BOOK REVIEW: CARNIELLI, Walter & MALINOWSKI, Jacek (eds.). *Contradictions, from Consistency to Inconsistency* (Trends in Logic 47, Springer International Publishing, 2018, VI+322 pages)

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Abstract: In this review I briefly analyse the main elements of each chapter of the book centred in the general areas of logic, epistemology, philosophy and history of science. Most of them are developed around a fine-grained investigation on the principle of non-contradiction and the concept of consistency, inquired mainly into the broad area of paraconsistent logics. The book itself is the result of a work that was initiated on the Studia Logica conference "Trends in Logic XVI: Consistency, Contradiction, Paraconsistency and Reasoning – 40 years of CLE", held at the State University of Campinas (Unicamp), Brazil, between September 12-15, 2016.

Trends in Logic is the conference series of the journal *Studia Logica*, covering contemporary formal logic and its relations to other disciplines. The works collected in this volume were initiated by the discussions that took place at the conference to commemorate the 40th Anniversary of the Centre for Logic, Epistemology and History of Science. The title of the event celebrates one of the three main areas of CLE – that has been called as the epicentre of a "Brazilian school of paraconsistency". The reasons for that are the original works of da Costa, followed by his pupils and collaborators that are part of CLE's history. Simply put their interest include the development of systems strong enough to encompass most of mathematics, while avoiding some well-known logical paradoxes.

Not surprisingly most of the works in this volume are developed around paraconsistent logics or, as it is explained by the editors in the introductory chapter, they are concerned about the subtle distinctions between consistency and non-contradiction, as well as among contradiction, inconsistency and triviality. There are many interesting problems discussed in the book, some of them are well-known among readers familiar with paraconsistency. Setting aside the introduction, the book itself does not intend to be a historical review on the main questions regarding the subject. Rather, the chapters help the reader to taste some information about where paraconsistency is now and where it is heading, as well as cast new lights on some old questions regarding the consistency of formal theories. In what follows I succinctly present the central elements of each chapter.

The introduction briefly presents some state-of-the-art discussion regarding the central questions that permeate the book. In the homonym chapter, Carnielli and Malinowski explain the title of the book and show the relevance of the subject in contemporary discussions in logic and philosophy of science – themes that are familiar to the authors. Walter Carnielli is full professor of Logic at the State University of Campinas (Unicamp) and served as the director of CLE, as well as editor and member of editorial boards of major journals. Some of his works encompass for instance combinations of logics, many-valued and paraconsistent logics – like the logics of formal inconsistency advanced by Carnielli and Marcos (2002) that systematises a large class of paraconsistent logics. Jacek Malinowski is the editor-in-chief of Studia Logica, Head of the Department of Logic and Cognitive Science at the Polish Academy of Science and Head of the Section of Logical Semiotics at Nicolaus Copernicus University in Torun, Poland. He has published works in several areas, for instance logical foundations of computer sciences, nonmonotonic and cognitive logic, just to name a few. The book reflects the multidisciplinary interest of the editors.

The second chapter (the first of 13 collaborative papers) brings Arenhart's investigation on an overlooked argument advanced by da Costa (1997) to the effect that there may be true contradictions about the concrete world. The novelty of the chapter "The Price of True Contradictions About the World" is bridging da Costa's argument to a well-known dialetheist understanding of paraconsistency. By advancing several objections to the argument, the daring conclusion drawn by the author is that the acceptance of true contradictions about the world comes with heavy prices to pay: for instance adopting an inconvenient conservative and pessimistic attitude towards change in science.

In "The Possibility and Fruitfulness of a Debate on the Principle of Non-contradiction", Estrada-González and del Rosario Martínez-Ordaz go back to the Aristotelian arguments regarding the principle of non-contradiction (PNC) originally advanced in his Metaphysics. The aim is to show how they can be used for a better understanding of the different standpoints that are present in the contemporary debate. The authors advance five major stances regarding the debate on the PNC, namely: Detractors, Fierce supporters, Demonstrators, Methodologists and Calm supporters. They suggest how we can find elements of those instances in several authors in the literature, from Aristotle up to the present. Maybe the main claim of this chapter is that one can find all the elements of Calm supporters already in Aristotle's works.

