Vague Existence

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Debates about what there is are common and often fascinating. If there were a black hole close enough to the solar system, we would have reasons to be worried. Had the Higgs Boson turned out not to exist, that would have meant bad news for the Standard Model of particle physics. Euclid's proof of the existence of infinitely many prime numbers was no small feat—and so on and so forth.

Ontology is the study of what there is, unrestrictedly. Ontologists can argue about the existence of things which are of concern to laypersons (macroscopical objects, values, fictional characters), scientists (fundamental particles, fields) and mathematicians (numbers, sets). In other cases, ontological disagreement will turn instead on more exotic items such as substances, possible worlds or spatiotemporally disconnected wholes.

Can existence, in the unrestricted sense of ontology, be vague? One popular construal of vagueness is defined by the method of *precisifications*. A precisification is a way of making precise all the terms of a language. A sentence is vague when some precisification makes it true and some other makes it false. According to an influential argument due to Sider [25] [26] [28], in ontology there is no such thing as vague existence, as long as vagueness is construed precisificationally.

I aim to show that existential vagueness is a coherent notion, albeit in a weaker form which I will refer to as *super-vague existence*. Section 1 exposes a gap in the alleged *reductio* of vague existence. I will wrap up the section by considering a potential objection. Section 2 develops and

defends a novel framework, dubbed *negative supervaluationary semantics*, which models super-vague existence and its logic. Two further objections will be anticipated at the end of Section 2.

1 Against 'Against vague existence'

1.1 Sider against vague existence

The question whether existence can be vague is relevant to both ontology and metaontology. Let us start with the latter.

One issue about ontological disputes is that it is often hard to identify the source of disagreement. This point is the main target of the recent debate in metaontology, which sees two opposing camps. Metaontological realists regard ontological disputes as genuine and substantive. When philosophers argue in the ontology room, it is claimed, their disagreement turns on different pictures of reality. This position, which took shape in Quine's work on ontological commitment, has been developed by Peter van Inwagen, Kit Fine and Ted Sider among others. The antirealist camp contends instead that ontological disputes are in some sense semantic or verbal. If the latter thesis is correct, the kind of disagreement taking place in the ontology room amounts to a sophisticated version of a dispute on the nature of tomatoes—whether they are fruits or vegetables. The core of this deflationist view, which originated with Carnap, was rejected by Quine and then revived by a group of philosophers including Hilary Putnam, Eli Hirsch, Amie Thomasson and, more recently, David Chalmers.[1]

Some popular forms of deflationism embrace *quantifier variance*, the Putnam-Hirsch view 'that quantifiers can mean different things, that there are multiple candidate meanings for quantifiers' (Sider [29, p. 391]). But if the quantifier used in the ontology room is semantically vague, there will be

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[1]A deflationary framework is also developed in Sider [27] on behalf of neo-Fregeans such as Bob Hale and Crispin Wright.

as many existence meanings as there are admissible precisifications of the quantifier. As a consequence, deflationism would likely be true of at least some ontological disputes. For instance, there may be no determinate fact of the matter as to whether, say, tables exist, provided that the meaning of 'exist' is compatible with multiple and equally good ways of carving the domain of the world at the relevant joints, so that tables will exist on some but not all such existence meanings.[2]

Whether existence could be vague has first-order ontological implications, as well. Restricted composition is the metaphysical view that not all collections of things have a mereological fusion. According to a particular brand of restricted composition advocated by van Inwagen [31], organisms represent the only case in which a collection of objects composes a whole. On that view, tables and chairs do not exist, whereas pluralities of simples arranged table-wise and chair-wise do.[3] Van Inwagen's *mereological organicism* is certainly not the only flavor of restricted composition. On a different way of constraining fusion, a collection of simples composes a whole just in case the members of the collection are topologically connected. Thus, organisms, tables and chairs will exist, whereas table-giraffes will not. This view is *prima facie* more consonant with common sense intuitions about the conditions under which something can be said to exist. At the end of the spectrum we have *mereological nihilism*, which only accepts the existence of simples.

It has been noted that restricted composition leads to vague existence.[4]

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[2] Truth be told, it is in principle possible that there be vague existence without ontological deflationism, namely if for every sortal P, $\exists x P x$ is true (false) on all candidate meanings for $\exists$. For in this case any existence question would have an objective and determinate answer, despite the existential vagueness. Thanks to an anonymous referee for the pointer.

