Noēsis: Plato on Exact Science

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Dedicated to Howard Stein on the occassion of his 70th birthday

1 Introduction

There are two places in Plato's *Dialogues* in which he discusses his conception of scientific explanation: the passages on the 'second best method' in the *Phaedo* and the passages on *noēsis* in the Divided Line simile in Book VI of the *Republic*. I have written about the first of these in [1986] and I want to discuss the second of them here. The conception in question is of what we would call *exact science*. Some exact sciences, the so-called *mathēmata*, were already in existence in the fourth century BC in Greece and Plato was concerned to argue for a proper foundations for them. The reason why is part of my story of the Divided Line. The Line itself, I will argue, is a rhetorical argument for foundations.[1] Plato was also concerned with extending the

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[1] My account of the Line, as an argument for foundations of the exact sciences, is in substantial agreement with the excellent discussion by Nicholas White in [1976, 95-99].

scope of exact science to other domains, including political science; but that is not part of my story, although it should be a substantial part of any accurate account of the *Republic* as a whole.

My reading of Plato, compared to most contemporary commentaries, is deflationary: I understand him to be saying things that we understand quite well, at least in the case of his conception of science, and can agree with, although they were novel in his time. But also, on my reading, and again in contrast with many contemporary commentaries, Plato was a brilliant man of his times. Whether or not he was first to see the need for foundations, he certainly understood it very well and was the one spokesman for it whose writings have come down to us.[2] Often when I read present-day discussions of Plato's conception of science, I am reminded of Marc Antony's funeral oration. Plato was indeed a very great man, a genius: they all affirm this; but then they go on to attribute to him views that would have been as foolish or unintelligible in his time as they are in ours.

In any case, my story begins with Book IV of the *Republic*. At 435d, in the course of discussing the nature of justice and concluding that it consists of the right proportions of courage, moderation and wisdom, Socrates remarks that the method of analysis that they had been employing up to that point is not entirely adequate for precise understanding, but that the correct way

The main difference, also substantial, in our accounts lies in the fact that, whereas White understands the new foundations to be a new and separate science of *dialectics*, with its own axioms and theorems, on my account the foundations is to consist in adequate first principles for, say, geometry, itself, to be found by a *process* of dialectic. The difference in our views reflects a difference in our judgement of the state of geometry itself at the time that Plato was writing.

[2]I am excluding Aristotle's theory of demonstrative science in the *Posterior Analytics* for two reasons: First, tied as it was to his conception of logic as syllogistic, it lost its relevance to exact science. This is manifest in the historical distinction between the geometric mode and the syllogistic mode of reasoning, surviving perhaps in Kant's distinction between demonstration and discursive reasoning. But my second reason is at least equally important: For Aristotle, the primary truths, the first principles, are general empirical truths. Aside from the difficulty that we shall raise in §2 with this in the case of exact science, it means that the original *motive* for foundations of exact science, for finding first principles, is completely lost. For Plato, as we shall see, the goal of foundations is to make explicit the rational structure we are studying and so to define what is true of that structure—namely, what can be deduced from those principles. For Aristotle, the goal of foundations can only be organizational: to organize empirical truths in a deductive system (where here I am ignoring the fact that deductive systems based on syllogistic are in any case puny things).

to proceed is more arduous. At that point he and Glaucon agree to continue the discussion at the level that they had been. But in Book VI, at 504b, while discussing the education of the future guardians, Socrates refers back to the more arduous way as the one appropriate for them. At this point in the dialogue they are interested not just in the nature of justice in the state and in the soul, but in the various subjects that the guardians should understand; and the longer way refers to a particular conception of knowledge or, better, science, which yields a deeper and more precise understanding of these subjects. Socrates is implying, both in the earlier passage in Book IV and in the present one in Book VI, that only by the canons of science in this sense will we really understand the nature of justice or anything else. The argument here reflects one in the *Phaedo*(95a6–97b), where he points out that even inexact science presupposes notions such as that of magnitude or quantity from exact science, and then goes on, in the passages on the 'second best method', to discuss exact science.

Socrates' argument that the guardian must take the arduous way is not just that he must obtain a precise understanding of justice and the like: at 504c9 he states that it is also necesary or else "he will never come to the end of the greatest study and that which most properly belongs to him". He is referring here to the study of the Good. The Sun Analogy then follows, interposed between his complaint about inexact science and his discussion of exact science—just as, in the *Phaedo*, the idea of the best order of things is interposed between his expression of dissatisfaction with inexact science and the 'second best method'. Plato's fullest account of the Good in the *Dialogues* is in the Sun Analogy, though perhaps the *Philebus* is the best place to look for a hint of how he proposed to give a rational account of it. But fascinating as this subject is, I will have to limit myself to a brief description of the role that the idea of the Good plays in Plato's conception of true knowledge.

Exact science presupposes a rational order or structure which the phenomena at least roughly exemplify. Why do the phenomena exemplify this rational structure and how is it that *we* should be able to discern just *this* structure in terms of which to understand the phenomena? In the *Phaedo*, the first question was answered in terms of the 'best order of things' and the second in terms of the doctrine of recollection. In the *Republic*, the doctrine of recollection is abandoned (it was never a very good idea[3] and the idea of

[3]See Leibniz, *New Essays*, pp. 78-79.)

the best order of things is incorporated into the idea of the Good, which is intended to answer both questions. Before the theory of evolution by natural selection, these questions seemed to admit of no naturalistic answer and they rightly taxed philosophers up to the time of Leibniz (whose solution resembles Plato's in many respects) and Kant (whose solution is quite different).

