

Frege's recognition criterion for thoughts and its problems

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Abstract According to Frege, we need a criterion for recognising when different sentences express the same thought to make progress in logic. He himself hedged his own equipollence criterion with a number of provisos. In the literature on Frege, little attention has been paid to the problems these provisos raise. In this paper, I will argue that Fregeans have ignored these provisos at their peril. For without these provisos, Frege's criterion yields wrong results; but with the provisos in place, it is of no use for Frege's purposes. This is connected to what Frege took to be the 'greatest difficulty for philosophy': natural language sentences don't just express thoughts; they convey evaluations and communicative hints. Because of this, Frege's recognition criterion for thoughts cannot be applied to them and we cannot make logical progress by 'recognising a thought in different linguistic guises'.

Keywords Frege · Thoughts · Equipollence · Sameness of sense · Difficulties in apprehending contents

1 Introduction

According to Frege, we need a criterion for recognising when different sentences express the same thought to make progress in logic. Frege hedged his own equipollence criterion with a number of provisos. In the literature on Frege, little attention has been paid to the problems these provisos raise. In this paper, I will argue that Fregeans have ignored these provisos at their peril. For without the provisos Frege's criterion yields wrong results; but with the provisos in place, the criterion is of no

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use for Frege’s purposes. This is connected to what Frege took to be the ‘greatest difficulty for philosophy’: natural language sentences don’t just express thoughts; they convey evaluations and communicative hints. Because of this, Frege’s recognition criterion for thoughts cannot be applied to them and we cannot make logical progress by ‘recognising a thought in different linguistic guises’.

The structure of the paper is as follows: In Sect. 2, I will outline what a recognition criterion for sameness of thought expressed is and why Frege held that we need such a criterion. In Sect. 3, I will introduce and clarify the role of the notion of sentence content in Frege’s equipollence criterion. It is often noted that there is a tension between Frege’s criterion and his metaphysics of thoughts. In Sect. 4, I will argue that the tension is spurious. Sections 5–8 are devoted to the role of the provisos in Frege’s criterion.

2 The need for recognition criterion for sameness of thought expressed

Thoughts, Frege tells us, are those things ‘for which the question of truth arises’.¹ The question of truth, for example, does not arise for the assertoric sentence ‘London is the largest city in Europe’: it is neither true nor false; at best it is true (false) *at a time* as a sentence in English. But some assertoric sentences in natural and formal languages express thoughts. For example, the thought expressed by the sentence ‘London is the largest city in Europe’ when it is uttered at a certain time can be assessed for truth and falsity and when we judge we acknowledge the truth of a thought.

Among the sentences that express thoughts, some express the same thought. Frege argues that it is a substantial task to establish which sentences express the same thought. It is also a task logic must tackle. Why?

Consider an example from Frege’s *Begriffsschrift*:

- (1a) Archimedes perished at the capture of Syracuse.
- (1b) The violent death of Archimedes at the capture of Syracuse is a fact.

In 1879, Frege did not yet have the notion of a thought at his disposal but he argued that (1a) and (1b) have the *same* judgeable content for which the question of truth and falsity arises. Logic is solely concerned with judgeable contents (thoughts), the relations between them and the laws that govern them. Hence, if (1a) and (1b) express the same judgeable content, logic should *not* distinguish between them. The distinctions between them are merely grammatical and can be ignored by logic. If we want to design a language like Frege’s *Begriffsschrift*—an ideography whose sentences only express thoughts—it should dispense with grammatical subject and predicate.²

In order to develop a language or script suitable for logic, we need to be able to recognise whether two different sentences express the same thought. Frege’s example suggests further that we must be able to recognise the same thought even if it is expressed by sentences that differ in grammatical form as (S1a) and (S1b) do and that seem to concern different objects and properties.

¹ Frege (1918/9, p. 292 [62]). Page references to German texts are in square brackets.

² For the question of what a *Begriffsschrift* is that takes the historical background into account, see Barnes (2002). I will come back to this in due course. I use ‘*Begriffsschrift*’ for Frege’s book and ‘*Begriffsschrift*’ for the ideography developed in the book.

In order to recognise whether two different sentences express the same thought, we need a criterion for sameness of thought expressed. Frege is clear about this:

[T]he task of logic can hardly be performed without trying to recognize the thought in its manifold guises [...] (Frege 1892b, pp. 171–172 [195]).

And in a later letter to Husserl, he wrote:

It seems to me that an objective criterion is necessary for a recognising a thought again as the same, for without it logical analysis is impossible (Frege 1906c, p. 70 [105]).

Frege uses ‘criterion’ in its standard sense. A criterion is a ‘distinguishing mark’.³ A distinguishing mark is a property or relation that allows one to recognise whether something has a distinct, further property. A property F or relation R is a distinguishing mark of a distinct property G if one can recognise that something or some things is (are) F (stand in R) independently of and prior to recognising whether it is (they are) G and the fact that an object is F (some objects stand in R) indicates that it is (they are) also G . There are different varieties of indication. For instance, if a mushroom looks and smells a certain way, it is likely that it is poisonous. However, here we will be mainly concerned with a criterion that specifies a relation whose holding *guarantees* that some sentences express the same thought. The relation specified may also figure in the identity conditions of thoughts, that is, an answer to the question under which conditions the thought T_1 is the same thought as the thought T_2 . But it need not do so. The primary role of the criterion is epistemic: it enables us to *recognise* whether two sentences express the same thought or not.

3 Frege’s recognition criterion

In the last section, we saw that Frege needs a criterion for sameness of thought expressed that allows him to decide whether sentences that differ in structure express the same thought or not. Frege’s ‘equipollence criterion’ (RecognitionEC) satisfies this need.⁴ It is worth quoting the passage in which he introduced the ‘equipollence criterion’ (RecognitionEC) in full:

[P1] When I use the word ‘sentence’ in what follows, I do not mean [optative, imperative, interrogative sentences], but assertoric sentences. [...] Now two sen-

³ For instance *Grimms Wörterbuch* gives ‘Unterscheidungskennzeichen’ for ‘Kriterium’.

