Geodesic Universality in General Relativity*

Michael Tamir

Abstract

According to (Tamir, 2012), the geodesic principle strictly interpreted is compatible with Einstein's field equations only in pathologically unstable circumstances and, hence, cannot play a fundamental role in the theory. In this paper it is shown that geodesic dynamics can still be coherently reinterpreted within contemporary relativity theory as a universality thesis. By developing an analysis of universality in physics, we argue that the widespread geodesic clustering of diverse free-fall massive bodies observed in nature qualifies as a universality phenomenon. We then show how this near-geodetic clustering can be explained despite the pathologies associated with strict geodesic motion in Einstein's theory.

1 Introduction

In Einstein's original conception of the general theory of relativity, the behavior of gravitating bodies was determined by two laws: The first (more fundamental) law consisted of his celebrated field equations describing how the geometry of spacetime is influenced by the flow of matter-energy. The second governing principle, referred to as the *geodesic principle*, then provides the "law of motion" for how a gravitating body will "surf the geometric field" as it moves through spacetime. According to this principle a gravitating body traces

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out the “straightest possible” or *geodesic* paths of the spacetime geometry. Not long after the theory’s initial introduction, it became apparent that the independent postulation of the geodesic principle to provide the theory’s law of motion was redundant. In contrast to classical electrodynamics and Newtonian gravitation, general relativity seemed special in that its dynamics providing principle could be derived directly from the field equations.

Though the motion of gravitating bodies is not logically independent of Einstein’s field equations, the geodesic principle canonically interpreted as providing a precise prescription for the dynamical evolution of massive bodies in general relativity does not follow from Einstein’s field equations. To the contrary, in (Tamir, 2012) it was argued that under the canonical interpretation, *not only does the geodesic principle fail to follow from the field equations, but such exactly geodetic evolution would generically violate the field equations for non-vanishing massive bodies.* In short, under the canonical interpretation the two laws are not even consistent.

Despite this failure, the widespread “approximately geodetic” motion of free-fall bodies must not be denied. The nearly-geodetic evolution of gravitating bodies is well confirmed within certain margins of error. Moreover, some of the most important confirmations of Einstein’s theory, including the classic recovery of the otherwise anomalous perihelion of Mercury, also appear to confirm the approximately geodetic motion of massive bodies. This abundance of apparent confirmation suggests that though the claim that massive bodies must exactly follow geodesics fails to cohere with Einstein’s theory, geodesic following may constitute some kind of *idealization* or *approximately correct* description of how generic massive bodies behave.

We must hence reconcile an apparent dilemma: On the one hand geodesic following appears illustrative as an ideal of the true motion of massive bodies. On the other hand the arguments against the canonical view in (Tamir, 2012) reveal that non-vanishing bodies that actually follow geodesics would be highly pathological with respect to the theory, suggesting that they are not suitable as ideal theoretical models. Moreover, even if we were to adopt such models as idealizations, in order to gain knowledge about the paths of *actual* bodies, it is unclear how to draw conclusions about the non-pathological cases by considering pathological models that are generically incompatible with the theory.

In this paper, we establish such a reconciliation by arguing that, in light of the failure of the canonical interpretation, the principle should instead be adopted as a *universality thesis* about the clustering of certain classes of gravitating bodies that exhibit *nearly*-geodetic motion. In section 2, we propose an analysis of the general concept of *universality phenomena* to designate a certain kind of similarity of behavior exhibited across a wide class of (ostensibly diverse) systems of a particular theory. Using this analysis, in section 3, we explain how the nearly geodetic behavior observed in numerous gravitational systems counts as such a clustering within appropriately close (topological) neighborhoods of *anchor models* that exhibit perfect geodesic motion. Finally, in section 4, we explain why such pathological anchor models can be employed to characterize this clustering of the realistic models, without having to reify the problem models or take them as representative of actual physical systems.

2 Universality in Physics

The arguments of (Tamir, 2012) reveal that the geodesic principle cannot be used to prescribe the precise dynamics of massive bodies in general relativity. Nevertheless, the geodesic principle, demoted from the status of fundamental law to a thesis about the general motion of classes of gravitating bodies, may still be of value to our understanding generic dynamical behavior in general relativity. The challenge is to find an appropriate way of characterizing such "nearly geodetic" motion in terms of closeness to perfect geodesic following motion in light of the fact that attempts to model gravitating bodies that could stably follow geodesics end up violating Einstein's field equations. If such a reinterpretation of the principle is well-founded, we must justify its endorsement in the face of the kinds of pathologies associated with actual geodesic motion. This can be done by interpreting the robust geodesic clustering patterns actually observed in nature as a *universality phenomena*. In this section, we begin with an explicit analysis of this concept's use in physics.

