

*Potentia, actio, vis*: the quantity  $mv^2$  and its causal role

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Abstract:

This article aims to interpret Leibniz's dynamics project (*circa* 1678-1700) through a theory of the *causation* of corporeal motion. It presents an interpretation of the dynamics that characterizes physical causation as the structural organization of phenomena. The measure of living force (*vis viva*) by  $mv^2$  must then be understood as an organizational property of motion conceptually distinct from the geometrical or otherwise quantitative magnitudes exchanged in mechanical phenomenon. To defend this view, we examine one of the most important theoretical discrepancies of Leibniz's dynamics with classical mechanics, the measure of *vis viva* as  $mv^2$  rather than  $\frac{1}{2}mv^2$ . This "error", resulting from the limits of Leibniz's methodology, reveals the systematic role of this quantity  $mv^2$  in the dynamics. In examining the evolution of the quantity  $mv^2$  in the refinement of the force concept (*vis*) from *potentia* to *actio*, I argue that Leibniz's systematic limitations help clarify the dynamical causality as neither strictly metaphysical nor mechanical but a distinct level of reality to which Leibniz dedicates the "*dynamica*" as "*nova scientia*".

## I. Introduction

Although a physical theory of corporeal motion was of central concern to Leibniz in his youth, leading him, under Cartesian, and more importantly, Hobbesian inspiration, to compose the two part *Hypothesis Physica Nova* (circa 1671), his mathematical maturation in Paris (1672-1676) provided a new stage for these investigations. As such, what began in the late 1670's as a refutation and reform of Cartesian laws of motion and collision grew into what we can retrospectively call a dynamics project terminating around 1700 when Leibniz ceased active work on the subject. The term "dynamics" understood as a "new science" was first privately used in a letter to Bodenhausen in 1689 during his year-long voyage to Italy and first publically presented in *De primae philosophiae emendatione et de notione substantiae* in 1694.<sup>1</sup> By the time of the publication of *Specimen Dynamicum* in 1695 and *De Ipsa Natura* in 1698, it is clear that Leibniz had a mature and systematic understanding of the dynamics, and employed its basic ideas to argue for a metaphysics of corporeal substances.<sup>2</sup> The *Essay de Dynamique* (circa 1699-1701) represents the last of Leibniz's systematic attempts to present the dynamics.<sup>3</sup>

In the roughly bidecennial dynamics project from 1678-1700, filled with detours and missteps, Leibniz was faced not only with the task of reworking a theory of motion

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Please refer to the bibliography for a table of abbreviations

<sup>1</sup> GP IV 469.

<sup>2</sup> This is the explicit central argument of "*De Ipsa Natura*". GP IV 504-516; AG 155-167.

<sup>3</sup> GM VI 215-231, A. Robinet, *Architectonique disjunctive, automates systémiques, et idéalité dans l'œuvre de G. W. Leibniz*, Paris: Vrin, 1986, 266-267. François Duchesneau, *La Dynamique de Leibniz*, Paris: Vrin, 1994, 244.

through the critique of the Cartesian position but also forced to provide it with a new systematic foundation. Although much has been made of this period of Leibniz's natural philosophy where he attempted to reinvent corporeal substance through a quasi-scholastic notion of substantial forms, the key motor pushing Leibniz along this path, the problem of physical causality, has received much less attention.<sup>4</sup>

My contribution to current debate on this aspect of Leibniz's interpretation is to provide an alternative interpretation of this main feature of Leibniz's dynamics project, with resonances in its accompanying metaphysics, as the refinement of a theory of structural causality. In the limited context of this article, I aim to provide grounds for interpreting Leibniz's dynamics as the development of a theory of structural causality. By "structural causality" I mean that the relationship of dynamical causation to empirical motion is a relationship between two strata of reality: a stratum of force and a stratum of (locomotive) phenomenon. This should not be understood in terms of an isomorphic mapping of one strata to another. Rather, as I shall argue, dynamical causes are expressed through the properties of a physical system, taken integrally, rather than through properties of individual bodies or their mechanical relation with other individual bodies. To take one of Leibniz's analogies, just as the Apollonian cone expresses a continuous multiplicity of curves, dynamical cause, *vis viva*, expresses a system of phenomena. This form of causality is termed "structural" because the effects of this cause (force *qua* cause) pertain to the distribution of proportional quantities (a system of effects) found in empirical motion.

The thesis of structural causation cannot be fully defended here and I aim only to provide some grounds for such a reading. However some implications of this thesis is concretely relevant for the analysis below. Most importantly, it frames the meaning of what it is for force (*vis*) to cause motion. That is, to assert that *vis viva* is the *cause* of motion is to say that *vis viva structures* the many groups of internally related and proportional phenomenal expressions of motion. Leibniz's mature metaphysical thesis of the non-interaction of substances and the more well-known anti-realism and relationalism concerning motion are related facets of this idea of structural causality. This means, above all, that Leibniz's conception of the cause of motion cannot be reduced to empirical factors

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<sup>4</sup> Jeffrey McDonough is one of the few interpreters who has been developing a systematic of Leibniz's theory of causality although he develops this from optics rather than a close reading of Leibnizian dynamics. Cf. Jeffrey McDonough, "Leibniz's Two Realms Revisited", *Noûs*, 42:4, 2008: 673-696.

understood along the model of efficient cause. This efficient causation model provides, at best, an account of the sequence of effects (the phenomenal properties of motion) rather than causes. Even as Leibniz continues to provide room for explanation by means of efficient (contact) causation, the scientific reduction of efficient causality to dynamical reality was aimed at providing a foundation to natural science by separating an infra- or non-phenomenal reality of dynamics from an empirical-phenomenal reality of effects (extended motion).

To be clear, I ultimately hold that the causal nature of Leibnizian *vis* should not be understood in terms of the operational powers involved in the interaction between moving bodies but should rather be seen as a higher-order, and hence structural, property of systems of bodies. Causality is structural when what *causes* and what is *caused* constitute two levels of reality. Leibnizian *vires* are causes and extended phenomena are effects. Although Leibniz does employ the term “*structura systematis*” explicitly in order to avoid a theory “that follow[s] *per se* from the bare laws of motion derived from geometry,” my use of structural cause is not itself an explicit aspect of Leibniz’s work.<sup>5</sup> For Leibniz’s own use of the term, he opposes a systematic structure against the reduction of the laws of motion to “pure geometry” (extension) but does not provide a more concrete elaboration of what is precisely “structural” in this causal account. What comes closest to an explicit statement of structural causality in Leibniz is found in the maturation of Leibniz’s natural science where he attempts to reintroduce final causes into the treatment of physical laws *selon les modernes*. This explicit use of final causes is given as a parallel mode of explanation to efficient causes in his argument for an interpenetrating and compatible reign of two kingdoms, the kingdom of power (efficient cause), and the kingdom of wisdom (final cause).<sup>6</sup> Leibniz here understands the interpenetrating reign of the two kingdoms as different aspects of the same reality yet certainly prioritizes the kingdom of wisdom in its determining role in explaining and formalizing the phenomenal expression of efficient causality.<sup>7</sup> In this, Leibniz thinks of efficient cause as a parallel yet secondary feature of the

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<sup>5</sup> Emphasis original. GM VI 241; AG 124.

<sup>6</sup> GM VI 242-243; AG 126.

<sup>7</sup> GP VII 279.

primary determination already laid out by final causes such that once the “general and distant” principles have been established, it need not be constantly referred to.<sup>8</sup>

Of course Leibniz until the very end of his life maintained a mechanistically informed account of corporeal motion. He unequivocally claims in his fifth letter to Clarke that “A body is never moved naturally, except by another body which touches or pushes it[...] Any other kind of operation on bodies is either miraculous or imaginary.”<sup>9</sup> Of course the two kingdoms of wisdom and power provide different *explicatae* for the same *explicans* and the phenomenon of contact can be taken as a necessary condition for motion’s well-foundedness rather than its ultimate cause. The difficult explication of the compatibility between these two kingdoms of wisdom and power is not my task here but a basic hierarchy certainly exists between them. That is, we know that the key metaphysical thesis that forms the root of the dynamics project as well as much of the systematic metaphysics of the mature Leibniz is the basic idea that “principles of corporeal nature and of mechanics itself are more metaphysical than geometrical, and belong to some indivisible forms or nature as the causes of appearances, rather than to corporeal mass or extension.”<sup>10</sup> Extended motion is thus in principle phenomenal and hence imaginary, as Leibniz often emphasizes, although certain conditions allow us to qualify them as “well-founded”.<sup>11</sup>

We also see the hierarchy of the two kingdoms at work in Leibniz’s theory of optics where the Cartesian theory, based on an efficient cause model of the hardness of the medium and the elasticity of light particles, is rejected for a teleological model of *optimized* geometrical proportions.<sup>12</sup> Despite this, Leibniz does not reject efficient causality but allows a “higher” determination through teleology to explain the mechanical or empirical level of reality. As such, my attribution of structural causality to Leibniz may be unfortunately misunderstood as an equivocation between an epistemological level *determining* natural laws, or their reason (*ratio*), and the ontic level of the *constitution* of their cause (*causa*). Indeed this identity of *ratio* and *causa* is an inherent problem for any

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<sup>8</sup> GM VI 242-243; AG 126.

<sup>9</sup> Leibniz 5<sup>th</sup> letter to Clarke, 18 August, 1716, §35; GP VII 398; L 702. Cf. “Antibarbarus Physicus”, GP VII 338; AG 313.

<sup>10</sup> GP VII 440; AG 52.

<sup>11</sup> GP VII 564; L 548.

<sup>12</sup> GP VII 274; L 479.

theory of final causality in natural science and this mode of reasoning was indeed the target of the famous rejection of final cause by the partisans of the “new science” from Descartes onwards. Nonetheless Cartesians and other mechanists also faced a version of this problem in reducing the *ratio* of corporeal motion to *causa efficiens* where the divine would inevitably have to be invoked to explain this “explanation” for physical causation.

