

# The World as Wavefunction

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There may be a way to maintain a monistic wavefunction ontology, but it is certainly not *trivial* to see what that way is

(Maudlin 2010, 129, his emphasis)

Murray Gell-Mann and James Hartle have made an original proposal for a version of decoherent histories quantum mechanics which is specifically tailored so as not to imply the emergence of multiple quasi-classical worlds. I argue that there is an explanatory gap in the proposal but that there remains an important insight which might be used to tackle what can be seen as an outstanding problem for Hugh Everett III's "relative state" interpretation of quantum mechanics, if it is understood as giving ontic priority to the wavefunction. The argument employs a recent novel analysis of self-location within any sort of multiverse.

## 1. All Coherence Gone.

"A quantum universe can be described by many decohering sets of alternative coarse-grained histories - many realms". So wrote Jim Hartle (2010, 94). Characterized in that way, decoherent histories quantum mechanics (DHqm) is suggestive of Hugh Everett III's "relative state" interpretation (RSqm) in the version currently championed by theorists such as Simon Saunders and David Wallace<sup>1</sup>. But Hartle and Murray Gell-Mann, both involved in the development of DHqm, have long been reluctant to concede that many realms, many emergent worlds, exist. And that's understandable; the idea that we have vast multitudes of doppelgangers living divergent lives on other planets which are constantly

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<sup>1</sup> For discussion and references see (Saunders 2010a, 8-12; Wallace 2010a).

emerging from an underlying “fine-grained” physical reality can seem a somewhat extravagant hypothesis which is to be avoided if possible.<sup>2</sup>

Of course, it may well be that our universe is spatially infinite and so contains a countable infinity of finite, causally isolated Hubble spheres whose past lightcones are isomorphic to the one which constitutes the visible hypersurface of our local region. That would also yield hosts of doppelgangers living parallel and divergent lives. But on our current understanding it is a contingent matter whether a spatially infinite universe contains infinitely many galaxies. In that case, maybe we have multitudinous doppelgangers and maybe not. But many see RSqm as entailing that similar legions exist *necessarily*. Furthermore, if all Hartle’s realms were real, that can seem to imply that we and our environment are constantly branching into myriad worlds which would conventionally be described as possible rather than actual. In trying to free DHqm of this specter Gell-Mann wrote:

To use the language we [he and Hartle] recommend is to address the familiar notion that a given system can have different possible histories, each with its own probability, it is not necessary to become queasy trying to conceive of many ‘parallel universes’, all equally real. (1994, 138)

Decoupling DHqm from RSqm has not proved so easy, but in a new paper Gell-Mann and Hartle (2011) claim to have achieved their goal of having DHqm without the many worlds which they take to be entailed by RSqm. Their argument turns on a concept of “extended probabilities”. These have values outside the range 0-1 but are precisely related to conventional probabilities with values within that range (ibid., 4, eqn. 2.2). The extended probabilities apply to fine-grained histories and it is the decoherent coarse-graining of sets of fine-grained histories which yield the conventional probabilities attaching to coarse-grained histories. The upshot is a novel account of, for instance, a two-slit experiment. For an

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<sup>2</sup> Jeffrey Barrett (2011) argues that RSqm is not committed to the existence of many worlds, in which case Gell-Mann’s and Hartle’s concern can be seen as misplaced.

electron arriving at the detector screen as part of the interference pattern it is true that it passed through only one of the slits. But it is not possible to assign probabilities to the electron having passed through the left slit or the right slit. Only extended probabilities can be assigned to these fine-grained alternatives, implying that it would be impossible to settle a bet on which of the alternatives actually occurred (ibid., 3).

Downstream from the slits, just before the electron reaches the screen, the characteristic waveform of the electron is to be understood as the coarse-graining of an ensemble of fine-grained histories. The squared modulus of amplitude yields conventional probabilities but the waveform is to be understood as describing a *notional* ensemble of fine-grained histories *only one of which exists*. Again, the single extant electron is somewhere in front of the screen but only an extended probability can be assigned to any possible region where that electron may be. The idea that the ensemble of fine-grained histories is notional and only one of them actually exists is central to Gell-Mann's and Hartle's attempt to avoid the many-worlds of RSqm. As they put it:

This real history is described by embedding it in an ensemble of comparable imagined fine-grained histories, not unlike the familiar ensemble of statistical mechanics. (ibid., 1)

Describing the fine-grained histories requires a choice of preferred variables and Gell-Mann and Hartle write:

The fine-grained histories are described by a preferred set of variables which we take to be those of a sum-over-histories formulation of quantum mechanics. They are histories of particle positions in the case of particles, four-dimensional field configurations — both bosonic and fermionic — in the case of quantum field theory, and histories of geometries and fields in the case of semiclassical quantum gravity. Histories of these variables

are assumed to be the most refined description of the system possible.