⁴ Frege himself proposed two different recognition criteria. One is framed in terms of provable sameness of truth-value. See Frege (1906b, pp. 67–69 [105–6]). The problem with this criterion is well-known: the criterion classifies logically equivalent sentences such as ‘ $2 + 2 = 4$ ’ and ‘ $7 + 5 = 12$ ’ as expressing the same thought. But by Frege’s own light, these sentences express different thoughts. I will therefore set this criterion aside and focus on the so-called ‘equipollence criterion’ (RecognitionEC). See, for example, Heijenoort (1977, pp. 105–6), Klement (2002, p. 92ff) and Schellenberg (2012, p. 164f). See Beaney (1996, p. 228ff) for a discussion of the difference between the criteria. Sander (2016) defends the provability criterion and takes it to be in harmony with the equipollence criterion if the thinkers that acknowledge contents are *ideal* thinkers. I take it that Frege is interested in a criterion that is usable for ordinary thinkers and will therefore focus on (RecognitionEC).

tences *A* and *B* can stand in such a relation that anyone who acknowledges the content of *A* as true must straightaway [acknowledge] the content of *B* as true and, conversely, that anyone who [acknowledges] the content of *B* must immediately [acknowledge] that of *A*. (*Equipollence*). [P2] It is here being assumed that there is no difficulty in apprehending [*Auffassung*] the contents of *A* and *B*. The sentences need not be equivalent in all respects. [...] [P3] I assume there is nothing in the content of either of the two equipollent sentences *A* and *B* that would have to be immediately acknowledged as true by anyone who grasped it properly [...] (Frege 1906a, p. 213, [197]).⁵

Frege supplements his criterion with three provisos that I have labelled above as (P1–P3). We can sum up Frege’s proposal as follows:

Provided that

- (P1) S_1 and S_2 are assertoric sentences,
- (P2) there is no difficulty in apprehending the content of S_1 and S_2 ,
- (P3) there is nothing in the content of S_1 and S_2 that is obviously true;
(RecognitionEC) the thought that is part of the content of S_1 = the thought that is part of the content of S_2
if, and only if,
[one must, if one acknowledges the content of S_1 as true, immediately acknowledge the content of S_2 as true (and *vice versa*)].

It is noteworthy that (RecognitionEC) does not specify a relation between two sentences S_1 and S_2 but between their *contents*. This is not merely a careless formulation, but it is essential that (RecognitionEC) appeals to a relation between the contents of sentences. In order to see why, consider Beaney’s criticism of Frege’s equipollence criterion, which he renders as follows:

The sense of the assertoric sentence S_1 = the sense of the assertoric sentence S_2
if, and only if,

Everyone who recognizes the content of S_1 as true, must immediately recognize the content of S_2 as true (and *vice versa*) (Beaney 1996, p. 229).

He takes this and similar criteria to be ‘useless’:

For if the ‘content’ of a proposition is precisely the ‘thought’ expressed [...], then, if two propositions do express the same thought, to grasp the ‘content’ of one is ipso facto to grasp the ‘content’ of the other, and if it is the ‘content’ that we recognize as true or false, then we automatically recognize the ‘content’ of one as true if we recognize the ‘content’ of the other as true. Not only does this make the

⁵ I have changed the translation. The original translation of Frege’s ‘Wunschsätze, Befehlssätze, Fragesätze’ as ‘sentences that serve to express a wish, a wish, or a command’ is misleading. An assertoric sentence can serve to express a wish or make a command. But Frege wants to distinguish assertoric sentences from these other kinds of sentences. Hence, we need the grammatical categories of optatives etc. that Frege himself mentions. Further, Frege uses ‘anerkennen’ which is not correctly translated as ‘accept’. The original translation has for ‘Auffassung’ ‘grasping’ (fassen). But one grasps thoughts, while Frege uses a distinct term for the cognitive act we bear towards a content. I translate therefore ‘Auffassung’ as *apprehension*.

criterion useless, but it also violates the constraints on an adequate criterion: it is unacceptable to presuppose on the right hand side of the biconditional precisely that notion that we are attempting to specify (Beaney 1996, p. 233; emphasis added).

Frege's criterion would indeed be useless and circular if the content of a sentence was the thought expressed. But Beaney's *modus ponens* is Frege's *modus tollens*. The content of a sentence is not the thought expressed; the thought is part of the content. As Frege put it, in 'The Thought', 'the content of a sentence often goes beyond the thought expressed by it' (Frege 1918/9, p. 296 [64]).

What is the content of a sentence if it exceeds the thought? The literature on this topic is vast. But for our purposes, we only need the basic idea that is nicely stated by Neale:

The phenomena noted by Frege, Grice, and others are, I think, quite natural once we take into account the nature of communication. We do not seek to transmit information only about the world; communication may also involve the transmission of information about our attitudes and emotions [...] (Neale 1999, p. 60).

For example, by putting a word into the subject position in a sentence, we express a thought but also communicate that that referent of the word commands our attention:

In ordinary language, the place of the subject in the sequence of words has the significance of a *distinguished* place, where we put that to which we wish especially to direct the attention of the listener (Frege 1879, p. 12 [3]).

The sentences (1a) and (1b) differ in what occupies the position of the grammatical subject. When understanding the sentences, the audience will direct their attention to different expressions and different referents. The content is different but the thought is the same. Further parts of the content are, for instance, evaluations conveyed by the words chosen ('dog' versus 'cur') or additional thoughts.

