Nominalism

Zoltán Gendler Szabó
The Sage School of Philosophy
Cornell University
218 Goldwin Smith Hall
Ithaca, NY 14853-3201
e-mail: zs15@cornell.edu
tel.: 607.272.6824

forthcoming in Michael Loux and Dean Zimmerman eds., *Oxford Handbook of Metaphysics.* Oxford: Oxford University Press.

Draft of May 22, 2001

0. Introduction

Are there numbers? What about directions, sets, shapes, animal species, properties, relations, propositions, linguistic expressions, meanings, concepts, rights, values, or any other abstract entities? There are two sorts of answers to such questions: straight ones and oblique ones. The straight answers are typically introduced by the expression "of course", as in "Of course there are, otherwise how could sentences like '2+2=4' and 'There is something Napoleon and Alexander have in common' be true?" and "Of course there aren't, for how could we even know or speak of things that are causally inert?" The oblique answers are usually headed by the locution "well, you know", as in "Well, you know that really depends on whether you take this to be an internal or external question" and "Well, you know that actually depends on whether you mean 'exist' in a thick or thin sense." Analytic philosophers tend to feel a strong inclination towards the clear-cut. But ontology — and especially the ontology of the abstract — is an area in which it is hard to dismiss oblique lines.

The *nominalist* sticks with straight negative answers: she unqualifiedly rejects abstract entities of any sort whatsoever.[1] The nominalist's equally straight opponent is the *anti-nominalist*,[2] who accepts at least one type of abstracta. On the face of it, their views are clear opposites. Nonetheless, both expend a good deal of effort fending off a variety of oblique answers seeking a middle ground between their views.

Nominalism is certainly not the most surprising eliminativist thesis — there are some who deny the existence of ordinary material objects, mental states, or persons — but it is among the most radical of those widely held. Nominalism does away with so many kinds of putative entities that the ontology it yields may not even be properly described as a desert landscape. After all, aren't landscapes, at least in one of the perfectly legitimate senses of this word, abstract?

Nominalism is a divisive doctrine. Proponents often concede that they are fighting an uphill battle, but justify their insistence with an appeal to ontological conscience; opponents tend to be skeptical about the sincerity of such appeals. They suspect that nominalism is indeed much like a desert: an uncomfortable place whose main attraction is that it is hard to be there. Some of this clash is no doubt the result of a genuine conflict in philosophical temperament, but there is another source as well. Contemporary nominalism grows out of a number of different traditions, each contributing its distinct

---

[1] 'Nominalism' is often used in another sense as referring to the doctrine that there are no universals. In the traditional medieval sense of the word, nominalism is the doctrine that whatever exists is particular, and nothing but particular. According to nominalists, generality belongs to certain *nominal expressions* alone, and it belongs to them only in the sense that they may apply to more than one particular. The origin of the term 'nominalism' is subject to serious scholarly dispute; cf. Courtenay (1992).

[2] The term 'Platonism' is occasionally used in the literature in the sense I employ 'anti-nominalism.' Unfortunately, is also often used in a richer sense, when it carries additional commitment to the mind-independence of abstract entities. Since neither sense of 'Platonism' has much to do with Plato's metaphysics, I have opted for a neutral term.

understanding of the key terms of the nominalist thesis. The intensity of many philosophers' belief in the *absurdity* of nominalism is partly the result of the seeming simplicity and underlying ambiguity of the position.

To bring out the perplexing character of nominalism, consider the often voiced concern that the view appears to be a self-undermining. For suppose that a nominalist — call him Nelson — just told you that there are no abstract entities. How should Nelson describe what he did? Did he say something? Certainly not, if saying something amounts to expressing a proposition. Did he utter something? Clearly not, if uttering something requires the articulation of a sentence type. Did he try to bring you to share his belief? Obviously not, if sharing a belief requires being in identical mental states.

Of course, Nelson is not likely to be moved by all this. After all, there is a nominalistically acceptable way of describing what happened: he produced meaningful noises and thereby attempted to bring you into a mental state relevantly similar to one of his own. There is no mention of propositions, sentences, or shareable beliefs here and still, in an important sense, we are told precisely what was going on. Nevertheless, that we can find such an alternative way of talking is by no means a *complete* response to the concern about self-undermining. For the questions raised were merely bypassed, not answered. We can raise them again: When Nelson produced those meaningful noises, did he say something? did he utter something? did he try to bring you to share his belief? If the answer is 'no', Nelson must tell us just how we ended up in a massive error in thinking the commonplace thought that Nelson *did* say something by uttering a sentence and that we might have ended up sharing his belief. If the answer is 'yes', he has to explain how that concession is supposed to be compatible with his renunciation of abstracta. How Nelson answers this challenge is crucial for a full understanding of his position.

I will begin (Section 1.) with a good deal of clarification. Participants in contemporary debates surrounding nominalism tend to share certain assumptions about what ontological commitment amounts to, how the abstract and the concrete are to be distinguished, and what objects in general are. It is good to have these assumptions on the table. Then (Section 2.) I turn to a discussion of nominalist attitudes towards the apparent commitment ordinary thinking and speech carries to abstracta. This is followed by a survey of some of the most influential arguments for nominalism (Section 3.) and against it (Section 4.). The essay ends (Section 5.) with a brief look at some oblique answers to the ontological question about abstracta. I will make no attempt to resolve the issues here but my anti-nominalist inclination will no doubt show throughout.

1. The nominalist thesis

The debate about nominalism concerns the question whether there are abstract entities. The terms of this question — 'there are', 'abstract' and 'entity' — are all subject to interpretative disagreements. I will start by examining them one by one.

1.1. Are there…

The standard view nowadays is that we can adequately capture the meaning of sentences like 'There are *F*s', 'Some things are *F*s' or '*F*s exist' through existential quantification. As a result, not much credence is given to the idea that we must distinguish between different kinds or degrees of existence.[3] When we talk about whether there are cheap hotels in New York and when we talk about whether possible worlds exist, there is no fundamental difference in logical form between the claims at stake. If this much is agreed upon, alternative conceptions of ontological commitment must be presented as alternative views about quantification.[4]

There are all sorts of exotic existential quantifiers in formal languages: some are interpreted substitutionally; some can bind predicate-, function-, or sentence-variables; some bind all variables within their scope unselectively; some contain only a finite number of variables. There are formal languages, for example those of intuitionistic, free, and quantum logic, where certain classical inferences are invalid. There is no serious

[3] Commitment to a univocal quantificational analysis of existence claims need not be taken as entailing the rejection of fundamental categories in metaphysics. But the distinction cannot be ontological: entities in the different categories exist in the same sense of the word.
[4] For reservations regarding the view that quantification and ontology are inextricably bound together, see Azzouni (1998) and Szabó (forthcoming b).

question about the coherence of the semantic rules governing such languages.[5] But this does not settle the deeper question whether these formal devices have anything to do with anything we ordinarily think or say.

The usual line of defense against employing non-standard quantification to capture our existential idioms goes back to Quine. It relies on two claims. First, that the interpretation *given* to the classical objectual first-order existential quantifier is just this: there are things that are thus and so. Second, that the ordinary existential idioms are *univocal*: there is only stylistic difference between saying that there are things that are thus and so and saying that thus and so's exist.[6] Both claims are widely endorsed, both are plausible, both are nonetheless questionable. We do tend to say when elucidating the meaning of the material conditional that we interpret $\varphi \rightarrow \psi$ as *if* φ *then* ψ, but there is good reason to suspect that we are wrong about that. The English 'if…then' seems to have a different semantics. This shows that the ordinary language glosses we give for sentences of first-order logic may not capture their correct interpretations. We do not bestow meaning upon our logical symbolism simply by insisting on a canonical paraphrase.[7] The univocality of our ordinary existential idioms is no less problematic. After all, it is a fact of ordinary language use that it is fairly *natural* to say that there is a good chance that the Supreme Court won't choose a president again and it is fairly *unnatural* to say that some thing is such that it is a good chance that the Supreme Court won't choose a president again. It is also a fact that many native speakers of English would balk at the inference from the first claim to the second. Is it really obvious, prior to any empirical investigation, that the proper explanation of this fact will not involve the postulation of ambiguity?

---

[5] Although claims of incoherence occasionally do surface in the philosophical literature. To get a sense how the coherence of non-standard quantification is to be defended, see for example Dummett(1973a), Boolos (1975), and Kripke (1976).

[6] Quine (1969): 106.

[7] It is of course true that we did not learn quantificational theory as our mother tongue. But this does not mean that its acquisition proceeds simply by establishing a translation-manual from ordinary language to the language of first-order logic. It would be hard to deny that the meaning of the standard existential quantifier is fixed by the way we use it. But it does not follow from this that a tiny aspect of this use – our willingness to offer the ordinary existential idioms as adequate translation – is by itself sufficient to determine what it means.

Quine has another argument for adopting his strategy of regimenting ontological disputes: just as he thinks we should believe in the existence of those things our *best theory* says there are, he also thinks we should interpret 'exist' to mean what our *best logic* says it means. And Quine thinks our best logic is classical first-order logic: he often praises it for its "extraordinary combination of depth and simplicity, beauty and utility."[8] No doubt, classical first-order logic is the best understood quantificational logic and it has remarkable meta-logical features, which distinguish it sharply from its alternatives. Still, it is by no means clear that this is enough to make sense of the claim that classical first order logic is *better* than the rest. And even if it is, couldn't it be that by regimenting our ordinary speech using our best logic, we end up misinterpreting it? Those of us who — unlike Quine — believe that typically there is a fact of the matter regarding the truth-conditions of sentences in ordinary language cannot simply dismiss this possibility.

Whether our ordinary existential idioms are well represented by the standard existential quantifier is an open empirical problem of linguistics. But this fact need not paralyze ontology. For even if it turned out that ordinary language does not employ the devices of classical first-order logic, there is no reason to doubt that we do *understand* those devices, and that we do find the use of '$\exists$' illuminating in articulating ontological problems. We want to know whether the sentence '$\exists x.x$ is an abstract entity' is true,[9] and we are prepared to say that the correct answer to this question would resolve the debate about nominalism. Once the semantic questions are bracketed, there is presumably no harm in the continued use of ordinary language. Even if it turns out that 'there are' or 'exist' mean something slightly different from what '$\exists$' does in classical first-order logic, the difference now appears immaterial to the debate at hand.[10]

1.2. … an abstract …

There is no generally accepted way to draw the distinction between the abstract and the concrete. Still, there is a rough agreement on the paradigms. Concrete entities are in some

[8] Quine (1969): 112 – 3.
[9] This, of course, is not a sentence in English. But we seem to have a pretty good grasp of its meaning anyway.

important aspect like pebbles (or donkeys, or protons), whereas abstracta are like numbers (or shapes, or propositions). To characterize the distinction this way is vague and unprincipled, but it is the natural starting point; discussions of the distinctions between the physical and the mental and between the descriptive and the normative begin the same way.

Tradition says that abstract entities are *abstractions* from concrete ones. Abstract entities lack specificity in the sense that an incomplete characterization of a complete entity may serve as a complete characterization of a correlated abstract entity. Geometrical shapes provide an obvious example: if we describe a large red wet circular patch of paint on a piece of paper in purely geometrical terms, we give on the one hand an incomplete description of the paint patch and on the other, a unique specification of an abstract entity, a circle of a certain size.

