Scientific Philosophy and Philosophical Science

Hourya Benis Sinaceur

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Directrice de recherche émérite
Institut d'Histoire et Philosophie des Sciences et des Techniques (IHPST)
Université Paris 1 Panthéon-Sorbonne – CNRS – ENS Ulm

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Abstract

Philosophical systems have developed for centuries, but only in the nineteenth century did the notion of scientific philosophy emerge. This notion presented two dimensions in the early twentieth century. One dimension arose from scientists' concern with conceptual foundations for their disciplines, while another arose from philosophers' appetite for more rigorous philosophy. In the current paper, I will focus on David Hilbert's construct of "critical mathematics" and Edmund Husserl' and Jules Vuillemin's systematic philosophy. All these three thinkers integrated Kant's legacy with the axiomatic method. However, they did so in different ways, with Hilbert's goal being the opposite of that of Husserl or Vuillemin. Specifically, I will show how the scientism of Hilbert's mathematical epistemology aimed at shattering the ambition of philosophy to submit mathematical practices and problems to philosophy's own principles and methods, be they transcendental or metaphysical. On the other hand, phenomenology promoted the idea of a non-exact philosophical rigour and highlighted the need of a point of view encompassing positive sciences, ontology, and ethical values in connection with the dominant category of sense/meaning, and Jules Vuillemin built on from the inseparability of thought - scientific or philosophical - from the metaphysics of free will and choice.

The rapid evolution, in many and varied ways, of the axiomatic approach in the 19th Century, coupled with the renewal of logic to which Gottlob Frege gave a decisive boost, shook ancient philosophical certainty concerning the status of and mutual relations between fundamental concepts such as intuition, concept, experience, object, subject and consciousness. At the same time the prestige of science triggered the revival of the idea of "philosophy as science". Physical sciences, mathematics and mathematical logic shaped the philosophical requirement for rigour, although there is neither test nor proof for philosophical assumptions.

The idea of philosophy as science firmly establishes itself in the first third of the 20th century. Most of its proponents share the desire to counteract the influence of Hegel's system that the author had presented as philosophical science and which had, in the 1830s, been a dominant doctrine of University of Berlin and the Prussian State. But they do not have a common vision of what should replace it. The idea of philosophy as science is not univocally determined. It includes distinct elements of varying composition, borrowing from both the philosophical

tradition and new scientific methods. Without being exhaustive, I will mention three or four of these compositions: the critical mathematics of Hilbert and Leonard Nelson, the scientific philosophy of Husserl and the systematic philosophy of Jules Vuillemin. Drawing from the common source of demand for rigour combining critical philosophy and the axiomatic method, the authors of these compositions develop very different, even antithetical, designs from the intersection of philosophy and science.

I. Science enters its critique phase: Hilbert, Hessenberg, Nelson

The first design consists of "critical" science, namely the adoption of the Kantian perspective for characterising the axiomatic renewal of science. Unlike dogmatism that takes its principles for granted, critical philosophy justifies its own principles. In this sense, a critical attitude itself appears to bring about rigorous standards for science. Reflexive reasoning effectively exercises control over its capacity and determines the conditions of its exercise. The axiomatic approach is credited with the ability to perform such a reflection back onto scientific objects and procedures to accurately delineate the extent of their validity. Exploring results previously acquired in different areas of mathematics, the axiomatic approach reflexively establishes objects of a new kind, namely structures, and it sets new standards of truth. Truth is no longer confused with evidence imposing itself on the so-called immediate knowledge (commonly known as intuition). The axiomatic approach organises scientific propositions into self-regulating systems of deduction based on axioms *set down as assumed truths.*[1]

From Descartes to Husserl, philosophy first conceived reflexiveness as self-conscious reflection. With Kant, self-consciousness abandons the certainty of cogito and the divine guarantee of clear ideas for the human tribunal of critique: the subject looks back on his own actions to first submit them to analysis with a view to revealing the a priori conditions for possibility of knowledge of objects. Pure consciousness, native and immutable, is a condition of empirical consciousness; it is a priori and necessary condition of both experience and objects of experience. Kant calls this pure consciousness "transcendental apperception." It is the relationship to this apperception that constitutes the *form* of all understanding of the object.

In the interpretation made by scientists, reflexiveness usually abandons self-consciousness, whether pure or merely empirical, so as to vindicate only the critical ingredient, and even then in a manner little in keeping with its Kantian origins. David Hilbert is well known for placing under the banner of Critique the two essential branches of his foundations of science programme[2]: the axiomatic method and proof theory. He therefore contributed to feeding, or even starting, discussions focused on whether and to what extent the framework of *Critique of Pure Reason* still has a legitimate claim to provide us with foundations of science after the scientific and epistemological revolutions of non-Euclidean geometry, relativity theory, and quantum physics.

1. *The axiomatic method and intuition*

[1] CF. the Frege-Hilbert correspondence (published by I. Angelelli in *Kleine Schriften*, Hildesheim, Olms, 1967): to Frege, who advocates the intuitive origin of geometrical axioms, Hilbert responds "Sobald ich habe ein Axiom gesetzt, ist es wahr und vorhanden" ("As soon as I have posed an axiom, it is true and available").

[2] When Hilbert was born in Königsberg in 1862, Kant had been dead for nearly 60 years (1804), but the considerable prestige of the thinker of the Enlightenment was far from being extinguished.

Hilbert's *Foundations of Geometry* (1899) presents and classifies the axioms of geometry in order to show various combinations generating different geometries: Euclidean geometry, Cartesian algebraic geometry, non-Archimedean geometry, projective geometry, etc. In his introduction Hilbert first highlights the famous words of Kant: "all human knowledge begins with intuitions, proceeds from thence to concepts, and ends with ideas".[3] Then he presents his axiomatic ordering as an "analysis of our intuition of space". Describing what the mathematician's intuition is, or what it consists of, is a theme that runs through Hilbert's whole work. Indeed, allegiance of the Göttingian mathematician to the Königsbergian philosopher is not merely decorative or transient and is not limited to this highlight, which indicates that Hilbert intentionally placed his axiomatic ordering of geometric propositions within Kant's perspective. The conclusion, the last section of which I will quote, explains what Hilbert meant by the analysis of the intuition of space.

> "The present work is a critical investigation of the principles [Prinzipien] of geometry. In this investigation the guiding precept [der leitende Grundsatz] is to examine each question so as to prove outright if the answer is *possible* when some limited means are imposed in advance. […]
>
> In modern mathematics the question of the *impossibility* [Hilbert's emphasis] of certain solutions or problems plays a leading role, and the attempts to answer such questions have often given the opportunity to discover new and fruitful areas of research. Examples of this include the demonstration by Abel of the impossibility of solving by radicals the 5th degree equation, the discovery of the impossibility of proving the axiom of Parallels, and the theorems of Hermite and Lindemann on the impossibility of constructing by algebraic means the numbers e and π.
>
> The precept by which one must always examine the principles of the *possibility* of proof is closely related to the requirement for "purity" of methods in proof, which in recent times has been considered of the highest importance by many mathematicians. This requirement is basically nothing more than a *subjective* version [Fassung] of the precept followed here. Indeed our present investigation attempts to explain generally what axioms, assumptions or auxiliary means are necessary to establish the truth of an elementary geometric proposition, and all that remains to be gauged is which method of proof is preferable in each case from the adopted point of view".[4]

Certainly, Hilbert's guiding precept is that of conditions of possibility. But, and this is essential, the precept is played out on a case by case basis and in each case it is circumscribed by both the problem to be solved and the limited resources previously allowed for the solution. Obviously those conditions of possibility are by no means universal and necessary, unconditional and unchanging principles of experience. Hilbert limits himself to the specific experience of mathematics, and furthermore to particular experiences concerning the demonstration of definite propositions in definite situations. Moreover, in mathematics, *proving* the *im*possibility of a solution does not establish a higher absolute domain, comparable to the Kantian realm of things-in-themselves, and does not close the door to exploring *other* possibilities by changing the way of formulating the problem and the means of proof.

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[3] *Critique of Pure Reason* (*CPR*), Transcendental Logic, Transcendental Dialectic, Appendix A702/B730. Another well-known phrase: "If all our knowledge begins with experience, it does not follow that it derives all of the experience."

[4] *Grundlagen der Geometrie*, 10. Auflage, Stuttgart, B.G. Teubner, 1968, Schlusβwort, 124-125 (my emphasis).

However, the parallel between the axiomatic approach and Kant's critical enterprise remains fixed in Hilbert's mind. In 1917, in "Axiomatisches Denken" (p. 148)[5], Hilbert says that critical examination [die kritische Prüfung] of certain proofs leads to new axioms being formed from more general and fundamental propositions than those previously held as such. This axiomatic deepening, also characterised as "proof critique" [Beweiskritik], represents the first stage of the critique of mathematical reason, which overturns the dogmatism of established evidence and practices[6]. In 1922 Hilbert links again the axiomatic method to Critique, saying on this, and to my knowledge only occasion, that axiomatising is nothing other than thinking in the light of consciousness [mit Bewußtsein denken][7], but Hilbert added that the most important thing is the *mathematical resolution* of questions of theory of knowledge posed by the axiomatic method. He then presents the work of Dedekind and Frege on arithmetic as the inauguration of "modern critique of Analysis" (p. 162). In 1930 in "Naturerkennen und Logik"[8] he briefly examines the Kantian a priori (to which I return below). The recourse to critical reason is constant, therefore. Let us examine whether and to what extent it is legitimate given Hilbert's actual mathematical practice.

According to Hilbert, showing which geometric theorems are logically derivable from a definite set of axioms, i.e. showing how different geometry systems are each related to a definite conjunction of axioms expressing the necessary and sufficient conditions for developing the whole system, thus showing the need for a deductive link between principles and consequences, is "the analysis of our intuition of space". Then this analysis displays different concepts or systems of space. To demonstrate the *logical* compatibility or deducibility between geometric propositions, to distinguish between assumed propositions (axioms) and demonstrated propositions (theorems), to ask whether an axiom, given the other axioms simultaneously admitted, is removable or indispensable[9] is to gain in mathematics the rigour acquired in philosophy by *critical attitude*.

The explanation seems Kantian, for Hilbert uses Kant's terminology: 'critique', 'condition of possibility', and 'intuition'. But the terminology can mislead, as notable philosophers have been. Hilbert's good faith is not in question, but one has to look closer the text of *The Foundations of Geometry*.

First and foremost it is clear that it is the *logical analysis* of the *objective* links of dependence between mathematical statements that is charged with taking on a critical attitude. This way is actually closer to the objectivist spirit of Bolzano, Dedekind, and Frege than to Kant's subjectivism. Hilbert's epistemological effort consists precisely in replacing the Kantian

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[5] *Mathematische Annalen* 78 (1918), 405-415, *Gesammelte Abhandlungen* III, Berlin, Springer, 1935, 146-156.

[6] In 1904, 5 years after the publication of Hilbert's *Foundations of Geometry*, the philosopher Leonard Nelson, whose habilitation and career Hilbert supervised and facilitated, would explicitly set out the programme for "transferring Critique to the axiomatic systems of mathematics in order to constitute a specific scientific discipline: critical mathematics"; cited by Volker Peckhaus in *Hilbertprogramm und Kritische Philosophie. Das Göttinger Modell interdisziplinärer Zusammenarbeit zwischen Mathematik und Philosophie*, Göttingen, Vandenhoeck und Ruprecht, 1990, p. 158. I return to critical mathematics below.

[7] Neubegründung der Mathematik, *Gesammelte Abhandlungen* III, p. 161.

[8] *Naturwissenschaften* **18**, 959-963; in *Gesammelte Abhandlungen* III, 378-387.

[9] Notable examples: Pascal's theorem cannot be proven in the simultaneous absence of the axioms of congruence and the axiom of Archimedes; Desargues' theorem is provable in space from the axioms of incidence, but in the plane it is necessary to add the 5 axioms of congruence; if we add just one more axiom which negates the existence of points outside the plane, we cannot construct projective geometry unless you also add the theorem of Desargues as an axiom, demonstrating the constitutive role thereof within the construction of planar projective geometry.

"subjectivist version" with an objectivist version or, if you will, of restoring the rights of formal logic over transcendental logic. Additionally Hilbert's understanding of 'formal logic' in the *Foundations of Geometry* and other works does not coincide with Kant's definition of formal logic. After Frege's *Begriffschrift* (1879) 'formal logic' had a definitely different meaning than before.

Let me explain in detail my arguments.

1. The scope of a geometrical proposition, its meaning/significance [Bedeutung], as Hilbert says, is shown by a set of variations governing the connection between axioms and theorems. The investigation concerns the conditions of *validity* of certain *mathematical content* according to different settings. So the possibility in question is material in as much as it concerns a formal logical structure.

2. Accordingly, the conditions in question operate *locally*. The very possibility of varying the choice of axioms (with or without the axiom of parallels, with or without the axiom of Archimedes, etc.) depending on the type of geometry that one wants to construct, itself attests to their regional (not universal) character and their relative necessity. To counter dogmatism is not to deny the universal, but to contextualise it. Indeed, Hilbert stresses that the axiomatic method does not change only the content but also the modality of our mathematical beliefs, and therefore dissolves dogmatism by explaining the logical connections between mathematical propositions. In this context Hilbert speaks of "necessary relativism"[10], which in this case, one should add, has nothing to do with Kant's demarcation between absolute things-in-themselves and knowable phenomena. Axiomatic relativism stems from the many systems corresponding to the same web of mathematical propositions and the many a priori possible interpretations for the same system. Dissolving dogmatism comes down here to abandon the idea of absolute truth of mathematical propositions in favour of the idea of truth relating to axiomatic systems.

3. Moreover, considering those desired conditions as axioms, which formally express the properties or relationships deemed fundamental, is completely different from designating an *empty form* of relationship between our understanding and things that appear to us only qua objects of experience or phenomena. In fact, Hilbert's description and advocacy of the axiomatic method relates to scientific practice which singles out sets of primitive propositions as the basis for proving theorems, it does not concern the theory of knowledge in general. Besides, philosophy plays only a peripheral role in *The Foundations of Geometry*, yet it is useful for piquing the interest of philosophers and bringing them into the mathematical school.

