A physicalist account of mathematical truth

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Realists, Platonists and intuitionists jointly believe that mathematical concepts and propositions have *meanings*, and when we formalize the language of mathematics, these meanings are meant to be reflected in a more precise and more concise form. According to the *formalist* understanding of mathematics (at least, according to the radical version of formalism I am proposing here) *the truth, on the contrary, is that a mathematical object has no meaning*; we have marks and rules governing how these marks can be combined. That's all.

Mathematics has nothing to do with the metaphysical concept of infinity. Mathematics does not produce and does not solve Zeno paradoxes. Mathematical structures are totally indifferent to our intuition about space, time, probability or continuity. The only truth in mathematics is "being formally proved". This is what the Tortoise said to Achilles.

Anybody, who believes in the power of mathematics, must recognize that its power is nothing more than the power of the rules of chess or rummy. To be sure, mathematics is not "applicable" to the real world, but we can construct *physical* theories that do refer to the elements of reality. A physical theory is a formal system plus a semantics pointing to the empirical world. Consequently, a sentence in a physical theory can be true in two different senses:

Truth$_1$: The sentence is proved in the formal system (which is a mathematical truth, a fact of the formal system itself).

Truth$_2$: According to the semantics, the sentence refers to an empirical fact (about the physical system described by the physical theory).

For example, the Coulomb law is a theorem of Maxwell's electrodynamics—one can derive it from the Maxwell equations. On the other hand, according to the semantics, it corresponds to an empirical fact (about the point charges).

Now, Truth$_1$ and Truth$_2$ are independent, in the sense that one does not imply the other—that is what the Tortoise said to Achilles.

The *ontology of formal systems* is crystal-clear: marks, say ink molecules diffused among paper molecules, or something like that. Of course, a derivation on paper is rather similar to a production line in a factory employing low-paid workers: at certain points of the technological process human hands (and brains) transpose the workpiece from one conveyor belt to the other.

This is, however, an unimportant technical problem. Since each step of manipulation is governed by strict rules, human beings can be replaced by trained animals, robots, etc. Also the marks can be of an entirely different nature, like the cybernetic states of a computer, supervening on the underlying electronic processes, etc.

Sometimes one executes simple formal derivations also in the head. If the signs and the rules of a formal system can be embodied in various physical states/processes, why not let them be embodied in the neuro-physiological, biochemical, biophysical brain configurations—more exactly, in the physical processes of the human brain? If this is the case, that one of the paths—as some rationalists believe, the only path—to trustworthy knowledge, the deductive/logical thinking, can be construed as a mere complex of physical (brain) phenomena, then this is, actually, a very strong argument for physicalism.

Wittgenstein's language game is one complex of physical interaction processes—interactions between *the builder's brain*, *the builder's body*, *the air* (with the sound-wave patterns), *the assistant's body*, *the assistant's brain*, and *the stone*. There is no "meaning" and no "intentionality" in this picture—just as there are no "abstract mathematical structures" and similar things accommodated in the Platonic realm or Popper's nth world or something like these. All these blue flowers are completely dissolved in physical reality. But we have ontological uniformity, instead. In this way, for example, a physical theory becomes intelligible for physics itself.

It is a widespread opinion that one cannot justify a general statement about the world by *induction*. According to this opinion, *deduction*, contrary to induction, provides secure confidence because it is based on pure reasoning, without referring to empirical facts. In Leibniz's formulation:

> There are ... two kinds of truths: those of reasoning and those of fact. The truths of reasoning are necessary and their opposite is impossible; the truths of fact are contingent and their opposites are possible.

As we have seen, however, a formal system is nothing but a physical system, and derivation is a physical process. The knowledge of a mathematical or logical truth is the knowledge of a property of the formal system in question—the knowledge of a fact about the physical world. In other words, *deduction is a particular case of induction.* So, contrary to Leibniz's position, we must draw the following epistemological conclusion: *The certainty available in inductive generalization is the best of all possible certainties!*