But although Plato thought that the efficacy of exact science pressupposed the Good, he did not think that it presupposed *knowledge* of the Good. This is clear from Socrates' disclaimer in the *Phaedo* to knowledge of the best order of things (which would yield the 'best' method of explaining the phenomena). For turning to the second best method would be of no avail if that method required knowledge of the Good. And in the *Republic*, for example at 533a, he is at least ambiguous on the question of whether he knows the nature of the Good. "But that something like this is what we have to see, I must affirm".

Having accounted for the possibility and efficacy of exact science in the Sun Analogy, Plato goes on to discuss it in more detail.

2 Opinion and Knowledge

At 509d-511d we consider a divided line segment

AC represents the sensible domain and CB the intelligible domain of Forms. Correlatively, AC represents the domain of the opinionable (*to doxaston*) and CB the domain of the knowable (*to gnoston*). Plato has already argued in Book V (477-8) that there is such a correlation between kinds of cognition and their objects: he speaks of the faculty or power of opining or knowing, and argues that such a faculty can be distinguished only by "that to which it is related and what it effects". Science or true knowledge is of that which is and opinion is intermediate between knowledge and ignorance: it is about that which both is and is not. He then argues that it is sensible things that both are and are not and it is Forms that are absolutely. Vlastos [1965] argues, correctly I think, that "are" here should not be interpreted intransitively as "exist", since Plato's argument in this connection is that a sensible thing S is both f and not-f, whereas the Form F corresponding to f is always simply f. *Sophist* 259a-b makes it clear that Plato regards "S is" as incomplete, just as "Simmias is small" (*Phaedo* 102b3-d2) is incomplete: the latter requiring completion to "Simmias is smaller than Phaedo", the former to "S is f". Plato is thus saying that true propositions about sensibles are never entirely true but true propositions about Forms are absolutely true.[4]

The tendency to read "is" or "are" as "exist" is closely connected with another tendency, namely to read Plato as holding that knowledge of the Forms is not propositional (knowledge about) but a kind of knowledge by acquaintance (knowledge of). Thus, belief and knowledge are of the existence of objects rather than of facts. Belief is of objects which change and which come into existence and pass out, and so belief is of that which both exists and does not and is both true and false. Knowledge is of objects which are changeless and eternal and so is of that which exists absolutely and hence is true absolutely. I have argued against this view in [1986] and won't repeat the argument here.

But it should be noted that the matter is capable of some confusion because of the ease with which the notion of a fact can be absorbed under that of an object—or perhaps it is a matter of the verb 'to be' in Greek having a wider scope than translators have respected. Thus, at 476e8-477a1, Glaucon asserts that to know is to know something, and then is asked whether 'it' is something that is or something that is not.[5] Another example is at *Theaetetus* 159b, where Socrates distinguishes between the objects Socrates-ill and Socrates-well. Thus, "Socrates is well" is true just in case, or perhaps better, to the extent that the object Socrates-well exists. In other words, in Plato's writings, *facts* or *states of affairs* seem to be easily included under the title "object". This observation will be important in §6, where we attempt to identify the objects corresponding to each of the four segments of the divided line.

Plato does not explicitly say in his initial description of the Line that the intelligible domain consists of the Forms—he merely refers to it as the intelligible domain. This is supposed to give some credence to the view that

[4]It has been pointed out, for example in [Owen, 1957, p.109], that Plato's case for propositions about sensibles never being perfectly true is favored by taking examples involving geometric concepts and looks less plausible when we consider propositions such as "Socrates is a man". But it was the context of exact science that was Plato's concern—although the issue is muddied by that fact that he clearly felt that all science, including political science, could be modeled on the exact sciences that he knew.

[5]See Paul Shorey's note *c*(3) to this passage in the Loeb edition of the *Republic*, Volume I.

Plato's ontology contained the so-called *intermediates* or *mathematicals* that Aristotle attributed to him. But the argument at 477-8 in Book V seems fairly clear on this point. "But in the case of a faculty, I look to one thing only—that to which it is related and what it effects, and it is in this way that I come to call each one of them a faculty, and that which is related to the same thing and accomplishes the same thing I call the same faculty, and that to another I call other." The faculties explicitly mentioned there are opinion and knowledge. Since the Forms are clearly objects of knowledge, I don't see that there is room for intermediates.

I do not want to discuss here precisely what Plato meant by the Forms. If one goes passage-hunting through the dialogues, as Ross did in his *Plato's Theory of Ideas* [1951], one will find references to Forms or, probably better, uses of the same terms that Plato used to refer to Forms throughout his writings. Plato himself showed signs in the later dialogues, e.g. the *Sophist* and the *Parmenides*, that he felt that the 'friends of the Forms' had gotten out of hand. But there is a central role that the Forms play in the *Phaedo* and *Republic* which does concern me here. True knowledge or exact science cannot have as its object sensible things. Plato argues for this in the *Phaedo* (e.g. "Two logs are never exactly equal"), but the conclusive basis for the argument was the discovery of incommensurable line segments in the late part of the preceding century. Reasoning in geometry cannot be founded on what we can see and measure, since measurements cannot distinguish between those lines commensurable with a given one and those which are not. More generally, as Whitehead was later on to put it, nature has ragged edges.[6] The terms in which we describe it in exact science don't literally apply. Then what *is* exact science about? What are the grounds for calling the theorems of geometry true, for example? Neugebauer, in his discussion of this situation in [1969], suggests with an almost charming innocence that the Greeks simply introduced axiom systems in which the phenomena were idealized and then based truth on provability from the axioms. A wonderful idea! But, unfortunately, not one available to the Greeks in fourth century BC: it was to be more than twenty-three centuries before the idea of a formal axiomatic theory would be invented. For example, Frege did not even understand it: for him, as for the Greeks, axioms have to be *true*. But what are they true *of*? What are, to use Plato's terms, the corresponding objects?

