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$(\mathcal{I},\mathcal{T})$ –Standard neutrosophic rough set and its topologies properties

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Abstract. In this paper, we defined $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets based on an implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on D^*; lower and upper approximations of standard neutrosophic sets in a standard neutrosophic approximation are defined. Some properties of $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets are investigated. We consider the case when the neutrosophic components (truth, indeterminacy, and falsehood) are totally dependent, single-valued, and hence their sum is ≤ 1.

Keywords: standard neutrosophic, $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets

1. Introduction

Rough set theory was introduced by Z. Pawlak in 1980s [1]. It becomes a useful mathematical tool for data mining, especially for redundant and uncertain data. At first, the establishment of the rough set theory is based on equivalence relation. The set of equivalence classes of the universal set, obtained by an equivalence relation, is the basis for the construction of upper and lower approximation of the subset of the universal set.

Fuzzy set theory was introduced by L.Zadeh since 1965 [2]. Immediately, it became a useful method to study the problems of imprecision and uncertainty. Since, a lot of new theories treating imprecision and uncertainty have been introduced. For instance, Intuitionistic fuzzy sets were introduced in1986, by K. Atanassov [3], which is a generalization of the notion of a fuzzy set. When fuzzy set give the degree of membership of an element in a given set, Intuitionistic fuzzy set give a degree of membership and a degree of non-membership of an element in a given set. In 1998 [22], F. Smarandache gave the concept of neutrosophic set which generalized fuzzy set and intuitionistic fuzzy set. This new concept is difficult to apply in the real appliction. It is a set in which each proposition is estimated to have a degree of truth (T), adegree of indeterminacy (I) and a degree of falsity (F). Over time, the subclass of neutrosophic sets was proposed. They are also more advantageous in the practical application. Wang et al. [11] proposed interval neutrosophic sets and some operators of them. Smarandache [22] and Wang et al. [12] proposed a single valued neutrosophic set as an instance of the neutrosophic set accompanied with various set theoretic operators and properties. Ye [13] defined the concept of simplified neutrosophic sets, it is a set where each element of the universe has a degree of truth, indeterminacy, and falsity respectively and which lie between [0, 1] and some operational laws for simplified neutrosophic sets and to propose two aggregation operators, including a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator. In 2013, B.C. Cuong and V. Kreinovich introduced the concept of picture fuzzy set [4,5], and picture fuzzy set is regarded the standard neutrosophic set [6].

More recently, rough set have been developed into the fuzzy environment and obtained many interesting results. The approximation of rough (or fuzzy) sets in fuzzy approximation space gives us the fuzzy rough set [7,8,9]; and the approximation of fuzzy sets in crisp approximation space gives us the rough fuzzy set [8, 9]. In 2014, X.T. Nguyen introduces the rough picture fuzzy set as the result of approximation of a picture fuzzy set with respect to a crisp approximation space [18]. Radzikowska and Kerre defined $(\mathcal{I},\mathcal{T})-$ fuzzy rough sets [19], which determined by an implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on [0,1]. In 2008, L. Zhou et al. [20] constructed $(\mathcal{I},\mathcal{T})-$ intuitionistic fuzzy rough sets determined by an implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on L^*.

In this paper, we considered the case when the neutrosophic components are single valued numbers in [0, 1] and they are totally dependent [17], which means that their sum is ≤ 1. We defined $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets based on an implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on D^*; in which, implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on D^* is investigated in [21].

2. Standard neutrosophic logic

We consider the set D^* defined by the following definition.

Definition 1. We denote:

$$D^* = \{x = (x_1, x_2, x_3) | x_1 + x_2 + x_3 \le 1, x_i \in [0,1], i = 1,2,3\}$$

For $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D^*$, we define:

$x \leq_{D^*} y$ iff $((x_1 < y_1) \wedge (x_3 \geq y_3)) \vee ((x_1 = y_1) \wedge (x_3 > y_3)) \vee ((x_1 = y_1) \wedge (x_3 = y_3) \wedge (x_2 \leq y_2))$, and $x = y \Leftrightarrow (x \leq_{D^*} y) \wedge (y \leq_{D^*} x)$.

Then $(D^*, \leq_{D^*})$ is a lattice, in which $0_{D^*} = (0,0,1) \leq x \leq 1_{D^*} = (1,0,0), \forall x = (x_1, x_2, x_3) \in D^*$. The meet operator $\wedge$ and the join operator $\vee$ on $(D^*, \leq_{D^*})$ are defined as follows:

For $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D^*$,

$$x \wedge y = (\min(x_1, y_1), \min(x_2, y_2), \max(x_3, y_3)),$$
$$x \vee y = (\max(x_1, y_1), \min(x_2, y_2), \min(x_3, y_3)).$$

On D^*, we consider logic operators as negation, t-norm, t-conorm, implication.

