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Erratum to: Free Algebras in Varieties of Glivenko MTL-Algebras Satisfying the Equation $2(x^2) = (2x)^2$

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Thanks to a question from Lluis Godo and Sara Ugolini, we realized that in Theorem 3.5 (ii) of [2] is erroneously stated that for each DL-algebra $\boldsymbol{A}$, the function $\psi_{\boldsymbol{A}} \colon S(\boldsymbol{A}^{\top}, \neg\neg) \to A$ defined for each $\langle x, i\rangle \in S(\boldsymbol{A}^{\top}, \neg\neg)$ by the prescription

$$\psi_{\boldsymbol{A}}(x,i) = \begin{cases} x & \text{if} \ \ i = 1, \\ \neg x & \text{if} \ \ i = 0, \end{cases}$$

is an injective homomorphism from $\boldsymbol{S}(\boldsymbol{A}^{\top}, \neg\neg)$ into $\boldsymbol{A}$. It is surjective (i.e., an isomorphism onto $\boldsymbol{A}$) iff $\boldsymbol{A}$ is directly indecomposable.

As a matter of fact, $\psi_{\boldsymbol{A}}$ is one-to-one and preserves the operations $\sqcup$, $\sqcap$ and $\Rightarrow$, but it fails to preserve $\odot$. Indeed, for $a, b \in A^{+}$,

$$\psi_A(b,1) * \psi_A(\neg\neg a, 0) = b * \neg a,$$
$$\psi_A(\langle b,1\rangle \odot \langle \neg\neg a, 0\rangle) = \psi_A(b \to \neg\neg a, 0) = \neg(b \to \neg\neg a).$$

Hence it is easy to corroborate that $\psi_{\boldsymbol{A}}$ preserves $\odot$, and hence it is an embedding, if and only if the following condition holds:

$$\textit{For any } a, b \in A^{\top}, \ \ \neg(b \to \neg\neg a) = b * \neg a. \tag{1}$$

Observe that $\neg(b \to \neg\neg a) = \neg\neg(b * \neg a)$. Since $\nabla(b * \neg a) = \bot$, (1) will hold if we require that $\nabla(x) = \bot$ implies $\neg\neg x = x$.

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Hence the statement of Theorem 3.5 is correct for DL-algebras $\boldsymbol{A}$ that satisfy the equation:

$$\nabla x \vee (\neg\neg x \to x) = \top. \tag{2}$$

Note that involutive MTL-algebras trivially satisfy (2). Pseudocomplemented MTL-algebras also satisfy (2), because (2) holds in all pseudocomplemented MTL-chains. Therefore all the results of [2] remain true for the subvariety of DL-algebras determined by (2).

The results of [2] concerning pseudocomplemented MTL-algebras and those concerning to involutive MTL-algebras have been extended to a wider class of residuated lattices in [1] and in [3], respectively.

References

[1] Cignoli, R., 'Free algebras in varieties of Stonean residuated lattices', *Soft Comput.* 12 (2008), 315–320.

[2] Cignoli, R., and A. Torrens, 'Free algebras in varieties of Glivenko MTL-algebras satisfying the equation $2(x^2) = (2x)^2$', *Stud. Log.* 83 (2006), 157–181.

[3] Cignoli, R., and A. Torrens, 'Varieties of commutative integral bounded residuated lattices admitting a boolean retraction term', *Stud. Log.* 100 (2012), 1107–1136.

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