[6] In *Metaphysics* I vi 2-3, Aristotle traces the motivation for Plato's doctrine to the influence of Heraclitus' view that "the whole sensible world is always in a state of flux".

We might take from Neugebauer the suggestion that they are true of an 'idealization' of the phenomena. But I think that if we try to spell out what this means, we are led to the view, which I think was essentially Plato's, that they are true of a certain structure which the phenomena in question roughly exemplify, but which, once grasped, we are capable of reasoning about independently of the phenomena which, in the causal sense, gave rise to it. The theorems of geometry are not literally true of sensible things: indeed, they do not even literally apply to them. No sensible figure can be a point or a line segment or a surface or solid in the sense of geometry. Yet the assumptions made in geometric proofs are also not arbitrary; something provides traction for them. We have the *idea* of a point, a line segment, a surface, whatever, which we can, by a process of analysis or, as Plato called it, *dialectic*, come to understand purely rationally, stripped free of its empirical source.

I believe that it is this which provided motivation for Plato's reference to Forms and against which attempts to understand his so-called 'doctrine of Forms' should be measured. I believe also that this conception of autonomous reason in the aid of natural science was Plato's great contribution. (Certainly, before him, Parmenides had emphasized the autonomy of reason; but the evidence suggests that he did not conceive it in aid of natural science.)

We have here the conception which Aristotle refers to as the *separate Forms*. A contrasting view is that the structure studied in exact science is obtained by *abstraction* from the phenomena. This is a very different idea: if I abstract the color of my shirt from my shirt, then what I say of the color is true just in case it is a true statement about my shirt, though it be restricted to the language of color. This seems to be Aristotle's view of geometry: when we speak of geometric figures, we are really speaking about sensible substances, except that what we say is restricted to the language of extension. This is a very different idea from Plato's; and his argument in the *Phaedo* already refutes it. If comparison in magnitude of sensible objects never yields exact results, then abstraction can't make the results more precise.[7]

[7]The most sophisticated attempt to found exact science and in particular geometry on abstraction is Whitehead's, in his method of 'extensive abstraction'. But, whatever version of this one takes, the fundamental relation of extensive connection between the objects of perception—events or regions, as he variously identified them—must be taken to be well-defined, in the sense that it is determined whether or not two such objects are extensively connected. Otherwise, contrary to Whitehead's claim, the ordinary Euclidean geometry of empirical space cannot be derived. But this determinacy of extensive connection is hardly

In any case, the faculty of opinion is our power to ascertain the truth of propositions about sensibles in the rough sense in which these are true. The faculty of knowledge is our power to ascertain the truth of propositions about Forms in the absolute sense.[8]

On Plato's conception, all scientific explanation of phenomena begins with the recognition that they exemplify a certain structure or, as he would say, 'participate in' a certain Form. The sensible thing S is f in virtue of participating in the corresponding Form Φ: that is the 'ignorant' or 'naive' explanation of why S is f (*Phaedo* 100d). As Aristotle put it: "The Forms are the causes of all other things" (*Metaphysics* 987b19). But for Aristotle—as it was to be later on for other philosophers, such as Leibniz and Whitehead—, this was a criticism of Plato's conception, whereas for us, it is precisely the way exact science proceeds.

Having recognized that S is f, one may be further warranted in asserting that S has some other property, e.g. that S is g. That is the 'sophisticated' kind of explanation (*Phaedo*). The sophisticated explanation rests on the naive explanation: the sophisticated explanation of why S is g is that S is f, i.e. that S participates in the Form Φ, together with the fact that Φ is g. For example, consider the propositions

(a) S is right triangular

S is f

(b) The squares on the sides of S equal the square on the hypotenuse

S is g

(c) The squares on the sides of a right triangle are equal to the square on the hypotenuse

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compatible with his view that nature has ragged edges.

[8]It is clear that Plato's use of the term *doxa* does not entirely correspond to our use of the terms 'opinion' or 'belief', since, as we use these terms, that concerning which we can have opinions or beliefs we can also have knowledge and, conversely, we can have opinions (short of knowledge) about what is knowable: one and the same proposition may be an expression of opinion and an expression of knowledge. However, I shall continue to translate 'doxa' as 'opinion.'

As we noted, the sensible figure S is not really a right triangle: indeed, the terms "point", "line" and "angle" in terms of which the notion of a right triangle is to be defined never perfectly apply to sensible things. Nor does the notion of equality (*Phaedo* 74-5). This does not mean, however, that (b) and (c) have no empirical content. The surveyor does indeed apply the Pythagorean theorem and gets good results. But the results, expressed in terms of empirical measurements and constructions, are only 'rough'. And one should not take 'rough' here to mean 'approximate'. For example, the circle can be approximated to any degree of accuracy by an inscribed regular polygon. But here the difference between the two figures is itself a precise magnitude, an area. But the sense in which the sensible figure S is roughly right triangular or in which the result of the empirical construction roughly corresponds to (c) is different from this. It is not a case of one geometric object (in our sense) differing from another by some precise amount: one of the terms in the correspondance is such that the geometric ideas do not perfectly apply to it. Thus it is in the nature of things that (c) applies only roughly to sensible figures and is never absolutely true of them. Plato's notion of *participation*, in spite of the logical positivists' attempts to analyze it in terms of co-ordinating definitions and the like, remains an essential ingredient in the story of how exact science works.