The content of the sentence contains the thought as well as the information about our attitudes and evaluations that is conveyed in the way outlined. Roughly speaking, everything a competent speaker can gather from understanding a particular sentence and its grammatical form is part of the content. But not everything that one can gather in this way is relevant for the truth of the content. (RecognitionEC) is supposed to 'separate off' the thought that is, properly speaking, true or false from other parts of the content of a sentence:

[O]ne has to separate off from the content of a sentence the part that can alone be accepted as true or rejected as false. I call this part the thought expressed by the sentence. It is the same in equipollent sentences of the kind given above. It is only with this part of the content that logic is concerned (Frege 1906a, p. 198, [213–214]).

With this in mind, let's illustrate the point and purpose of (RecognitionEC).

(RecognitionEC) allows us to recognise sameness of thought expressed independently of the structure of S_1 and S_2 . It allows, to take a limiting case, a one-word ('Gavagai') and a many-word sentence ('There is a rabbit') to express the same thought. It also justifies Frege's judgements about sameness of thought expressed. For example, Frege (1923–6, p. 15 [49–50]) provided some striking examples of sentences that differ in grammatical form and contain expressions that differ in sense, yet express, according to (RecognitionEC), the same sense. Consider the following sentences:

- (S2a) The earth is a planet.
- (S2b) The earth is a planet and the earth is planet.
- (S2c) The earth is a planet or the earth is planet.

One cannot understand (S2a) 'The earth is a planet' and (S2b) 'The earth is a planet and the earth is a planet' and endorse the first content while rejecting the second.

4 Frege's recognition criterion, the picture thesis and the Begriffsschrift

According to (RecognitionEC), sentences that are *not* composed of simple expressions with the same senses in the same way can express the same thought. Now this seems to conflict with Frege's metaphysics of thoughts. For he also held that thoughts are made up of parts ('Gedankenbausteine') in a way that corresponds, by and large, to the way the sentence expressing the thought is made up of parts:

It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the earth for the first time, a form of words can be found in which it will be understood by someone else to whom the thought is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought (Frege 1923–6, p. 1 [36]).

I will call the conjunction of the thesis (i) that thoughts contain senses as parts and (ii) that the parts of thoughts correspond to the parts of the sentence expressing it the *Picture Thesis*.⁶ If the structure of a sentence can serve as a picture of the structure of the thought it expresses, two sentences S_1 and S_2 can only express the same thought if they have the same structure.⁷ Otherwise, one of them would express the thought under consideration, yet its structure would not be a picture of the thought expressed.

(RecognitionEC) is incompatible with the Picture Thesis. For example, the thought expressed by (S2b) seems to contain the sense of 'and' not contained in the thought expressed by (S2a). Hence, the thoughts differ in constituents and the Picture Thesis pronounces them different, while (RecognitionEC) justifies us in saying that the thoughts expressed are the same.

⁶ See Frege (1919, p. 275 [254]); Frege (1923, p. 259 [279]). For further discussion, see Dummett (1981b, p. 261) and Heck and May (2011, p. 144f).

⁷ See Dummett (1981a, pp. 378–379). Levine (2002, p. 210ff) argues that Frege is not committed to the thesis Dummett ascribes to him.

Is Frege's theory of thoughts then inconsistent?⁸ No, for if we look at the Picture Thesis in other remarks by Frege, we see that we need to take it with a pinch of salt. To see this, let us have another look at Frege's reason for the Picture Thesis in the last quote. Does Frege claim that any language can express an incalculable number of thoughts with a few syllables? This reading of Frege's quote about the astonishing nature of language would make the Picture Thesis plausible. But the reading of the quote can't be right if we take into account that, already in 1880/1, Frege had observed that in natural language, the way concept words are composed out of constituents only imperfectly matches the way concepts are composed:

The words 'lifeboat' and 'deathbed' are similarly constructed though the logical relations among the constituents are different. So the latter isn't expressed at all, but is left to guesswork. [Language] often only indicates by inessential marks or by imagery what a Begriffsschrift should spell out in full (Frege 1880/1, p. 13 [13]).⁹

The words 'lifeboat' and 'deathbed' are composed out of words in the same way, yet the relation between the senses expressed by their constituents is different. A lifeboat is a boat whose dedicated use is to save lives; a deathbed is not a bed whose dedicated use is to save deaths. Since the construction of these expressions from parts does not correspond to the relation between the sense and the reference of the expressions, one grasps the thought expressed by 'There is a lifeboat on the beach' and 'He was on his deathbed', without the combination 'lifebed' even having a sense. Such examples are easily multiplied. The expressions 'Earl Grey tea' and 'breakfast tea' are constructed in the same way out of parts, yet the corresponding complex senses are not constructed in the same way out of their parts¹⁰. Therefore one can grasp the senses of the parts 'Earl Grey', 'tea' and 'breakfast' and their mode of combination without thereby being able to grasp the complex senses. I might, for instance, grasp the sense of 'Breakfast is in the morning' and the sense of 'Earl Grey tea is reviving' without being able to grasp the sense of 'Breakfast tea is reviving'. More often than not, knowledge of how a complex expression is composed out of words, together with knowledge of the senses of the components, is insufficient to grasp the thought expressed by the complex expression. Frege anticipated with these observations the contemporary critique of the

⁸ This alleged tension has sparked a lot of interest in the literature on Frege. Some authors reject the Picture Thesis and want to preserve (RecognitionEC) (see, for example, Bell (1996, pp. 595–596) and Kemmerling (1990, 2011)). Thoughts, they say, have no parts; they have parts relative to a sentence. Others hold on to the Picture Thesis and, in turn, reject (RecognitionEC): two sentences can only express the same thought if they 'involve' the same senses (see, for example, Dummett's principle K in his 1988, 1 and Künne (2004, pp. 47–48)). Since (S2b) 'involves' the concept of conjunction and (S2a) doesn't, (S2a) and (S2c) don't express the same thought. Neither response is entirely satisfactory. Penco (2003) takes Frege, therefore, to operate with two different conceptions of sense. I outline a third response above.