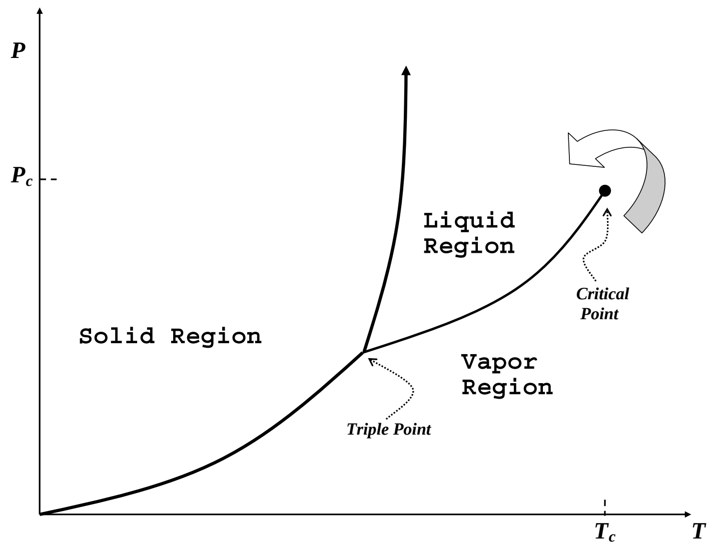

Figure 2.1: *Phase diagram of a generic material at fixed density.*

2.1 The Paradigm Case: Universality in Phase Transitions

The notion of a universality phenomenon was initially coined to characterize a remarkable clustering in the behavior of thermal systems undergoing phase transitions, particularly the behavior of systems in the vicinity of a thermodynamic state called the "critical point." In thermodynamics the state of a system can be characterized by the three state variables pressure, temperature, and density. According to the thermodynamic study of phase transitions, when the state of a system is kept below the particular "critical point" values (P_c, T_c, ρ_c) associated with the substance, phase transition boundaries correspond to discrete changes in the system (signified in figure 2.1 by the thick black lines). If, however, a system is allowed to exceed its critical values, there exist paths available to the system allowing it to change from vapor to liquid (or back) without undergoing such discrete changes. These paths involve avoiding the vapor-liquid boundary line by navigating around the critical point as depicted by the broad arrow in figure 2.1.

There exists a remarkable uniformity in the behavior of different systems near the critical point. One such uniformity is depicted in figure 2.2. In this figure we see a plot of data recovered by Guggenheim (1945) in a temperature-density graph of the thermodynamic

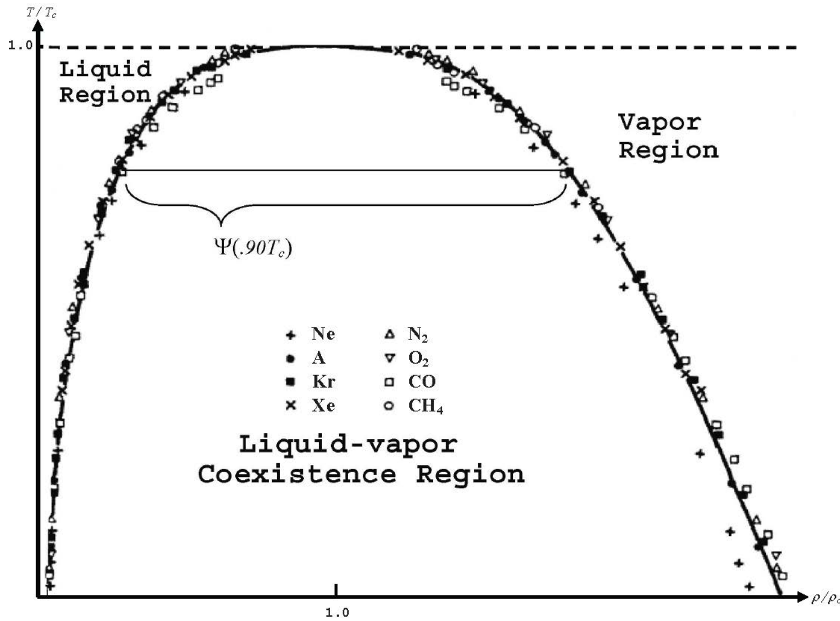

Figure 2.2: *Adapted plot of (Guggenheim, 1945) data rescaled for criticality.*

states at which various fluids transition from a liquid or vapor state to a "two phase" liquid-vapor coexistence region. Systems in states located in this latter region can be in liquid or vapor phases and (according to thermodynamics) maintains constant temperature as the density of the system changes. An important feature exhibited in figure 2.2 is that (after rescaling for the ρ_c and T_c of the respective molecules) the transition points of the each of the distinct substances near criticality appears to be well fit by a *single curve* referred to as the *coexistence curve.* This similarity in the coexistence curves best fitting diverse molecular substances can be characterized by a particular value β referred to as the *critical exponent* found in the following relation:

