Misunderstood," *Analysis*, XLVIII, 3 (June 1988): 128–30.

[53] See Vann McGee, "Kilimanjaro," *Canadian Journal of Philosophy*, Supplementary Volume XXIII (1997): 141–63.

distinct "precise" states-of-affairs, which we might describe as *Harry's having less than 20,000 hairs*, *Harry's having less than 20,001 hairs*, etc. But, if there is no vagueness in reality – no positive metaphysical vagueness – then it's not as if there is some distinct "vaguely obtaining state-of-affairs" over-and-above all of these precise states-of-affairs. Rather there are *only* these precise states-of-affairs, and it's simply indefinite which one is the state-of-affairs of *Harry's being bald*.

The upshot is the existential generalization schema must be restricted to cases where the terms are variables or don't appear in the scope of the indefiniteness operator:[54]

Restricted Existential Generalization $\vdash \phi(t) \rightarrow \exists x \phi(x/t)$ if t is a variable or t occurs outside the scope of the indefiniteness operator.

IV.1.2. Higher-Order Logic. Williamson's proposal is that we can understand vagueness in reality in terms of vaguely obtaining states-of-affairs. He adds one more twist to this suggestion. Instead of reifying states-of-affairs and applying a predicate 'obtains' to them, we can use higher-order logic, understood primitively, to regiment such talk. Here's what he says:

> ...let us suppose that expressions of different grammatical categories are all correlated with different elements of reality, their ontological correlates. The ontological correlate of a sentence is a state-of-affairs, just as the ontological correlate of a singular term is an object and the ontological correlate of a predicate is a property or relation.... Since states-of-affairs are the ontological correlates of sentences, the most natural way to generalize over states-of-affairs is by quantifying into sentence position.[55]

Following Williamson, let's add to our regimented language quantification directly into predicate position for any *n*-place predicate, including 0-place predicates. We'll add to our language quantifiers $\exists^n$ and variables $X^n, Y^n, ..$ for quantification into the position of an *n*-place predicate. So, sentences like '$\exists^1 X^1(X^1 a)$' or '$\forall^0 X^0 \exists Y^0(X^0 \wedge Y^0)$' are well-formed. (Where there is no risk of ambiguity, we'll drop the superscripts.) Such quantification is not to be understood substitutionally nor should it be understood as disguised first-order quantification over

[54] Similarly for the version of Leibniz's Law that applies to sentences with names occurring in the scope of the definiteness operator. Instead, we can accept a universally-generalized version of the law or a version that applies only to names that don't fall in the scope of a definiteness operator. β-Conversion must be similarly restricted in the lambda calculus.

[55] Williamson, "Vagueness in Reality," *op. cit.*, pp. 699-700.

an ontology of properties, sets, or states-of-affairs. Instead, I will assume that we have a primitive grasp of higher-order quantification. So, although I will sometimes translate sentences of higher-order quantification by reifying properties or states-of-affairs, such talk is not to be taken strictly and literally. Understanding quantification over states-of-affairs as quantification into sentence position, Williamson understands the claim that there is vagueness in reality as the claim $\exists S \triangledown(S)$.[56]

One advantage to construing the debate over vaguely obtaining states-of-affairs in higher-order terms is that this debate is no longer held hostage to traditional disputes over the existence and nature of objects like states-of-affairs or properties. Separating such disputes is particularly helpful in the context of assessing ontological deflationism because the answer to the traditional disputes (over the existence and nature of states-of-affairs and properties) often turns on whether one accepts the deflationism that one is assessing.

Another advantage to construing the debate over vaguely obtaining states-of-affairs in higher-order terms is that it allows us to regiment the distinction between non-qualitative and qualitative states-of-affairs. Recall that a non-qualitative state-of-affairs is, loosely speaking, a state-of-affairs of a particular object instantiating a particular property, such as the state-of-affairs of *Williamson's being a philosopher.* These states-of-affairs are 'the ontological correlates' of sentences composed of a directly-referring term and a one-place predicate. So, if we want to quantify over just these non-qualitative states-of-affairs and say something $\phi()$ about them, we can quantify over objects and one-place properties: $\forall X \forall x \phi(Xx)$. So, the claim that some non-qualitative state-of-affairs indefinitely obtains can be formulated in higher-order terms as: $\exists X \exists x \triangledown(Xx)$.

This last claim is the important upshot for our purposes: we have a sufficient condition for positive metaphysical vagueness – namely a vaguely obtaining non-qualitative state-of-affairs – which we can state in purely logical terms:

Positive Metaphysical Vagueness A sufficient condition for positive metaphysical vagueness is: $\exists X \exists x \triangledown(Xx)$

[56] Of course, even construed in these terms, the banal claim $\triangledown(Bh)$ should not entail that there are vague states-of-affairs. Because we're working in a higher-order setting, we must extend our restriction on existentially generalizing on non-variables from the first-order quantifier to higher-order quantifiers.

V. SECOND STEP: VAGUE EXISTENCE REQUIRES POSITIVE METAPHYSICAL VAGUENESS

We are now in a position to present the second step of our argument for the Subtle Answer: the variantist's commitment to vague existence requires a commitment to positive metaphysical vagueness. That argument relies on two additional claims, which we can also state in purely logical vocabulary.

The first claim is Everything Instantiates Something (EIS):

EIS $\forall x \exists X(Xx)$

According to EIS, for any individual, there is some property that it has. This claim is independently plausible. It also follows from the relatively weak[57] comprehension schema: $\vdash \exists X \forall x(Xx \leftrightarrow \phi)$. When we instantiate ϕ with '$x = x$', the resulting axiom claims that there is a property that is instantiated by everything that is self-identical. But everything is self-identical. So, there is a property that everything instantiates, which entails EIS.

The second premise is what I call Existence is Easy (EE):

EE $\forall X \forall x \triangle(Xx \rightarrow Ex)$

In English: every property and every individual is such that it's definite that, if that individual has that property, then that individual is something. According to EE, existence (understood in the thin sense of merely *being something*) is the easiest property to have, so the property of existence comes along with any other property. The modal analogue of this principle is sometimes called the claim of *serious actualism*, proponents of which include both "contingentists" like Plantinga and Stalnaker as well as "necessitists" like Williamson.[58]

When evaluating EE, we have to distinguish it from two other claims that are much less controversial. On the one hand, the claim '$\triangle \forall X \forall x(Xx \rightarrow Ex)$' is uncontroversially true (the embedded claim is

[57] A stronger comprehension schema, analogous to the modal schema advocated by Williamson, *Modal Logic as Metaphysics*, *op. cit.* would be '$\vdash \exists X \triangle \forall x(Xx \leftrightarrow \phi)$'. Endorsing this stronger schema, however, begs the question: the schema implicitly assumes that there is positive metaphysical vagueness. Letting 'Bx' instantiate ϕ in the strong schema, the resulting theorem asserts the existence of a particular property that is definitely instantiated by all and only the bald items that exist. Note, however, that there is some particular thing that definitely exists but is neither definitely bald nor definitely not bald. So, that particular thing will indefinitely instantiate that particular property.