2.1. Standard neutrosophic negation

Definition 2. A standard neutrosophic negation is any nonincreasing $D^* \to D^*$ mapping n satisfying $n(0_{D^*}) = 1_{D^*}$ và $n(1_{D^*}) = 0_{D^*}$.

Example 1. For all $x = (x_1, x_2, x_3) \in D^*$, we have some standard neutrosophic negations on D^* as follows:

+ $n_0(x) = (x_3, 0, x_1)$
+ $n_1(x) = (x_3, x_4, x_2)$ where $x_4 = 1 - x_1 - x_2 - x_3$.

2.2. Standard neutrosophic t-norm

For $x = (x_1, x_2, x_3) \in D^*$, we denote

$$\Gamma(x) = \{y \in D^* : y = (x_1, y_2, x_3), 0 \leq y_2 \leq x_2\}$$

Obviously, we have $\Gamma(0_{D^*}) = 0_{D^*}$, $\Gamma(1_{D^*}) = 1_{D^*}$.

Definition 3. A standard neutrosophic t-norm is an $(D^*)^2 \to D^*$ mapping $\mathcal{T}$ satisfying the following conditions

(T1) $\mathcal{T}(x, y) = \mathcal{T}(y, x), \forall x, y \in D^*$

(T2) $\mathcal{T}(x, \mathcal{T}(y, z)) = \mathcal{T}(\mathcal{T}(x, y), z)), \forall x, y, z \in D^*$

(T3) $\mathcal{T}(x, y) \leq \mathcal{T}(x, z), \forall x, y, z \in D^*$ and $y \leq_{D^*} z$

(T4) $\mathcal{T}(1_{D^*}, x) \in \Gamma(x)$.

Example 2. Some standard neutrosophic t-norm, for all $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D^*$

+ t-norm min: $\mathcal{T}_M(x, y) = (x_1 \wedge y_1, x_2 \wedge y_2, x_3 \vee y_3)$
+ t-norm product: $\mathcal{T}_P(x, y) = (x_1 y_1, x_2 y_2, x_3 + y_3 - x_3 y_3)$
+ t-norm Lukasiewicz: $\mathcal{T}_L(x, y) = (\max(0, x_1 + y_1 - 1), \max(0, x_2 + y_2 - 1), \min(1, x_3 + y_3))$.

Remark 1.

+ $\mathcal{T}(0_{D^*}, x) = 0_{D^*}$ for all $x \in D^*$. Indeed, for all $x \in D^*$ we have $\mathcal{T}(0_{D^*}, x) \leq \mathcal{T}(0_{D^*,} 1_{D^*}) = 0_{D^*}$

+$\mathcal{T}(1_{D^*}, 1_{D^*}) = 1_{D^*}$ (obvious)

2.3. Standard neutrosophic t-conorm

Definition 4. A standard neutrosophic t-conorm is an $(D^*)^2 \to D^*$ mapping S satisfying the following conditions

(S1) $S(x, y) = S(y, x), \forall x, y \in D^*$

(S2) $S(x, S(y, z)) = S(S(x, y), z)), \forall x, y, z \in D^*$

(S3) $S(x, y) \leq S(x, z), \forall x, y, z \in D^*$ and $y \leq_{D^*} z$

(S4) $S(0_{D^*}, x) \in \Gamma(x)$

Example 3. Some standard neutrosophic t-norm, for all $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D^*$

+ t-conorm max: $S_M(x, y) = (x_1 \vee y_1, x_2 \wedge y_2, x_3 \wedge y_3)$
+ t-conorm product: $S_P(x, y) = (x_1 + y_1 - x_1 y_1, x_2 y_2, x_3 y_3)$
+ t-conorm Luksiewicz: $S_L(x, y) = (\min(1, x_1 + y_1), \max(0, x_2 + y_2 - 1), \max(0, x_3 + y_3 - 1))$.

Remark 2.

+ $S(1_{D^*}, x) = 1_{D^*}$ for all $x \in D^*$. Indeed, for all $x \in D^*$ we have $S(0_{D^*}, 1_{D^*}) \in \Gamma(1_{D^*}) = 1_{D^*}$ so that $\leq S(0_{D^*}, 1_{D^*}) \leq S(0_{D^*,} x) \leq 1_{D^*}$.

+ $S(0_{D^*}, 0_{D^*}) = 0_{D^*}$ (obvious).