[3] On plural quantification, see Boolos [5].

[4] Or, to be precise, that all interesting ways of constraining the relation of composition lead to vague existence. Some philosophers, however, beg to differ: Carmichael [8] and

Assume that composition is restricted *à la* van Inwagen. On December 7, 43 BC, Marcus Tullius Cicero was on his deathbed. There is a time t_0 at which he was definitely still alive and a time t_1 at which he was definitely dead. However, there was arguably no cut-off point in the series starting with "Cicero is alive" at t_0 and ending with "Cicero is dead" at t_1. For some time t_k between t_0 and t_1 it is vague wether Cicero was dead or alive, therefore it is vague whether the simples arranged Cicero-wise lying on the bed would constitute a whole at time t_k. Since there were times at which it was vague whether Cicero still existed, dying Cicero constitutes a case of vague existence. Or consider the case of viruses. Because it is vague whether a virus is an organism, it is vague whether it constitutes a whole or it is just a collection of simples arranged virus-wise.

Existential vagueness also arises if we think, as most people arguably do, that objects must be connected. For, although the mathematical concept of connectedness is a sharp one, things get tricky when we apply it to physical objects. Suppose we are soldering two pieces of metal m_1 and m_2. What is the minimum threshold of subatomic interaction that must occur between m_1 and m_2 so that the two pieces will count as connected? It does not look as though a unique non-arbitrary answer to that question could be provided. If so, there must be a time at which it is vague whether there exists a whole composed of m_1 and m_2.[5]

Lewis [18, p. 213] objected to restricted composition on the assumption that vague existence is incoherent. This strategy was further developed by

Donnelly [11] have proposed precisificational accounts of composition that do not entail existential vagueness.

[5]Topology in fact distinguishes between multiple notions of connectedness. That fact seems to suggest that, if our concept of object is subject to a connectedness constraint, existential vagueness arises on multiple dimensions: relative to a notion of connectedness and relative to the application of such a notion to the physical realm. (But it might be argued, as well, that the first dimension of vagueness should instead be construed as a case of ambiguity, rather than vagueness.)

Sider [25, pp. 120–32], who appealed to unrestricted composition in his proof of four-dimensionalism. A full-blown attempt at discharging the assumption that existence cannot be vague appears only in Sider [26, pp. 138–43]. The main claim is that

(V) if vagueness is given a precisificational account and existence is expressed by the unrestricted existential quantifier, then existence cannot be vague.

The first conjunct of the antecedent leaves out some theories of vagueness, most notably all degree-theoretic construals (viz., fuzzy logic) but includes the supervaluationism of Fine [13], as well as the epistemicism of Williamson [35] and the semantic nihilism of Braun and Sider [6]. Since there arguably are independent reasons to reject degree-theoretic semantics for vague language,[6] Sider's proof, if sound, would suffice to cover what happens to be the *de facto* standard representation of vague talk.

But what does it mean to precisify a language $\mathcal{L}$? Precisifications are ways of making precise all of the terms in $\mathcal{L}$.[7] Typically, a precisification is interpreted *extensionally*, in the sense that it (i) specifies a domain of quantification and (ii) assigns extensions over the domain to the non-logical constants in $\mathcal{L}$.

According to the precisificational framework, a statement of $\mathcal{L}$ is true (false) if it is true (false) in every precisification of $\mathcal{L}$; it is vague if it is neither true nor false. The precisifications of $\mathcal{L}$ are assumed to be *admissible*—roughly, they must be compatible with the linguistic practice of the competent speakers of $\mathcal{L}$. For instance, a precisification of English in which the extension of 'hirsute' contains people with zero hairs will be inadmissible. Likewise for a precisification in which the extension of 'bald' contains

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[6]It has been shown that degree-theoretic interpretations of vagueness violate classical logic, misrepresent penumbral connections (i.e., logical connections among indefinite sentences) and fail to account for higher-order vagueness. See Williamson [38], Keefe [16].

[7]Varzi [32] considers a number of construals of a precisification. In the present context, however, such distinctions will not matter.

people with ten hairs and the extension of 'hirsute' contains people with nine hairs. The task of determining the admissible precisifications of a language is far from obvious, but for present purpose it will suffice to employ a primitive notion of admissibility.