Outside of our immediate context, we should also note that the Leibnizian legacy of final cause that eventually contributed to development of the *principle of least action* in the work of Maupertuis, Euler and Lagrange became stripped of its metaphysical sense and instrumentalized without much need for the metaphysics of substance that accompanied such a notion.<sup>13</sup> Whereas mechanists such as Boyle famously articulated a place for final causes as an extrinsic and extra-scientific level of accounting for the laws of nature, Leibniz not only insisted but became more and more convinced of the immanence of final causes within physical reality as a fruitful means for a scientific treatment of the nature of corporeal substances.<sup>14</sup> This immanence of teleology in corporeal substance is no doubt one of the key dimensions of Leibniz’s mature metaphysics. As such, at least for Leibniz, the dangers of epistemological and metaphysical equivocation inherent in the assertion of final causality remain certainly problematic in general but are nonetheless an inevitable feature of Leibniz’s interpretation.

Any general discussion of final causes and the kingdom of wisdom can ultimately be reduced to an argument about the “best of all possible worlds”, but this metaphysical understanding cannot be adequate to Leibniz’s concrete treatment of dynamical causality. That is, the application of final causality to natural science always requires an analogical structural concept for the form of “harmony” at stake in nature.<sup>15</sup> A particularly alluring example of this is Leibniz’s 21 January 1704 letter to De Volder where he makes an analogous use of number series, say for example the Leibniz series for  $\pi$ , to describe the

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<sup>13</sup> J. Christiaan Boudri, *What was Mechanical about Mechanics*, trans. Sen McGlinn, Dordrecht: Springer-Science and Business Media Dordrecht, 2002, 126-133.

<sup>14</sup> See discussion in the *Theodicy*, GP VI 321. Cf. Margaret J. Osler, “From immanent natures to nature as artifice: The reinterpretation of final causes in Seventeenth Century Natural Philosophy”, *The Monist*, Vol. 79, No. 3, July 1996, 388-407.

<sup>15</sup> Leibniz provides a general presentation of this bridge between harmony and geometric order in “*Quid sit idea*”, A VI, iv, 1370; GP VI 263; L 207.

evolution of (derivative) forces under the invariant of a conserved primitive force.<sup>16</sup> In this argument, primitive force is “pregnant” or “preinvolved”, as Leibniz likes to say, with the totality of its discrete moments of evolution (the instances of derivative force in a motion) just as a law of the series gives the  $n$ -th term of its expansion. This analogical suggestion also seems to resonate with his work on optics such as the *Tentamen Anagoricum* as well as his famous criticisms of Descartes’ laws of collision in *Animadversiones in partem generalem Principiorum Cartesianorum*. In both of these key cases, Leibniz’s arguments do not privilege empirical adequateness, albeit verificationally useful, but draw primarily from structural features (like optimization, in the case of optics, and continuity in the case of collision) of the proportions resulting from the provided measurements. My thesis of structural causation is thus metaphysically reducible, like any instance of final cause, to a general discussion about the “best of all possible worlds”. Yet insofar as principles are empty without concrete instances, this metaphysical thesis has no meaning in Leibniz’s dynamics other than structural causation.

Taking up the immanent features of Leibniz’s dynamics and leaving aside the broader metaphysical problems, in what follows I defend this principle of structural causality through the mathematical structure of the dynamics. If dynamic causality is structural then the mathematics of the dynamics would not only reflect this structure but also demonstrate the irreducibility of this structure to more basic empirical factors. More precisely, since the mathematical structure of the dynamics is centered on the conservation of *vis viva*, expressed quantitatively as the invariance of  $mv^2$  in motion, I argue that understanding the role of this quantity in Leibnizian dynamics demands an understanding of cause as structural. In doing so I show that what Leibniz reveals in the concrete context of his methods of measurement and the treatment of dynamic causality is often more enlightening than what he explicitly remarks *about* them.

## II. The measurement of *vis viva* and its conservation

In order to grasp the structural character of dynamical causality, we need first to be explicit about the limitations and shifts in Leibniz’s measurement of *vis viva* through the

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<sup>16</sup> GP II 262; DeV 452

quantity  $mv^2$ . We know that this quantity, borrowed from his mentor Huygens' work, provided the cornerstone for his eventual dynamics project and remained a constant in his dynamics project from his first attempt at a treatise in the 1678 *De Corporum Concorso* until his final works on the subject.<sup>17</sup> Nonetheless, there are in fact two different problems in evaluating the role of this quantity in Leibniz's work. The first concerns measurement and corresponds to the problem of how Leibniz justified this quantity  $mv^2$  as the measure of *vis viva*. The second concerns Leibniz's concept of conservation of this quantity and corresponds to the conceptual relation between the quantity conserved  $mv^2$  and the proportional relation between the conserved quantity, mass and velocity. By distinguishing between these two problems, we move closer to understanding how Leibniz's limitations in the measurement can help clarify the concept of causality intended by the conservation of *vis viva*.

Starting with the first problem of measurement, we examine how Leibniz accounts for the measurement of *vis viva* as  $mv^2$ . It is important to note that though *vis viva* directly translates to "living force", the notion is not what we canonically understand by "force". The historical uses of the term "vis" as "force", "Kraft", "power" (*potentia*) or pressure remained vague in the 17<sup>th</sup> to the 19<sup>th</sup> century in the history of mechanics. Since a sufficiently generalized and completed formalization of classical mechanics was only accomplished in the 19<sup>th</sup> century, ambiguity over the referents of force, energy and work as well as their systematic relations should not surprise us.<sup>18</sup> Our standard classical-Newtonian term "force"  $F$  refers to  $F=m \cdot a$  (where mass is  $m$  and acceleration  $a$ ). Force also possesses, in the classical understanding, both scalar (the magnitude of force) and vector ( $\vec{F} = m\vec{a}$ ) expressions, which are not in the Leibnizian understanding of *vis viva*. Hence although Leibniz uses the term *vis* here, we should clarify that the following uses of *vis* (*vis viva*, *vis mortua*, *vis activa primitiva*, etc.) should not be confused with force as we understand it in

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<sup>17</sup> Christiaan Huygens, "The motion of colliding bodies", trans. Richard J. Blackwell, *Isis*, Vol. 68, No. 4, Dec., 1977, 574-597.

<sup>18</sup> Cf. Yehuda Elkana, *The Discovery of the Conservation of Energy*, Cambridge MA: Harvard University Press, 1974, 22-51.



its standard use.<sup>19</sup> Leibnizian *vis* should then be understood independently and only analogically with another fundamental concept in classical mechanics, work.

Thinking of *vis* in analogy to work, or the quantity energy-work, allows us to step directly into Leibniz's own account of the measurement of *vis*.<sup>20</sup> It is worth recalling here that Leibniz's initial entry into the dynamics project was motivated by the attempt to "reform" Cartesian-styled mechanics by refuting the conservation of the quantity of motion  $mv$  in nature. This refutation of  $mv$  as conserved quantity is probably the most famous aspect of Leibniz's dynamics, seen in general metaphysical works like the 1686 *Discours de Métaphysique* (§17-18). However, as early as January 1678, in an early treatise *De Corporum Concursu*, Leibniz had already argued, in view of the perceived error of the Cartesians,

I now see where the error is to be found. The force in bodies should not be estimated [*aestimanda est*] from speed and the size of bodies but from the height from which it falls. Hence the heights from which bodies fall are as [a proportion of] the square roots of the speeds in question. [...] Thus generally, the *vires* are in a ratio composed of the simple product of the bodies and the square of the speeds.<sup>21</sup>

Indeed by 1678 the quantity  $mv^2$  was already a systematic part of Leibniz's treatment of body, motion and *vis*. Of course the origin of this very quantity comes from his mentor Huygens who had, in 1669 already published his argument for the conservation of  $mv^2$  in the British Royal Society's *Philosophical Transactions*. Henry Oldenburg, the founding secretary of the Royal Society, had invited J. Wallis, C. Wren and C. Huygens to publish on the conservation of  $mv$ ,  $m|v|$  and  $mv^2$  in order to settle the conservation controversy and this publication was read by a young Leibniz in early 1670s while still in Mainz.<sup>22</sup> It is however

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<sup>19</sup> In the following all references to Leibnizian "force" will be made by the use of the term "*vis*" while "force" will refer to the general classical-Newtonian concept.

<sup>20</sup> Rene Dugas provides the standard account of Leibnizian *vis viva* as energy and provides an account of the quantity in classical-Newtonian terms. Rene Dugas, *A History of Mechanics*, trans. by J.R. Maddox, London: Routledge and Kegan Paul, 1955, 219-221.

<sup>21</sup> [Author's translation] Leibniz, "De Corporum Concursu", in *La réforme de la dynamique*, ed. Michel Fichant, Paris: Vrin, 1994, 134.

<sup>22</sup> Eric J. Aiton, *Leibniz: A Biography*, Bristol and Boston: Hilger Alexander, 1985, 30. Cf. Philip Beeley, "A Philosophical Apprenticeship: Leibniz's Correspondence with the Secretary of the Royal Society, Henry Oldenburg" in *Leibniz and his Correspondents*, edited by Paul Lodge, Cambridge: Cambridge University Press, 2004, 47-73, 55.

important to note that while the original Huygensian context remained squarely limited to the problem of the laws of (elastic) collision, aimed against the Cartesian formulation, Leibniz sought to extend and generalize this conservation principle as the quantity conserved in nature as such.<sup>23</sup>

Although Leibniz's systematic development of the concept of *vis* had only begun in earnest during the late 1670s and thus lacking in its eventual metaphysical and scientific sophistication, we can already identify the continuity of his justification for the quantity  $mv^2$  in this early work with the later. Here we first look at the "negative" argument for  $mv^2$  conservation then turn to the "positive" in order to grasp what is at stake in the transition between different uses for this same quantity.

