Flipping Arrows

Bryan W. Roberts: Reversing the Arrow of Time,
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Time, so it appears, goes one way. Not the other. Why? One would expect the theories of modern physics would provide us with something like an answer to such a basic and fundamental question about the universe we live in. And, although the matter is variously contested, to a reasonable extent they do. What physics does not provide, however, is a unified and systematic account of the representation of time reversal symmetry that is applicable across physical theories. It is in this regard that Bryan Roberts has taken a decisive step forwards. As such, this book mines the rich but underexploited seam of natural philosophical ideas that deepen our understanding of the structure of physical theory in a manner valuable to philosophers and physicists alike. Much of the book is highly formal, but the philosophical and scientific value of what is done are difficult to overstate.

The centre piece of Roberts's account is the simple, yet hugely powerful, observation that the *meaning* of time reversal operation, T, on the state space of a physical theory, S, can be fixed via the *representation* of a spacetime symmetry group, G, that contains a sub-structure corresponding to the one-parameter Lie group of time translations, $(\mathbb{R}, +)$. This 'representation view' on time reversal can be established in three steps. First, it is demonstrated that if τ is a non-trivial automorphism of $(\mathbb{R}, +)$ that is also an involution, then we have that $\tau(t) = -t$ (Roberts 2022, Proposition 2.1). Second, we note that a representation of G is given by a homomorphism $\phi : G \to \mathrm{Aut}(\mathfrak{A})$, where $\mathrm{Aut}(\mathfrak{A})$ are the automorphisms of the state space structure $\mathfrak{A}$ and where the restriction of ϕ to the sub-structure of time translations is also a homomorphism (Roberts 2022, Definition 2.3). Finally, we define the instantaneous time reversal operation T via the image of τ under the homomorphism, $T = \phi(\tau)$. Remarkably, these sparse ingredients alone are almost sufficient to recover the standard form of the T operation recognised in physical practice. Most significantly, if we add the further assumption of half-bounded Hamiltonians we can recover the anti-symplectic operation for Hamiltonian systems and anti-unitary operation for unitary quantum theories (Roberts 2022, §3). That is, we get the operations that take the form $T : (q, p) \to (q, -p)$ with the qs and ps understood as variables, fields, or operators as appropriate. We then recover T invariance of the dynamical laws under the expected conditions. For example, we find that Hamilton's equations and the von Neumann equation are T invariant if and only if the respective classical and

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quantum Hamiltonians are T invariant. The view also provides an interesting frame for some valuable work on the foundations of thermodynamics (which we unfortunately will not have space to discuss). Finally, and most satisfactorily, Roberts's representation view serves to isolate the one physical context (electroweak theory) in which we *do* find violation of instantaneous time reversal symmetry, both in theory and in experiment. So far so good.

But given we arrive at such unequivocal support for the physics orthodoxy, why is all this so valuable? Steven Weinberg once quipped that that benefits of philosophers' insights to physicists are generally that of 'protecting them from the preconceptions of other philosophers' (Weinberg 1994, p. 166). This is at least partially true in this context. Here the relevant preconception is the (intuitive) view that an instantaneous time reversal operation in a physical theory is essentially a time reflection: what it *is* to be a time reversal transformation is to 'just flip t'. That is, we should define the operation time reversal operation applying the transformation $t \to -t$ within a spacetime coordinatization of set of physical events and any further transformations of equations of motion or state space variables must then follow from 'logic or definition alone' (Callender 2023, p.11).[1] Significantly, the standard anti-unitary time reversal operation in quantum mechanics *does not* follow form such a view, and on that basis it could be argued that we should apply a unitary definition of time reversal and reach the conclusion that quantum mechanics is *not* time reversal invariant (*pace* the physics orthodoxy). The 'just flip t' view makes a degree of sense in the context of the force law representation of Newtonian mechanics. However, it is evidently inadequate in the context of richer physical theories. In particular, as Roberts notes, if we are representing the world via a state space we should require that any spacetime symmetry transformations defined on that space is a representation (in the formal sense) of a spacetime symmetry structure. Whilst this may not follow from logic and definition alone, it is an unavoidable precondition of coherent physico-mathematical practice. And once we are in the business of representing time translation transformations on state spaces, Roberts shows that minimal further ingredients are needed to recover the familiar time reversal operations and restore the time reversal symmetry of quantum theory. Though framed in a charmingly ecumenical manner, Roberts's book represents a concrete refutation for the 'just flip t' view and is a substantial achievement for that alone.