[58] See Alvin Plantinga, "On Existentialism," *Philosophical Studies*, XLIV (1983): 1–20; Robert Stalnaker, *Mere Possibilities* (Princeton: Princeton University Press, 2012); Williamson, *Modal Logic as Metaphysics*, *op. cit.*.

a logical truth and the unembedded claim is derivable by the rule of definitization), as is its modal analogue. On the other hand, the variantist is under no pressure to accept '$\forall X \forall x(Xx \rightarrow \triangle Ex)$' because they don't think that everything definitely exists, just as the contingentist is under no pressure to accept its modal analogue. Assuming the latter claim would straightforwardly beg the relevant question. EE, by comparison, must be assessed on the basis of highly-theoretical considerations.

Some arguments in favor of EE repurpose those given for its modal analogue, serious actualism – the premises remain plausible when the operators are read as expressing definiteness instead of metaphysical modality. The following, for instance, is based on an argument given by Stephanou for serious actualism.[59] The argument rests on two claims. The first claim is that for any object, definitely, if it is in a set s, there is a subset s' of s that doesn't contain that object but contains everything else in s. The second claim is that for any object and property, definitely, if the object has the property, then there is a set that contains that object (for example, the extension the property might have).[60] (For an opponent of EE, this second claim implies that there is some object such that it might have been in a set even though it isn't anything. But this does not beg the question against such an opponent – indeed it would be terribly ad hoc for them to deny it. Given that the opponent of EE thinks that the object might have exemplified a property without being anything, surely it might be in a set without being anything – if it is "robust enough" to exemplify a property, it is robust enough to be in a set!)

Using these two claims, we have our desired reductio. Suppose existence isn't easy so that, for some x and property X, x might have X without being anything. By the second claim, this means it might be in a set s without being anything. With the first claim, x might be in s without being anything, *and fail to be in a subset of s, s′, that contains everything in s but x*. But that's just to say x might be in some set s and fail to be in a subset s' where subset s' contains *everything* in s (because it might be that, although x is in s and not s', x isn't anything!). But according to (the definiteness of) the axiom of extensionality: definitely if a subset of s, s', contains everything in s, then s is identical to s'. So, the denier of EE is committed to the absurd claim that there is some object that might be and not be in the very same set!

[59] Yannis Stephanou, "Serious Actualism," *Philosophical Review*, CXVI, 2 (2007): 219–50.
[60] In the face of recherche counterexamples, we can limit the principle to concrete objects.

There are also considerations in favor of EE that trade on unique features of indefiniteness. Here's one. Recall the plausible assumption IPKA according to which, of a particular object, we can know that it is a certain way only if it is definitely that way. So, if a precise hunk of rock *r* in the vicinity of Mt. Kilimanjaro is not definitely a mountain, we cannot know that it is a mountain, even if we can know that it is a large rock. Similarly with assertion: it would be inappropriate to assert of *r* that it is a mountain. In slogan form: *de re knowledge and assertibility require definite instantiation.*

Now, note that, while we can have de re knowledge about *definitely* existing objects and are in a position to make various assertions about them, we cannot have de re knowledge about *indefinitely* existing objects and it's inappropriate to make assertions about them. In so far as we think that there are tables in our table-sorites that indefinitely exist (and we've already argued that there are), of these indefinitely existing objects, we cannot know *anything* about them. A mutually reinforcing observation can be made about assertion: it would be inappropriate to point to the indefinitely existing table and say of it that, for example, it is sturdy. (Our reaction to such an assertion would be similar to someone who asserted that *r* is a mountain.) At best, we can make our assertion conditional on the table's existence ('if there is a table there, then it is sturdy') (compare the appropriateness of 'if *r* is a mountain, that it is a tall mountain'). In slogan form: *de re knowledge and assertibility require definite existence.*

EE offers the best explanation of these latter norms, by explaining them in terms of the former. According to EE, a vaguely existing object cannot definitely instantiate a property, and because de re knowledge and assertibility require definite instantiation, we can explain why de re knowledge / assertions require definite existence.

Of course, a resolute skeptic of EE can deny these arguments (compare the longstanding debate over serious actualism). In response to the first, for instance, they could hold that some objects might be in a set without there being a subset that excludes them. And in response to the second argument, they could hold that we can know about particular indefinitely existing objects. But, on their face, these are strange things to say. So together these arguments create significant pressure to accept EE.

V.1. The Argument. With EIS and EE we can show that BE requires positive metaphysical vagueness. We'll show this by taking as a premise that there is no positive metaphysical vagueness, and arguing that existence is not vague (the negation of BE). Here's the argument:[61]

[61] See Appendix §?? for a proof and the required axioms and inference rules.

B1. $\forall x \forall X(Xx \rightarrow \triangle(Xx))$ (No Positive Metaphysical Vagueness)

B2. $\forall x \exists X(Xx)$ (EIS)

B3. $\forall x \exists X \triangle(Xx)$ (from B1 and B2)

B4. $\forall X \forall x \triangle(Xx \rightarrow Ex)$ (EE)

B5. $\forall x \triangle(Ex)$ (from B3 and B4)

The argument's informal line of thought is as follows. We assumed there was no positive metaphysical vagueness. That means that any state-of-affairs that obtains – including non-qualitative states-of-affairs – will *definitely* obtain (B1). But, every individual instantiates some property and so participates in some non-qualitative state-of-affairs (EIS). Thus every individual is part of a state-of-affairs that definitely obtains (B3). That's another way of saying every individual definitely instantiates some property. But, definitely, instantiating a property requires existence, because existence is the easiest property to have (EE). So, because every individual definitely instantiates a property, which in turn requires existence, every individual definitely exists – existence is not vague! In short: every individual instantiates some property and, if there is no metaphysical vagueness, definitely instantiates that property and thus definitely instantiates the weakest property of existence.

Abstracting from the details, the underlying idea of the argument for Step 2 is surprisingly simple. When we carve the world up into individuals, we thereby carve the world up into states-of-affairs. So, vague existence leads to vagueness in which states-of-affairs obtain – vagueness in existence infects states-of-affairs. But vaguely obtaining states-of-affairs is a sort of metaphysical vagueness.[62]

This completes our argument for the Subtle Answer. Quantifier variance requires positive metaphysical vagueness – if quantifier variance is correct, reality is infused with vagueness in the sense that there is a state-of-affairs that indefinitely obtains. Indeed, *any* view that accepts common-sense mereology (CM) is committed to this form of metaphysical vagueness. Thus, even if a common-sense mereologist can avoid the

[62] Compare the point made by John Hawthorne, "Superficialism in Ontology," in David Chalmers, David Manley, Ryan Wasserman, eds., *Metametaphysics* (Oxford: Oxford University Press, 2009), pp. 213–30 at pp. 220–24 that, given certain assumptions of de re modality, when we carve the world up into individuals, we thereby carve modal space up into possibilities – which individuals exist partially determines which de re possibilities there are. See also Rohan Sud and David Manley, "Quantifier Variance," in Ricki Bliss and J.T.M. Miller, eds., *The Routledge Handbook of Metametaphysics* (New York; Routledge, 2021), pp. 100–117 at pp. 109–110.