4. Finally, proposing an "analysis of our intuition of space" is to state that space is an object of our intuition and thus an intuitive datum - real or conceptual. Yet for Kant space is not a datum but the *form* of sensory data provided by perception. It is "the *subjective* condition of sensitivity under which alone external intuition is possible for us".[11] And besides, Kant distinguishes between sensory intuition (empirical intuition) and pure intuition[12], the latter

[10] Neubegründung der Mathematik, *Gesammelte Abhandlungen* III, p. 169.

[11] *CPR*, Transcendental Aesthetic I, § 2, A26/B42 (I have highlighted 'subjective').

[12] "Space and time are pure forms [of perception], sensation in general its matter. We can cognize only the former *a priori*, i.e., prior to all actual perception, and they are therefore called pure intuition; the latter, however, is that in our knowledge that is responsible for its being called *a posteriori* knowledge, i.e., empirical

conditioning the former. Space is pure intuition (and not a pure concept), it is a formal a priori condition of experience, "the basic form of all external sensation".

> "*Space is not something objective and real*, [non aliquid objectivi et realis]; nor a substance, nor an accident, nor a relation; instead, it is *subjective* and ideal, and originates from the mind's nature in accord with a fixed law [natura mentis stabili lege profiscens] as a scheme for coordinating everything sensed externally." [13]

It follows that on the requirement for the a priori representation of space rests "the apodictic certainty of all geometric principles, and the possibility of their construction a priori."[14] The properties of a triangle, for example, are constructed a priori in pure intuition. Similarly, the three-dimensional Euclidean space is pure a priori intuitive evidence. Space as a form of both experience and objects of experience is a priori and necessary representation, it is one of the principles of a priori knowledge[15]; in particular it is a necessary subjective principle of geometric propositions, which are "always apodictic, that is, *united with the consciousness of their necessity*" emphasis added).

Thus, for Kant space as pure intuition is the principle of the axioms of geometry (Euclidean geometry, the only known then). On the contrary, for Hilbert sets of axioms are the principles of the geometry they determine. In Kantian language, we can say that Hilbert totally disregards "transcendental ideality" of space, that is, the fact that for Kant space *is nothing* from the point of view of things, their properties or relationships, and has but formal reality as a condition of possibility of phenomena, a condition belonging to the subjective constitution of the mind. In fact, the misunderstanding or confusion derives from the very meaning of the term 'space'. Kant holds that "the original representation of space is an intuition a priori, and not a concept"[16]. By contrast, since Gauss', Riemann's, and Dedekind's works, geometric space is not the space of external experience, it is neither empirical intuition nor pure intuition but a body of mathematical properties, that is to say a "concept" in Dedekind' and Hilbert's wording. In *Stetigkeit und irrationale Zahlen* Dedekind clearly maintains the conceptual nature of geometric space and Hilbert will explicitly recognise that the axioms defining a geometry form a "conceptual framework" [ein Fachwerk von Begriffen][17] to formalise a structure, among several possible structures, expressing in a synthetic and coherent way experimental data collected in the real world by physical instruments. For the mathematician Hilbert, conceptual/axiomatic frameworks capture geometric intuitions, which refer less to the sensory world than to the world of scientific (physical, biological, astronomical, etc.) experiments and they do not presuppose pure a priori intuition as their formal condition of possibility.

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intuition. The former adheres to our sensibility absolutely necessarily, whatever sort of sensations we may have". *CPR*, Transcendental Aesthetic I, § 8, A42-43/B59-60.

[13] Dissertation de 1770, Paris, Vrin, 1951, p. 55 (Kant's emphasis), English W.J. Eckoff, Columbia College, 1894, p. 65 (https://archive.org/details/cu31924029022329). *CPR*, Transcendental Aesthetic I, § 2, A28/B44: "We maintain […] the empirical reality of space in regard to all possible external experience, although we must admit its transcendental ideality; in other words, that it is nothing, so soon as we withdraw the condition upon which the possibility of all experience depends and look upon space as something that belongs to things in themselves".

[14] *CPR*, Transcendental Aesthetic I, § 2, A25.

[15] *CPR*, Transcendental Aesthetic I, § 1, A22.

[16] "Also ist die ursprüngliche Vorstellung vom Raume Anschauung a priori, und nicht Begriff". *CPR,* Transcendental Aesthetic I, § 2, B40.

[17] 'Fachwerk' literally means half-timbering. The concepts are therefore the visible structure of the theoretical edifice. Hence, Cavaillès' and Bourbaki's insistence on the "architecture" of mathematics.

Yet Kant's shadow continues to hang over Hilbert as over other German and non-German mathematicians (Poincaré and Brouwer in particular). In a seminar held in 1905 Hilbert presented similar material to that of his lecture at the 3rd International Congress of Mathematicians, "On the foundations of logic and arithmetic". There, preceding logical calculation, an "axiom of thought" [Axiom des Denkens] is meant to represent "the a priori of philosophers".[18] Kant clearly constitutes the philosophical horizon of Hilbert and the Göttingen mathematicians, discussions focusing on thought, a priori, and the division between analytical judgements and synthetic a priori judgements.

2. *Critique of reason and proof theory*

In 1917, in "Axiomatisches Denken", Hilbert once again explains the contribution of the axiomatic approach. This time he sees it as the instrument of transformation of a set of facts within a given scientific field into a *unified theory* of this field. Thus the theory of arithmetic, the Galois theory, the theory of heat, the theory of gases, the theory of money, etc. A theory is therefore a conceptual framework [Fachwerk von Begriffen] such that a concept corresponds to a particular object of the scientific field being studied and that logical relations between concepts correspond to relationships between facts within the field. At the base of the framework a few concepts and their interrelations enable the reconstruction of the entire framework, at least that is how it was envisaged before Gödel's incompleteness theorem (1931).

It is also in 1917 that Hilbert, to complete his work on the axiomatic method, sets up the project to build a new branch of mathematics, named metamathematics, whose specific object is the concept of mathematical proof. A similar approach to the physicist's theory of his technological equipment and the philosopher's critique of reason, he writes, will supply a Critique of proof [Beweiskritik]. A little later (1922) this metamathematical project will begin to be realised under the name of proof theory: the Beweiskritik becomes Beweistheorie, i.e. both method and meta-content. This crowns the whole *critique of mathematical reason* enterprise begun with the axiomatic method. An important inflection appears in this 1917 paper. Hilbert now puts the problem of non-contradiction of axioms, that of a criterion for simplicity of mathematical proof, that of the relationship between contentuality and formalism [Inhaltlichkeit und Formalismus] in logic and mathematics and that of decidability of a mathematical question by a finite number of steps in the set of "questions of the theory of knowledge with a specific mathematical coloration". Therefore, Hilbert has in mind a mathematical theory of knowledge, highlighting that the Kantian theory of knowledge can no longer prescribe the new mathematics, the new physics and the new logic.

In a lecture delivered in 1919-1920 on "The role of intuition and experience", Hilbert intends to deliver a "kind of preparation for the construction of a theory of knowledge" which promises, as far as mathematics is concerned, to be a "far greater success than Kant's" (!)[19] The example of the construction of the mathematical continuum that he sets out in detail shows that Hilbert, unlike Kant, makes little distinction, or only one of degree, between intuition and perception. Indeed, intuition begins with perception and leads to a concept, which frees us from intuition as Einstein's theory shows us.[20] By speaking here on intuition

[18] Cited by Peckhaus, p. 62.

[19] Hilbert, *Natur und mathematisches Erkennen*, herausgegeben von David. E. Rowe, Basel, Birkhäuser Verlag, 1992, p. 3-4.

[20] Ibid. p. 50-51. Thus intuition is rather preliminary than a priori.

Hilbert wishes above all to show that mathematics is not an empty game but "a conceptual system constructed according to an internal requirement". Echoing the view taken by F. Klein and especially R. Dedekind[21], whose essays on numbers so impressed him, Hilbert substitutes the Kantian requirement for forms of experience with the constraint imposed by *the content of mathematical problems*. His fight against dogmatism and intuitionism leads him far from Kant: mathematical constraints are not uniquely formal, nor universal, nor unchangeable.

In 1930, in the article entitled "Naturerkennen und Logik" mentioned above, Hilbert proposed treating the old epistemological problem of the relationship between thought and experience in the light of advances in physics due to Planck, Bohr, Einstein, the Curies, Röntgen, etc. After flatly rejecting Hegel's absolute rationalism and invoking Leibniz' pre-established harmony to account for the correlation between logical axiomatics and experience, Hilbert turns to Kant, who adds, according to him, an a priori element consisting of "some knowledge of reality". Here is the text:

> In fact philosophers have argued that Kant is the classic representative by stating that in addition to logic and experience we still have some a priori knowledge. I acknowledge that certain a priori *views* [Einsichten] are required for the construction of the theoretical structure and constitute the basis of our knowledge. I believe that mathematical knowledge also rests, ultimately, on a kind of intuitive view of this nature. And even to build arithmetic we need a certain a priori intuitive attitude [eine gewisse a priori Anschauliche Einstellung]. It is here, therefore, that the most fundamental thinking of Kant's theory of knowledge lies, namely the philosophical problem of establishing this a priori intuitive attitude and thereby examining the condition of possibility of conceptual knowledge and at the same time that of experience. I think this is essentially achieved by my studies on the principles *of mathematics*. The a priori is nothing more nor less than a fundamental attitude or the expression of certain indispensable preconditions [der Ausdruck für gewisse unerläßliche Vorbedingungen] for thought and experience. But we must draw differently from Kant the border between, on one hand, that which we possess a priori and, on the other, that for which experience is needed. Kant has overrated the role and scope of the a priori [...] We can say that today science has produced a safer result from the point of view expressed by Gauß and Helmholtz about the empirical nature of geometry […] The Kantian a priori includes anthropomorphic scoria from which we must be freed; once these have been cleared out, all that remains is this a priori attitude, which is also the basis of pure mathematical knowledge: it is the sum and substance of what, in my various writings, I have characterised as finitist attitude" (emphasis added).

This text picks up on the considerations already present in the more technical articles of 1922, 1923, 1927, 1928 and 1930.[22] It shows Hilbert's interpretation of Kant's pure intuition, not

[21] Über die Einführung neuer Funktionen in die Mathematik (The introduction of new functions in mathematics), *Gesammelte mathematische Werke* III, Vieweg & Sohn, Braunschweig, 1932, 428-438, French trans. in *La création des nombres*, Paris, Vrin, 2008, 221-233.

[22] In chronological order: Neubegründung der Mathematik (reprint in *Gesammelte Abhandlungen* III, 155-177), Die logischen Grundlagen der Mathematik (*Gesammelte Abhandlungen* III, 178-191), Die Grundlagen der Mathematik (*Abhandlungen aus dem mathematischen Seminar der Universität Hamburg*, 6, 65-85), Probleme der Grundlegung der Mathematik (*Mathematische Annalen* 102, 1-9), Die Grundlegung der elementaren Zahlenlehre (*Math. Ann.* 104, 485-494). Most of those papers are translated into French by par J. Largeault, *Intuitionisme et théorie de la démonstration*, Paris, Vrin, 1992.

present in his previous defence of the axiomatic method, which was rather more a question of "analysing intuition" and gathering facts into a theory. Hilbert now recognises that building axiomatic frameworks rests on "a priori intuitive attitude" as *the* condition of possibility of any conceptual knowledge and simultaneously of all experience. However he believes that a priori is only "a fundamental attitude or the expression of certain indispensable preconditions of thought and experience", and he says later in his presentation that these preconditions may change or prove to be mere prejudice, as demonstrated by the example of the concept of absolute time, defined by Newton as an a priori datum and identical for all observers. The notion of absolute time was accepted "without critique by the philosopher of Critique", quips Hilbert, while it is refuted by Einstein's gravitational theory. For that reason Hilbert agrees now with the empiricist conception of geometry as espoused by Gauss and Helmholtz. The Kantian a priori is still, according to him, encumbered with "anthropomorphic scoria" that must be eliminated to supply an objective version. Out of this is born his finitist conception[23] of the foundations of pure mathematics, which turns a *subjective "view"* into an *objective method* for determining the conditions of acceptability of mathematical proofs. Mathematics is therefore the mediation [Vermittlung] between theory and praxis, between thought and observation. Formalism materialised as symbolic procedures renders superfluous the hypothesis of pure apperception as empty form. It is amusing to note the aid Hilbert finds in the Hegelian term and concept of mediation, which differs from the Kantian "Verbindung" [conjunctio].[24] Evidence, if it were needed, of the pervasiveness of this Hegelian figure of reflection, which will be significant in modern philosophy.

What about the Kantian conjunctio? Conjunctio is "the highest principle in all human knowledge"; it is not given by objects and is only "an accomplishment of the understanding, which is itself nothing more than the power of conjoining a priori and of bringing the variety of given representations under the unity of [pure] apperception".[25] By contrast, Hegel's Vermittlung rejects the dualism of understanding and sensitivity and expresses the reciprocal immanence of thought and being. According to the latter perspective, the laws of thinking, that Hilbert regards as expressed by the rules of his proof theory, would give us the mathematical intelligence of reality in coincidence with reality itself. But Hilbert seems to use 'Vermittlung' by chance, he does not avail himself of Hegel's decisive contribution consisting of internal identity of being and thought. On the contrary, he stands by the externality of the traditional relationship, and even goes back from Kant to Leibniz and Galileo. Indeed, instead of speaking of subjective representation and pure apperception (Kant) he invokes Leibniz' pre-established harmony and confirms Galileo's statement: mathematics is indeed the *expression* of reality. Quite remarkably language rather than consciousness functions as mediation. One cannot reproach a great mathematician for not being a coherent philosopher, let alone for ultimately preferring language, a public and controllable vehicle, to the interiority and opacities of consciousness.

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[23] Finitism consists of establishing propositions involving infinity upon an analysis of finite sequences of formulas constituting proofs of the propositions in question.