The second conjunct of the antecedent in (V) serves to rule out restricted quantification, which is obviously open to vagueness. For instance, the statement 'There are over 21 million people' will be vague if uttered by someone referring to the population of the Greater Mexico City, due to the unsharp nature of its urban sprawl. A quantifier is unrestricted iff it ranges over absolutely everything that exists. As a consequence, only the unrestricted quantifier, i.e. the one employed in the ontology room, is relevant to Sider's argument.

The strategy adopted by Sider in proving (V) consists in showing that, if an object-language quantifier $\exists$ is vague and absolute, a contradiction can be derived in its metalanguage. By an application of *reductio ad absurdum*, it is concluded that object-language quantification cannot be vague.[8] The argument against vague existence can be reconstructed as follows. Assume by way of *reductio* that

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[8]In Sider's own words:

> Suppose '$\exists$' has two precisifications, '$\exists_1$' and '$\exists_2$', in virtue of which '$\exists x \phi$' is indeterminate in truth value, despite the fact that ϕ is not vague. '$\exists x \phi$', suppose, comes out true when '$\exists$' means '$\exists_1$', and false when '$\exists$' means '$\exists_2$'. How do '$\exists_1$' and '$\exists_2$' generate these truth values? A natural thought is:
>
> > **Domains** '$\exists_1$' and '$\exists_2$' are associated with different domains; some object in the domain of one satisfies ϕ, whereas no object in the domain of the other satisfies ϕ
>
> But the natural thought is mistaken. If **Domains** is assertible, it must be determinately true. But **Domains** entails that some object satisfies ϕ (if "...some object in the domain of one satisfies ϕ...", then some object satisfies ϕ). And so '$\exists x \phi$' is determinately true, not indeterminate as was supposed. (Sider [28, pp. 557–58])

1. $\ulcorner \exists x \phi \urcorner$ is vague

By the underlying semantics for vagueness, (1) is equivalent to

2. In some precisification $\ulcorner \exists x \phi \urcorner$ is true and in some precisification $\ulcorner \exists x \phi \urcorner$ is false

hence,

3. In some precisification $\ulcorner \exists x \phi \urcorner$ is true

Truth in a precisification is truth in a precise language. So, by the standard extensional truth-conditions for quantified statements in a precise language, (3) is equivalent to

4. There is something such that, in some precisification, it belongs to the domain of $\exists$ and satisfies ϕ

and, *a fortiori*,

5. There is something such that, in some precisification, it satisfies ϕ

Now, ϕ is intended to be precise. For, otherwise, the main premise (1) would not amount to the assumption that the quantifier $\exists$ is vague. But the interpretation of a precise expression coincides across all precisifications. Therefore, (5) entails

6. There is something such that, in every precisification, it satisfies ϕ

Recall that $\exists$ is unrestricted—it ranges over all there is. So, from (6) it can be inferred that

7. There is something such that, in every precisification, it belongs to the domain of $\exists$ and satisfies ϕ

By the truth-conditions for quantified statements in precise languages, (7) entails

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8. In every precisification $\ulcorner \exists x \phi \urcorner$ is true

which, in a precisificational framework, is tantamount to

9. $\ulcorner \exists x \phi \urcorner$ is true

Since a statement is vague just in case it is neither true nor false, (1) and (9) are jointly inconsistent. By *reductio ad absurdum*, we can discharge the main premise (1) and infer its negation, namely

10. $\ulcorner \exists x \phi \urcorner$ is not vague

which concludes the proof.[9]

1.2 Super-vague existence

Before we take a closer look at the merits of the argument, some preliminary remarks are due. As was pointed out, the *reductio* requires that, whereas $\exists x \phi$ is vague, ϕ should be precise. Accordingly, there must exist at least one instance of ϕ meeting such conditions. What language could be such that all of its terms are sharp except at most the quantifiers? Sider [26, 139–40] originally formulated the argument in the vocabulary of mereology, so that the main premise would be

(E) $\exists x (x$ is composed of the F and the G).

