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Research Article

Unified Framework for Optimal Routing Choice under Guidance Information

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In order to satisfy the diverse demand of travel service in the context of big data, this paper puts forward a unified framework for optimal routing choice under guidance information. With consideration of the influence of big data, the scenario analysis of routing choice is implemented, and the routing choice under guidance information is discussed. The optimal routing choice problem is abstracted into the collaboration optimization model of travel route choice, departure time choice, and travel mode choice. Based on some basic assumptions, the collaboration optimization model is formulated as a variational inequality model. The method of successive averages is applied to solve the proposed model. A case study is carried out to verify the applicability and reliability of the model and algorithm.

1. Introduction

With the rapid development of mobile internet, Internet of Things, cloud computing, and so on, the era of big data is coming and new data are generated day by day. Handling big data is a significant problem. A special issue on *Big Data* was published for the first time by *Nature* in 2008 [1] and *Dealing with Data* by *Science* in 2011 [2]. In 2012, the United States launched *the Big Data Research and Development Initiative* [3]. China issued *A Survey on China's Big Data Development* in 2015 [4]. In recent years, the technologies and applications of urban traffic data have grown rapidly. Recently, a lot of new technologies and applications of traffic have come into the market. There is a big issue to handle the data of traffic. The traditional methods to collect traffic data are induction coil [5], microwave radar [6], and floating car system [7], and nowadays, new ways for the collection of the data are new compass navigation system [8] and smartphone [9].

(A) Multisource data: big data technologies, such as smartphone [10], social media [11], Internet of Vehicles [12], variable message sign [13], and vehicle navigation system [14], provide more approaches of information detection and release, not only enriching the reliable sources of information but also expanding the scope of information dissemination. Therefore, the optimal routing choice can be implemented when the travelers fully understand the traffic condition based on big data.

(B) New travel modes: the deep integration of big data technology and the concept of sharing economy have created some new travel modes with a new system, for example, the online car-hailing system [15], the bike-sharing system [16], and the car-sharing system [17]. These new travel modes based on the sharing system expand the range of combined travel mode, which is becoming one of the most popular and common travel modes these days.

Table 1: Ways and types of information publishing.

Ways of publishing	Real-time traffic condition	Traffic incidents	Traffic control	Traffic guidance	Query service
Static traffic sign			○	○	
Variable message sign	○	○	○	○	
Vehicle navigation system	○	○	○	○	○
Cell phone APP	○	○	○	○	○
Internet	○	○	○	○	○
TV service	○	○	○	○	○
Radio broadcast		○	○		○

Recently, there is a vivid discussion in the literature concerning about optimal routing choice. For example, Chen and Hsueh [18], Abdul Aziz Ukkusuri [19], and Long et al. [20] studied the combination selection of travel route and departure time. Nakayama et al. [21], Meng et al. [22], and Shi et al. [23] studied the combination selection of travel route and travel mode. Moreover, travelers are influenced by the guidance information. Ren et al. [24], Huang et al. [25], and Sun et al. [26] divided travelers into two groups according to whether they assembled the advanced traffic information system (ATIS). Zhong et al. [27], Liao and Chen [28], and Zhong et al. [29] studied the route choice behavior in the ATIS context.

The optimal routing choice problem is abstracted into the collaboration optimization model of travel route choice, departure time choice, and travel mode choice. However, there is still a problem in the optimization model. In addition, influenced by big data, many new problems are created in the mathematical modeling. The following are examples:

(A) The mathematical modeling is very complex. The collaboration optimization involves multiple decision variables, such as travel route, departure time, and travel mode, which have a close connection of mutual influence and mutual cause and effect.

(B) The travel mode needs to be redefined under big data. These new mode technologies to be merged and combined together are very popular and common in current life. The map of trips has changed under this background.

(C) The guidance efficiency should be considered in the mathematical modeling, because of the increase of information publishing ways in the context of big data. Having different information types and format, these information publishing ways affect the guidance efficiency.

With the consideration of these problems, this paper establishes a unified framework for optimal routing choice under guidance information. The rest of the paper is organized as follows: in Section 2, the problem statement is put forward. In Section 3, the optimal routing choice problem is formulated as a variational inequality model. In Section 4, the method of successive averages is used to solve the proposed model. In Section 5, a case study is discussed to verify the applicability and reliability of the model and algorithm.

Table 2: Information format.

Ways of publishing	Information format			
	Script	Image (live)	Image (status)	Numerical value
Static traffic sign	○	○		
Variable message sign	○		○	○
Vehicle navigation system			○	○
Cell phone APP	○	○	○	○
Internet	○	○	○	○
TV service	○	○	○	○
Radio broadcast	○			○

Finally, in Section 6, there is conclusion and discussion for future research.

2. Problem Statement

2.1. Analysis of Routing Choice Based on Big Data

2.1.1. Guidance Information. Based on a variety of information detection methods, abundant traffic data can be obtained, as long as the supply and demand characteristics of the traffic system. Moreover, by means of a variety of information publishing methods, the guidance information is issued to travelers. There are lots of ways to information publishing, such as static traffic sign, variable message sign, vehicle navigation system, cell phone app, internet, TV service, and radio broadcast. The first two ways are usually laid out on the key sections of the urban traffic network. Comparatively speaking, the latter five ways cover a wider range. The types of information publishing include real-time traffic condition, traffic incidents, traffic control, traffic guidance, and query service. Different ways of information publishing, limited by a hardware device, have different information types, as shown in Table 1.

Different ways of information publishing have different formats as shown in Table 2.

2.1.2. Combined Travel Mode. In addition, the emergence of the online car-hailing system, bike-sharing system, and car-sharing system greatly enriched the choices of travel mode, which perfects the chain of travel. Based on the descriptions above, the map of trips in the context of big data can be plot as shown in Figure 1.