A strong case can be made that these histories are adequate for the prediction of all observable quantities. (ibid., 3)

But, accepting that assumption, there still seems to be a problem with Gell-Mann's and Hartle's proposal. Their one true fine-grained history is rather like a lone ant in an anthill. The erratic path which the ant follows may well be identical to that of an ant interacting with a host of fellows, the ant may move as if it were one of a community even if it's all alone, but if no such community exists any *causal explanation* of why the ant moves thus and so looks obscure. There's a fundamental difference between the proposed notional ensemble and the "familiar ensemble of statistical mechanics". It is that for the classical system there is a simple dynamics internal to one instance of the ensemble. But Gell-Mann's and Hartle's unique fine-grained history has no such simple internal dynamics. As they put it:

The set of all fine-grained histories between an arbitrary pair of times  $t_0$  and  $t_f$  is the set of all such paths  $\{q^i(t)\}$  between these two times. They are continuous but typically non-differentiable. (ibid., 3)

The unique fine-grained history is only calculable because it behaves as if it were interacting with other members of the ensemble. Histories which have not decohered *interfere* non-negligibly. But if Gell-Mann and Hartle were right quantum interference would not exist as a physical process. It would be purely notional. Now, it may be logically possible that there is one fine-grained history with an internal dynamics which is extremely complex and only describable as if it were one of an ensemble of histories mutually interacting in a way described by the evolution of the wavefunction. But, on the face of it, isn't it more plausible that the ensemble *exists*?

I shall not attempt to press that point further. What I'm concerned to show here is that *if* the ensemble of fine-grained histories is regarded as extant rather

than notional then Gell-Mann's and Hartle's work may help to resolve what some have seen an outstanding problem for RSqm. It has long been overlooked by many working with Everett's relative state interpretation that it provides no coherent realist account of what's going on in the paradigm two-slit experiment. According to RSqm what passes through the slits is the *wavefunction* of the electron. But what, precisely, is that? Tim Maudlin (2010) stresses that this fundamental ontological question remains unanswered. In what follows I'm going to argue that Gell-Mann's and Hartle's characterization of fine-grained histories in terms of extended probabilities, coupled with a novel interpretation of the location of an observer within a multiverse, can yield an answer to the question which Maudlin poses. This need not in any way threaten Hartle's claim that "Extended probabilities give a simple and general way of formulating quantum theory" (2008, 1).<sup>3</sup>

## **2. Self-Location in a Multiverse.**

Maudlin's problem has to do with the constitution of philosophers' tables and chairs. It can seem that according to RSqm they are nothing but seething masses of wavefunction. But if Gell-Mann's and Hartle's one fine-grained history really were all that exists then a table could, for instance, be understood as being constituted by a fragment of a history involving particles with definite, albeit non-differentiable, trajectories. The wavefunction would be banished, so to speak, to the domain of the notional ensemble.

However, if Gell-Mann's and Hartle's ensemble were taken as extant rather than notional then the wavefunction would become instantiated by that multiplicity of fine-grained histories. In that case our local environment would seem to be constituted by a sort of minimal slice of wavefunction. The idea would be this. If the ensemble exists then the one coarse-grained history which we inhabit would consist of a multitude of mutually interacting fine-grained histories. But presumably our own bodies and brains would be constituted by elements of

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<sup>3</sup> For an alternative approach to providing a fundamental ontology for RSqm see (Wallace and Timpson 2010).

only one of those fine-grained histories. Each of us would have a multitude of doppelgangers in environments which were isomorphic at the coarse-grained level but anisomorphic at the fine-grained level. For a bunch of such doppelgangers about to observe an electron detected at the screen of a two-slit apparatus there would be some doppelgangers relative to whom the electron had gone through the left slit and some relative to whom it had gone through the right slit.

On this picture our philosophers' table is constituted by a minimal slice of wavefunction because we as observers are only ever instantiated by one fine-grained history, even if the other fine-grained histories of the ensemble exist. The table which a given observer sees would be a table sharing a particular fine-grained history with that observer. But a change of perspective is possible which brings the wavefunction closer to home. The inspiration comes from reflection on a genre of thought experiments which have been prominent in analytic philosophy of mind in recent decades and are often called Twin Earth thought experiments<sup>4</sup>. We are asked to imagine two matched doppelgangers, each on one of two isomorphic Earthlike planets. Because these thought experiments are directed towards understanding the relation between a creature's mentality and its environment, an anisomorphism is usually introduced but for the moment let's think about a pair of planets which are perfectly isomorphic.