The first, negative argument, remains in the mood of a refutation of the Cartesian quantity  $mv$ .<sup>24</sup> This is notably found in Leibniz's 1685 *Brevis Demonstratio erroris memorabilis Cartesii* and repeated in the 1686 *Discours de Métaphysique* (§17-18). In both accounts Leibniz considers two bodies A and B with masses of one unit and four units raised to four units length and one unit length respectively. We notice that the heights and masses are inversely proportional. Leibniz argues to establish a quantity, call this  $w$ , as the same quantity needed (*quanta opus*) to carry (*elevandum*) the two bodies to their respective heights.<sup>25</sup> We might analogically understand this as work and insofar as the heights and the masses are inversely proportional, the work is the same to elevate (under similar conditions) both bodies to their respective heights. Now Leibniz does not give an explicit reason for why this *quanta opus*  $w$  is equal in both cases of elevation but assumes that readers understand this to be the case. From statics we can understand the two bodies with their respective inversely proportional weights and distances to be in equilibrium. This results from the consideration that their heights are inversely proportional to the ratio of the mass A and B. As such the notion of conservation at work here is the result of a statical

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<sup>23</sup> Cf. Christiaan Huygens, "The motion of colliding bodies". Herman Erlichson, "The young Huygens solves the problem of elastic collisions, *American Journal of Physics*, 65,2, February 1997, 149-154.

<sup>24</sup> Although the distinction between speed and velocity plays a role in the 17<sup>th</sup> century, they do not play a particular role in Leibniz's work even in his critique of Cartesian laws of motion and collision. Due to the uses of these terms in the cited texts, this article will use these terms without particular distinction. The exception will be in my account of Leibniz's argument in the figure 2 where velocities are represented by positive and negative quantities.

<sup>25</sup> A VI 4, 2027-2030. Cf. A VI 4, 1556-1558.

consideration of the two bodies with respect to the ratio of mass and height. To say that the “quantity needed” to raise body A one foot and body B four feet is equipollent is simply to say that the body A and body B at their respective heights are in equilibrium.

To refute the Cartesians, Leibniz argues that this same conserved *quanta opus w* will be conserved in some way in the fall of each body A and B from their respective heights. The refutation is simple. Appealing to Galileo’s law of falling bodies, the refutation follows simply by noting that the final speed of the falling body is independent of the mass of the body but dependent on the duration (proportional to height) of free fall. Hence, the body falling from the greater height, B, will endure a longer duration of fall and hence achieve a greater final speed. Leibniz’s argument is simply that since the final speeds of each falling body is proportional to their heights and not their masses, the original *quanta opus w*, posited as the quantity needed to carry the two bodies to their respective heights is not conserved by  $mv$ , the product of the quantity of mass and final speed. That is:

$$w = \text{mass} \cdot \text{unit of height, or, } w = m \cdot h,^{26}$$

$$w_A = 1 \cdot 4 = 4$$

$$w_B = 4 \cdot 1 = 4$$

And if we calculate for the Cartesian quantity of motion  $= m \cdot v$  (bulk  $\cdot$  speed) and we assume through an analogue of Galileo’s law that  $v(\text{at the base of fall}) = \sqrt{2 \cdot h}^{27}$

$$v_A = \sqrt{2 \cdot 4} = \sqrt{8}$$

$$v_B = \sqrt{2 \cdot 1} = \sqrt{2}$$

Hence for  $mv$ ,

$$mv_A = 1 \cdot \sqrt{8} = \sqrt{8} = 2\sqrt{2}; \text{ or the quantity of motion of A}$$

$$mv_B = 4 \cdot \sqrt{2} = 4\sqrt{2} = 4\sqrt{2}; \text{ or the quantity of motion of B}$$

$$\text{and thus } mv(A)/mv(B) = 1/2$$

We also note that the same example, calculated for  $mv^2$  results in the following:

$$mv^2_A = 1 \cdot 8 = 8$$

$$mv^2_B = 4 \cdot 2 = 8$$

As we know, the quantity of motion  $mv$ , interpreted as momentum, is indeed conserved but this quantity is not conserved in this example across the two bodies since gravity

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<sup>26</sup> The gravitational constant is not included here as a factor of the quantity for work or free fall due to the fear of anachronism.

<sup>27</sup> Leibniz interprets Cartesian bulk as mass in many of his demonstrations. Although the concept of mass was not yet fully developed, Leibniz does understand it as the product of volume and density. GM VI 298-299.

accelerates the body in free fall. Whether we understand the Cartesian quantity of motion  $mv$  as momentum or not, it is indeed different from the quantity  $w$ . Leibniz understood the problem differently. His refutation of Descartes is established insofar as the quantity of motion of A and B are different despite the equivalence of the quantity necessary to raise them to the height of 4 feet and 1 foot respectively. It is for this reason that the fundamental concept of conservation in Leibniz is most closely related to energy-work.

Now the context of Leibniz's argument here was aimed at making a more general metaphysical claim. Leibniz's work on the reform of Cartesian mechanics in the 1670's had clearly evolved into an outright rejection of the metaphysical foundations of the latter's natural science. The refutation of the quantity of motion  $mv$  had evolved into a rejection of the more general Cartesian thesis of the reducibility of corporeality and corporeal motion to size, shape and motion, that is, a rejection of the mechanistic foundations of natural science.<sup>28</sup> Hence, Leibniz's argument here should be understood, following commentators such as Lodge, as arguing, not primarily for the conservation of  $mv^2$  but rather the inadequacy of  $mv$  as a measurement for the quantity conserved in nature.<sup>29</sup> The "negative" nature of such an argument is aimed at heuristically arguing for a different scientific foundation that turns toward understanding motion in terms of its cause *qua vis* than establishing the quantity  $mv^2$  as such. Ultimately Leibniz would have made the same metaphysical claim even if  $mv^4$  or  $mv^3$  or some other quantity were indeed conserved. In essence all Leibniz needed to provide was a principle of conservation that was not  $mv$ . The refutation of the conservation of  $mv$  was the occasion to make a larger metaphysical point about the inadequacy of the reduction of bodies to extension.

Bracketing the larger metaphysical issue, we see that the role played by  $mv^2$  here only serves to introduce a problem of the distinction between the Cartesian quantity of motion and the quantity conserved in nature. Indeed, the conservation principle that Leibniz intends exploits the ratio between the final velocity of the falling body and the work needed to lift the body up to its respective height. Interpreting the example in analogy with the conservation of energy-work, the Cartesian conservation of the quantity of motion is clearly inadequate. This is perhaps indeterminate for ultimately judging between the two thinkers

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<sup>28</sup> GP VII 280-283; AG 245-250.

<sup>29</sup> Paul Lodge, "Force and Nature of Body in Discourse on Metaphysics §§17-18", *Leibniz Society Review*, 7, 1997, 116-124.

because Descartes is understood by Leibniz to be in the business of measuring the work of a system and it is not at all clear that such an interpretation is fair. Of course, the referents of these different sorts of quantities (energy, work, force) had not yet been stabilized and Leibniz's criticisms of Descartes can easily be understood as a conflict over the referent of distinct conservation principles. This ambiguity certainly echoes what D'Alembert would later call "a dispute of words".<sup>30</sup>

In order to draw something out from this ambiguity, we turn to the second problem concerning the quantity  $mv^2$ , that of a concept of conservation drawn from the proportional relations to velocity and mass. We examine this through a "positive" argument for  $mv^2$  as articulated in Leibniz's later work such as the 1695 *Specimen Dynamicum*. This "positive" argument also includes a criticism of Descartes but is more ambitious in providing a direct account of the structural features of this quantity in the account of *vis*. The argument once again relies on the inference, seen already in *De Corporum Concursu*, from Galileo's law of falling bodies correlating the height of the falling body with the square of the final velocity of fall. A span of ten years exists between *Brevis demonstratio* and *Specimen* but I refer to the argument in *Specimen* for the sake of its simplicity rather than provide a larger developmental account.

Unlike his earlier example in *Brevis Demonstratio* and *Discours de Métaphysique*, Leibniz's example in the *Specimen* echoes his earlier work in *De Corporum Concursu* insofar as a pendulum (rather than a simple falling body) is also employed. The idea in the *Specimen* as well as in *De Corporum Concursu* is simply that a pendulum allows us to demonstrate conservation by isolating a determinate ratio between the height attained by the pendulum bob and its maximum speed at the base. In the *Specimen*, Leibniz argues that two pendulums, side by side, of equal mass, A and C, will present maximum speeds proportional to their respective maximum heights.<sup>31</sup> Again, with respect to each pendulum, Leibniz employs the proportion of height to square of velocity drawn from his

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<sup>30</sup> Cf. Thomas L. Hankins, *Jean D'Alembert: Science and the Enlightenment*, New York, Gordon and Breach, 1990, 207. Although the work of many historians of science like Hankins have shown D'Alembert's judgment here to be rather hasty with respect to the eventual developments of the *vis viva* controversy in the 18<sup>th</sup> century, this problem of terminological confusion does indeed apply to the conflict between Leibniz and the Cartesians.

<sup>31</sup> GM VI 245; AG 128.

understanding of Galileo's law. In this example, Leibniz varies the velocity of each pendulum A and C with equal mass. Pendulum A with velocity of 1 unit will attain the height of 1 foot and pendulum C with velocity of 2 units will attain the height of 4 feet.<sup>32</sup> Again, we see the direct application of the proportion of height and square of the velocity as the main means of calculating these quantities. What is different from the previous argument is Leibniz's aim to establish that the quadratic increase of velocity is proportional to the linear increase of height or  $\Delta v^2 \propto \Delta h$ . We can extend the example such that a third or fourth pendulum with different velocities is compared to the two in Leibniz's example such that the principle holds.