But if 'F', 'G' or 'compose' are vague, (E) could be vague without there being any vagueness at the quantificational level. The use of those terms is inessential, however, since (E) can be replaced with a sentence containing only logical vocabulary. Thus, instead of attempting to disprove the vagueness of (E), we could attempt to disprove, at a world containing exactly the two simples F and G, the vagueness of the sentence expressing the existence of a third object:[10]

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[9]See Section 2.4 for an attempt at rephrasing Siders' argument without appealing to *reductio*.

[10]Sider [29, p. 390].

(E') $\exists z(x \neq y \wedge y \neq z \wedge x \neq z)$

where x and y refer to F and G, respectively. This solution faces two challenges. If the quantifiers are truly unrestricted, they will have to range over abstract as well as concrete objects. Since it is necessarily the case that there definitely exist infinitely many *abstracta*, (E') would come out trivially true at the world in question. In order to obtain the desired restriction in the object language, we must introduce a concreteness predicate 'C' and replace (E') with:

(E") $\exists z(C(x) \wedge C(y) \wedge C(z) \wedge x \neq y \wedge y \neq z \wedge x \neq z)$.

But then, as Korman [17, p. 893] pointed out, the source of vagueness in (E") could be 'C' rather than the quantifiers. If so, Sider would have failed to provide a *reductio* of vague existence.

I find this strategy to block the argument unconvincing. Even if the concreteness predicate were vague, its vagueness would be irrelevant to the present argument. For if the vagueness in (E") were due only to the vagueness of 'C', there would have to be an admissible precisification in which the mereological sum of F and G is concrete and one in which it is not. But we can make the reasonable assumption that the sum (if any) of a collection of definitely concrete objects is definitely concrete. Since it was assumed that F and G are concrete simples, F and G will then have to be definitely concrete and therefore their sum (if any) must also be definitely concrete. Hence, relative to the world at which we evaluate (E"), there is no admissible precisification of the language according to which the sum of F and G—the third object in question—fails to be concrete.

A related challenge concerns the semantic status of identity, for there is no point in replacing (E) with (E'), unless identity is precise. This extra assumption could be discharged by piggybacking on the well-known *reductio* of vague identity offered in Evans [12]. Nevertheless, the validity of Evans's argument has been disputed.[11] Be that as it may, for the time being I will

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[11]For a discussion, see Williams [33], Barnes [3], Heck [15], Akiba [1] [2]. Cf. Lewis [19].

concede that all logical vocabulary is precise, with the sole possible exception of quantifiers, and therefore that any *reductio* of the vagueness of (E') amounts indeed to a *reductio* of quantifier vagueness.

A separate and more crucial issue concerns the *reductio* step at the very end of Sider's argument. Indeed, *reductio ad absurdum* is a valid form of inference for bivalent languages: an inconsistent condition cannot be true, hence it must be false. When the language is not bivalent, on the other hand, *reductio ad absurdum* fails, since being non-true is a weaker condition than being false. In particular, *reductio* is invalid in the case of vague languages: if a condition is inconsistent, it must be either false or indeterminate. It follows that Sider's final *reductio* step is valid insofar as the language in which the argument is formulated, viz. the metalanguage of $\exists x \phi$, is *perfectly precise*. Since the main premise (1) is equivalent to its precisificational truth-condition (2), one is vague just in case the other is. Now, (2) does two things: it quantifies over precisifications, and it says of the object-language sentence $\exists x \phi$ that it is true in some but not all of them. By definition, for every precisification s, truth-in-s is determinate; therefore, if there is any vagueness at all in (2), it must come down to what precisifications there are. The moral is that, if quantification over precisifications is precise, so that there is exactly one candidate set of precisifications for the object-language, Sider's *reductio* goes through—if not, not.[12]

Let me flesh out this point a bit further. It is not at all uncommon for a vague term (and, *a fortiori*, a vague language) to be associated with multiple sets of precisifications. Take for instance the color adjective 'blue'. This could refer to the spectrum of visible light with a wavelength between 450 and 490 nanometers—each single value specifying a precisification. But there is nothing about that very interval that makes it the right one. If the set of precisifications of 'blue' can be 450–490nm, then the intervals 455–

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[12]The idea that some assumption in Sider's argument could be indeterminate has also been explored in Barnes [4].