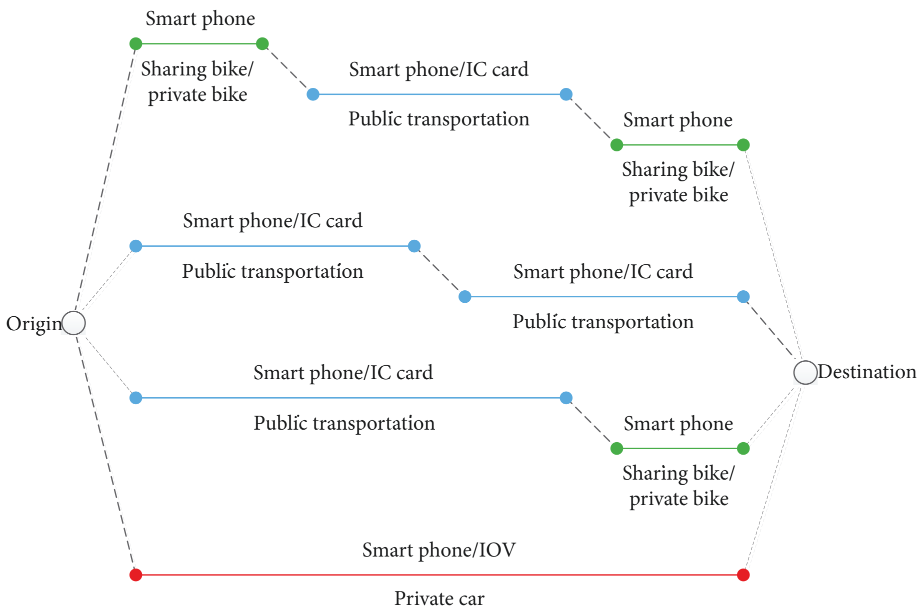

Figure 1: Map of trips in the context of big data.

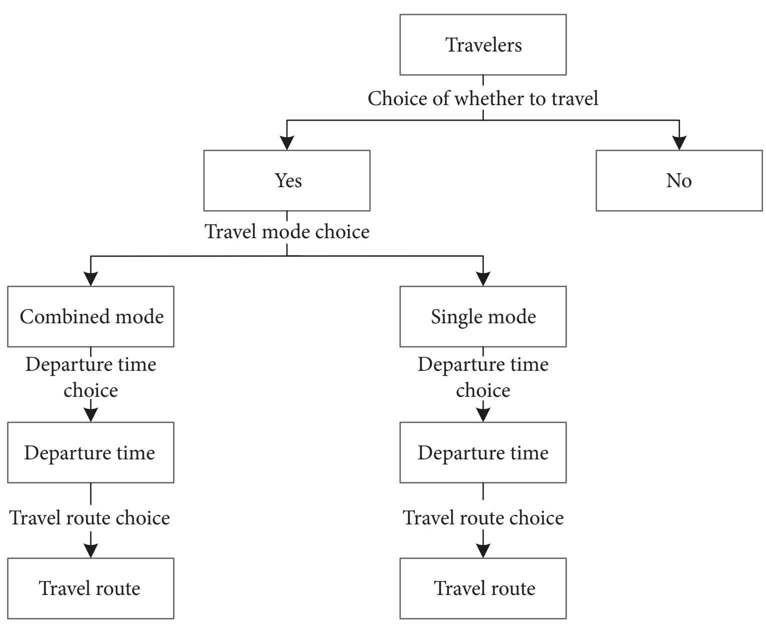

Figure 2: Sustainable routing choice considering guidance effects.

2.2. Routing Choice considering Guidance Information. Considering the guidance information, the optimal routing choice can be described as a series of choices with a tree structure [30], as shown in Figure 2.

Optimal routing choice involves the choice of whether to travel, travel route, departure time, travel mode, and so on. It has a wide range and many influencing factors. In this paper, the latter three choices are discussed. Then, the optimal routing choice problem is abstracted into a collaboration optimization model of travel route choice, departure time choice, and travel mode choice (TRC-DTC-TMC collaboration optimization model), which is the main job of this paper.

2.2.1. Travel Route Choice (TRC). In the case of travel route choice, the traveler chooses a route with the minimum negative utility. Assuming that $c_p^{rs}(t)$ denotes the negative utility of pth route of OD (origin-destination) pair rs at time interval t. The optimization model of travel route choice is given below:

$$\begin{aligned} \min_{p} \quad & c_p^{rs}(t), \\ \text{s.t.} \quad & \sum_p f_p^{rs}(t) = q^{rs}(t), \quad f_p^{rs}(t) \geq 0, \end{aligned} \tag{1}$$

where $f_p^{rs}(t)$ is the arrival rate of the pth route of OD pair rs at time interval t and $q^{rs}(t)$ is the traffic demand of the OD pair rs at time interval t.

There is an equilibrium state for the optimization problem of travel route choice, satisfying dynamic user optimal (DUO) condition. That is to say, for any OD pair rs, in any time interval t, the actual impedance of the route used by the traveler is equal and minimal; simultaneously, the actual impedance of all routes not used is not less than the minimum actual impedance. Mathematical expressions are given below:

$$\begin{aligned} f_p^{rs}(t)\left[c_p^{rs}(t) - c_{\min}^{rs}(t)\right] &= 0, \\ c_p^{rs}(t) - c_{\min}^{rs}(t) &\geq 0, \\ f_p^{rs}(t) &\geq 0, \end{aligned} \tag{2}$$

where $c_{\min}^{rs}(t)$ is the minimum of $c_p^{rs}(t)$. That is,

$$c_{\min}^{rs}(t) = \min_p c_p^{rs}(t). \tag{3}$$

2.2.2. Departure Time Choice (DTC). In the case of departure time choice, the traveler chooses the departure time with the minimum negative utility. Assuming that $\varphi^{rs}(t)$ denotes the negative utility of the traveler of the OD pair rs at time interval t, the optimization model of departure time choice is given below:

$$\begin{aligned} \min_t \quad & \varphi^{rs}(t), \\ \text{s.t.} \quad & \sum_t q^{rs}(t) = D^{rs}, \quad q^{rs}(t) \geq 0, \end{aligned} \tag{4}$$

where D^{rs} is the traffic demand of the OD pair rs.