Proposition (b) is a consequence of (a) and (c) (the sophisticated explanation of (b)). (a) and (b) are about sensibles. What about (c)? It reads to us as a general proposition, about all right triangles. But what, for Plato, is a right triangle? As we noted, Aristotle and, following him, many later commentators attribute to him the view that, besides sensibles and Forms, there are perfect instances of the Forms, the so-called 'intermediates'; and, in particular, there are perfect instances of the geometric Forms, such as the Form Right Triangle. On this view, (c) could be read as a general statement about all perfect instances of Φ. Indeed, the doctrine of intermediates is frequently made an integral part of the interpretation of the Divided Line: intermediates are taken to be the objects corresponding to CE. But, as I have already indicated, I think that there is little merit in the view that Plato held that there are such things. (We shall shortly encounter another objection to the idea of the intermediates inhabiting CE.) So, what are the right triangles, the instances of Φ? The only instances there can be for Plato are the imperfect ones, the sensible figures. But now the notion of (c) as a universal proposition in our sense breaks down. For its scope, the right triangles, would have to consist of the imperfect exemplifications of Φ; but when does a sensible figure count as an exemplification of Φ? How 'straight' do the sides have to be and how thin, for example? For this reason, incidently, it seems to me seriously misleading to speak of Forms as universals: for they determine no precise extensions. But, moreover, a universal proposition is true because each of its instances is true. On the other hand, the instances of (c), namely (b), are *never* perfectly true: they are in the domain of opinion. And, however one interprets the Line and, in particular, CE, mathematics is assigned to CB, the domain of knowledge. It is therefore impossible to understand (c) as a universal proposition in our sense: its truth does not derive from the truth of its instances. Rather, it is true of the Form Φ and the imperfect truth of the instance (b) is *explained* by the fact that S exemplifies Φ. (This is a sophisticated explanation.) Let me remark that it is on this point precisely that we have the starkest contrast with Aristotle's philosophy.

Thus true knowledge for Plato is of the Forms, and the faculty of knowledge is our power to discern truth about the Forms. This faculty is reason and does not involve sensible things. It is important to note here, though, that Plato does not deny a *causal* role to sense experience in our coming to know the Forms: the doctrine of recollection in the *Phaedo* and the Sun Analogy explicitly affirm this role. His point is only that, once given the Forms, they have their own internal logic which is the source of truth about them.

Notice that (c), which I assert to be about the Form *Triangle*, is an ordinary geometric proposition. Thus, as I am reading Plato, the propositions of exact science *are* the propositions about Forms. In this respect I am in disagreement with a tradition according to which the doctrine of Forms exists as a separate theory, distinct from and superior to the exact sciences. For example, a common reading of the 'second best method' passage in the *Phaedo* has it that the doctrine of Forms is an *example* of the method, rather than, as I have indicated, its underpinning. But I would challenge anyone to make any real sense of that reading.

It follows then from my reading that, when we say that geometric propositions are *about* the Forms, "about" cannot be understood in terms of the usual corrspondance theory of truth. The theorem that very line segment contains two distinct points, if true of the Forms in the sense of correspondance, would demand the existence of two different Forms ‘Point’. Indeed, we would clearly be led to the existence of infinitely many such Forms by Euclid’s postulates. Aristotle’s *Metaphysics*, Books *M* and *N*, bears witness that there may have been some attempt to understand ‘aboutness’ in the sense of the correspondance theory of truth in the Academy, either by postulating that there are many Forms Point, Line, etc, or by postulating the existence of infinitely many perfect ‘mathematicals’ or ‘intermediates’, to serve as the reference of the terms occuring in the theorem of geometry. But there is simply no indication in his writings that Plato himself held such a view.

3 THE FIRST PROPORTION

The segments AC and CB are to be unequal. I have chosen to take AC<CB, expressing the higher status accorded knowledge over opinion. (Plato, realizing that this is arbitrary, does not specify which alternative we should choose.) Socrates says that AD represents images and DC the corresponding sensibles of which they are images. At 510a-b he asks if Glaucon would be willing – and Glaucon agrees – to express the ratio between AC and CB “in respect of truth or falsety” by the proportion

(1) AD:DC = AC:CB.

Earlier, at 509d-e, Socrates describes (1) as an expression of relative clarity or obscurity. At 511e, he explains that the comparison with respect to clarity concerns the kinds of cognition corresponding to the four subsegments of the Line and the comparison with respect to truth has to do with their objects. But what does this mean? One difficulty is that, if AC and CB each correspond to two kinds of cognition and to two kinds of objects, then which of the four possibilities is the right hand side, AC:CB, of (1) supposed to represent either with respect to clarity or with respect to truth? This, indeed, is a difficulty for *any* view which would have the subsegments of either AC or CB consist of distinct objects. (This is another difficulty for the view that CE consists of intermediates.)[9]

[9]There is a tendency to regard AB as a ‘continuous scale’—an ordered set of points like a thermometer, whose points correspond to degrees of reality or truth or knowledge. This picture is sometimes embellished by placing the sun at C and the Good at B. (Cf.

