Talking past each other: Mach and Husserl on thought economy

Iulian D. Toader*

(forthcoming in *Vienna Circle Institute Yearbook*)

In a letter to Mach from June 18, 1901, right after the publication of a new edition of *The Science of Mechanics*, Husserl claimed that his analysis of pure logic in the *Logical Investigations* neither should, nor could be taken to invalidate or make obsolete Mach's methodological views on science, especially his doctrine of thought economy.[1] This claim contrasted rather sharply with Husserl's published remarks on this doctrine, as we will see, and it raises a question about his actual understanding of Mach's views. The doctrine of thought economy had been introduced in the 1882 paper "On the economical nature of physical inquiry" and then expanded upon in "The economy of science" – chapter IV, section 4 of *The Science of Mechanics*, published in 1883. Husserl reacted to it in 1900, in his "Logic and the principle of the economy of thought" – chapter 9 in the "Prolegomena to Pure Logic," the first volume of his *Logical Investigations*. Mach responded directly in Appendix XXVII of *The Science of Mechanics*, the 1901 edition, which triggered Husserl's letter, and then again indirectly in *Knowledge and Error*, published in 1905.

In this note, I revisit this debate and argue that, to a considerable extent, Mach and Husserl talked past each other, insofar as the latter rejected thought economy as a principle of *theoretical* rationality, whereas the former conceived of it as a principle of *practical* rationality. My argument is further supported, as I will show, by their correspondingly different readings of the so-called principle of the permanence of forms, explicitly formulated by the Cambridge algebraist George Peacock in the first half of the 19th Century, and later propagated by the German mathematician Hermann Hankel.

According to Mach's methodological views on science, let us briefly recall, thought economy is, on the one hand, an ideal of science, and on the other hand, an adequate description of (at least part of) science. This was emphasized by Mach, himself, in the 1882 paper: "The goal which [science] has set itself is the simplest and most economical

*The Research Institute, University of Bucharest; Descartes Centre for the History and Philosophy of the Sciences and the Humanities, Utrecht University. Contact: itoad71@gmail.com or i.d.toader@uu.nl

[1]See "Ein Brief Edmund Husserls an Ernst Mach", edited in 1965 by Joachim Thiele, *Zeitschrift für philosophische Forschung*, 19, 134–138.

conceptual expression of facts. [...] The greatest perfection of mental economy is attained in that science which has reached the highest formal development, and which is widely employed in physical inquiry, namely, in mathematics." He further specifically indicated abstraction, idealization, and symbolization as thought economical methods in mathematics: "The use of the signs of algebra and analysis, which are merely symbols of operations to be performed, stems from the observation that one can disburden the mind and spare it for more important and more difficult functions by transferring a part of the mechanically repetitive operations to the hand." But in fact, Mach referred to anything that goes beyond the flux of irrepeatable experiences as thought economical. Most famously, perhaps, in chapter IV of the 1883 book, he noted with respect to mathematical methods: "Even a total disburdening of the mind can be effected in mathematical operations, for operations of counting hitherto performed are symbolised by mechanical operation with signs, and our brain energy, instead of being wasted on the repetition of old operations, is spared for more important tasks."[2]

In his discussion of Mach's doctrine of thought economy, in the *Logical Investigations*, Husserl agreed that this provides an adequate description of some scientific methods, e.g., symbolization in mathematics: "Mathematical disciplines [...] overcome the defects of our mental constitution, and permit an indirect achievement by way of symbolic processes from which the intuitive element, as well as all true understanding and inner evidence are absent. [They] have the character of devices which economize thought. They arise [...] out of certain natural processes of thought-economy. [...] Such methods can be used without insight, so to say mechanically." However, Husserl strongly emphasized that this does not mean that these methods *should* be used in this manner. One reason for this was that, according to him, thought economy does not provide an adequate epistemic ideal for science, since no such ideal could admit of a merely psychological grounding. More generally, one cannot derive an adequate epistemic ideal for science from a description of scientific practice, as Mach had allegedly done with thought economy. Another reason was that Husserl saw thought economy as an obstacle to verificationism – the epistemic theory of truth developed in the *Logical Investigations*.[3]

As an adequate epistemic ideal for science, Husserl proposed the ideal of maximum rationality – the supreme goal of all rational sciences: "If all matters of fact obey laws, there must be some minimum set of laws, of the highest generality and maximum deductive independence, from which all other laws can, by mere deduction, be derived. These 'basic laws' are, accordingly, laws of supreme coverage and efficacy, whose knowledge yields the absolute maximum of insight in some field, which permits

[2]For a recent discussion of Mach's doctrine of thought economy, see Eric Banks' book *Ernst Mach's World Elements: A Study in Natural Philosophy*, esp. chapter 8, and his article "The Philosophical Roots of Ernst Mach's Economy of Thought", *Synthese*, 139, 23–53.