Until recently it has been universally assumed that in a Twin Earth setup there are two numerically distinct subjects, one on each planet, and the discussion focuses on whether or not their mentality differs. However, it turns out that it is possible to interpret the setup as involving a single subject with a single mental life which is doubly instantiated, just as any matched pair of objects on the planets instantiate one and the same physical form. The full details of this novel interpretation of doppelgangers are to be found in (Tappenden 2011b) and it would be inappropriate to set out the arguments at length here, but I shall outline the salient points.

Call the single subject instantiated by a pair of doppelgangers Una and imagine that Una is thinking of a table that it is round. On a standard representational view of mind it is supposed that Una entertains the thought

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<sup>4</sup> See (Putnam 1975) for the seminal paper.

because she deploys a mental representation. It is perhaps most plausible that that representation is manifest as a cerebral process<sup>5</sup>. On that assumption, in Una's case what constitutes the *single* mental representation she deploys is the *pair* of relevant isomorphic cerebral processes going on in the heads of the two doppelgangers.

Ordinarily, for a mental representation to represent a table's being round it is supposed to be the vehicle of a representational content which is expressed by the proposition "T is round" where T is the table in question. How the representation comes to bear its content is controversial but need not concern us. However, in Una's case it appears that what she's thinking about is not a table but a pair of isomorphic tables. So in order to coherently interpret Una as a single subject it is necessary to understand her as thinking not about a table but about what might be called a 2-table, where a 2-table is not just any pair of isomorphic tables but rather a corresponding pair of isomorphs in matched worlds of the Twin Earth genre. Clearly that genre of multiverse may contain any number of isomorphic worlds of the sort Una's doppelgangers inhabit so there's scope for 3-tables, 4-tables and so on, assuming that worlds are understood as finite and causally isolated, like Hubble spheres.

So we might attribute the content expressed by "<sub>2</sub>T is round" to Una's mental representation, where <sub>2</sub>T is the 2-table in question. But this raises a problem. It makes no sense to say of a *pair* of tables that it is round. What is needed is to modify the predicate. The appropriate modification is to introduce the idea that an n-table is n-round if and only if each of its component tables is round. So we attribute the content "<sub>2</sub>T is <sub>2</sub>round" to Una's mental representation. Where the conventional "plural" interpretation of doppelgangers would suppose that there were matched doppelgangers in a pair of isomorphic worlds each thinking of a local table that it is round, on the novel "unitary" interpretation we have Una thinking of a single 2-table that it is 2-round.

The unitary interpretation of doppelgangers can be applied to understanding the thoughts, speech and intentional actions of a single subject spanning any

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<sup>5</sup> "Extended mind" theorists currently claim that some mental representations may be constituted by elements of an organism's environment but that is controversial and anyway does not affect the

number of isomorphic worlds. But this is from a third-person point of view. We imagine the interpreter as somehow standing outside that multiverse and looking down on it; the god's eye perspective. But how might we think of ourselves as inhabiting a multiverse, each of us spanning a multitude of isomorphic worlds? What we see as a table is a table, not an n-table. And a round table in our observed environment cannot be round because it is an *aggregate* of round objects. If we observed a table to weigh a dozen kilograms that table could not be an aggregate of a multitude of objects each weighing a dozen kilograms. The weight of an aggregate is the sum of the weights of its parts.

The way forward is to suppose that an object in our environment may be a *set* of isomorphic *elements*. Willard Van Orman Quine once proposed that any particular, such as a table, could be understood as being a self-membered singleton set (1969, 31-2). The idea now is that a table in our environment may be a multipleton set, reducing to being a self-membered singleton if there is no multiverse. But if we span a number  $n$  of isomorphic worlds we can imagine that any number of them might suddenly disappear, leaving us spanning any sample of the original collection but still faced with a table. So any *subset* of the original table must also be a table. Any table we're left with could then be understood to have some *observable* property if and only if each of its subsets had that property. It's necessary to introduce the qualification "observable" because clearly the number of elements constituting a table which is a multipleton set cannot be the same as for each of its subset tables. However, it is plausible that if we were to span a multitude of isomorphic worlds the magnitude of that multitude would not be observable; if the number of worlds should change we would not notice any difference. This leads to the following proposal for the constitution of objects in our environment if we are to coherently understand ourselves as being single subjects spanning a multitude of worlds. Corresponding matched objects in isomorphic worlds can be called "parallel world counterparts".

