The Problem of Time[*]

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1 Introduction

The 'problem of time' is a cluster of interpretational and formal issues in the foundations of general relativity relating to both the representation of time in the classical canonical formalism, and to the quantization of the theory. The problem was first noticed by Bergmann and Dirac in the late 1950s, and is still a topic of intense debate in contemporary physics and philosophy of physics. The purpose of this short chapter is to provide an accessible introduction to the problem, and this will inevitably mean that many signifiant technical details will be obscured or over simplified. The most significant simplification that we will make is to focus exclusively on the *global* aspect of the the problem of time. That is, we will, for the most part, restrict ourselves to the 'disappearance of time' in theories invariant under global time reparametrizations. This restriction inevitably means that the important and philosophically rich subtleties relating to local time reparametrizations (refoliations) will not be considered in much detail. Furthermore, in the presentation below I have chosen to focus on a particular dialectic, drawn from the contrast between the views of Barbour and Rovelli, as a means to illustrate my own views. As such the treatment here is claimed to be neither comprehensive, nor entirely neutral. The reader in search of discussions of the problem of time seen from a wider viewpoint and described in its full technical splendour, is directed towards the research literature in physics and philosophy of physics.[1]

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[1]A comprehensive monograph devoted to the problem is (Anderson 2017). The three main physics review articles are (Isham 1992; Kuchař 1991; Anderson 2012). Two exemplary modern discussion are (Pons et al. 2010) and (Gambini et al. 2009). The best physics 'textbook' dicussions are in (Rovelli 2004; Thiemann 2007). Techni-

2   Leibniz and the Problem of Time

Like many of the deepest conceptual problems in modern physics, important aspects of the problem of time in quantum gravity can be traced back to debates within early modern natural philosophy. Of particular importance is Leibniz's critique of the Newtonian absolute conception of time. In particular, Leibniz's assertion that 'instants, consider'd without the things, are nothing at all; and that they consist only in the successive order of things' (Alexander 1998, p.26-7). The first half of this quote contains a negative critique of absolute time along the lines of what has been called 'Aristotle's principle' – there cannot be changeless duration. The second half of the quote then puts forward the kernel of Leibniz's positive view – that time is an 'order of successions'. Although it is remarkably subtle, and arguably not entirely consistent, it will prove instructive to our discussion to consider Leibniz's positive metaphysics of time in a little detail. In particular, rather surprisingly, Leibniz's metaphysical vocabulary will be found to be well suited to distinguishing between modern approaches to the problem of time advocated by Julian Barbour and Carlo Rovelli. For the most part our discussion of Leibniz's metaphysics of time draws upon the magisterial scholarship of Richard Arthur (1985, 2014) other relevant sources will be indicated as appropriate.

An instructive starting point distinction made in the contemporary literature (although not endorsed by Arthur) is between three 'levels of reality' in Leibniz's mature metaphysics.[2] At the most basic level, what is real for Leibniz are simple substances which alone have true unity. These are the famously obscure monads. Next, we have the 'phenomenal level' that is made up of *phenomena bene funda* – well founded phenomena – that, due to pre-established harmony, are accurate reflections of the real and actual monadic states. Finally, we have the ideal level which, by contrast, is made up of *entia rationis* – abstract or fictional things – that include 'phenomena' founded upon possible but non-actual monadic states. Crucially, although both the phenomenal and the ideal levels can include things which are infinite, all concepts that depending upon the continuum are only applicable to the ideal realm. Thus, if we were to define time as the real line, $\mathbb{R}$, then this concept of time could only be represented for Leibniz as an entia rationis and thus ideal. Furthermore, phenomenal things for Leibniz can only acquire their status as phenomena bene funda by their grounding upon the actual. They must always be understood as representations or perceptions of the monads of the actual world.

Leibniz's view of the ontological status of relations is subtle and

cally informed philosophical dicussions are (Belot and Earman 2001; Earman 2002; Maudlin 2002; Belot 2007; Thébault 2012; Gryb and Thébault 2015; Pitts 2014a; Gryb and Thébault 2016) and (Rickles 2007, §7).

[2]See for example (Winterbourne 1982; Hartz and Cover 1988).

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significant for his view of time. Following Arthur (2014) (who is following (Mugani 2012)) we can take Leibniz to believe that relations such as situation and succession supervene on intrinsic modifications of the monads: changes in their perceptions and appetition. Thus, with regard to time, real temporal relations of succession, both between the states of the same monad and the states of different monads, are founded upon the appetitive activity of individual monads. We can then consider notions of time relevant to each of the 'three levels' defined above. On the fundamental level of the real monads or the actual world, all we have are changes of the monadic states which are coordinated via the principle of pre-established harmony. The phenomenal level consists of well founded phenomena arise as representations (perceptions) of the actual monads. At this level we have both actual time ordering and also actual durations. Finally, at the ideal level of entia rationis we have possible time orderings and also possible durations. Rich and sophisticated though it may be, the Leibnizian metaphysics of time runs into two immediate problems. Each of which foreshadows an important aspect of the problem of time in quantum gravity. First, what determines the 'order of succession' of phenomenal states needed to fix the actual time ordering? Second, what determines the duration measure needed to fix the 'quantity of time' between actual phenomenal states?