[24] According to Kant, "any combination [Verbindung/conjunctio] is either composition [Zusammensetzung/compositio] or connection [Verknüpfung/nexus]". Both are synthesis of the manifold, but only the second sort is "the synthesis of a manifold, in so far as its parts do belong necessarily to each other; for example, the accident to a substance, or the effect to the cause. Consequently it is a synthesis of that which though heterogeneous is represented as connected a priori." *CRP*, Transcendental Analytic, Book II, chapter 2, section 3, B202, Footnote.

[25] *CRP* B135.

So far from being inspired by Hegel, and dismissing Kantian orthodoxy, Hilbert performs a kind of naturalisation of the mind. In effect he brings the a priori back to the linguistic sphere (a little like Quine will later): "thought", he writes, "is parallel to language and writing."[26] This naturalisation stems from his conception of arithmetic signs as constituting, in a material and visible way, the fundamentals of building formal axiomatic systems. Signs do not have a representative function they are themselves the necessary exteriority of thought, its material.[27] An equation such as $f(x) = x^2$ is both the symbolic expression of a curve and a mathematical material (a quadratic equation). Just as f and x here, mathematical signs have no predetermined content, but are the material building blocks, "concrete objects of intuitive experience [Erlebnis] preceding any thought,"[28] which nevertheless "liberate us from the subjectivism already inherent in Kronecker's intuitions and which reached a peak with [Brouwer's] intuitionism." So, Hilbert admits that intuition is rooted in perceiving sensory signs outside of the mind, but he rejects the subjective synthesis of perception. Hilbert explains that the set of formulas attacked by Brouwer for their alleged lack of content, of meaning, is actually the instrument that allows us to express the whole thought content [Gedankeninhalt] of mathematics in a uniform (standard) way so that the interconnections between the formulas of symbolic language and mathematical facts become clear. For Hilbert this ordered set of formulae not only has mathematical value but also an important philosophical meaning/significance because "it is carried out according to certain rules in which the technique of our thought is expressed." His proof theory, he says, has no other purpose than to "describe the activity of our understanding, to draw up a set[29] of rules by which our thinking actually performs".[30] Thought is a mechanism whose elements are sequences of signs connected by deductive links. In 1927 Hilbert therefore replaces lucid thought [mit Bewußtsein denken] by technical thought: formality is not about consciousness but about language and symbol writing. Intuition is not a priori but sensory, it is not subjective but formally, that is expressly, objectified. Symbolic expressions are really an objective starting point and a material support for mathematical practice. There is no need to presuppose a native intuition of number (Brouwer) or a specific faculty of pure understanding accounting for the principle of complete induction (Poincaré's "intuition of pure number"[31]).

Intuitionism denies the autonomy of meaning from its psychological actualisation. In contrast, Hilbert, logical positivism, the Vienna Circle, model theory, and Quine deny the autonomy of meaning from its linguistic expression. Hilbert wants to save the infinite involved in mathematical abstractions through the materially expressed formality of finite sequences of signs. This is why he claims that his "finitism" is the expression of a conception of the a priori that is free from the anthropomorphic aids preserved in Kant's doctrine, and thus offers the basis for *pure* mathematical knowledge. The regulated manipulation of sequences of signs,

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[26] Die Grundlagen der Mathematik, lecture delivered in 1927 and published in 1928 in the *Abh. Math. Sem. Hamburg* 6, p. 79-80. French trans. in J. Largeault, *Intuitionisme et théorie de la démonstration*, Paris, Vrin, Mathesis, 1992, p. 145-163.

[27] Neubegründung, p. 163: "In the beginning is the sign".

[28] Die Grundlagen der Mathematik, p. 65. Hilbert writes 'Erlebnis' which indicates an empirical experience whereas the Kantian 'Erfahrung' is a synthetic unity of sensory perceptions produced by the understanding.

[29] "Protokoll", which literally means the minutes or report of proceedings.

[30] Die Grundlagen der Mathematik, p. 79: "The thrust of my proof theory is nothing but a description of the activity of our understanding, an inventory of rules under which our thought proceeds effectively. Thought is parallel to language and writing…".

[31] Du rôle et de l'intuition de la logique en mathématiques, reprint in *La valeur de la science*, chapter I, Paris, Flammarion, 1905 ; Sur la nature du raisonnement mathématique, reprint in *La science et l'hypothèse*, chapter I, Paris, Flammarion, 1902.

figures or bars (see Hilbert 1905) is the mediation between thinking and observing. It is likely that Hilbert knew that Fries maintained that *Critique of Pure Reason* was a psychological or anthropological attempt to build a base for a priori knowledge.[32] What we can say with more certainty is that by "anthropomorphic scoria" Hilbert is referring to the subjective side of the Kantian reflexive perspective. The literality and formal legality of proof theory are meant to save us from this subjectivity. Thus science is substituting for philosophy of conscience, it says what the a priori consists of. If Hilbert the mathematician entertained a real interest in philosophy it was in order to depose its supremacy rather than to recognise it as the "primary science", that which determines the conditions of intelligibility of objects of any particular science. In this respect, Hilbert certainly played a role in the philosophical rejection of philosophy as primary science and the call often issued by the proponents of historical epistemology to place it within the school of science, i.e. to learn philosophically by the practice of science. Even today proponents of the philosophy of mathematical practice intend to base philosophical insights on mathematical material.[33]

As was his wish, Hilbert's perspective has given much food for philosophical thought. Philosophers have variously used it for flatly rejecting the transcendental (Cavaillès[34]), interpreting it differently from Kant (Husserl, Vuillemin, Granger), or diminishing it on a real and empirical level (Foucault's historical a priori). With his structuralist objectivism, Hilbert's perspective contributed – in cooperation or in conflict with other elements resulting, in particular, from the promotion of philosophy of history accomplished by Hegel and from the development of human sciences (Dilthey's Geisteswissenschaften) – to the abandonment of exclusive attention to the subject (Husserl) or to the preponderant privilege of the object (Cavaillès, linguistic, philosophical structuralism).

In the following section, after a brief presentation of the philosophy of Leonard Nelson, who, as we have seen, sparked some reflections in Hilbert, I will explain more fully that of Edmund Husserl, whose ambitions for philosophy was at least as great as Hilbert's ambition for mathematics and who maintained a deeper relationship with Kantian philosophy. In the third part I will turn to Jules Vuillemin who wrote *La Philosophie de l'Algèbre*, as a dialogue with Leibniz, Kant, and Husserl in the light of modern mathematics.

3. *Critical mathematics of Gerhard Hessenberg and Leonard Nelson*

Hilbert believed that the foundation of mathematics on the axiomatic method and proof theory was of great interest to philosophy. And indeed, from the philosopher's point of view the idea of considering the axiomatic approach as a critical step in the development of mathematics was very attractive, since it allowed bridges to be built between philosophy and mathematics, bridges which would stretch over and ignore the speculative Spirit of Hegel, the dialectical history of philosophy redistributed among successive figures of growing rationality, and the conquering empiricism of experimental sciences and humanities (psychophysics, statistical sociology, etc.).

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[32] Jakob Friedrich Fries, *Neue oder anthropologische Kritik der Vernunft,* 3 Band, Heidelberg, Christian Friedrich Winter, 1828-1831.

[33] CF. for example P. Mancosu (ed.), *The Philosophy of Mathematical Practice*, Oxford University Press, 2008, and P. Mancosu, *Infini, logique, géométrie*, Paris, Vrin, 2015, third part.

[34] See my work, in particular *Jean Cavaillès. Philosophie mathématique*, Paris, PUF, 1994 and *Cavaillès*, Paris, Les Belles Lettres, 2013.

In fact, "critical mathematics"[35] was a rallying point for mathematicians and philosophers around Hilbert. In 1904 the mathematician Gerhard Hessenberg published a short article, Über die kritische Mathematik[36], whose theme is more widely developed in "scientific philosophy" [wissenschaftliche Philosophie] by the philosopher Leonard Nelson.

Hilbert had initiated an exchange with Edmund Husserl who taught at Göttingen from 1901 to 1916. Husserl had already published *Philosophie der Arithmetik* (1891) and *Logische Untersuchungen* (1900-1901), which showed his strong interest in the relationship between mathematics and psychology versus logic, a relationship that was precisely the purpose of Hilbert's technical work. Husserl and Hilbert developed a close and intellectual friendship, and the latter put all his weight behind support for Husserl against the hostility of the university professors of psychology and history and for his continued employment in Göttingen, in the hope of seeing him occupy a chair of "systematic philosophy of the exact sciences".[37] The phenomenologist might not have endorsed the perspective to commit to the programme, which was not his own. Testament to this are his lectures and publications during his years in Göttingen, which were far from being appropriate to Hilbert's positivistic and empiricist views: *Die Idee der Phänomenologie. Fünf Vorlesungen* (1907), *Philosophie als strenge Wissenschaft* (1911), and *Ideen zu einer reinen Phänomenologie und phänomenologische Philosophie I Allgemeine Einführung in die reine Phänomenologie* (1913). Indeed the first and third works mark a turning point of phenomenology towards transcendental idealism while the second fiercely criticises scientific positivism, which is non-philosophical by nature. After Husserl's departure to Freiburg in 1916, Hilbert pursued his goal by supporting Nelson who, after many obstacles, was appointed professor in 1919 and put in charge of imparting lessons in the aforementioned "systematic philosophy of the exact sciences" programme.[38] In 1917 Hilbert joined the Neue Fries'sche Schule founded by Nelson in 1903. The school was a focal point for exchanges between scientists and philosophers in Göttingen: among its members there were Kurt Grelling, Richard Courant, Max Born and Paul Bernays, editor of the complete works of Nelson in 9 volumes.

With the support of Gerhard Hessenberg Nelson created a journal titled *Abhandlungen der Fries'schen Schule. Neue Folge*. In the first volume Nelson defines the project to develop a philosophy "whose method is as rigorously scientific as the method of mathematics and natural sciences".[39] He argues against Hegelian "scholasticism" and its powerful influence, against historicism, against the Platonism of Schelling, and against empiricism that, he considers, was refuted definitively by Kant. The axiomatic approach is the paragon of rigor to follow.

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[35] See K. Herzog, *"Kritische Mathematik"— ihre Ursprünge und moderne Fortbildung*, Dissertation (1978) Düsseldorf.

[36] *Sitzungsberichte der Berliner mathematischen Gesellschaft* 3, 21-28.

[37] A detailed historical study, exploring many unpublished documents can be found in the excellent book by Volker Peckhaus, cited in note 6.

[38] Hilbert wrote: "Among the philosophers who are not primarily historians and experimental psychologists, Husserl and Nelson are the most remarkable personalities, it seems, and for me it is not a coincidence that these two found themselves on the mathematical ground of Göttingen. […] Neither is it a coincidence that I speak out on this subject. Without me, Husserl would have been caught earlier, without me Nelson would never have been seen here in Göttingen. Göttingen is predestined for this task - a huge cultural task." (cited by Peckhaus 1990, p. 223).

[39] Die kritische Methode und das Verhältnis der Psychologie zur Philosophie. Ein Kapitel aus der Methodenlehre, *Abhandlungen der Fries'schen Schule*, *Neue Folge* 1 (1906), p. 1–88. On line: http://archive.org/stream/abhandlungenderf01gtuoft/abhandlungenderf01gtuoft_djvu.txt

But Nelson aligned himself equally with Fries, who was a philosophical adversary of Hegel. Nelson attributes to Fries the first transfer of *Critique* to mathematics and the constitution of the "philosophy of mathematics" as an autonomous discipline in *Die mathematische Naturphilosophie*, published in 1822. Like Fries, Nelson understands mathematics as a set of *a priori synthetic propositions* and wishes to provide a critical foundation for mathematical axioms by subjecting them to transcendental deduction in the manner of Kant as revisited by Fries. Absent from Hilbert's terminology, 'consciousness', 'a priori synthetic judgements' and 'transcendentalism' come into play. Nelson distinguishes between demonstration and deduction: the former works in mathematics and science by showing the intuitions that underlie our basic judgements, the latter is the foundation, through Fries' reprise of Kant's "regressive method", of fundamental metaphysical judgements and applies equally to mathematics.[40] For Nelson deduction is the most important task of critique. Indeed he writes:

> "Unlike demonstrable judgements, judgements that are only deductible are not based on intuition, that is, the immediate knowledge on which they rest is not immediate to us, but mediated by reflection, led by judgements to consciousness."[41]

And a little further on:

> "A critical deduction of mathematical axioms must also be possible. This transfer of *Critique* to the axiomatic system represents a scientific discipline it its own right: the philosophy of mathematics or, by better description, critical mathematics."

For Nelson the axiomatic is but the first task of Critique; it is logical in nature. The second, real task falls within the theory of knowledge and involves the question of "origin and validity of axioms", a task whose total absence in Hilbert's work he deplores. We have seen, however, that Hilbert, probably prompted at least partly by the reproach that he could not ignore, turned from 1917 to questions of the theory of knowledge which he understood and resolved by way of the mathematical filter. For Nelson, on the contrary, it is a metaphysical deduction of axioms that achieves the aim of Hilbert's finitist programme.

No doubt it was by way of obstruction of this view and reaffirmation of his own understanding that Hilbert, who ignored a priori synthetic judgements and transcendental vs. metaphysical deduction, returned, in his 1930 article, to the problem of the relationship between thought and experience, *in order to assign to mathematics itself* the power to link thought and experience. Finitism is presented as intuitive a priori attitude: so a *method* of proof plays the role of Kant's *pure consciousness or apperception* which is the a priori form of any synthetic representation of objects. In other words, Hilbert maintains and reiterates the view which was at the heart of his proof theory: mathematical treatment of epistemological issues and not, like Nelson, use of axiomatic method as a model to build a metaphysical theory of knowledge. Hilbert's programme for cooperation between mathematics and

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[40] Hilbert picked up on Nelson's distinction between progressive and regressive methods in his lecture "Die Rolle der Voraussetzungen", in *Natur und mathematisches Erkennen* (David Rowe ed.), Birkhäuser, 1992, p. 17-18. But for him the regressive method finds its most perfect expression in the axiomatic method. This means that for him transcendental deduction is advantageously replaced by mathematical proof.