Lewis-Sider argument against vague existence (as the variantist can), they are nevertheless still saddled with *one sort* of metaphysical vagueness.

VI. NEGATIVE METAPHYSICAL VAGUENESS

What about the negative element of the Slogan of metaphysical vagueness? Given that the variantist must accept positive metaphysical vagueness – in the sense of indefinitely obtaining states-of-affairs – must the variantist also think that there is vagueness that is "not in our descriptions or knowledge"? We've already discussed how the variantist dodges the Lewis-Sider argument against vague existence. I'll continue that discussion here by looking at two regimentations of this negative element of the Slogan, and argue that quantifier variantism does not commit us to either of these regimentations.

Let's start by considering a characterization of metaphysical vagueness from Elizabeth Barnes which focuses on the negative element of the Slogan. She begins by noting:

> vagueness has three potential sources – how we represent the world (representational or semantic vagueness), the limits of our knowledge of the world (epistemic vagueness), or the way the world is in and of itself (ontic vagueness). [...] So if we know that there's (non-epistemic) indeterminacy and we know that our representations are wholly blameless, then we can conclude that the source of the indeterminacy is the world itself.[63]

This leads her to propose (what she calls) a "negative definition" of metaphysical vagueness, according to which (roughly) vagueness would remain even in a perfectly precise language. As she puts it:

Negative Counterfactual There is a sentence S such that "were all representational content precisified, there is an admissible precisification of S such that according to that precisification the sentence would still be non-epistemically indeterminate [i.e. it would be indefinite whether S is true] in a way that is Sorites-susceptible."[64]

Is the variantist committed to Negative Counterfactual? Recall our discussion from §??. The variantist posits various tribal languages (e.g. Nihilese, 0.43-ese, Universalese) and, in order to avoid the Lewis-Sider argument, claims that among these languages are precisifying interpretations of vague counting sentences in English. One precisification

[63] Barnes, "Ontic Vagueness," *op. cit.*, pp. 603-4.
[64] *Ibid.*, 604

might be 0.43-ese; another might be 0.44-ese. These languages "precisify" the representational content of our vague language in the sense that the corresponding tribes' linguistic dispositions are more settled than ours: they treat sentences like "The table base and table top that are stuck together to degree 0.435 compose something" as unproblematically true or false whereas we hem and haw. But sentences – at least those related to composition – in these precisifying interpretations are either definitely true or definitely false. Suppose again that we have a table top and a table base that are alone in a room and stuck-together to degree 0.435; and suppose it's indefinite whether the English sentence "there is a table in the room" is true. The 0.43-ese sentence homophonic to that sentence is definitely true and the 0.44-ese sentence is definitely false. (Of course, we might have to further precisify the languages spoken by our various tribes to determine definite truth-conditions for sentences involving, say, terms like 'tall' or 'bald', but that raises no special problem for the variantist.) So, once we've precisified the English sentences, no vagueness need remain: the variantist is not committed to Negative Counterfactual.

Barnes' "negative definition" of metaphysical vagueness is counterfactual: we are supposed to consider a non-actual world in which the sentence is made precise, and then ask whether that precisified sentence is indefinite. We might have reservations, however, about using a counterfactual to articulate the negative element of our Slogan.[65] The important thesis underlying the negative element seems to do with (as Barnes herself says) "the source" or explanation of vagueness, but counterfactuals are notoriously poor renditions of explanatory notions. Instead, we might prefer to state the thesis of negative metaphysical vagueness directly in such explanatory terms:

Negative Explanation (Rough) There is some fact involving the indefiniteness operator (e.g. the fact that it's indefinite that Harry is bald) that is not explained (even in part) by facts involving our language or mental states.

Negative Explanation uses the term "explain", which we might in turn choose to regiment in various ways, for instance in terms of partial ground. And we might decide to tweak Negative Explanation in other ways, for instance by nominalizing our quantification over facts. What follows won't depend on how Negative Explanation is further specified or tweaked. Instead, I'll give an indirect argument. I'll first consider the most popular strategy for avoiding Negative Explanation (however it's

[65] Thanks to an anonymous referee for raising this worry.

ultimately cashed out) outside the context of vague existence: Lewis's *semantic indecision* account. I'll then explain why, given quantifier variance, that account can be extended to accommodate facts of vague existence as well. So I'll conclude that the quantifier variantist is under no particular pressure to accept Negative Explanation.

Consider Lewis's semantic indecision account of indefiniteness, as applied outside the context of vague existence. According to this account, the source of vagueness is the failure of our linguistic dispositions (or any other meta-semantic factors) to privilege one among several assignments of linguistic objects to meanings. Let a *language-assignment* be a mapping from uninterpreted sentences to truth-conditions. Based on meta-semantic factors (such as our linguistic dispositions), different language-assignments will be better or worse candidates for assigning meanings to the sentences in our language. For instance, a language-assignment that pairs the sentences we assert with meanings that are reasonable to believe will, ceteris paribus, be a better candidate assignment than one that has us asserting obviously false sentences. Vagueness results when several assignments are tied for "best".

Consider the uninterpreted sentence "Bob is bald". Suppose one of several best assignments maps that sentence to the condition that Bob has less than 50,000 hairs. And suppose another best assignment maps that sentence to the condition that Bob has less than 50,001 hairs. According to the semantic indecision model, this is supposed to explain why it's indefinite that Bob is bald. More generally, the fact that it's indefinite that ϕ is supposed to be explained by the fact that on one best assignment 'ϕ' is mapped to a condition that obtains (a set of worlds that includes the actual one), while on another most-privileged assignment 'ϕ' is mapped to a condition that does not obtain.

This model certainly has its critics.[66] Despite these criticisms, semantic indecision accounts of vagueness remain overwhelmingly popular. Assume that they can be made to work for relatively mundane facts like that it's indefinite that Harry is bald. Our concern here is whether quantifier variance – and in particular, their commitment to vague existence – raises any *special* problem for semantic indecision accounts.

As we already discussed, Lewis and Sider argued that it does. That argument, you will recall, relied on Domains – roughly, that the candidate meanings for an unrestricted quantifier will be domains – which

[66] See especially Andrew Bacon, *Vagueness and Thought* (Oxford: Oxford University Press, 2018) at pp. 47–68. For some discussions of the difficulty of specifying a notion of semantic indeterminacy (in particular, one that doesn't subtly appeal to "metaphysical" indeterminacy) see also David E. Taylor and Alexis Burgess, "What in the World is Semantic Indeterminacy?" *Analytic Philosophy*, LVI, 4 (December 2015): 298–317.