There is an equilibrium state for the optimization problem of the departure time choice, satisfying dynamic user optimal (DUO) condition. That is to say, for any OD pair rs, the departure time chosen by the traveler should minimize the negative utility of the traveler, and the negative utility of the other departure times is no less than the minimum negative utility. Mathematical expressions are given below:

$$\begin{aligned} q^{rs}(t)[\varphi^{rs}(t) - \varphi_{\min}^{rs}] &= 0, \\ \varphi^{rs}(t) - \varphi_{\min}^{rs} &\geq 0, \\ q^{rs}(t) &\geq 0, \end{aligned} \tag{5}$$

where $\varphi_{\min}^{rs}$ is the minimum of $\varphi^{rs}(t)$. That is,

$$\varphi_{\min}^{rs} = \min_t \varphi^{rs}(t). \tag{6}$$

2.2.3. Travel Mode Choice (TMC). In the case of travel mode choice, the traveler chooses the travel mode with the minimum negative utility. Assuming that $\mu_m^{rs}(t)$ is the negative utility of the mth travel mode of the OD pair rs at time interval t, the optimization model of travel mode choice is given below:

$$\begin{aligned} \min_m \quad & \mu_m^{rs}(t), \\ \text{s.t.} \quad & \sum_m q_m^{rs}(t) = q^{rs}(t), \quad q_m^{rs}(t) \geq 0, \end{aligned} \tag{7}$$

where $q_m^{rs}(t)$ is the traffic demand of the mth travel mode of the OD pair rs at time interval t.

There is an equilibrium state for the optimization problem of the travel mode choice, satisfying dynamic user optimal (DUO) condition. That is to say, for any OD pair rs at any time t, the traveler's choice of travel mode should minimize the traveler's negative utility, and the negative utility of other travel modes is no less than the minimum negative utility. Mathematical expressions are given below:

$$\begin{aligned} q_m^{rs}(t)[\mu_m^{rs}(t) - \mu_{\min}^{rs}(t)] &= 0, \\ \mu_m^{rs}(t) - \mu_{\min}^{rs}(t) &\geq 0, \\ q_m^{rs}(t) &\geq 0, \end{aligned} \tag{8}$$

where $\mu_{\min}^{rs}(t)$ is the minimum of $\mu_m^{rs}(t)$. That is,

$$\mu_{\min}^{rs}(t) = \min_m \mu_m^{rs}(t). \tag{9}$$

2.3. Basic Assumptions for Mathematical Modeling. Based on big data, a unified framework for route choice under guidance information is put forward in this paper. Some basic assumptions are considered.

2.3.1. Model Simplification considering Super Networks. Travel mode choice, especially the choice of combed mode trips, needs consideration of transferring. However, the traditional traffic network studies the bus network, car network, bicycle network, and so on independently. Only one mode is used in each path. Therefore, it is difficult to adapt to multimodal traffic assignment problems. Considering these problems, Sheffi proposed the concept of the super network [31], in which each path is a super path with both travel route and travel mode. In recent years, the traffic assignment problem of combined mode [32] and the optimal design of bus lane network [33] is studied by using the super network. If the travel time is taken into account without considering the complex factors such as comfort and cost, the super network can simplify the TRC-DTC-TMC collaboration optimization problem into a TRC-DTC collaboration optimization problem.

2.3.2. The Introduction of Utility Function at Arrival Time. At peak hours, the ideal arrival time range is the same for the travelers with the same destination. If $\tilde{t}_s$ indicates the ideal arrival time and Δ_s indicates the flexible time, then the ideal arrival time range is $[\tilde{t}_s - \Delta_s, \tilde{t}_s + \Delta_s]$. Arriving in the ideal arrival time interval results in no negative utility; otherwise, there is negative utility. The negative utility of the early arrival and late arrival is expressed by $\tau_p^s(\cdot)$.

$$\tau_p^s\left[t+c_p^{rs}(t)\right]=\begin{cases}\alpha_s\left[\tilde{t}_s-\Delta_s-t-c_p^{rs}(t)\right], & t+c_p^{rs}(t)<\tilde{t}_s-\Delta_s,\\ 0, \quad \tilde{t}_s-\Delta_s\le t+c_p^{rs}(t)\le\tilde{t}_s+\Delta_s, & \\ \beta_s\left[t+c_p^{rs}(t)-\tilde{t}_s-\Delta_s\right], & \tilde{t}_s+\Delta_s<t+c_p^{rs}(t),\end{cases} \tag{10}$$

where α_s, β_s, and γ_s are penalty coefficients.

Then, $\varphi_p^{rs}(t)$, the negative utility of the traveler of the pth route of the OD pair rs at time interval t, can be expressed as

$$\varphi_p^{rs}(t)=\gamma_s c_p^{rs}(t)+\tau_p^s\left[t+c_p^{rs}(t)\right]. \tag{11}$$

Literature [34] assumes that $\tilde{t}_s$ and Δ_s and penalty coefficients α_s, β_s, γ_s are only related with destination s. And literature [35] found that

$$\beta_s>\gamma_s>\alpha_s>0. \tag{12}$$

2.3.3. Routing Choice Behavior under Guidance Information. Considering stochastic user equilibrium (SUE), the nested-logit function is applied to express the choice behavior of departure information. The departure time choice behavior can be expressed as

$$q^{rs}(t)=D^{rs}\frac{\exp\left[-\theta_T\varphi^{rs}(t)\right]}{\sum_t\exp\left[-\theta_T\varphi^{rs}(t)\right]}, \tag{13}$$

where θ_T is the correction parameter, which reflects the degree of perceived error of departure time-negative utility of travelers.