⁹ Frege's example is 'Berggipfel' and 'Baumriese'. The original translation gives 'Sprache' as 'Speech'. This suggests that the problem concerns only spoken words. This suggestion is misleading and should be resisted. I have therefore changed the translation.

¹⁰ Thanks to Jessica Leech for suggesting this example.

view that the sense of a complex expression is constructed from the sense of its parts according to grammatically encoded forms of construction.¹¹

Frege's examples make it plausible that, more often than not, the structure of a sentence of natural language is not a picture of the structure of the thought expressed (if a thought is expressed at all). Hence, in order to grasp the sense of a complex expression, it is often necessary to know the senses of its parts, the mode of combination and, as Frege put it in the last quote, make guesses. This is not a problem: we can easily guess which thought is expressed by an utterance of such a sentence because we can rely on common knowledge, tacit conventions and contextual clues. He goes on to repeat again and again that 'our everyday language leave a good deal to guesswork' (Frege 1914, p. 230 [213]).¹²

In contrast, a Begriffsschrift shall express thoughts in such a way that the construction of complex expressions from parts mirrors the construction of their senses and thereby makes such guesses superfluous. This feature of a Begriffsschrift is part and parcel of the conception of a Begriffsschrift.¹³ The authors before Frege who worked on the development of a Begriffsschrift held that it is a necessary condition for a script to be a Begriffsschrift that it satisfies the Picture Thesis. Barnes (2002, p. 78) quotes Krug, who says that an ideography is a pasigraphy if

it is a truly universal language (a lingua characteristica universalis), i.e. a script which expresses, in a universal intelligible fashion, not only concepts but also connexions and relations, and further all their possible modes of combination in judgements or propositions (Krug 1832-38 II, pp. 500–501).

In this sense, Frege's Begriffsschrift is not only an ideography, but a pasigraphy because its signs express concepts and the combinations of the signs are pictures of combinations of the concepts they express. Frege's Begriffsschrift is designed to satisfy the Picture Thesis. In § 32 of *Basic Laws of Arithmetic*, after taking the reader through the construction of truth-value names from names of objects and function expressions, Frege can correctly say that if a Begriffsschrift name 'n' is part of a truth-value name, the sense of 'n' is part of the sense of the truth-value name¹⁴. Hence, if one knows how a Begriffsschrift sentence is constructed from signs and knows the method of construction, one grasps the thought expressed. No guesswork is necessary and therefore no unwanted assumptions can be smuggled in when we prove a truth in Begriffsschrift. While such assumptions are unwanted when proving mathematical truths, they are frequently made and largely harmless in natural language communication.

So we need to take Frege's remark about what language can do with a pinch of salt. In a language *in which the logical relation between senses and concepts is fully expressed and never merely hinted at*, the structure of the sentence will mirror the

¹¹ See Recanati (2004, p.131ff) and the works cited there.

¹² In an undated letter to Peano, Frege goes even as far as saying that the sentences of natural language don't need to be composed out of words that have sense and reference on their own if these sentences are essentially vehicles of communication. See PMC, 115; BW, pp. 182–183.

¹³ See Barnes (2002). Frege is aware of Leibniz's and Trendelenburg's projects to develop Begriffsschriften. See, for example, Frege 1880/1, pp. 9–10 [9–10].

¹⁴ See Frege 1893, p. 51 [51]

structure of the thought expressed. Languages like English only approximate such a language. More often than not, we need to guess which thought is expressed by a sentence of such a language. Hence, the Picture Thesis is, in general, not true of natural language sentences. For this reason, we are not entitled to assume that the way natural language sentences are built up from (simple) expressions mirrors or is even a good guide to the way the thought expressed is built up from senses.

At the same time, we need to start from natural language to develop a *Begriffsschrift* that fully expresses what many natural sentences only hint at. This constitutes, Frege argued, the greatest difficulty for philosophy:

Just here I see the greatest difficulty for philosophy: the instrument it finds available for its work, namely ordinary language, is little suited to the purpose, its formation having been governed by requirements wholly different from those of philosophy. So also logic is first of all obliged to fashion a useable instrument from those already to hand. And for this purpose it initially finds but little in the way of useable instruments available (Frege 1923–6, p. 11 [45]).

We can only start to develop a *Begriffsschrift* or any other kind of symbolism for the expression of thoughts if we can separate merely grammatical features of sentences from logical ones. For this task, we need a criterion that allows us to ‘detect’ whether sentences express the same thought or not, *independently of whether the sentences differ in structure*. For, as we have seen, when it comes to natural language sentences, we are not justified in taking a difference in structure to suffice for a difference in sense. Frege’s recognition criterion (RecognitionEC) seems to be a good candidate for the mark of sameness of thought expressed that Frege needs. There is, then, no clash between the Picture Thesis and (RecognitionEC): Frege employs (RecognitionEC) to develop a script for which the Picture Thesis is true and he needs to employ (RecognitionEC) because the languages in use only approximate a *Begriffsschrift*. If we have developed such a script, we are entitled to hold the Picture Thesis with respect to its sentences (formulae). Even if we have such a script, we will still need (RecognitionEC) to regiment and translate sentences into it.

In the following sections, I will look more closely at (RecognitionEC). I will set aside the problem with obvious truths to which (proviso3) responds aside and focus on sentences whose *wording* expresses a thought; that is, I will not discuss sentences with indexical and demonstratives. In the following sections, I will discuss the problems that arise in connection with (proviso1) and (proviso2). (Proviso2) will lead to a fundamental problem for Frege’s philosophy.

5 The ‘Assertoric sentence’ proviso

Prima facie, (proviso1) is unproblematic. Thoughts are supposed to be the senses expressed by *assertoric* sentences. Hence, (proviso1) simply narrows the range of application to the only kind of sentence that Frege takes to express thoughts. But (proviso1) is for internal reasons problematic. To see this, let’s go back to *Begriffsschrift*:

We can imagine a language in which the proposition “Archimedes perished at the capture of Syracuse” would be expressed thus: “The violent death of Archimedes at the capture of Syracuse is a fact”. To be sure, one can distinguish between subject and predicate here, too, if one wishes to do so, *but the subject contains the whole content*, and the predicate serves only to turn the content into a judgement (Frege 1879, p. 12 [3–4]; emphasis added).