A standard neutrosophic t-norm $\mathcal{T}$ and a standard neutrosophic t-conorm S on D^* are said to be dual with respect to (w.r.t) a standard neutrosophic negation n if

$$\mathcal{T}(n(x), n(y)) = nS(x, y) \quad \forall x, y \in D^*,$$
$$S(n(x), n(y)) = n\mathcal{T}(x, y) \quad \forall x, y \in D^*.$$

Example 4. With negation $n_0(x) = (x_3, 0, x_1)$ we have some t-norm and t-conorm dual as follows:

a. $\mathcal{T}_M$ and S_M
b. $\mathcal{T}_P$ and S_P
c. $\mathcal{T}_L$ and S_L

Many properties of t-norms, t-conorms, negations should be given in [21].

2.4 Standard neutrosophic implication operators

In this section, we recall two classes of standard neutrosophic implication in [21].

A standard neutrosophic implication off class 1.

Definition 5. A mapping $\mathcal{I}: (D^*)^2 \to D^*$ is referred to as a standard neutrosophic implicator off class 1 on D^* if it satisfying following conditions:

$$\mathcal{I}(0_{D^*}, 0_{D^*}) = 1_{D^*}; \mathcal{I}(0_{D^*}, 1_{D^*}) = 1_{D^*}; \mathcal{I}(1_{D^*}, 1_{D^*}) = 1_{D^*}; I(1_{D^*}, 0_{D^*}) = 0_{D^*}$$

Proposition 1. Let $\mathcal{T}, S$ and n be standard neutrosophic t-norm $\mathcal{T}$, a standard neutrosophic t-conorm S and a standard neutrosophic negation on D^*, respectively. Then, we have a standard neutrosophic implication on D^*, which defined as following:

$\mathcal{I}_{S,\mathcal{T},n}(x, y) = S(\mathcal{T}(x, y), n(x)), \forall x, y \in D^*$.

Proof.

We consider border conditions in definition 5.

$\mathcal{I}(0_{D^*}, 0_{D^*}) = S(\mathcal{T}(0_{D^*}, 0_{D^*}), n(0_{D^*})) = S(0_{D^*}, 1_{D^*}) = 1_{D^*}$,

$\mathcal{I}(0_{D^*}, 1_{D^*}) = S(\mathcal{T}(0_{D^*}, 1_{D^*}), n(0_{D^*})) = S(0_{D^*}, 1_{D^*}) = 1_{D^*}$,

$\mathcal{I}(1_{D^*}, 1_{D^*}) = S(\mathcal{T}(1_{D^*}, 1_{D^*}), n(1_{D^*})) = S(1_{D^*}, 0_{D^*}) = 1_{D^*}$, and

$\mathcal{I}(1_{D^*}, 0_{D^*}) = S(\mathcal{T}(1_{D^*}, 0_{D^*}), n(1_{D^*})) = S(0_{D^*}, 0_{D^*}) = 0_{D^*}$.

We have the proof.□

Example 5. For all $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D$, we have some standard neutrosophic implication of class 1 on D^* based on proposition 1 as follows

a. If $\mathcal{T} = \mathcal{T}_M, S = S_M$ and $n_0(x) = (x_3, 0, x_1)$ then $\mathcal{I}_{S_M,\mathcal{T}_M,n_0}(x, y) = (\max(\min(x_1, y_1), x_3), 0, \min(\max(x_3, y_3), x_1)$.

b. If $\mathcal{T} = \mathcal{T}_P, S = S_P$ and $n_1(x) = (x_3, x_4, x_1)$ then $\mathcal{I}_{S_P,\mathcal{T}_P,n_1}(x, y) = (x_1y_1 + x_3 - x_1y_1x_3, x_2y_2x_4, x_1(x_3 + y_3 - x_3y_3))$.

A standard neutrosophic implication off cals 2.

Definition 6. A mapping $\mathcal{I}: (D^*)^2 \to D^*$ is referred to as a standard neutrosophic implicator off class 2 on D^* if it is decreasing in its first component, increasing in its second component and satisfying following conditions:

$$\mathcal{I}(0_{D^*}, 0_{D^*}) = 1_{D^*};\ \mathcal{I}(1_{D^*}, 1_{D^*}) = 1_{D^*};$$
$$\mathcal{I}(1_{D^*}, 0_{D^*}) = 0_{D^*}$$

Definition 7. A standard neutrosophic implicator $\mathcal{I}$ off class 2 is called boder standard neutrosophic implication if $\mathcal{I}(1_{D^*}, x) = x$ for all $x \in D^*$.