[41] "Die nur deducierbaren Urteile aber haben ihren Grund nicht, wie die demonstrierbaren, in der Anschauung; d.h. die ihnen zugrunde liegende unmittelbare Erkenntnis kommt uns nicht unmittelbar, sondern nur durch *Vermittlung* der Reflexion, nur durch das Urteil zum Bewusstsein." (Die kritische Methode). I highlight the word 'Vermittlung', a Hegelian term which encroached on the author's Critique. As I have highlighted above, Hilbert also uses the term 'Vermittlung'. We will find further on, with Husserl and Vuillemin, more traces of the resonance of Hegelian thought on those very people who reject it outright.

philosophy did not lead in Nelson's work - nor in Husserl's, as we will now see - to a resorption of philosophy within mathematics through a complete objectification of thought by means of symbolic processing.

II. Philosophy as rigorous science: Husserl

Husserl held in high esteem the work of Hilbert, at whose request he had made two presentations in 1901 to the Mathematical Society of Göttingen. In contrast he found Nelson devoid of "scientific seriousness" and disliked his programme of "Systemphilosophie" and his metaphysical deduction. Husserl intended to proceed "from below" and to develop a purely phenomenological method.[42] In fact, the two philosophers held antagonistic positions on the place of metaphysics in philosophy and the founding status of intellectual intuition. In 1908 Nelson had severely criticised categorial intuition, as it had appeared in *Logical Investigations*, and the descriptive (i.e. non-critical) nature of Husserl's analyses.[43] This played a part in the evolution of descriptive phenomenology into transcendental phenomenology, and Husserl's programme of "Philosophy as rigorous science"[44] which he conceived as a definitive eradication of the metaphysics of the things-in-themselves.

> "We must eliminate any 'metaphysical' thing-in-itself and with it any Spinozist ontological metaphysics, which derives a *science of being* from a system of *pure concepts*".[45]

1. *Need for critique*.

Philosophy as rigorous science is a science, a "philosophical science" [philosophische Wissenschaft], very different from Nelson's scientific philosophy [wissenschaftliche Philosophie]. The reversal of the terms is not without significance, although Husserl also sometimes uses the term 'wissenschaftliche Philosophie', for example when he describes and critiques the worldview [Weltanschauung] philosophy. Husserl, like Nelson, considers as decisive both the critical revolution brought about by Kant, as well as the axiomatic turn of mathematics initiated by Hilbert - we will see how Vuillemin also maintained this dual allegiance. But while they share a vindication of critical attitude and an appropriation of the axiomatic method, Husserl and Nelson proceed differently.

Husserl takes the critique of reason as the primary condition of philosophical scientificity. In 1903 in Göttingen, Husserl devoted a few lessons of a course on the history of philosophy to Kant. And in 1906 he wrote in his *Persönliche Aufzeichnungen*[46]:

> "The general problem I have to solve for myself if I want to call myself a philosopher.

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[42] I refer to the Correspondence Nelson-Hessenberg cited by Peckhaus 1990, p. 203-204. Cf. also P. A. Varga, Ein bisher unbekanntes Portrait von Edmund Husserl, http://hiw.kuleuven.be/hua/Media/mitteilungsblatt/portrait, 2010, revised in Sept. 2013.

[43] See Nelson, *Geschichte und Kritik der Erkenntnistheorie*, Part 2, Section XII entitled "Husserls phenomenological Methode und die intellektuale Anschauung", *Complete Works*, Volume II, p. 171-177.

[44] Philosophie als strenge Wissenschaft (PSW), *Logos - Zeitschrift für Philosophie und Kultur*, Bd. 1, Tübingen 1910/11. Online http://www.gleichsatz.de/b-u-t/archiv/phenomeno/husserl_streng1.html.
English trans. M. Brainard: *The New Yearbook for Phenomenology and Phenomenological Philosophy* II (2002): 249–95.Online:https://fr.scribd.com/doc/63651073/Husserl-Philosophy-as-a-Rigorous-Science-New-Translation.
French trans. Q. Lauer, Paris, Presses Universitaires de France, 1955.

[45] Letter from Husserl to Dilthey cited by Q. Lauer, p. 175 of his French translation (emphasis added).

[46] Published by W. Biemel in *Philosophy and Phenomenological Research* XVI, 1956, p. 297.

I mean a *Critique of reason"*.

Husserl believes that this prerequisite has been present in philosophy from Plato to Kant and Fichte while it is absent from the Hegelian system. – And yet Hegel had conceived his system as *the* philosophical science totalling all knowledge. – It is therefore Husserl's ambition to construct a philosophical science that combines the rigour and autonomy of exact science with the critical dimension of Kant's philosophy. While rejecting the traditional idea of *philosophical system* [Systemphilosophie][47], "springing like Minerva fully armed from the head of a creative genius so as in later times to be preserved in the silent museum of history"[48], Husserl is searching for a *systematic philosophy* endowed with the "seriousness of science". The "true path to a scientific theory of reason"[49] will lead to the "primary science", which Kant could not stumble upon, Husserl thinks, for lack of pushing his analysis of pure intuition far enough. The steps along the path to the primary science: epoché, reduction, eidetic variation should be *systematically* followed to answer the fundamental question, which is no longer the Kantian question of knowing *what* the conditions of possibility of experience and of knowledge of the objects of experience are, but that of knowing *how*, in concrete terms, these conditions give the conscious mind access to objectively valid concepts, how the a priori *encounters* experience, how it details and organises *structurally* this encounter. The question is the following:

> "How are we to understand the fact that the "in itself" of objectivity reaches 'representation' and even 'apprehension' in knowledge, and yet eventually becomes subjective again? What does it mean to say that the object has 'being-in-itself' and is 'given' in knowledge?" [50]

In the name of serious science Husserl tackles historicism and naturalism, which were then also in the sights of philosophers working in Göttingen in contact with Hilbert and willing to take into account new developments in mathematics, logic and physics.[51] Mathematicians and physicists reciprocally supported Husserl and Nelson against the faculties of philology, history and psychology of their university. These historical data confirm the understanding of Hilbert's theory of mathematical signs as being a linguistic objectivism rather than a psychological or cognitive naturalism.

More than the Hegelian system itself, Husserl fights the avatars of Hegel's philosophy of history, that is, the worldview philosophies, which propagate a sceptical and relativistic historicism, though contrary to the absolute Spirit of Hegel. He holds post-Hegelian historicism responsible for the weakening, indeed perversion, of the "drive towards the establishment of a rigorous philosophical science". The criticism mainly concerns the concept of human sciences [Geisteswissenschaften] introduced by Dilthey, with whom Husserl was connected in Göttingen. Husserl recognises that Dilthey rejects historical scepticism but he

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[47] "A truly living and truly scientific philosophy can not be possible but as free philosophy, as a philosophy that questions not philosophers and their systems, but things themselves," (Hua XXVII, 198-199).

[48] The allusion to Hegel is transparent. Paradoxically, according to Husserl, building a philosophical system concurs with the "worldview philosophies" he intends to replace because they amount to subordinating thought to a particular view of the whole.

[49] *PSW*, English trans. p. 278.

[50] *Logical Investigations*, Volume II, Part 1, Investigations into Phenomenology and theory of knowledge, Introduction, English trans. J. N. Findlay, Routledge & Kegan Paul Ltd. Reprint 2001, p. 169.

[51] In 1916, Nelson wrote: "philosophy is in deep distress [...] since experimental psychologists on the one hand, and historians on the other, dispute their place ever more successfully and have begun to share the inheritance", cited by Peckhaus, p. 219.

does not understand how one could derive decisive reasons against scepticism from Dilthey's rich analyses of the structure and typology of "worldviews".

Psychologism and positivism, which were then asserting themselves in response to Hegelianism, were even more responsible for weakening the drive for scientific philosophy in favour of sceptical relativism and naturalism. Husserl has in his sights the natural science of consciousness and ideas, that is to say experimental psychology and psychophysics which were thriving tanks to Fechner, Wundt, von Helmholtz and others.[52] In fact Husserl was reacting against his own earlier leaning. Nonetheless Husserl retains from experimental psychology the postulate of the primacy of perception and the idea of the inseparable connection between mind and body.

2. *Radicalisation of critique: from scientific explanation to philosophical understanding*

The second condition of scientificity is the "radicality" of critique. By 'radical' Husserl refers to the very first origin. The question of origin has a different meaning for Husserl than for Nelson. It is not a question of metaphysical deduction, but a kind of Cartesian undertaking: not accepting anything as pre-given, not taking anything bequeathed as beginning, not being blinded by any great figure from philosophy or science, starting from the problems themselves and their demands. As in science[53] we will begin by precisely formulating the problems and methods. But we must go beyond scientific positivity, beyond the positive character of the experimental sciences, natural sciences and sciences of the mind [Geisteswissenschaften], beyond mathematics and even beyond the available corpus of logic. We cite as examples extracts from two different texts.

> "It is true that natural science is, *in its own way*, very critical. The experience of a mere individual, even when multiplied, still has very little value to it. It is in the methodical arrangement and combination of experiences, in the interplay of experience and thought which has its logically firm rules, that valid [gültig] and invalid experiences are distinguished, that each experience gains its degree of validity and that, from there, objectively valid knowledge (knowledge of nature) is worked out. Yet however much this type of critical experience may satisfy us as long as we stand *in* natural science and share its way of thinking, a totally different critique of the experience is still possible and necessary, a critique that simultaneously calls into question the whole experience as such and the way of thinking proper to experiential science".[54]

> "Logic must be more than simply a positive science of logico-mathematical idealities" […]. Transcendental logic must understand "*how the ideal objectualities* [ideale Gegenständlichkeiten] that originate purely in our subjective activities of judgement and knowledge, that are present *originaliter* in our field of consciousness

[52] Fechner (1801-1887) and von Helmholtz (1821-1894) were lecturers at Göttingen, later followed by Georg Elias Müller (1850-1934), who succeeded Lotze in the chair of philosophy and in 1887 founded an Institute of Psychology (in the image of the one founded by Wundt in Leipzig in 1879) and in 1904 the Experimental Psychology Society. Müller was a bitter opponent of Husserl and Nelson. Husserl attended Wundt's philosophical lectures in Leipzig in the late 1870s and wrote under Carl Stumpf's supervision Über den Begriff der Zahl (Halle, 1886), which is the basis of *Philosophie der Arithmetik*.

[53] "Starting from the problems themselves" was a leitmotiv of Felix Klein and Richard Dedekind. Hilbert popularised the theme in his Parisian 1900 lecture.

[54] *PSW*, English trans. slightly modified (Husserl's emphasis).

> purely as formations of our spontaneity, *acquire the being-sense of "objects"*, existing in themselves, being not contingent on actions and subjects. How does this sense "come about", how does it originate in us? And where could we get it if not from our own sense-constituting power? Can what makes sense for us receive sense ultimately from somewhere other than ourselves? This question, once posed about one sort of object, becomes immediately general: is not each and every objectivity, in every sense which is valid for us, an objectivity that is winning or has won validity in us with the sense that we ourselves have acquired for it? Accordingly, the *transcendental problem* that the *objective logic* […] has to pose with respect to its field of ideal objectualities runs *in parallel with the transcendental problems of the sciences of reality,* namely the problems that must be raised with reference to the regions of reality to which those sciences pertain, therefore in particular with the transcendental problems concerning Nature, which were treated by Hume and by Kant.*"*[55]

The latter text shows that Husserl aims to supply positivist scientific *explanations* with a philosophical framework for the *elucidation* of the being-sense of the phenomena that science explains. Husserl is integrating in his own way the paradigm of understanding that Dilthey opposes with the explanation. Moreover he gives a transcendental status to the sense component of objects and simultaneously does not include the hermeneutical dimension that will predominate among authors such as Hans-Georg Gadamer and Paul Ricoeur.

Husserl reproaches Kant for having kept the natural sciences as a model of rigorous science and of having removed logic from his transcendental enterprise. Is he aware of the semi-similarity between his reproach and Hegel's critique of scientific positivity?[56] Indeed, like Hegel, Husserl does not intend to subordinate philosophical thought to the "dry" abstraction of scientific truths, but, unlike Hegel, he makes the utmost of the objectivity of constituted science while assigning to philosophy the task of questioning the meaning. For contrary to the wisdom of worldviews, science, or more accurately the *Idea of science*, is not bounded by the spirit of a time [Zeitgeist], it is marked with the seal of eternity: "worldviews may conflict, science alone can decide and its decision bears the seal of eternity."

Philosophy as rigorous science will even have to abandon opinions and points of view and strive for perfect objectivity which is necessary and universal to all human beings at all times. Husserl values science, not completed science but the scientific spirit, while he rejects scientism which limits the Idea of science to pre-known realisations. Phenomenology will be

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[55] *Formale und Transzendentale Logik*, § 100 : "Denn nun stand man vor der Unverständlichkeit, wie *ideale Gegenständlichkeiten*, die rein in unseren subjektiven Urteils- und Erkenntnistätigkeiten entspringen, rein als Gebilde unserer Spontaneität in unserem Bewußtseinsfeld originaliter da sind, *den Seinssinn von „Objekten" gewinnen*, an sich seiend gegenüber der Zufälligkeit der Akte und Subjekte. Wie „macht" sich, wie entspringt dieser Sinn in uns selbst, und woher sollen wir ihn anders haben als aus unserer eigenen Sinn-konstituierenden Leistung; kann, was für uns Sinn hat, letztlich anders woher Sinn selbst haben als aus uns selbst? Diese Frage, einmal an einer Art von Objekten gesehen, wird sofort zur allgemeinen: ist nicht alle und jede Objektivität, mit allem Sinn, in dem sie uns je gilt, in uns selbst zur Geltung kommende oder gekommene, und das mit dem Sinn, den wir uns selbst erworben haben?
Danach tritt das transzendentale Problem, das die objektive Logik in welch enger oder weiter Fassung immer in Bezug auf ihr Feld idealer Gegenständlichkeiten zu stellen hat, in Parallele zu den transzendentalen Problemen der Realitätenwissenschaften, nämlich den in Bezug auf ihre Regionen der Realitäten zu stellenden, also insbesondere den von Hume und Kant behandelten transzendentalen Problemen der Natur" (Husserl's emphasis), Halle, Max Niemeyer Verlag, 1929.