[3]For the historical context of Husserl's verificationism, see Kevin Mulligan's article "Brentano's Knowledge, Austrian Verificationisms, and Epistemic Accounts of Truth and Value", *The Monist*, 100, 88–105. See also my *Objectivity Sans Intelligibility: Hermann Weyl's Symbolic Constructivism*, PhD diss., University of Notre Dame, esp. chapter 2.

the explanation of all that is in any way explicable in that field." It is the verificationist element constitutive of this ideal, i.e., the claim that a maximally rational science requires maximum insight, that Husserl believed collided with thought economy. This is because he took thought economy to require minimization of insight and, thus, to go against verificationism. However, as an argument against Mach, this is puzzling, because as we will see below Mach also endorsed verificationism.

Lest one considered that one could found the ideal of maximum rationality on thought economy, Husserl further argued that this would be rather absurd: "The thought-economist turns the *ideal* tendency of logical thinking towards rationality into a *real* tendency of actual thinking. ... [But] our actual thinking does not in fact conform to its ideals – as if ideals were some sort of natural forces." And he continued: "The ideal validity of this norm [of maximum rationality] is *presupposed* by all meaningful *talk* of an economy of thinking; it is not therefore a possible explanatory outcome of a theory of such economy. We *measure* our empirical by our ideal thinking, and we then say that the former to some extent runs as if guided by insight into these ideal principles." On Husserl's view, one must distinguish between "blind", thought-economical reasoning in science and the logical thinking of pure logic, the latter being the quintessential expression of the ideal of maximum rationality and fundamentally prior to all thought economy.

In his response, Mach first noted that Husserl's "animadversions on my theory of mental economy ... are in part answered in my reply to Petzoldt" in the 1896 *Principles of the Theory of Heat*. Petzoldt's earlier criticism, as quoted by Mach, had attempted to reduce the importance of thought economy for understanding science, while relegating it to other principles: "Not maxima, minima and [thought] economy, but uniqueness and stability are brought into relief by those aspects of reality which must stand in the foreground of our interest." To this Mach replied that "Economy cannot be predicated of physical processes, since there is no choice between the actual happening and another. For this very reason I have not used the notion of economy in any way in this domain." Basically, he says that, in relation to science, he spoke of mental, not physical, economy. So he thought that Petzoldt was somewhat confused.

Mach might not have thought that Husserl, too, was confused, but he did think that Husserl was rather audacious, and noted with frenzy in the 1901 Appendix: "I am perfectly able to distinguish between psychological and logical questions." Most certainly, he did not appreciate Husserl's "temerity" to say that he couldn't make the distinction between "blind", thought-economical reasoning and logical thinking. For what is worth, Mach insisted that even a maximally rational science should deploy thought economical methods: "Even if the logical analysis of all the sciences were complete, the biologico–psychological investigation of their development would continue to remain a necessity for me... Thought economy is [...] a very clear logical ideal which retains its value even after logical analysis has been completed." The claim here is that the ideal of thought economy does not go against the ideal of maximum rationality. In particular, even if verificationism were satisfied, thought economy would still be a requirement: "The systematic form of a science can be deduced from the same prin-

ciples in many different manners, but some one of these deductions will answer to the principle of economy better than the rest. [...] What appears to Husserl as a degradation of scientific thought, the association of it with vulgar or "blind" (?) thinking, seemed to me to be precisely an exaltation of it." Mach's suggestion here is that even if a minimal set of most general laws in a given domain provided maximum insight in that domain, this may be obtained on various deductive routes, some more economical than others.