Those who prefer to distinguish between the abstract and the concrete in this way will often say that abstract entities are *given to us* through abstraction, a mental process whereby we selectively attend to some, but not other features of a concrete thing.[11] But this should not be taken as an invitation to psychologism. Even if one thinks that abstraction is nothing but the formation of abstract ideas, those abstract ideas themselves will not be abstract entities. They are concrete representational states of concrete minds. If there *are* abstract entities, they are things that are *uniquely* represented by abstract ideas. Like John Locke, one can believe in abstract ideas and be a wholehearted nominalist.

Even if one steers clear of the psychologistic connotations of the traditional distinction, it is hard not to read some sort of ontological dependence into the doctrine that certain entities are abstractions from others. It is natural to think that a length is necessarily a length *of* something, that a direction is necessarily the direction *of* something, that a set is necessarily the set *of* some things, etc. Following up on this insight, one might suggest that *criteria of identity* for abstract entities must be spelled out

[10] No doubt this quick argument will not convince everyone. In section 5 I will briefly return to this issue.
[11] There is another, closely related mental process often referred to as 'abstraction'. Abstraction of this second kind is a kind of generalization: we attend to features that a number of distinct concrete things have in common. For a criticism of the idea that certain concepts are acquired through the mental process of abstraction, see Geach (1957).

in terms of concrete ones. The length of *a* is identical to the length of *b* iff a Euclidean transformation maps the endpoints of *a* to the endpoints of *b*; the direction of *a* is identical to the direction of *b* iff *a* and *b* are parallel; the set of *Fs* is identical to the set of *G*s iff all *F*s are *G*s and all *G*s are *F*s, etc. It has even been suggested that we could bypass the traditional notion of abstraction and *define* the distinction between abstract and concrete in terms of the sort of criteria of identity associated with them.[12] But if we do so, we commit ourselves to a modal claim: that abstract entities could not exist without their concrete correlates. (How could 'the direction of *a*' denote something if *a* does not refer?) This sounds plausible in some cases; if there were no lions, there would not be such a thing as the genus *Panthera leo*, if the Earth didn't exist, there would not be such a thing as the Equator, and if there were no tokens of the English word 'house' then the word itself would fail to exist. But not all abstracta seem to be like this. Should we really believe that if there were no circular patches, circular geometrical shapes would also fail to exist? If we say that propositions are abstractions from sentences, which are, in turn, abstractions from pencil marks and human noises, should we also insist that before there were those marks and noises there were no propositions either? It seems better not to include in the definition of the abstract entities that they ontologically depend on their concrete correlates.[13]

The real problem with the traditional way of drawing the line between abstract and concrete is not that talk about abstraction carries dubious connotations. One can

[12] Chapter 14 of Dummett (1973b) makes the proposal that an abstract object is such that it is essential to the understanding of any of its names that the referent be recognized as lying within the range of a functional expression, such as 'shape of …' or 'direction of…'. Dummett recognizes that his distinction is not precise, but he insists on the importance of the insight behind it. He claims that the sense in which a shape or direction must be 'of' something is "very akin to the conception of logical dependence which Aristotle expresses by the preposition 'in' when he gives as part of his characterization of a substance that it is not 'in' anything else." (487) Dummett's distinction has been contested on the grounds that it characterizes abstract entities purely extrinsically, and hence, does not tell us about their nature. (Cf. Lewis (1986): 82) Even if it is true that we could not understand the name of a direction unless we recognize that the direction is a direction of some line, one could raise the question why this is so. One answer to this, suggested in Chapter 3 of Hale (1987) is that in order to understand a name of a direction we must understand the sortal predicate '…is a direction', in order to understand this predicate we must know the criterion of identity for directions, and the criterion of identity of directions is spelled out in terms of the relation of parallelism between lines.

[13] Rejecting the idea that abstract entities ontologically depend on *their concrete correlates* is not the same as rejecting that they ontologically depend on the *totality of concreta*. Rosen (1993) calls this latter claim 'the supervenience of the abstract' and he argues that it is part of the commitments of ordinary thought. He

resist those connotations: the core of the traditional division is nothing more than the claim that abstract entities can be fully characterized in a vocabulary that would be insufficient to fully characterize concrete entities. The vocabulary of geometry is sufficient to identify the circle, but could not be used to identify any circular paint patch. If this is so, the reason must be that the circle *lacks* certain properties that can distinguish paint patches from one another. The traditional story fails to tell us what these properties are. Photons don't have rest mass, black holes don't emit light, points in space don't have extension, so they all lack properties that are standardly used to distinguish among concrete things.[14] Nonetheless, they are all classified as concrete. It is hard to see how the traditional division can explain this.

This leads us to the way abstract and concrete entities are usually distinguished in current discussions. Abstract entities are supposed to lack observational, causal and spatio-temporal properties, i.e. they are (i) in principle imperceptible, (ii) incapable of causal interaction, and (iii) not located in space-time. These features are typically not taken to be independent; in fact the first is often explained through the second, which in turn is explained by the third. These explanations are not beyond doubt. One might certainly hold that one could see that a cat is on a mat, that *that cat is on the mat* is a singular term referring to a proposition, and that propositions do not enter into causal relations. Or one might hold that in understanding the English word *cat* we must enter into a causal relation with the word, while denying that the word *cat* occupies some region (or regions) of space-time. Of course, those who deny that we must be causally related to what we perceive or that causal relations must hold between spatio-temporally located entities may well be wrong. It is, nevertheless, a good idea not to try to smuggle substantive doctrines into the explication of a distinction. So, I will simply drop the first

---

also notes that the asymmetry of this dependence cannot be adequately captured modally: the relevant global supervenience claim holds in the opposite direction as well.

[14] One might argue that photons have zero rest mass, that black holes emit light of zero intensity and points have zero extension, and so they all possess properties abstract entities lack. But the difference between lacking a property and possessing it to degree zero is even less clear than the difference between abstract and concrete.

two criteria and stick with the third: an entity is abstract just in case it is not in space-time.[15,16]

1.3. … entities?

It is best to understand 'entity' as it occurs in the nominalist thesis as a predicate whose extension is all encompassing. Given that 'there is' is construed as the first-order existential quantifier, this decision amounts to taking the quantification in the nominalist thesis to be absolutely unrestricted.[17]

For some, the debate over nominalism is not about the existence of abstract *entities*, but about the existence of abstract *objects*. Such a distinction is usually motivated on broadly Fregean grounds: objects can be referents of genuine singular terms, functions can be the referents only to other kinds of expressions.[18] Since it is usually emphasized that genuine singular terms are all and only those expressions that can flank the identity sign in a meaningful sentence, this distinction is closely connected with another one, according to which objects are entities that possess determinate identity conditions.[19]

---

[15] There are putative entities that are intuitively in time, but not in space. It is, for example, quite natural to say that words or animal species came to being and will cease to exist, though they are nowhere. Vendler (1967) claims that events fall in this category. (Compare: 'The collapse of the Germans was sudden' and 'The collapse of the Germans was 2000 miles long'.) Disembodied spirits might be another example. If these proposals are coherent, we must recognize an ambiguity in the above characterization of abstract entities. Not having spatio-temporal location can be construed as lacking *both* spatial *and* temporal properties, or as lacking *either* spatial *or* temporal properties.

[16] According to some, impure sets (if they exist) are where their members are; according to some, God is outside space and time. Given the spatio-temporal characterization of the abstract/concrete distinction, these views entail respectively that impure sets are concrete and that God is abstract. These conclusions are no doubt in conflict with our initial intuitions. But I would be reluctant to blame the definition for the conflict.

[17] Whether wholly unrestricted quantification even makes sense is a matter of some controversy. For arguments against the coherence unrestricted quantification, see Dummett (1973b): 530 – 1, 567 – 9 and Dummett (1991): 232 – 5, 313 – 9. For a response to Dummett, see Cartwright (1994).

[18] Cf. Dummett (1973b), Wright (1983), Hale (1987).

[19] Cf. Lowe (1995). Lowe's distinction does not coincide with the way Fregeans draw the line between objects and non-objects. Lowe believes that there are vague entities (e.g. elementary particles or ordinary waves) that can be referred to by expressions, that may well pass all the syntactic tests Fregeans might posit for genuine singular terms. Identity statements involving such terms would be meaningful, but would lack determinate truth-value. Cf. Lowe (1994).

Why think that the distinction between objects and non-objects drawn *within* the category of entities bears ontological significance? Because not all existential quantification appears to have the same kind of ontological significance. The inferences from 'Peter kicked a stone' to 'Peter kicked something' and 'Peter did something' are equally irresistible. But while 'This thing Peter kicked in the morning is identical to that thing peter kicked in the afternoon' makes perfect sense, 'This thing Peter did in the morning is identical to that thing Peter did in the evening' is rather dubious. Perhaps we can have entities without identity, but surely not objects without identity.[20]

The intuition that 'Peter did something' does not have the same ontological significance as 'Peter kicked something' is worth taking seriously. But it is not clear that the best way to accommodate it involves distinguishing objects from other sorts of entities. The second order formula '$\exists X$. Peter *X*ed' has plausibly the same content as the sentence 'There are some agents and Peter is one of them', and if this is so it only entails the existence of agents. The sentence involves plural quantification and hence, it ought to be distinguished from its singular counterparts: 'There is something whose instances are agents and Peter is an instance', 'There is something whose members are agents and Peter is a member' and 'There is something some of whose parts are agents and Peter is such a part.'[21] That there are true sentences apparently expressing higher-order existential quantification does not show that we must distinguish between two kinds of entities; rather it indicates that some existential quantification carries no commitment to a value (as opposed to value*s*) of its bound variable.[22]

If the decision made in section 1.1. to interpret the nominalist thesis as involving classical first-order existential quantification was correct, considerations about the ontological commitments of higher-order quantification are beside the point anyway. The sentence '$\exists x.x$ is an abstract entity' is true just in case an abstract entity is included in the

[20] Although followers of Davidson (1967) do tend to point to the validity of such inferences as providing support for postulating quantification over events in the logical form of action sentences, they do not regard this as a decisive issue. The crucial evidence comes from the logic of adverbial modification.

[21] The quickest way to see the need for distinguishing between plural and singular quantification is to compare the sentences 'There are some sets of which every set that is not a member of itself is one' and 'There is a set of which every set that is not a member of itself is a member'. The first is a truism, the second a contradiction. Cf. Boolos (1984): 66.

[22] For a detailed argument that monadic second-order quantification is ontologically innocent, see Boolos (1985). For a dissent, see Resnik (1988).

domain of quantification. If the domain is unrestricted an internal division in it can make a difference for ontology, no matter how metaphysically important it is.

2. How to be a nominalist

Nominalism is nothing more than the thesis that there are no abstract entities. But to be a nominalist is more than to accept nominalism. Despite their occasional rhetoric, no nominalist thinks that abstracta are *exactly* on a par with ghosts, sea serpents, and other figments of our imagination. Since there are no ghosts or sea serpents, stories that are told about them are plainly false and should not be propounded as factual. But nobody — well, almost nobody — thinks that we should demote our talk about numbers, probabilities, languages, species, concepts, or virtues to that of fairy tales. What is the difference?