[56] See in particular *Phenomenology of Mind* on the abstract, ineffective, fixed and non-living nature of mathematical truths, French trans. Bernard Bourgeois, Paris, Vrin, 2006, p. 86-89.

a new type of science and transcendental analysis will not be limited to determining the conditions of possibility of positive science.[57]

Philosophical science cannot stop at the causal explanation of phenomena. Husserl deplores that naturalism is transmitted from the natural sciences to the sciences of the mind, to psychophysics and experimental psychology in particular, whose motivation is the naturalisation of consciousness, ideas and reason. Naturalists certainly challenge post-Hegelian historicism but they also fight for a worldview philosophy[58] and share the same "superstition of the fact". They interpret Ideas as facts and transform all reality "into an incomprehensible mix of 'facts' from which any idea is absent." Factualism is thus the main obstacle to a more comprehensive critique of reason. Yet, it goes without saying that *empirical science* ["Tatsachenwissenschaften"] cannot provide the foundations for the *normative principles* specific to theoretical disciplines such as logic, epistemology and axiology. Natural sciences and sciences of the mind have the pre-Kantian naivety to accept the given as given, as self-evident, and have no other goal than transforming this given into an object of knowledge. However, upstream of knowledge, we must question *how* the object is given and elucidate the essential content [Wesensgehalt]. For objective knowledge of the given is not sufficient, one must grasp *the sense/meaning*[59] which emerges in it. Scientific knowledge represents the highest degree of objectivity, but it is about asking what 'objective' means. "To speak as Lotze", wrote Husserl, "*calculating* the trajectory of the world does not mean *understanding* it".[60]

What did Husserl think of Hilbert's presentation of his Beweistheorie as a positive, objective inventory of the technical rules expressing the laws of thought? One can easily guess, because for Husserl scientific objectivism equals philosophical naivety. Husserl executes a radical disconnection between scientific philosophy and positive science. One has the tendency, he observes, to conceive as rigorous science only positive sciences, and as rigorous philosophy only that based on such a science. But it is a prejudice that diverts philosophy away from its scientific future. It is not about emulating either the natural sciences or mathematics or positive logic. It is about developing an autonomous philosophical science that brings its own set of problems, methods and theories. Rigour and scientific objectivity can and should be conceived more broadly than simply in a positive way. At any rate it is the task of scientific philosophy to implement this new meaning. The concepts of science and rigour must be extended to hold not only for the enterprise of objectification of phenomena, which was as much the goal of Hilbert as of the physical and empirical sciences, but also for the genetic

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[57] I cannot go further into the characterisation of phenomenology here. Jocelyn Benoist, for example, explains that the freeing of the meaning of 'consciousness' (as the pure sphere of appearance) involves the suspension of the very purpose of any science… namely the world. Science is more interested in the a priori of actuality, and remains guided by this actuality of the world. Phenomenological reduction serves to subvert this attitude by the very questioning of the direct or indirect reference of science to the world (this 'reference' gives science its scientific meaning)." *Autour de Husserl*, Paris, Vrin, 1994, p. 175. Epoché is intended to put the world in brackets so as to reveal what knowledge, as such, means.

[58] *PSW*: "The advent of the new *worldview philosophy* [*Weltanschauungsphilosophie*] is essentially determined by the transformation of Hegel's metaphysical philosophy of history into sceptical historicism. Worldview philosophy appears to be spreading rapidly nowadays, and, incidentally, even with its mostly anti-naturalist and sometimes even anti-historical polemics, it wants nothing less than to be sceptical". English trans. modified (Husserl's emphasis). For the critique of Weltanschauungen see also *Logical Investigations*, I.

[59] Husserl uses 'Sinn' and 'Bedeutung' as synonymous. For a detailed account see D. Pradelle, On the notion of Sense in Phenomenology: Noematic Sense and Ideal Meaning, *Research in Phenomenology* 46, 2016, 184-204.

[60] Emphasis added. Husserl nevertheless makes a side note about the determination by Lotze of the task of philosophy: "it is essentially a worldview philosophy ". Cited in Lauer's French trans, p. 179.

makeup of the meaning of phenomena. Without having to preserve "the apparatus of conclusions and proof" of the experimental or deductive disciplines and "without all indirect methods of symbolisation and mathematisation" so advocated by Hilbert, Phenomenology must assume complete responsibility for the demands of scientific rigour specific to philosophy and leave no room for opinions or points of view, be they as they might defended by geniuses.

> "It lies precisely in the essence of philosophy, insofar as it returns to the ultimate origins, that its scientific work moves in spheres of direct intuition, and it is the greatest step our age has to make to see that with philosophical intuition in the right sense, the phenomenological *seizing upon essences* opens up an endless field of work and a science which, without any indirect method of symbolisation and mathematisation, without the devices of deductions and demonstrations, nevertheless obtains an abundance of the most rigorous and decisive cognitions for *all* future philosophy. "[61]

He further states:

> "I hope nevertheless to have shown that this is not the old rationalism, which was absurd and in general unable to grasp the problems of the mind concerning us the most, and which can be renewed here. The *ratio* which is now in question is none other than the truly radical and truly universal understanding of the mind by itself, as a responsible, universal science by which *an entirely new mode of scientificity* is undertaken, where all conceivable questions, questions of being and questions of the norm, questions of so-called existence, find their place." (Emphasis added)

Thus, distancing himself from classical rationalism, where the philosopher insists on taking the mathematical or physical sciences as a model and ideal, and criticising empiricism which believes the answer to questions of law raised by the facts of experience can be found *in* the experience of facts, Husserl wishes to promote a philosophy whose rigour is of a different kind than that of the positive sciences. Against the positivity of the constituted theoretical sciences, against the platitude (factuality) of empirical science ["Tatsachenwissenschaften"] which cannot found logical, epistemological or ethical *norms*, against the depth of Romanticism which loses the sense of conceptual clarity and exactness in favour of the subjective and also secretes scepticism and relativism, Husserl argues for the purpose of a philosophy dedicated to the search for an unconditioned truth through the radical critique of the presuppositions not of this or that science (arithmetic, geometry, the theory of electromagnetism, etc.) but of the enterprise of scientific objectification itself. We must systematically challenge every fact, including the scientific fact (results, theories and methods). For the problems of being and value can neither be treated according to a positive logic nor be dissolved in the perspectivism of worldviews. Philosophy has no other given than that which it constitutes itself. It follows, in particular, that the questions of the theory of mathematical knowledge and the mathematical solution given to them by Hilbert, the scientific value of which is undeniable, fall short of or outside the proper field of philosophy and cannot claim to invalidate the specifically philosophical questioning about them. Science cannot be a substitute for philosophy. Whatever the strength of scientific data, they cannot serve as building blocks to construct a rigorous philosophy. The axiomatic method, if

[61] *PSW*: last sentence (Husserl's emphasis). Husserl further asserts in Krisis that "transcendental phenomenology overcomes naturalistic objectivism and all objectivism in general." (French. trans. N. Depraz, p. 112, online: http://www.ac-grenoble.fr/PhiloSophie/wp-content/uploads/ebooks/husserl_depraz.pdf.)

paradigmatically useful for analysing what a "unit of meaning" is in contrast to a unit of something[62], does not, however, escape the need for a phenomenological reevaluation. And Beweistheorie, by the admission of its author, is the formal expression of the *techniques of thought*, and not a clarification of what thought is in its essence. "As instigating philosopher", wrote Husserl in *Cartesian Meditations*, "we do not ascribe value to any normative ideal of science; and we cannot have a normative ideal to the extent we create it ourselves." Husserl insists that mathematical standards are to be found in mathematics, logical standards in logic, ethical standards in ethics, etc. The autonomy of science as science is not in question. The purpose is not to subordinate science to philosophy, but to make philosophy an autonomous science.

3. *The essence of consciousness*

Philosophy is not *in principle* a "theory of science" [Wissenschaftslehre], whose purpose would be to build the foundations of science, that is to determine the principles from which one can derive valid scientific propositions or explain phenomena, a goal shared by Hilbert and Nelson who pursued it in two different ways. Philosophy has as subject the "fundamental sphere", brought about by the problem of the foundation of science but treated as such neither by scientists nor by previous philosophers; it is the sphere of the primordial givenness of the object, be the object real (physical or mental) or ideal like scientific objects. This is a long way from Hilbert's programme of an axiomatic or metamathematical foundation and far, also, from Nelson's metaphysical deduction of mathematical principles.[63] Husserl aims to discover and reveal in its very origin [Ursprung] the processes which beginning in nature engenders the theoretical interest to which the unfinished building of science responds. One must clarify "the origin of all our formal-logical and natural-logical principles and any other normative principles, and all issues intrinsically related to the correlation of the being (natural being, being of value, etc.) and consciousness."[64] Philosophical science or phenomenology views origin in a different way than in the sense of regressing to non-conditioned principles or of describing the course of a series of historical facts. It is meant neither as metaphysical deduction nor as actual history, but as a genetic constitution of meaning[65]. It is concerned with this radically originary sphere in which all objectification and all scientific practice are rooted, the sphere of "things themselves" – as totally opposed to the unknowable things-in-themselves. Things themselves are the phenomenal foundations upon which every scientific and cultural superstructure is constructed. They are the *pure*[66] experiences of consciousness "in which the appearance of the object resides", and from where arises the dual demand for an

[62] "Every physical thing has *its* nature - as quintessence [Inbegriff] of *what it is*: identity - by dint of being the meeting point between causal series inside a total nature and one," *PSW*, English trans. p. 268 modified slightly. In *Logical Investigations*, Husserl shows that the unity of meaning is ideal and, as such, the prototype for all "ideal objects" of science, particularly mathematical ones, for which Hilbert, following Leibniz and many others, tried to set down the rules.

[63] It is also far from what we call today "metaphysics of science", or even "meta-science", understood as the fundamental analysis of concepts structuring scientific research, for example the analysis of concepts of cause, individual, disposition, etc. Thus the 5th Congress of the Philosophy of Science Society was held in June 2014 at the University of Lille 3, with the theme Meta-physics in Science and the primary objective of "considering ontological problems arising from science and, more specifically, from their discoveries, concepts, models and theories. From there, Metaphysics in Science aims to reconstruct a "scientific picture of the world." The project is one of metaphysical naturalism, revealing the ontology that results from scientific theories and practices. More broadly, metaphysics of science is interested in "any study of the links between science and metaphysics."

[64] *Philosophie als strenge Wissenschaft*, English trans. p. 275 (modified).

[65] Husserl's genetic constitution is not creation but elucidation.

[66] This does not refer to empirical experiences of psychology but to theoretical experiences of phenomenology.

originary relationship to the pre-scientific world and a reflexive, rational understanding of this relationship. Research must therefore be directed at determining scientifically not the *fact* of consciousness, which is the concern of experimental psychology, but the *essence* of consciousness, that is the objective structures of the givenness of the object to consciousness. Those structures are, shall we say, independent of the hypothesis of the thing-in-itself and consequently render it useless. It is a matter of studying structures, as with the axiomatic method, but applied to consciousness and here the reflexive method consists not of the axiomatic construction of philosophical concepts, but of asking *how* data enter consciousness. The answer should surpass both subjective factuality and objective factuality.

> "Inquiry must be aimed at a scientific eidetic knowledge of consciousness, at what consciousness *itself*, by its *essence*, "is" in all its distinguishable formations, but at the same time at what it "signifies' [bedeutet], as well as at the different ways in which it—in accordance with the essence of these formations (now clearly, now unclearly, now presentiatingly or representiatingly, now signitively or pictorially, now simply, now mediated by thought, now in this or that attentional mode, and so on in innumerable other forms)—intends *something objectual* [Gegenständliches] and perhaps "shows" [erweist] it to be a "valid," "actual" being [Seiendes]. [67]

The essence of consciousness consists of intentionality. To analyse the essence of consciousness is to discover the intentional relationship to every kind of objectualities [Gegenständlichkeiten] and with respect to every modality of this relationship. As mentioned in the above passage, essence therefore consists of a family of distinct binary relations, mathematically understood as correspondence between a source domain: the consciousness modalities and a target domain: the set of all possible objectualities. Hence there is nothing empirical about this analysis, neither in its object – *pure* experiences of consciousness – nor in its approach, which is mathematical. Husserl makes a remarkable philosophical interpretation of the mathematical concept of relation, whose particular case is the concept of function[68]. The broad definition we know today of a function as an *arbitrary correlation* between two numbers was established by Johan Peter Gustav Lejeune-Dirichlet (1805-1859) [69] and broadened to include elements of any sets by Richard Dedekind, who between 1863 and 1894 published four editions of Dirichlet's *Lectures* from the winter of 1856-57 at Göttingen University. It is more than likely that Husserl came across this famous work during the mathematical stage of his education. Husserl likewise endowed the concept of the invariant with a philosophical destiny. To use his language, one might say some kind of invariance characterises any mathematical ideality, whenever and wherever we might place it historically and geographically. In the 19th century, the concept of invariant was at the heart of an important trend in research, notably illustrated by Hilbert's theory of algebraic invariants. In 1888, while a young lecturer at Königsberg (and while Husserl, assistant lecturer at Halle, was working on his *Philosophy of Arithmetic*, published in 1891), Hilbert proved the existence of a finite basis for generating invariants of an algebraic form of any number *n* of variables. This theorem, already remarkable for its generality, reverberated like a clap of thunder in the mathematical sky. Hilbert posed the problem in Kantian terms, wondering,

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[67] *PSW*, English trans. p. 259 (Husserl's emphasis).

[68] In a binary relation an element of the source set (the domain) may be associated with several elements in the target domain (the range). A function is a binary relation such that every element in the domain is related to *only one* element in the range.