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central idea here. See (Menary 2010) for discussion of this issue.



## Concrete Sets

Any environmental object  $O$  is a set of self-membered singletons which are parallel world counterparts and  $O$  possesses some observable property  $F$  if and only if all the subsets of  $O$  possess  $F$ . (Tappenden 2011b, 133)<sup>6</sup>

According to Concrete Sets, if we inhabit a multiverse consisting of a number  $n$  of isomorphic worlds then any table we observe in our local environment has  $n!$  subsets which are all isomorphic tables and our table is round because all its subset tables are round. Note that the idea has so far been applied to thought experiments of the Twin Earth genre where quantum mechanics is not an issue. I shall come to applying the idea to an ensemble of fine-grained histories in the final section. But first it will be useful to pursue these thoughts further in a classical context.

### 3. A Classical Model for Linear Superposition.

Return to the god's eye view of our two isomorphic planets and imagine that Una reports seeing a black box before her. What we see from our detached perspective is two matched black boxes and we can understand Una as speaking truly if we take her to be referring to a 2-black 2-box. But suppose that, unbeknown to Una, the interiors of the matched black boxes are causally isolated from their surrounding environments allowing a contained anisomorphism between the two planets. One box contains a black cat and the other contains a white cat. Una's 2-box thus harbors a 2-cat since each component box contains a cat, but the 2-cat is neither 2-black nor 2-white since the component cats are neither both black nor both white. However, we could say that Una's 2-cat is  $\frac{1}{2}$ -black and  $\frac{1}{2}$ -white in the sense that half the component cats of the 2-cat are black and half are white.

Imagine now what would plausibly happen if Una were to unmask the contents of her 2-box. On one planet a doppelganger's retinas would register an

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<sup>6</sup> This proposal is incompatible with David Lewis's Main Thesis in his *Parts of Classes* (1991, 7)

image of a black cat and on the other planet the corresponding doppelganger's retinas would register an image of a white cat. We could expect Una to undergo personal fission into an observer seeing a black cat and an observer seeing a white cat. The analogy with the RSqm fission account of what would happen to an observer unveiling Schrödinger's proverbial boxed cat is unmistakable. There has been much discussion of how an informed subject about to make an observation using, say, a Stern-Gerlach apparatus to measure spin should understand their predicament if believing RSqm<sup>7</sup>. Here I shall simply assume that such a subject can intelligibly suppose that their situation is in some sense like that of a subject who believes that a stochastic process takes place and expects to see either one or the other possible outcome with a given probability. Since it is controversial whether such analyses can be understood in terms of genuine probabilities I shall speak of \*probability (star probability) for branching contexts. To be clear about this, the idea is that if a subject knows that they will undergo personal fission into subjects observing different outcomes A, B, C... then prior to fission the subject can intelligibly assign \*probabilities to observing A, observing B, observing C and so on. There is then a question of what values the \*probabilities should take. For RSqm much work has gone into attempting to show that those values should be equal to the relevant squared moduli of amplitude.<sup>8</sup>

So, if Una were to be informed about her situation prior to opening the 2-box, the analogy with RSqm would suggest that she should regard her situation as somehow being like that of being able to assign probabilities to the seeing of a black or a white cat. Since Una is to fission into two subjects a first thought might be that she should assign a \*probability of  $\frac{1}{2}$  to each outcome. But on further reflection this becomes problematic. For suppose Una spanned three isomorphic planets and were faced with a 3-box where two component boxes contained matched black cats and the third contained a white cat. We should still expect that Una would fission into one observer seeing a black cat and one seeing a white cat. Which might again suggest that if informed she should suppose the situation as being somehow like having a fifty-fifty chance of seeing black or white. And yet

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<sup>7</sup> For a variety of views see (Saunders and Wallace 2008; Greaves and Myrvold 2010; Saunders 2010b; Papineau 2010; Tappenden 2011a).

it's tempting to think that the *proportions* of parallel worlds containing black cats and white cats might somehow be analogous to the squared moduli of amplitude in the quantum-mechanical case.

This suspicion can in fact find some backing. So as not to be restricted to a finite model and the concept of proportion, suppose Una spans an infinite number of parallel worlds and now, into the classical model, let's introduce the idea of an irreducibly stochastic process which selects whether a boxed cat on a given planet is black or white. We can imagine that Una's multi-box is equipped with a multi-button and when she presses the multi-button a stochastic process takes place on each planet. And suppose that on each planet the stochastic process is such that there's a two-thirds chance of the cat being black and a one-third chance of its being white. The limit frequency measures across the set of worlds will be two-thirds for black cats and one-third for white cats.