Proposition 2. Let $\mathcal{T}, S$ and n be standard neutrosophic t-norm $\mathcal{T}$, a standard neutrosophic t-conorm S and a standard neutrosophic negation on D^*, respectively. Then, we have a standard neutrosophic implication on D^*, which defined as following:

$$\mathcal{I}_{S,n}(x, y) = S(n(x), y), \forall x, y \in D^*.$$

Example 6. For all $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in D$, we have some standard neutrosophic implication of class 1 on D^* based on proposition ? as follows

a. If $S = S_M$ and $n_0(x) = (x_3, 0, x_1)$ then $\mathcal{I}_{S_M,n_0}(x, y) = (\max(x_3, y_1), 0, \min(x_1, y_3))$

b. If $S = S_P$ and $n_1(x) = (x_3, x_4, x_1)$ then $\mathcal{I}_{S_P,n_1}(x, y) = (x_3 + y_1 - x_3y_1, x_4y_2, x_1y_3)$

Note that, we can define the negation operators from implication operators, such as, the mapping $n_{\mathcal{I}}(x) = \mathcal{I}(x, 0_{D^*}), \forall x \in D^*$, is a standard negation on D^*. For example, if $\mathcal{I}_{S_P,n_1}(x, y) = (x_3 + y_1 - x_3y_1, x_4y_2, x_1y_3)$ then we obtain $n_{I_{S_P,n_1}}(x) = \mathcal{I}_{S_P,n_1}(x, 0_{D^*}) = (x_3, 0, x_1) = n_0(x)$.

2.5 Standard neutrosophic set

Definition 8. Let U be a universal set. A standard neutrosophic (PF) set A on the universe U is an object of the form $A = \{(x, \mu_A(x), \eta_A(x), \gamma_A(x)) \mid x \in U\}$ where $\mu_A(x)(\in [0,1])$ is called the "degree of positive membership of x in A", $\eta_A(x)(\in [0,1])$ is called the "degree of neutral membership of x in A" and $\gamma_A(x)(\in[0,1])\gamma_A(x)(\in [0,1])$ is called the "degree of negative membership of x in A", and where μ_A, η_A μ_A, γ_A and γ_A η_A satisfy the following condition:

$$\mu_A(x) + \eta_A(x) + \gamma_A(x) \le 1, (\forall x \in X)\ \mu_A(x) + \gamma_A(x) + \eta_A(x)) \le 1, (\forall x \in X).$$

The family of all standard neutrosophic set in U is denoted by PFS(U).

3. Standard neutrosophic rough set

Definition 9.
Suppose that R is a standard neutrosophic relation on the set of universe U. $\mathcal{T}$ is a t −norm on D^*, $\mathcal{I}$ an implication on D^*, for all $F \in PFS(U)$, we denote $F(v) = (\mu_F(v), \eta_F(v), \gamma_F(v))$. Then (U, R) is a standard neutrosophic approximation space. We define the upper and lower approximation set of F on (U, R) as following

$$\bar{R}^{\mathcal{T}}(F)(u) = \bigvee_{v\in U} \mathcal{T}(R(u, v), F(v)), \forall u \in U$$

and

$$\underline{R}_{\mathcal{I}}(F)(u) = \bigwedge_{v\in U} \mathcal{I}(R(u, v), F(v)), u \in U.$$

Example 7. Let $U = \{a, b, c\}$ be an universe and R is a standard neutrosophic relation on U

$$R = \begin{pmatrix} (0.7,0.2,0.1) & (0.6,0.2,0.1) & (0.5,0.3,0.2) \\ (0.5,0.4,0.1) & (0.6,0.1,0.2) & (0.5,0.1,0.2) \\ (0.3,0.5,0.1) & (0.4,0.2,0.3) & (0.7,0.1,0.1) \end{pmatrix}$$

A standard neutrosophic on U is $F = \{\langle a, 0{,}6{,}0.2{,}0.2\rangle, \langle b, 0.5{,}0.3{,}0.1\rangle, \langle c, (0.7{,}0.2{,}0.1)\rangle\}$. Let $\mathcal{T}_M(x, y) = (x_1 \wedge y_1, x_2 \wedge y_2, x_3 \vee y_3)$ be a t-norm on D^*, and $\mathcal{I}(x, y) = (x_3 \vee y_1, x_2 \wedge y_2, x_1 \wedge y_3)$ be an implication on D^*, forall $x = (x_1, x_2, x_3) \in D^*$ and $y = (y_1, y_2, y_3) \in D^*$, We compute

$$\mathcal{T}(R(a, a), F(a)) = \mathcal{T}((0.7,0.2, 0.1), (0.6,0.2,0.2)) = (0.6,0.2,0.2)$$
$$\mathcal{T}(R(a, b), F(b)) = \mathcal{T}((0.6,0.2,0.1), (0.5,0.3,0.1)) = (0.5,0.2,0.1)$$
$$\mathcal{T}(R(a, c), F(c)) = \mathcal{T}((0.5,0.3,0.2), (0.7,0.2,0.1)) = (0.5,0.2,0.2)$$

Hence $\bar{R}^{T}(F)(a) = \bigvee_{v\in U} \mathcal{T}(R(a, v), F(v)) = (0.6,0.2,0.1)$.