The value of this book, and this approach in general, goes much beyond settling internal debates in the philosophy of physics, however. What we get by adopting the representation view of time reversal is a unifying framework for understanding the interrelation of a class of state space operations across physical contexts. There are multitude of payoffs from such a framework, not least in better understanding the intricate interplay between state space structures and spacetime symmetries both within and between theories. Moreover, what is achieved in this remarkable book also serves as an exemplification of a general and powerful philosophy of symmetry that offers new insights into centuries

[1] For the two most prominent and influential expressions of this sentiment see in particular (Albert 2000, p. 20) and (Callender 2000, p.254). In a similar vein, (Castellani and Ismael 2016, p. 1011) suggest that '[i]n physical terms, time reversal should leave the states intrinsically untouched and just change their order'. Here it is interesting to note the view adopted by López (2021) that there is an implicit appeal to substantivalism underlying the intuition of the 'just flip t' viewpoint. For further philosophical arguments towards physical orthodoxy see Malament (2004); Roberts (2017, 2021).

old debates concerning direct empirical significance, symmetry-to-reality-inferences, and the relation between spacetime and dynamical symmetries.

What is not clear, however, is whether the violation of time reversal symmetry fully captures the physical content of the arrow of time. In fact, it is plausible to argue that violation of time reversal symmetry, in Roberts's representation, is neither necessary nor sufficient to establish an arrow of time. That is, there can be situations both where we have: (A) violation of time reversal symmetry but no arrow of time; and (B) an arrow of time but no violation of time reversal symmetry.

Let us first consider the prime candidate for an (A) situation. Here we can draw upon the interpretation of electroweak theory offered by Price (2011) in response to Maudlin (2007). The key idea is to focus upon the residual freedom to choose a temporal orientation of Minkowski spacetime in electroweak theory that exists despite the violation of time reversal symmetry. Following this view, the fact that we are forced to choose one or the other temporal orientation is understood as a *much weaker condition* than there being a fundamental asymmetry between past and future. The move is, in essence, built upon the existence of a gauge freedom within the theory such that there is a descriptive redundancy as to which orientation we pick as representing past-future. Roberts offers some perceptive commentary on the substance of the Price-Maudlin exchange and does not dispute Price's central contention that the requirement to pick an orientation is weaker than there being an objective past-future distinction. The existence of an electroweak arrow is perfectly compatible with absence of an 'arrow of time' that uniquely distinguishes the past from the future. Furthermore, it is also physically plausible that the existence of an electroweak arrow is compatible with the absence of the usual phenomenology of directed temporal processes such as the cooling of cups of coffee, the spreading of ripples in ponds, and so on. In particular, it remains to be seen what connection, if any, there is between the violation of time reversal symmetry in electroweak theory and the low entropy initial conditions that are standardly understood to be the basis of the gravitational, radiative, statistical mechanical arrows of time. Nothing in our present physical knowledge rules out a universe in which time reversal symmetry is violated in electroweak interactions and yet the phenomenological arrow of time, outside the electroweak sector, is absent. Without such connections being better understood it is not clear that the functional role of an arrow of time can be fulfilled by the failure of time reversal invariance in electroweak theory.[2]

Let us move on to consider a candidate for a (B) situation. This is the case where we have an arrow of time but no violation of time reversal symmetry. Here we can consider a simple mechanical system with damping forces. Intuitively, we would expect that such a system will be time reversal invariant and yet temporally directed when the damping force is an even power of velocity. More formally, we can consider a geometric representation of a damped system in terms of *contact Hamiltonian mechanics*, which is a generalisation of symplectic Hamiltonian mechanics characterised by an odd-dimensional manifold. Let us assume, in the spirit of Roberts's view, that we require time reversal to

[2]It should also be noted that since the electroweak sector of the standard model is not expected to be UV complete, there are reasons to doubt arguments based upon renormalization theory that the standard model provides a 'complete enough' representation to establish fundamental facts about the temporal structure of spacetime. See (Roberts 2022, §7.2) for the argument and Antoniou and Thébault (2023) for more details on the status of renormalization theory arguments in the standard model.

an automorphism of the contact phase space and that we would like contact time reversal to coincide with the symplectic time reversal in the appropriate circumstances. Consider a contact manifold with coordinates (q, p, S) and a differential one-form $\eta = ds - pdq$ (Bravetti et al. 2017). Time reversal will be the contactomorphism $T\eta = -\eta$ and it is then straightforward to show that the contact Hamilton's equations are invariant under T if and only if the contact Hamiltonian is.[3] Thus, we find that the contact Hamiltonian theory is time reversal invariant in precisely the same situations as the symplectic Hamiltonian theory. However, such models clearly *do* have an arrow of time in the physical sense of time directed dissipation. Formally this is encoded in the compression of the volume form along the Reeb vector field given by $\frac{\partial}{\partial S}$. And indeed, the generalisation of such models provides a possible basis for a cosmological arrow of time (Sloan 2021; Gryb 2021; Gryb and Sloan 2021). Thus, it seems that we can have an arrow of time without violation of time reversal symmetry. As such, if we want to understand the full content of the asymmetry of time in physics, it is plausible that we will need to look beyond violation of time reversal symmetry.

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[3] An interesting project would be to explicitly recover the contact Hamiltonian mechanics T operation based upon the representation theory of time translations.

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