[69] Leonard Nelson was the great grandson of J.P.G. Lejeune-Dirichlet, who succeeded to Gauß' chair at Göttingen on his death in 1855.

given an infinite system of f_i forms of a finite number of variables x_1, x_2, …, x_n, under what conditions a finite set of forms exists in terms of which all others are expressed as linear combinations whose coefficients are rational functions of the same variables as the starting forms. He actually proved that a finite base *necessarily* exists as suggesting otherwise would lead to contradiction. Hilbert did not present an effective basis nor indicated a way of calculating one. This was a famous launch pad for lively and interminable mathematical and philosophical controversies about the validity of reductio ad absurdum and about what 'existence' means in mathematics. Kronecker, the master mathematician of Berlin decreed that there is no existence without construction. To confirm the validity of its non-constructive approach, Hilbert drew on Kronecker's theory of algebraic numbers to produce, in 1892, an algorithmic proof of this theorem. Presumably, Husserl ignored neither these discussions nor the theory of invariants from which they originated. Furthermore, as a reader and supporter of Bolzano, he knew the variation technique Bolzano applied to non-logical parts of a proposition to test its logical validity.

The concepts of relation/function and invariant very likely sustained, if not inspired Husserl's willingness to turn his back on the notion of substance, dominant from Aristotle to Descartes and Hegel, and to promote the Platonic notion of essence as disconnected from that of substance. Husserl resumes the dialogue with Kant who banished metaphysics of substance from the domain of human reflection[70] in favour of a transcendental subject, and he radicalises this banishment. For one does not speak of the essence of substance but of the essence of the phenomenon itself, and essence is not what hides behind the phenomenon (since there is nothing behind the phenomenon), but it is *the invariant of the modes in which the phenomenon enters consciousness*. The experimental method is, of course, essential, recognises Husserl, when it comes to establishing factual connections, but it presupposes what no experiment could achieve, the analysis of consciousness itself. Only phenomenology can reach the ultimate components of acts of consciousness. For the "answers to the questions of knowing how givens achieve objective ascertainment, what 'objectivity' and 'ascertainment of objectivity' mean depend on the meaning of the given itself, that is *of the meaning given to it by consciousness, in its essence*".[71] The method for accessing the systematic connections of consciousness with its immanent correlations is *eidetic seeing* [Schauen], which is unlike sensory perception, but has no other "mystical secrets" than sensory perception.[72] Indeed, since essence is nowhere except in the phenomenon itself and since the phenomenon is only as much as its correlation to consciousness, I can *see* essence *directly* with no need for "signitive representation" [signitiv vorgestellt][73], as it is used in Hilbert's proof theory. Essence is not the being itself, but the "being-given" of the phenomenon, which is simultaneously being and sense.

Originary givenness takes place in eidetic intuition, which originates in sensory intuition without being limited to it. And Husserl's sensory intuition is no longer limited to passively receiving raw data from the senses as Locke, and to a certain extent Hume, Kant and the neo-Kantians, thought. Husserl's eidetic intuition is none of the Kantian intuitions: neither sensory intuition nor pure intuition as an a priori form of sensibility. Let me stress once more that Kant's sensory intuition is not bare sensation it is already synthesis at the sensory level. At

[70] Kant understood functionally the concept of substance as being the permanence which allows change to be grasped. "All we know about substances is force", *Reflexions* 4824.
[71] *PSW*, (emphasis added).
[72] *PSW*, English trans. p. 272.
[73] *PSW*, English trans. p. 273.

every level, Kantian intuition indicates the *power of form* over the matter that it informs. In contrast, Husserl's eidetic intuition is not an *empty form*, but a full view of essence. It is an ability of the understanding, which is applied equally to different modalities of being, essential as much as existential. Sensory perception encompasses a sort of primitive categorial activity by which *things give themselves* in their identity. Eidetic intuition has to do with *ideal identities of the meaning* of phenomena; it does not seize upon what exists but upon its essence, turning variably towards the invariant that arises from all the correlations between noema and noesis. I perceive the thing itself and I seize eidetically upon the essence of a thing, the being of a thing, which is given to me not as a thing but as meaning.

> "The seeing seizes upon the *essence as being an essence* and in no way does it posit *existence*. Accordingly, knowledge of essence is no *matter-of-fact* knowledge, and includes not the least assertive content regarding an individual (e.g. natural) existence." [74]

The "true" critique of reason is intentional constitution where both meaning and the presence of the object are led to the state of givenness.

Hilbert totally ignored the thing-in-itself and wished to nullify the exteriority of the relationship between thought and experience through semio-linguistic objectification of the form of mathematical demonstration. Hilbertian intuition is materially represented by the symbols of formal, algebraic or logical calculations. Husserl in turn also eliminated the thing-in-itself, but not consciousness - transcendental consciousness, valid for all subjects at all times. Consciousness reigns even more supremely than in Kant's conception: not only does it constitute the noematic *structure* of the object in correlation with its noetic acts, but moreover it aims and can achieve, by eidetic reduction, the essence, that is the *being-sense* of the appearance of the phenomenon. Phenomenology will go beyond noumenon-phenomenon dualism and nullify in a certain sense the reciprocal exteriority of consciousness and object, since essence is given within the phenomenon. As eidetic inquiry, phenomenology, wrote Husserl, is "in the genuine sense a priori inquiry, and it simultaneously takes full account of all the legitimate motifs of aprioricism".[75] The true a priori is both form and sense, and not just form, as Kant says, or principle of formal constructivity for blind thought, thought by symbols, as in Hilbert's metamathematics.

Husserl promoted the concept of sense/meaning as a non-material/ideal entity, the essence of things themselves, which is independent from its linguistic expression and yet has an objective reality, not reducible to a subjective psychological experience.

III. Systematic philosophy: Jules Vuillemin

1. *The "analogies of mathematical knowledge"*

Following the animosity towards the Hegelian system, coming from diverse quarters and for diverse reasons and fed in particular by the offensives of the experimental credo of science (notably psychophysics) and positive philosophy (A. Comte), and after the spread of historical consciousness (Hegel, Dilthey), of the existential movement (Kierkegaard, fierce opponent of

[74] *PSW*, English trans. p. 272.
[75] *PSW*, English trans. p. 278.

"empty palaces"), of deconstructive back-worlds (Nietzsche), of hermeneutics (Dilthey, Heidegger), of historic-scientific rationalism (Léon Brunschvicg and the tenants of French historic epistemology), the idea of a philosophical system was once again taken up by Jules Vuillemin.

Rejecting as strongly as Husserl the worldview perspective, Vuillemin applies analogically a method inspired by the axiomatic approach and logic to "theoretical philosophy". Contrary to psychology and history, including history of science, "theoretical philosophy" aims, he says, to determine non-contingent and objective connections, comparable to those established by logic and exact science. The idea is not to develop a philosophy which reflects on the emergence or characteristic nature of modern mathematics, nor a philosophy so well suited to it that a mathematician might agree with or endorse it[76], but rather to use new mathematical tools to analogically build a systematic philosophy both in its content and in its architectural analysis of previous systems. Clearly presented in *La Philosophie de l'Algèbre* [*The Philosophy of Algebra*], dedicated to the mathematician Pierre Samuel, with whom he was in dialogue[77], J. Vuillemin's persistent idea is to draw a parallel between deductive theories constructed in mathematics as sets of consequences derived from explicitly stated *axioms*, and the project of a philosophy as a coherent system of compatible propositions based on a small number of *principles*. The system is to philosophy what deductive theory is in mathematics. Hence the will to use "the analogies of mathematical knowledge to critique, reform and define, as much as possible, the Method of Theoretical Philosophy"[78], and consequently to build a systematic history of philosophy.

"Analogies of mathematical knowledge" are meant to replace Kant's "analogies of experience" in order to achieve a "*general* critique of reason". What does Vuillemin mean by 'general'? Actually he aims to rewrite *Critique of Pure Reason* in a totally a priori manner independent as much from the sensory experience as from historical considerations. He tracks a priori conditions of possibility of knowledge freed from the limiting relationship to experience which was the fulcrum of Kant's theory. According to Kant indeed, analogies of experience are fundamental principles [Grundsätze] – of substantiality, causality, and coexistence or reciprocity – which must be presupposed in the experience of the world, and "experience is possible only through the representation of a *necessary connection* of perceptions". "An analogy of experience is … *a rule* according to which a unity of experience may arise from perception" (emphasis added). So experience "contains in one consciousness the *synthetic unity* of the manifold of perceptions. This synthetic unity constitutes the essential in any knowledge of objects of the senses, that is, in experience as distinguished from mere intuition or sensation of the senses".[79]

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[76] Presumably conversely to the positive reception of André Weil to the philosophy of Jean Cavaillès and that of Charles Ehresmann to that of Albert Lautman.

[77] Both were teachers in Clermont-Ferrand, then in the early 1960s, at the École Normale Supérieure de Jeunes Filles. One can see the connection between Vuillemin's point of view and those, respectively, of Bourbaki and P. Samuel in Sébastien Maronne's article: Pierre Samuel et Jules Vuillemin: mathématiques et philosophie, in Thierry Lambre (éd.), *Des mathématiques en Auvergne: histoire, progrès, interactions*, t. 1, Clermont-Ferrand, Revue d'Auvergne, p. 151-173, 2014 (I am grateful to the author for providing me with a copy of his article).

[78] *La Philosophie de l'algèbre I. Recherches sur quelques concepts et méthodes de l'Algèbre moderne*, Paris, Presses Universitaires de France, 1962, Introduction, §2, p. 5. See also the passage extracted from *Critique of judgement* that Vuillemin quotes on p. 510.

[79] *Critique of pure Reason*, Transcendental Logic, The Analytics of Principles B 219 (emphasis added).

Now, how should we understand Vuillemin's analogies of mathematical knowledge? That is how should we understand the role of mathematical knowledge in the reform of theoretical philosophy? If we follow Kant's general line of thought but leave aside the central status of consciousness, we would say that mathematical knowledge supplies rules for gathering and ordering diverse mathematical results or propositions in synthetic theories. Of course it's particularly about rules of the axiomatic method. And then, how should we link mathematics to philosophy? The answer is "by analogy", taking this term in a broader sense rather than in its specific use in the architecture of *Critique of Pure Reason*. Once again, according to Kant's accurate account, "analogy does not mean, as is commonly understood, an imperfect similarity between two things, but a perfect similarity of relations within two quite dissimilar things". [80] So Vuillemin's project seems quite similar to the traditional search for philosophical rigour by using *mutatis mutandis* a deductive pattern. Indeed, Vuillemin aims to elaborate a systematic philosophy whose propositions would be ordered in roughly the same way as mathematical propositions in an axiomatic theory, even if the main ordering relation would be logical compatibility rather than strict logical deduction. And of course Vuillemin takes as guide the bare principle of the axiomatic method regardless of its embodiment in a symbolic language and its framing in a logical formal system.

There is yet another important difference between systematic philosophy and axiomatic theories. Systematic philosophy must be built on a priori principles, and those principles *are not* axioms. The analogy between theoretical philosophy and axiomatic theories concerns uniquely the respective ordering relations which state explicitly what propositions are set as primitive and what propositions are compatible with them versus are deducible from them. Philosophical a priori principles do not have the same epistemological status as mathematical axioms. Mathematical axioms are constitutive of the set of propositions derived from them, while philosophical principles have only a regulative function: they are rules for establishing an "ideal" system in accordance with the unifying and guiding role of reason's ideas. In Kant's interpretation an idea implements the necessary role for reason's demand for systematic unity, but refers to no object of experience[81]. An idea has no objective reality it has a "transcendental subjective reality". Vuillemin holds that an "ideal" system refers to no actual system but he does not accept that it has a transcendental subjective reality, precisely because he calls transcendental subjectivism into question.

2. *Abstract algebra and systematic philosophy*

In Chapter III of *The Philosophy of Algebra*, the study of Abel's "general method" and his proof of the impossibility of solving algebraically general equations of degree higher than the fourth degree inspires Vuillemin's aim to radicalise Kant's *Critique* through a "*general* critique of pure reason".

Vuillemin quotes (p. 208-209) Abel's prescription about general algebraic equations of the fifth degree: "One must formulate a problem in a form that always allows the possibility of solving it…Instead of trying to find a relation without knowing whether it exists or not, one

[80] *Prolegomena to any future Metaphysics* 58.

[81] Kant distinguishes between reason and understanding. Reason does not itself provide us with concepts of objects but only orders the concepts produced by the understanding. Reason takes the understanding and its concepts as its object while the understanding, by contrast, relates to objects themselves.

must ask whether such a relation is actually possible"[82]. Vuillemin comments in Kantian terms (p. 221):

> "Particular proofs are real: they presuppose the principle of the possibility of experience given in sensation. General proofs concern what is possible and start from concepts alone, ignoring the limitative conditions of the senses".

There is an ambiguity here. In this comment, indeed, mathematical experience seems to rely on sensation or *empirical* intuition, whereas in Kant's *Critique* mathematical experience relies on *pure* intuition. Anyway, general algebraic methods provide us with the paradigm of the purely formal exercise of reason, an exercise independent of the principle of the possibility of experience. Vuillemin's version of Kant's theory of knowledge is, in fact, very close to Dedekind's purely conceptual construction of natural numbers in *Was sind und was zollen die Zahlen?* Like Dedekind, Vuillemin rejects the Transcendental Aesthetic. He writes:

> "When the critical philosopher says…that all our knowledge begins with experience, although it does not derive from it, and from the fact that in reality he mixes origin and beginning by subordinating the deduction of concepts to the principle of experience, he makes the very task he proposes impossible, proceeding as he does from individuals whose experience is at least the occasion… The limits which it [classical philosophy, otherwise known as genetic idealism] attempts to assign to the faculties of knowledge are born not *internally* of the structure of these faculties, but are suggested from the outside, by the individual objects to which it applies itself through encounter."[83]

And further on:

> "Kant borrows these limits [of our knowledge] from sensory intuition, which is a faculty external to reason".[84]

An finally, to eliminate the assumption of possible experience

> " is not to weaken but strengthen the Kantian objective to free itself from *extrinsic limitations* under which it was restricted by the principle of the possibility of experience.[85]

Therefore "the g*eneral* critique of pure reason" undertakes "to define knowledge as the relationship of reason with itself, and not as a relationship between this faculty and external data"[86]. From a philosophical point of view this internalisation is a legacy from Hegel's process of concept, which develops internally with no external reference. Hegelian conceptual process is not definable through a relation to something external to it. In mathematics the instrument for such an internalisation is provided by the notion of structure, which reorganises

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[82] Niels Henrik Abel, Beweis der Unmöglichkeit der algebraischen Gleichungen von höheren Grade als dem vierten allgemein aufzulösen*, Journal für die reine und angewandte Mathematik* 1, 1826, p. 65-84.