The subject of

(1b) The violent death of Archimedes at the conquest of Syracuse is a fact

is the singular term ‘The violent death of Archimedes at the conquest of Syracuse’. In Frege’s *Begriffsschrift*, ‘[t]he entire specific content of a sentence is localized within a component singular term’ (Rumfit 2011, p. 7). The predicates ‘is a fact’ or ‘is true’ don’t contribute to the content of (1b). While Frege will change his view about the role of the predicate ‘is true’, he will maintain that sentence-nominalisations express thoughts:

If I attach the word ‘salty’ to the word ‘sea-water’ as a predicate, I form a sentence that expresses a thought. To make it clearer that we have only the expression of a thought, but that nothing is meant to be asserted, I put the sentence in dependent form ‘that sea-water is salty’ (Frege 1915, p. 217 [251]).

The that-clause ‘that sea-water is salty’ is obviously not an assertoric sentence. But if it only expresses a thought, it expresses a thought.

No criterion of content-identity that is prefixed by (proviso1) can be used to decide whether Frege is right or wrong about the content of ‘The violent death of Archimedes at the conquest of Syracuse’, etc. For ‘The violent death of Archimedes at the conquest of Syracuse’ is not an assertoric sentence. In order to justify a central claim of *Begriffsschrift*, Frege needs to be able to detect sameness of thought across grammatical categories; and (proviso1) makes this impossible.

6 The ‘Not difficult to apprehend’ Proviso and psychological limitations

(Proviso2) may seem like a mere unimportant aside. But, as I will now argue, it is not. We first need to ask how one ought to understand (proviso2): which contents are difficult to apprehend and why? In this section, I will look at Frege’s suggestion that a content may be difficult to grasp because of our psychological limitations.

Frege writes:

We simply do not have the mental capacity to hold before our minds a very complex logical structure so that it is equally clear to us in every detail. For instance, what man, when he uses the word ‘integral’ in proof, ever has clearly before him everything that appertains to the sense of the word! (Frege 1914, p. 222 [240]; in part my translation.)

If the thought that is part of the content of the assertoric sentence S_1 and the thought that is part of the content of the assertoric sentence S_2 are the same, but at least one of them

is not clearly grasped, one can accept the content of S_1 without *eo ipso* accepting the content of S_2 and without incurring blame or criticism for being irrational. Complex senses are difficult to grasp for us and if a part of a content is difficult to grasp, the whole content will be difficult to grasp. Hence, we need to add to (EC) the proviso about ‘no difficulty in apprehending a content’. When are the conditions for applying (proviso2) satisfied? Frege said little about unclearly grasping senses:

we do not have a clear grasp of the sense of the simple sign, but that its outlines are confused as if we saw it through a mist. The effect of the logical analysis will then exactly be this—to articulate the sense clearly (Frege 1914, p. 211 [228]).

We have a clear grasp if we can articulate the sense clearly; we have an unclear grasp if we can’t. This is of little help in applying (proviso2). For we have no independent grip on an incomplete grasp and therefore cannot know whether (proviso2) is satisfied or not. We have an unexplained distinction between a clear and an unclear grasp and no guide for applying it.

Burge tries to help Frege out by connecting clear grasp with ideal theory:

The senses of expressions could be fully grasped only by grasping equivalences given by ideal scientific explications, or by otherwise understanding the contribution of those expressions to a theory. Of course, since most of the envisioned ideal explications would come as discoveries, it is possible to doubt them (even if the doubt depends on less than full analytic mastery of the senses), while no doubting the corresponding self-identities. So by Frege’s test for the identity of senses, the senses of the explicans and explicandum would be different (Burge 1990, p. 265, Fn. 16).

However, Frege should reject Burge’s help. Frege is concerned with a lack of mental power that prevents us from grasping the sense of some words fully. My cognitive deficiency in fully grasping the sense of, say, ‘number’ is not addressed by the development of ideal arithmetic, but by removing psychological barriers.

Schellenberg (2012, p. 168) has a different take on the complexity reading of ‘easily graspable’. She says:

The qualification “where the content of both A and B is easily graspable” constrains the applicability of the criterion to exclude complex sentences. With a complex sentence, I mean a sentence that is not easily graspable (Schellenberg 2012, p. 168).

This sounds empty. But, charitably construed, Schellenberg has the following problem in mind. Consider for illustration the sentence pair:

(3a) $2 + 2 = 4$.

(3b) It is not the case that it is not the case that $2 + 2 = 4$.

(3a) and (3b) may express the same thought. I may nonetheless assent to what (3a) says, but not immediately to what (3b) says, because it takes time to parse (3b).

Schellenberg proposes finessing this problem by replacing ‘immediate acknowledgement’ in Frege’s criterion with ‘rational commitment’:

Two sentences S_1 and S_2 have the same sense iff

- (i) anyone who understands S_1 and S_2 and takes S_1 to have a certain truth-value is rationally committed to recognize that S_2 has the same truth-value as S_1 and *vice versa*;
- (ii) the understanding of S_1 is constitutively connected to the understanding of S_2 and *vice versa* (Schellenberg 2012, p. 170).

Schellenberg does not explain ‘rational commitment’ further. But the examples she gives suggest that a speaker’s taking S_1 to have a certain truth-value rationally commits her to recognising that S_2 has the same truth-value if, and only if, S ought to recognize that S_2 is true (false) if she has accepted S_1 as true (false).¹⁵ She may be excused in various ways for not living up to this demand—she was not thinking carefully, was distracted, the command of a word was shaky—yet the demand applies to her. More needs to be said about the force of this ‘ought’, but for our purposes these remarks will suffice.