According to expected utility theory,

$$\begin{aligned}\varphi_{\min}^{rs}&=-\frac{1}{\theta_T}\ln\left\{\sum_t\exp\left[-\theta_T\varphi^{rs}(t)\right]\right\}\\&=\varphi^{rs}(t)+\frac{1}{\theta_T}\ln\frac{q^{rs}(t)}{D^{rs}}.\end{aligned} \tag{14}$$

Routing choice behavior can be expressed as

$$f_p^{rs}(t)=q^{rs}(t)\frac{\exp\left[-\theta_P\varphi_p^{rs}(t)\right]}{\sum_p\exp\left[-\theta_P\varphi_p^{rs}(t)\right]}, \tag{15}$$

where θ_P is the correction parameter, which reflects the degree of perceived error of route negative utility of travelers.

According to expected utility theory,

$$\begin{aligned}\varphi^{rs}(t)&=-\frac{1}{\theta_P}\ln\left\{\sum_p\exp\left[-\theta_P\varphi_p^{rs}(t)\right]\right\}\\&=\varphi_p^{rs}(t)+\frac{1}{\theta_P}\ln\frac{f_p^{rs}(t)}{q^{rs}(t)}.\end{aligned} \tag{16}$$

Substitute (15) and (16) into (14), and (17) is obtained as

$$\begin{aligned}\varphi_{\min}^{rs}&=\varphi_p^{rs}(t)+\frac{1}{\theta_P}\ln\frac{f_p^{rs}(t)}{q^{rs}(t)}+\frac{1}{\theta_T}\ln\frac{q^{rs}(t)}{D^{rs}}\\&=\varphi_p^{rs}(t)+\frac{\theta_T-\theta_P}{\theta_P\theta_T}\ln\frac{f_p^{rs}(t)}{q^{rs}(t)}+\frac{1}{\theta_T}\ln\frac{f_p^{rs}(t)}{D^{rs}}\\&=\frac{\theta_P}{\theta_T}\varphi_p^{rs}(t)+\frac{\theta_P-\theta_T}{\theta_P\theta_T}\ln\left\{\sum_p\exp\left[-\theta_P\varphi_p^{rs}(t)\right]\right\}\\&\quad+\frac{1}{\theta_T}\ln\frac{f_p^{rs}(t)}{D^{rs}}.\end{aligned} \tag{17}$$

According to (17), for $\varphi_p^{rs}(t)$, universal negative utility of the pth route of the OD pair rs at time interval t can be expressed by $\widehat{\varphi}_p^{rs}(t)$ as

$$\begin{aligned}\widehat{\varphi}_p^{rs}(t)&=\varphi_p^{rs}(t)+\frac{1}{\theta_P}\ln\frac{f_p^{rs}(t)}{q^{rs}(t)}+\frac{1}{\theta_T}\ln\frac{q^{rs}(t)}{D^{rs}}\\&=\frac{\theta_P}{\theta_T}\varphi_p^{rs}(t)+\frac{\theta_P-\theta_T}{\theta_P\theta_T}\ln\left\{\sum_p\exp\left[-\theta_P\varphi_p^{rs}(t)\right]\right\}\\&\quad+\frac{1}{\theta_T}\ln\frac{f_p^{rs}(t)}{D^{rs}}.\end{aligned} \tag{18}$$

3. Mathematical Model

3.1. Variational Inequality Model. Based on basic assumptions, the super network can simplify the TRC-DTC-TMC collaboration optimization problem into a TRC-DTC collaboration optimization problem. That is, the traveler chooses the travel route and departure time with the minimum negative utility. The optimization model is given below:

$$\begin{aligned}&\min_t\left[\min_p\widehat{\varphi}_p^{rs}(t)\right],\\&\text{s.t.}\quad\sum_{rs}\sum_t\sum_p f_p^{rs}(t)=\sum_{rs}D^{rs},\quad f_p^{rs}(t)\ge 0.\end{aligned} \tag{19}$$

There is an equilibrium state for TRC-DTC collaboration optimization problem, satisfying dynamic user optimal (DUO) condition. That is to say, for any OD pair rs, changing departure time and travel route cannot reduce negative utility of travelers, and the departure time choice and the travel route choice are met with (13) and (15), respectively, which can be expressed as

$$f_p^{rs}(t)\left[\widehat{\varphi}_p^{rs}(t)-\widehat{\varphi}_{\min}^{rs}(t)\right]=0, \tag{20}$$

$$\widehat{\varphi}_p^{rs}(t)-\widehat{\varphi}_{\min}^{rs}(t)\ge 0, \tag{21}$$

$$f_p^{rs}(t)\ge 0, \tag{22}$$

where $\widehat{\varphi}_{\min}^{rs}(t)$ is the minimum of $\widehat{\varphi}_p^{rs}(t)$:

$$\widehat{\varphi}_{\min}^{rs}(t)=\min_t\left[\min_p\widehat{\varphi}_p^{rs}(t)\right]. \tag{23}$$

The set Ω represents the feasible region of $f_p^{rs}(t)$.

$$\Omega = \left\{ f_p^{rs}(t) \geq 0 \mid \sum_{rs}\sum_{t}\sum_{p} f_p^{rs}(t) = \sum_{rs} D^{rs} \right\}. \tag{24}$$

The optimization model (19) clearly reflects the physical meaning of TRC-DTC collaboration optimization. However, it is difficult to be solved. Therefore, we need to find an equivalent model, which is easy to be solved. With the consideration of the advantages of the problem-solving process, variational inequality (VI) is used in this paper.