$$\Psi(T) \propto \left|\frac{T - T_c}{T_c}\right|^{\beta} \tag{1}$$

where the parameter $\Psi(T)$, called the *order parameter* tells us the width of the coexistence curve at a particular temperature value T. As depicted in figure 2.2, as T gets closer and closer to the critical temperature T_c from below, this width drops down eventually vanishing at criticality. We can think of the critical exponent β as telling us about how rapidly such a vanishing occurs. As confirmed by the above data, this number turns out to be similar (in the neighborhood of $\beta \simeq .33$) for vastly different fluid substances.[1]

What is fascinating about examples such as this is not the universal (or "nearly" universal) regularity in physical systems. That uniform reliable regularities (viz. "universal laws") can be found to apply to numerous physical systems (though remarkable) is nothing new. The interesting part is that such uniform reliable behavior occurs *despite the fact that at least at one level of description the systems are so incredibly dissimilar*. From a level of description thought to be perhaps more "fundamental" than the gross state variables (P, T, and ρ) used to characterize thermodynamic systems, the various substances exhibiting similar critical exponent values have quite diverse descriptions: At the quantum mechanical level, for instance, the state vectors or density matrices representing the respective quantum mixtures will be incredibly distinct (e.g. close to orthogonal). Moreover, we need not go down to a quantum level of description to recognize the vast diversity. From a chemical perspective monotonic neon is different from a diatomic oxygen molecule, or an asymmetrical carbon monoxide molecule. We might hence expect surprise from a physicist or chemist since despite such vast differences in the ostensibly pertinent details at these levels of theorizing, the substances still share this observed similarity. This similarity despite such (speciously relevant) differences is what distinguishes the behavior across thermal systems as a kind of *universality phenomenon*. In the next section we begin a more explicit analysis of the concept's general application in physics.

Though the usage of the term originated in the study of thermal systems, universality has now been identified in a multitude of other domains. Over the past decade, Robert Batterman has argued in the philosophical literature that "while most discussions of universality and its explanation take place in the context of thermodynamics and statistical mechanics,... universal behavior is really ubiquitous in science" (Batterman, 2002). A (far from comprehensive) list of vindicating examples includes the clustering behavior found in contexts including non-thermal criticality patterns exhibited in avalanche and earthquake

[1] This similarity in the value of the critical exponent exists not only for thermal fluid systems, but also in describing the behavior of ferromagnetic systems in the neighborhood of a thermal state that can be analogously characterized as the critical point.

modeling (Kadanoff et al., 1989; Lise and Paczuski, 2001), extinction modeling in population genetics (Sole and Manrubia, 1996), and belief propagation modeling in multi-agent networks (Glinton et al., 2010). Batterman has discussed many examples of universality phenomena distinct from criticality phenomena, including patterns in rainbow formation, semi-classical approximation, and drop breaking(Batterman, 2002, 2005). Numerous non-criticality examples of universality have also been discovered in contexts such as the study of chaotic systems exhibiting "universal ratios" in period doubling (Feigenbaum, 1978; Hu and Mao, 1982), or the clustering similarities in models of cold dark matter halos found in astronomical observations (Navarro et al., 2004), to name a couple. In the next section we offer an explicit analysis of the concept's general application in physics.

2.2 The Same but Different: Analyzing Universality

The term *universality* is used in physics to describe cases in which broad similarities are exhibited by classes of physical systems despite possibly significant variations according to apparently "more fundamental" representations of the systems. Kadanoff (2000, p225) describes the term most generally as applying to those patterns in which "[m]any physically different systems show the same behavior." Berry (1987) has characterized it as the "way in which physicists denote identical behavior in different systems." Batterman (2002, p4) explains that the "essence of universality" can be found when "many systems exhibit similar or identical behavior despite the fact that they are, at base, physically quite distinct." Characterizations such as these reveal that the concept hinges on the satisfaction of the two seemingly competing conditions of displaying a particular *similarity* despite other (evidently irrelevant) *differences* in the systems at some level of description. To make this conceptual dependency explicit, we propose the following analysis of *universality phenomena*.