In essence, the first question relates to the the requirement for a monotonic parametrization of states. That is, an undirected labelling of temporal states by a parameter which is either always increasing or always decreasing. An earlier literature (Rescher 1979) follows Russell (1900) in convicting Leibniz a vicious circularity that requires a non-relational concept of time at the basic monadic level. In contrast, Arthur (1985, 2014) convincingly argues that we can understand time at the monadic level purely in terms of a (non-circular) inter-monadic notion of temporal succession based upon compossibility. Temporal succession at the monadic level could then be taken to ground a total ordering of temporal states in the phenomenal realm.[3] This is sufficient to give us a monotonic parametrization of states and thus a non-directed model of time as an order of successions. Compossibility is not, however, sufficient to ground a directed ordering temporal states. Such a notion depends on further structure implicit in the monadic appetition. Since the question of directed time ordering is rather tangential to the problem of time we will set it aside and implicitly assume that 'succession' has its undirected connotation. See Arthur (1985, 2014) for discussion in the context of Leibniz and (Kiefer and Zeh 1995) for discussion in the context of the problem of time.

Let us now turn to the second question regarding duration. This is-

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[3]Arguably, compossibility seems to underdetermine the actual temporal ordering since it does not give us grounds to distinguish it from merely possible temporal orderings. See (Cover 1997; De Risi 2007).

sue seems particularly pressing for Leibniz. In fact, in the famous correspondence with Clarke, arguably the strongest critique Clarke gives of Leibnizian time is that 'the order of things succeeding each other in time is not time itself, for they may succeed each other faster or slower in the same order of succession, but not in the same time' (Alexander 1998, p.52). Leibniz's response in the correspondence is not entirely clear: he claims that the quantity of time could not become greater and yet the order of successions remain the same since 'if the time is greater, there will be more successive and like states interposed' (Alexander 1998, pp.89-90). On the one hand, although it does seem consistent for Leibniz to assert Aristotle's principle and flatly deny that the distinction Clarke is making corresponds to a difference. However, on the other, there is still an appreciable explanatory burden upon Leibniz to provide an means by which his phenomenal notion of duration can be quantified. A relational notion of time still requires a *determinate metric structure* in order for time to play its functional role in mechanics (De Risi 2007, p.273). There is a strong hint towards a more satisfactory resolution in Leibniz's late writings. Arthur (Arthur 1985; Arthur 2014), in particular, suggests that *Initia rerum mathemat carum metaphysica* (Loemker 1969) contains a line of response via the definition of temporal distances in terms of 'maximally determined' or 'simplest path' through interposed constituents (see also (Vailati 1997, p.136)). Even more tantalisingly, Leibniz wrote in 1680 that 'the basis for measuring the duration of things is the agreement obtained by assuming different uniform motions (like those of different precise clocks)' (quoted in (Arthur 2014, p.206)). Furthermore, (Rescher 1979, p.66) suggests that based upon the principle of perfection we might expect that 'nomic harmony' sufficient to establish a nature phenomenal measure of duration is a contingent feature of the actual world. These hints not withstanding, Leibnizian relationalism about time seems to have insufficient resources to give a relational basis for temporal distances.

Three points from our discussion of Leibniz's metaphysics of time will be of particular significance in what follows. First we have the idea that a relationalist about time, such as Leibniz, may still consistently assert that time orderings are fundamental. That is, a modern Leibnizian style relationalist about time will look to retain a monotonic parametrization of temporal states. Second, we have the Aristotle's principle that asserts that duration is inseparable from change, and thus that denies that changeless duration can exist. Finally, we have the metricity problem, notwithstanding Aristotle's principle, a relational notion of time still requires means to fix a determinate metric structure in order for time to play its functional role in mechanics.

3 Reparametrization Invariance

The formalism of Newton's system of mechanics is one of differential equations. In particular, Newtonian theory features equations between *rates of change of velocity* (i.e. acceleration) and forces (e.g. between gravitating bodies). When all goes well, these equations can be solved and the solutions are usually expressed as functions for the position of a body *over time*. Most iconically one can derive the elliptical orbits of the celestial bodes. Newtonian mechanics as written in terms of force laws and differential equations is extremely cumbersome in practice. One of the most important developments in eighteenth and nineteenth century mathematics was in *reformulating* Newtonian mechanics as a theory of *variational principles*. The essential idea is to represent the possible states of a mechanical system in an abstract high-dimensional space (a possibility space) and to represent possible histories as curves in this space. Physical possibilities are then picked out via restrictions on the curves. One of the most important variational formulations of mechanics is the 'Lagrangian formulation'. In this formulation the possibility space, labelled $T\mathcal{C}$, is made up of $6n$-dimensions where n is the number of particles in the system. If we have three particles then we would have 18 dimensions. To describe a particle, labelled with an index $i = 1, ..., n$, we specify the spatial position, $\boldsymbol{q}_i$ and velocity, $\dot{\boldsymbol{q}}_i$. Since space is three dimensional each of these quantities requires three numbers to be specified – it is *vectorial* – this is indicated by the bold typeface. All together we end up with $2 \times 3 \times n = 6n$. Paths, γ, through this $6n$ dimensional possibility space are mappings between the set of real numbers and $T\mathcal{C}$; i.e. we have that $\gamma : \mathbb{R} \to T\mathcal{C}$. We can pick out a privileged group of physical paths by use of an *action functional*, $S(\gamma)$, which is defined via the integration of the *Lagrangian functional* along the path:

$$S(\boldsymbol{q}_i, t)] = \int_\gamma L(\boldsymbol{q}_i, t) dt \tag{1}$$

The physical paths are those that have an *extremal* action, $\delta S = 0$. This idea of an extremal action is a subtle one, and lies at the heart of all variational approaches to mechanics. Most significantly, variational principles of extremal action supply us with a nomological restriction of which curves in the possibility space are physically possible.[4]