Furthermore, in his 1905 book, Mach strongly endorsed the verificationist doctrine that genuine scientific knowledge and understanding requires the possibility to verify that there is something real and intuitive corresponding to symbols and concepts: "Thought does not proceed in empty forms, but according to a vividly presented content, either directly or through concepts. [...] Empty logical formulae cannot replace a knowledge of the facts. Nevertheless a look at algebra and mathematical symbolism in general shows that attention to thought as such and the symbolic representation of the abstract forms of intellectual operations are by no means devoid of all merit. Anyone who could not carry out these operations without such help would however gain no profit from these methods." This indicates that thought economical methods are practically, although not theoretically, necessary. Scientists should be capable of dispensing with them, however advantageous they may be otherwise. So Mach warned against the unverifiable use of symbolic reasoning: "Symbolic representation has likewise the disadvantage that the object represented is very easily lost sight of, and that operations are continued with the symbols to which frequently no object whatever corresponds. [...] Are we not subject here to an illusion, in that we operate with symbols to which perhaps nothing real corresponds, or at least nothing intuitive, by means of which we can verify and rectify our concepts?" If one were unaware of this illusion, Mach implied, one might wrongly think that atomistic physics is science, rather than metaphysics.

Mach had expressed similar verificationist ideas already in the 1882 paper: "One must say that there is no scientific result that in principle could not have been found without any [thought economical] methods." In other words, thought economical methods are profitable only if they are in principle dispensable. Also, with regard to the scientific character of atomistic physics, Mach had noted in his 1897 book, *The Analysis of Sensations*: "The [hypothetical atoms and molecules of physics and chemistry] remain economical ways of symbolizing experience. But we have as little right to expect from them, as from the symbols of algebra, more than we have put into them, and certainly not more enlightenment and revelation than from experience itself. We are on our guard now, even in the province of physics, against overestimating the value of our symbols."

All this textual evidence, then, indicates that Mach considered thought economy to be a principle of practical rationality, whereas Husserl understood thought economy, and rejected it, as a principle of theoretical rationality. This explains why he believed that thought economy could be in conflict with the ideal of maximum rationality. Furthermore, if this is true, and I think there is good reason to believe it is, then Mach

and Husserl really talked past each other in this debate.

Interestingly, however, in his letter to Mach, which I mentioned at the outset, Husserl denied that he actually reacted to Mach's views, and claimed that his criticism was rather directed against the school of Avenarius and especially against Cornelius. Husserl added that he did not think that his analysis of pure logic should, or even could, invalidate or make obsolete Mach's views on science. Moreover, Husserl highly deferentially emphasized the extraordinary fruitfulness of these views for the methodology of science, and then added that Mach's name came up in his critical remarks merely because Cornelius had referred to him (though not always with justification) in his own discussion of thought economy. The letter remains silent, though, on why Husserl thought that his critical remarks should not, and could not, despite all appearances, be seen as extending to Mach's doctrine of thought economy. There is no hint that Husserl actually realized that, for Mach, thought economy was a principle of practical, rather than theoretical, rationality. In any case, it's hard to imagine how such a letter could have eased Mach's unhappiness with the remarks in the *Logical Investigations*.

The view suggested here is, I believe, indirectly supported by Mach's and Husserl's correspondingly different readings of the principle of the permanence of forms (henceforth, PPF). As I show in the balance of this note, Husserl rejected the PPF as a principle of theoretical rationality, for the reason that it unjustifiedly assumes an inference from consistency to truth, while on Mach's view, permanence, of which Hankel's permanence of forms is just a special case, is a thought-economical principle of practical rationality, just like consistency. Husserl first considered the PPF for a public *Disputatio* at the University of Halle, in 1887, with the ocassion of his *Habilitation*, and then in his *Doppelvortrag* at the Mathematical Society of Göttingen, in 1901. For Mach's views on permanence, I will refer again to his 1905 book, *Knowledge and Error*, but also to Musil's 1908 doctoral dissertation.