(UP): A class $X_{\mathcal{T}}$ of models of physical systems in a theoretical context $\mathcal{T}$ will be said to exhibit a *universality phenomenon* whenever the class can simultaneously meet the following two conditions:

(Sim) There exists a robust similarity in some observable behavior across the physical systems modeled by members of $X_{\mathcal{T}}$.

(Var) This similarity in the behavior of members modeled in $X_{\mathcal{T}}$ is stable under robust variations of their state descriptions according to context $\mathcal{T}$.

The first thing to specify is what counts as a "class of models of physical systems in a theoretical context." In order to avoid complications associated with multiple (possibly not entirely equivalent) formulations of a full physical theory, **(UP)** is best analyzed in terms of the more restrictive notion of a theoretical context $\mathcal{T}$ which identifies within a given theory a particular formulation and variety of studied phenomena. Examples of different theoretical contexts in classical mechanics include the Hamiltonian versus the Lagrangian formulations, or in quantum mechanics we might distinguish between wave mechanics and operator mechanics.[2] A theoretical context may also restrict the phenomena considered by the total theory. For example, *source free* classical electrodynamics might be considered a distinct theoretical context within the full theory of classical electrodynamics which also models the effects of sources. In some cases it is possible for a theoretical context $\mathcal{T}$ to specify an entire theory uniquely, in other cases, a specification in terms of (potentially nonequivalent) formulations and specific phenomena types may be appropriate.

Given a particular theoretical context $\mathcal{T}$ of a universality phenomena, the expert will typically be able to identify pertinent state descriptions "according to context $\mathcal{T}$." For example, in classical electromagnetism the relevant state description may come in the form of fields specifying the flow of the source charges and the electromagnetic field values throughout a spacetime; in general relativity the metric and energy-momentum tensors might play this role; in thermodynamics, state descriptions may be parametrized by P, T, and ρ (or perhaps V and N), whereas in quantum statistical mechanics one may use density operators.

Satisfaction of (Sim) is primarily an empirical question. In order to claim that something universality-like is occurring, there must be an evident similarity in the class of systems exhibiting the phenomenon. This evident similarity need not be (directly) in terms

[2] Note, in both dichotomies there exist occasional circumstances or conditions such that the respective formulations can cease to be equivalent.

of any of the state descriptions used to characterize elements of $X_{\mathcal{T}}$. So for the paradigm example of the universality of phase transitions, (Sim) is satisfied once physicists recover sufficient empirical data of the kind depicted in figure 2.2. The robust similarity of (Sim) can be quantified in terms of the remarkable closeness of the critical exponents of these various systems even though the critical exponent parameter β may not necessarily be put in terms of the state quantities of $\mathcal{T}$ (e.g. chemistry or statistical mechanics).

Satisfaction of (Var) depends primarily on the size and most importantly the diversity of the models in class $X_{\mathcal{T}}$. The larger and more varied the members of class $X_{\mathcal{T}}$ with respect to the relevant state descriptions of $\mathcal{T}$, the more "stable under variations." If $X_{\mathcal{T}}$ is suitably rich with diverse members, then a member $x \in X_{\mathcal{T}}$ may be "mapped" to a rich variety of other members of $X_{\mathcal{T}}$ while still maintaining the very similarity shared by all members of $X_{\mathcal{T}}$ that allowed the class to satisfy (Sim). In the paradigm example of thermal universality, (Var) is satisfied by the fact that at the chemical or the statistical mechanics levels of description, the members in our class sharing this similar critical behavior are so diverse.

We note that the central concepts of *robust variation* and *robust similarity* on which (Var) and (Sim) respectively depend are not binary. Some universality phenomena may be "more robust" than other instances, in terms of both the "degree" of similarity displayed and the "degree" of variations that the systems can withstand while still exhibiting such similar behavior. The greater the robustness of the pertinent similarity in behavior across the class of systems and the more ($\mathcal{T}$-state) variation in the class, the more robust the universality is.[3] This non-binary dependence means universality may be subject to vagueness challenges in some cases. While certain examples, such as thermal criticality behavior and, as we argue, the clustering behavior of free-fall massive bodies around geodesic paths may be identified as determinant cases of universality, penumbral cases where it is unclear whether a candidate universality class is sufficiently similar and robust under variations may exist.