The Lagrangian description of mechanics makes use of a temporal parameter in two senses: first, within the definition of the velocities, $\dot{\boldsymbol{q}}_i = \frac{d\boldsymbol{q}_i}{dt}$; and second, within the time labelling of the curves in the possibility space – their *parametrization*. This representation of time conflicts with Leibniz's view since in this formalism time is something *more* than order of successions: the formalism allows us to represent distinct possibilities that have the same sequence of states of affairs but a different

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[4]See (Smart and Thébault 2015) for discussion of extremal action principles and the metaphysics of laws of nature.

rate at which the states are passed through. This is precisely the possibility that Clarke asserts and Leibniz denies. We can think about this notion of duration is in terms of the existence of a *privileged temporal metric*. That is, an absolute temporal distance measure. Such a structure also implies an order of successions in terms of a monotonic parameterisation of instantaneous states. Thus, the problem with Lagrangian mechanics from a Leibnizian relational viewpoint is one of excess temporal structure.

Fascinatingly for our purposes, not long after the Lagrangian formalism was developed (principally by Lagrange himself but also by Hamiltonian) a modification of the theory by Jacobi was made that allows more naturally for a Leibnizian viewpoint.[5] The first step is to expand our possibility space and treat time as an additional coordinate, $q_0 = t$, in a $6n + 2$ dimensional *extended possibility space*. Velocities in this space are then defined for all of the q_μ by differentiation with respect to an arbitrary parameter τ so we have that $q'_\mu = \frac{dq_\mu}{d\tau}, \mu = 0, ..., n$. This arbitrary parameter is also taken to vary monotonically along curves in extended configuration space: it is an arbitrary label for an (undirected) ordered succession. An important property of extended mechanics is that it is physically invariant under re-scalings of the parameter τ. Theories which display such a dynamic insensitivity to parameterisation are said to be *reparametrization invariant*.

We can associate the time coordinate t (q_0) in extended mechanics with the value taken by a clock external to our mechanical system. In the case of an open system such an interpretation would seem appropriate; but what about if the system is a closed subsystem of the universe? – or even the universe as a whole? In this case there is clearly no physical basis for an external clock and as such we would look to eliminate q_0 as an independent variable. We can do this by the process of *Routhian reduction*.[6] Applying Routhian reduction to extended mechanics leads to a new *Jacobi formalism* that has a possibility space of the same dimensions as the Lagrangian formalism we started with, i.e. $6n$. The Jacobi formalism also features the same set of possible instantaneous states. The difference is that this process of expansion and reduction has lead to a further *constraint* that the total energy is zero. This constraint is directly related to the fact that our new *Jacobi formalism* has retained the reparametrization invariance of the extended formalism. In fact, it can be show that for *any* theory that is reparametrization invariant, the *Hamiltonian function*, which represents the total energy, must be zero.[7].

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[5]The classic formal treatments of the Jacobi formalism in the literature are are (Lanczos 1970, §5), (Johns 2005, §11-12) and (Rovelli 2004, §3.1).

[6]A fuller discussion of Routhian reduction in general, and in this case in particular, is given in (Lanczos 1970, §5) and (Arnold, Kozlov, and A.I. 1988, §3.s2).

[7]More precisely, reparametrization invariance of the action by definition implies that the Lagrange density is homogeneous of order 1 in the velocities. This, via Euler's homogeneous function theorem, then implies that the Hamiltonian density must

These two features – zero Hamiltonian and reparametrization invariance – are at the heart of the problem of time in quantum gravity. They are also, of course, directly related to the Leibnizian viewpoint on time. If time is *only* relational order of successions then we should demand that a theory of mechanics is reparametrization invariant and thus has a zero Hamiltonian.

4   The Global Problem of Time

A standard, and rather misleading, way of introducing the problem of time in quantum gravity is to make reference to a 'deep conceptual conflict' between the treatment of time within the two great pillars of modern physics: quantum theory and general relativity. The problem, it is supposed, arises from forcing a background independent theory of spacetime onto the Procrustean bed of quantization with respect to a background time. For some vague, and rather mysterious, reason it is supposed that time simply *disappears* when we attempt to understand gravity within a quantum framework. Although such a view is difficult to countenance in any substantive sense, it does contain an important kernel of truth. Neither quantization nor gravity are fundamentally at the heart of the problem of time. Rather the problem arises generically from the manipulation of a particular class of classical theories, that includes general relativity, according to the standard formal steps that are preparatory for quantization. That is, the problem of time becomes apparent in the process of preparing *any* reparametrization invariant theory for quantization.

Here we have most in mind two different paths towards quantization: i) Schrödinger's early and rather heuristic route via the Hamiltonian-Jacobi formalism; and ii) the more rigorous canonical quantization techniques which were first developed by Dirac and von Neumann in the 1920s, and subsequently extended to the case of theories with constraints by Dirac in the 1950s (Dirac 1964a). In each case one chooses a particular classical formulation of a theory and then applies a standard recipe to transform to the quantum domain. In each case, when we consider a reparametrization invariant theory the problem of time's disappearance becomes apparent *before* the transformation is applied.