2.1. "Speak with the vulgar…"

According to most nominalists, there is *nothing wrong* with serious utterances of sentences like 'Caesar uttered the same sentence over and over again', 'The number of planets in the solar system is nine', or 'After the Jurassic period many dinosaur species went extinct' *despite the fact* that there are no sentences to be uttered twice, no numbers to count planets and no species to go extinct. To bolster their case, they might point out, for example, that there is similarly nothing wrong with saying that the sun rises, sets, or moves above the meridian. We all say such things, even though most of us are no longer in the grips of Ptolemaic astronomy. We can "think with the learned, and speak with the vulgar."[23]

But things cannot be left at this. Like any radical eliminativist, a nominalist owes us a story of why we can speak in just about any setting — except for the one of philosophical inquiry — as if there were certain entities out there to be referred to when we believe no such thing. As Carnap puts it, a philosopher with such a disposition seems to speak with an uneasy conscience, “like a man who in his everyday life does with qualms many things which are not in accord with the high moral principles he professes on Sundays.”[24] Now we have analogy set against analogy: the nominalist insists that his talk about abstracta is like everyone else’s talk about the rising and the setting of the sun, while his opponent contends that it is more like the faint hypocrisy of a Sunday Christian’s prayers. Which analogy is more apt?

When pressed about this matter, nominalists tend to invoke the notion of a *paraphrase.* When we say that the Sun is rising, our words could be paraphrased roughly as ‘The Sun appears to be raising’ or, perhaps as ‘Some straight lines between our eyes and points on the surface of the Sun no longer intersect with the surface of the Earth.’ Since the paraphrases clearly do not require the truth of Ptolemaic astronomy, we may go ahead and use the original, less clumsy sentences in our speech. Similarly, the story goes, since (1) can be paraphrased as (2), and since (2) does not carry ontological commitment to chances, talk about possibilities in (1) is unproblematic, even for the nominalist.

(1) There is a good chance that it will snow tomorrow.
(2) It will most likely snow tomorrow.

As it stands, this line of defense is rather murky.[25] Although it is intuitively clear that the existence of a paraphrase somehow legitimizes the use of a sentence that appears to carry an unacceptable commitment, it is unclear both what this legitimization amounts to and how it is accomplished. This is because the very notion of a paraphrase involves a crucial ambiguity. One can think of a paraphrase either as a way of bringing out what a sentence really means by providing an approximate synonym, or as a way of replacing the sentence with another that has quite a different meaning but could nonetheless be

[23] The phrase and the example are from Berkeley’s *Principles* §51. There they serve to defend his immaterialism against the charge of verbal impropriety.
[24] Carnap (1950): 205.
[25] Things are not helped by the fact the Quine, the source of the paraphrase defense, is often rather elusive on what he means by paraphrase. In Quine (1948), he uses all of the following expressions to characterize the relationship between an expression and its paraphrase: ‘translate’, ‘explain’, ‘rephrase’, ‘analyze’, ‘identify’, ‘interpret’, and ‘expand’.

reasonably employed in its stead. I will call the first type of paraphrase *semantic*, the second *pragmatic*.[26]

How semantic paraphrases are supposed to legitimize sentences like (1) is fairly clear. If what (1) really means is something like (2), then perhaps it does not, after all, commit us to the existence of chances. The trouble is that it is not easy to believe that (1) and (2) are near synonyms. The nominalist claims there are no such things as chances. If she is right, it sure *seems like* (1) would have to be false, but (2) could still be true. But how could semantic paraphrases differ so obviously in their truth-conditions? In ordinary contexts, where we don't much care whether our words carry commitments to chances and other abstracta, (2) may count as a semantic paraphrase, but how can we sustain such a judgement once we shift our attention to the problem of nominalism? Furthermore, even if we grant that (2) is an adequate semantic paraphrase of (1) in any context, and that consequently the intuitions that (1) entails the existence of a chance and that (2) does not cannot *both* be correct, we still don't know which one to jettison. Why interpret the alleged equivalence in a deflationary, rather than an inflationary way; why assume that *neither* of them entails the existence of chances, rather than that *both* of them do?[27]

These worries are by no means decisive against nominalists who wish to make use of semantic paraphrase. The usual answer to the first worry is that our willingness to explain the meaning of either of these sentences with the other is sufficient evidence for the claim that they are near synonyms. In responding to the second worry, nominalists may suggest that we break the symmetry by appeal to intuition, or the principle that *ceteris paribus* ontology ought to be as slender as possible. Whether the claims that (2) is a near synonym of (1) and that neither entails that there are chances is ultimately acceptable depends on whether they can find their place among the consequences of our best and most comprehensive semantic theory. In matters of meaning, it is hard to see how there could be a higher authority to appeal to.

Semantic paraphrases are usually given in a piecemeal fashion. The anti-nominalist throws a number of sentences at her opponent, each of which apparently

[26] Burgess and Rosen (1997) call a nominalist strategy that provides semantic paraphrases *hermeneutic*, and a nominalist strategy that is aimed at pragmatic paraphrases *revolutionary.*
[27] Cf. Alston (1957) and Wright (1983): 31 – 2.

quantifies over abstracta. The nominalist throws their semantic paraphrases back. As the anti-nominalist's sentences get more sophisticated, so do the nominalists's paraphrases. (For example: 'There are more cats than dogs' is paraphrased by Goodman and Quine as follows: 'Every individual that contains a bit of each cat is larger than some individual that contains a bit of each dog.' A bit of something is defined as a part of that thing whose size equals that of the smallest of the cats and dogs; officially: *x* is a bit of *z* iff for every *y*, if *y* is a cat or a dog and is bigger than no other cat and dog, neither is *x* bigger than *y* nor is *y* bigger than *x* and *x* is part of *z*.'[28]) As the game advances, the claim that these paraphrases do nothing more than uncover what the ordinary sentences *really* mean becomes more and more baffling. Given the unsystematic character of the project, the idea that the real meaning of a large (probably infinite) set of sentences of our language are given this way is a threat to systematic semantics. Still, it is possible that semantic theory will come up with truth-conditions of the relevant sentences that match the truth-conditions of their suggested paraphrases. Whether we should *expect* this could only be assessed on a case-by-case basis.

Pragmatic paraphrases work very differently. Semantic paraphrases are approximate synonyms, and hence, can hardly diverge in truth-value. But if paraphrases are nothing more than suitable replacements, all we need to insist on is that *most ordinary consequences* of pragmatic paraphrases have the same truth-values. So, we can concede that if there are no such things as chances, (1) is false even though (2) may well be true, without thereby undermining the claim that typically (2) is a good replacement for (1). The existence of a pragmatic paraphrase does not legitimize the use of the original sentence in *all* contexts, but it may do so in *some* where we are not concerned about certain entailments. The question is, how?

At this point *fictionalism* comes to the rescue. Philosophers who are fictionalists about *F*s believe that sentences that entail 'There are *F*s' are literally false but fictionally true.[29] When we use literally false but fictionally true sentences, our practice is

---

[28] Goodman and Quine (1947): 180.

[29] This is not *the* standard definition of fictionalism, because there is no standard definition. Mine is fairly narrow. Some would regard fictionalism about *F*s to be compatible with the claim that 'There are *F*s' lacks truth-value (e.g. Field (1989): 4, fn.4); others think fictionalists can be agnostic regarding the existence of *F*s (e.g. the "third grade of metaphorical involvement" in Yablo (forthcoming).

legitimate, as long as it is clear in the context that we are immersed in the fiction, that we do not intend to question the constitutive assumptions of the fiction. When I utter ‘Odysseus was set ashore at Ithaca while sound asleep’ my utterance is unobjectionable as long as it is clear that I merely recount how things are according to Homer’s epic. The same sort of thing occurs, according to the fictionalist nominalist when we utter ‘There are prime numbers larger than 100.’ The appropriate pragmatic paraphrase for the first sentence is ‘According to Homer’s *Odyssey*, Odysseus was set ashore at Ithaca while sound asleep’; for the second sentence ‘According to the Peano Arithmetic, there are prime numbers larger than 100.’

The fictionalist can even provide paraphrases in a reasonably uniform fashion. The algorithm is roughly as follows: Suppose *S* is a sentence that carries commitment to abstract entities of a certain type. Suppose further that our best theory about entities of that type is *T*. Then the pragmatic paraphrase of *S* is ‘According to *T*, *S*.’ There are, of course, a number of problems with this. We don’t know how to select *T*, we are not told what we should do if *S* carries commitment to more than one type of abstract entity, and — most importantly — we don’t have a precise understanding of ‘according to *T*’ for arbitrary *T*.[30] Nonetheless, the approach looks promising.

Non-literal use is an unquestionably pervasive feature of natural language. Even in the middle of our most serious theoretical discussions, even when we are using straightforward declarative sentences in a way that is indistinguishable from their assertoric use, we *may* in fact speak metaphorically and we *may* in fact convey the content of some fiction.[31] Still, when we do this, we tend to be aware, or at least easily made aware, that we are speaking figuratively. The surprising suggestion here is that in the philosophically interesting cases this is not so: we are *wholly* immersed in a fiction

[30] If we take *T* to occupy referential position in ‘According to *T*’ the paraphrases will carry commitment to theories. If theories are taken to be abstracta, this does not help the nominalist. If they are taken to be contingent concrete entities (e.g. linguistic tokens), we face the problem that our paraphrases will be contingent truths. This is a problem if we are paraphrasing sentences about mathematical entities, which according to most exist necessarily. So, it seems that the fictionalist nominalist should not accept that *T* occupies a referential position in ‘according to *T*’. Also, we must surely insist that ‘According to Frege’s *Basic Laws of Arithmetic*, there are numbers’ is true and ‘According to Frege’s *Basic Laws of Arithmetic*, there are unicorns’ is false, even though the system of Frege’s *Basic Laws of Arithmetic* is inconsistent. These facts make a semantic theory about ‘according to *T*’ hard to come by.

[31] In fact, exploiting non-literal talk can be theoretically advantageous; cf. Melia (1995), Balaguer (1998), and especially Yablo (1998).

and it takes serious reflection to notice that our words are not to be taken literally. According to the fictionalist nominalist, with regard to mathematics and other disciplines deep into commitment to abstracta, we are much like children lost in the game of make-believe.[32]

2.2. "… think with the learned"

Suppose we have nominalistically acceptable paraphrases for every sentence we would wish to maintain in our ordinary and scientific discourse. This does not mean that nominalism has won the debate about abstract entities. After all, as Quine remarks, we could paraphrase each closed sentence S of a theory T as '$True(n)$', where n is the Gödel number of S and '$True$' is the truth-predicate for T, and in this way reduce our ontology to that of the natural numbers.[33] But not even a modern day Pythagorean would believe that this *shows* that there is nothing beyond the world of numbers. By itself, paraphrase settles no ontological question. Still, one might suggest, even if the nominalist has not won the debate, by providing paraphrases he has certainly done enough to *explain* what his position is.

Not so. For the nominalist must face a query regarding the status of the nominalist principle itself. If ordinary sentences about abstracta are in need of paraphrase, it seems that we could paraphrase the nominalist thesis as well. According to the nominalist, when someone says that there are prime numbers larger than 100 she should not be taken as quantifying over numbers. Why should then she be taken as quantifying over abstracta, were she to say that there are abstract entities? But if she is not, why on earth would the nominalist object? Nominalist paraphrase, when applied across the board, does not help the nominalist. Rather, it leads to a thorough elimination of the metaphysical debate concerning nominalism.

[32] Stanley (forthcoming) argues that this consequence of fictionalism sits badly with what we know about the way children acquire the ability to comprehend mathematical discourse and the way they learn about games of make-belief.