495nm or 445–485nm will work just as fine, as well as many other intervals in that neighborhood. Which is to say, none of them is a better candidate *set* of precisifications for the meaning of 'blue'. We must conclude that the term 'blue' is *second-order vague*, insofar as the extent of its vagueness is itself vague. Could it be the case that second-order vagueness affects the language of $\exists x\phi$? From what we have seen so far, nothing prevents such a possibility. In a scenario of this sort, quantifying over precisifications in Sider's argument would be vague and the *reductio* step, therefore, unwarranted.[13]

Let's take stock. As it should now be clear, Sider has conclusively shown us that

1. $\ulcorner \exists x\phi \urcorner$ is vague

cannot be true, provided that ϕ is precise and $\exists$ is unrestricted. If the metalanguage of $\exists$ is not perfectly precise, however, there is no guarantee that (1) is false and, therefore, that vague existence has been disproved. In particular, even though the quantifier $\exists$ cannot be definitely vague, there is a *prima facie* possibility that it might be second-order vague. In this case, there would be a set of precisifications that makes $\exists x\phi$ vague, and another set that makes $\exists x\phi$ precise. At this point Sider might reply by offering a second *reductio*, this time around of the vagueness—not the truth—of (1). The conjunction of this new argument with the original one would then amount to the desired disproof of vague existence.

The second *reductio* goes as follows. (Note that the object language in the present argument coincides with the metalanguage in the previous one. Accordingly, what 'precisification' means in this argument is not what it used to mean in the previous one. I will use the term 'precisification$_2$' to refer to precisifications of the meta-language of $\exists x\phi$, i.e. second-order precisifications of the language of $\exists x\phi$.) Assume

1* $\ulcorner\ulcorner \exists x\phi \urcorner$ is vague$\urcorner$ is vague

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[13] xxx

Hence,

2* In some precisification$_2$ ⌜⌜∃$x\phi$⌝ is vague⌝ is true, and in some precisification$_2$
⌜⌜∃$x\phi$⌝ is vague⌝ is false

Now, by virtue of Sider's first argument, the first conjunct of (2*) entails

3* In some precisification$_2$ ⌜⊥⌝ is true

On the other hand, precisifications$_2$ being classical, it is the case that

4* In all precisifications$_2$ ⌜⊥⌝ is false

hence a contradiction. If we could now apply *reductio ad absurdum*, we
would be able to infer

5* ⌜⌜∃$x\phi$⌝ is vague⌝ is not vague

thus disproving the second-order vagueness of ∃$x\phi$. But we are back to
square one. For, the *reductio* step is licensed as long as the language in
which the argument is formulated (i.e., the meta-metalanguage of ∃$x\phi$) is
perfectly precise. In particular, quantification over precisifications$_2$ in (2*)
will be vague if the quantifier ∃ is third-order vague. In such a scenario,
there will be a set of sets of precisifications of the language of ∃$x\phi$ according
to which ⌜⌜∃$x\phi$⌝ is vague⌝ is vague, and another set of sets of precisifications
of the language of ∃$x\phi$ according to which ⌜⌜∃$x\phi$⌝ is vague⌝ is sharp.

Sider's argumentative strategy could be iterated at any order—and so the
relevant rejoinder. Consequently, as long as quantifying over precisifications$_n$
is vague at every order n, neither side will get the upper hand, and the pos-
sibility of vague existence will remain neither proved nor disproved. The
dialectics prompts the following moral. A precisificational semantics is a
framework in which truth, falsity and vagueness for a language $\mathcal{L}$ are for-
mulated, within some relevant metalanguage $\mathcal{L}'$, via the notion of *truth-in-
a-precisification-of-$\mathcal{L}$*. Since precisifications are ways of making a language
precise, it cannot be vague whether an $\mathcal{L}$-statement is true or false in a

given precisification. This account is perfectly compatible, however, with there being a vague set of precisifications of $\mathcal{L}$. When *that* happens, the precisificational truth-conditions for $\mathcal{L}$-statements can be vague, insofar as they are formulated by means of $\mathcal{L}'$-statements of the form 'there is a precisification...' Crucially, Sider's *reductio* of vague existence requires that the truth conditions for $\exists x \phi$ be formulated in a perfectly precise metalanguage. (Or at least that a precise metalanguage could be found somewhere up in the hierarchy.) The existence of such a language is neither guaranteed nor required by a precisificational account of vagueness. Therefore, *pace* Sider, if what precisifications there are is vague at all orders, vague existence will remain an open possibility.