[83] *La Philosophie de l'Algèbre*, p. 217 (emphasis added).

[84] *La Philosophie de l'Algèbre*, p. 220. Cf. also p. 463 : "The principle difficulty of Kantian doctrine is its notion of intuition".

[85] *La Philosophie de l'Algèbre*, Conclusion, p. 474-475 (emphasis added).

[86] *Leçon inaugurale au Collège de France*, p. 28.

axiomatically the mathematical material. Like others philosophers (especially Jean Cavaillès[87] whose influence in France was strong), Vuillemin uses the axiomatic method as a tool to reform Kant's theory of knowledge and to achieve "the revival of the problems posed by philosophy"[88].

Vuillemin endorses Hilbert's distinction between contentual axiomatic theories and formal axiomatic theories.[89] Euclid's *Elements* exemplifies contentual axiomatics, whereas Hilbert's *Foundations of Geometry* exemplifies formal axiomatic theories, that is theories where one abstracts from concrete mathematical content[90]. Let me quote Hilbert's characterisation of both kinds of deductive theories:

> "Contentual axiomatics introduces its fundamental concepts by reference to known *acts of experience* and its basic principles either as *obvious facts*, which one can make clear to oneself, or as extracts from complexes of experiences, thereby expressing the belief that one is on the track of *laws of nature* and at the same time intending to support this belief through the success of the theory" (emphasis added). [91]

> " [Formal axiomatics] cannot get a foundation through a reference to either the evident truth of its axioms or to experience; rather such a foundation can only be given when the idealization is performed, i.e. when the extrapolation through which the concept formations [Begriffsbildungen] and the principles [Grundsätze] of the theory come to overstep the reach either *of intuitive evidence* or *of the data of experience*, is understood to be consistent" (emphasis added).[92]

As one can note, reference to external experience is what makes the difference between contentual and formal axiomatics. Consequently the concept of truth and the meaning of intuition must be changed in order to fit the new standard of idealization achieved by pure reason. Vuillemin is not completely content with Hilbert's formal axiomatics. He revives Leibniz' *logical-metaphysical* rationalism as contrasted with Kantian intuitionism. His philosophy is partly a dialogue between Kant and Leibniz, developed notably in the chapter on Galois of *The Philosophy of Algebra*. Indeed, Vuillemin uses Leibniz' philosophical concepts, suitably altered, to enlighten substitution groups. According to his philosophical reading, Galois theory furnishes a method "to construct individual elements, no longer in intuition and following flawed schemes, but in the concepts themselves, in a completely *a priori* and general way, now without reference to any data or owing anything to luck" (p. 288-289). So the question: is one entitled to apply the concepts of group and structure in the philosophical method? (§ 34, p. 292-300). Besides, *What are philosophical systems?* (Cambridge University Press, 1986) presents in a Leibnizian style a systematic and a priori classification of the various possible philosophical systems.

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87 See Hourya Sinaceur, *Jean Cavaillès. Philosophie mathématique*, Paris, Presses Universitaires de France, 1994.

88 *Mathématique et métaphysique chez Descartes*, Paris, Presses universitaires de France, 1960, p. 141. For an example of the application of the axiomatic method to philosophy see "On two cases of the application of the axiomatic approach to philosophy: Zeno of Elea's analysis of motion and Diodorus Cronus' analysis of freedom", *Fundamenta Scientiae*, 6, 1985, p. 209-219.

89 D. Hilbert - P. Bernays, *Grundlagen der Mathematik* I, Springer, 1934, zweite Auflage, 1968, § 1, p. 1-8.

90 D. Hilbert - P. Bernays, *Grundlagen der Mathematik* I, §1, p. 20.

91 D. Hilbert - P. Bernays, *Grundlagen der Mathematik* I, §1, p. 2.

92 D. Hilbert - P. Bernays, *Grundlagen der Mathematik* I, §1, p. 3.

Vuillemin also endorses in history of philosophy and to a large extent applies in human sciences (linguistics, psychology, anthropology, etc.), the Hilbertian distinction between genetic and structural points of view[93]. He attributes the first to philosophers who ascribe priority to the mind, that is, German idealists in general and Fichte[94] in particular, and the second to Martial Gueroult, arguing that only the latter allows a "*general* critique of pure reason".[95] Adopting the structural method that Gueroult applied to make explicit the internal coherence of a philosophical system (Descartes, Spinoza, Leibniz, Malebranche, Fichte) Vuillemin classifies philosophical systems without true consideration of any historical philosophical doctrine (Platonic, Aristotelian, Epicurean, etc) and with the aim to highlight the rational a priori links between different effective systems, links which therefore have nothing to do with effective links and can be unearthed only by structural analysis. Granger qualified this combinatorial enterprise as "metasystematic"[96]. I will not go into the details of this metasystematic approach, which reminds us of Leibniz[97]. It was the subject of the last part of Baptiste Mélès' thesis, which leads to a volume now published.[98] I just will comment briefly in the part 4 of this section devoted to Vuillemin's views.

3. *Mathematical reflexivity and philosophical reflexivity: what is critique?*

The Philosophy of Algebra can be read as a long and patient effort to set out the possible meanings of the concept of critique, in mathematics and philosophy, and to add a new sense to existing meanings. At the same time the various figures of dogmatism are explained.

Critique is a "reflexive activity", attributed by philosophers to the ego (Descartes, Husserl) or to the thinking Subject (Kant). The question arises of what the subject of this attribution is when it comes to mathematical methods. Is it still the conscious subject, who investigates first the limits of his understanding? Or is about the objective limits of this or that mathematical method?

The answer to this question depends basically on the relations one sets between mathematics and philosophy. We can advocate a separation of method between mathematics and philosophy, and consider mathematics as an autonomous positive science apt to provide us with exact methods for a mathematical treatment of philosophical or at least epistemological

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93 D. Hilbert - P. Bernays, *Grundlagen der Mathematik* I, §1, p. 1-2.

94 Fichte's work was the subject of Martial Gueroult's doctoral thesis.

95 *La philosophie de l'Algèbre*, chapter 3, §25, p. 218-221 ; Conclusion, § 60, p. 517 (emphasis added).

96 Axiomatic Method and the idea of system in the work of Jules Vuillemin, in *Causality, Method and Modality, Essays in Honor of Jules Vuillemin* (Gordon G. Brittan ed.) Dordrecht/Boston/ London, Kluwer Academic Publishers, 1991, cited by Jacques Bouveresse, Vuillemin between intuitionism and realism, in *Philosophie des mathématiques et théorie de la connaissance*, (R. Roshdi & P. Pellegrin eds.), Paris, Librairie Scientifique et Technique, 2005, p. 77.

97 "necessary truths, such as we find in pure mathematics and particularly in arithmetic and geometry, must have principles whose proof doesn't depend on instances (or, therefore, on the testimony of the senses), even though without the senses it would never occur to us to think of them" wrote Leibniz in *New Essays on Human Understanding*, Preface. The idea of a priori combinatorics returns many times to analyses by Vuillemin of the structural method, and of course it is common also among mathematicians, as, for example, in the fine text of Poincaré summarizing the work of Sophus Lie, cited in *La Philosophie de l'Algèbre*, p. 426.

98 Paris, Vrin, October 2016. See also the article by Elisabeth Schwartz "History of mathematics and philosophy," and that of Stéphane Chauvier, "Philosophy of the classification of philosophical systems: criticism and decision-making", in *Philosophie des mathématiques et théorie de la connaissance. L'Œuvre de Jules Vuillemin*, R. Rashed & P. Pellegrin eds., Paris, Albert Blanchard, 2005, p. 1-28 and 187–204 respectively.

questions. We can also admit a separation of content between mathematics and philosophy, and consider that philosophy can simultaneously use structural methods and keep its own ability to address very general philosophical problems in a way that leads to round off the narrower views growing in the field of mathematical practice. It is in virtue of this alternative that Vuillemin distinguishes between intrinsic intuitionism, that of Kronecker and Brouwer, and extrinsic intuitionism, that of Kant, who demands "of concepts of understanding that they relate to necessarily sensory intuitions, and whose pure form thus relates to the externality of space and time."[99] In Vuillemin's opinion intrinsic intuitionism incorporates a metaphysical choice into mathematical practice, whereas extrinsic intuitionism holds that metaphysics is beyond the reach of human knowledge.

Anyway, some issues call into question the analogy between axiomatics and critique, as is done by both Hilbert and Vuillemin. Concerning Hilbert one will remember the analysis in the first part of this article. I now turn to Vuillemin and his *Philosophy of Algebra*.

3.1. It is clear that Vuillemin adopts Hilbert's idea that the axiomatic approach represents mathematics entering its critique phase. He bases this idea on the distinction between a posteriori and a priori and between genetic and structural methods, and he continually uses cross-referencing mathematical and philosophical texts in a way that often makes it difficult to follow his account. So Lagrange is paralleled with Fichte (Chapter I, § 13) to show how their shared a genetic method, where form is never separated from content, joins the rudiments of a thoughtful critical analysis that nevertheless has difficulty disengaging from its empirical origin. "Any genetic method is latent empiricism" says Vuillemin[100]. The study of the phase represented by the works of Gauss gives Vuillemin the opportunity to show how the prejudices of extrinsic intuitionism underlie the notion that geometric constructions are proof of existence in mathematics. Such a prejudice delayed full acceptance of complex numbers until their geometric representation by Gauss. Thank goodness for Abel, one might say! For it was he who was astonished that one would want to solve particular algebraic equations, without solving first the general question of the possibility or impossibility of some solution. In Vuillemin's opinion Abel's reasoning is comparable to Kant's critical method. And critical method is thought to be generally applicable in mathematics through the axiomatic method. However this opinion would be unquestionable only if one assumes Hilbert's view of the axiomatic method as the critical step in mathematics and if the term and concept of possibility had the same meaning in Abel's mathematical work and in Kant's transcendental philosophy.

Vuillemin adjusts Husserl's distinction between the Aristotelian "abstraction by generalisation" and an "abstraction by formalisation" to interpret Abel's perspective (p. 216) and Galois' theory (p. 288). The new kind of abstraction "is irreducible to the old one in that it requires that we first expose the general *conditions* of a problem and therefore the postulates on which the theory depends"[101]. Notably with Galois, then with Dedekind and Kronecker, it appears that the concept of group or the concepts of field and domain of rationality "are a typical example of the idea of *critique* in algebra"[102], which consists, according to Vuillemin, in using an a priori method, illustrated for example by the construction of Galois resolvent. In the conclusion of his book, Vuillemin reaffirms his belief in this filter of critique in mathematics, writing

[99] *La Philosophie de l'Algèbre*, p. 172.
[100] *La Philosophie de l'Algèbre*, p. 118.
[101] Ibid., p. 216 (emphasis added).
[102] Ibid., p. 232 (Vuillemin's emphasis).

> "[In] material mathematics, […] one can prove what is, but not the impossibility of what is not. Thus the long-held illusion under which algebraists remained concerning the solution of equations beyond the fourth degree could only be dispelled once, by a change of method, they sought to build not the solutions to equations they could solve, but the formal conditions rendering the solutions possible or impossible. Material mathematics remained necessarily dogmatic. Formal mathematics right away became critical.[…] It sought, indeed, […] to define *a priori* the types of structures on which the solution to a problem depends, and consequently the *intrinsic* limits these structures entail…"[103]

One can easily understand that the abstract axiomatic approach determines a priori the conditions for validity of operations defining a structure. But could one say inasmuch as it falls within the philosophical framework established by Kant's transcendental idealism? Let me try to address this question.

3.2. In footnote 1) of p. 262 of *The Philosophy of Algebra*, Vuillemin contrasts classical mathematics, which "defines an object independently of reflection", and modern mathematics, which "internalises reflection in its own methods". Vuillemin immediately adds a warning which seems to contradict somehow the aforesaid internalisation: "I will return later to the *confusions* insinuated here between the mathematical and the philosophical concept of operation, which consist in inferring untenable conclusions from the correct idea of *two levels of pure knowledge*" (emphasis added). The philosophical conception of operation is reflection[104], it constitutes one of the two levels, the other being the mathematical and logical notion of operation. Does Vuillemin mean that reflection, qua "active operation of the soul", is internalised in mathematical methods?

It recently became a commonplace to stress the "reflective turn" of modern mathematics regardless of different levels of knowledge. Vuillemin's view is deeper and more subtle though complicated and limited to an inceptive state. According to it, the answer to the question posed above would demand a theory of operations of knowledge (conceiving, judging, reasoning) in relation to mathematical structures. This theory will be based on the difference between objective operators, such as conjunction, disjunction, negation or modal operators, and transcendental operations of the sort "I think that…" which, inter alia, do not have certain properties common, for example, to conjunction and disjunction such as symmetry and associativity.