What is crucial to notice here is that Leibniz argues starting from speed rather than work of lifting the mass. Leibniz's account thus attends to the eventual work done in each pendulum given the starting position of the pendulum bob at the base possessing a certain (maximum) speed. He then argues for the proportion  $\Delta v^2 \propto \Delta h$  by reasoning that the linear difference of speed between mass A and mass C will produce a quadratic difference in "future effect". That is, he establishes this proportion by treating the difference between the work accomplished in A and C in terms of their velocities rather than vice versa. As Leibniz remarks to a Bayle in a 1687 letter, crucial to the dynamics, "[F]orce should not be measured by the composition of speed and mass but by future effect. However it seems that force or power is something real in the present and the future effect is not."<sup>33</sup> This is important since Leibniz's argument in the *Specimen* supposes that the speed of the pendulum at the base will produce a certain amount of future effect proportional to its *vis qua*  $mv^2$  at the height of the swing even though its temporal evolution is not taken into account.

With an even cursory understanding of energy-work, Leibniz's example here might appear trivial. That is, it might be inconsequential to contrast the earlier example of treating  $mv^2$  starting from the point of view of the work done by lifting up different masses and the later example of the conservation of energy from the perspective of a pendulum's maximum velocity. Other than the shift from a negative to a positive form of argumentation, the more important difference however is the analysis of motion that this

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<sup>32</sup> GM VI 245; AG 128.

<sup>33</sup> GP III 48.

later conservation argument provides. Although Leibniz does not go into more detail in the passages of the *Specimen*, this later example provides a key insight into the role played by  $mv^2$ . Here Leibniz establishes a description of motion as a function of the conserved quantity  $mv^2$ . That is,  $mv^2$  allows us to establish the principle  $\Delta v^2 \propto \Delta h$  by reasoning from the maximum velocity to the total amount of “future effect” that the system is capable of accomplishing as this “intensity”, registered as velocity, is exhausted in the upward swing. The crucial difference is that Leibniz does not merely rely on the static comparison between the work of raising two masses and comparing their speeds at fall but turns to a more dynamic methodology of describing the motion of the body in terms of the conserved quantity. Leibniz reasons that  $\Delta v^2 \propto \Delta h$  implies that  $mv^2$  is conserved. This, at least, is Leibniz’s reasoning in this example. We shall next look at some problems with this reasoning and move toward establishing the limits of Leibniz’s methodology and its insights for grasping the structural nature of dynamic causality.

### III. Error in calculation or systematic limitation?

The themes surrounding the measurement of *vis viva* only intimates something of a structural understanding of causality. In his criticism of the Cartesians, Leibniz relies on a statical methodology to bring together the quantities of maximum speed and maximum height. Although a proportion is certainly determined such that it establishes a generalized ratio between the linear growth of velocity with the quadratic growth of height across systems, it is still not clear in what sense this organizational principle is more than mere measurement. The ambiguity here is that the pendulum example relies on the idea of an exchange between quantities. Velocity transforms into height as this “intensity” is exhausted. This conservation rule is established only across different motions, that is, across the comparisons of different pendulums with varying maximum velocities and maximum heights. Although it served Leibniz’s purposes in the *Specimen* to present the basics of his conservation principle, it is insufficient to isolate the causal principle at work. Nonetheless we are given a basic sketch of how conservation is related to the laws of motion.

What is inadequate in this account is that the particular effects measured in those pendulums implicitly rely on the interpretation of *vis viva* under the model of the power or

*potentia* of a body to bring about a certain effect. Like the rebounding of a compressed (or stretched) spring, the maximum height of a pendulum is understood as the effect caused by the exhaustion of the intensity in the pendulum's maximum velocity. Nonetheless in the pendulum example we already grasp how Leibniz was informed by a structural understanding of *vis viva* qua cause. With the conservation of  $mv^2$ , Leibniz was already on a *conceptual* move beyond a reliance on the intensity-extension model measured through statical means. As we shall examine in the following, although Leibniz still relied on a statical method to provide the general proportions between maximum velocity ( $v$  at lowest point of the pendulum) and maximum height, the linear growth of velocity with respect to the quadratic growth of height demonstrates a conservation principle that stepped outside of confines of mere equilibrium. As such, we will draw attention to the fact that Leibniz's thinking of *vis viva* as cause (or force as cause) eventually develops beyond the correspondence of cause qua *potentia* (intensity) with effect (extension). In order to grasp the conceptual importance here, we must turn to a major problem with this ambiguity of structure for which the conservation quantity  $mv^2$  holds the key: the omission of the  $\frac{1}{2}$ . By making this clear, we will also see why Leibniz's use of  $mv^2$  provides the key for the structural notion of *vis* qua cause of motion.

Our analysis above has made use of the close association of Leibnizian *vis*, to be more precise *vis viva*, with work-energy, in the immediate context of Leibniz's work and its influence on successive generations in the history of mechanics. However it is along this same interpretation that Leibniz's account here has been susceptible to two lines of criticisms. The first line, represented by figures like Iltis, argues quite fairly that Leibniz has not argued for the conservation of work-energy but simply assumes it in examples such as the ones above. Leibniz's accounts for  $mv^2$  are thus not demonstrations in the strict sense but rather examples of the application of  $mv^2$  as conserved quantity.<sup>34</sup> An important aspect of this form of criticism is the fact that Leibniz does not supply much by way of arguing for the generalization of such a conservation principle from the case of the pendulum swing to other cases like the compressed (or stretched) spring or the varieties of collision.<sup>35</sup> This is

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<sup>34</sup> Carolyn Iltis, "Leibniz and the Vis Viva Controversy", *Isis*, Vol. 62, No. 1, 1971, 21-35, 26.

<sup>35</sup> It is true that although Leibniz does not provide a demonstration of this generalization, he did explicitly conceive of these cases as equivalent in the 1689 *Tentamen de Legibus Naturae Mundi*. LH 35, 10, 4, f. 1v-4.



true although defenders of Leibniz on this account, like Duchesneau, also compellingly argue that such a demonstration would be asking too much of Leibniz since the theoretical and empirical aspects of the eventual work-energy theorem came together in a piecemeal fashion such that there is simply no reason to suppose that empirical results could give rise to the theorem without the prior (and *a priori*) “faith” in such a conservation principle that remained metaphysical in character.<sup>36</sup> Nonetheless, this first form of criticism can allow us to turn our focus on Leibniz’s methodology. That is, Leibniz argues for a mathematical proportion in the conservation of *vis viva* that establishes the relation between a certain kind of work quantity (height in the example) and the extended motion of the body (speed in the example). This point would be relatively trivial if not for its implications for the mathematical structure implied by this idea of conservation at work here.

The non-triviality of these geometrical proportions between quantities of height and speed can be opened up by looking at the second form of criticism that Leibniz’s account has traditionally provoked. This criticism more directly concerns the question of the omission of the  $\frac{1}{2}$  in Leibniz’s *vis viva* as  $mv^2$ . Having seen where Leibniz gets his quantity  $mv^2$ , we now look briefly at why energy in standard classical mechanics is  $\frac{1}{2} mv^2$ . The short answer here is the calculus of integration which Leibniz certainly had a large role in developing.

The short answer for why energy-work is  $\frac{1}{2} mv^2$  then is:

For Energy= $E$ ,  $F$ = (Newtonian) force,  $s$ =displacement,  $m$ =mass  $a$ =acceleration and  $v$ =velocity

Hence if we take energy as the product of force across the displacement:

$$\Delta E = F \Delta s$$

And since  $F = ma$ :

$$\Delta E \approx ma \Delta s$$

Since the velocity in time is  $\Delta s$  and acceleration in time is  $\Delta v$ :

$$\Delta E \approx m \cdot v \cdot \Delta v$$

On the other hand, if we *integrate* over the changes of  $v$  then:

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See Bertoloni Meli’s discussion in *Equivalence and Priority*, Oxford: Oxford University Press, 2002, 123-124.

<sup>36</sup> Duchesneau, *La Dynamique de Leibniz*, 137.

$$E = \int_{s_0}^{s_1} m \frac{dv}{ds} \frac{ds}{dt} ds = \int_{v_0}^{v_1} mv dv = 1/2 mv^2 \Big|_{v_0}^{v_1}$$

Or simply:

$$\Delta E \approx 1/2 m \Delta v^2 = \Delta(1/2 mv^2)^{37}$$

We easily see that how this simple mathematical result stems from understanding energy in terms of  $mv\Delta v$  and gives us an integration of  $1/2 mv^2$ . We have done this without respecting the limits of Leibnizian methodology since the  $F=ma$  concept above is Newtonian. We can nonetheless appreciate that the motion of the falling body (pendulum) requires that speed changes in time across the duration of the fall (acceleration), the simple integration of this path of fall implies the factor of  $1/2$  as a direct result of calculation. At the same time, we can also notice that one does not necessarily require the method of integration to achieve the same results. Evaluating  $\Delta E$  through  $m \cdot v\Delta v$  means treating  $v\Delta v$  through average velocity:

$$\Delta s / \Delta t = (v_{\text{initial}} + v_{\text{final}}) / 2$$

$$F \Delta s = m (v_{\text{final}} - v_{\text{initial}}) \cdot (v_{\text{initial}} + v_{\text{final}}) / 2$$

With initial velocity = 0 and final velocity =  $v$ :

$$F \Delta s = mv^2 / 2$$

In either case, what is missing pertains to operational forces (such as Newtonian forces) that determine the path of the body with respect to distance (or time) traveled.

The longer, and more Leibnizian, answer requires that we add an additional layer of complexity to Leibniz's example above. We know that the rising pendulum achieves its work by acting against gravity and, in turn, the falling pendulum accumulates speed by accelerating due to the force of gravity. Leibniz's own acknowledgement of Galileo's law of falling bodies as one of the only principles cited in this account in these very passages of the early *De Corporum Concorso*, discussed above, indicates Leibniz's clear awareness, from an early period, that it is acceleration that gives rise to the proportion of linear speed to quadratic height (or work).