The above discussion is complicated by the use of a hierarchy of metalanguages, each expressing whether sentences of the relevant object language are true, false, or indeterminate. Matters can be simplified as follows. Given sentence ϕ in an object language $\mathcal{L}$, in lieu of the metalanguage expression

$\ulcorner \phi \urcorner$ is true

we will use

$\Delta \phi$

where Δ is an object-language sentential operator with the intended meaning 'it is definitely the case that'. Also, let's define the expression $I\phi$ ('it is vague whether ϕ') as $\neg \Delta \phi \wedge \neg \Delta \neg \phi$. By iterating these newly introduced sentential operators, it is now possible to reduce the hierarchy of metalanguage truth/falsity/vagueness predicates to the object language $\mathcal{L}$. For instance, the above condition

1* $\ulcorner \ulcorner \exists x \phi \urcorner$ is vague $\urcorner$ is vague

translates into $\mathcal{L}$ as

$II \exists x \phi$

In general, we can express that ϕ is n-th order vague simply by iterating the I operator n times. Now, let I^n be short for the concatenation $\underbrace{I...I}_{n}$. What the generalized *reductio* has shown is that, for all n, $\ulcorner I^n \exists x \phi \urcorner$ is not true. Which is to say, existence cannot be said to be definitely vague, no matter what the vagueness order is (provided that ϕ is precise). I will henceforth refer to this condition as *Sider-determinacy*.

Let us stipulate that an existence statement $\ulcorner \exists x \phi \urcorner$ is *anti-Sider-determinate* just in case, for all n, $\ulcorner I^n \exists x \phi \urcorner$ is not false. I have argued in the foregoing discussion that precisificational truth conditions need not be given in a precise metalanguage and, in particular, that the set of precisifications can in principle be vague at each order in an infinite hierarchy of metalanguages. Insofar as, for all n, $\ulcorner I^n \exists x \phi \urcorner$ can be vague, it follows that Sider-determinacy is compatible with anti-Sider-determinacy. When an existence statement is both Sider-determinate and anti-Sider-determinate, I will say that it expresses an instance of *super-vague existence*. For example, it might be that it is vague whether there exists the sum of F and G, and it is vague whether it is vague whether there exists the sum of F and G, and it is vague whether it is vague whether there exists the sum of F and G, etc.

In the next Section, I will show how super-vague existence can be accommodated within a precisificational model theory. But before that, I wish to address a potential objection.

1.3 Metalanguage objection

In the above reconstruction of Sider's argument against vague existence, I pointed out that the final *reductio* step requires the metalanguage of $\exists x \phi$ (viz., the language in which the argument is formulated) to be perfectly precise. The same problem cropped up, *mutatis mutandis*, in the higher-order generalization of the argument.

Sider touches on the issue of a precise metalanguage as he describes what it means to give an account of vague statements such as 'S is bald':

> When confronted with vagueness, I retreated to a relatively pre-
> cise background language to describe the relevant facts. *In this*
> *background language I quantified over the various sets* containing
> persons with different numbers of hairs, and said that the refer-
> ent of S was in some but not all of these sets. [...] Moreover,
> *in principle one could describe the sets with perfect precision* by
> retreating to a background language employing only the vocab-
> ulary of fundamental physics. (Sider [26, p. 139], my emphasis.)

It is tempting to interpret the passage as entailing that the precisificational truth-conditions of $\exists x \phi$ can be assumed to be perfectly sharp and, therefore, that Sider's *reductio* of vague existence is valid. Consequently, existence cannot be super-vague, since it is definitely not vague.

The above passage, however, does not license this conclusion. Recall from the discussion in Section 1.2 that one thing is to say that (i) 'true-in-s' is sharp, for s a given precisification; and another thing is to say that (ii) 'true in some precisification' is sharp. Indeed, (i) is true by definition, insofar as each precisification is a classical interpretation described in the metalanguage with perfect precision. But (i) can obtain in the absence of (ii), namely, if it is indeterminate what precisifications there are. In the particular case of 'S is bald', every precisification for 'bald' will be a sharp set of persons. Nevertheless, it could still be indeterminate what set of sets is the extension of the metalanguage term 'precisification'. And since the truth conditions of a vague statement require quantification over precisifications, the language of Sider's argument can be vague, hence the *reductio* is invalid. As I argued in Section 1.2, the assumption that the language be perfectly precise is not part and parcel of a precisificational account of vagueness.