[33] Quine (1964). There is a catch: to be able to define the truth-predicate, the language of paraphrases would typically need higher-order quantification. This, in turn, depending on one's views on ontological commitments of higher-order logic, may bring extra commitments to properties, sets or whatever one

So, nominalists need a story about *when* paraphrase is to be applied. One rather dismissive but nonetheless widespread reaction to this worry is to say that nominalist paraphrase is to be applied when we conduct serious business. Mathematics is the queen of sciences, so we must strive to interpret the results of mathematics as truths; metaphysics is the handmaid of the sciences, so we need not bother.

But this response is unsatisfactory. First of all, defenders of semantic paraphrases cannot simply say that the anti-nominalist credo 'There are abstract entities' *need not* undergo nominalist paraphrase; they must insist that it *should not.* Otherwise, the anti-nominalist credo is nominalistically acceptable. Second (and this applies to defenders of semantic and pragmatic paraphrases alike), if the anti-nominalist's claim that there are numbers is to be rejected, so is his argument that there must be numbers because there are primes over 100 and primes are numbers. And this argument cannot be rejected unless the nominalist is willing to concede that *in this context*, mathematical sentences are not to be paraphrased.

The obvious retreat is that sentences should only be paraphrased in contexts where such a paraphrase does not defeat the very purpose of their utterance. When the anti-nominalist says that there are abstract entities, the aim of his utterance is to make an *assertion* that is true just in case there are abstract entities. When the mathematician says that there are prime numbers that are larger than 100, her purpose must be something else. Either she does not assert anything (she only *quasi-asserts*[34] that there are prime numbers that are larger than 100), or if she does assert something, it must be something nominalistically acceptable (for example, the nominalistic paraphrase itself). Or so the nominalist must believe.

This is a significant empirical hypothesis about what mathematicians actually do when they make sincere utterances in the context of doing mathematics. One way it can be defended is by asking mathematicians what they think they are doing when they make those utterances. If this yields a result unfavorable for the nominalist (as I suspect it will), the nominalist must insist that the real purpose of the mathematician's utterance is hidden

---

thinks predicate-variables are assigned as semantic values. Still, we would have here an ontological reduction of the concrete to the abstract.

[34] The term is from Rosen (1994). He introduces it in discussion Van Fraassen's notion of acceptance. Quasi-assertion stands to genuine assertion as acceptance in van Fraassen's sense stands to genuine belief.

from her. If she insists that all she ever wanted to do in uttering sentences like 'There are primes that are larger than 100' is to assert the proposition her sentence expresses, she is in error about her own speech acts. This proposal boils down to the suggestion that there are numerous truths about the way we *should* interpret each other's ordinary utterances that are hidden from linguistically unsophisticated but otherwise well-informed and able native speakers. In other words, interpretation — the process whereby a competent hearer determines what a particular utterance of the speaker is supposed to convey — is a radically *non-transparent* matter.[35]

If the non-transparency of interpretation is to hard to swallow, the nominalist may opt for a different strategy. He may concede that the mathematicians really do intend to assert things that entail the existence of abstract entities, but insist that the aims of individual mathematicians should not be confused with the aim of mathematics. Perhaps, when individual mathematicians make their utterances, they really do intend to commit themselves to abstracta. But that does not mean that they do this *qua* mathematicians. If, for example, we could say that the aim of mathematics is not truth, only the enhancement of empirical science, and that the aim of empirical science is also not truth, only empirical adequacy, we could maintain that in going beyond the aim of their discipline, mathematicians who intent to make genuine assertions in doing mathematics are trespassing the bounds of their trade.[36] And there might be other, less radical ways to argue that the aims of science would not be frustrated by nominalist paraphrase.

To be a nominalist one must do two things besides accepting the truth of nominalist thesis. One must explain why speaking *as if* there were abstracta is an innocent thing to do, and one must also explain why the innocence of such speech does not entail the innocence of anti-nominalism.

3. Arguments for nominalism

[35] This concern is raised in Szabó (forthcoming a).
[36] Van Fraassen (1980) advocates such a view. For his discussion of the relation between the aims of science and the intentions of individual scientists, see Van Fraassen (1994).

There are a number of considerations that had been advanced in favor of nominalism. But before they are surveyed, it is useful to remember what Goodman and Quine have to say about their own motives for refusing to admit abstract entities in their ontology: "Fundamentally this refusal is based on a philosophical intuition that cannot be justified by an appeal to anything more ultimate."[37] If the arguments seem weak, that may be because they are not the real grounds for the *horror abstractae*.

3.1. Intelligibility, physicalism, and economy

Goodman has argued that sets are unintelligible because set theory embraces a distinction between entities without a genuine difference. The metaphysical principle (he calls it the principle of nominalism), violation of which is supposed to result in unintelligibility is this: if *a* and *b* are *made up* of the same constituents, then *a*=*b*.[38] This is a fairly restrictive principle. Besides sets, it also excludes linguistic types (the sentence types 'Nelson admires Van' and 'Van admires Nelson' are distinct, even though they are made up of the same word types), Russellian propositions (the Russellian propositions expressed by 'Nelson admires Van' and 'Van admires Nelson' are also distinct, despite the fact that they are made up of the same individuals and the same relation) and events (the event of Nelson's admiring Van and Van's admiring Nelson are distinct, despite the sameness of their participants), etc. It also forces us to give up the distinction between the statue and the clay it is made of. So, how can anyone believe that Goodman's principle is not only true, but that putative entities that violate it are unintelligible?

One reason Goodman cites is the connection between his principle of nominalism and the principle of extensionality. Let $\in^*$ be the ancestral of $\in$, then the principle of extensionality can be stated as (3), and Goodman's nominalist principle as (4):[39]

(3) $\forall x \forall y \forall z((x \in y \leftrightarrow x \in z) \rightarrow x = y)$
(4) $\forall x \forall y \forall z((x \in^* y \leftrightarrow x \in^* z) \rightarrow x = y)$

---

[37] Goodman and Quine (1947): 174.
[38] Cf. Goodman (1956).
[39] For Goodman, the constituents of a set are exactly the members of its transitive closure. One might protest that Goodman confuses '$\subseteq$' and '$\in$', but since the issue is precisely whether we can fully understand set-theoretic membership as a non-mereological notion, this objection would not be dialectically helpful.

So, Goodman's nominalism is nothing more than a natural strengthening of Quine's strictures against intensional entities. Of course, this may not be a lot of help nowadays: contemporary philosophers who reject properties, relations, or propositions typically do not do so on account of the mere fact that they violate extensionality.

There is another way to get a feel for nominalistic qualms about set theory. Set theory dictates that we must distinguish between an object and the singleton set containing that object. As David Lewis has argued, if we have a primitive singleton function, plural quantification and mereology, then we have set theory in its full glory.[40] Since everyone agrees that plural quantification and mereology together cannot generate abstracta from concreta, the culprit must be singleton function. And the relationship between an object and its singleton is indeed puzzling. Our intuitive conception of a set is that it is a collection of *objects* — if we have but a single object, what are we to make of the collection constituted by *that object alone*?[41]

Even if we accept unintelligibility as a good *prima facie* reason to deny existence, we are still far from nominalism in the contemporary sense. Rejecting sets, sentence types and propositions is not the same as rejecting *all* abstracta. There are all sorts of putative abstract entities that are intuitively atomic (e.g. numbers, word types, basic properties, etc.) and Goodman-style reasons are insufficient for rejecting them.[42]

Will physicalism come to the nominalist's help? Some have felt that the real problem with abstracta that their existence conflicts with the view that everything is physical. Unfortunately, although it is quite intuitive to say that spatio-temporal location is a *sine qua non* of physicality, it is not clear how this intuition can be backed up. If we say that physical entities are the ones whose existence is guaranteed by contemporary physical theories, then – as proponents of indispensability arguments tirelessly emphasize – a wide array of mathematical entities turn out to be physical. If we say that

[40] Cf. Lewis (1991), chapter 3.

[41] Couldn't we use the lasso-metaphor and say that a set is what one gets by "lassoing" a number of objects together? Singleton sets would then be single objects with a "lasso" around them. The problem, of course, is exactly how to make sense of this metaphor. Lewis himself is deeply puzzled by singletons, but refrains from rejecting set theory on account of metaphysical misgivings. See esp. Lewis (1991): 57 – 9.

[42] Goodman (1956) insists that the ordinary distinction between abstract and concrete is vague and unstable, and claims that for him nominalism is nothing beyond the rejection of classes. (156)

physical entities are the ones that are subject to physical laws, abstracta may or may not qualify depending how the laws are stated. For example, if the law of gravity implies that gravitational force is exerted on absolutely everything, then the number 2 is probably not physical; if the law of gravity only implies that gravitational force is exerted on everything *that has mass*, it may well be. If physical entities are those capable of entering into causal interactions, abstracta will count as non-physical depending on how strict we are on what 'entering' amounts to. If entering into causal relations requires being actually causally related to something, then plausibly nothing but events will qualify as physical. If it means something weaker, then material objects and elementary particles are physical, but so are perhaps lots of abstracta. Why could one not say, for example, that properties can be involved in causal relations because they can be instantiated and their instances can participate in events that are causally related to others? If physical entities are those that are made up of elementary particles, abstracta would probably not qualify as physical, but then neither would concrete events. Even if Brutus and Caesar are constituted by elementary particles, it is hard to see how Brutus's killing of Caesar could be. And if causal relata are events, this sort of view paves the road to causal nihilism.

I am not saying that it would be impossible to define 'physical' in a non *ad hoc* manner that justifies the intuition that abstracta are non-physical without causing trouble elsewhere. But I do think that the failure of the most obvious definitions is indicative of a serious difficulty with this line of thinking. There are two ways in which the existence of entities of a certain sort may be in conflict with a world-view based on physics. The entity may get in the way of physical explanations, or it may be superfluous for physical explanations. But abstract entities are not likely to get in the way: unlike phlogiston, ether, angels, or the unmoved mover, they are not used in physical explanations as providing the casual source of some observable phenomenon. They may be superfluous, but they are not obviously so. And *if* they are superfluous, they are not only superfluous for physical explanations, but for all sorts of blatantly non-physical ones as well. Why would a theist, a phenomenalist, or an idealist need properties or numbers any more than a physicalist in trying to explain natural phenomena? The source of the intuition that there is a conflict between the acceptance of abstract entities and physicalism may have nothing to do with physics, and everything to do with ontological economy.

Is there a powerful argument from ontological economy to nominalism? There might be. Why multiply entities beyond necessity? If abstracta are indeed superfluous for explanatory purposes, most of us would gladly banish them from our ontology.[43] The problem is that most scientific theories do appear to quantify over abstracta, and consequently, the claim that they are superfluous in explanation is rather shaky. Even if we can reconstruct our usual theories in such a way that they are no longer committed to anything but concrete entities, it remains a serious question whether the explanations provided by the reconstructed theory are *all things considered* preferable to the explanations we had before. Ontological economy is not a free lunch: typically theories with slimmer ontologies are vastly more complex in other regards. This sort of complexity cannot always be dismissed as inessential. Otherwise what should we say to those who stubbornly maintain their belief in the Ptolemaic astronomy arguing that by postulating enough epicycles all the experimental evidence supporting the heliocentric view could be accommodated?

Besides, ontological economy does not clearly favor the nominalist. If the size of one's ontology is such a powerful concern, why not try to reconstruct our scientific theories so that they no longer quantify over concreta? Reducing the physical world to the world of numbers is technically no harder than proceeding the other way around. If we are to prefer nominalism to Pythagoreanism, there must be something besides a desire for desert landscapes that motivates us.