Because the equilibrium conditions (20), (21), and (22) describe the equilibrium state of optimal routing choice, namely, TRC-DTC collaboration optimization, we should put forward a variational inequality (VI) problem, which is equal to the three equilibrium conditions.

We find that equilibrium conditions (20), (21), and (22) equal the variational inequality (VI) problem with a feasible region Ω:

$$\sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] \left[f_p^{rs}(t) - f_p^{rs*}(t) \right] \geq 0, \tag{25}$$

where variables with * need to be resolved.

Therefore, the VI problem (25) is the mathematical model for the collaboration optimization. In order to show the equivalence property, the following 3 problems need to be proved.

(A) Optimum solution $f_p^{rs*}(t)(f_p^{rs*}(t) \in \mathbf{\Omega})$, which satisfies equilibrium conditions (20), (21), and (22), is also the optimum solution of VI problem (25).

Proof 1. The following formula can be conducted from (21) and (22):

$$f_p^{rs}(t) \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] \geq 0. \tag{26}$$

And the following formula can be conducted from (20):

$$f_p^{rs*}(t) \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] = 0. \tag{27}$$

Subtract (26) from (27), and sum it by rs, t, and p; then VI problem (25) can be obtained.

(B) The optimum solution $f_p^{rs*}(t)(f_p^{rs*}(t) \in \mathbf{\Omega})$ of VI problem (25) also satisfies equilibrium conditions (20), (21), and (22).

Proof 2. Since $f_p^{rs*}(t)(f_p^{rs*}(t) \in \mathbf{\Omega})$ is the optimum solution of VI problem (25), then

$$\begin{aligned} &\sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs}(t) \\ &\quad \geq \sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs*}(t). \end{aligned} \tag{28}$$

Define that

$$\omega\left[\mathbf{f}_p^{rs}(t) \right] = \sum_{rs}\sum_{t}\sum_{p} f_p^{rs}(t) - \sum_{rs} D^{rs}, \tag{29}$$

where $\mathbf{f}_p^{rs}(t)$ is the vector form of $f_p^{rs}(t)$, which satisfies

$$\omega\left[\mathbf{f}_p^{rs}(t) \right] = 0. \tag{30}$$

Introduce a sufficiently large positive number M, which satisfies

$$\begin{aligned} &\sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs}(t) + M\left\{ \omega\left[\mathbf{f}_p^{rs}(t) \right] \right\}^2 \\ &\quad \geq \sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs*}(t) \\ &\quad + M\left\{ \omega\left[\mathbf{f}_p^{rs}(t) \right] \right\}^2. \end{aligned} \tag{31}$$

Then, the optimum solution $f_p^{rs*}(t)(f_p^{rs*}(t) \in \mathbf{\Omega})$ of VI problem (25) is also the optimum solution of the following minimization problem:

$$\begin{aligned} \min \quad & \sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs}(t) + M\left\{ \omega\left[\mathbf{f}_p^{rs}(t) \right] \right\}^2, \\ \text{s.t.} \quad & f_p^{rs}(t) \geq 0. \end{aligned} \tag{32}$$

The following Lagrange function is constructed:

$$L = \sum_{rs}\sum_{t}\sum_{p} \left[\widehat{\varphi}_p^{rs*}(t) - \widehat{\varphi}_{\min}^{rs}{}^*(t) \right] f_p^{rs}(t) + M\left\{ \omega\left[\mathbf{f}_p^{rs}(t) \right] \right\}^2. \tag{33}$$

According to Karush-Kuhn-Tucker (KKT) conditions, the following conditions are satisfied at the extreme points of Lagrange function (33):

$$f_p^{rs}(t) \frac{\partial L}{\partial f_p^{rs}(t)} = 0, \qquad \frac{\partial L}{\partial f_p^{rs}(t)} \geq 0. \tag{34}$$

The equilibrium conditions (20), (21), and (22) are further obtained.

(C) Existence and uniqueness of solutions for VI problem (25) are proved as follows:

Proof 3. Considering the monotone condition and that the feasible domain is a bounded closed convex set, with the help of Brouwer's fixed-point theory, it is easy to prove the existence and uniqueness of the solution.

Besides, after substituting (18) into (25) and with transformation, the following VI problem is obtained:

$$\sum_{rs}\sum_{t}\sum_{p}\left\{\left[\varphi_p^{rs*}(t)+\frac{1}{\theta_{\mathrm{P}}}\ln\frac{f_p^{rs*}(t)}{q^{rs*}(t)}\right]\left[f_p^{rs}(t)-f_p^{rs*}(t)\right]+\frac{1}{\theta_{\mathrm{T}}}\ln\frac{q^{rs}(t)}{D^{rs}}\left[q^{rs}(t)-q^{rs*}(t)\right]\right\}\geq 0. \quad (35)$$

VI problem (35) is transformed from VI problem (25). There is no need to prove again its equivalence with equilibrium conditions (20), (21), and (22) and the existence and uniqueness of its solutions.

3.2. Dynamic Constraint Condition

3.2.1. Section State Equation. The discretized section state equation can be expressed as

$$x_a(t+1)=x_a(t)+f_a(t)-v_a(t), \quad (36)$$

where $x_a(t)$ is the flow (state variable) of section a at the tth time interval, $f_a(t)$ is the arrival rate, and $v_a(t)$ is the departure rate.