The Hamiltonian formulation of mechanics makes use of a $6n$ dimensional possibility space: 'phase space'. Each point is made up of a pairing of a position and canonical momentum variable. Canonical momentum is a vectorial quantity possess by each particle and given by the expression, $\boldsymbol{p}_i = \frac{\partial L}{\partial \dot{\boldsymbol{q}}_i}$ with $i = 1, ..., n$. Each point in our phase space can then be specified as $\Gamma \in x = (\boldsymbol{q}_i, \boldsymbol{p}_i)$ and curves, as before, can be

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vanish; or, rather, that it be proportional to a constraint (Dirac 1964a)

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taken to represent histories of physical systems.

It is possible within the Hamiltonian framework to provide the nomological restriction to physical curves that represent physically possible histories in terms of a variational principle. However, it is also possible to provide the relevant nomology in terms of the evolution of an *algebra of observables*. Any quantity that can be measured can be represented as a function that maps between points in phase space and the space of real numbers, $f : \Gamma \rightarrow \mathbb{R}$. These functions are called observables and together they form the algebra of observables, $\mathcal{O}(\Gamma)$. The most important observable is energy, and the function that represents total energy is the Hamiltonian function, H, that we met earlier. Now consider *any* other observable, f. If we want to know how f changes with time, in Hamiltonian mechanics all we have to do is calculate the *Poisson bracket* between H and f. This is simply given by:

$$\{f, H\} = \sum_{i=1}^{n} \frac{\partial f}{\partial \boldsymbol{q}_i} \frac{\partial H}{\partial \boldsymbol{p}_i} - \frac{\partial f}{\partial \boldsymbol{p}_i} \frac{\partial H}{\partial \boldsymbol{q}_i} = \dot{f} \qquad (2)$$

The observables form an algebra precisely because the Poisson bracket is a binary operation that takes any pairing of observable functions and returns a third. The Poisson bracket has a deep physical and mathematical significance and there is an important sense in which it is one of the key 'heuristic structures' upon which quantum mechanics was constructed (Saunders 1993) – we will meet another in terms of the Hamilton-Jacobi principal functional shortly.

Essentially the idea is that a classical algebra of observable functions, $\mathcal{O}(\Gamma)$, can be used as a platform upon which to construct a quantum algebra of observable operators, $\hat{A} \in \mathcal{A}(\mathcal{H})$. The former being defined upon phase space, Γ, (a smooth manifold), the latter being defined upon Hilbert space, $\mathcal{H}$ (a normed vector space equipped with an inner product). Just as points, x, in the phase space classical states, vectors in the Hilbert space, $|\psi\rangle$, represent quantum states. Whereas the Poisson bracket, $\{,\}$ plays the role of the binary operation in the classical observable algebra, the *commutator*, $[,]$, plays the role of the binary operation in the quantum observables algebra. The commutator takes two quantum observable operators and returns a third:

$$[\hat{A}_1, \hat{A}_2] = i\hbar \hat{A}_3 \qquad (3)$$

where $\hbar$ is Planck's constant dived by 2π. The crucial connection between the two algebras is encoded in the relation:

$$[\hat{A}_f, \hat{A}_g] = i\hbar \hat{A}_{\{f,g\}} \qquad (4)$$

Formally speaking quantization, the process of constructing a quantum from a classical theory, takes many forms. One of the best understood and most widely used is canonical quantization. This is the method

of quantization that starts from the Poisson bracket and the Hamiltonian formalism and proceeds to the quantum regime by this identification between the two bracket structures (technically this is a Lie algebra morphism).

As mentioned above, canonical quantization in its original form is not applicable to theories where there are constraints on the phase space, and this of course includes reparametrization invariant theories, within which the Hamiltonian is itself a constraint. Dirac's methodology for the quantization of constrained Hamiltonian theories is rather too complicated to go into full detail in this short chapter.[8] However, by reference to the concept of an algebra of observables introduced above we can outline one key ingredient, and in doing so give a first, rather schematic, presentation of the problem of time. The idea is that for theories with constraints, all elements of the algebra of observables must have zero Poisson bracket with the constraints – they must *commute* with the constraints.[9] For reparametrization invariant theories, the idea is thus to consider a sub-algebra, $\mathcal{P} \subset \mathcal{O}$, made up of functions with *vanishing* Poisson bracket with the Hamiltonian $g \in \mathcal{P}$, where $g : \Gamma \to \mathbb{R}$ such that,

$$\{g, H\} = \dot{g} = 0. \tag{5}$$

Such functions correspond to observable quantities that do not change over time. The restriction to observables that commute with the Hamiltonian amounts to a restriction that *anything that is physically measurable cannot change*. We will call these functions *perennials* after the coinage of the Czech physicist Karel Kuchař – see (Kuchař 1999) – they are also often referred to in the literature as Dirac observables. According to most standard accounts, the problem of time can be explained in terms the equivalence between the observables and perennials within the Hamiltonian formulations of reparametrization invariant theories. That is, when we recast theories such as the Jacobi theory – or in fact general relativity – into a Hamiltonian form, so the argument goes, there are 'good formal reasons' to believe that the set of observables is equivalent to the set of perennials. In this sense, change is no longer part of our Hamiltonian theory! Clearly, whether or not we accept this argument depends very much upon detailed analysis of these 'good formal reasons'. Such an analysis would requires us to take a rather large detour into the technicalities of constrained Hamiltonian mechanics and we will not proceed in this direction here.[10] Rather we will consider the

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[8]For the classic textbook dicussions see (Dirac 1964b; Henneaux and Teitelboim 1992).