Does the appeal to ‘rational commitment’ help Frege? No, for now sentences turn out to express the same thought that, by Frege’s own light, should express different thoughts. Consider

$$(4a) \quad 2 + 1 + 3 = 2 + 1 + 3$$

$$(4b) \quad 2 + 1 + 3 = 1 + 1 + 1 + 1 + 1 + 1$$

If one understands and accepts (4b), one can extend one’s knowledge of arithmetic, while one cannot extend one’s arithmetical knowledge by accepting (4a). Hence, these sentences ought to express different thoughts. But if one understands (4a) and (4b) and takes (4a) to be true, one is rationally committed to taking (4b) to be true (and *vice versa*). One ought to acknowledge the truth of (4b) if one has acknowledged the truth of (4a), whether one has worked out that they have necessarily the same truth-value or not. Is one’s understanding of (4a) constitutively connected to (4b) and *vice versa*? This question is difficult to answer because Schellenberg does not give the reader a good grip on the constitutive connection she has in mind. But it seems plausible to say that it is not possible to understand (4a) without understanding (4b) and *vice versa*. After all, they contain the same numerals and arithmetical symbols or numerals that can be defined in terms of each other. Hence, Schellenberg’s recognition criterion makes (4a) and (4b) express the same thought. This is an unwanted consequence; the criterion overgeneralises.

A general problem for Frege starts to emerge: if we want to recognise the same thought in different linguistic guises, we need to be able to tell whether the non-immediacy of recognising the sameness of truth-value of two sentences S_1 and S_2 is due to difficulties in grasping the content of these sentences or not. If I acknowledge the truth of the content of S_1 without immediately acknowledging the truth of S_2 , is this due to the fact that a separate act of judgement is required and the thoughts expressed are different or because at least one of the contents is not easy to apprehend?

¹⁵ See Schellenberg (2012, p. 171).

Frege did not answer this question, but simply excluded sentences whose content is not easy to apprehend by adding (proviso2). This move severely restricts the usefulness of the criterion. It does not allow us, for example, to decide whether a complex tautology and a simple sentence express the same thought. Schellenberg tried to preserve the applicability of the criterion by framing it without the proviso. Yet the proposal turned out to be implausible.

In the next section, I will argue that this problem arises independently of the complexity of the sentences under consideration. The problem is pervasive and makes Frege's criterion useless.

7 Thought's wrapping

If two simple sentences S_1 and S_2 express the thought P and something else, it is possible that a speaker will grasp P , yet while she acknowledges the content of S_1 as true, she might not immediately acknowledge the content of S_2 as true. The reason for this is that the speaker does not distinguish between the thought P and other parts of the content. Frege took the described possibility to be actual. The sentences of the language we use express thoughts in disguised form. For example, he wrote in the introduction to *Begriffsschrift*:

If it is one of the tasks of philosophy to break the domination of the word over the human spirit by laying bare the misconceptions that through the use of language almost unavoidably arise concerning the relations between concepts by freeing the thought from that with which only the means of expression of ordinary language, constituted as they are, saddle it, then my ideography, further developed for these purposes, can become a useful tool for the philosopher (Frege 1879, p. 7 [vi–vii]).

Frege notes that a *Begriffsschrift* will not achieve pure expression, but only limit the 'damage' done by expressing a thought to a minimum.

The same idea can be found at the beginning of *Foundations of Arithmetic*. There, he characterises logical progress as follows:

Often it is only after immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in its pure form, in stripping off the irrelevant accretions [*fremden Umhüllungen*] which veil it from the eyes of the mind (Frege 1884, p. xix).

Austin translated Frege's 'fremde Umhüllungen' as 'irrelevant accretions'. This is unfortunate. For example, Frege's 'Logik (1897)' has a section called 'Trennung des Gedankens von den *Umhüllungen*' which is translated in Frege's *Posthumous Writings* (138) as 'Separating a Thought from its *Trappings*'. A better translation that captures the meaning of 'Umhüllungen' is 'wrappings'. A content or thought can be wrapped up in such a way that it is hard to recognise. One makes progress in logic by distinguishing the wrapping of the thought from the thought itself. If we have managed to free the thought from its wrapping, we express it in pure form.

The most detailed discussion of the opposition between expressing a thought in pure form and in disguised or wrapped up form can be found in the section of ‘Logik (1897)’ just mentioned. Here, Frege uses an analogy between chemistry and logic to make his point. A chemist might be faced with the practical question of establishing whether two samples are samples of the same substance. He might have a test for doing so, say, testing how each sample responds to litmus paper. But the test will only work if the samples are pure. If the samples are contaminated, that is, if they contain other substances, samples of different substances may pass the test or samples of the same substance may fail it. Hence, the test can only be applied to *pure* samples. The same goes for thoughts:

The purity of the object of one’s investigation is not just important for the chemist. How should he be able to securely recognize that he arrived by different routes at the same result if the difference might have its ground in impurities of the applied substances? (Frege 1897, pp. 141–142 [153].)

Just as two impure samples of the same substance may fail the sameness test, two sentences that express the same thought, but impurely, may not be detected by Frege’s criterion for sameness of thought expressed. So the logician has to require the objects of investigation to be presented in pure form:

[The first and most important task is to set out the objects of investigation in pure form]. Only by means of this is one able to make the acts of recognition that probably constitute the fundamental discoveries in logic, too. Therefore let us never forget that two different sentences can express the same thought, that we are only concerned with only that part of the sentence’s content which can be true or false (Frege 1897, p. 143 [154]; in part my translation).¹⁶

In pure form, a thought is purged of impurities. What are the impurities Frege has in mind? Many grammatical features of natural language count, by Frege’s light, as ‘contaminating’ the expression of a thought. In *Begriffsschrift*, Frege lists such features. By stressing a phrase or word, we draw our audience’s attention to this part of a sentence. We can stress an expression in different ways: by intonation or by putting the expression in a distinguished position in the sentence we use.¹⁷ Changes in word-order that put a word or phrase into the subject position direct the attention of the audience, but, argued Frege, do not change the thought expressed:

Whereupon the attention is directed, what is stressed, can otherwise be very important but does not concern logic (Frege 1897, p. 141 [153]; my translation).