Now if what Leibniz relies on to provide his argument for  $mv^2$  is the notion that the maximum velocity of the falling body (pendulum) is the result of the acceleration of the body in the duration of fall, it seems unlikely that his calculation would ignore the fact that this maximum velocity should be the integration of the acceleration of the body in the duration of fall. Indeed, drawing from earlier analysis, we know why Leibniz was

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<sup>37</sup> Approximations here are used to outline reasoning through proportions and dimensions where equations outline how the issues might look from the perspective of calculation.

indifferent to such a calculation by integration because Leibniz takes the formula  $\Delta v^2 \propto \Delta h$  as the means to understand the pendulum example above. With or without the added coefficient  $\frac{1}{2}$ , the exchange between maximum velocity and maximum height preserves the same proportions. Recall that in the previous example we have bodies A and C of equal mass, their maximum velocities at 1 unit and 2 units, and their maximum heights attained 1 foot and 4 feet respectively. As such:

For  $w = \text{mass} \cdot \text{height}$ ,

$$w_A = 1 \cdot 1 = 1$$

$$w_C = 1 \cdot 4 = 4$$

For a calculation of  $mv^2$ , we have

$$mv^2_A = 1 \cdot 1 = 1$$

$$mv^2_C = 1 \cdot 4 = 4$$

Hence if take the energy formula  $\frac{1}{2} mv^2$ , this does not alter the proportions set out by Leibniz since, calculating for  $\frac{1}{2} mv^2$ , we get  $\frac{1}{2} mv^2_A = \frac{1}{2}$  and  $\frac{1}{2} mv^2_B = 2$ . Both proportions satisfy the conservation quantity established through  $\Delta v^2 \propto \Delta h$ .

Standard responses to this interpretive problem argue that Leibniz was simply in the habit of dropping constants in his calculations.<sup>38</sup> We find this understanding implicit in much of the important commentaries of Leibniz's dynamics from Dugas, Gueroult and, more recently, Duchesneau.<sup>39</sup> This coefficient  $\frac{1}{2}$  is also in principle eliminable from perspective of the statical method of Leibniz's measurement. Yet without directly contradicting these established commentaries, I argue that we can have a more concrete and systematic understanding of this problem in Leibniz by looking at harsher criticisms. In this I follow Szabó's assessment of Leibniz's *vis viva* argument as one of systematic failure rather than an error due to the convention of calculation.

Szabó's argument is that Leibniz's dynamics should never have been called that because the latter did not understand (Newtonian) force and the problem of *vis viva* (and its measurement) remained essentially tied to that of statics rather than that of dynamics.<sup>40</sup> As we saw in our discussion of Leibniz's pendulum examples, the measure given to his

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<sup>38</sup> Bertoloni Meli, 153. E.J. Aiton, "Leibniz on Motion in a Resisting Medium", *Archive for History of Exact Sciences*, Vol. 9, Issue 3, 1972, 257-274, 264.

<sup>39</sup> Cf. Martial Gueroult, *Dynamique et métaphysique Leibniziennes*, Paris, Les Belles Lettres, 1934, 38-39.

<sup>40</sup> Istvan Szabó, *Geschichte der mechanischen Prinzipien*, Basel: Birkhauser, 1987, 3<sup>rd</sup> Ed., p. 70-71.

conservation principle treated only maximum height and the final velocity in geometrical terms, with the extrapolation from this methodology that the quadratic increase of height as proportional to the linear increase in speed. That is, Leibniz did not describe the path of the fall in algebraic terms as a function of force to acceleration. Following Szabó's rather harsh commentary, we recognize the fact that Leibniz, despite his theoretical intentions for a dynamics, remained methodologically limited by the statical means of evaluation. What was ironically lacking from Leibniz's dynamics was a dynamical view of motion. This fact of methodological limitation explains why the integration of  $mv dv$  into  $\frac{1}{2}mv^2$  could have escaped Leibniz. It is clear then that the origin of  $mv^2$  does not rely on integration at all and the quantity  $mv dv$  was not part of Leibniz's conception. Hence although Leibniz, citing Galileo, explicitly relied on the kinematic figure of the path of a falling body accelerating with respect to the duration of fall (proportional to height), the way in which the quantity of velocity increases in time because of the solicitation of gravitational force is not an aspect of Leibnizian dynamics. What is omitted by Leibniz is much more than the constant  $\frac{1}{2}$ . Rather what is omitted from the quantity of *vis viva*  $mv^2$  is the systematic understanding of the *dynamic* problem of the acceleration in time of a falling body due to the interplay of gravitational solicitation and inertial resistance.<sup>41</sup>

Szabó does not equivocate on how many great minds and how much precious time was wasted on Leibniz and the eventual *vis viva* controversy. Yet regardless of how we evaluate this period of the formation of classical mechanics, we can nonetheless grasp two different interpretations for this omitted " $\frac{1}{2}$ ". The first interpretation, one that understands Leibniz as making an error, sees the omission of the  $\frac{1}{2}$  as the error of Leibniz's *measurement* of the path of the falling body. The second interpretation, one that understands Leibniz as limited by his methodology, sees the omission of  $\frac{1}{2}$  as a systematic and conceptual limitation. I follow Szabó here in arguing that the omission of  $\frac{1}{2}$  in  $mv^2$  is a

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<sup>41</sup> This is not due to Leibniz's ignorance of either Keplerian, Cartesian or Newtonian inertia which play a role in the development of Leibniz's account of *vis passiva* in the resistance and impenetrability of body. I follow Bernstein in holding that Leibniz develops his own idiosyncratic view of inertia qua *vis insita* that relates to the persistence of *vis* in body rather than the state of motion-rest. In this reading Leibniz consciously, rather than confusedly, ignores the Newtonian innovation over the Scholastic "*inclinatio ad quietem*". Cf. GP VII 280-293; AG 245-250. Cf. Letter to De Volder 24 March/3 April 1699, GP II 170; DeV 313; AG 172. Cf. Howard Bernstein, "Passivity and Inertia in Leibniz's Dynamics", *Studia Leibnitiana*, Bd. 13, H. 1, 1981, 97-113. Cf. GP IV 510; AG 161.

result of his limited methodology and from this it is implied that Leibniz was not in error about the integration of  $mvdv$  since there was simply no integration problem at all. But I temper Szabó's conclusion by maintaining that this methodological limitation does not tell the whole story.

Although Szabó does not go into further detail about his considerations of Leibniz's methodological limitation, one could nonetheless argue against the conclusions that he draws. Leibniz did in fact consider infinitesimal quantities of soliciting "force" in terms of a dynamics. The problem is that Leibniz did not engage in his thinking about infinitesimal quantities of "force" in an algebraic way. In other words, Leibniz's scientific method with respect to the mathematization of dynamics was limited insofar as this dynamic conception of the path of the motion of the falling body was not rendered mathematically. The *Specimen* renders this point sufficiently clear. Leibniz argues here that,

From this it follows that force is also twofold. One force is elementary, which I also call dead force, since motion [*motus*] does not yet exist in it, but only a solicitation to motion [*motus*], as with the ball in the tube, or a stone in a sling while still being held by the rope. The other force is ordinary force, joined with actual motion.<sup>42</sup>

Leibniz's idea here is that the outward path of the moving body in the tube, as motion due to centrifugal force, is the result of a series of impressed solicitations to move. In turn, the calculation of the final velocity of a moving body, solicited by a constant force, would certainly result from the sum of these transformations of velocity in time while receiving such solicitations. As such he states that,

[J]ust as the numerical value of a motion extending through time derives from an infinite number of impetuses, so, in turn, impetus itself (even though it is something momentary) arises from an infinite number of increments successively impressed on a given mobile thing. And so impetus too has a certain element from whose infinite repletion it can only arise.<sup>43</sup>

Leibniz here seems to be arguing for a conception of the path of motion that we have just set aside. But Leibniz, in the next paragraph remarks further that,

[W]hen we are dealing with impact, which arises from a heavy body which has already been falling for some time, or from a bow that has already been restoring its shape for some time, or from a

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<sup>42</sup> GM VI 238; AG 121.

<sup>43</sup> GM VI 239; AG 121.

similar cause, the force in question is living force which arises from an infinity of continual impressions of dead force.<sup>44</sup>

This problem of the successive impression or *nisus* on a moving body is correlate to the consideration of the moments of the force of gravitation on an accelerating body. To dispel the alleged errors of integration, I refer to Bertoloni Meli's comment concerning this problem. I follow his argument that,

When he talks of a 'heavy body which has been falling for some time', he does not mean that the integral of dead force is multiplied by some element of time, but is simply providing a general description of the phenomenon.<sup>45</sup>

In these terms, Bertoloni Meli argues, as Leibniz himself makes clear, that this conception of the infinitesimal-finite difference allows us to compare quantities correlated to dead and living force. This comparison, although important for understanding how Leibniz conceives of the continuity between dead and living force, does not mean that the infinitesimal quantity assigned to dead force integrates into living force. In fact there is no such correlation between the solicitations (*nisus*) of dead force such as to integrate in mathematical terms into the integrated sum of final velocity of a body "falling for some time". We should thus not let the infinitesimal-finite comparison of dead and living forces mislead us into thinking that this relation also implies that one integrates into the other.

Through the same interpretation, we can also understand another salient remark on this problem found in Leibniz's 27 December 1698 letter to De Volder where Leibniz, on this same subject of the relation between solicitation and motion remarks that,

Of course the speed increases in equal amounts according to time, but the absolute force itself increases according to distance or the square of the times, i.e., in accordance to the effect. So by analogy with geometry, or my analysis, solicitations are as  $dx$ , speeds are as  $x$ , and forces [*vires*] are as  $xx$  or  $\int xdx$ .<sup>46</sup>

Again here, Leibniz is dealing with the comparability of the kinds of quantities involved. Hence rather than analyzing the actual (analytical) relation between *nisus* and motions, or speeds and forces, Leibniz uses his infinitesimal analysis *analogically* to compare the

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<sup>44</sup> GM VI 238; AG 122. In a different context, Leibniz also argues for the same distinction between dead and living force to De Volder in a letter of 27 December 1698. Here Leibniz straightforwardly claims that the analogy to the distinction between finite and infinite for the distinction between dead and living force is made to argue for the continuity between the terms on the model that *natura non facit saltum*. GP II 154, DeV 286.

<sup>45</sup> Bertoloni Meli, 90.