3.2. Causal isolation

The arguments from intelligibility, physicalism or ontological economy are not at the center of contemporary debates about the ontological status of abstract entities. The arguments that move most contemporary nominalists tend to be variations on a single theme. Since abstract entities lack spatio-temporal location, they cannot have any sort of causal impact on us. Their causal isolation makes our *access* to them deeply problematic.

[43] Not everyone, though. Prior (1954) writes: "I simply do not possess the sheer zeal for waving Ockam's razor about which seems to burn within so many of my contemporaries; my motto is *entia non sunt subtahenda praeter necessitatem*, and even the property of non-self-inherence I have given up with a sigh and only under extreme compulsion." (31)

Without causal links, nominalists contend, our knowledge about and reference to abstract entities becomes mysterious. This sort of consideration against abstract entities of one kind or another is probably very old, but contemporary versions go back only to Paul Benacerraf's paper, 'Mathematical Truth.'[44]

Even before considering how to state this objection more precisely a caveat is required. This sort of argument is applied all the time across the board against all sorts of abstracta, but the fact that it was originally presented in the context of the philosophy of mathematics is of utmost importance. For, as I noted in Section 1.2. it is by no means clear that all abstract entities are causally isolated from us. The novel *The Good Soldier Šweik* is presumably an abstract entity, but one that is causally dependent on a host of concrete ones. It could never have existed without the efforts of the Czech writer Jaroslav Hašek and he would never have written it were it not for the involvement of the army of the Austro-Hungarian Monarchy in the World War I. Furthermore there are a host of other concrete events — among them the writing of this very passage — that causally depend on the novel itself. Entities of this sort are often called *dependent abstracta*. If they exist, the argument from causal isolation cannot establish nominalism.[45]

Let us set dependent abstracta aside and focus on the preferred targets of the causal argument: abstract entities that exist independently of how the concrete world happens to be. To state the argument from causal isolation properly, we would need to spell out the exact sense in which causal connection is required for knowledge and reference. This is not a trivial task; simple versions of the causal constraint are likely to exclude future events from what is knowable or available for reference. The nominalist may bite the bullet and accept presentism or the growing block model of time, but intuitively, the case against abstracta is stronger than the case against the reality of the future. After all, even if future events cannot cause anything in the present, they will be causally *connected* with events occurring now. Whatever meteorologists know about the

[44] Benacerraf (1973).

[45] It seems plausible that linguistic expressions, if they exist, are dependent abstracta. I assume that we speak truthfully when we say that Lewis Carroll coined many words and I assume that we are also correct in thinking that coining is a matter of creation, not of discovery. Had Lewis Carroll not written *Jabberwocky* the English word 'chortle' would not exist. Many philosophers reject this common sense view and regard linguistic expressions as existing eternally and necessarily.

weather tomorrow, they know it because they know about past events, which are likely to causally contribute to future events.

Although the epistemological and the semantic challenges to anti-nominalism are both based on the causal isolation of (non-dependent) abstracta, it is important to keep them apart. The claim that some appropriate causal connection is required for knowledge is more secure that the claim that it is required for reference.

Causation may enter the theory of reference at two points: at initial baptism and at the chain of reference-preserving uses. The problem for the anti-nominalist is presumably at the first of these points: abstract entities, not being in space or time, cannot be objects of initial baptisms. But an initial baptism, at least as Kripke originally conceived it, does not require causal contact.[46] We can introduce a name by means of a meaningful description that uniquely fits the object to be named. A frequent response to this is that predicates we could use in constructing the appropriate reference-fixing description for an abstract object are as problematic as the names themselves.[47] To fix the reference of '9' through 'the number of planets' or the reference of 'sphere' through 'the shape of the tennis ball' works only if the predicates 'number' and 'shape' stand in appropriate causal connections to entities they apply to. But how could they, given the abstract character of numbers and shapes?

As it stands, this is argument is not convincing. First of all, couldn't the anti-nominalist drop the problematic predicate? What if we say that '9' refers to the result of counting all the planets and 'sphere' refers to what the end of a stick can touch when the other end is fixed? One might object that the result is a description that can only be properly interpreted by someone who already knows that the intended denotation is a number or a shape, respectively. I have no idea whether this hypothesis is true, but it is in sharp conflict with our common experience with young children. Second, it is far from clear what exactly the causal requirement for predicates amounts to. Clearly, there are meaningful predicates with empty extensions, so we cannot demand that *all* meaningful predicates be causally connected to things they apply to. One might explicitly restrict the

---

[46] Kripke (1972) explicitly allows for the introduction of names through initial reference fixing via descriptions. Versions of the causal theory of reference that disallow this would have a difficult time explaining how the name 'Neptune' came to refer.
[47] Cf. Jubien (1977).

requirement to nonempty predicates, but this seems *ad hoc.* Why could nonempty predicates be meaningful in virtue of the same sort of facts — whatever they might be — that account for the meaningfulness of empty predicates?[48] Finally, even if this line of argument is correct, it is hard to see how it could help the nominalist. If due to the lack of causal connections between the numbers and us, the predicate 'number' lacks a semantic value, then presumably so do the sentences 'There are numbers' and 'There are no numbers.' So, instead of establishing his thesis, the nominalist pushing the semantic challenge has instead made a great step towards dissolving the metaphysical debate.[49]

So, nominalists must agree than in some way or other we *can* denote abstract entities, otherwise their own view is inexpressible. But even if semantic considerations are of not much use for them, nominalists can still make a strong case against abstracta on epistemological grounds. Typically the argument assumes that knowledge requires causal connection and if stated this way the debate may well get bogged down in debates about what exactly constitutes knowledge. But the most treacherous minefields of epistemology can be bypassed, as Hartry Field has emphasized.[50] All the argument requires is two rather obvious premises: that mathematicians have by and large *reliable* beliefs about mathematics (i.e. that if they believe a mathematical theorem that that

---

[48] A possible suggestion is that all semantically simple predicates are causally connected to what they apply to and insist that all empty predicates are semantically complex; e.g. 'unicorn' is equivalent to 'horse-shaped animal with a horn on its forehead' or something like that. But the failure of earlier proposals involving lexical decomposition should give us a pause. See Fodor (1999) for an argument that we are unlikely to find such equivalencies.

[49] Hodes (1984) argues along a different line that no definite description we can come up with could adequately fix the reference of numerals. Benacerraf (1965) and Putnam (1967) already argued that reference-fixing via descriptions is problematic in mathematics. The idea behind Hodes's argument is that reference-fixing through a description cannot work, because we can systematically reinterpret whatever we can say about the natural numbers in such a way that (i) we preserve the truth-values of all our claims, but (ii) the numerals will now pick out different objects. The argument is similar to Putnam's "model-theoretic" argument, so at first it seems that the anti-nominalist can claim that whatever move saves radical indeterminacy in the general case can be applied in the mathematical case as well. But, according Hodes, what saves the determinacy of reference in the general case is some sort of causal relation, and since that is unavailable when the referent is a number, the anti-nominalist does have a real problem here. This argument is immune to the first two objections above. As an argument *against anti-nominalism*, it is also immune to the third: the opponent of anti-nominalism may opt for a oblique answer to the question whether there are numbers, as opposed to embracing the straight nominalist line. This is exactly the view taken in Hodes (1990).

[50] Cf. Field (1988) and Field (1989) .

theorem is *ceteris paribus* true[51]), and that this is not a brute fact resisting all explanation. If these are granted, the epistemological challenge is to give the rough outlines of what this explanation could be. This is a hard task: for the nominalist, the claim that despite the lack of causal connections there are reliability links between the beliefs of mathematicians and the entities those beliefs are supposed to be about is as mysterious as if someone claimed to have reliable beliefs about "the daily happenings in a remote village in Nepal."[52]

The analogy is gripping, but suspicious. Clearly, we could not have reliable beliefs about the happenings in remote places without being somehow causally linked with the events that take place there. But the world of abstracta is not remote — its denizens are not in space and time at all. More importantly, it is not a world where anything happens, for (non-dependent) abstract entities do not undergo change. The predictable reaction to Field's charge from anti-nominalists is that although causal connections are required to track daily happenings, their role is less clear when we are talking about beliefs whose subject-matter is eternal and necessary.[53] This should not be taken as a full-blown response to the epistemological challenge, though. The point is merely that reliability of mathematical beliefs is on a par with the reliability of other beliefs of necessary truths.

There is, however, a legitimate concern about declaring the epistemology of mathematics a mere chapter in a yet undeveloped general modal epistemology. Mathematical necessity seems importantly different form both physical and logical necessity. On the one hand, most philosophers regard physical necessity as necessity given the fundamental but contingent laws of physics. In other words, they believe that what is physically necessary could still have been otherwise. A similar view about mathematics would not be very plausible. On the other hand, most philosophers regard logical necessity as absolute, but not existence-involving. Logic tells us nothing about

[51] As Field has pointed out, this way of spelling out the claim that mathematicians are by and large reliable in forming mathematical beliefs does not require an inflationary notion of truth. The claim boils down to nothing more than that the schema 'If mathematicians accept 'p' then p' holds in nearly all its instances.
[52] Field (1989): 26.
[53] If one believes, as David Lewis does, that causal influence requires counterfactual dependence, one must conclude that necessary facts are all causally inefficacious. Counterfactuals with necessary antecedents are

what sort of things there are, or even about how many things there are.[54] By contrast, assuming standard semantics for mathematical theories, mathematics does require the existence of an enormous number of entities. And because mathematical necessity is *sui generis* in this way, concerns about explaining the reliability of our mathematical beliefs may well be more severe. We can think of physical necessities as physical facts of maximal generality, and if we do so we can perhaps still make sense of the idea that in forming beliefs about them we are causally influenced by those facts themselves. And we can think of logical necessity as arising from the meanings of logical constants and explain the reliability of our logical beliefs by an appeal to our conceptual mastery. But these explanations will not easily carry over to the mathematical case.

It is worth pointing out that although Field often talks as if the task of explaining the reliability of mathematical beliefs required giving an account of a pervasive correlation, this is certainly an overstatement. We could account for the reliability of mathematical beliefs by pointing out that mathematical theories tend to be axiomatized and hence, belief in the theories is reliable if belief in the axioms is. So the epistemological challenge boils down to a demand of explaining how belief in the axioms can be reliable.

Anti-nominalists have three ways to counter this challenge. They can follow Gödel, accept the apparent *sui generis* character of mathematical necessity and postulate an equally *sui generis* faculty we have that ensures the reliability of our beliefs in the axioms of mathematics.[55] They can follow Frege in epistemologically assimilating the axioms of mathematics to the truths of logic[56] and swallow the consequence that our knowledge of the meanings of key mathematical terms somehow guarantees that they successfully refer. Or they can follow Quine in epistemologically assimilating

vacuous. This is one of the reasons Lewis rejects this sort of epistemological argument against mathematical entities as well as possible worlds. Cf. Lewis (1986): 111.

[54] Classical logic, as it is usually stated, requires the non-emptiness of the domain of quantification and hence it is not ontologically innocent. The common view nowadays seems to be that this is only a matter of convenience, not of substance.

[55] Such a view need not follow Gödel (1947) in thinking that this rational faculty is relevantly similar to perception.