[3]Often this can be rigorously assessed by an appropriately natural norm, metric, topology, etc. defined on the state descriptions of $\mathcal{T}$. E.g. we might use some integration norm to quantify the difference between two (scalar) fields found in $X_{\mathcal{T}}$. The choice of appropriate norm, topology, etc. identifying differences in the members of $X_{\mathcal{T}}$ is directly dependent on the context $\mathcal{T}$.

3 The Geodesic Universality Thesis

In this section we reconsider the case of near-geodesic clustering observed in nature in terms of the **(UP)** analysis. In 3.1 we examine why such clustering qualifies as an example of a universality phenomenon. In 3.2 we then identify how the limit operation result of Ehlers and Geroch offers what we identify as a *universality explanation* of this clustering.

3.1 The Similarity and Diversity of Geodesic Universality

Consider a sequence of classes $(X^{\epsilon}_{\mathcal{GR}})_{\epsilon \in (0,s)}$ indexed by some sufficiently small error parameter $\epsilon \in (0, s)$. For fixed ϵ, the class $X^{\epsilon}_{\mathcal{GR}}$ consists of (local) solutions to Einstein's field equations:

$$T_{ab} = G_{ab} \tag{2}$$

where the energy-momentum field T_{ab} describes the flow of matter-energy and G_{ab} describes the "Einstein curvature" determined by the metric field g_{ab}. Moreover, each member of $X^{\epsilon}_{\mathcal{GR}}$ models some massive body whose spacetime path comes close to following a (timelike) curve γ that is close to actually being a geodesic (where these two senses of closeness are parametrized by respective functions monotonically dependent on the smallness of ϵ). With the **(UP)** analysis in hand, for a given degree of "ϵ-closeness" we can now ask if such a class $X^{\epsilon}_{\mathcal{GR}}$ satisfies the (Sim) and (Var) conditions in the context of general relativity theory purged of the canonical commitment to geodesic dynamics argued against in (Tamir, 2012).

The satisfaction of (Sim) is an empirical matter apparently well confirmed by centuries of astronomical data recovered from cases in which a relatively small body (a planet, moon, satellite, comet, or even a star) travels under the influence of a much stronger gravitational source. Examples involving non-negligible relativistic effects (like the Mercury confirmation) are of particular importance, but even terrestrial cases including Galileo and leaning towers or other (nearly) free-fall examples in determinately Newtonian regimes can count as confirming instances for certain ϵ-closeness values. Since observational precision is inevitably bounded, it is often claimed that the satellite, moon, planet, etc. indeed "follows a geodesic," despite the results of (Tamir, 2012). In such instances, the body is actually observed to come "close enough" to following a geodesic to warrant such equivocation. These instances hence confirm membership in a class $X^{\epsilon}_{\mathcal{GR}}$ for some ϵ threshold below the level of experimental precision or attention.

In order to appreciate the satisfaction of (Var), we must consider the relevant theoretical context of general relativity theory. State descriptions of physical systems according to the theory come in the form of the tensor fields T_{ab} and g_{ab}, related by the equations (2). Assuming we only consider (local) solutions to Einstein's equations, there exist six independent field components describing g_{ab} and so the matter-energy flow T_{ab}. In other words, from a fundamentals of relativity theory perspective, there are six physical degrees of freedom to how these bodies are described at each spacetime point.

Given the wealth of evident confirming instances falling under a class $X^{\epsilon}_{\mathcal{GR}}$ with suitable ϵ, there will be significant variation in terms of these degrees (even after rescaling) once we consider the significant differences in the density, shape and flow of the matter-energy of a planet, versus a satellite, asteroid, anvil, etc. In these "fundamental state description" terms, the diversity of the bodies in a given class $X^{\epsilon}_{\mathcal{GR}}$ will be quite significant. Despite this diversity, such bodies still satisfy the defining requirement of ϵ-closeness to following a geodesic. It is with respect to this diversity in these degrees of freedom (of the energy-momenta/gravitational influences of the "near-geodesic following bodies" of members in $X^{\epsilon}_{\mathcal{GR}}$) that a "robust stability under variations" can be established in accordance with (Var).