Now let's for a moment abandon the unitary interpretation of doppelgangers and revert to the standard plural interpretation. In that case, we have one subject in each world where a stochastic process takes place and if each of those doppelgangers, prior to opening the box, were to state their chance of seeing a black cat they would be speaking truly if they were to say that the probability of seeing a black cat was two-thirds. But for the unitary interpretation of doppelgangers to be consistent Una must be taken to be speaking truly if she says that her situation is somehow like that of there being a probability of two-thirds that she will see a black cat. In other words, she should assign a \*probability of two-thirds to that outcome.

If talk of \*probabilities in branching scenarios is acceptable then it should be possible on the unitary interpretation to take Una as speaking of \*probabilities rather than probabilities. But this alternative interpretation would collapse if the \*probabilities of the unitary interpretation were not numerically equal to the probabilities of the standard plural interpretation. One might say, "so much the worse for the unitary interpretation" but that wouldn't be justified because in the situation where Una fissions into two subjects making different observations *there is no prior reason* to suppose that she should assign a \*probability of  $\frac{1}{2}$  to seeing

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<sup>8</sup> For recent work on this issue see (Wallace 2010b).

one or the other outcome. And so there is no overriding justification for throwing out the unitary interpretation of doppelgangers on those grounds.

The classical setup does not *require* that Una has a multi-button which initiates a stochastic process. I just used that idea for ease of exposition. Stochasticity could simply be introduced in determining the “initial conditions” for classical worlds, as in any classical deterministic theory. In that case the picture which emerges for the classical model is that if we have a large collection of worlds which are parallel up to a point but with differing initial conditions then an observer spanning a fiber-bundle of such worlds will be subject to personal fission when observing differing phenomena as that fiber-bundle differentiates into sub-bundles. And the limit frequency measures of worlds in branching sub-bundles will be numerically equal to the informed subject’s \*probability assignments.

We can now apply this idea to an extant ensemble of fine-grained histories where a fine-grained history is understood in terms of extended probabilities, always remembering that the picture relies on the acceptability of the choice of preferred variables.

#### **4. A Constitution for Wavefunction.**

Return to the two-slit experiment and consider a part of an electron’s wavefront just prior to impinging on the detector screen. It contains the characteristic interference pattern. And suppose that Gell-Mann’s and Hartle’s notional ensemble of fine-grained histories actually exists. Now apply the unitary interpretation of doppelgangers and Concrete Sets. In that case the single electron in the observer’s environment is composed of a multitude of subset electrons. Each fine-grained history contains a relevant self-membered singleton subset electron.

Relative to the observer, one subset electron has passed through the left slit and one has passed through the right slit (for the sake of simplicity I ignore marginal pathways such as those tunneling through the barrier in which the slits are made). Each of those subset electrons has a host of singleton electrons as

elements. For the chosen region of wavefront about to impinge on the screen there is a distribution of those fine-grained singleton subset electrons, each with an associated extended probability in relation to being at a particular finite area of location. As decoherence takes place with the interaction between the observer's single environmental electron and the screen, the ensemble of fine-grained histories branches into many coarse-grained histories, each composed of a sub-ensemble of fine-grained histories. For each of those coarse-grained histories the squared modulus of amplitude yields a \*probability associated with the electron being detected in a particular finite area of the screen and for RSqm those \*probabilities are to be interpreted as being the so-called branch weights.

The observer fissions into multiple observers, each seeing an electron activate a different detector in the array which makes up the screen. The single electron in each of the downstream observers' environments is a subset of the single electron in the upstream observer's environment. And the single electron in each of the downstream observers' environments has a host of singleton electrons as elements. Each downstream observer spans a multitude of fine-grained histories which is a subset of the original ensemble and branch weight is a measure of the subset of histories constituting a branch relative to the pre-branching set of histories.

What we have here is an account of branching for RSqm which puts flesh on wavefunction. In general, any object in an observer's environment, such as a table, can be understood to be *constituted* by wavefunction in the sense that the wavefunction of the table is the manifestation of an ensemble of temporal segments of fine-grained histories which the observer inhabits as a single subject. The observer's body and brain spans the same ensemble of fine-grained histories as are spanned by the observed table at the time it is observed, that table being understood as a set with many tables as subsets. This account makes metaphysically precise a picture of the structure of the quantum multiverse towards which David Deutsch has been working (1985, 20; 1997, 43-5; 2002).

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