[9]Being more precise, the observables must 'weakly commute', meaning that the Poisson bracket must be zero only on the sub-manifold of phase space that the constraints define.

[10]In essence, the question is whether of not we should understand Hamiltonian constraints as generating unphysical 'gauge' transformations that do no change the physical state. Whilst, the majority opinion dating back to Dirac is that we should

classical global problem of time in the context of the Hamilton-Jacobi formalism, such that the connection with quantum problem of time is immediately apparent.

The Hamiltonian system of mechanics that we introduced earlier has behind it a rich and beautiful geometrical structure. Much of this structure is in fact encoded in the Poisson bracket itself, and relates to ideas from symplectic geometry which we will not discuss here.[11] The most important geometrical idea is that of a *generating functional*. These are objects based upon which we can generate transformations of the *entire phase space* into different, but physically equivalent, canonical co-ordinates.[12] Such transformations are particularly useful when they are chosen such that the dynamics in the new coordinates takes a particularly simple form. More specifically, if we label the old coordinates $(\boldsymbol{q}_i, \boldsymbol{p}_i)$ and the new coordinates $(\boldsymbol{Q}_i, \boldsymbol{P}_i)$, then what we would like to find is a transformation such that $\dot{\boldsymbol{Q}}_i = 0$. That is, the new position coordinates are constants of the motion. It can be proved that the generating functional that performs this task is given by a time dependent function of a mix of the old and new coordinates, $S_1(t, \boldsymbol{q}_i, \boldsymbol{Q}_i)$ that solves the *Hamilton-Jacobi equation*:

$$H\left(\boldsymbol{q}_i, \frac{\partial S_1(t, \boldsymbol{q}_i, \boldsymbol{Q}_i)}{\partial \boldsymbol{q}_i}\right) = \frac{\partial S_1(t, \boldsymbol{q}_i, \boldsymbol{Q}_i)}{\partial t} \tag{6}$$

The function S_1 is called the *principal functional*. The usual trick to solve the Hamiltonian equation is use the *Ansatz* for the principal functional, $S_1(t, q, Q) = Et + W(q, Q)$. This reduces the problem to one of calculating the *characteristic functional*, $W(q, Q)$, that solves the equation:

$$H\left(\boldsymbol{q}_i, \frac{\partial W(\boldsymbol{q}_i, \boldsymbol{Q}_i)}{\partial \boldsymbol{q}_i}\right) = 0, \tag{7}$$

The last two equations give us a means to characterise the problem of time. According to one view the hallmark of reparametrization invariant theories is that the Hamilton-Jacobi principal functional, $S_1(t, \boldsymbol{q}_i, \boldsymbol{Q}_i)$, should be identified with the characteristic functional $W(\boldsymbol{q}_i, \boldsymbol{Q}_i)$.[13] That is we only have a 'timeless' equation of the form:

$$H\left(\boldsymbol{q}_i, \frac{\partial S_1(\boldsymbol{q}_i, \boldsymbol{Q}_i)}{\partial \boldsymbol{q}_i}\right) = 0, \tag{8}$$

Given this equation as basic to the Hamilton-Jacobi formalism of reparametrization invariant theories there is then a precise sense in

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– see for example (Rovelli 2004) – various authors have also put forward arguments that we should not (Kuchař 1991; Barbour 1994; Pons 2005; Pooley 2006; Barbour and Foster 2008; Gryb and Thébault 2014; Pitts 2014b).

[11]See (Thébault 2011) for a discussion of the problem of time specifically in the context of symplectic mechanics.

[12]Here and below we are following (Arnold 2013, §9). See also (Lanczos 1970, §8).

[13]See in particular (Rovelli 2004, §3.2).

which even a Leibnizian notion of ordered succession is unavailable. In a theory of mechanics described by (6), the parameter t marks out a (one dimensional) ordered family of canonical transformations that trivialise the dynamics. In a theory of mechanics described by (8) there is only one such transformation, and thus all time itself is trivialised by our canonical transformation. Which of these two formalisms is a more adequate rendition of reparametrization invariant mechanics is thus a question of crucial importance and marks the divide between the two responses to the global problem of time as discussed in the next section.

