

Constraint Semantics and its Application to Conditionals

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We can think of ordinary truth-conditional semantics as giving us constraints on cognitive states.

But constraints on cognitive states can be more complicated than simply *believing a proposition*. And we communicate more complicated constraints on cognitive states. We also communicate constraints that seem to bear on affective and conative states.

For a given *prima facie* communicated constraint, truth-conditional semantics generally either

1. does not aim to deliver it, or
2. tries to cram it into the ‘belief in a proposition’ model.

Constraint semantics aims to deliver *whatever* constraints we communicate, and to do so compositionally: constraint semantics handles quantifiers that scope over (what it takes to be) constraint operators:

- (1) Every inch of the floor might have paint on it.

Crucial questions:

1. How would a semantic interpretation function that delivers ‘constraints’ work?
2. To what extent should (and could) a constraint semantics be compositional?
3. Can constraint semantics give a better model for a fragment of natural language than truth-conditional semantics can?

How constraint semantics works

Constraints are *sets of inadmissibles*. Those might be

- inadmissible credences in a proposition
- inadmissible credences / ratios for credences in a pair of propositions
- inadmissible selection functions / similarity orderings
- inadmissible preference orderings
- or whatever you like ...

The semantic value of a sentence is the characteristic function of a set of inadmissibles. A set of inadmissibles represents a constraint by representing the conditions that are, *as far as that constraint is concerned*, inadmissible.

Semantic types of subsentential expressions:

Referring expressions	$\langle e \rangle$
Predicates	$\langle e, \langle i, t \rangle \rangle$
Quantifiers	$\langle \langle e, \langle i, t \rangle \rangle, \langle i, t \rangle \rangle$
Constraint operators	$\langle \langle i, t \rangle, \langle i, t \rangle \rangle$

(‘ $\langle e, \langle i, t \rangle \rangle$ ’: a function from entities (e) to functions from inadmissibles (i) to truth values (t).)

I take the semantic value of a predicate to be a function from individuals to sets of inadmissible *credences*.

Constraint operators change one kind of constraint into another. Consider

(2) John will rest.

The semantic value of the predicate ‘will rest’ is a function from an individual x to the set consisting of credences that are inadmissible given the belief that x will rest. Now consider

(3) John may rest.

Given the logical form [may [John will rest]] for (3), $\llbracket \text{may} \rrbracket$ takes the semantic value of (2)—a *credal* constraint—and delivers the semantic value of (3)—a *normative* constraint.

More concretely (but making some unforced choices):

$\llbracket \text{John will rest} \rrbracket$ = the characteristic function of the set of all functions that take the proposition that John will rest to a value in $[0, 1)$ and are otherwise undefined.

$\llbracket \text{may} \rrbracket$ takes sets of functions from propositions into values in $[0, 1]$ to sets of functions from propositions into $\{\text{IMPERMISSIBLE}, \text{PERMISSIBLE}, \text{OBLIGATORY}\}$.

In particular, $\llbracket \text{may} \rrbracket$ takes a constraint that makes credences in $[0, 1)$ to ϕ inadmissible, and returns a constraint that makes assignments of IMPERMISSIBLE to ϕ inadmissible.

So $\llbracket \text{John may rest} \rrbracket$ = the characteristic function of the set consisting of the function that takes the proposition that John will rest to IMPERMISSIBLE and is otherwise undefined.

How compositional a theory do we need?

We need at least enough compositionality to explain *sub-clausal* linguistic phenomena:

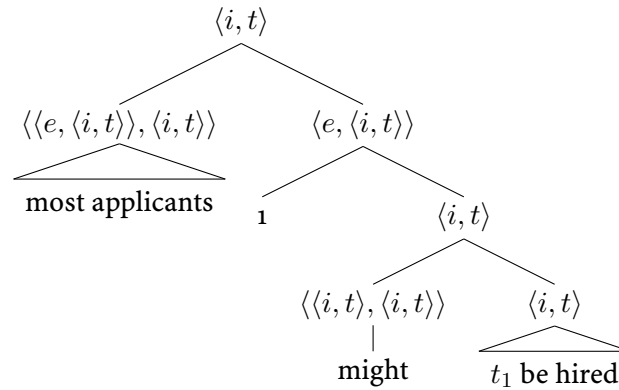
(4) Most of the applicants might be hired ...

‘For most of the applicants x , x might be hired.’ (MOST OF THE APPLICANTS > MIGHT)

‘It might be that most of the applicants are hired.’ (MIGHT > MOST OF THE APPLICANTS)

(5) ...but we will hire only one.

Logical form of (4), with wide-scope quantifier:



The indexed predicate here takes an object x to the set consisting of credences that take the proposition that x is hired to values in $[0, \mu)$. (μ = the least credence sufficient for a ‘might’ belief)

In constraint semantics, a quantifier combines with a predicate whose semantic value is P to yield the same set of inadmissibles as

$$\bigvee_{X \in Q} \left(\bigwedge_{x \in X} P(x) \right)$$

(For ‘most of the applicants,’ Q = the set of sets consisting of more than half of the applicants.)

$\llbracket \text{most of the applicants} . 1 \text{ (might } t_1 \text{ is hired)} \rrbracket$ is the same set of inadmissibles as is yielded by

$$\bigvee_{X \in Q} \left(\bigwedge_{x \in X} \lambda t_1 (\text{might } t_1 \text{ is hired})(x) \right)$$

Put another way: $\llbracket \text{most of the applicants} \rrbracket$ is a function that takes an $\langle e, \langle i, t \rangle \rangle$ object (call it $P(\cdot)$) and yields the constraint corresponding to ‘ $(P(\text{applicant 1}) \wedge P(\text{applicant 2}) \wedge \dots) \vee (P(\text{applicant 2}) \wedge P(\text{applicant 3}) \wedge \dots) \vee \dots$ ’

So constraint semantics allows that

(4) Most of the applicants might be hired ...

can be followed by

(5) ...but we will hire only one.

without inconsistency.

The application to conditionals

Conditionals can also scope under quantifiers:

(6) Most bubbles pop if touched.

This is *not* an assertion of ‘most bubbles pop’ conditional on their being touched.

Do linguistic phenomena like this force us to hold that indicative conditionals express propositions?

(7) If the bubble is touched, it will pop.

(7) expresses the constraint that one’s probability that the bubble’s popping conditional on its being touched is high; that one’s cognitive state not be such that $Pr(C|A) \approx 1$.

So $\llbracket \text{if } A, C \rrbracket = \{ \text{credences making } Pr(C|A) \approx 1 \}$

Now, quantifying in:

$\llbracket \text{most bubbles } . 1 \text{ (if } x \text{ is touched, then } x \text{ pops)} \rrbracket$ is the same set of inadmissibles as is yielded by

$$\bigvee_{X \in Q} \left(\bigwedge_{x \in X} \lambda t_1 (\text{if } t_1 \text{ is touched, then } t_1 \text{ pops})(x) \right)$$

So we can explain how quantifiers can scope over modals and conditionals, without having to say that all modalized statements and conditionals express propositions; they express constraints.

What about (8) and (9)?

(8) Most of the bubbles would have popped if they had been touched.

(9) If the bubble had been touched, it would have popped.

Do they demand a semantics radically different from that proposed for (6) and (7)?

Not necessarily. We can make progress on this question by asking how the cognitive constraints associated with different kinds of conditionals are similar, and how they are different.