(as even Sider admits) a variantist would not accept. Thus, there is no obvious barrier to the variantist extending semantic indecision to quantificational language. As Hirsch writes:

> I accept Lewis's assumption that vagueness is a matter of semantic indecision....but what is the problem Lewis is raising about the vagueness of the quantifier? Since the meaning of the quantifier is given by its role in determining the truth-conditions of certain sentences [contra *Domains*], the vagueness of the quantifier would consist in our semantic indecision with respect to the truth conditions of certain sentences, for example, the sentence, "There exists something composed of the top and the leg"...in a situation in which the top and the leg are borderline attached.[67]

Suppose again the top and the leg are attached to degree 0.435 and therefore it's indefinite whether there is something composed of the top and the leg. Recall that, according to the variantist, the sentence 'Something is composed of the top and the leg' is true in the language-assignment for 0.43-ese and false in the language-assignment for 0.44-ese. The variantist can therefore extend the semantic indecision account to this case of indefiniteness. The indefiniteness is explained by the facts that (i) that these language-assignments assign divergent truth-values to the sentence and (ii) the meta-semantic facts induce an eligibility ordering on which these assignments are both best. In favor of (ii), it's plausible that nothing about our linguistic dispositions support assigning a particular truth-value for the uninterpreted sentence. And, given the variantist's claim that neither languages' interpreted term 'something' is "metaphysically privileged", it's plausible that no other meta-semantic factors would privilege one language-assignment over the other.

(We've just seen how the variantist can extend semantic indecision accounts for "de dicto" indefiniteness claims. In Appendix **??**, I consider semantic indecision accounts of quantified-in indefiniteness claims.)

Of course, semantic indecision is not the only strategy for giving a representational explanation of vagueness that avoids Negative Explanation. But the variantist can extend other suggestions in analogous ways. For instance, on Williamson's epistemicist explanation of vagueness, it's indefinite whether Harry is bald because there are semantically indiscriminable worlds (see §**??**) where the sentence "Harry is bald" would have been true and semantically indiscriminable worlds where the sentence would have been false. That's because the truth-condition for the sentence is extremely sensitive to minor changes in our linguistic

[67] Hirsch, "Quantifier Variance and Realism," *op. cit.*, p. 66.

dispositions. The variantist can extend this story: it's indefinite whether something is composed of the top and the leg because there are semantically indiscriminable worlds where the prejacent has divergent truth values. That's because which of the various tribal languages (the languages 0.43-ese, 0.44-ese, etc.) we are speaking is extremely sensitive to minor changes in our linguistic dispositions.

Summing up: variantism does not commit us to "negative metaphysical vagueness" as articulated by either Negative Counterfactual or Negative Explanation. That shouldn't be particularly surprising. It was the purported lack of "precisifying" interpretations for quantificational language that supposedly blocked representational explanations of vagueness from being extended to cases of vague existence. As explained in §??, the variantist posits such interpretations. So, assuming variantism, it's no surprise that representational explanations of vagueness can be seamlessly extended to the cases of vague existence to which the variantist is committed.

VII. RESISTANCE AND REFLECTION

I've argued that the quantifier variantist is committed to one sort of metaphysical vagueness: positive metaphysical vagueness – vagueness "in the world" – in the sense of vaguely obtaining states of affairs. And I've argued that this is so even though they are not committed to another sort: negative metaphysical vagueness – vagueness that is not "in our representations" – in the sense of Negative Counterfactual or Negative Explanation.

The argument presented here – like most arguments in metaphysics – is not a knockdown one: it can be resisted. To my mind, the most promising avenue for developing this resistance is motivated by the obscure deflationary metaphor of "the world as amorphous dough". Here's what I have in mind. Earlier, I claimed that quantifying over individuals and 1-place properties ($\forall x \forall X \phi(Xx)$) is (loosely speaking) a way of quantifying over non-qualitative states-of-affairs – the states-of-affairs of particular individuals instantiating particular properties. From there I concluded that the claim $\exists X \exists x \triangledown(Xx)$ would represent an indefinitely obtaining non-qualitative state-of-affairs. Arguably, this conclusion subtly assumes a sort of fine-grained conception of states-of-affairs – a conception a variantist may very well want to reject. In other words, the variantist might reject that $\exists X \exists x \triangledown(Xx)$ is a sufficient condition for "vagueness in reality" in any theoretically interesting sense. Instead, they may claim that quantification over states-of-affairs is *only* to be understood as quantification into 0-place predicate position. Then, they might accept the formal results of this paper – $\exists X \exists x \triangledown(Xx)$ – but somehow retain the claim that $\forall S \neg \triangledown(S)$.

Logical theories on which these two claims are consistent are surely awkward, but they can be constructed. And more investigation would be required to decide whether we should accept such theories. For example, we would have to decide whether the theories have unintuitive consequences and whether they are simple and strong when compared to rival logical theories. So, the argument I presented here has, at least, shifted the burden for those that want to deny my result to construct and defend such a proposal. Most importantly, it has established a framework for that debate to take place in.

For now, let's set aside this avenue of resistance and ask: assuming the conclusion of this paper holds up, what is its significance?[68]

The main result of the paper – that the variantist is committed to a sort of "vagueness in the world" – has one fairly straightforward dialectical upshot. The variantist took pride in their ability to avoid the Lewis-Sider argument from vague existence to metaphysical vagueness. Our result shows, however, that even if they can dodge arguments linking vague existence with *negative* metaphysical vagueness, their commitment to vague existence still comes saddled with *some* metaphysical vagueness – in particular, *positive* metaphysical vagueness. So, those that oppose metaphysical vagueness in all its guises should reject quantifier variance. And, conversely, fans of quantifier variance must accept at least some form of metaphysical vagueness.

If, in addition to this main result, we accept the results of the previous section, a more subtle upshot of our investigation emerges. On the variantist's picture, the positive metaphysical vagueness – the vagueness in the world – seems to be explained by facts about our language. Here's what I mean. In the last section, we argued that the variantist can extend representational explanations of vagueness to claims of vague existence: vague existence (like vague baldness) is explained by semantic indecision with respect to a plenitude of subtly different candidate meanings for quantificational language. But, Step 2 showed that our notion of a state-of-affairs is intimately bound up with our notion of existence (unlike our notion of baldness): vagueness in what exists induces vagueness in which states-of-affairs obtain. As a result, semantic indecision with respect to '$\exists$' (as opposed to 'bald') seems to explain vagueness in which states-of-affairs obtain – positive metaphysical vagueness. How should we react to this more subtle upshot? I can think of three reactions.

First: we might take this to show that there is something confused about semantic indecision accounts of vagueness in general. Facts about

[68] Thanks to Tom Donaldson for helping me think through this question.

language, we might think, cannot properly explain even mundane facts involving indefiniteness (e.g. that it's indefinite that Harry is bald). Although they remain overwhelmingly popular, semantic indecision (and more generally "linguistic") accounts of vagueness have become increasingly beleaguered.[69] And if indefinite baldness isn't properly explained by our language, then (pace Hirsch) indefinite existence needn't be either – despite the variantist's posit of a plentitude of semantic candidates. In this case, our main result holds, and the variantist still must accept a sort of metaphysical vagueness (vaguely obtaining states-of-affairs). But they can at least dodge the puzzling additional result that vagueness in the world is arising from our language.