And

$$\mathcal{T}(R(b, a), F(a)) = \mathcal{T}((0.5,0.4, 0.1), (0.6,0.2,0.2)) = (0.5,0.2,0.2)$$
$$\mathcal{T}(R(b, b), F(b)) = \mathcal{T}((0.6,0.1,0.2), (0.5,0.3,0.1)) = (0.5,0.1,0.3)$$
$$\mathcal{T}(R(b, c), F(c)) = \mathcal{T}((0.5,0.1,0.2), (0.7,0.2,0.1)) = (0.5,0.1,0.2)$$

Hence $\bar{R}^{\mathcal{T}}(F)(b) = \bigvee_{v\in U} \mathcal{T}(R(b, v), F(v)) = (0.5,0.1,0.2)$

$$\boldsymbol{\mathcal{T}}\big(R(c,a),F(a)\big) = \boldsymbol{\mathcal{T}}\big((0.3,0.5,0.1),(0.6,0.2,0.2)\big) = (0.3,0.2,0.2)$$
$$\boldsymbol{\mathcal{T}}\big(R(c,b),F(b)\big) = \boldsymbol{\mathcal{T}}\big((0.4,0.2,0.3),(0.5,0.3,0.1)\big) = (0.4,0.2,0.3)$$
$$\boldsymbol{\mathcal{T}}\big(R(c,c),F(c)\big) = \boldsymbol{\mathcal{T}}\big((0.7,0.1,0.1),(0.7,0.2,0.1)\big) = (0.7,0.1,0.1)$$

So that $\bar{R}^{\mathcal{T}}(F)(c) = \bigvee_{v\in U} \boldsymbol{\mathcal{T}}\big(R(c,v),F(v)\big) = (0.7,0.1,0.1)$.

We obtain the upper approximation $\bar{R}^{T}(F) = \frac{(0.6,0.2,0.1)}{a} + \frac{(0.5,0.1,0.2)}{b} + \frac{(0.7,0.1,0.1)}{c}$.

Similarly, computing with the lower approximation set, we have $\boldsymbol{\mathcal{I}}\big((0.7,0.2,0.1),(0.6,0.2,0.2)\big) = (0.1,0.2,0.7) \vee (0.6,0.2,0.2) = (0.6,0.2,0.2)$

$$\boldsymbol{\mathcal{I}}\big(R(a,b),F(b)\big) = \boldsymbol{\mathcal{I}}\big((0.6,0.2,0.1),(0.5,0.3,0.1)\big) = (0.1,0.2,0.6) \vee (0.5,0.3,0.1) = (0.5,0.2,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(a,c),F(c)\big) = \boldsymbol{\mathcal{I}}\big((0.5,0.3,0.2),(0.7,0.2,0.1)\big) = (0.2,0.3,0.5) \vee (0.7,0.2,0.1) = (0.7,0.2,0.1)$$

$\underline{R}_{\mathcal{I}}(F)(a) = \bigwedge_{v\in U} \boldsymbol{\mathcal{I}}\big(R(a,v),F(v)\big) = (0.5,0.2,0.2)$.

And

$$\boldsymbol{\mathcal{I}}\big(R(b,a),F(a)\big) = \boldsymbol{\mathcal{I}}\big((0.5,0.4,0.1),(0.6,0.2,0.2)\big) = (0.6,0.2,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(b,b),F(b)\big) = \boldsymbol{\mathcal{I}}\big((0.6,0.1,0.2),(0.5,0.3,0.1)\big) = (0.5,0.1,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(b,c),F(c)\big) = \boldsymbol{\mathcal{I}}\big((0.5,0.1,0.2),(0.7,0.2,0.1)\big) = (0.7,0.1,0.1)$$

$\underline{R}_{\mathcal{I}}(F)(b) = \bigwedge_{v\in U} \boldsymbol{\mathcal{I}}\big(T(b,v),F(v)\big) = (0.5,0.1,0.1)$.

$$\boldsymbol{\mathcal{I}}\big(R(c,a),F(a)\big) = \boldsymbol{\mathcal{I}}\big((0.3,0.5,0.1),(0.6,0.2,0.2)\big) = (0.6,0.2,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(c,b),F(b)\big) = \boldsymbol{\mathcal{I}}\big((0.4,0.2,0.3),(0.5,0.3,0.1)\big) = (0.5,0.2,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(c,c),F(c)\big) = \boldsymbol{\mathcal{I}}\big((0.7,0.1,0.1),(0.7,0.2,0.1)\big) = (0.7,0.1,0.1)$$

Hence $\underline{R}_{\mathcal{I}}(F)(c) = \bigwedge_{v\in U} \boldsymbol{\mathcal{I}}\big(R(c,v),F(v)\big) = (0.5,0.1,0.1)$.