Extended to the route level, the discretized route state equation can be expressed as

$$x_p^{rs}(t+1)=x_p^{rs}(t)+f_p^{rs}(t)-v_p^{rs}(t), \quad (37)$$

where $x_p^{rs}(t)$ is the flow (state variable) of the pth route of the OD pair rs at the tth time interval, $f_p^{rs}(t)$ is the arrival rate, and $v_p^{rs}(t)$ is the departure rate.

3.2.2. Section Impedance. Taking the travel time as the section impedance, the point queuing model considers that the travel time function of the section can be expressed as

$$c_a(t)=c_a^0+\frac{x_a(t)}{Q_a}, \quad (38)$$

where $c_a(t)$ is the impedance of section a at the tth time interval, c_a^0 is the free flow travel time of section a, and Q_a is the capacity of a.

As explained by "Buckets Effect," the section with the minimum capacity of the whole route affects the capacity of the entire path; that is,

$$Q_p^{rs}=\min_a Q_a, \quad (39)$$

where Q_p^{rs} is the capacity of the pth route of the OD pair from rs.

Then, the section impedance is

$$c_p^{rs}(t)=\sum_a c_a^0+\frac{x_p^{rs}(t)}{Q_p^{rs}}. \quad (40)$$

3.2.3. Propagation Characteristic Function. The method for calculating the departure rate of a section is

$$v_a(t)=\begin{cases} x_a(t)+f_a(t), & Q_a>x_a(t)+f_a(t),\\ Q_a, & Q_a\leq x_a(t)+f_a(t). \end{cases} \quad (41)$$

Extended to the route level, section $a-1$, the immediate predecessor of section a, should satisfy the conservation constraints:

$$v_{a-1}(t)=f_a(t). \quad (42)$$

The method for calculating the departure rate of a route is

$$v_p^{rs}(t)=\begin{cases} x_p^{rs}(t)+f_p^{rs}(t), & Q_p^{rs}>x_p^{rs}(t)+f_p^{rs}(t),\\ Q_p^{rs}, & Q_p^{rs}\leq x_p^{rs}(t)+f_p^{rs}(t). \end{cases} \quad (43)$$

3.2.4. General Constraints. The traffic conservation constraint assumes that the traffic demand is fixed and known, as shown in (3), (6), and (23). The boundary condition assumes that there is no traffic at the initial moment, as shown in (44). Nonnegative constraints guarantee the nonnegativity of inflow for routes and sections, as shown in (21) and (45).

$$x_a(1)=0, \quad (44)$$

$$x_a(t)\geq 0. \quad (45)$$

4. Solution Algorithms

Taking $[0,\delta T]$ as a research time interval, with the help of the basic idea of the method of successive average (MSA) algorithm, VI problem (25) is resolved with the following algorithm.

Step 1. In initialization, with iteration number $h=0$, Q^{rs} is equally allocated to routes and time intervals to get the initial value of $f_p^{rs}(t)^{(h)}$.

Step 2. According to $f_p^{rs}(t)^{(h)}$, dynamic random distribution and load on the network are performed to obtain $\varphi_p^{rs}(t)^{(h)}$.

Step 3. According to $\varphi_p^{rs}(t)$ and with the help of (13) and (15), $f_p^{rs*}(t)^{(h)}$ is calculated.

Step 4. Set $h=h+1$, and the traffic volume of each route is updated:

$$f_p^{rs}(t)^{(h)}=f_p^{rs}(t)^{(h-1)}+\left[f_p^{rs*}(t)^{(h-1)}-f_p^{rs}(t)^{(h-1)}\right]/h \quad (46)$$

Step 5. In convergence judgment, if the results of the two iterations are not much different, then stop the algorithm. Otherwise, $h=h+1$ and return to Step 2.
The convergence criterion is

$$\sqrt{\frac{1}{\eta}\sum_{rs}\sum_{p}\sum_{t}\left(f_p^{rs}(t)^{(h)}-f_p^{rs}(t)^{(h-1)}\right)^2}\leq\varepsilon, \quad (47)$$

where η is the number of $f_p^{rs}(\cdot)$ and ε is the threshold of the standard deviation of $f_p^{rs}(\cdot)$ for two consecutive iterations; generally, $\varepsilon=0.01$.

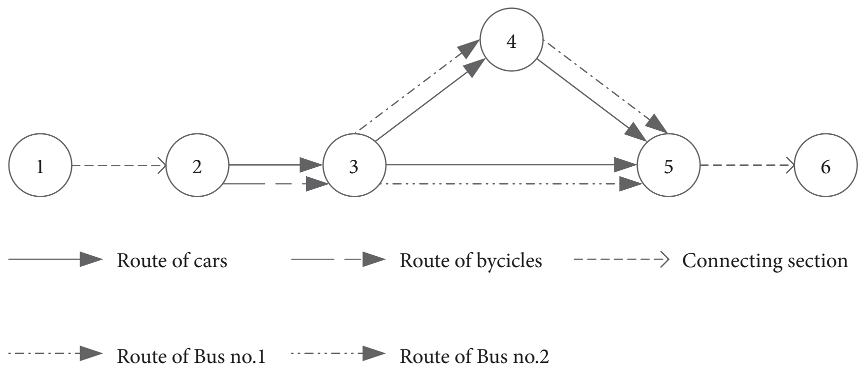

Figure 3: Multimode traffic network.