Starting from the Hamilton-Jacobi formalism there is a reliable heuristic, dating back to Schrödinger, that takes us from the Hamilton-Jacobi principal functional to the wavefunction (Rund 1966, pp. 99-109).[14] Essentially, one considers families of hypersurfaces of constant value of the characteristic function as wavefronts propagating in configuration space with respect to the time parameter. The crucial step is then to interpret these wavefronts as surfaces of constant phase of a complex valued wavefunction on configuration space evolving with respect time. This means one takes the principal functional $S_1(t, q, Q)$ as the basis for a complex wavefunction $\Psi(t)$, defined on a Hilbert space labelled by the eigenvalues of a complete observables and the time parameter t. Applying this heuristic to Equation (6) leads directly to the Schrödinger equation:

$$\hat{H} \ket{\Psi} = i\hbar \frac{\partial \ket{\Psi}}{\partial t}. \tag{9}$$

On the other hand, starting from Equation (8), we are lead to an equation of the form:

$$\hat{H} \ket{\psi} = 0. \tag{10}$$

This is a simple form of the famous Wheeler-DeWitt equation.[15] The Wheeler-DeWitt equation provides us with a 'frozen formalism' for quantum theory, and thus a problem of recovering time evolution. It describes a quantum system trapped in an energy eigenstate with the wavefunction a time independent function. How should we respond to this problem? Below we will discuss two options.[16] The first is to attempt to abstract an internal notion of time evolution based upon the Wheeler-DeWitt type formalism together with *classical* internal clocks. The second is to avoid passage to the frozen formalism in the first place. Each of these options will be discussed in the following section.

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[14]See (Butterfield 2005) for philosophical discussion.

[15]The full Wheeler-DeWitt equation for general relativity can be derived via an exactly analogous line of reasoning based upon the the Einstein-Hamilton-Jacobi equation (Peres 1962; DeWitt 1967).

[16]See (Isham 1992; Kuchař 1991; Anderson 2012) for discussion of further approaches.

5 Finding Time Again

The staring point in our search for lost time is an iconic quote from the great nineteenth century German thinker Ernst Mach (1883):

> It is utterly beyond our power to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected.

Following the Mittelstaedt–Barbour (Mittelstaedt 1976; Barbour 1993) *interpretation* of Mach, we can take such quotes to motivate a view in which a consistent notion of time can be abstracted from the 'changes of things' in a manner such that the inherently interconnected nature of every possible internal measure of time is accounted for. According to the Mittelstaedt–Barbour interpretation, we can understand this 'second Mach's principle' as motivating a relational notion of time that is not merely ontologically parasitic on change, but also equitable, in that it can be derived uniquely from the motions of the entire system taken together. Thus, any isolated system – and, in fact, the universe as a whole – should have its own natural clock emergent from the dynamics. This form of relationalism involves a relative notion of duration as abstracted from change. For there to be a notion of time in the this sense it is not enough to be merely a structure of temporal relations: our emergent time must also be unique and equitable. We cannot, therefore, merely identify an isolated subsystem as our relational clock, since to do so is not only non-unique but would also lead to an inequitable measure, insensitive to the dynamics of the clock system itself. Such sentiments are, to a large extent, consistent with the Leibnizian view of time discussed in the first section. Most obviously, we have Aristotle's principle of inseparability of duration from change. Furthermore, in assuming that there is a unique method for abstracting duration from change, we also assume that there is an absolute ordering within the change; otherwise the abstraction process would be underdetermined. This means that the second Mach's principle ultimately involves the assumption of temporal ordering structure equivalent to a monotonic time parametrization.

Recall from the first section, that a crucial problem that we identified was for the relationalist to fix a determinate metric structure such that time can play its functional role in mechanics. Fascinatingly, it is precisely in addressing this question that a relationalist response to the global problem of time can be formulated. In particular, in various formal and philosophical treatments spanning five decades Barbour (together with collaborators) has put forward a relationalist programme for mechanics that is self-identified as in the spirit of both Leibniz and Mach.[17] We do not have space here to conduct a lengthly analysis of

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[17]A selection of Barbour's key works are: (Barbour 1974; Barbour and Bertotti 1982;

Barbour's views, rather we shall consider a particular formal step made in response to the classical global problem of time and discussed in (Barbour and Foster 2008). First we re-consider the evolution equation for an observable function in a theory invariant under time reparameterizations but this time re-writing the total differential in terms of infinitesimal changes:[18]

$$\frac{\delta g}{\delta t} = \{g, H\} \tag{11}$$

Barbour and Foster insist that, contra Dirac, this equation should *not* be set to zero. Rather, we take the change in the observable to be real even though it may be arbitrarily parameterized. Furthermore, Barbour and Foster show that can express the change in the observable without reference to the parametrization at all by rewriting the equation as:

$$\delta g = \sqrt{\frac{\Sigma_i \delta \boldsymbol{q}_i . \delta \boldsymbol{q}_i}{2(E - V)}} \{g, H\} \tag{12}$$

with E the total energy and V the potential energy. This realises exactly the idea that Leibniz seems to have had in mind: duration emerges as a harmonious aggregation of motions. Furthermore, in constructing a temporal metric from change we arrive at precisely the structure needed to complete Leibniz's relational project.

In contrast to the more moderate species of Leibniz-Barbour temporal relationalism, we can consider a more radical variant of relationalism which does not involve commitment to temporal ordering. In radical relationalism about time we assert that what it means for a physical degree of freedom to change is for it to vary with respect to a second physical degree of freedom; and there is no sense in which this variation can be described in absolute, non-relative terms. This radical relationalism about time is closely associated with the work of Rovelli[19] and to a large extent the mainstream view in the community of physicists working on problem of time in quantum gravity. The attractiveness of the view lies in its connection to a proposal for abstraction of internal notions of change within fundamentally timeless systems of equations (both classical and quantum). This proposal in its modern form is based upon the idea of 'partial observables' and 'complete observables' and can be illustrated explicitly using the Hamilton-Jacobi formalism discussed above.