So that

$\underline{R}_{\mathcal{I}}(F) = \frac{(0.5,0.2,0.2)}{a} + \frac{(0.5,0.1,0.1)}{b} + \frac{(0.5,0.1,0.1)}{c}$.

Now, we have the upper and lower approximations of $F = \frac{(0,6,0.2,0.2)}{a} + \frac{(0.5,0.3,0.1)}{b} + \frac{(0.7,0.2,0.1)}{c}$ are

$$\bar{R}^{\mathcal{T}}(F) = \frac{(0,6,0.2,0.1)}{a} + \frac{(0.5,0.1,0.2)}{b} + \frac{(0.7,0.1,0.1)}{c}$$

and

$$\underline{R}_{\mathcal{I}}(F) = \frac{(0.5,0.2,0.2)}{a} + \frac{(0.5,0.1,0.1)}{b} + \frac{(0.5,0.1,0.1)}{c}$$

Example 8. Let $U = \{a,b,c\}$ be an universe set. And R is a standard neutrosophic relation on U with

$$R = \begin{pmatrix} (1,0,0) & (0.6,0.3,0) & (0.6,0.3,0) \\ (0.6,0.3,0) & (1,0,0) & (0.6,0.3,0) \\ (0.6,0.3,0) & (0.6,0.3,0) & (1,0,0) \end{pmatrix}$$

Let $F = \frac{(0.4,0.3,0.3)}{a} + \frac{(0.5,0.2,0.3)}{b} + \frac{(0.4,0.4,0.1)}{c}$ be standard neutrosophic set on U. A $t-$ norm $\boldsymbol{\mathcal{T}}(x,y) = (x_1 \wedge y_1, x_2 \wedge y_2, x_3 \vee y_3)$, and an implication operator $\boldsymbol{\mathcal{I}}(x,y) = (x_3 \vee y_1, x_2 \wedge y_2, x_1 \wedge y_3)$ for all $x = (x_1,x_2,x_3) \in D^*$, $y = (y_1,y_2,y_3) \in D^*$, we put

$$\boldsymbol{\mathcal{T}}\big(R(a,a),F(a)\big) = \boldsymbol{\mathcal{T}}\big((1,0,0),(0.7,0.2,0.1)\big) = (0.7,0,0.1)$$
$$\boldsymbol{\mathcal{T}}\big(R(a,b),F(b)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.5,0.2,0.3)\big) = (0.5,0.2,0.3)$$
$$\boldsymbol{\mathcal{T}}\big(R(a,c),F(c)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.4,0.4,0.1)\big) = (0.4,0.3,0.1)$$

Then $\bar{R}^{\mathcal{T}}(F)(a) = \bigvee_{v\in U} \boldsymbol{\mathcal{T}}\big(R(a,v),F(v)\big) = (0.7,0,0.1)$.

$$\boldsymbol{\mathcal{T}}\big(R(b,a),F(a)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.7,0.2,0.1)\big) = (0.6,0.2,0.1)$$
$$\boldsymbol{\mathcal{T}}\big(R(b,b),F(b)\big) = \boldsymbol{\mathcal{T}}\big((1,0,0),(0.5,0.2,0.3)\big) = (0.5,0,0.3)$$
$$\boldsymbol{\mathcal{T}}\big(R(b,c),F(c)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.4,0.4,0.1)\big) = (0.4,0.3,0.1)$$

Hence $\bar{R}^{\mathcal{T}}(F)(b) = \bigvee_{v\in U} \boldsymbol{\mathcal{T}}\big(R(b,v),F(v)\big) = (0.6,0,0.1)$.

$$\boldsymbol{\mathcal{T}}\big(R(c,a),F(a)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.7,0.2,0.1)\big) = (0.6,0.2,0.1)$$
$$\boldsymbol{\mathcal{T}}\big(R(c,b),F(b)\big) = \boldsymbol{\mathcal{T}}\big((0.6,0.3,0),(0.5,0.2,0.3)\big) = (0.5,0.2,0.3)$$
$$\boldsymbol{\mathcal{T}}\big(R(c,c),F(c)\big) = \boldsymbol{\mathcal{T}}\big((1,0,0),(0.4,0.4,0.1)\big) = (0.4,0,0.1)$$
$$\bar{R}^{\mathcal{T}}(F)(a) = \bigvee_{v\in U} \boldsymbol{\mathcal{T}}\big(R(a,v),F(v)\big) = (0.6,0,0.1).$$

We obtain the upper approximation set $\bar{R}^{\mathcal{T}}(F) = \frac{(0.7,0,0.1)}{a} + \frac{(0.6,0,0.1)}{b} + \frac{(0.6,0,0.1)}{c}$.