Since there is no space here for a detailed discussion of the historical and conceptual significance of this principle, let us just note that Peacock formulated the PPF in his 1833 "Report on the recent progress and present state of certain branches of analysis" as follows: "Whatever form is algebraically equivalent to another when expressed in general symbols, must continue to be equivalent, whatever those symbols denote." In particular, it must continue to be equivalent when the symbols specifically denote numbers, as they do in arithmetical algebra. The converse states that "Whatever equivalent form is discoverable in arithmetical algebra considered as the science of suggestion, when the symbols are general in their form, though specific in their value, will continue to be an equivalent form when the symbols are general in their nature as well as in their form." Thus, for example, if m, n and a denote integers, then $ma + na$ is equivalent to $(m + n)a$ in arithmetical algebra. This is true in virtue of the previously given definitions of basic operations. As Peacock put it, an arithmetical equivalence like $ma + na = (m + n)a$ has a "necessary" existence. The PPF demands that $ma + na$ remains equivalent to $(m + n)a$ in symbolic algebra as well, where m, n and a may denote anything whatsoever. An algebraic equivalence has, however, only a "conventional" existence. For it cannot be true in virtue of the definitions of basic

operations, since no such definitions are previously given. The meaning of the basic operations is only determined by algebraic rules like $ma + na = (m + n)a$.

In advocating the PPF as "the real foundation of all the rules of symbolic algebra", one of Peacock's main concerns was the applicability of symbolic algebra. He pointed out that algebraic rules are sufficient for deducing equivalent forms, but argued that symbolic algebra would be a science of mere symbols if it amounted to a set of arbitrary rules having a conventional existence but admitting no applications. To ensure that this was not the case, Peacock adopted arithmetical algebra as "a science of suggestion", i.e., he required that all algebraic equivalences allow an arithmetical interpretation. Another concern was the generality of the PPF, which seems to further require that all equivalences in arithmetical algebra should be transferable to symbolic algebra. As Peacock was well aware of, however, some arithmetical equivalences are essentially connected to the specific value of some of their symbols, and so they are not transferable. Such are, for example, Euler's inexplicable functions. In this case, arithmetical equivalences are only hypothetically transferable in the sense that their transferable forms have only a "hypothetical" existence and degenerate into the actual forms of the inexplicable functions for some specific values of its symbols.

It is doubtful that Peacock considered this maneuver completely satisfactory. For he clearly recommended extreme caution when applying the PPF. But what is important for my discussion in this note is that he seems to have conceived of it as a principle of theoretical rationality, just like logical consistency. What I mean by this is that, as I understand Peacock's position, he seems to have thought that the PPF was indispensable to the development of symbolic algebra as a genuine science. He considered the PPF as the "proper guide" that "must guide us" in the development of symbolic algebra. Peacock implied that without taking the PPF as our guide, we might end up with a set of merely arbitrary symbolic rules with no application whatsoever. That, according to him, would hardly be deserving of the name of science.

A similar conception of the PPF was later defended by Hermann Hankel in his 1867 book, *Vorlesungen über die complexen Zahlen und ihre Functionen*. His proposal and development of a purely formal mathematics, completely disconnected from intuition and constrained only by the conditions of logical consistency and mutual independence of its rules, was to be similarly guarded against potential meaninglessness. This required stipulating that the formal rules for operations with objects of thought admit the actual rules for operations with objects of intuition (e.g., the rules of universal arithmetic) as subordinate. This provision was meant to ensure that the results in formal mathematics would have an interpretation and applicability. In Hankel's own words, the PPF was given the following formulation: "If two forms expressed in the general signs of universal arithmetic are equal to one another, they should remain equal even if the signs cease to denote simple quantities and the operations thereby take on a different content as well." This corresponds to the converse of Peacock's formulation of the principle. Just like Peacock, Hankel conceived of the PPF as an indispensable guide for the development of formal mathematics. He also warned against its incautious general application. For example, in developing the formal theory of complex

numbers, he determined its rules via the PPF, but duly noted that not all rules that are valid for real numbers are transferable. $a^2 > 0$, for instance, loses its meaning for complex numbers.

Meanwhile, others unequivocally denied the validity of the PPF. For instance, Russell famously did so in his 1903 book, *The Principles of Mathematics*: "The principle of the Permanence of Form [...] must be regarded as simply a mistake: other operations than arithmetical addition may have some or all of its formal properties, but operations can easily be suggested which lack some or all of these properties." Russell denied that one can develop symbolic algebra only if guided by the PPF. He rejected both its indispensability and its generality, though as we have suggested above, both Peacock and Hankel had recommended caution with respect to the latter. Somewhat later than Russell, in 1910, Peano also expressed skepticism about the PPF in his *Foundations of Analysis*: "This principle of permanence reached its apogee with Schubert, who, in the *Encyclopädie der mathematischen Wissenschaften*, affirmed that one must 'prove that for numbers in the broad sense, the same theorems hold as for numbers in the narrow sense.' Now, if all the propositions which are valid for the entities of one category are valid also for those of a second, then the two categories are identical. Hence – if this could be proved – the fractional numbers are integers! In the French edition of the *Encyclopédie* these things are put to rights. There it says that one must be 'guided by a concern for keeping the formal laws as much as possible.' Thus the principle of permanence acquires the value of a principle, not of logic, but of practice, and it is of the greatest importance in the selection of notation." While rejecting the PPF, whether with good reason or not, Peano clearly endorsed it as a principle of practical rationality. Indeed, he conceived of the PPF as a thought-economical principle, in Mach's sense: "The principle of permanence [is] a particular case of what Mach called the principle of economy of thought."