[56] Of course, Frege proposed such assimilation only for the truths of arithmetic, not for all mathematical truths.

mathematical and physical truth and downplaying key modal intuitions, perhaps rejecting the idea of absolute necessity altogether.[57]

In the end, what considerations of causal isolation give to the nominalist is an important epistemological challenge for the mathematical anti-nominalist. It boils down to the demand to account for the epistemic credentials of our beliefs in the fundamental axioms of mathematics. The force of this challenge is somewhat weakened by the fact that we don't have particularly convincing views about why other general beliefs — beliefs in the laws of physics and logic — are reliable, so it is hard to see whether mathematics really poses special difficulties. Nonetheless, to the extent that we find mathematical axioms to be quite different from other claims of maximal generality, the challenge stands.

4. Arguments against nominalism

The most popular objection to nominalism stresses the extent to which rejection of all abstract entities flies in the face of common sense. A nominalist must deny either that the sentence '2 is a prime number' is true, or must insist that it does not commit one who believes in its truth to the existence of the number 2. Either way, she must reject a well-entrenched belief. Those who are firmly convinced that philosophy can never hold surprises will no doubt find this objection decisive. Those of us who have doubts about the ultimate wisdom of the folk may need arguments.

4.1. Indispensability

But what if we replace the folk with the experts of the scientific community? Then we arrive at what is currently the most influential argument against nominalism, the *indispensability argument*. Versions of the argument go back to Quine and Putnam.[58] It can be stated as follows: Certain mathematical theories, such as arithmetic or real

[57] Again, this need not involve acceptance of Quine's epistemological holism or a wholesale denial that there are important differences in how we assess mathematical claims and the claims of empirical sciences.
[58] The standard references are Quine (1951) and Putnam (1971).

analysis, are indispensable for modern physics in the sense that the physical theories cannot be stated in a from that would be compatible with the falsehood of those mathematical theories. But these mathematical theories are ontologically committed to abstract entities: the quantifiers used in stating them range over domains that must include mathematical entities that are not in space or time. So the physical theories themselves carry commitment to abstracta. And since we have no adequate grounds for rejecting these physical theories — they are part of our overall best theory of the world — we should acquiesce to the existence of abstracta.

There is an easy way for the nominalist to reject such considerations. He could simply deny that we must believe what our best scientific theory says. This need not involve an outright rejection of the results of science, only suspension of belief. One might suggest, for example, that it is enough if we *accept* our best scientific theory, where acceptance is an attitude that requires that we act, at least when we theorize, as if we believe.[59] But the easy way out is not very popular nowadays. It requires a willingness to override the usual standards by which our scientific theories are evaluated and hence, it is in conflict with *naturalism*, even in the least demanding sense of that word. Most analytic philosophers (and certainly the overwhelming majority of the usually scientifically hard-nosed nominalists) are reluctant to announce that the best scientific theory we have is unworthy of belief, unless they can point at reasons that would be recognizably scientific.[60] So, nominalists tend to look elsewhere for a reply to the indispensability argument.

If one takes the indispensability argument seriously, there are only two strategies for resisting its conclusion: one could either deny that mathematical theories are committed to the existence of abstracta, or that they are indispensable for physics. The former involves a program in semantics: show that despite appearances mathematical theories could be true without there being mathematical entities that they are true of. The latter involves a program in physics: develop new physical theories that fail to entail the

[59] Van Fraassen (1980) advocates such a shift of attitudes towards empirical science that would do away with ontological commitments to unobservables.

[60] Although note that almost no physicist would argue that the fact that we employ standard analysis in our theories about most physical quantities *settles* the question whether these entities are in fact continuous.

existence of mathematical entities, but nonetheless have the level of empirical adequacy and explanatory power that our current theories do.[61]

Those who take the former route typically introduce modality into their interpretation of mathematical theories. The key idea goes back to Hilary Putnam, who suggests that mathematical claims could be interpreted as involving modality, where the relevant notion of possibility is taken as a primitive.[62] Mathematical existence claims are then to be understood not as claiming that there are mathematical entities of one kind or another, but only that the existence of such entities is possible.

There are many ways this basic idea can be turned into a full-blown sentence-by-sentence reinterpretation of mathematical theories.[63] For the sake of illustration, let me sketch briefly and informally a particular example of such an approach, the so-called *modal structuralist* line.[64] Consider a sentence S of second-order Peano-arithmetic. We will provide a new interpretation for S by associating it with another sentence S' of second-order Peano-arithmetic and declaring that the new interpretation of S is identical to the old interpretation of S'. S' is a conjunction of a hypothetical claim and a categorical one. The categorical component ensures that it is possible that there is an ω-sequence, i.e. a set isomorphic to the natural numbers.[65] The hypothetical component tells us that if there were any ω-sequence then S would hold in it. Given the usual rendering of subjunctive conditionals, this latter claim says that necessarily, if X is an ω-sequence then S holds in X.

There *might be* scientific arguments against the claim that our best scientific theory is strictly speaking true. Cf. Cartwright (1983).

[61] It is important to notice that insofar as a nominalist is attempting to respond to the indispensability argument, she need not be concerned with the entirety of mathematics. For it is arguable that only a small fragment of contemporary mathematics has any application outside of mathematics. As a result, nominalists tend to focus primarily on the theory of real numbers, which is undoubtedly widely employed in the empirical sciences.

[62] Putnam (1967).

[63] The first idea one might pursue is to reinterpret mathematical sentences as making claims not about what sorts of mathematical entities there are, but about what sorts of physical marks there could be; e.g. 'There are prime numbers larger than 100' would be rendered roughly as 'There could be numerals for prime numbers larger than 100.' Such a strategy is pursued in Chihara (1990). For a survey of modal strategies in reinterpreting mathematical theories, see Burgess and Rosen (1997), II.B and C.

[64] Due to Hellman (1989). For more details see Chapter 1.

[65] Since the nominalist does not want to conclude from $\Diamond\exists X$(X is an ω-sequence) that $\exists X\Diamond$(X is an ω-sequence), she has to employ a modal logic without the Barcan formula.

The proposal that arithmetical sentences have such elaborate semantics is a radical claim. This fact is often overlooked because nominalists are often vague about the status of such reinterpretations. Saying that arithmetic *could be* interpreted in a nominalistically acceptable fashion is not the same as saying that such an interpretation captures what arithmetical sentences *mean.* (We can all agree with Quine that 'Lo, rabbit!' could be interpreted as 'Lo, undetached rabbit-parts!' but we can also reject semantic nihilism and insist that these sentences don't mean the same thing.) Besides working out the precise details of these nominalistic interpretations, nominalists must also provide some reason for us to believe that these interpretations are correct.

Let us now consider the prospects of the other nominalist program aimed at responding to the indispensability argument. Proponents of this second program concede that there is no mathematics without numbers (or other mathematical entities) and that consequently current scientific theories are nominalistically unacceptable. Their aim is to show that we could do science without numbers (or other mathematical entities). The most prominent advocate of this idea is Hartry Field.[66]

Field's strategy for nominalizing science has two components. First, he suggests that we can develop alternatives to current scientific theories that are nominalistically acceptable. In support of this claim he develops a version of the Newtonian gravitational theory, where no quantifier rangers over mathematical entities. The entities that play the role of surrogates for real numbers are space-time points and regions. He argues that this is not a violation of nominalism: parts of space-time are concrete entities instantiating contingent physical properties and participating in causal interactions.[67] The ontology of the theory is not small: the axioms entail that there are continuum many space-time points and the powerset of continuum many space-time regions, and hence, that there are this many concrete physical objects. Second, he argues that adding mathematical theories to a nominalistically acceptable scientific theory has no bad effect on the nominalistic portion of the resulting theory. That is, mathematical theories are *conservative* over nominalistically acceptable theories in the sense that if a sentence within the nominalistic

[66] Field (1980).
[67] Of course, talk about 'properties' and 'instantiation' here are not to be taken seriously. Field regards the eliminability of apparent reference to properties from the language of science as a forgone result.

language of the original theory was not a theorem of the original theory, then neither is a theorem of the expanded theory. This means that although mathematical theories are *false* (due to their commitment to the existence of abstract mathematical entities) they are nonetheless instrumentally *good*, for we can use them to provide shortcuts for tedious deductions within the nominalistically acceptable theories.

Both steps of Field's program have been subjected to detailed criticism. It is an open question whether all scientific theories are amenable to the sort of nominalization Field performs on Newtonian gravitational theory.[68] And it is a matter of controversy whether informal statements of the conservativity of mathematics over nominalistically acceptable theories adequately capture the technically much more involved results given in the appendix to Field's book.[69]

4.2. The Context Principle

There is simple, but influential argument against nominalism, which is sometimes referred to as the 'Fregean argument'. It is goes as follows.[70] Consider a simple sentence of arithmetic, say, the sentence '2+2=4'. Observe two things about this sentence. First, that it is an obvious truth. Second, that the numeral '2' functions as a singular term within this sentence. But if an expression functions as a singular term within a sentence then there must be an object denoted by the singular term, if the sentence is true. Therefore, there is an object denoted by '2', or in other words, the number 2 exists.

The last step in this argument is supported by Frege's *Context Principle*, the thesis that only in the context of a sentence does a word have meaning. This principle has been subject to a variety of interpretations; the one that this premise derives from was largely motivated by a reading of § 60 of the *Foundations of Arithmetic* developed by Michael Dummett and Crispin Wright.[71] The main ideas behind this reading are as

[68] Malament (1982) argues that the strategy does not extend to quantum mechanics. Balaguer (1998) attempts to give a nominalistic version of quantum mechanics.
[69] Cf. Shapiro (1983), Field (1985) and Burgess and Rosen (1997) Chapter III.B.i.
[70] Cf. Hale (1987): 11.
[71] Cf. Dummett (1973b): 494ff and Wright (1983), section iii.

follows: (i) the principle should be construed to be about reference,[72] (ii) it denies that once we settle on how a certain expression contributes to the truth-conditions of sentences in which it occurs, we can have any further questions about the semantic value of the expression, and hence (iii) it subordinates questions of ontology to questions of truth and logical form.

As stated the Context Principle appears too strong: it neglects the possibility of expressive limitations. For if it is plainly possible for a language to have non-codesignating singular terms that are intersubstitutable in every sentence without change of truth-value. But it seems that the current reading of the Context Principle excludes this: since there is no difference in how they affect the truth or falsity of any sentence in the language, there should be no difference in their semantic values either. Demanding that the language under consideration pass certain minimal criteria of expressive power (e.g. requiring that it should contain the identity predicate) can eliminate the problem. Alternatively, we can weaken the principle. Perhaps settling all questions about how an expression contributes to the truth-conditions of sentences in which it occurs does not settle *all* questions about the semantic value of the expression, but it may well settle the question whether the expression *has* a semantic value. For the purposes of the Fregean argument, this is all we need. The idea is that by determining that a certain expression functions as a singular term within a true sentence we have settled how it contributes to the truth of that sentence, and so given the Context Principle, we can no longer hesitate in accepting that it has the appropriate semantic value.