Some commentators would explain all the ratios involved in the Line, including (1), simply in terms of the image metaphor: just as the things in AD are images of things in DC, so sensibles (AC) are images of Forms (CB). Thus, each segment and subsegment must be understood to represent a kind of object and the ratios express a comparison of image to model. We have already mentioned one difficulty with this view: with which of the image and sensible object is the Form being compared in AC:CB? But another difficulty is that it requires that the relation between CE and EB be understood as one between image and model. But the only image/model relation that Plato suggests with the Forms as models has sensible objects as the images; and sensible objects do not inhabit CE.

So lets think about (1) in a different way. Given that we have agreed to correlate length of line segment with degree of truth, the rough truth of "S is g" compared with the absolute truth of "Φ is g" yields AC<CB.

On the other side of the equation (1), AD:DC could reasonably be thought to concern the relation between a *particular* image I and *its* sensible model S, e.g. the reflection of a sensible figure on a surface and the figure itself. But, in what respect are they being compared? In analogy with the principle division, we might suppose that we are comparing "I is g" with "S is g"; but there are objections to that. If "is g" is predicable only of solid objects, then "I is g" is either absolutely false or meaningless; and in either case, it is hard to see what sense can be given to (1) (since in AC:CB, AC is represented by "S is g", which is neither absolutely false nor meaningless). If "is g" is also predicable of images, as in the case of "is right triangular", then there is no reason to think that in general "I is g" is less true that "S is g". The most serious objection, however, is this: if AD is represented by "I is g" and DC by "S is g", then with which of these two is "Φ is g" being compared in AC:CB?

In view of these considerations, it seems to me that, in both AD and DC, we must be considering the sensible object S and, although we shall only discuss this later on, in both of CE and EB we must be considering the Form Φ. The difference in both cases between the two subsegments must concern

[Grube, 1974, p. 164, fn. 16].) But I agree with [Fogelin, 1971] that no sense can be made of this picture. A segment is not a set of points representing a range of degrees; rather, it is a geometric object representing, in ratio at least, one degree. Otherwise, no sense can be made of the ratios. AD:DC is a ratio of *magnitudes*, not quantities. If the segments are not sets of points, then it follows that the sun and the Good do not occupy points on it.

how we judge the objects in question. Namely, in AC we are concerned with sentences (b) about the sensible figure S. But the grounds for (b) may be of two kinds: in DC we observe S directly, making the necessary constructions and measurements, and in AD we make the observations on an image I of S, say a reflection of S on some surface. Thus, I am suggesting that the difference between AD and DC does not reflect a difference in the sentence in question but rather a difference in the *evidence* for the same sentence. In other words, Socrates is distinguishing two ways of judging in AC, judging about a sensible directly and judging about it on the basis of an image. So if we agree to also correlate length of line segment with degree of *evidence*, then we have the inequality AD $<$ DC.

So we have AC$<$CB and AD$<$DC. But why do Socrates and Glaucon go on to agree to (1), to the assertion that the two pairs are in the same ratio? It is clear that no literal sense can be given to this equation: the left hand term is the 'ratio' of evidence for (b) as judged from the reflection of S and the evidence for (b) as judged from S itself. On the right hand side we have the 'ratio' of the truth of (b) and the truth of (c). On neither side do we in any sense have ratios between like magnitudes. Socrates secures Glaucon's explicit agreement to (1) at 510*a* on the basis of the image/model metaphor: I is to S as S is to Φ. But according to the best sense we can give to this metaphor, in AD:DC we are comparing the evidence I with the evidence S for the same proposition (b); whereas in AC:CB we are comparing different propositions (b) with (c) with respect to truth. Thus, the relation of image to model plays quite different roles in the two sides of (1). [10]

[10]There is another difficulty with the image/model metaphor: in the sense that I is an image of S, they share a form of structure. For example, if I is the reflection of S in a pool, then they share the form of structure that is preserved by the projection of S on the pool from the sun. If S is a sensible triangle, then it has vertices and sides, and I contains images of these. Φ, on the other hand, *is* a form of structure and, as I have argued in [1986] quite independently of the arguments in the *Parmenides*, it is unlikely that Plato held that Φ was also an exemplification of itself. The Form, Triangle, is triangular in a different sense than S is triangular. S participates in Φ (i.e. has the Form Φ), but Φ does not participate in itself. In particular, it does not have vertices and sides. Thus, I is an image of S in a quite different sense than S is an image of Φ.

One might want to argue that Plato simply failed to distinguish two different senses of *paradigme* , namely our sense of 'paradigm' and the sense of a form of structure which a paradigm perfectly exemplifies. This would be analogous to attributing confusion to him because he asserts both that Φ is f and that S is f, which I have argued in [1986b] would be unjust. There might be some grounds for attributing confusion to him if the equation (1) otherwise made literal sense. But we have already noted that it does not; and so it

It seems more reasonable to look for a reason for asserting (1) which is independent of the fact that, literally, it makes no sense. It is true that the Line is just a simile; but it is a very elaborate one and, if it is not just a pretentious bit of mathematical nonesense, then it – and in particular (1) – must have a point. I believe that it indeed does, namely a *rhetorical* point: from (1) we shall derive

(2) AC:CB = CE:EB

so that AC:CB is a measure of the superiority of *noēsis*, EB, over *dianoia*, CE. Thus, I propose to read the Line simile not as an illustration of certain relationships between kinds of cognition and truth, which are meaningful in their own right and need only to be pointed out, but as a dramatic argument: *To the extent that the superiority of DC over AD is a measure of the superiority of CB over AC, it is also a measure of the superiority of EB over CE.* The superiority of exact science, i.e. of knowledge of the Forms, is not the issue here: that is generally agreed upon. The issue is *how* to reason about the Forms; and Plato is arguing rhetorically for the superiority of *noēsis* over *dianoia*.