Second: we might take this as a reductio, that shows that the variantist is committed to a sort of absurd "anti-realism". Anti-realist slogans about "reality depending on language" are notoriously difficult to cash out. But if semantic indecision with respect to quantificational language gives rise to vaguely obtaining states-of-affairs involving tables and their parts, this starts to smack of one sort of objectionable "dependence". And we arrived at this accusation of anti-realism using only the uncontroversial and formally tractable notion of indefiniteness, thereby avoiding the pitfalls that plagued regimentations of realism that deployed more heavy-duty metaphysical notions.

There is, however, a third interpretation (which I suspect proponents of quantifier variance will take up) of our results. According to the variantist, our notion of existence is parochial, flexible, and metaphysically undistinguished. Step 2 of our argument showed that our notions of existence and states-of-affairs are intimately bound up. Thus, our notion of a state-of-affairs is also parochial, flexible, and metaphysically undistinguished. And, of course, the notion of positive metaphysical vagueness is intimately bound up with (indeed analyzed in terms of) our notion of a state-of-affairs. Thus, our notion of positive metaphysical vagueness is similarly parochial, flexible, and metaphysically undistinguished.

Seen in this light, it's natural for the variantist to deflate the significance of our notions of states-of-affairs and positive metaphysical vagueness (and even claims of "anti-realism," in so far as they are glossed using those notions). After all, the above argument merely showed that the claim that there is a vaguely obtaining state-of-affairs ('$\exists X \exists x \nabla(Xx)$') is true in *our* mouths. However, UQLs in the mouths of one of the tribes in our thought experiment – say speakers of 0.43-ese – are not vague. So there is no reason to accept the proposition expressed by the homophonic 0.43-ese sentence '$\exists X \exists x \nabla(Xx)$' – in *their* mouths, the claim

[69] See the citations in fn. ??.

is false. Similarly, in their mouths, sentences like 'semantic indecision gives rise to vagueness in states-of-affairs' are false. Speaking loosely and provocatively: according to the 0.43-ese notion of states-of-affairs, there are not any vaguely obtaining states-of-affairs. According to their notion of positive metaphysical vagueness, there is none. And of course, their notions of states-of-affairs and positive metaphysical vagueness are no less metaphysically distinguished than ours. So, why care whether there are vague states-of-affairs in our parochial sense of the notion?

From this perspective, our investigation has shown that deflationism with respect to our notion of existence should be extended to our notion of states-of-affairs and subsequently defined notions such as that of positive metaphysical vagueness. Accordingly, although the variantist is committed to vagueness in states-of-affairs *as we understand them*, perhaps that's a result that they can live with.

APPENDIX A. SIMPLE FIRST ORDER FRAGMENT

Ultimately, the language we are theorizing in is a higher-order modal language with identity and non-rigidly referring singular terms and primitive predicates. Systems for such languages are poorly understood. Fortunately, the claims in §§??-?? of the text can be formulated in a simple fragment $\mathcal{L}_{SF}$ of that language: a first order modal language without identity or constants, with a primitive existence predicate E (which thereby allows us to predicate existence without the use of an identity predicate).

A.1. Axioms. We'll use a minimal contingentist axiomatic system and model theory as given by Kripke which we'll call $KC+T$.[70] Let a closure of ϕ be any formula without free variables that results by prefixing universal quantifiers and '$\triangle$', in any order, to ϕ. The axioms of $KC+T$ include all closures of the following:[71]

(Norm) All instances of truth functional tautologies, the K-axiom, and the T-axiom

(VQ) $\phi \to \forall x\phi$, where x is not free in ϕ

($\forall^{\to}$) $\forall x(\phi \to \psi) \to (\forall x\phi \to \forall x\psi)$

($\forall$K) $\forall y((\forall x\phi(x)) \to \phi(y))$

(E1) $\forall x E(x)$[72]

[70] Saul Kripke, "Semantical Considerations on Modal Logic," *Acta Philosophica Fennica*, XVI (1963): 83–94.

[71] *Ibid.*, 89

[72] *Ibid.*, 90

$(E2)$ $(\forall x\phi(x) \wedge E(y)) \rightarrow \phi(y)$

The only inference rule is modus ponens ('MP'). Necessitation is a derived rule (because the axioms include all closures of the above schemas). Note that all theorems are closed sentences. We'll follow the conventions in the main text, letting '$\nabla\phi$' abbreviate '$(\neg\triangle\phi) \wedge (\neg\triangle\neg\phi)$' and subscripted quantifiers (e.g. $\exists_A x\phi(x)$) be quantifiers that are restricted by the subscript (e.g. $\exists x(Ax \wedge \phi(x))$).

A.2. Models. Let a *model structure* be a tuple $\langle w^*, W, R, D_G, Q\rangle$ where W is a set of points ('worlds'), R is a reflexive binary relation on W, w^* is some world in W, D_G a set of objects ('the global domain') and Q a function from worlds to subsets of D_G (we'll write $Q(w)$ as 'D_w'). Let a *valuation* be a function V from worlds and n-place predicates to sets of n-tuples of objects in the global domain (for $n > 0$) or truth values (for $n = 0$) ('the extension of the predicate at that world') with the requirement that $V(w, E(x)) = D_w$. A *model of* $KC + T$ is a model structure together with a valuation function $\langle w^*, W, R, D_G, Q, V\rangle$.

Let an assignment function a map each variable to a member of D_G and let a μ-variant of a ('a_μ') be an assignment function that differs only with respect to some variable μ. Satisfaction on a model $\mathcal{M}$ at world w relative to a is defined for atomic sentences as follows.

- For 0-place atomic sentence, ϕ: $\mathcal{M}, w, a \models \phi$ iff $V(w, \phi) = T$
- For atomic sentence with n-place predicate ($n > 0$), $\phi(x_1, x_2, ...x_n)$: $\mathcal{M}, w, a \models \phi(x_1, x_2, ...x_n)$ iff $\langle a(x_1), a(x_2), ..., a(x_n)\rangle \in V(w, \phi)$

The inductive clauses for complex sentences are the typical ones for the connectives and $\triangle$. According to the clause for the quantifier, at world w the universal quantifier only quantifies over entities in the domain of w:

- For sentence $\forall\mu\phi(\mu, y_1, y_2...)$: $\mathcal{M}, w, a \models \forall\mu\phi(\mu, y_1, y_2...)$ iff $\mathcal{M}, w, a_\mu \models \phi(\mu, y_1, y_2...)$ for all μ-variants of a such that $a_\mu(\mu) \in D_w$.

Finally, $\mathcal{M}, a \models \phi$ just in case $\mathcal{M}, w^*, a \models \phi$

For convenience, we'll exploit completeness of the above system and conduct our proofs using the model theory for $KC + T$. But the model theory is performing the purely instrumental function of demonstrating, in a definiteness-operator-free language, the existence of a proof in that system. I am not making any claim about how the model theory relates to the compositional meanings of terms in our language.[73]

[73] Thanks to an anonymous referee for encouraging me to get clear on this.