And:

¹⁶ In *Posthumous Writings*, Frege’s ‘Die erste und wichtigste Aufgabe ist, die Gegenstände der Untersuchung rein darzustellen’ is translated as ‘The first and most important task is to set out clearly what the objects of investigation are’ (143). This is a significant mistranslation. Frege is concerned with the purity (rein darzustellen), not with clarity. The translation makes Frege’s use of the ‘purity’ metaphor unrecognisable.

¹⁷ See Hofweber (2007, p. 12).

Naturally such transformations are not indifferent in every respect; *but they do not touch the thought, they do not touch what is true or false*. If the inadmissibility of such transformations were generally admitted then all deeper logical investigation would be hindered (Frege 1918/9, p. 296 [64]; emphasis added).

For illustration, consider (S1a) and (S1b) again. (S1b) is a case where a whole sentence is in focus:

- (1a) Archimedes perished at the capture of Syracuse.
- (1b) The violent death of Archimedes at the capture of Syracuse is a fact.

The focus effect is achieved by nominalising (1a), putting the nominalised sentence in the subject position and adding ‘is a fact’ as the grammatical predicate to make a sentence. Consider the question ‘Is it a fact or is it merely probable that Archimedes perished at the capture of Syracuse?’ Answering with (1b) stresses ‘The violent death of Archimedes at the capture of Syracuse’. While Frege takes ‘is true’ to create a focus effect and nothing more, Ramsey has ‘It is true that’ in mind:

[I]t is evident that “It is true that Caesar was murdered” means no more than that Caesar was murdered, and “It is false that Caesar was murdered” means no more than that Caesar was not murdered. They are phrases which we *sometimes use for emphasis or for stylistic reasons, or to indicate the position occupied by the statement in our argument* (Ramsey 1927, p. 157; my emphasis).

Central examples of sentences that Frege takes to express the same thought are ones in which the focus is changed.¹⁸ Consider a further example:

- (5a) There is at least one square root of 4.
 - (5b) The number 4 has the property that there is something whose square it is.
 - (5c) The concept *square root of 4* is realised.
- (Frege 1892b, p. 174 [199–200].)

For Frege, (5a–c) express the same thought. If (5a–c) are different ways of expressing the *same* thought, they all have the same truth-value. However, in (5c) ‘the concept square root of 4’ does not refer to a concept, but to an object of a special kind that is related to the concept referred to by the predicate ‘ ξ is a square root of 4’:

The sentence ‘there is at least one square root of 4’, we have an assertion, not about (say) the definite number 2, nor about -2 , but about a concept, square root of 4; viz., that it is not empty. But if I express the same thought thus: ‘The concept square root of 4 is realized’, then the first six words form the proper name of an object, and it is about this object that something is asserted. (Ibid, 1892b, p. 174 [199])

Frege does not say much about the special kind of objects that correspond or represent concepts.¹⁹ But (5c) is true if, and only if, the object referred to by ‘the concept square

¹⁸ ‘represents’ is Parsons’s (1986, p. 52) term. Burge (1984, pp. 283–284) speculates that the object of a special kind are the extensions of the concept.

¹⁹ See Parsons (1986, pp. 52–53).

root of 4' falls under the first-level concept referred to by 'ξ is realized'.²⁰ Only objects of 'the special kind' can fall under this concept.

The difference between (5a), (5b), and (5c) concerns which expression or phrase is in the subject position and thereby in focus. In (5b), '4' is in the subject position and thereby in focus; in (5a), it is not in focus; in (5c), 'square root of 4' has been nominalised by combining it with 'the concept' and the new expression has been moved into the subject position. In fact Frege talks as if (5a–5c) are the same sentence:

It need not then surprise us that the same sentence may be conceived as an assertion about a concept and also as an assertion about an object; only we must observe that what is asserted is different. (1892b, p. 175 [200])

The fact that (5a–5c) have different grammatical subjects and predicates changes their content; they predicate different concepts of different things. This difference in content masks that the same thought is expressed:

We shall have no truck with the expressions 'subject' and predicate' of which logicians are so fond, *especially since they not only make it more difficult for us to recognize the same as the same*, but also conceal distinctions that are there (Frege 1897, p. 143 [155], my emphasis).

Seeing a sentence as containing subject and predicate makes it difficult to realise that it shares its senses with sentences of other form. Here we have, then, the most fundamental kind of difficulty in apprehending contents that affects our ability to detect sameness of sense.²¹

8 'The greatest difficulty for philosophy'

Frege made a good case that such examples involve a change of focus that makes the thought difficult to detect. However, he also assumed that the same thought is expressed and that the examples *only* involve a change of focus. Can he justify this assumption by relying on (EC)? No, for if we apply (EC), (5a) and (5b) will come out as expressing different thoughts. I may grasp the content expressed by (5a) and (5b) and take the content expressed by (5a) to be true, but I will not *immediately* take the content expressed by (5b) to be true. It takes *reflection and some reasoning* to work out that (5b) is true if, and only if, (5a) is true. Hence, (EC) will tell us that (5a) and (5b) express contents that contain different thoughts. This conclusion generalises for similar sentence-pairs. A further problem concerns (5c). According to Frege, concepts cannot be designated by singular terms. They are essentially predicative and their

²⁰ See Hofweber (2007, p.23). Hofweber argues that many sentence-pairs with which Frege is concerned are distinguished in terms of focus.