So, according to our **(UP)** analysis, such near-geodesic clustering observed in nature constitutes a *geodesic universality phenomenon*. However, meeting the conditions of the analysis depends entirely on the truth of the above made *empirical* claims about the existence of bodies well modeled by members of the respective $X^{\epsilon}_{\mathcal{GR}}$ classes for a suitable range of ϵ values, and that the bodies in each class are so fantastically diverse from the perspective of their T_{ab} (g_{ab}) fields. In the next section we turn to the more *theoretical* question of understanding how such geodesic universality is possible in general relativity, by considering the properties of the classes $(X^{\epsilon}_{\mathcal{GR}})_{\epsilon\in(0,s)}$ in terms of an important geodesic result of Ehlers and Geroch (2004).

3.2 Explaining Geodesic Universality

We have now formulated the geodesic universality thesis in the context of general relativity as an empirically contingent claim about classes of the form $X^{\epsilon}_{\mathcal{GR}}$ whose members model a physical system such that the path of some body counts as ϵ-close to being geodetic without violating Einstein's field equations. We have also given a plausibility argument suggesting why observational data already obtained by experimentalists confirms this empirical hypothesis. Moreover, given such confirmation and the diversity of the energy-momenta of the respective bodies, membership in some $X^{\epsilon}_{\mathcal{GR}}$ will be sufficiently stable under significant variations of the fundamental state descriptions of the theory to satisfy (Var). A remaining theoretical question must now be answered: *How can the systems exhibiting this universality phenomenon behave so similarly while being so different at the level of theoretical description fundamental to general relativity?*

Geodesic universality can be explained by appealing to an important "limit proof" of the geodesic principle discussed in (Tamir, 2012). It was argued there that Ehlers and Geroch (2004) are able to deduce the "approximate geodesic motion" of gravitating bodies with relatively small volume and gravitational influence, by considering sequences of energy-momentum tensor fields with positive mass of the form $(\underset{(i,j)}{T}{}_{ab})_{i,j\in\mathbb{N}}$, referred to as "EG-particles." The spatial extent and gravitational influence of these EG-particles can be made arbitrarily small by picking sufficiently large i and j values respectively. The theorem of (Ehlers and Geroch, 2004) entails that if for a given curve γ there exists such an EG-particle sequence, then by picking a large enough j, γ comes arbitrarily close to becoming a geodesic in a spacetime containing the $\underset{(i,j)}{T}{}_{ab}$ instantiated matter-energy.

Specifically, let $(\underset{(i,j)}{g}{}_{ab})_{i,j\in\mathbb{N}}$ be the sequence of metrics that couple to these $(\underset{(i,j)}{T}{}_{ab})_{i,j\in\mathbb{N}}$ according to (2) in arbitrarily small neighborhoods $(\mathcal{K}_i)_{i\in\mathbb{N}}$ of γ, containing the support of the respective $\underset{(i,j)}{T}{}_{ab}$. Then if for each i, as $j\to\infty$ the $\underset{(i,j)}{g}{}_{ab}$ approach a "limit metric" g_{ab} in the $\mathscr{C}^1(\mathcal{K}_i)$ topology, which keeps track of differences in the metrics and their unique connections, then the curve γ approaches geodicity as $j\to\infty$.

To understand the impact of the theorem for our universality classes $(X^{\epsilon}_{\mathcal{GR}})_{\epsilon\in(0,s)}$, we need to appreciate the kind of limiting behavior established by Ehlers-Geroch. The limit result essentially establishes a kind of "ϵ-δ relationship" between, **(a)** how "nearly-geodetic" we want the curve γ to be, and **(b)** how much we need to bound the gravitational effects of the body on the background spacetime.[4] That is to say, the Ehlers-Geroch limit result can be thought of as telling us that "for every degree of ϵ-*closeness* to geodicity we want the bodies' path to be, there exists a δ-*bound* on the gravitational effect of the body that will keep the path at least that close to geodicity." The important thing to observe about this ϵ-δ interplay is that though the limiting relationship *does* require imposing a δ-bound on the perturbative effects of the body, it does not impose any *specific constraints on the details* of how the matter-energy of the body flows within the ϵ-close spatial neighborhood of the curve, nor how the metric it couples to specifically behaves. So though the metric is "bounded" within a certain δ-neighborhood of the limit metric, the particular details of the tensor values, the corresponding connection, and especially the curvature have considerable room for variation so long as they stay "bounded in that neighborhood."