Similarly, computing with the lower approximation, we have

$$\boldsymbol{\mathcal{I}}\big(R(a,a),F(a)\big) = \boldsymbol{\mathcal{I}}\big((1,0,0),(0.7,0.2,0.1)\big) = (0,0,1) \vee (0.7,0.2,0.1) = (0.7,0,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(a,b),F(b)\big) = \boldsymbol{\mathcal{I}}\big((0.6,0.3,0),(0.5,0.2,0.3)\big) = (0,0.3,0.6) \vee (0.5,0.2,0.3) = (0.5,0.2,0.3)$$
$$\boldsymbol{\mathcal{I}}\big(R(a,c),F(c)\big) = \boldsymbol{\mathcal{I}}\big((0.6,0.3,0),(0.4,0.4,0.1)\big) = (0,0.3,0.6) \vee (0.4,0.4,0.1) = (0.4,0.3,0.1)$$

$\underline{R}_{\mathcal{I}}(F)(a) = \bigwedge_{v\in U} \boldsymbol{\mathcal{I}}\big(T(a,v),F(v)\big) = (0.4,0,0.3)$.

Compute

$$\boldsymbol{\mathcal{I}}\big(R(b,a),F(a)\big) = \boldsymbol{\mathcal{I}}\big((0.6,0.3,0),(0.7,0.2,0.1)\big) = (0,0.3,0.6) \vee (0.7,0.2,0.1) = (0.7,0.2,0.1)$$
$$\boldsymbol{\mathcal{I}}\big(R(b,b),F(b)\big) = \boldsymbol{\mathcal{I}}\big((1,0,0),(0.5,0.2,0.3)\big) = (0,0,1) \vee (0.5,0.2,0.3) = (0.5,0,0.3)$$

$\mathcal{I}(R(b,c),F(c)) = \mathcal{I}((0.6,0.3,0),(0.4,0.4,0.1))$
$= (0,0.3,0.6) \vee (0.4,0.4,0.1)$
$= (0.4,0.3,0.1)$

$\underline{R}_{\mathcal{I}}(F)(b) = \bigwedge_{v\in U} \mathcal{I}(T(b,v),F(v)) = (0.4,0,0.3)$.

and

$\mathcal{I}(R(c,a),F(a)) = \mathcal{I}((0.6,0.3,0),(0.7,0.2,0.1))$
$= (0,0.3,0.6) \vee (0.7,0.2,0.1)$
$= (0.7,0.2,0.1)$

$\mathcal{I}(R(c,b),F(b)) = \mathcal{I}((0.6,0.3,0),(0.5,0.2,0.3))$
$= (0,0.3,0.6) \vee (0.5,0.2,0.3)$
$= (0.5,0.2,0.3)$

$\mathcal{I}(R(c,c),F(c)) = \mathcal{I}((1,0,0),(0.4,0.4,0.1))$
$= (0,0,1) \vee (0.4,0.4,0.1) = (0.4,0,0.1)$

$\underline{R}_{\mathcal{I}}(F)(c) = \bigwedge_{v\in U} \mathcal{I}(T(c,v),F(v)) = (0.4,0,0.3)$.

Hence

$$\underline{R}_{\mathcal{I}}(F) = \frac{(0.4,0,0.1)}{a} + \frac{(0.4,0,0.3)}{b} + \frac{(0.4,0,0.3)}{c}$$

Now, we have the upper and lower approximation sets of $F = \frac{(0.4,0.3,0.3)}{a} + \frac{(0.5,0.2,0.3)}{b} + \frac{(0.4,0.4,0.1)}{c}$ as following

$$\bar{R}^{\mathcal{T}}(F) = \frac{(0.7,0,0.1)}{a} + \frac{(0.6,0,0.1)}{b} + \frac{(0.6,0,0.1)}{c}$$

and

$$\underline{R}_{\mathcal{I}}(F) = \frac{(0.4,0,0.3)}{a} + \frac{(0.4,0,0.3)}{b} + \frac{(0.4,0,0.3)}{c}.$$

Remark 3. If R is reflexive, symmetric transitive then $\underline{R}_{\mathcal{I}}(F) \subset F \subset \bar{R}^{\mathcal{T}}(F)$.

4. Some properties of standard neutrosophic rough set

Theorem 1. Let (U,R) be the standard neutrosophic approximation space. Let $\mathcal{T}, S$ be the t-norm , and t –conorm D^*, n is a negative on D^*. If S and T are dual w.r.t n then

(i) $\sim_n \underline{R}_{\mathcal{I}}(A) = \bar{R}^{\mathcal{T}}(\sim_n A)$

(ii) $\sim_n \bar{R}^{\mathcal{T}}(A) = \underline{R}_{\mathcal{I}}(\sim A)$

where $\mathcal{I}(x,y) = S(n(x),y), \forall x,y \in D^*$.