Of course, there will be some class of "intended models" which are models $\mathcal{M}$ for which ϕ is true simpliciter just in case it is true on $\mathcal{M}$ (relative to all assignment functions). (Because it will be vague which claims are true simpliciter, it will be vague which models are intended, but, nevertheless there will definitely be some!) We might be tempted to think that, for some such model, the subdomains D_w are domains over which precisifications of our quantifier range. A variantist should resist the temptation, just as they resisted Sider's Domain principle (§??). For temptation quickly leads to paradox: if subdomains are precisifications of our quantifier, D_{w^*} must include everything there is, but we want to allow objects in D_G that are not in D_{w^*}. Our situation is thus not unlike that of an actualist contingentist who nevertheless appeals to Kripke semantics in order to investigate the consequences of modal claims.[74]

Is there *anything* we can say about how the model theory relates to the meanings of the terms in our language? In §??, I gave two strategies the variantist can take towards the demand to specify the meanings of the UQLs in the tribal languages that precisify our quantifier. On the first, we decline to give, in our own language, a compositional semantics for these UQLs, and instead rest content to give a semantics in Universalese. Once we're speaking Universalese, it's easy to give a compositional semantics of our formalized English language using Kripke model-theory. That is, we can truly say *in Universalese* things like: "Among the intended models will be one where the subdomains D_w of the accessible worlds are domains over which precisifications of the formalized English UQL range. That model gives the meaning of formalized English." Perhaps we can even translate this theory into our own language (as an initial attempt, following Dorr, we can attach the counter-possible operator "If universal composition were the case" to the start of each sentence in that theory). This strategy is analogous to that of contingentists who are willing to talk *as if* there are mere possibilia – including when engaging in compositional linguistics – but then offer translation schemes from possibilist talk into the more austere language of Modalese.

A second strategy followed Dorr in taking the meanings of the UQLs that precisify the English quantifier to be second-order properties. With meanings of precisifications in hand, Dorrians can give a semantic theory for definiteness claims which appeals to those precisification, including, for instance, theorems like: '$\triangle(\exists Fx)$' is true iff '$\exists Fx$' is true on all precisifications, and '$\exists Fx$' is true on a precisification p iff the semantic value of 'F' on p instantiates the semantic value of '$\exists$' on p. And we

[74] Cf. Williamson, *Modal Logic as Metaphysics*, *op. cit.*, p. 138.

can start to see the connection to Kripke models. Even though the subdomains aren't literally domains over which the precisifications of our quantifier range, the subdomains D_w and the valuation function jointly represent precisifications of our predicates and quantifiers. (E.g. take a world w accessible from w^* at which '$\exists xFx$' is true. The fact that the subdomain D_w overlaps with $V(w,$'F'$)$ represents there being a precisification p for which the semantic value of '$\exists$' on p is instantiated by the semantic value of 'F' on p.) And so the models represent indefiniteness facts by representing facts about precisifications.

These remarks linking the model-theory with compositional meanings are admittedly sketchy. But the contingentist isn't much better off: no explanation of how Kripke model theory relates to the compositional semantics of modal terms has garnered wide-spread consensus. And we shouldn't hold the variantist to a higher standard, especially given the relative paucity of attention paid to vague existence compared to contingent existence. Most importantly we shouldn't demand a fully worked out interpretation of the model-theory before we are willing to use it to show proof-theoretic facts, which is all I am using it for here.

A.3. Proofs. **Proposition 1**: The claim that definitely something is barely* tall ($\triangle(\exists x(Tx \wedge \triangledown(Tx)))$) is logically consistent

Proof: Consider the following model. Let there be three worlds, $1, 2, 3$ and let $D_G = D_1 = D_2 = D_3 = \{a, b, c\}$ where xRy just in case $y = x$ or $x + 1$. And let the extension of 'T' at 1 be $\{a, b, c\}$, at 2 be $\{b, c\}$ and at 3 be $\{c\}$. Finally let $w^* = 1$. The claim '$\exists x(Tx \wedge \triangledown(Tx))$' is true at 1 (with a as the witness) and at 2 (with b as the witness). So, the claim $\triangle(\exists x(Tx \wedge \triangledown(Tx)))$ is true at 1, so it is true on the model.

Proposition 2: The claim that definitely something indefinitely exists ($\triangle(\exists x \triangledown(Ex))$) is logically consistent

Proof: Consider the following model. Let there be two worlds, w^* and w' and let $D_G = \{a, b\}$ where $D_{w^*} = \{a\}$ and $D_w' = \{b\}$. And let each world be R-related to one another. Then, the sentence '$\exists x \triangledown(Ex)$' is true at both w^* (with a as the witness) and w' (with b as the witness), so the sentence $\triangle(\exists x \triangledown(Ex))$ is true at w^* and thus true on the model.

Proposition 3: $\triangle \forall x(Sx \leftrightarrow C'x), \forall_A x \triangle Ex \vdash \forall_A x \triangle(Sx \leftrightarrow C'x)$

Proof: Consider some arbitrary model $\mathcal{M}$ and arbitrary assignment function a that satisfies $\triangle \forall x(Sx \leftrightarrow C'x)$ and $\forall_A x \triangle Ex$. Consider some arbitrary world w such that $w^* Rw$ and some arbitrary item t in D_{w^*} that is in the extension of A. Because $\mathcal{M}, a \models \forall_A x \triangle Ex$ we know t is in D_w and because $\mathcal{M}, a \models \triangle\forall x(Sx \leftrightarrow C'x)$ we know anything in D_w is in the extension, at w, of Sx just in case it's in the extension of $C'x$. So, t is in the extension of Sx just in case it's in the extension of $C'x$ at w. Because w and t were arbitrary we have $\mathcal{M}, a \models \forall_A x\triangle(Sx \leftrightarrow C'x)$.

Proposition 4: $\forall_A x\triangle(Sx \leftrightarrow C'x), \exists_A x\triangledown(Sx) \vdash \exists_A x\triangledown(C'x)$

Proof: Consider some arbitrary model $\mathcal{M}$ and arbitrary assignment function a that satisfies $\forall_A x\triangle(Sx \leftrightarrow C'x)$ and $\exists_A x\triangledown(Sx)$. Because $\mathcal{M}, a \models \exists_A x\triangledown(Sx)$, there is some item t in D_{w^*} and the extension of A at w^* and some worlds w and w' such that: w^*Rw and w^*Rw' such that t is in the extension of Sx at w but not at w'. Because $\mathcal{M}, a \models \forall_A x\triangle(Sx \leftrightarrow C'x)$, we know that t is in the extension of Sx at w just in case it's in the extension of $C'x$ at w. Similarly for w'. So, we know t is in the extension of $C'x$ at w and is not in the extension of $C'x$ at w'. So, $\mathcal{M}, a \models \exists_A x\triangledown(C'x)$.