<sup>46</sup> GP II 156; DeV 289.

magnitudes. Leibniz exploits here only the linear and quadratic difference between speed and *vis* rather than a dynamic account of the path of motion through solicitation.

Leibniz's failure to provide a mathematical account of the relation between soliciting forces and motion further clarifies the limits of his dynamics and the incompleteness of his physical theory. Nonetheless we also see that Leibniz possessed a conception of what he was unable to formalize. It is in treating the gap between Leibniz's methodological limits and conceptual aims that we shall clarify the nature of structural causality in Leibniz's dynamics.

#### IV. From *potentia* to *actio*: Leibniz's refinement of the *vis viva* concept

Our analysis above has interpreted Leibniz's omission of  $\frac{1}{2}$  from  $mv^2$  by characterizing this "error" as a methodological limitation. Judging from Leibniz's own arguments, we have established that Leibniz calculates the dynamical properties of motion through statical methods. That is, the relation  $\Delta v^2 \propto \Delta h$  is inferred from a series of statical proportions between maximum speed and "work", leaving the consideration of the acceleration of the path of the motion  $v\Delta v$  (across a distance) unaccounted for except in a general description of phenomenon.

The idea of "vindicating" Leibniz is not my aim here. Rather, I argue that treating Leibniz's omission as a concrete methodological limitation can help us separate the different aspects of Leibniz's dynamics and help us grasp how the quantity  $mv^2$  functions within his dynamics. We have mentioned briefly above that Leibniz's maturation in the dynamics project consists of a conceptual move beyond the limited methodology of the measurement of *vis viva* as  $mv^2$ . Leibniz's methodological limitations, as we have seen, constrain him to a model of measurement that can only take up static proportional relations between maximum and final quantities of velocity and height. This model is based on the equipollence between a quantity such as velocity and its "exhaustion" in height. In other words, a body's velocity at a time  $t_n$  indicates its power or *potentia* to achieve some future effect such as maximum height. The conceptual framework for understanding *vis viva* then is thus the model of the *potentia* of a certain intensive power to achieve some extended motion in its exhaustion: the translation of full intensity to completed extension.

The limits of this intensity-extension model can be directly seen in our critique of Leibniz's statical methodology. However in our analysis we have also seen Leibniz conceptually reaching beyond these limitations. Although the conserved quantity  $mv^2$  serves nicely as a treatment of the intensive-extensive equipollence between velocity and height  $\Delta v^2 \propto \Delta h$ , this quantity  $mv^2$  also allows Leibniz to go much further. Separating the concrete problems of the measurement of  $mv^2$  from its eventual structural role in the dynamics will enable us to grasp the true role of  $mv^2$  as a conserved quantity and see how Leibniz moves beyond a treatment of corporeal causation based on the equipollence of intensity and extension.

In what follows I argue for a conceptual distinction between the understanding of *vis viva* as power (*potentia*) from action (*actio*). Both aspects of *vis viva* are present in Leibniz's theory but whereas understanding *vis viva* as power emphasizes the concept of intensity (*intensio* or *longitudines*) exhausted or otherwise translated into another quantity in a moving body through time, *actio* emphasizes the immanent realization or the organization of the properties of a moving body at any time  $t_n$  in temporal evolution. On the one hand, we have seen the limitations of understanding *vis viva* through the concept of power insofar as Leibniz failed to provide some analogous notion of (Newtonian) force where the final velocity of the falling body would be the expression of a series of compounded attractions (or solicitations to motion) and inertial resistances to these attractions (or solicitations to motion) in its path of motion. Though Leibniz saw the need to describe the expression of power in motion in just this way, it remained a vague description far from any direct mathematical treatment of the path of the body  $vdv$  (across a distance). We could thus say that Leibniz's methodology limits him in the account of understanding *vis viva* as the translation of intensive *potentia* to extensive motion. On the other hand, understanding the quantity conserved through the concept of *actio* would allow us to grasp Leibniz's conservation principle through  $mv^2$  by another means. *Actio* allows us to treat *vis viva* as an organizational principle that works structurally over the extended properties of corporeal motion. In other words, the conservation principle  $mv^2$  would serve to provide a principle of invariance over a field of quantitative transformations within a physical system of one or many bodies. As I argue below, Leibniz's methodological limitations allow us to more coherently grasp the refinement of *vis viva* from the model of *potentia* to that of *actio*, a maturation that makes clear the structural nature of dynamical causality.



Before examining *actio* more closely, we first provide some grounds by first briefly looking at the context for this later stage of Leibniz's dynamics. The major transformation of Leibniz's dynamics project is his turn from a largely *a posteriori* mode of justification to an *a priori* one. This transformation can be dated to 1690, towards the end of the year-long sojourn to Italy (through Austria). I follow Duchesneau in seeing the discrepancies between the *Phoronomus seu de potentia et legibus naturae* of 1689 and the *Dynamica de potentia et legibus naturae corporeae* of 1690, written a few months between each other, as definitive in rendering a "before and after" picture of this shift.<sup>47</sup> Concerning the same transition, Fichant notes that the later *Dynamica*, drawing his title from the Latinized Greek, should be understood as a distinct shift towards a new science concerning *actio*, a turn that subordinates previous mechanical concepts such as *potentia* under a new formulation.<sup>48</sup> Hence although the title is often referenced as *Dynamica de potentia*, it should really be *Dynamica: de potentia et legibus naturae corporeae*. That is, in English, "Dynamics: On power and the laws of natural bodies" rather than "Dynamics of power and the laws of natural bodies". The concept of power is subsumed under a new understanding of dynamics through *actio*. Although Fichant makes an alluring case, I am unsure that the shift, considering the other documents of the period, can be made so neatly. Nonetheless a shift towards the privileging of *actio* circa 1690 is clear and although it is difficult to underline the exact cause of this shift, roots of this can be traced to the public criticism of his earlier published *Brevis Demonstratio* (1685) by the Cartesians F. Catelan and N. Malebranche (1687), a debate later on resumed by D. Papin (1696), and to Leibniz's own increasing critical engagement with the physical theory of Newton's *Principia* which Leibniz claims only to have actually read in Rome in late 1689.<sup>49</sup>

Regardless of the fascinating historiographical details here, we shall content ourselves only with the general context that they provide for the conceptual shift. Now, we can see the turn from *a posteriori* to *a priori* as coinciding with the turn from a negative

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<sup>47</sup> François Duchesneau, "Leibniz's Theoretical Shift in the Phoronomus and Dynamica de Potentia", *Perspectives on Science*, 1998, vol. 6, 1-2, 77-109.

<sup>48</sup> Michel Fichant, "De la puissance à l'action : la singularité stylistique de la Dynamique", *Revue de Métaphysique et de Morale*, 100<sup>th</sup> year, No. 1, Jan-Mar 1995, 49-81, 50-53.

<sup>49</sup> For an account of the ambiguities concerning the date of Leibniz's encounter with Newton's *Principia* see Bertoloni Meli, 8.

project of the critique and reform of Cartesian mechanism to that of a positive project of Leibniz's own systematic articulation of the laws of motion. Catelan's critique of Leibniz as ignoring the factor of the time of the free fall of bodies in the critique of Descartes made clear that the project of "reform" had to reach beyond a mere rejection of the conservation of the Cartesian quantity of motion.<sup>50</sup> Hence Leibniz faced the task of generalizing the basic notions at work in his reform of mechanics. Secondly an "internal" drive within Leibniz's own conception of the ideals of scientific demonstration during this period meant that the maturation of his project should be understood along the methodology of the reciprocal correspondence between an analytic and synthetic mode of demonstration. This turn to an *a priori* presentation can thus be seen as an eventuality resulting from the crystallization of Leibniz's general ideals about the method of scientific argumentation rather than from specific mechanical or dynamical problems. As such, while the analytic mode of determining the basic elements of dynamics could be developed from a modeling of empirical features of motion, only an *a priori* presentation of these elements could allow for a synthetic form of demonstration starting from the scientific ideal of real, rather than nominal, definitions and a concretely syllogistic form.<sup>51</sup>

To avoid confusion it is also worth noting that this *a priori* turn here does not mean a demonstration of the laws of motion on the basis of the *synthetic a priori* in Kantian terms. What is *a priori* about Leibniz's method cannot be mapped onto the necessary-contingent distinction in Kant but owe its status to the earlier Scholastic tradition of distinguishing between argumentation from real and nominal definitions. An *a priori* scientific demonstration, for Leibniz, is synthetic because it begins with elemental *a priori* (real) definitions building toward the demonstration of a complex proposition via syllogism. Conversely an *a posteriori* method for scientific demonstration dissects (*analyses*) phenomena according to nominal definitions.

What then governs this transition between the *a posteriori* analytic phase and the turn to an *a priori* and synthetic phase is the attempt to solve the problem of how to characterize and bridge the difference between causes and effects in corporeal motion. The basic model for the analysis of motion in the earlier phase was based on the interpretation

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<sup>50</sup> GP III 41-42

<sup>51</sup> Leibniz provides a clear discussion of the relation between the distinction of real and nominal definitions and causation in §24 of his *Discours de Métaphysique*. GP IV 450.

of the equipollence of cause and effect as the extensional expression of a certain intensity (*potentia*) in motion. That is, although *vis* is non-phenomenal, we can nonetheless measure the “quantity” of this intensity, *potentia*, by comparing the motion of different bodies in order to indirectly draw out a proportional relation between them. Hence, the measure of the differences between different elements of an *a posteriori* experiment allows us to infer an indirect measure of the causes responsible for these phenomenal effects. As such, although the architectonic principle of the equipollence of effect and cause is both *a priori* and essential, the actual account of the laws of motion remained dependent on *a posteriori* elements (nominal): the various proportions that hold between empirical effects. In order to overcome this *a posteriori* orientation, Leibniz introduced, between the *Phoronomus* and the *Dynamica*, the notion of *actio* that would introduce a third term alongside power and effect.<sup>52</sup>

What does the introduction of *actio* change in Leibniz’s capacity to render a satisfactory account of the laws of motion? We shall examine this in more detail below. For the moment we note that the relation between power and effect provided Leibniz at an earlier phase with the capacity not only to refute the Cartesian conservation of quantity of motion but also to indicate that there is something in corporeal motion that does not simply reduce to its extensional features. Now, although the indication of this non-extensional intensity (*potentia*) in motion allowed Leibniz to argue for something in motion beyond the geometric features of size, shape and magnitude, the very treatment of this “something” that is *vis*, remained dependent on the capacity to take a measure of the extensional expression of *vis* in motion. These cases depend on what Leibniz called, drawing on Aristotelean terminology, “violent” motion since it is only in cases of, say, collision or the exhaustion of motion that quantities such as final velocities and heights, could be determined and brought into comparison. This is a direct feature, as we have analyzed above, of Leibniz’s methodological limitations. With the notion of *actio*, empirical phenomenal effect is replaced by the concept of an immanent activity in corporeal motion that constantly expresses the properties of *vis* in space and time. As such, the measure of *actio* does not have to rely on the “violent” cases of motion since the expression of *vis* in terms of *actio* does not require terminal maximum measures or the efficient exchange of velocities in collision. We shall look at this more closely in the treatment of Leibniz’s *Essay*

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<sup>52</sup> GM VI 291-292.