In any case, if we distinguish between mathematical or logical operations and operations of the mind, that is to say between objective and subjective operations, then it becomes hard to interpret the axiomatic method as the critical or Kantian stage of mathematics. Aware of this difficulty, Cavaillès and Granger used the Husserlian concept "thematisation" to describe the specific mathematical process which goes from an operation, let us say addition, to *the theory* of its possible properties: associativity, commutativity, invertibility, etc. and from the set of those properties to the concept of additive structure. Vuillemin does not discuss "thematisation", though this concept might have shed some light on his purpose. But he rightly reproaches post-Kantians for unduly associating the structural approach and idealism (p. 273), and, I would add, the objective method and the subjective act. However to break this very common but undue association, one must, I believe, explicitly and rigorously distinguish

[103] Ibid., p. 471 (Vuillemin's emphasis).
[104] See for example p. 290, end of 1st paragraph.

between *philosophical reflection*, which necessarily refers to an empirical or transcendental subject, and *mathematical reflexivity*, which refers to the superimposed planes of ideational objects. If one truly admits this distinction, the Kantian interpretation of the axiomatic method becomes highly questionable, unless one wishes to refer mathematical reflexivity and any other mathematical process as objective products to a subjective, empirical or transcendental, thinker or producer. Making this reference or not is a philosophical choice. In his last metamathematical writings Hilbert, for instance, moved away from his initial Kantian interpretation of the axiomatic method in order to advocate a more objectivist point of view.

But if one agrees with Vuillemin that the structural approach is not idealistic and not subjective, what philosophical sense could we give to the research of the mathematical general conditions of possibility for solving some problem? Certainly not, I think, a sense consistent with the Kantian *Critique*, which is inseparable from its transcendental setting.

First of all, as I have observed in my analysis of Hilbert, in mathematics the conditions of possibility are neither universal nor immutable, but local and variable depending on the particular problem to solve, its formulation and its links with known results and conjectural hypotheses. The a priori and formal character of the conditions posed at the outset, that is to say of the axioms, is not external to the content they determine: they are necessary *and sufficient* conditions, constitutive of the content of a deductive theory rather than only regulative. Vuillemein is perfectly aware of this fact, and the mathematical material analysed by him illustrates these points abundantly. It follows that a non-Kantian conception of reason is required, since according to Vuillemin reason is "the faculty of thinking about structure" independently of the objects to which it is applicable and free from the constraints of intuition.[105]

Actually, Vuillemin does not totally abandon Kant. Like many scientists and philosophers of the early 20th century who sought to adapt Kant to new developments in mathematics and physics (Hilbert, Poincaré, Brouwer, Cassirer, to name but a few), Vuillemin wanted only to redefine the extension of critical philosophy "by relating it to the necessary plurality of the choices between axiomatic systems and not to the internal[106] limitation of knowledge required by the fact of experience". However, the relationship between axiomatic systems and critical philosophy remains problematic. For it rests on a shaky analogy, and that for at least three reasons: philosophy and mathematics are two heterogenous domains with different aims and ways, their operations are fundamentally dissimilar, and the vocation of philosophy, according to Vuillemin, is to seek legitimacy *by right* and to pose foundational questions that the positive sciences usually push aside.

The last trait engaged Vuillemin in an analysis of Husserl, who posed exactly the same question about the legitimacy of *every* judgement's "being claim", and who in his *Logical Investigations* studied the structures of meaning corresponding to different formal levels. But the *objective structures of intentionality* cede to the *constituent power of intentional consciousness* and to the *ultimate authority of categorial intuition*, which reestablishes the power of intuition and the conception of truth as adequacy between knowledge and Being. Vuillemin very justifiably concludes that phenomenology is dogmatism.[107] To respect the criticism in spirit, if not in letter, Vuillemin decided to overturn the interdiction which since

[105] *La Philosophie de l'Algèbre*, p. 467.

[106] *La Philosophie de l'Algèbre*, p. 476 (here one would have to substitute "external" for "internal" in order to remain consistent with the analysis Vuillemin made previously in his book).

[107] Ibid. p. 493 and following pages.

Kant had weighed heavily on metaphysics and assumes that "all understanding - whatever it may be - is metaphysical through and through in that, at its core, it entails decisions and choices which do not in themselves belong to the internal jurisdiction of this understanding". Working on this assumption, the task of philosophy consists not of "ignoring metaphysical choices, but studying motives in relation to man's free will". If one cannot build a formal ontology founded on Kantian principles, which actually forbid any kind of ontology, it is worth examining *the hypothesis* that the "*general* critique of pure reason" is compatible with formal ontology. As a philosopher of human free will, Vuillemin shines a light on the metaphysical choices and decisions which preside, independently of all experience, he stresses, over every thought - including those in concepts and scientific theories - and every action. But given his intention to substitute "human cogito in a universe of gods with human doing in the world of man", Vuillemin claims he respects the spirit of Kantian philosophy, if not the letter, in his removal of formal ontology from theology without undermining its metaphysical foundations. Thus, "Critique should study the convenience of these general decisions with beings and values", that is to say with basic ontological and ethical assumptions.

4. *From abstract algebra to metaphysics*

In contrast with classical algebra, which deals with theories, abstract algebra allows us to determine classes of theories, whose axioms systems may have multiple interpretations. For example, abstract field theory is instantiated by the field of real numbers, the field of complex numbers, the finite fields $\mathbf{Z}/n\mathbf{Z}$ where n is a prime integer, the field of p-adic numbers – completion of the $\mathbf{Q}$ field of rational numbers supplied by a so-called ultrametric distance, etc. Analogically Vuillemin defines classes of philosophical systems based on sets of compatible structural properties. He writes:

> "When we encounter the new and worn notion of *structure*, its use in the study of philosophical systems no doubt produces effects analogous to those suffered by pure reason at the hands of formal algebra. Distinguished both from a chronicle where thoughts are narrated as events and a philosophy of history where they constitute the materials of a theodicy, the technological history of systems of philosophy[108] abandons the intolerable idea of an absolute rule…"[109]

The possibility of varying mathematical structural settings according to the problem to be resolved or to a preferred method of solving it is frequently realised. A familiar example is that of divisibility, treated differently by Dedekind and Kronecker. The first introduces the set theoretical concept of ideal, defining it by the conjunction of two very simple axioms, the second considers an algebraic number (the root of a polynomial with rational coefficients) as a "divisor"[110]. Vuillemin states analogically that it is the fundamental plurality of *standards*

[108] Martial Gueroult gave the name "History and technology of systems of philosophy" to the chair he occupied at the Collège de France from 1951 to 1962.

[109] *Leçon inaugurale* [Inaugural lecture] *au Collège de France*, 5 December 1962, p. 20 (emphasis added). Current mathematical usage of the qualifier 'formal' is now reserved for a theory written in logical language, that is, which specifies the primitive logical constants and the permitted rules of inference.

[110] The two axioms defining an ideal can be found in the second edition of the *Vorlesungen über Zahlentheorie von Dirichlet*, §§ 159-170 (1871), and in the article on algebraic theory (1877), partially reproduced in Dedekind's *La création des nombres*, Paris, Vrin, 2008, in particular p. 248. Kronecker's theory of divisors can be found in the Grundzüge einer arithmetischen Theorie des algebraischen Zahlen, *Journal für die reine und angewandte Mathematik*, vol. 92, 1882, 1-123. It is summarised in an accessible manner in the excellent article

that drives the various possible philosophical systems. Drawing up a rational distribution of philosophical systems shows that there is a choice at the root of every system. Now it will not be a question of saying which set of standards is the best but of laying out the range of standards with the purpose of providing the base for rational choice. Vuillemin gives a personal interpretation to the Kantian distinction between understanding and reason and replaces intuition by decision, that is to say seeing or thinking by doing.

> "For the intuitions of *the understanding*, it [formal algebra] substitutes the order according to which *reason* combines operations, which do not in themselves have representative value and which, rather than ideas that one can see, embody the *decisions* which one can take and for which truth, without any substantive appropriateness, comes down to formal compatibility" (emphasis added).

Moreover, Vuillemin's aim is to join together thinking and choosing, reflection and will. Hence the consequence for Fichte's *Theory of Science* [Wissenschaftslehre], which

> "necessarily loses its absolute character and becomes permeable to the presence of different and relative interpretations that assign new content to the task of critique, in relating to the necessary plurality of *choices* between axiomatic systems, not to the internal[111] limitation that *the fact* of experience imposes on knowledge" (emphasis added).

On the one hand, considered not only from the internal perspective of mathematics, but also and ultimately from the autonomous and overlooking standpoint of philosophy, the choice engages metaphysics. But on the other, annulling the limitations imposed by the Kantian principle of possible experience permits the reintroduction of metaphysics to the very heart of Critique. In Vuillemin's hands the structural point of view has a paradoxical consequence in giving the leading and ultimate role to metaphysics.

> "In the new critical metaphysics, the being addresses itself[112] over and above all exterior affection, to the level of structures which define the relationship of reason with itself."[113]

This is a typically anti-Hilbert and anti-positivist programme with a Hegelian flavour. Far from dismissing metaphysics and even philosophy itself in favour of science, which has according to Hilbert critical aspirations thanks to the axiomatic method and proof theory, Vuillemin uses the tools of science, principally the axiomatic method, to return philosophy its autonomy and its truly critical specificity with regards to theology and science as well, and to confer to metaphysics new credence in its contribution not as a thought of the One or of the Absolute, but as a thought of multiplicity, without it necessarily leading to ontological relativism. Finally, the true philosophical lesson to be learned from the mathematical concept

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by Alain Michel, Après Jean Cavaillès, l'histoire des mathématiques, *Philosophia Scientiae*, vol. 3, cahier 1, 1998, p. 113-137; the two principal theorems of the theory are set out in note 22, p. 131.

[111] Read 'external'.

[112] 'The being addresses itself' would not be understood without the contribution of Hegelian dialectics, finally rejected by Vuillemin for the finalist nature of its dynamic, mesmerised by divine Absolute. Vuillemin preserves certain souvenirs from his foray into Hegelian territory.

[113] *Leçon inaugurale au Collège de France*, p. 30. The last sentence belongs also to Hegel's legacy. "The relationship of reason with itself" is a consequence of the Hegelian reduction of instant sensory experience to an illusion.

of structure is an invitation to revive and invert the Platonic relation between the One and the Multiple.

It is true that Vuillemin believes that an updating of scientific methods, mathematics in particular, offers the opportunity to rethink the mission of philosophy. But as tight as this bond between science and philosophy may be, it precisely remains *a bond* between two visions whose intransigent duality prohibits one being absorbed into the other - as it happens in the opposite views of Hilbert and Husserl - and allows only external and retrospective enlightenment of science by philosophy, which simultaneously illuminates the philosophical systems chosen by Vuillemin as candidates for his analogical analysis.

In Vuillemin's mind the analogy with science does not by any account mean that philosophy is or could be a kind of science, not even of a different type from that of the positive sciences as Husserl wished. Indeed, a choice, when it concerns not scientific axioms, which are always more or less local, confined in scope to a particular domain of a particular science, but general principles of interpretation of the constitution of the world as a whole, is a question of ethical decision, that is, of problems falling under *The Critique of Practical Reason*. Stéphane Chauvier[114] sheds light on what Vuillemin's reflections have in common with those of Cavaillès, which makes them both heirs of Pascal: the human being as man of science and man of action is basically forced to gamble. In Vuillemin's ultimate ontological view the gamble concerns the being as being, whereas Cavaillès' urgent scientific and ethical worries are about gambling on the future[115].

Conclusion

In this paper, I have tried to analyse in some detail the *mutual* relationships between philosophy and mathematics in the modern era.

The goal of the Lisbon Congress was to explain what philosophers thought about mathematics of their time and how they used mathematical innovations in the development of their own doctrines. As examples, I have thus examined the question of the "intrinsic possibility of pure knowledge in relation to the faculty of thought" that philosophers - in this case Husserl and Vuillemin - pose in thinking about the autonomy acquired by abstract mathematics from sensory intuition. Would it not be possible, indeed, to revive Kantian Critique through a theory of objective structures of reason whose origins or motivation are not to be found in sensory experience and whose keystone does not lie in formal subjectivity of pure apperception? This question was vital to French philosophers, Cavaillès then Vuillemin and Granger. This paper deals with Vuillemin point of view, while I have studied Cavaillès' and Granger's contributions in previous works[116].

Besides this I have also wished to study here the inverse impact of philosophical doctrines not on the mathematical methods, concepts and techniques themselves, but on the ideas used by mathematicians to introduce them, while explaining their novelty and their advantages. Hence Richard Dedekind intended to break the Kantian link of pure thought to empirical experience, while Henri Poincaré, David Hilbert and L.E.J. Brouwer thought, each in his own way, that

[114] Article cited above in note 98.

[115] Cf. Hourya Benis Sinaceur, *Cavaillès*, Paris, Les Belles Lettres, 2013.

[116] Formes et concepts, *La connaissance philosophique, Essais sur l'œuvre de Gilles-Gaston Granger*, Joëlle Proust et Elisabeth Schwartz eds., Paris, PUF, 1995, p. 93-120 ; Style et contenus formels chez Gilles Gaston Granger, *La pensée de Gilles-Gaston Granger*, A. Soulez et Arley R. Moreno eds., Paris, Hermann, 2010, p. 161-206

intuition supplies initial inspiration, a beginning or an origin, i.e. a fundamental material basis versus a metaphysical justification for elementary mathematical concepts.

It is to the crossroads of lines of reasoning drawn by Kant, Husserl and Hilbert that I have dedicated this essay. The choice of this triad was motivated by my re-reading of *La Philosophie de l'Algèbre* by Jules Vuillemin, whose reflections interweave, in an original and sometimes confusing fashion, the results of abstract algebra with the philosophies of Descartes, Leibniz, Kant, Fichte and Husserl.

In summary one might say that in the classical era (Descartes, Spinoza, Leibniz, etc.), philosophy and science went hand-in-hand. In the modern era, by contrast, each protagonist, fascinated by the other, nevertheless battles to remain undisputed captain of the ship. On the one hand, the scientism of Hilbert's mathematical epistemology aimed to reduce, if not to eliminate the ambition of philosophy to submit mathematical practices and problems to its own principles and methods, be they transcendental or metaphysical. On the other, phenomenology has promoted the idea of a non-exact philosophical rigour and advocated un point of view encompassing positives sciences, ontology, and ethical values in connection with the dominant category of sense/meaning, while Jules Vuillemin assumed the inseparability of thought - scientific or philosophical - from the metaphysics of free will. The battle has been, and still is in some circles, about the meaning and effective content of the idea of scientific philosophy and its putative links to metaphysics. A "disputatio" practiced from the beginnings of philosophy.

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