Coming back to the debate between Mach and Husserl, let us recall that one of the several theses that Husserl set out to defend in Halle, in 1887, was the following: "Hankel's 'Principle of the permanence of formal laws' in arithmetic is neither a 'metaphysical' nor a '*hodegetische*' [i.e., methodological] principle." We don't know whether he did actually speak about this, or in case he did, what his argument looked like. But we do know his critical remarks in the 1901 *Doppelvortrag* in Göttingen. There he said the following: "We rise, according to the principle of permanence, above the particular domain, pass over into the sphere of the formal, and there can freely operate with [signs like] $\sqrt{-1}$. Now the algorithm of the formal operation is indeed broader than the algorithm of the narrower operations. But if the formal arithmetic is internally consistent, then the broader operating can exhibit no contradiction with the narrower. Therefore what I have formally deduced in such a way that it contains only signs of the narrower domain must also be true for the narrower domain." The view that Husserl ascribed here to mathematicians who adopt the PPF is that consistent extensions of arithmetic can prove arithmetical truths. But he immediately denied that this inference from consistency to truth is justified. For he saw here yet another tension with verificationism, which was made evident by the fact that that formal proofs, i.e., proofs

of formal arithmetic, are "symbolic," "blind," or "mechanical."

As we have seen above, Mach's own verificationism comprised the requirement that there be something real and intuitive corresponding to symbols and concepts. So it seems fair to say that Mach would have agreed with Husserl that the PPF, conceived of as a principle of theoretical rationality, is to be rejected. But Mach never thought of the PPF as a principle of theoretical rationality. Rather, he took it to be a principle of practical rationality. On his view, as expressed in the 1905 book, *Knowledge and Error*, permanence of forms, just like logical consistency, is not indispensable to the development of a scientific theory; both permanence and consistency are merely thought-economical principles: "In the service of life, thoughts adapt to each other and to facts, and if the thinking process has become sufficiently strong, disagreement between thoughts is in itself disturbing, so that one will try to solve the conflict if only to remove intellectual unease ... The mutual adaptation of thoughts is not exhausted in the removal of contradictions: whatever divides attention or burdens the memory by excessive variety, is felt as uncomfortable, even when there are no contradictions left. The mind feels relieved whenever the new and unknown is recognized as a combination of the known, or the seemingly different is revealed as the same, or the number of sufficient leading ideas is reduced and they are arranged according to the principles of permanence and sufficient differentiation." As particular cases of a general principle of permanence, Mach spoke about the permanence of ideas, of relations, as well as of the permanence of basic laws and equations. This was emphasized also by Musil, in 1908, in his doctoral dissertation on Mach: "It is in [...] constant laws and equations [...] that thought seeks to grasp those ideas which can be held on to permanently whatever individual changes may occur."

To conclude, then, in light of what has been said in this brief note, one can maintain with confidence that Husserl was not only "audacious," but also rather confused about Mach's doctrine of thought economy. For whereas the latter defended thought economy as a principle of practical rationality, the former rejected it as a principle of theoretical rationality. This indicates that they really talked past each other in the debate on thought economy. This conclusion is further supported by Mach's and Husserl's correspondingly different interpretations of Hankel's permanence of forms. For to endorse the PPF as a principle of practical rationality, like Mach and Peano did, is to uphold certain goals like convenience of notation and thought economy. To deny its validity and indispensability for the development of a genuinely scientific theory, like Husserl and Russell did, is to reject the PPF as a principle of theoretical rationality.[4]

[4]The author acknowledges support by the PN-III-P4-ID-PCE-2016-0531 project at the University of Bucharest and by the MTA BTK "Morals and Science" Lendület Research Group.