Proof.

(i) $\sim_n \bar{R}^{\mathcal{T}}(\sim_n A) = \underline{R}_{\mathcal{I}}(A)$.

Indeed, for all $x \in U$, we have

$$\bar{R}^{\mathcal{T}}(\sim_n A)(x) = \bigvee_{y\in U} \mathcal{T}[R(x,y),\sim_n A(y)]$$
$$= \bigvee_{y\in U} nS[nR(x,y),n(\sim_n A(y))]$$
$$= \bigvee_{y\in U} nS[nR(x,y),A(y)] .$$

Moreover,

$$\underline{R}_{\mathcal{I}}(A)(x) = \bigwedge_{y\in U} \mathcal{I}(R(x,y),A(y))$$
$$= \bigwedge_{y\in U} S[nR(x,y),A(y)]$$

Hence

$$\sim_n \underline{R}_{\mathcal{I}}(A)(x)(x) = n(\bigwedge_{y\in U} S[nR(x,y),A(y)])$$
$$= = \bigvee_{y\in U} nS[nR(x,y),A(y)]$$

and $\bar{R}^{T}(\sim_n A)(x) =\sim_n \underline{R}_{\mathcal{I}}(A)(x), \forall x \in U$.

(ii) $\underline{R}_{\mathcal{I}}(\sim_n A) =\sim_n \bar{R}^{\mathcal{T}}(A)$

Indeed, for all $x \in U$ we have

$\underline{R}_{\mathcal{I}}(\sim_n A)(x) = \bigwedge_{y\in U} \mathcal{I}(R(x,y),\sim_n A(y)), x \in U = \bigwedge_{y\in U} S[nR(x,y),\sim_n A(y)]$

And $\sim_n \bar{R}^{T}(A)(x) = n(\bigvee_{y\in U} \mathcal{T}[R(x,y),A(y))]) = \bigvee_{y\in U} n\mathcal{T}[R(x,y),A(y)]$

$$= \bigwedge_{y\in U} S[nR(x,y),\sim_n A(y)]$$

It means that $\underline{R}_{\mathcal{I}}(\sim_n A)(x) =\sim_n \bar{R}^{\mathcal{T}}(A)(x), \forall x \in U$. □

Theorem 2. a) $\bar{R}^{\mathcal{T}}((\widehat{\alpha,\beta,\theta})) \subset (\widehat{\alpha,\beta,\theta})$, where $(\widehat{\alpha,\beta,\theta})x = (\alpha,\beta,\theta)$, $\forall x \in U$

b) $\underline{R}_{\mathcal{I}}((\widehat{\alpha,\beta,\theta})) \supset (\widehat{\alpha,\beta,\theta})$, where I is a border implication in class 2.

Proof.

a) We have
$\bar{R}^{\mathcal{T}}((\widehat{\alpha,\beta,\theta}))(u) =$
$\bigvee_{v\in U} \mathcal{T}\left(R(u,v),(\widehat{\alpha,\beta,\theta})(v)\right) =$
$\mathcal{T}\left(\bigvee_{v\in U} R(u,v),(\alpha,\beta,\theta)\right) \leq_{D^*} \mathcal{T}(1_{D^*},(\alpha,\beta,\theta))$
$= (\alpha,\beta,\theta) = (\widehat{\alpha,\beta,\theta})(u),\ \forall u \in U$

b) We have
$\underline{R}_{\mathcal{I}}((\widehat{\alpha,\beta,\theta}))(u) =$
$\bigwedge_{v\in U} \mathcal{I}\begin{pmatrix} R(u,v), \\ (\widehat{\alpha,\beta,\theta})(v)\end{pmatrix} = \bigwedge_{v\in U} \mathcal{I}\begin{pmatrix} R(u,v), \\ (\alpha,\beta,\theta)\end{pmatrix} \geq_{D^*} \bigwedge_{v\in U} \mathcal{I}(1_{D^*},(\alpha,\beta,\theta)) =$
$(\alpha,\beta,\theta) = (\widehat{\alpha,\beta,\theta})(u),\ \forall u \in U$□

5. Conclusion

In this paper, we introduce the $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets based on an implicator $\mathcal{I}$ and a t-norm $\mathcal{T}$ on D^*, lower and upper approximations of standard neutrosophic sets in a standard neutrosophic approximation are first introduced. We also have some notes on logic operations. Some properties of $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets are investigated. In the feature, we will investigate more properties on $(\mathcal{I},\mathcal{T})-$ standard neutrosophic rough sets.

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