Proposition 5: $\exists_A x(Sx \wedge \triangledown(Sx)), \forall_A x\triangle(Sx \leftrightarrow C'x) \vdash \exists_A x(C'x \wedge \triangledown C'x)$

Proof: Consider some arbitrary model $\mathcal{M}$ and arbitrary assignment function a that satisfies $\exists_A x(Sx \wedge \triangledown(Sx))$ and $\forall_A x\triangle(Sx \leftrightarrow C'x)$. Because $\mathcal{M}, a \models \exists_A x(Sx \wedge \triangledown(Sx))$, we know there is some item t in D_{w^*} and in the extension of A and S at w^* but is not in the extension of S at some other world w where w^*Rw. Because $\mathcal{M}, a \models \forall_A x\triangle(Sx \leftrightarrow C'x)$ we know that t is in the extension of S iff it is in the extension of C' at both w^* and w. So, we know t is in the extension of C' (and A) at w^* and not in the extension of C' at w. Thus, $\mathcal{M}, a \models \exists_A x(C'x \wedge \triangledown C'x)$

Proposition 6: $\exists_A x(\exists y(xCy) \wedge \triangledown\exists z(xCz)), \forall_A x\forall y(xCy \rightarrow \triangle(Ey \rightarrow xCy)) \vdash \exists y\triangledown Ey$

Proof: Consider some arbitrary model $\mathcal{M}$ and arbitrary assignment function a such that $\mathcal{M}, a \models \exists_A x(\exists y(xCy) \wedge \triangledown\exists z(xCz))$ and $\mathcal{M}, a \models \forall_A x\forall y(xCy \rightarrow \triangle(Ey \rightarrow xCy))$. Because $\mathcal{M}, a \models \exists_A x(\exists y(xCy) \wedge \triangledown\exists z(xCz))$ we know there is some items c and t in D_{w^*} such that c is in the extension of A at w^* and $\langle c, t\rangle$ is in the extension of C at w^*. And we know that for some world w' such that w^*Rw' for all items x in $D_{w'}$, $\langle c, x\rangle$ is not in the extension of C at w'. Thus, if t were in $D_{w'}$, then $\langle c, t\rangle$ would not be in the extension of C at w'. But, because $\mathcal{M}, a \models \forall_A x\forall y(xCy \rightarrow \triangle(Ey \rightarrow xCy))$, we know that if t is in $D_{w'}$, then $\langle c, t\rangle$ is in the extension of C at w'. Thus, t must not be in $D_{w'}$. So, $\mathcal{M}, a \models \exists y\triangledown(Ey)$

APPENDIX B. SIMPLE HIGHER-ORDER FRAGMENT

We were able to formulate the argument for §§??-?? in a simple first-order fragment of that language. However, the argument in §?? of the paper cannot be formulated in $\mathcal{L}_{SF}$. So, we must move to a more expressive language.

Once again, the language we're ultimately theorizing in is a higher-order modal language with identity and non-rigidly referring singular terms and primitive predicates. But systems for such languages are poorly understood. Fortunately, once again, it suffices to formulate our argument in a simple fragment of such a language, without identity, singular terms, or primitive predicates. We'll call that fragment $\mathcal{L}_{SH}$.

B.1. Grammar. Our aim is to keep the language $\mathcal{L}_{SH}$ as sparse as possible, while still being expressive enough to formulate the argument in §??. In addition to the connectives and definiteness operator, let $\mathcal{L}_{SH}$ includes first-order quantifiers $\forall x$ and variables $x, y, \ldots$ and higher-order quantifiers for 1-place predicate variables $\forall X$ and a stock of such variables $X, Y, \ldots$. (Existential quantifiers are defined as the dual of the universal quantifiers.) We'll also include a primitive one-place existence predicate E. No other expressions are in the language (no identity symbol, names, other primitive predicates, etc.). Open or closed sentences of the language are defined in the usual way. So, the language contains closed sentences like '$\forall x \forall X \triangle (Xx \vee \neg Xx)$' and '$\neg \forall X \exists x (Xx \rightarrow \neg Xx)$'. The language does not include sentences like '$\forall x(Fx)$', '$\forall x(x = x)$', '$\exists X(Xa)$', and '$\forall x \forall y \exists X(Xxy)$'.

B.2. Axioms. We can lay down an incomplete axiomatic logical theory for this language, which will suffice for the argument in §?? while avoiding thorny questions about the model theory. Let the system $KC + HO$ include as axioms the closures of the following:

(Norm) All instances of truth functional tautologies, and the K-axiom

($\forall^{\rightarrow}$-HO) $\forall x(\phi \rightarrow \psi) \rightarrow (\forall x \phi \rightarrow \forall x \psi)$ and $\forall X(\phi \rightarrow \psi)) \rightarrow (\forall X \phi \rightarrow \forall X \psi)$

(VQ-HO) $\phi \rightarrow \forall X \phi$, where X is not free in ϕ

The only inference rule we'll use is modus ponens ('MP').

B.3. Proofs. This minimally specified theory is enough to give us the desired result. Note three simple propositions that we'll make use of:

Proposition 7: $\vdash \forall x \forall X(\phi \rightarrow \psi) \rightarrow \forall x(\forall X \phi \rightarrow \forall X \psi)$ where ϕ and ψ have no free variables other than x and X

1. $\vdash \forall x(\forall X(\phi \rightarrow \psi) \rightarrow (\forall X \phi \rightarrow \forall X \psi))$ (Closure of Instance of $\forall^{\rightarrow}$-HO)

2. $\vdash \forall x \forall X(\phi \to \psi) \to \forall x(\forall X\phi \to \forall X\psi)$ (MP on 1 and Instance of $\forall^{\to}$-HO)

Proposition 8: Where the closure of $\phi \to \psi$ is a theorem and ϕ and ψ have no free variables other than x and X, $\vdash (\forall x \forall X\phi) \to (\forall x \forall X\psi)$. (Similarly, where ϕ and ψ have no free variables other than x, $\vdash (\forall x\phi) \to (\forall x\psi)$.)

1. $\vdash \forall x \forall X(\phi \to \psi)$ (Assumed)
2. $\vdash \forall x(\forall X\phi \to \forall X\psi)$ (MP on 1 and Proposition ??)
3. $\vdash (\forall x \forall X\phi) \to (\forall x \forall X\psi)$ (MP on 2 and Instance of $\forall^{\to}$-HO)

Proposition 9: $\vdash \forall x((\forall X\phi) \to \psi) \to \forall x(\phi \to \psi)$ where ϕ and ψ are free only in x.