Once the justification for the last step in the Fregean argument is made clear, one possible reaction from the nominalist is to say something like this: "I agree that if '2' were a *genuine* singular term then there would be no denying that it purports to refer to the number 2. And if we grant that 2+2=4, there is no way to banish numbers from our ontology. But why say that '2' functions as a genuine singular term? Why not insist that, despite superficial appearances, its real work in the semantics of this sentence is something quite different?" In defense of the claim that '2' is really a singular term, the

[72] What makes the interpretation of Frege's dictum particularly difficult is that the *Foundations* predate the distinction between sense and reference. The word actually used by Frege is '*Bedeutung*', which normally means 'meaning' but will come to mean 'reference' in Frege's later writings.

anti-nominalist appeals to formal — syntactic and inferential — criteria. Since numerals as they occur in arithmetical sentences are noun phrases, and since they support the right kinds of inferences, they are singular terms by ordinary criteria. And these cannot be overridden by extraordinary criteria formulated by an appeal to intuitions about what there *really* is. Syntax over ontology, as the spirit of the Context Principle dictates.

Ordinary criteria exclude from the class of singular terms a variety of idiomatic or quasi-idiomatic expressions, and so anti-nominalists who take the Fregean argument on board are not saddled with ontological commitment to sakes and their kin. Nonetheless, there are plenty of expressions that are troublesome: 'the existence of a proof', 'the identity of the murderer', 'the occurrence of the explosion', and 'the whereabouts of every student' are just a few. In response to these, proponents of the Fregean argument must refine their criteria or bite to ontological bullet.[73] This is a genuine balancing act, and it may well prompt doubts about the viability of the whole program. Are we really sure that our intuitions about validity of certain patterns are independent of our intuitions about the referential status of expressions within the arguments? If not, there is a danger that our judgments used in determining whether an expression passes all the tests for singular termhood are already tainted by our views on whether the expression purports to designate.[74] And if this is so, nothing is left of the primacy of broadly formal considerations over ontological ones.

Let us set these concerns aside and assume that the balancing act can succeed. Can the nominalist then resist the force of the Fregean argument? Well, he can do so by denying that the sentence '2+2=4' is true. This certainly amounts to going against one of the best entrenched beliefs we have, but as Hartry Field has shown, one can make a case. The question is whether there is any way the anti-nominalist can bypass a direct appeal to intuition here, whether he could make an argument that could rationally compel a nominalist of Field's stripe.

[73] For a detailed discussion, see Hale (1987), chapter 2.

[74] There is another concern here worth mentioning: one might worry whether the intuitions regarding the validity of the relevant inferences are reliable. In Szabó (2000) I argue against the Russellian view that definite descriptions carry semantic uniqueness implications. If the semantic contribution of the definite article is simply existential quantification and if uniqueness implications associated with many uses of sentences containing definite descriptions arise pragmatically, then definite descriptions *tout court* fail the inferential tests for singular termhood. They appear to pass them due to a pragmatic illusion.

Here is an attempt. Forget about sentences of pure arithmetic and concentrate on the arrangement of plates and forks on a big dining room table. Suppose you see that the forks and the plates are paired perfectly: each fork is immediately adjacent to exactly one plate and there is no plate without a fork immediately adjacent to it. Then you can immediately conclude that the number of forks is identical to the number of plates. But if this inference is indeed valid, then the sentence 'if the plates and the forks are perfectly paired then the number of plates is identical to the number of forks' must be true. And since expressions like 'the number of plates' pass the formal tests for singular termhood as well as '2' does, this sentence can replace '2+2=4' in the Fregean argument.

But what if the nominalist disputes the validity of this inference? Couldn't he argue that since a claim about perfect pairing of plates and forks does not commit one to the existence of numbers, but the claim that the number of plates is identical to the number of forks does, the inference is illegitimate?[75] Well, the anti-nominalist can point to the fact that the relevant conditional is simply one direction of an instance of *Hume's Principle*, the claim that the number of *F*s = the number of *G*s iff there is a one-to-one correspondence between the *F*s and the *G*s. And Hume's Principle is something that is arguably constitutive of our grasp of the concept of a natural number, so rejecting it is tantamount to embracing conceptual confusion. In other words, the anti-nominalist may insist that this principle is analytic.

The nominalist is going to resist this move. He may concede that the conditional 'if natural numbers exist then Hume's Principle is true' is analytic, or at least (in case he is an opponent of the analytic/synthetic distinction) that it is obvious to the point of indisputability. But he will not yield on the categorical formulation. After all, we know that second-order logic together with Hume's principle entails the Peano axioms,[76] and consequently the existence of infinitely many objects. Should we really believe that *analytic consequences of pure logic* exclude a finitistic ontology?[77]

[75] This is the line taken in Field (1985).
[76] This fact was first noted in Parsons (1965) and independently rediscovered in Wright (1983).
[77] For an enlightening exchange on the question whether it makes sense to declare Hume's principle analytic, see Boolos (1997) and Wright (1997).

One way to strengthen the anti-nominalist argument is to drop the talk of analytic connections and resort to *explicit stipulation*. Here is the idea.[78] (The case is much easier to make for directions than it is for numbers, so I will here switch examples.) Start with a classical first-order language L with a restricted vocabulary understood as talking only about concrete inscriptions of lines, not about their directions. Then extend this language into L^* in such a way that all the sentences of L retain their meanings in L^*. In addition to the vocabulary of L, L^* contains a unary primitive function symbol 'the direction of' and new predicates 'F^*', 'G^*', … indexed to the old ones 'F', 'G',… . We give meaning to the sentences of L^* containing new symbols by stipulating that they are synonymous with certain sentences of L. For example, we stipulate that 'the direction of a = the direction of b' means in L^* what 'a is parallel to b' does, that if it is true that whenever 'F' applies to a line, it applies to every line parallel to it, then 'the direction of a is F^*' means in L^* what 'a is F' does, and otherwise it is meaningless, etc. By making the appropriate stipulations, we can ensure both that '$\exists x.x$ = the direction of a' is true in L^* and that expressions of the form 'the direction of x' pass the inferential tests for singular termhood. If the context principle is correct, this entails that we now have *bona fide* singular terms in true sentences, so we must accept ontological commitment to directions.[79]

Now, it is probably best not to think that we can expand a theory's ontology through appropriate linguistic stipulation. It's the other way around: the argument is best construed as an attempt to show that languages that apparently carry no commitment to abstracta are actually up to their necks in that sort of commitment. Still, this is a striking conclusion, a result that not only nominalists find puzzling. The natural suspicion is that the nominalist cannot have it both ways: he cannot insist that 'the direction of a' functions as a genuine singular term in 'the direction of a = the direction of b' *and* that this sentence means by stipulation nothing more or less than 'a is parallel to b'. But it is not clear how exactly the conclusion is to be resisted without giving up on the Context Principle. To bolster his case here, the anti-nominalist needs only a special case of the

[78] The argument is from Rosen (1993).
[79] Actually, all we get is ontological commitment to *something* all concrete line inscriptions have and all and only parallel line inscriptions share. There is a further step in concluding that these entities are directions. But this step is probably not particularly controversial.

principle, according to which there is nothing more to being a classical first-order language with identity than behaving in all inferential respects like such a language.[80]

The moral is that the fate of the Fregean argument rests on a thorough defense of a precisely stated version of the context principle. No anti-nominalist believes that this is an easy task, or that it has been done. But some hope that it can be done.[81]

5. A middle way?

None of the arguments for or against nominalism is, it seems to me, conclusive. By itself, this is not surprising: philosophical arguments are rarely have that character. But the elusiveness of the problem has tempted some philosophers to entertain less than straightforward answers to the question whether there are abstract entities. The first and most famous of these oblique answers is due to Rudolf Carnap.[82]

According to Carnap, to understand the problem of the existence of abstract entities, we need to distinguish between two kinds of questions, or more precisely, between two ways of interpreting a question. We can construe a question as *internal* or *external* with respect to a particular linguistic framework. A linguistic framework is the totality of conventions specifying the syntax and semantics of a language together with certain rules of confirmation; a question is taken as internal with respect to it just in case it is interpreted as asking for an answer to be established within that framework. Carnap's verdict is that questions regarding the existence of abstracta tend to be trivial when taken as internal and deeply problematic when taken as external. The problematic character of external questions of existence is supposed to be the result of our lack of established rules for providing an answer. For example, if we construe the question 'Are there numbers?' as internal to the framework of arithmetic, the answer is a straightforward 'yes'. But an interpretation of this question that is external to any particular linguistic framework can only be regarded as a query about whether we should adopt the framework of arithmetic.

[80] Rosen (1993): 162. Rosen continues the argument by showing that there is a way out for the nominalist, even if he accepts the context principle. The escape works when the existence of directions is at stake, but unfortunately fails when the Fregean argument is run for sets or numbers.

[81] For criticism and ultimate rejection of the Context Principle, see e.g. Hodes (1990) and Lowe (1995).

[82] Carnap (1950).

But for Carnap, this is not a theoretical question. It is a practical issue to be settled on the basis of considerations of expediency and fruitfulness.

Carnap’s view has been out of fashion for a while. This is largely due to the fact that philosophers were convinced by the gist of Quine’s famous criticism. There is something deeply unappealing about the thought that means of rational justification arise from limited frameworks we might adopt and that consequently there is no rational way to justify the choice of any one of these particular frameworks. For a naturalist of Quine’s stripe, there is no sharp boundary between theoretical questions and practical ones. Every question must be answered as part of a project of designing the best overall theory of the world, where the criteria of evaluation *include* Carnap’s expediency and fruitfulness along with other considerations normally used in comparing scientific theories. But the aspect of Carnap’s view Quine so forcefully criticized could be separated from the rest. One could maintain the idea that ontological questions are in some way fundamentally ambiguous, while giving up of Carnap’s insistence that one of their readings is rationally unapproachable.

One can ask the question ‘Are there infinitely many prime numbers?’ or the question ‘Are there *really* numbers?’ One would be naturally inclined to regard the former as a sort of question that is appropriately raised in the context of high school mathematics class and fully answered by the Euclidean proof. One would not be naturally inclined to think that the fact that the positive answer to the first question entails that there are numbers that the Euclidean proof answers the second question as well. The word ‘really’ seems to be doing some sort of work in the latter question, it indicates somehow that we are looking for a different sort of answer. This is the source of Carnap’s intuition. He then goes on, analyzes the difference in a conventionalist way, and repudiates the latter question as asking for a sort of answer that could not be given. One might abandon his solution to the problem, while respecting the underlying insight.

What sort of alternative explanation could there be for the alleged fundamental ambiguity in ontological questions? There are a number of possibilities. The one that departs least from Carnap’s original thought is the fictionalist line. According to a fictionalist about abstracta, when we speak about numbers, models, properties and the rest we immerse ourselves in pretense. *Within* this fiction all those entities exist, but *outside* the fiction in the real world they don't. When we ask a question like 'Are there infinitely many prime numbers?' we are not looking for a true answer, only for an answer that in true in the mathematical fiction. The word 'really' marks that we wish to drop the curtains and go after truth *simpliciter*.[83]

Now, this is not really an oblique answer: it is only fictional worlds that have abstracta, the real one is just as the nominalist thinks it is. But there are other ways to argue for a fundamental ambiguity in ontological questions, which make it harder to say whether the adopted view is a version of nominalism or anti-nominalism. One might, for example, argue that we have two notions of *reference* — one requiring full-blown epistemic contact and another one that does not — and that terms for certain abstracta refer in one, but not the other sense. Or one might claim that there is no unique *structure* associated with sentences talking about certain abstracta and that the truth-conditions determined through one of these structures commit one to the existence of those entities, while the truth-conditions determined through the other don't. Or one might insist that we have two notions of *belief* — believing that *F*s exist and believing in *F*s — and that these two beliefs involve different kinds of commitment, one appropriate for certain abstracta, while the other inappropriate.[84]

It may be that each of these attempts to strike a middle ground between straightforward acceptance and straightforward rejection of abstract entities fails. With enough foot stamping one can usually convince people that there is no conceivable source of ambiguity in asking the simple question 'Are there abstract entities?.' (I myself performed some of the foot stamping at the end of section 1.1. above.) Be it as it may, a convincing answer does not seem to loom large on the horizon.[85]

ACKNOWLEDGEMENTS

[83] Cf. Yablo (1998) and Yablo (forthcoming).
[84] For the first view, see Azzouni (1994) and (1997), for the second, Hodes (1984), (1990), (1991), for the third Szabó (forthcoming b).
[85] For the record: Balaguer (1998) argues that the question is univocal, that there is an optimal version of nominalism and an optimal version of anti-nominalism, that these versions are immune from refutation, and that the best way to think of this is to admit that there is no fact of the matter.