²¹ Formal languages like the language of first-order predicate logic do not contain ambiguous and context-dependent expressions and structural ambiguities are resolved by brackets, etc. A formal language of this kind is still no Begriffsschrift in which only thoughts are expressed. For example, the mere order in which expressions are written on paper can make the content of a sentence richer than the thought expressed.

designations cannot be in subject-position.²² But in (5c), ‘the concept *square root of 4*’ is in the subject position. This expression refers to an object; but the sentence seems false to us for ‘the concept square root of 4’ seems to refer to a concept. Hence, the grammatical transformation not only changes the focus of the sentence; it takes us from a sentence that expresses a true thought to one that seems to express a false thought.²³ This puts pressure on the idea that grammatical transformations that affect focus leave the thought unchanged.

We don’t need to accept Frege’s view about concept designation to put pressure on his view that grammatical transformations concerning focus leave the thought untouched. Consider the sentence ‘ $3 + 5 = 8$ ’.²⁴ We can transform it into ‘8 is the result of adding 3 to 5’ and ‘3 is what when added to 5 makes 8’ by putting ‘8’ and then ‘3’ in the subject position. Is it possible to apprehend the content of these sentences and accept the content, of ‘8 is the result of adding 3 to 5’ without immediately accepting ‘3 is what when added to 5 makes 8’? Yes, it is. You might accept what ‘8 is the result of adding 3 to 5’ says without *eo ipso* accepting what ‘3 is what when added to 5 makes 8’ because you have learned that $3 + 5$ makes 8, but you don’t yet know what has to be added to 5 to make 8. You need to work out that the second content is true by subtracting 5 from 8, a different process from the one that lead to assent to what ‘8 is the result of adding 3 to 5’ says. Hence, application of (EC) yields the result that the thoughts expressed are different. Yet the only difference between the sentences concerns which expression is in the subject position and thereby in focus. Frege can always blame this result on a difficulty in apprehending the content of sentences. But then we have no criterion at all by which to decide the question of whether they express the same thought or not.

Frege is therefore in a difficult position: we need to make sure that a grammatical transformation *only* affects the focus. But we cannot use (EC) to ensure that the thought expressed is the same. Frege exhorted us that ‘we must not let ourselves be seduced by adhering impurities to see distinctions where none are’ (Frege 1897, p. 142 [154]). However, if thoughts are expressed in impure form, we can no longer apply (EC) to distinguish the thought from the impurities. For the difference in grammatical form interferes with our disposition to immediately accept the content of one sentence if we accept the content of another.

One can sum Frege’s problem up as follows: If we have ‘set out’ thoughts in pure form, that is, expressed *only* the thought and not additional clues, etc. that guide the audience, (EC) can be applied. But in order to set out a thought in pure form, we

²² See 1892b, p. 172 [197].

²³ See Picardi (1993, p. 77). See also Glanzberg (2005, section II.1), for examples in which a change of focus seems to change truth-conditions. These examples involve adverbs of quantification or modality.

²⁴ I take this example from Stout (2010, p. 396ff); see also the references there. On the basis of the example, he comes to the opposite conclusion: ‘We could also apply Frege’s test for the distinctness of two thoughts – *whether it is possible to grasp both but have different attitudes to each*. Is it possible for Rebecca to know that what $3 + 5$ makes is 8 without knowing that what when added to 5 makes 8 is 3, while grasping both thoughts? I think it is not possible’ (Stout 2010, p. 398; emphasis added). Stout comes to the opposite conclusion because he misconstrues Frege’s criterion. Frege’s criterion concerns apprehending the content of assertoric sentences, not grasping two thoughts and assessing whether one can know one without knowing the other. Otherwise, the criterion would be open to the problems discussed by Beaney.

need to have prior knowledge of which assertoric sentences express this thought and which sentences express the same thought. Only then can we express the thought in pure form, that is, regiment the sentences that express the same thought in the same way and disregard their grammatical differences. However, to do so we need to apply a criterion to recognise the same thought expressed *in impure form*. But (EC) cannot be applied to such sentences. The fact that we express the thought in a natural language sentence with its grammatical peculiarities and a particular intonation makes the thought and thereby the content difficult to apprehend. Given (proviso2), (EC) cannot be used to identify the same thought in different sentences whose content is difficult to apprehend. There is no other criterion available that can be taken to be a criterion of *thought* recognition. Hence, Frege cannot recognise the same thought in different guises.

Frege is caught in a vicious circle: in order to apply his criterion of thought recognition to assertoric sentences, these sentences need to express thoughts in pure form. But natural language sentences express thoughts only in impure form; and yet they are the only possible starting point to develop sentences that express thoughts in pure form. In order to regiment natural language sentences in such a way that they express thoughts in pure form, we need to have already applied a criterion of thought recognition to detect sentences that express the same thought, recognise mere grammatical difference and improve our regimentation accordingly. And this is what Frege tells us we can't do. We need to present the thoughts in pure form first.

If logic can only free our thinking from language by relying on a criterion of thought recognition, we will forever be ruled by language. We can't make progress in logic by recognising a thought in its manifold guises. Frege was right to think that the fact that natural language sentences express more than just thoughts and are not suited for the purposes of logic poses the 'greatest difficulty for philosophy'. However, he provided no answer to this difficulty. The provisos to (RecognitionEC) make the difficulty hard to spot; they don't solve it.²⁵

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²⁵ This paper is based on a talk I gave at the workshop *Frege@ Sterling Workshop III: Sense-Identity and the Status of Definitions in Frege* in June 2015. I am grateful to the audience for feedback. Special thanks go to Mike Beaney, Philip Ebert, Robin Jeshion, Colin Johnston, Walter Pedriali, William Taschek and Crispin Wright. Many thanks go to Keith Hossack, Nils Kürbis, Jessica Leech, Guy Longworth, Eliot Michaelson and Ian Rumfitt for comments and criticism. I am grateful to Bob Hale for written comments on a previous version. My thanks go also to the referees for feedback.

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