1. $\vdash \forall x((\phi \to (\forall X\phi)) \to (((\forall X\phi) \to \psi) \to (\phi \to \psi)))$ (Closure of Instance of Truth-functional Tautology)
2. $\vdash \forall x(\phi \to (\forall X\phi)) \to \forall x(((\forall X\phi) \to \psi) \to (\phi \to \psi))$ (MP on 1 and Instance of $\forall^{\to}$-HO)
3. $\vdash (\forall x(((\forall X\phi) \to \psi) \to (\phi \to \psi))) \to (\forall x((\forall X\phi) \to \psi) \to \forall x(\phi \to \psi))$ (Instance of $\forall^{\to}$-HO)
4. $\vdash \forall x(\phi \to (\forall X\phi)) \to (\forall x((\forall X\phi) \to \psi) \to \forall x(\phi \to \psi))$ (2 and 3 and classical reasoning[75])
5. $\vdash \forall x(\phi \to (\forall X\phi))$ (Closure of VQ-HO)
6. $\vdash \forall x((\forall X\phi) \to \psi) \to \forall x(\phi \to \psi)$ (MP on 4 and 5)

This is enough to generate the desired results:

1. $\forall x \forall X(Xx \to \triangle Xx)$ (Assumption: No Positive Metaphysical Vagueness)
2. $\forall x \forall X(\neg\triangle Xx \to \neg Xx)$ (MP on 1 and Instance of Proposition ?? (where ψ is the contrapositive of ϕ))
3. $\forall x(\forall X\neg\triangle Xx \to \forall X\neg Xx)$ (MP on 2 and Instance of Proposition ??)
4. $\forall x(\neg\forall X\neg Xx \to \neg\forall X\neg\triangle Xx)$ (MP on 3 and Proposition ?? (where ψ is the contrapositive of ϕ))
5. $(\forall x\neg\forall X\neg Xx) \to (\forall x\neg\forall X\neg\triangle Xx)$ (MP on 4 and Closure of Instance of $\forall^{\to}$)
6. $\forall x\neg\forall X\neg Xx$ (Assumption: EIS rewritten with universal quantifiers)
7. $\forall x\neg\forall X\neg\triangle Xx$ (MP 5 and 6)
8. $\forall x \forall X\triangle(Xx \to Ex)$ (Assumption: EE)

[75] We use the inference from $\vdash \phi \to \psi$ and $\vdash \psi \to \chi$ to $\vdash \phi \to \chi$, which is derivable from truth-functional tautologies and MP.

9. $\forall x \forall X \triangle (Xx \rightarrow Ex) \rightarrow \forall x \forall X (\triangle Xx \rightarrow \triangle Ex)$ (Proposition ?? and Closure of Instance of K)

10. $\forall x \forall X (\triangle Xx \rightarrow \triangle Ex)$ (MP on 8 and 9)

11. $\forall x \forall X (\neg \triangle Ex \rightarrow \neg \triangle Xx)$ (MP on Instance of Proposition ?? (where ψ is the contrapositive of ϕ) and 10)

12. $\forall x (\forall X \neg \triangle Ex \rightarrow \forall X \neg \triangle Xx)$ (MP on Instance of Proposition ?? and 11)

13. $\forall x (\neg \triangle Ex \rightarrow \forall X \neg \triangle Xx)$ (MP on Instance of Proposition ?? and 12)

14. $\forall x (\neg \forall X \neg \triangle Xx \rightarrow \triangle Ex)$ (Proposition ?? (where ψ is the contrapositive of ϕ) and MP on 13)

15. $(\forall x \neg \forall X \neg \triangle Xx) \rightarrow (\forall x \triangle Ex)$ (MP 14 and Instance of $\forall^{\rightarrow}$)

16. $(\forall x \triangle Ex)$ (MP on 15 and 7)

APPENDIX C. DE RE SEMANTIC-INDECISION FOR QUANTIFIER VARIANTIST

In the main text, I explained how a semantic indecision account of vagueness is supposed to explain *de dicto* indefiniteness facts of the form '$\triangledown(\phi)$'. Here, I explore semantic indecision accounts of *de re* or *quantified in* indefiniteness facts. For considerations of space, I will restrict my attention to simple quantified-in indefiniteness facts of the form '$\exists x \triangledown Fx$'. And what I have to say will be largely exploratory: applying semantic indecision accounts to quantified-in indefiniteness facts are rarely discussed, even outside the context of quantifier variance.[76]

Recall that, according to semantic indecision accounts, de dicto indefiniteness claims of the form '$\triangledown\phi$' were explained by the fact that ϕ is true in some, but not all, of the precise language-interpretations that do "best" with respect to meta-semantic considerations. If we allow precise language-interpretations to assign semantic values to predicates, we can say that '$\exists x \triangledown Fx$' is explained by the fact that there is some x for which some but not all of the semantic values expressed by 'F' in the best language-interpretations apply. I'll denote the semantic value of an expression 'ϕ' in precise language interpretation L as $[\![\phi]\!]_L$. And to say that the semantic value, in L, of an expression ϕ applies to x, I'll simply write $[\![\phi]\!]_L x$. Then, on this theory, $\exists x \triangledown Fx$ is explained by the fact that there is some x such that, for some but not all of the best Ls, $[\![F]\!]_L x$.

In order for the quantifier variantist to make use of this strategy, they must give an account of the semantic values of the predicates in the various languages spoken by the tribes (languages like 0.45-ese). Inspired by Dorr, we can say that the semantic values for n-place predicates in

[76] Thanks to Tom Donaldson for pressing me to think more about this.

any language are simply n-place properties.[77] Then, we can say that the UQLs (i.e. '$\exists$') in the various languages express different second-order properties of 1-place properties. So, a sentence like '$\exists xFx$' in 0.45-ese is understood as saying that the second-order property picked out by '$\exists$' in that language applies to the first-order property picked out by 'F' in that language. In other words, '$\exists xFx$' is true in tribal language L just in case $[\![\exists]\!]_L[\![F]\!]_L$.[78] Given this Dorrian semantic theory for our tribal language, we can more-or-less extend the standard semantic indecision account from de re indefiniteness to the context of quantifier variance and vague existence without much change. We simply allow that among the precise language-interpretations between which we are semantically undecided are some corresponding to the tribal languages posited by the variantist. At least to a first approximation, we can continue to say that $\exists x \triangledown Fx$ is explained by the fact that there is some x such that, for some but not all of the best Ls, $[\![F]\!]_L x$.

This is only an approximation: we will need to make further tweaks to the core account in order to accommodate features of de re indefiniteness that we notice only in the context of vague existence. For instance, recall that existence is supposed to be easy (EE). Suppose there is a table top and table base that barely* compose, so there is a table that barely* exists. And suppose further that there are no other table tops and table bases. In this case, $\exists x \triangledown Tx$. But if, for any tribal language L, 'T' expresses in L the property of *being a table*, then, on our current account, this claim remains unexplained. Instead we should tweak the account: letting 'E' be the existence predicate ('$\lambda x.\exists y = x$'), we can say that $\exists x \triangledown Fx$ is explained by the fact that there is some x such that, for some but not all L, $[\![F]\!]_L x$ and $[\![E]\!]_L x$. The original unmodified account is just the special case where for any L, $[\![E]\!]_L$ is the property of *existing*.

With this initial discussion in view, there is no immediate reason why the variantist cannot extend semantic indecision accounts to explain quantified-in indefiniteness claims.

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[77] Cf. Dorr, "What we Disagree about," *op. cit.*, pp. 234–86.

[78] To generalize the account to more complex locutions, we need to introduce a λ operator that binds variables. So, strictly speaking, '$\exists xFx$' is true in tribal language L just in case $[\![\exists]\!]_L[\![\lambda x.Fx]\!]_L$, although it's plausible that $[\![\lambda x.Fx]\!]_L = [\![F]\!]_L$ given that the two terms are η-equivalent.