This relationship established by the Ehlers-Geroch theorem hence gives us a kind of *details-free* way of understanding the diverse populations of our respective universality classes $(X^{\epsilon}_{\mathcal{GR}})_{\epsilon\in(0,s)}$. In effect the Ehlers-Geroch limiting relationship highlights that for each $X^{\epsilon}_{\mathcal{GR}}$ class, there exists a particular δ-bound around a limit metric with some geodesic anchor γ such that any body coupling to a metric that stays within that bound (in addition to remaining spatially close enough to γ) satisfies the relevant ϵ-closeness part of the requirements for membership in $X^{\epsilon}_{\mathcal{GR}}$. But as we just emphasized, *falling under this δ-bound does not impose specific constraints on the detailed values of the energy-momenta or metric fields.* In other words, membership in the universality class $X^{\epsilon}_{\mathcal{GR}}$ is possible as long as the body is a massive solution to Einstein's equations, and its gravitational effect and extent are sufficiently bounded in the right way, but beyond these requirements the specific details concerning "what the gravitational effect does below those bounds" are

[4] For purposes of exposition, we characterize the established relationship as an "ϵ-δ relationship," suggesting that the closeness relations in question have been quantified, the actual Ehlers-Geroch result is formulated (primarily) in topological terms. See (Gralla and Wald, 2008, §3-5) for a more explicitly quantified approach.

irrelevant. Hence, the limit behavior established by the Ehlers-Geroch theorem explains how the ϵ-clustering near geodesic anchors is possible despite significant differences in the energy-momenta of our near-geodesic following bodies: So long as the bodies' gravitational influences are bounded in the right way their (positive) matter-energy can vary as much as we like under those bounds.

4 Explanation without Reification

Before concluding there remains a potential challenge concerning how we can endorse any kind of geodesic "idealization" thesis if the actual geodesic motion of massive bodies is incompatible with Einstein's theory. Recall, while explaining how the classes $X^{\epsilon}_{\mathcal{GR}}$ whose respective members are "ϵ-close" to geodesic following models could be so diverse, we needed to take the "geodesic limit" of the metrics $(\underset{(i,j)}{g}{}_{ab})_{i,j\in\mathbb{N}}$ coupling to the EG-particles $(\underset{(i,j)}{T}{}_{ab})_{i,j\in\mathbb{N}}$ in accordance with the equations (2).[5] By taking such a "geodesic limit" to identify the diversity of our $X^{\epsilon}_{\mathcal{GR}}$ classes, haven't we made an "essential" appeal to the kind of pathological models precluded by Einstein's field equations?

The answer to this challenge is that though appreciating the kind of ϵ-δ interplay in the appropriate neighborhoods of the geodesic limit was essential to our explanation of geodesic universality, the role played by the limiting *geodesic anchor model* does not require us to reify the idealization or make it representative of any physical system in Einstein's theory. Even though there are significant complications associated with what happens *at* the geodesic limit **(1)** the ϵ-δ behavior of the systems has a well-defined mathematical structure (the $\mathscr{C}^1$ topologies defined for each spacetime neighborhood of γ) describing the *approach* to the limiting anchor model, and **(2)** the behavior of the models in $X^{\epsilon}_{\mathcal{GR}}$, which are "close but not identical to" a geodesic anchor model, still obey Einstein's theory. A geodesic anchor model establishes (as the name suggests) a kind of *anchor* for the (topological) neighborhoods within which the elements of the respective

[5]Note, though the $({}_{(i,j)}g_{ab})_{i,j\in\mathbb{N}}$ converge to a well defined "geodesic limit" (in the $\mathscr{C}^1$ topologies) the coupled energy-momentum tensors $({}_{(i,j)}T_{ab})_{i,j\in\mathbb{N}}$ may not. Moreover, even if they do converge in a physically salient and independently well-defined way, at the limit they must either fail to obey (2) or vanish. For a detailed discussion see (Tamir, 2012, §4).

$X^{\epsilon}_{\mathcal{GR}}$ can be said to cluster. However, using these models as anchors to identify the points around which the *actual solutions* to Einstein's equations cluster does not require that the anchors themselves be admitted in $X^{\epsilon}_{\mathcal{GR}}$.

In contrast to more traditional "idealizations," universality phenomena are about the *group behavior of classes of* $X_{\mathcal{T}}$ not *individual systems*. For non-universality idealizations severe pathologies can be detrimental because they render *the sole idealized model* theoretically inapposite. With universality, however, the existence of a pathologically idealized model "close to but excluded from" a universality class need not entail that members of the class are likewise poorly behaved. Moreover, if a topological clustering "near to" an idealized model has physical significance (as with the $\mathscr{C}^1$ topologies), such proximity may allow inferences about the well-behaved classes without molesting their admissibility according to the laws of $\mathcal{T}$.