*de dynamique* (circa 1699-1701) in the following. It is important here to note that the continuous action of a moving body in time allows us to take its velocity and work achieved at any time  $t_n$ . Hence *actio* allows Leibniz to further distance the phenomenal from the metaphysical or essential aspects of motion by allowing *actio* to stand between, on the one side, the non-phenomenal causality of *vis* and, on the other side, the phenomenal effects of size, shape and motion.

There is significant overlap between concrete models of *potentia* and models of *actio*. The use of “early” and “late” here does not correspond, as noted above, to a neat developmental distinction. Nonetheless whereas the earlier model, privileging the equipollence of effect and cause, made use of empirical measurements between effects to instantiate the comparison of causes (*vis*) and effects (motion), the later model starts by instantiating *vis qua actio* in bodies (*vis* immanent in bodies insofar as acting in space and time) and treats effects as the empirical expressions of *actio*. Actions are thus the causes of the properties of motion in a system of one or many bodies. With the introduction of *actio* as the product of mass, distance traveled and speed, Leibniz takes a move in a different direction. As I will argue, this conception of the quantity of *actio* steps beyond the static model and allows us to interpret the conservation of *vis viva* as an invariant whose role is the structural organization of these extensional properties of a system of bodies in motion and helps clarify the nature of *vis viva* as structural cause of motion.

#### V. *Actio* and structural causation

The *Essay de Dynamique* (circa 1699-1701) is the last comprehensive contribution to Leibniz’s dynamics project. It followed the turn in the mode of presentation brought forth after the *Dynamica* where *actio* was introduced to provide a new presentation of *vis* in terms of its embodiment in the immanent evolution of corporeal motion in space and time.

Leibniz’s own example from the *Essay* is complicated and I have simplified it here for the sake of clarity. Leibniz here takes an initial system of three bodies A, B and C along three axes, M, L and N respectively. A moves along axis M to strike the resting bodies B and C, causing them to move along the axes L and N respectively. Now, since A moves towards the origin to strike the two bodies B and C (at rest), the two bodies B and C will move away from the origin and they will move in proportion to the mass, speed and angle of collision of A moving toward the origin.

From this basic scenario, Leibniz adds two additional bodies, D and E, on axes L and N respectively, moving toward the origin. Hence as bodies B and C move away from the origin, they will each strike the bodies D and E respectively. In this scenario, after the meeting of bodies BD and bodies CE, B and D will continue in the same direction away from the origin while C and E will be rebounded leaving C moving back to the origin while E away from the origin along axis N.<sup>53</sup>

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<sup>53</sup> GM VI 223.

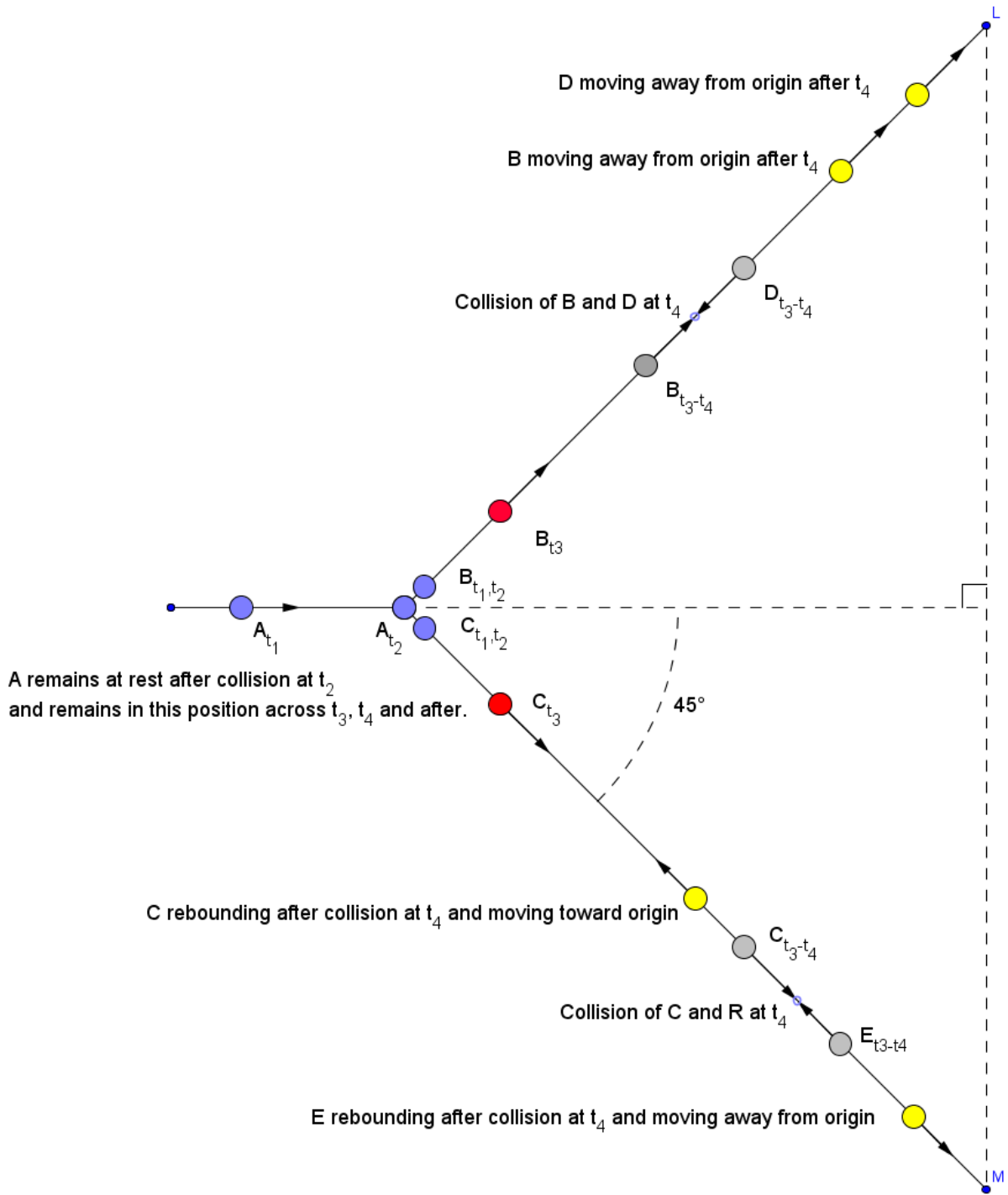


Figure 1<sup>54</sup>

<sup>54</sup> This figure is redrawn consulting Gerhardt's figure 22 in the appendix to GM VI. The bodies are color-coded to indicate time. Blue represents the events that occur between  $t_1$  and  $t_2$ . Red represents the events that occur between  $t_2$  and  $t_3$ . Grey represents the events that occur between  $t_3$  and  $t_4$  and yellow represents the events after  $t_4$ . Since A remains in the same position after  $t_2$ , no color coding is made.

Leibniz’s aim here is to take this system of five bodies in order to demonstrate an invariance at each time  $t_n$  as the system evolves. By assigning different speeds and masses to each body and in tracking their speeds at time  $t_n$ , Leibniz seeks to make his case for a conservation of what he calls “motive action” (*actio motrix* or *action motrice*). Rather than enter into the description that Leibniz provides at each time  $t_n$  of the system, perhaps a chart might better serve our purposes here.

		A	B	C	D	E
	mass	1	1	1	2	$\frac{1}{2}$
$t_1$	velocity	0	0	0	0	0
	formal effect=mass·distance	0	0	0	0	0
	<i>actio</i> =formal effect·velocity	0	0	0	0	0
$t_2$	velocity	$\sqrt{2}$	0	0	$-\frac{1}{2}$	$-\frac{2}{3}$
	formal effect=mass·distance	$\sqrt{2}$	0	0	1	$\frac{2}{6}$
	<i>actio</i> =formal effect· velocity	2	0	0	$\frac{1}{2}$	$\frac{2}{9}$
$t_3$	velocity	0	1	1	$-\frac{1}{2}$	$-\frac{2}{3}$
	formal effect=mass·distance	0	1	1	1	$\frac{2}{6}$
	<i>actio</i> =formal effect· velocity	0	1	1	$\frac{1}{2}$	$\frac{2}{9}$
$t_4$	velocity	0	$\frac{1}{3}$	$-\frac{1}{9}$	$\frac{5}{6}$	$\frac{14}{9}$
	formal effect=mass·distance	0	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{5}{3}$	$\frac{7}{9}$
	<i>actio</i> =formal effect· velocity	0	$\frac{1}{9}$	$\frac{1}{81}$	$\frac{25}{18}$	$\frac{98}{81}$

Figure 2<sup>55</sup>

The aim of the demonstration is primarily to note that the *actio* of the system of bodies ABCDE as the sum of their respective *actio* at and after time  $t_2$  is invariant, the quantity in here is  $\frac{49}{18}$  units.