Of course, in Socrates' original description of the Line at 509d, he specifies that both (1) and (2) are to hold. But at that point he is simply describing the geometric structure of the Line: he has not yet told us how to interpret it. When he explains the interpretation of the segments AC, CB, AD and DC, he immediately gains Glaucon's acceptance of (1) on the basis of the imprecise image/model metaphor. Then, on the basis of (1) and the interpretation of the other segments, he gains his acceptance of (2). When Glaucon agrees to the appropriateness of the simile at 511e, he is agreeing to an argument that began with and depends on the *stipulation* of (1).

4 *Dianoia* and the 'Hidden Equality'

But, to substantiate this reading, we have to go on to consider CE, the domain of *dianoia*. Concerning CE Socrates says

> ... there is one section of it [CE] which the soul is compelled to investigate by treating as images the things imitated in the

would be entirely gratuitous to assume that Plato was confused on this point.

> former division [DC], and by means of assumptions from which it proceeds not up to a first principle but down to a conclusion ...

Glaucon does not fully understand and Socrates tries again:

> ... students of geometry and reckoning [i.e. algebra] and such subjects first postulate the odd and the even and the various figures and three kinds of angles and other things akin to these in each branch of science, regard them as known, and, treating them as absolute assumptions, do not deign to render any further account of them to themselves or others, taking it for granted that they are obvious to everybody. They take their start from these, and pursuing the inquiry from this point on consistently, conclude with that for the investigation of which they set out. ... they further make use of the visible forms and talk about them, though they are not thinking of them but of those things of which they are a likeness, pursuing their inquiry for the sake of the square as such and the diagonal as such, and not for the sake of the image of it which they draw ...(510b3-e1).

Thus, CE is concerned with the Forms, with propositions 'F is g' and not with 'S is g'. This is in conformity with Socrates' general description of the Line, according to which all of CB is concerned with the Forms. What distinguishes CE from EB is that, in the former and not in the latter, we make use of sensibles and we reason from hypotheses without giving an account of them. As I read the above passage, the reasoning from the hypotheses is entirely rigorous; and so we may assume that the appeal to sensibles arises only in the choice of hypotheses. It is reasonable to suppose that Plato has in mind here the sort of reasoning illustrated by the slave's proof of a special case of (c) in the *Meno*. Starting with a drawn figure, further constructions are made from which the equality of certain areas becomes evident, e.g. by appeal to certain symmetries. But these are sensible constructions which, being special, cannot prove a general proposition and, being sensible, in any case do not perfectly exemplify the kind of structure in question. For example, the argument involves the construction of the square on a line segment (in a given half-plane): how do we know that this square exists? (Cf. Euclid's *Elements*, Book I, Proposition 46.) The assumption that it does is the kind of 'absolute assumption' to which Socrates is referring here. This assumption is extracted from the drawn figure, which is inadequate on the two grounds that we have noted. The construction of the square on a given side is an example of what Socrates means by the postulation of a figure. Note also that it is precisely such constructions that the postulates in Euclid's *Elements* provide for.

In DC, (b) is about sensible figures and we establish it by appeal to such figures. Of course, (b) is not about sensible objects *simpliciter*: it is about them as right triangles, with 'sides' and 'virtices' distinguished. But, as we have already noted, no sensible object is really a right triangle. So, with respect to the comparison of (b) and (c), we have that DC is less than CE. Indeed, in this respect, the ratio is identical with the ratio between AC and CB: for this comparison has to do with the kind of object, sensible or Form, that the proposition is about. However, in CE, although (c) is about Φ, we again establish it by appeal to sensible figures in framing our hypotheses. We may reason ever so carefully, but the starting point of our reasoning is tainted by reference to the phenomena. So, although the two sections, DC and CE, are concerned with the different propositions 'S is g' and 'Φ is g', we invoke the same evidence in both of them. Thus, in exactly the same sense in which we have AD<DC with respect to the evidence we invoke, we have the equation

(3) DC = CE.

In other words, given (1) and the interpretation of the segments AC, CB, AD, DC and CE, the point E is determined. This 'hidden equation,' so called because it follows from (1) and (2) but is not explicitly stated, does not seem to me to be adequately accounted for in the literature on the Line. It is either assumed that Plato did not notice it or that he noticed it but it plays no role in his simile, convicting him of a considerable inelegance. But in Book VII at 533e-534a Socrates explicitly states the equations CB:AC = EB:DC = CE:AD as though he were simply restating (1) and (2). And to see this, he would have had to know (3): indeed, the first of these equations and (2) immediately imply (3); and the conjunction of (1) and (2) follow from these equations only under the assumption of (3). It therefore makes no sense at all to suppose that Plato was unaware of (3). And to propose a reading of the Line which makes no sense of (3) is to attribute to Plato a simile that, as he must have been perfectly aware, *immediately* breaks down. It would seem preferable to infer from the fact that, on a given reading, no sense can be made of (3) that one does not have the right reading.[11]