Friend and del Rosario Martínez-Ordaz explore a formal method to model the fact that sometimes mathematicians and scientists reason with inconsistent premises while denying that this is possible or makes any sense – a tooll called *Chunck and Permeate* (C&P) advanced by Bryson and Priest (2004). Roughly speaking, C&P divides a given proof with inconsistent premises into consistent subsets, called *chunks*, and allows only some information to permeate from one chunk to the next. In "Keeping Globally Inconsistent Scientific Theories Locally Consistent", the authors extend C&P by adding a visual representation of chunks in the form of bundle diagrams. By extending it, they apply the method to analyse a case in physics and discuss the implications of inconsistency toleration in science, possibly opening up avenues for other discussions in the role of logic in science.

In "What is a Paraconsistent Logic?", Barrio, Pailos and Szmuc recall some canonical definitions of paraconsistent logics (advanced for instance by Priest, Tanaka and Weber (2016); Carnielli and Coniglio (2016); and Ripley (2015)) in order to suggest a new one. By taking into account a meta-inferential notion of explosion, the authors bring into the light the fact that some logical systems might validate the *Explosion Principle* but invalidate a meta-inferential version of it. Relaying on some well-formulated logical and philosophical reasons, this chapter advances the novel thesis that a logic is paraconsistent if it invalidates either the inferential or the meta-inferential notion of Explosion.

Being so, a number of systems in the literature turn out to be, in that sense, paraconsistent logics.

Gaytán, D'Ottaviano and Morado present a system motivated by the problems of modelling explanation from the point of view of Philosophy of Science. In the chapter "Provided You're not Trivial: Adding Defaults and Paraconsistency to a Formal Model of Explanation", the authors advance the so-called GMD framework. Within that formal system it is possible to make an analysis of the interaction between rules and a minimal conception of *context* – composed by a *set of beliefs* (a minimal idea of a *theory*) in interaction with an *inferential engine* (a *logic*). In order to illustrate this novel epistemic system, the authors adopt it to analyse the concept of explanation using Reiter's default theories and a specific paraconsistent logic of da Costa.

In the chapter "Para-Disagreement Logics and Their Implementation Through Embedding in Coq and SMT", Woltzenlogel Paleo advances a novel approach to para-disagreement logics. The basic language is the usual propositional one, extended with *box* and *diamond* operators from modal logics and the @ operator from hybrid logics. The semantics are very similar to *possible worlds* for modal logics with small differences regarding the representation of world reachability. This framework allows a fine-tuned approach regarding information source, so that conflicting information from different sources can be consistently combined. By suggesting some possible semantical embeddings in Coq and SMT, the author advocates the implementation of automated reasoning tools for these logics.

Džamonja and Panza, in "Asymptotic Quasi-completeness and ZFC", put forward a thesis that the axioms ZFC of first order set theory is actually very powerful at some infinite cardinal, contrary to what it could be stated. Since ZFC axioms are subject to Gödel's

Incompleteness Theorems (cf. Gödel (1931)), if they are assumed to be consistent then they are necessarily incomplete – a fact that can be supported by various concrete statements, including the celebrated Continuum Hypothesis. In order to illustrate their thesis, it is explained that by looking at limits of uncountable cardinals, such as $\aleph_\omega$, and working with singular cardinals (which are necessarily limits, cf. Kojman (2011)), at such cardinals there is a very serious limit to independence. Furthermore, many statements which are known to be independent on regular cardinals become provable or refutable by ZFC at singulars. The thesis then follows by the fact that the behaviour of the set-theoretic universe is asymptotically determined at singular cardinals by the behaviour that the universe assumes at the smaller regular cardinals. Being so, ZFC foundationally provides an asymptotically univocal image of the universe of sets around the singular cardinals.