Moreover, the assumption that the precisificational truth-conditions of a vague statement be formulated in a perfectly precise meta-language can lead to paradox. The statement

(*) ⌜Ted is bald⌝ is vague

is tantamount to

(**) in some precisification ⌜Ted is bald⌝ is true, and in some precisification ⌜Ted is bald⌝ is false

If (**) is perfectly precise, as per hypothesis, it must be either definitely true or definitely false. It follows that ⌜Ted is bald⌝ is either definitely vague or definitely not vague. Therefore, although Ted—or anybody else, for that matter—could be a borderline case of baldness, he could not be a borderline borderline case of baldness. Sider's hypothesis that the metalanguage is perfectly precise rules out the possibility of higher-order vagueness. However, there are independent reasons to admit the possibility of higher-order vagueness. If 'bald', for example, is only first-order vague, there will be a clear-cut border between the definitely tall and the not definitely tall, which is no less absurd than there being a clear-cut border between the tall and the not tall. Moreover, the absence of higher-order vagueness can be exploited to generate higher-order sorites paradoxes. I conclude that Sider's assumption is unwarranted and unwelcome

2   Super-vague existence and its logic

What is the logic of a language whose quantifiers are super-vague? This Section attempts to provide a model-theoretic answer to that problem.

2.1   Finean supervaluationism

The most popular precisificational model theory is arguably the supervaluationary semantics put forward in Fine [13]. A *specification space* is a model defined by (i) a set of *specification points*; (ii) a binary admissibility relation defined on the set of specification points; and (iii) a selected specification point called '*base point*'.

Specification points are identified with partial models, each representing a way of making the language more precise. A partial model is defined by

a domain of individuals and an interpretation function assigning to each predicate a positive and a negative extension such that the two extensions are disjoint but do not necessarily exhaust the domain. (Classical models are the degenerate case of partial models in which the positive and negative extension of any predicate are jointly exhaustive.) Since quantification is assumed here to be absolute, a quantifier must range over the whole domain of a specification point.

The admissibility relation is a reflexive, antisymmetric and transitive ordering R such that (1) if uRv and p is true (false) at u, then p is true (false) at v; (2) every specification point bears the ancestral of the admissibility relation to a *complete specification point*, which is a classical model (intuitively, a precisification of the language).

A sentence p is said to be true in a specification space if it is true at the base point. It is said to be true at a specification point if it is true at all accessible specification points. 'Δp' is said to be true at a specification point if p is true at the base point. It follows from the definitions that *a sentence is true at the base point iff it is true at all complete specification points*.

A sentence ϕ is said to be a supervaluationary consequence of a set Γ of sentences if every specification space which makes Γ true makes ϕ true. Sentence ϕ is said to be valid if it is a supervaluationary consequence of the empty set.

Fine's framework immediately yields some desired results concerning vagueness phenomena. Bivalence fails, since it is not the case that any given p is either true or false in a specification space. This is equivalent to the object-language fact that $\Delta p \vee \Delta \neg p$ is invalid, as it should be. On the other hand, $p \vee \neg p$ is valid (and so is any classical tautology) in virtue of the classicality of complete specification points.

The feature of Fine's supervaluationary semantics relevant to the present discussion is that there is no vague existence at any order (provided that

there definitely are finitely many objects). To see that, it suffices to show that (i) existence is definite, i.e., $\exists x(x = y) \to \Delta \exists x(x = y)$, and that (ii) definite statements cannot be indefinitely definite, i.e., $\Delta \phi \to \Delta \Delta \phi$. It follows that Fine's framework, in its basic form, is not suitable for modeling super-vague existence.[14]

2.2   Variable domain frames

I just argued that the Finean supervaluationism presented here is unable to account for higher-order vagueness at the object-language level (cf. Appendix A). Since the failure of higher-order vagueness famously leads to higher-order sorites paradoxes, an alternative precisificational framework has been proposed by Timothy Williamson in which statements can be higher-order vague. In the remainder of Section 2, I aim to show that a suitable generalization of Williamson's semantics admits of models for super-vague existence, i.e., in which both of the following conditions are met:

Sider-determinacy: for all n, it is not definitely the case that existence is n-th order vague

anti-Sider-determinacy: for all n, it is not definitely the case that existence is not n-th order vague

The resulting model-theory will give us an idea of the logic of super-vague existence.