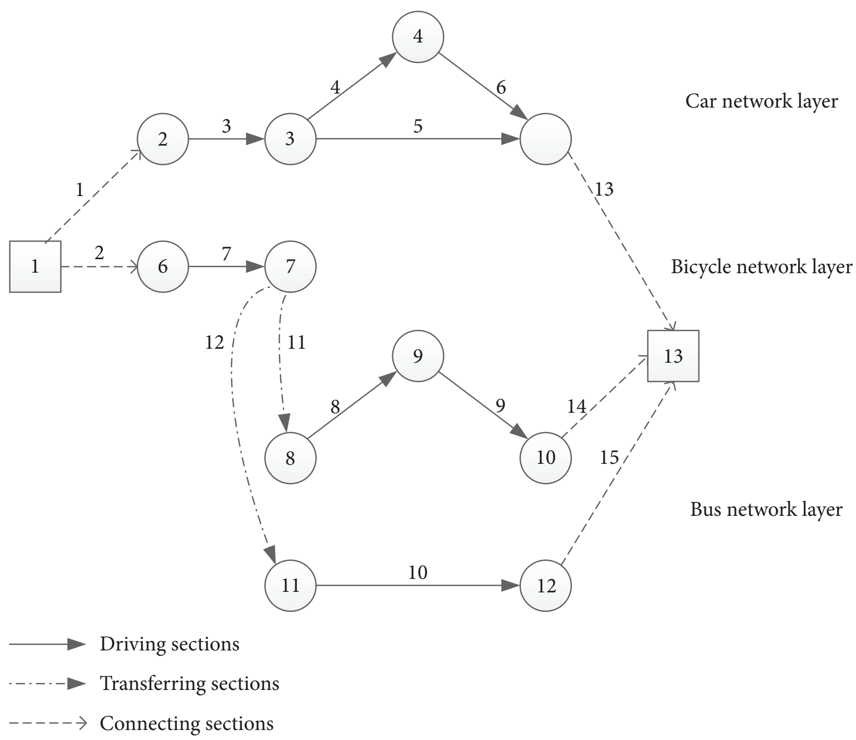

Figure 4: Super network.

5. Case Study

5.1. Traffic Network. According to the map of trips in the context of big data, the multimode transportation network in Figure 3 is designed as the research object, and a case study is conducted. The connecting sections of the network only indicate the trip process without travel time. In addition, it is assumed that there is no transferring between cars and buses.

Based on big data, the following known conditions can be achieved:

(A) The traffic demand of each OD (origin-destination) pair (D^{rs})

(B) The value of penalty coefficients and correction parameters for routing choice (α_s, β_s, γ_s, θ_T, and θ_P)

(C) The multimode traffic network (as shown in Figure 3)

(D) Others

This paper focuses on the mathematical model under the background of big data and does not pay attention to the data collection method.

5.2. Calculation Process. The multimodal traffic network is converted into a super network, as shown in Figure 4. The super route is used to express the combined travel mode, and the route 2-7-11-8-9-14 is the super route of bicycle-bus transferring travel.

The valid route set is shown in Table 3.

The basic parameters of the road sections are shown in Table 4.

Table 3: Valid route set.

Number	Routes	Travel mode
1	1-3-4-6-13	Car travel
2	1-3-5-13	Car travel
3	2-7-11-8-9-14	Bicycle-bus transferring travel
4	2-7-12-10-15	Bicycle-bus transferring travel

Table 4: Basic parameters of the road section.

Sections	c_a^0	Q_a
3	100	6
4	350	3
5	600	6
6	200	4
7	150	2
8	380	12
9	220	12
10	690	15
11	50	60
12	10	60

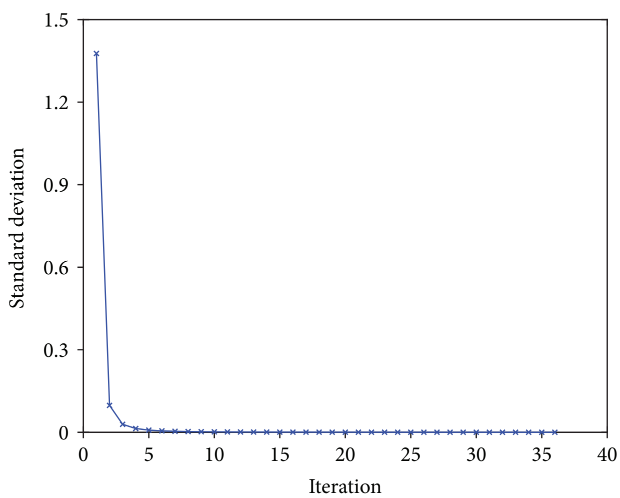

Figure 5: Iterative process.

Taking $[0, 2400]$ s as the research interval, $T = 240$ and $\delta = 10$ s. If $\tilde{t}_s = 2000$ s and $\Delta_s = 100$ s, then the ideal arrival time interval is $[1900, 2100]$ s. Besides, $\alpha_s = 0.5$, $\beta_s = 1$, $\gamma_s = 0.8$, $\theta_T = 0.005$, and $\theta_P = 0.010$.

By solving the variational inequality model based on the MSA algorithm, the results are obtained within 5 iterations. The standard deviation is stable after 5 iterations, as shown in Figure 5.

5.3. Results and Discussion. The relationship between departure time choice and travel route choice is shown in Figure 6. The departure time choice functions of different routes have the same trends. The travel peak is influenced by the ideal arrival time interval $[1900, 2100]$ s, with the overall situation of the trip volume shown in Figure 7. Comparatively speaking, travel peak of car travel (route 1 and route 2) is later than that of bicycle-bus transferring trip (route 3 and route 4).

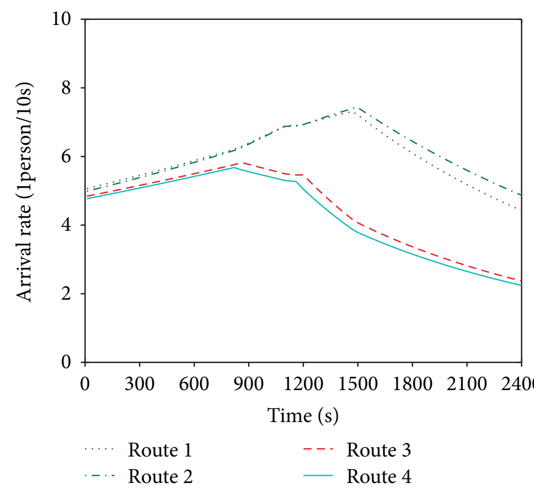

Figure 6: The relationship between departure time choice and travel route choice.