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<sup>55</sup> This chart is created from Leibniz’s argument from GM VI 223-225. Each row of time  $t_n$  marked here represents what happens (immediately) *after*  $t_n$ .

The example in question also occurs in Leibniz's mature work on the dynamics in different places. In his correspondence with Bernoulli and De Volder on 27 December 1698, a simpler version was employed.<sup>56</sup> What is important to note is that the principal crux of his argument with De Volder surrounded the establishing of the proportion of the speeds of the three bodies (A, B and C) at various times rather than the larger quantitative exposition.

In the *Essay de dynamique* Leibniz attempts to explain the quantities chosen here in detail but for simplicity we will stick closely with the aim of the argument, the determination of an invariant. However, a few things require some explanation. First in the initial meeting of bodies ABC at the origin, we note that the bodies are of equal mass. Hence the speed of body A at  $t_2$  is distributed between bodies B and C in proportion to the relation to the hypotenuse of the two equal sides of L and M. Hence the speed of  $\sqrt{2}$  is decomposed into two speeds (B and C) each of 1 unit. This is as if the motion of B and C can be equated to the decomposition of the motion of A along two orthogonal axes.

Secondly, we must note that C rebounds after the meeting of CE at time  $t_3$  while B and D does not rebound after the meeting of BD. Here, although Leibniz does address the problem of elastic and inelastic collisions in his explanation following his experimental scenario, he does not address the specific case of the meeting of bodies BD. This further problem of elastic collision is not addressed by Leibniz. Nonetheless the general picture seems to be clear. If we take the subset of bodies A, B and C, the quantity of motive *actio*, the invariant, is 2 units after collision at  $t_2$ . If we take the entire set of bodies A, B, C, D, and E, the quantity of motive *actio* is 49/18 units across times  $t_2$  to  $t_4$  (and after) and continue as such without the addition of new bodies in the system.

The goal of an invariant calculated in this way allows us to return to the analysis of  $mv^2$  in a different way. We know that since the respective speeds of each body in a system are proportional to the distance traveled by each body, the speed is also linearly proportional to the "formal effect" of each body considered. As such, the quantity  $mv^2$  is thus proportional to the quantity of motive *actio* of each body in the system at each time  $t_n$ . This demonstration in the *Essay de dynamique* was meant to answer those who "persist in disputing this definition of motive action".<sup>57</sup> This might refer to De Volder who expressed

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<sup>56</sup> GP II 159-169; DeV 292-295

<sup>57</sup> GM VI 221-222.



heavy reservations concerning the calculation of motive *actio* through the formal effect of motion. In his correspondence with Bernoulli and De Volder on 3 April 1699, Leibniz had provided the quantity of formal effect as the product of mass and distance.<sup>58</sup> Since distance is the product of velocity and time, the quantity of formal effect is the product of mass, velocity and time. The quantity of *actio* is, in turn, the product of mass, time and velocity squared or  $mv^2t$ . Since velocity is distance  $s$  over time  $t$ , *actio* can also be understood as  $ms^2/t$ . Hence for the calculation for an invariant in a system of bodies in motion, the calculation of  $mv^2$  at time  $t_n$  is equivalent to the calculation of *actio* at time  $t_n$ . What we know from the analysis of the formula can be seen more directly in the following:

Sum of various quantities for bodies A, B, C, D and E.			
	formal effect	actio	$mv^2$
$t_1$	0	0	0
$t_2$	$4/3 \cdot \sqrt{2}$	49/18	49/18
$t_3$	10/3	49/18	49/18
$t_4$	26/9	49/18	49/18

Figure 3

What is perhaps most important here is to grasp the difference between this invariance of *actio* and  $mv^2$  in light of the non-invariance of the quantity of formal effect. Of course, it is clear that formal effect or  $m \cdot s$  (product of mass and distance traveled) is not conserved since it evolves as a function of velocity in time. But as distance increases and *actio* remains constant, velocity must then also diminish at an inversely proportional rate. That is insofar as *actio*  $A$  can be understood as  $A = m \cdot s \cdot v$ , then:

$$\Delta A/v \approx \Delta m \cdot s$$

The formal effect  $m \cdot s$  increases inversely proportional to (diminishing) velocity. As such the relation between the invariance of *actio* and the quantity of formal effect translates the conservation of *actio* into the relation between velocity and the instances of the distance

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<sup>58</sup> GP II 172-174. DeV 317-318

traveled of each body in the system in time  $t_n$ . Leibniz does establish something close to a dynamics here. With *actio* (measured by energy-work) as a constant given in a physical system, the formula establishes a proportion between the velocity at time  $t_n$  and the distance covered  $s$  at  $t_n$ . This translates the conservation of *actio* into the proportional (inverse) relation between the actual velocity of a system of bodies and their “formal effect” actualized by that moving body at any time  $t_n$ . Hence the conceptual theme underlying this measurement of the proportion between these quantities in terms of conserved *actio* is that which might designate something close to the proportion between the quantity of the work achieved by a system and the *potential* energy of each body in the system through its temporal evolution. Of course the direct identification of the system of *actio* and energy is anachronistic. Nonetheless this perspective of the quantity of *actio* might allow us some insight into Leibniz’s method.

In our examination of *potentia*, we see that the quantity of conserved *vis*,  $mv^2$ , was inferred indirectly from a comparison of quantities. Leibniz’s use of *actio* attempted to remove itself from a method of indirect measure of *potentia*. It stands as the quantity regulating the structural translation of *potentia* and effect. Further, the quantity of *actio* has no direct correlate to the “mechanical” notions of external sollicitation, internal impetus, static counter-balancing or any such physical models, its role is purely a systematic one that concerns the formal organization of these proportions in temporal evolution.

The calculation of *actio* here then provides us with a structural interpretation of  $mv^2$ . Our original interpretation of  $mv^2$  shifted the emphasis away from the quantity as the sum of the moments of forces acting on and in the body as it falls and focused understanding this expression via statics as the proportion of work and maximum velocity. From the analysis of the limits of this statical method, we saw how this revealed Leibniz’s conceptual aims for a structural understanding of *vis*. This structural understanding is reinforced by seeing how Leibniz places the invariance of  $mv^2$  at work in a very different kind of example where the proportions are not drawn from the exhaustion of intensity into extension but rather played the role of governing the evolution of the motion or a system of motions in time. This allows us to clarify Leibniz’s refinement of the concept of *vis* through the structural character of the use of  $mv^2$  qua *actio* and emphasize the importance of the status of *vis viva* as “action” under which the concept of “power” is subsumed.

## VI. Concluding remarks

As a final note, we acknowledge that, for most of the mature period of Leibniz's metaphysics, he affirmed the scholastic axiom that "*actiones sunt suppositorum*".<sup>59</sup> This notion most famously applies to the metaphysical thesis concerning the containment of predicates (in temporal evolution) for a subject such as Julius Cesar, Genghis Khan or, abstractly, the present author, but it also importantly concerns the constant action of individual substances. This constant *metaphysical* action thus translates into the *physical* thesis, adopted at least as early as *circa* 1678, of bodies in constant micro-motions and the rejection of "perfect rest" in bodies.<sup>60</sup> Of course this metaphysical notion of action, though metaphysically suggestive for the foundation of the conservation of *vis viva*, is far from what we have considered here. Indeed, Leibniz's metaphysical concept of *actiones* requires much more investigation in order to make clear its convergence with his intended systematic theory of motion.

Leaving the metaphysics of *actio* aside, we nonetheless have gained some clarity on the role of *actio* in dynamics where it provides a bridge between the intensity of *vis* and the extendedness of motion. *Actio* plays a direct role in providing a concrete picture of the causal nature of *vis* through the demonstration of its conservation of the quantity  $mv^2$ . Whereas the earlier model of cause and effect required Leibniz to bring together, for example, a proportional organization of colliding speeds or terminal velocities and heights, *actio* allows Leibniz to treat the quantity conserved in a system of moving bodies during their motion at any time  $t_n$ . This allows us to treat a system of bodies in the course of motion with the quantity conserved in their motions with respect to their evolutions at given time intervals. With the notion of the intensity of force understood as power, on the one hand, and a certain expression of that intensity unfolding in terms of distance traveled by the moving body, on the other hand, *actio* structurally regulates these transformations of intensity and its expression. Most importantly, Leibniz is also able to overcome the implication of the simple image of conservation as that which translates one quantity into another (maximal height to maximal velocity). The conservation of the quantity of *actio* no

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<sup>59</sup> Leibniz provides a clear exposition of this position in §8 of the *Discours*. GP IV 432-433; AG 40-41. The axiom comes directly from Thomas Aquinas, ST II-II 58, Art. 2.

<sup>60</sup> A IV 267, 1400; LC 249.

doubt applies to these simpler cases but generalizes conservation to stand as a structural cause governing the entire range of the properties of motions. In conceptual terms, we move from a static understanding of the equipollence of cause and effect closer to a dynamic one of the expression of action in the path of motion at given time intervals.

Of course, Leibniz did not provide the means to enter into the field of dynamics as we now understand it. But the development of the concept of *actio* puts the interpretation of *vis qua* cause on the right footing. The measure of *vis* through its conservation plays a structural role that serves to provide the cause of extended motion not as the passing over of one configuration of quantities into another. Rather, it structures the extended properties of motion across its activity in time with the quantity of *actio* composed of the properties of a number of bodies in a given system. This activity is theorized as *actio*, the expression of *vis* as the immanent property of a body in space and time. *Actio* does not transfer between bodies but remains conserved across the collisions of bodies, the exhaustion of a motion, the motion of a system of bodies (at motion or rest according to a given center of gravity) whether there is or is not contact or “violent” motion. As such, the notion of causality at work in Leibniz dynamics, moving from an earlier notion of *vis* and the later notion of *actio*, is structurally causal rather than reducible to *causa efficiens*.

§§§

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