Consider a globally reparametrization invariant description of n free particles. The Hamiltonian will take the simple form, $H = \Sigma_i \frac{\boldsymbol{p}_i^2}{2m_i}$ for

Barbour 1994; Barbour 2001a; Barbour 2001b; Barbour 2009; Barbour, Koslowski, and Mercati 2014).

[18]Again, we neglect the 'weak equality' for simplicity of exposition.

[19]See for example (Rovelli 1990; Rovelli 1991; Rovelli 2002; Rovelli 2004; Rovelli 2007; Rovelli 2014).

$i = 1, ..., n$. Solving the Hamilton-Jacobi equation gives us an expression for the position variables, q_i, as functions of the constants of motion, Q_i and P_i, and time, t. This takes the form:

$$q_i(t) = Q_i + \frac{P_i}{m_i}t \tag{13}$$

These position variables do not commute with the Hamiltonian and so are clearly *not* perennials. This means that, on the standard view, they should not be considered observables. They do, however, prima facie, seem to have obvious physical significance since they represent the spatial degrees of freedom of our system of particles. To emphasise that such variables are physically significant but not fully observable, Rovelli calls them 'partial observables'. By definition a partial observable is 'a physical quantity with which we can associate a (measuring) procedure leading to a number' (Rovelli 2002, p.2). The essence of the Rovelli internal clock prescription for dealing with the problem of time is to designate a sub-set of partial observables as internal clocks, and then use these clocks to construct 'complete observables', that are both predicable and measurable, and which correspond to perennials.

We can illustrate the Rovelli prescription using our simple system as follows.[20] First, restrict to 1D so that each particle is represented simply by a single scalar position and momentum variable. Next, choose particle $i = 1$ as our clock and invert Equation (13) for t to get

$$t = \frac{m_1}{P_1}(q_1 - Q_1) \tag{14}$$

Re-insert this into (13) we get

$$q_a(q_1) = Q_a - \frac{P_a}{m_a}\frac{m_1}{P_1}(q_1 - Q_1), \tag{15}$$

for $a = 2, ..., n$. We then take $q_1 = \tau$, where $\tau \in \mathbb{R}$, to be the value of an internal clock, and define members of a family of 'complete observables' in terms of $q_a(\tau)$ for some specified value of τ. Crucially, for any specification of τ we have, $q_a(\tau) : \Gamma \to \mathbb{R}$ and $\{H, q_a(\tau)\} = 0$, which means that the complete observables are perennials. Given this, one can proceed to construct a quantum theory based upon the classical algebra of complete observables. In the context of this quantum formalism the complete observables will be constructed as operators on a 'physical' Hilbert space made up of states that solve the Wheeler-DeWitt equation.

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[20]See (Dittrich 2006; Dittrich 2007) for formal refinement of the procedure. Following the work of Dittrich the partial and complete observables proposal can be generalized to systems of multiple constraints via the idea of 'partially invariant partial observables'. This idea, combined with the notion of 'weakly Abelian' constraints, allows for expression of complete observables of an arbitrary constrained system as an infinite power series.

There are number of conceptual and formal difficulties attached to the Rovelli proposal.[21] Perhaps the most philosophical interesting is whether the idea of things that are 'measurable but not predictable' is coherent. Some authors[22] think not:

> "...a measurable quantity is always a complete observable, even pointers of a clock are observables and not partial observables. Now complete observables are defined with respect to nonmeasurable quantities...which we will simply call non-observables..." (Thiemann 2007, p.78)

The problem is that, if Thiemann is right and the partial observables are non-measurable, then we seem to loose our ability to use different values of the internal clocks to describe change. Rather, all we have are measurements of the complete observables which are (in a precise sense) temporally non-local. Furthermore, in denying the measurability of partial observables the internal time view arguably runs into the Leibnizian relationalist problem in explaining the determinate (local) metric structure of time at the functional level needed for mechanics. On the other hand, if Rovelli is right and the partial observables are measurable, although we do seem to have a good response to the metricity problem, we still need a more precise way of making sense of the ontological status of things that are 'measurable but not predictable'. These fascinating questions have received rather too little philosophical attention and are still, to a large extent, open. An important exception to this relative neglect are the various dicussions of Rickles,[23] who in the context of advocating for his own structuralist position, makes the highly valuable observation that in essence the debate turns on the old metaphysical question of the relative ontological status of relations and relata – with Rovelli asserting (and Thiemann denying) the independent measurability of the relata.[24]

A further conceptual issue relates to the fact that the complete observables may be multivalued. That is, we are not-guaranteed that the partial observable chosen as the internal clock will be monotonically increasing. This is of course in direct conflict with the Leibniz-Babour form of relationalism discussed above. Given that we want to preserve temporal ordering structure, an alternative prescription for dealing with the problem of time is needed. Recent steps in this direction have been presented in a series of papers by Gryb and Thébault.[25] On the moderate temporal relationalist view defended by Gryb and Thébault, there is

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[21]The most important formal difficulties relate to invertibility and integrability. See (Bojowald, Höhn, and Tsobanjan 2011; Dittrich, Hoehn, Koslowski, and Nelson 2015).

[22](Rickles 2005, p.26) contains a similar observation, made a little earlier

[23]See in particular (Rickles 2007, pp.161-171) and also related remarks in (Rickles 2005; Rickles 2006a; Rickles 2006b; Rickles 2008; Rickles 2016).