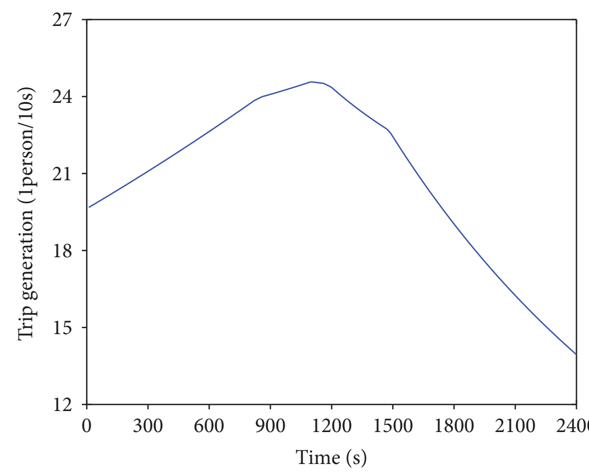

Figure 7: The overall trend of trip generation.

The relationship between departure time choice and travel negative utility is shown in Figure 8. The travel cost of different routes decreases first and then increases, because of the negative utility of arriving early and late. The relationship between departure time and $\tau_p^s(\cdot)$, the negative utility of early/late arrival, is shown in Figure 9. A traveler who set off within $[1100, 1470]$ s happens to arrive within the ideal time interval when choosing route 1. If choosing route 2, he needs to set off within $[1100, 1490]$ s, route 3 within $[860, 1190]$ s, and route 4 within $[820, 1160]$ s.

From the results of the calculation, it is not difficult to find that collaboration optimization makes the route choice, departure time choice, and travel mode choice of travelers more reasonable, the significance of which on urban traffic management is embodied in the following aspects:

(a) Travel mode choice is more reasonable, which reduces the traffic flow under signal control and relieves the pressure on city traffic management.

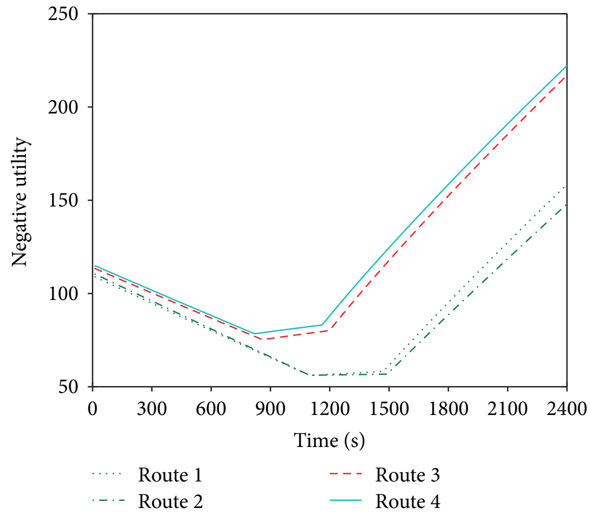

FIGURE 8: The relationship between departure time and negative utility of the travel.

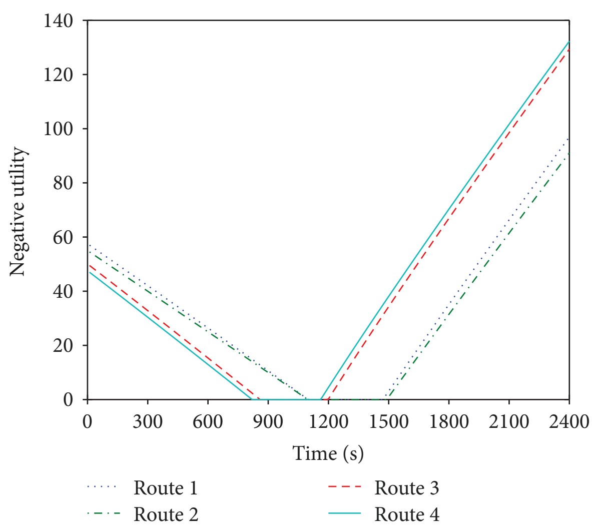

FIGURE 9: The relationship between departure time and negative utility of early/late arrival.

(b) The choice of departure time is more reasonable, which stabilizes the traffic flow under signal control, avoids the sudden increase of traffic flow, and reduces the pressure on urban traffic management.

(c) The comprehensive consideration of the choice of travel route, departure time, and travel mode can improve the accuracy of OD estimation and then assists urban traffic management.

6. Conclusion

The significance of this research lies in an explanation-unified framework for optimal routing choice under guidance information.

(A) Discussing the guidance information and combined travel mode, the analysis of routing choice based on big data is employed to study the map of trips in the context of big data. Moreover, routing choice considering guidance information is analyzed and some basic assumptions are proposed for mathematical modeling.

(B) The optimal routing choice problem is abstracted into the collaboration optimization model of travel route choice, departure time choice, and travel mode choice. Based on the basic assumptions, the collaboration optimization model is formulated as a variational inequality model. With the help of the MSA algorithm, a case study verifies the model and algorithm.

As this paper is devoted to the research of a unified framework, the collaboration optimization model presented in this paper is relatively simple, and a traffic network is abstracted for case study. In future research, the proposed model needs to be further deepened based on actual data.

Conflicts of Interest

The authors declare no conflict of interest.

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