But, in any case, it seems to me that (3) has a very important meaning for Plato. It is clear from the passage just quoted from the Line and from Book VII that he was assuming agreement, at least among his immediate audience, that the source of truth in arithmetic, algebra and geometry is not in the objects of sense but in the Forms. In this respect these subjects are contrasted in Book VII (530-1) with astronomy and music theory, against the practitioners of which Plato complains that they mistake the objects of their study to be the paths of the stars and, say, the string on the monochord, rather than the curves and line segments in which these participate.[12] But what (3) tells us is that it doesn't matter what we take science to be about, the phenomena or the forms of structure which they (imperfectly) exemplify: if we draw on the sensible figure as evidence for (c), then it is no more reliably established than is (b). The point is not that (3) is a consequence of (1) and

[11] It is sometimes argued that Plato cannot have intended the hidden equation because, right before the above cited passage, at 533d, Socrates indicates that *dianoia* involves more clearness than opinion and more obscurity than *noēsis* . Socrates actually uses the term *episteme* here; but he immediately makes it clear that he means *noēsis* . But note that the comparison of *dianoia* , of CE, is not with DC but with AC; and this makes sense only as a comparison with respect to truth and falsety of (c) with (b); for, with respect to the kinds of evidence, AC includes two cases, AD and DC. Thus, in the only reasonable sense of comparison, we have that AC is less than CE. On the other hand, CE and EB are compared with respect to the kind of evidence invoked; and in that sense, we have CE<EB. So the passage at 533d makes perfectly good sense and is compatible with the hidden identity which compares DC with CE with respect to the kind of evidence invoked.

[12] This passage is widely misinterpreted as a call to give up empirical science in favor of 'mathematics'. But, as I noted in [1986b] and above, this is anachronism: Plato had no conception of mathematics in our sense. For example, geometry *was* for him the study of sensible figures and measurement of them. What he understood, put in terms natural for us, is that this subject advanced by idealizing the phenomena that it studied, and he was advocating that astronomy and music theory proceed in the same way – by studying the structure imperfectly exemplified by the phenomena in question.

In fact, this is a slight oversimplification, since Plato was also advocating the study of a more general form of structure than that exemplified by the phenomena: steriometry in general and numerical proportions. Perhaps in part this may be explained by his recognition of the added insight gained from studying the general case: think of Newton studying the dynamics of central forces in general in developing his theory of gravitation. But there is a strong hint (531c-d) that Plato also believed that this study would lead to the discovery of connections among the forms of structure exemplified by the various phenomena – e.g. between the proportions exemplified by harmonies and those exemplified by the periods of the superimposed circular motions of the heavenly bodies – and so to the discovery of the proportions which express the best order of things: the Good.

(2). Rather it is that, given the interpretations of the subsegments of the Line, (3) stands on its own feet and that (2) follows from (1) and (3). This, I believe is the point of an otherwise contrived appearing simile: once Socrates and Glaucon agree on (1), all else follows. If we agree that CB:AC measures the superiority of proving (b) about S by considering S itself over proving it by considering an image of S, then we are forced to accept CB:AC as the measure of the superiority of *noesis* over *dianoia*. And it is at this that the Divided Line simile aims. It is, as I suggested in the beginning of my paper, an argument for foundations of exact sciences. To understand the sense of 'foundations' I have in mind, we should consider what Plato has to say about *noēsis*.

5 *Noēsis*

What is EB, the domain of *noēsis*? At 510b, after the first description of *dianoia* , Socrates continues

> ... there is another section in which [the soul] advances from its assumptions to a beginning or principle that transcends assumptions, and in which it makes no use of the images employed by the other section [i.e. CE], relying on ideas only and progressing systematically through ideas.

This is the point at which Glaucon confesses his failure to fully understand. Indeed, it is not quite clear what the above passage means. Is Socrates saying that, in the advance to a principle, no use of images is made? Grube translates the passage as explicitly saying that. But it doesn't make sense read in that way. From what point would the advance begin, if not with sensible images? The matter is clarified when Socrates reformulates his description of EB at 511b:

> ... by the other section [EB] of the intelligible I mean that which the reason itself lays hold of by the power of dialectic, treating its assumptions not as absolute beginnings but literally as hypotheses, underpinnings, footings, and springboards so to speak, to enable it to rise to that which requires no assumption and is the starting point of all, and after attaining to that again taking hold of the first dependencies from it, so to proceed downward to

> a conclusion, making no use whatever of any object of sense but only of pure ideas moving on through ideas to ideas and ending with ideas.

It is clear from this formulation that Socrates has in mind two stages:

UP STAGE. Up, by means of dialectic, from hypotheses to a principle which transcends hypotheses.

DOWN STAGE. Down from the principle to a conclusion, by means of rigorous reasoning and without appeal to sensibles.

Thus, the answer to the question raised by Socrates' first description of EB seems to be that it is in the Down Stage that there is no appeal to sensibles. There is no reason to suppose that Plato conceived the process of dialectic as not at least beginning with hypotheses that are suggested by sensory experience or 'recollected' from it. Indeed, if I am right that Plato is expanding here on his discussion of the second best method in the *Phaedo*, then it is clear from the latter discussion that he does understand the process of dialectic to begin with what is suggested by empirical experience. The difference between the hypotheses in EB and in CE has to do with the fact that, in the latter case, they are treated as absolute; whereas, in the former case, we analyze them, pushing them back until we come to propositions that transcend hypotheses (or, in the *Phaedo*, until we come to something 'adequate'). Notice that, on this analysis of the passage, it is a mistake to identify, as van der Waerden does [1963], dialectic with deductive reasoning. The former, although it may involve deduction, is the method of the Up Stage, the latter is the sole method of the Down Stage.