[24]For further discussion in the context of the physics literature see (Tambornino et al. 2012; Rovelli 2014)

[25]See in particular, (Gryb and Thébault 2011; Gryb and Thébault 2014; Gryb and

always assumed to exist a monotonically increasing time parametrization, but this parametrization is taken only to be defined up to smooth rescallings, and thus we do not have an absolute notion of duration.

The Gryb and Thébault view depends upon a particular interpretation of the Hamilton-Jacobi formalism discussed above. In particular, the view relies upon noting that the difference between Equations (6) and (7) above is entirely due to an extra time boundary term, namely: the transformation $S \to S + Et$. This does not affect the local equations of motion. At the classical level the two formalisms are observationally indistinguishable, the difference between them reducing to an interpretational choice regarding the energy being a constant of motion or constant of nature.[26] With this in mind, we are then free to *choose* which of the two Hamilton-Jacobi formalisms to base our quantum theory upon depending on the form of relationalism about time we which to adopt. Choosing the more moderate relationalism, and starting from (6), as discussed above, the resulting quantum formalism will retain a fundamental notion of time evolution, and we end up with a unitary evolution equation of the Schrödinger-type,

$$\hat{H} \left| \Psi \right\rangle = i\hbar \frac{\partial \left| \Psi \right\rangle}{\partial t}. \tag{16}$$

The classical algebra of observables through which the quantum formalism is defined are given by the partial observables. For our simple system these can be expressed in terms of equation (13). Clearly this equation traces out dynamical curves labelled by the arbitrary parameter t, which is of course itself not an observable. Rather, t is an independent parameter, and, as such, can be specified independently of quantities which are deemed measurable within the theory. The curves defined by (13) are reparametrization invariant even if the equation makes reference to the unphysical labelling parameter. Thus, the observables *are* invariant under the relevant global reparametrization symmetry. The 'relational quantization' procedure developed by Gryb and Thébault can be motivated in the context of an analysis of globally time reparametrization theories via Faddeev-Popov path integral (Gryb and Thébault 2011), constraint quantization (Gryb and Thébault 2014) or Hamilton-Jacobi techniques (Gryb and Thébault 2016). In each case, the resulting quantum formalism retains a fundamental notion of Schrödinger time evolution and in this sense the attractiveness of the proposal is obvious.

A severe limitation of relational quantization relates to the 'local' problem of time that we have thus far been avoiding in our discussion. Theories such as the Jacobi theory are *globally* time reparametrization invariant and have a single Hamiltonian constraint that generates *global*

Thébault 2015; Gryb and Thébault 2016).

[26]See (Gryb and Thébault 2016) for extensive discussion of this point.

time evolution. Re-foliation invariant theories such as general relativity, by contrast, are *locally* time reparametrization invariant, and have an an infinite family of Hamiltonian constraints that generate *local* 'many fingered' time evolution. Imagine a loaf of bread that we can irregularly cut up into a sequence of slices. The loaf is spacetime and the slices are instantaneous spatial surfaces. A *foliation* is then a parameterization of a spacetime by a time ordered sequence of spatial slices. Such a parametrization is local in the sense that it is defined for every point on every spatial slice. The symmetries of general relativity imply that all spacetimes that are related by *re*-foliations are physically equivalent. In practice, application of the partial and complete observables programme to general relativity also suffers a number of limitations, such as those relating to integrality (Dittrich, Hoehn, Koslowski, and Nelson 2015). However, in principle, there is no bar to applying the Rovelli prescription for constructing observables to theories invariant under local time reparametrization transformations. Thus, the partial and complete observables approach is a prospective solution to the local and global problem of time. On the other hand, relational quantization is geared specifically towards the solution of the global problem of time, and arguably in principle inapplicable to the local problem.

The viability of relational quantization as an approach to the the problem of time in any full theory of quantum gravity thus rests upon the adoption of a re-description of gravity in terms of a formalism that features a notion of preferred foliation. One attractive possibility along these lines is suggested by the *shape dynamics* formalism.[27] Within this formalism, the principle of local (spatial) scale invariance is introduced with the consequence of favouring a particular notion of simultaneity. This selects a unique global Hamiltonian and thus allows for relational quantization to be applied. Shape dynamics is based upon a re-codification of the physical degrees of freedom of general relativity via exploitation of a duality between the two relevant sets of symmetries. In the class of spacetimes where it is possible to move from one formalism to the other (those that are 'CMC foliable') the physical degrees of freedom described by the two formalisms are provably equivalent, they are merely clothed in different descriptive redundancy.

Our two options for 'finding time' thus both come with a mix of attractive and unattractive features. Arguably, relational quantization is more attractive than partial and complete observables on account of clearer ontological categories for the observables. And arguably partial and complete observables is more attractive than relational quantization on account of flexibility it gives in dealing with local time reparametrization invariance. In the end, the choice between the two

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[27]Shape dynamics was originally devloped by Barbour and collaborators (Barbour 2003; Anderson, Barbour, Foster, and O'Murchadha 2003; Anderson, Barbour, Foster, Kelleher, and O'Murchadha 2005) and then brought into modern form in (Gomes, Gryb, and Koslowski 2011).

is underdetermined by the choice between the relational ontologies of time, one with temporal ordering structure, one without. As is so often in science, future theoretical and empirical development is the only real prospect to decisively break such underdetermination.

Acknowledgments

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