"Interpretation and Truth in Set Theory" also presents an inquiry on some fundamental questions of set theory. In this chapter, Freire grasps concrete axiom systems in terms of a double-layer schema: respectively containing the conceptual and the deductive components of the system. The conceptual component is identified with a criterion given by directive principles, supposable bounding the subject matter of the system. After advancing two lists of directive principles for the set theory, the set-theoretic truth and the fixation of truth-values in each double-layer picture that emerged from these lists are then analysed. It is worth noticing that the general approach that is forwarded in this chapter can be applied to other mathematical theories with interesting results.

In the short but sturdy chapter "Coherence of the Product Law for Independent Continuous Events", Mundici demonstrates a formal result regarding probability theory: the product law for logically independent events (for Boolean as well as for continuous MV-algebraic events) follows from de Finetti's fundamental notion of a coherent set of betting odds, in the same sense that it was originally demonstrated for the additivity law by de Finetti's 1932 Dutch Book theorem.

In the chapter "A Local-Global Principle for the Real Continuum", Magossi and Rioul present a logical flow of proofs in the most influential undergraduate and graduate textbooks on Real Analysis in the U.S.A., France and Brazil in order to start a discussion regarding the local-global principle (LG) as a new efficient and enjoyable tool for proving the basic theorems of real analysis. Both, LG (any local and additive property is global) and the related principle of global-limit (GL: any global and subtractive property has a limit point) could be used as basis for a new presentation of the integral, just as Cousin's lemma was used to build the Kurzweil-Henstock integral – what the authors intend to advance in future works.

The chapter "Quantitative Logic Reasoning" by Finger brings an unifying approach on some logical systems, namely propositional Probabilistic Logic (classical propositional logic enhanced with probability assignments over formulas); first-order logic with counting quantifiers over a fragment containing unary and limited binary predicates; and propositional Łukasiewicz Infinitely-valued Probabilistic Logic (a multi-valued logic for which there exists a well-founded probability theory). From the viewpoint of Quantitative Logic Reasoning, the author shows that analogous properties hold throughout that class of systems, and presents for each one a language, semantics and decision problem, followed by normal form presentation and satisfiability characterization. Furthermore, complexity results and decision algorithms are also advanced.

Carnielli, Mariano and Matulovic advance an algebraic method based on the polynomial representation of first-order sentences in order to introduce algebraic semantics for first-order logic, departing from modern forms of "algebraizing a Logic" tradition like presented by Blok and Pigozzi (1989). In "Reconciling First-Order Logic to Algebra" the authors employ the notion of M-rings, rings equipped with infinitary operations that can be naturally associated to the first-order structures and each first-order theories. It is shown that infinitary versions of the Boolean sums and products are able to express algebraically first-order logic from a new perspective. This chapter also suggests an unifying algebraical approach to logic by opening-up avenues for possible generalizations of the method to n-valued and other non-classical logics.

In the book's last chapter, Marcelino, Caleiro and Rivieccio clarify the efficiency of some novel techniques in the study of Hilbert-style logics. In "Plug and Play Negations" the authors focus on the negation fragments of logics which result from different possible choices of well-known rules involving the connectives $\{\rightarrow,\bot\}$, with a few variations – in fact negation is usually introduced as a derived connective by making $\neg p := p\rightarrow\bot$ (that is, using the material implication $\rightarrow$ and the *falsum* constant $\bot$). In turn the degree of tolerance to inconsistencies of a logic (degree of paraconsistency) can be determined by the interaction among these three connectives. The techniques used allow for a modular analysis of the logics, providing complete semantics based on (non-deterministic) logical matrices and complexity upper bounds.

Albeit the great diversity of themes discussed in the chapters, all of them can be subsumed into a broadly understood study of consistency – the book perfectly demonstrates how issues surrounding that study go well beyond traditional inquiries on paraconsistent logics, taking novel perspectives that are not too far away from such inquiries. The selection helps the reader to perceive how those works intersect with core traditional mathematical and philosophical questions. *Contradictions, from Consistency to*

Inconsistency nicely supplements the existing literature on the subject. This is a volume that it is well worth reading!

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