This is precisely what occurs with geodesic universality. Members of a class $X^{\epsilon}_{\mathcal{GR}}$ can take advantage of their closeness to the geodesic anchor models without "contracting" the pathologies occurring *at* the actual geodesic limits. Moreover, we were able to *explain* such ϵ-closeness by appealing to what we characterized as the "specific details irrelevant" δ-closeness in the $\mathscr{C}^1$ topologies. Since we are talking about geodesic universality, we are able to infer directly from such ϵ-closeness that the relevant bodies modeled by the members of $X^{\epsilon}_{\mathcal{GR}}$ are *close* to following a geodesic in the relevant physical senses defined when we constructed the classes.

5 Conclusion

While the incompatibility result of (Tamir, 2012) entails that the geodesic principle strictly interpreted must be rejected at the fundamental level, in this paper we have argued that reinterpreting the role of geodesic dynamics as a universality thesis is both viable and coherent with contemporary general relativity. By developing an analysis of universality phenomena in physics, we saw that the widespread geodesic clustering of a rich variety of gravitating, free-fall, massive bodies actually observed in nature qualifies as a geodesic universality phenomenon.

Not only can this approximation of geodesic dynamics be recovered in the form of such a geodesic universality thesis, but by reconsidering the implications of limit operation proofs of the principle, we were able to generate a universality explanation for why we can expect such a remarkable clustering of these gravitating bodies despite the fact that from the perspective of their more fundamental relativistic descriptions (the energy-momentum field and its gravitational influence) they may be incredibly dissimilar. We concluded with a defense of our appealing to pathological geodesic anchor models in explaining the universality clustering. Unlike more traditional forms of approximation or idealization, as revealed by the **(UP)** analysis, when it comes to universality phenomena, the claim is about *the group behavior* of entire classes of models, not individual idealizations. Hence, in the case of universality, it is possible to take advantage of relevant types of mathematical proximity to pathological anchors without actually infecting the members of the class with the illicit behavior. Moreover, when the right kind of (topological) closeness is employed it may be possible to draw inferences and gain knowledge about the physical properties of modeled systems thanks to this proximity of their models to the pathological anchor.

References

Batterman, R., 2002. The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction, and Emergence. Oxford University Press, USA.

Batterman, R., 2005. Critical Phenomena and Breaking Drops: Infinite Idealizations in Physics. Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 36 (2), 225–244.

Berry, M., 1987. The Bakerian Lecture, 1987: Quantum Chaology. Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences 413 (1844), 183.

Ehlers, J., Geroch, R., 2004. Equation of Motion of Small Bodies in Relativity. Annals of Physics 309 (1), 232–236.

Feigenbaum, M., 1978. Quantitative Universality for a Class of Nonlinear Transformations. Journal of Statistical Physics 19 (1), 25–52.

Glinton, R., Paruchuri, P., Scerri, P., Sycara, K., 2010. Self-Organized Criticality of Belief Propagation in Large Heterogeneous Teams. Dynamics of Information Systems 40, 165–182.

Gralla, S., Wald, R., 2008. A Rigorous Derivation of Gravitational Self-force. Classical and Quantum Gravity 25, 205009.

Guggenheim, E., 1945. The Principle of Corresponding States. The Journal of Chemical Physics 13, 253.

Hu, B., Mao, J., 1982. Period Doubling: Universality and Critical-point Order. Physical Review A 25 (6), 3259.

Kadanoff, L., 2000. Statistical Physics: Statics, Dynamics and Renormalization. World Scientific Publishing Co.

Kadanoff, L., Nagel, S., Wu, L., Zhou, S., 1989. Scaling and Universality in Avalanches. Physical Review A 39 (12), 6524–6537.

Lise, S., Paczuski, M., 2001. Self-organized Criticality and Universality in a Nonconservative Earthquake Model. Physical Review E 63 (3), 036111.

Navarro, J., Hayashi, E., Power, C., Jenkins, A., Frenk, C., White, S., Springel, V., Stadel, J., Quinn, T., 2004. The Inner Structure of Λ CDM Haloes–III. Universality and Asymptotic Slopes. Monthly Notices of the Royal Astronomical Society 349 (3), 1039–1051.

Sole, R., Manrubia, S., 1996. Extinction and Self-organized Criticality in a Model of Large-scale Evolution. Physical Review E 54 (1), 42–45.

Tamir, M., 2012. Proving the principle: Taking geodesic dynamics too seriously in Einstein's theory. Studies in History and Philosophy of Modern Physics 43, 137–154.