Thanks to a lot of people, including Tamar Szabó Gendler, Benj Hellie, Harold Hodes, and Scott Spiker. I will be more specific in the final draft.

BIBLIOGRAPHY

Alston, William (1957) 'Ontological Commitments.' *Philosophical Studies* **9**: 8 – 17.

Azzouni, Jody (1994) *Metaphysical Myth, Mathematical Practice: The Ontology and Epistemology of Exact Sciences.* New York: Cambridge University Press.

Azzouni, Jody (1997) 'Thick Epistemic Access: Distinguishing the Mathematical from the Empirical.' *Journal of Philosophy* **94**: 427 – 84.

Azzouni, Jody (1998) 'On "On What There Is".' *Pacific Philosophical Quarterly* **79**: 1 – 18.

Balaguer, Mark (1998) *Platonism and Anti-Platonism in Mathematics.* Oxford: Oxford University Press.

Benacerraf, Paul (1965) 'What Numbers Could Not Be.' *Philosophical Review* **74**: 47 – 73.

Benacerraf, Paul (1973) 'Mathematical Truth.' *Journal of Philosophy* **70**: 661 – 79.

Berkeley, George (1711) *A Treatise concerning the Principles of Human Knowledge.* In A. A. Luce and T. E. Jessop eds., *The Works of George Berkeley, Bishop of Cloyne*, 9 vols. London: Thomas Nelson and Sons, Ltd. 1948 – 57.

Boolos, George (1975) 'On Second-Order Logic.' *Journal of Philosophy* **72**: 509 – 27.

Boolos, George (1984) 'To Be is to Be a Value of a Variable (or to Be Some Values of Some Variables).' *The Journal of Philosophy* **81**: 430 – 49.

Boolos, George (1985) 'Nominalist Platonism.' *Philosophical Review* **94**; 327 – 44.

Boolos, George (1997) 'Is Hume's Principle Analytic?' In Richard Heck ed., *language, Truth and Logic: Essays in Honour of Michael Dummett.* Oxford: Clarendon Press, pp. 245 – 262.

Burgess, John and Gideon Rosen (1997) *A Subject with no Object.* Oxford: Clarendon Press.

Carnap, Rudolf (1950) 'Empiricism, Semantics, and Ontology.' Reprinted as a supplement to *Meaning and Necessity: A Study in Semantics and Modal Logic.* Chicago: University of Chicago Press, 1956, pp. 205 – 21.

Cartwright, Nancy (1983) *How the Laws of Physics Lie.* Oxford: Oxford university Press.

Cartwright, Richard. 1994. Speaking of Everything. *Nous* **28**: 1 – 20.

Chihara, Charles (1990) *Constructibility and Mathematical Existence.* Oxford: Clarendon Press.

Courteney, William J. ed., (1992) *Vivarium* **30**. Special issue devoted to the papers of a conference on 'The Origin and Meaning of Medieval Nominalism' held at the University of Wisconsin-Madison, October 3 – 5, 1991.

Davidson, Donald (1967) 'The Logical Form of Action Sentences.' Reprinted in *Essays on Actions and Events.* Oxford: Clarendon Press, 1980, pp. 105 – 23.

Dummett, Michael (1973a) 'The Philosophical Basis of Intuitionistic Logic.' In *Truth and Other Enigmas.* Cambridge, MA: Harvard University Press, 1978, pp. 215 – 47.

Dummett, Michael (1973b) *Frege: Philosophy of Language.* Cambridge, MA: Harvard University Press.

Dummett, Michael (1991) *Frege: Philosophy of Mathematics.* Cambridge, MA: Harvard University Press.

Field, Hartry (1980) *Science without Numbers: A Defense of Nominalism.* Oxford Basil Blackwell.

Field, Hartry (1984) 'Platonism for Cheap? Crispin Wright on Frege's Context Principle.' In *Realism, Mathematics and Modality.* Oxford: Basil Blackwell, 1989, pp. 147 – 70.

Field, Hartry (1985) 'On Conservativeness and Incompleteness.' In *Realism, Mathematics and Modality.* Oxford: Basil Blackwell, 1989, pp. 125 – 46.

Field, Hartry (1988) 'realism, Mathematics and Modality.' *Philosophical Topics* **19**: 57 – 107.

Field, Hartry (1989) 'Fictionalism, Epistemology and Modality.' In *Realism, Mathematics and Modality.* Oxford: Basil Blackwell, 1989, pp. 1 – 52.

Geach, Peter (1957) *Mental Acts: Their Content and Their Objects.* London: Routledge & Kegan Paul.

Goodman, Nelson (1956) 'A World of Individuals.' Reprinted in *Problems and Projects.* Indianapolis: Bobbs-Merrill Co., 1972, pp. 155 – 72.

Goodman, Nelson and Willard Van Orman Quine (1947) 'Steps towards Constructive Nominalism.' *Journal of Symbolic Logic* **12**: 97 – 122. Reprinted in Nelson Goodman, *Problems and Projects.* Indianapolis, IN: Bobbs-Merrill Co., 1872, pp. 173 – 99.

Gödel, Kurt (1947) 'What is Cantor's Continuum Problem?' Revised version in Paul Benacerraf and Hilary Putnam eds., *Philosophy of Mathematics: Selected Readings.* 2nd edition, 1983. Cambridge: Cambridge University Press, pp. 470 – 85.

Hale, Bob (1987) *Abstract Objects.* Oxford: Blackwell.

Hellman, Geoffrey (1989) *Mathematics Without Numbers.* Oxford: Clarendon Press.

Hodes, Harold (1984) 'Logicism and the Ontological Commitments of Arithmetic.' *Journal of Philosophy* **81**: 123 – 49.

Hodes, Harold (1990) 'Ontological Commitment: Thick and Thin.' In G. Boolos ed., *Essays in Honor of Hilary Putnam.* Cambridge: Cambridge University Press, pp. 235 – 60.

Hodes, Harold (1991) 'Where Do Sets Come From?' and 'Corrections to *Where Do Sets Come From?*' *Journal of Symbolic Logic* **66**: 150 – 75 and 1486.

Jubien, Michael (1977) 'Ontology and Mathematical Truth.' *Nous* **11**: 133 – 50.

Kripke, Saul (1976) 'Is There a Problem about Substitutional Quantification?' In Gareth Evans and John McDowell eds., *Truth and Meaning.* Oxford: Clarendon Press, 325 – 419.

Kripke, Saul (1972) *Naming and Necessity.* Cambridge, MA: Harvard University Press.

Lewis, David (1986) *On the Plurality of Worlds.* Oxford: Blackwell.

Lewis, David (1991) *Parts of Classes.* Oxford: Basil Blackwell.

Lowe, E. J. (1994) 'Vague Objects and Quantum Indeterminacy.' *Analysis* **45**: 110 – 4.

Lowe, E. J. (1995) 'The Metaphysics of Abstract Objects.' *Journal of Philosophy* **92**: 509 – 24.

Malament, David (1982) 'Review of Hartry Field's *Science Without Numbers.*' *Journal of Philosophy* **79**: 523 – 34.

Melia, Joseph (1995) 'On What There Isn't.' *Analysis* **55**: 223 – 9.

Parsons, Charles (1965) ‘Frege’s Theory of Number.’ Reprinted in *Mathematics in Philosophy.* Ithaca, NY: Cornell University Press, 1983, 150 – 72.

Prior, Arthur (1954) ‘Entities.’ Reprinted in *Papers in Logic and Ethics.* Amherst, MA: University of Massachusetts Press, 1976.

Putnam, Hilary (1967) ‘Mathematics Without Foundations.’ *Journal of Philosophy* **64**: 5 –22.

Putnam, Hilary (1971) *Philosophy of Logic.* New York: Harper.

Quine, Willard Van Orman (1948) ‘On What There Is.’ *Review of Metaphysics* **2**: 21 – 38.

Quine, Willard Van Orman (1951) ‘Two Dogmas of Empiricism.’ *Philosophical Review* **60**: 20 – 43.

Quine, Willard Van Orman (1964) ‘Ontological Reduction and the World of Numbers.’ In *The Ways of Paradox.* Cambridge, MA: Harvard University Press, revised and enlarged edition 1976, 212 –20.

Quine, Willard Van Orman (1969) ‘Existence and Quantification.’ In *Ontological Relativity and Other Essays.* New York: Columbia University Press, pp. 91 – 113.

Resnik, Michael (1988) ‘Second-Order Logic Still Wild.’ *Journal of Philosophy* **85**: 75 – 87.

Rosen, Gideon (1993) ‘The Refutation of Nominalism(?)’ *Philosophical Topics* **21**: 149 – 86.

Rosen, Gideon (1994) ‘What is Constructive Empiricism?’ *Philosophical Studies* **74**: 143 – 78.

Shapiro, Stewart (1983) ‘Conservativeness and Incompleteness.’ *Journal of Philosophy* **80**: 521 – 31.

Stanley, Jason (forthcoming) ‘Hermeneutic Fictionalism.’ forthcoming in *Midwest Studies in Philosophy* **25**: Figurative Language, H. Wettstein, ed., Oxford, Blackwell Press.

Szabó, Zoltán Gendler (2000) ‘Descriptions and Uniqueness.’ *Philosophical Studies* **101**: 29 – 57.

Szabó, Zoltán Gendler (forthcoming a) ‘Fictionalism and Moore’s Paradox.’ *Canadian Journal of Philosophy.*

Szabó, Zoltán Gendler (forthcoming b) 'Believing in Things.' *Philosophy and Phenomenological Research.*

Van Fraassen, Bas (1980) *The Scientific Image.* Oxford: Clarendon Press.

Van Fraassen, Bas (1994) 'Gideon Rosen on Constructive Empiricism.' *Philosophical Studies* **74**: 179 – 92.

Vendler, Zeno (1967) 'Facts and Events.' In *Linguistics in Philosophy.* Ithaca, NY: Cornell University Press, 122 – 46.

Wright, Crispin (1983) *Frege's Conception of Numbers as Objects.* Aberdeen: Aberdeen University Press.

Wright, Crispin (1997) 'On the Philosophical Significance of Frege's Theorem.' In Richard Heck ed., *Language, Truth and Logic: Essays in Honour of Michael Dummett.* Oxford: Clarendon Press, pp. 201 – 244.

Yablo, Steven (1998) 'Does Ontology Rest on a Mistake?' In *Proceedings of the Aristotelian Society*, supp. vol. **72**: 229 – 61.

Yablo, Steven (forthcoming) 'A Paradox of Existence.'www.mit.edu/~yablo/apex.html.