So, to sum up my reading of the Divided Line: It is the embodiment of an argument for the deductive method in exact science, for finding the first principles, i.e. the definitions and axioms, which define the structure in question, and then proceeding purely deductively to investigate it. He is arguing that the practice of beginning deductions with premises drawn from the consideration of empirical examples is inadequate, because the empirical examples do not adequately represent the structure.

Probably the most serious objection to this interpretation arises from the question of motivation: why should Plato at that time have been concerned with foundations? I have mentioned one motive, the discovery of incommensurable line segments. The earliest evidence of the discovery of these is about 435 B.C. One might suppose that the reaction to the discovery would have come somewhat earlier than Plato's middle dialogues; but it should be pointed out that the most important reaction to the discovery, a geometric theory of proportion had likely still not been discovered by the time of the *Republic*.

Behind the question of motivation lies a fairly common assumption that the deductive method was already in place at the time that Plato wrote. But the evidence for this assumption is exceedingly thin. It is certainly true that proof, in the sense of deriving something less obvious from something more obvious, had been around for a very long time and was not the invention of the Greeks. Cut-and-paste proofs of the kind found in Books I and II of Euclid existed long before fourth century B.C. and in other cultures besides Greek. One thing characteristic of Greek mathematics is the definition of terms and the ordering of theorems according to dependance, so that earlier ones may serve as lemmas in the proofs of later ones. But when Proclus, who is the main source of information about the development of geometry in classical Greece, wrote of others before Euclid who wrote 'elements', there is no reason to think that he was referring to more than this. In particular, there is no reason to think that the idea of geometry as a deductive science based on *primary truths*, as represented by the Common Notions and Postulates in Euclid's *Elements* preceded the *Republic*.[13]

Indeed, there is evidence in the *Republic* itself to the contrary. First, there is Plato's criticism of the geometers at the end of Book VI and in Book VII (533b-d). This is often taken to indicate that Plato believed geometry, along with the other exact sciences, to be intrinsically inferior to 'real knowledge', namely knowledge of the Forms, and so confined to the realm of *dianoia*. But it is far more plausibly taken to be an indication that geometry had not yet been sufficiently founded on primary truths. Further evidence for this may be found in Book VII (527a), where the geometers' use of terms such as "squaring", "adding", etc., which suggest physical construction, is criticized, although Socrates states that "they cannot help it". A plausible interpretation of this is that Plato is calling for a precise foundation for the notion of geometric construction. The plausibility is moreover reenforced

[13]Indeed, he refers to 'elements' written by the fifth century geometer Hippocrates of Chio; who would have been too early to have been motivated by the discovery of incommensurable lines.

by the fact that the Common Notions and Postulates in Euclid's *Elements*, written some fifty years later, *do precisely that.*

6 The Four Kinds of Objects

There are two places where Plato refers to the objects corresponding to the subsegments of the Line. One is at the very end of Book VI (511e) where he is speaking of the segments as representing affections of the soul, and then says that they should "participate in clearness and precision in the same degree as their objects partake of truth and reality". For, indeed, all of the ratios are equal to AC:CB. But the passage at 534a in Book VII seems unambiguously to associate with each of the four subsegments its own object. He speaks of the objective correlates of AC and CB and then of the "division into two of each of these." In other words, he seems to be saying that both the domain of the sensibles and the domain of Forms are to be subdivided corresponding to the subsegments of the Line. But he also says "let us dismiss [this], lest it involve us in discussion many times as long as the preceding". What precisely Plato had in mind here is perhaps unimportant in relation to the main point of simile, since he does, after all, leave it aside. But also, since he leaves it aside, it is reasonable to suspect that there is a twist in the answer. In view of all this, the most straightforward possibility seems to me to be that the objects in AD are the same objects as in DC, but qualified. Thus, in DC, we judge the sensible object S on the basis of S itself; but in AD, we judge on the basis of S-as-imaged-by-I. That is, in AD the basis, the 'objective correlate' of the judgement about S is not S as it is in itself, nor is the basis of judgement simply the image I. (For example, I may be elliptical; but we nevertheless judge that S is circular.) In the same way, the objects in CE are the same as those in EB, the Forms; but in EB we are judging about Φ as it is in itself, i.e. our judgements are based on deductions from first principles. But in CE we are judging, not on the basis of Φ as it is in itself. Nor are we judging about the sensible S which images Φ. For example, if Φ is the Form, Triangle, the ratio of the sides of S are irrelevant to its representation of Φ. (For example, the sides of S may be uneven, though what we are judging must apply to isosceles triangles, too.) The objective correlate is Φ-as-imaged-by-S.[14]

[14] This approach to the problem of 'objective correlates' is similar in some respects to that of N.D. Smith in his excellent paper [1981]. What I am calling 'S-in-itself', 'S-as-

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imaged-by-I', etc., Smith refers to as *intensional* objects. An essential difference is that he takes CE to be inhabited, not by 'Φ-as-imaged-by-S', but by 'S-as-image-of-Φ'. He considers but rejects my alternative on grounds of the image metaphor: He writes (p. 134) "Unless it can be shown that the poorly conceived Forms of the geometer image the properly viewed Forms of the dialectician, we have not, by making this move [viz. adopting my alternative], generated a contrast analogous to the one between *eikasia* and *pistis*." But that is so because Smith fails to carry the analysis through to the subsegments of AC. For him, the objective correlate of AD is not S-as-imaged-by-I, but I. Thus, he not only admits an asymmetry in his interpretation of the Line, but in my opinion, fails to solve the problem of what the ratio AC:CB is to mean: Which of S and I in AC is being compared with which of Φ and S-as-image-of-Φ/ in CB?

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