The semantics developed in Williamson [35] [37] is designed for a sentential language with a definiteness operator. The simplest generalization to the first-order case can be defined as follows. A frame $\mathcal{F}$ is a structure

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[14]The proof is provided in Appendix A. (The finiteness condition can be dropped if the language is infinitary.)

$\langle S, U, R \rangle$ where S is a set of points (the specifications), U a set of individuals (the universe of discourse) and R a relation over S (the admissibility relation). Let $\mathcal{L}_\Delta$ be a first-order language with a definiteness operator Δ. A model $\mathcal{M}$ for $\mathcal{L}_\Delta$ is a pair $\langle \mathcal{F}, \sigma \rangle$ where, σ is an interpretation function such that, for every point $s \in S$, (i) $\sigma(=, s)$ is the identity relation over U and (ii) for P an n-ary predicate, $\sigma(P, s) \subseteq U^n$.

The truth condition for an atomic formula at a point given a value assignment for the variables is classical. Truth conditions for connectives and quantified formulas are as expected. Given a value assignment, $\Delta \phi$ is true at s if, for every point t such that sRt, ϕ is true at t. A formula ϕ is true (false) in a model $\mathcal{M}$ under a value assignment if it is true (false) at every point in $\mathcal{M}$. A formula ϕ is a supervaluationary consequence of a set Γ of formulas if, given a model and a value assignment, if Γ is true then ϕ is true. A formula ϕ is valid if it is a supervaluationary consequence of the empty set.

The admissibility relation R is intended to be reflexive and symmetric (so as to validate $\Delta \phi \rightarrow \phi$ and $\Delta \phi \rightarrow \Delta \neg \Delta \neg \phi$) but intransitive. This choice allows for the possibility of higher-order vagueness.[15]

It will be useful to stress a crucial difference between Fine's and Williamson's precisificational frameworks. Recall that being definitely true means being true in all precisifications. On the Finean approach, what counts as a precisification of the language is an *absolute* matter, since precisifications are identified with complete specification points (i.e., the points to which the base point bears the ancestral of the admissibility relation). As a consequence, facts about definiteness and vagueness are absolute, as well—there can be no instances of higher-order vagueness. In Williamson's models, on

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[15]Statements of higher-order vagueness are satisfiable if either $\Delta \phi \rightarrow \Delta \Delta \phi$ or $\neg \Delta \phi \rightarrow \Delta \neg \Delta \phi$ fails. On the model-theoretic side, the failure of either condition corresponds to a non-transitive or non-euclidean admissibility relation: see Williamson [37, p. 133]. In fact, higher-order vagueness has its modal counterpart in the contingency of contingency, which also takes place only in systems weaker than S5.

the other hand, there is no such thing as *the* set of precisifications of the language. For every specification point s there is a set of admissible specification points—intuitively, those precisifications that are in the neighborhood of s. Therefore, an expression of the form 'there is a precisification so-and-so' is always *relativized*. This important feature is meant to capture the idea, discussed in Section 1.2, that the language can have multiple sets of precisifications, and that it can be vague which of those sets is the correct one. Consequently, Williamson's models make room for second-order vagueness. Likewise, distinct sets of sets of specifications can be admissible to distinct sets of specifications, thus making room for third-order vagueness—and so on and so forth. This kind of higher-order vagueness can be put to use in order to model super-vague existence.

Nevertheless, the Williamson-style semantics just sketched does not represent a generalization over Fine's framework in the desired direction, since all specification points in a model have constant domain and, as a result, constrain existence to be definitely not vague at all orders.

I submit that, in order to be able to model super-vague existence, we should proceed as follows. A *variable domain frame* $\mathcal{F}^*$ is a quadruple $\langle S, U, R, Dom \rangle$ such that

(i) specification points S, universe U and admissibility relation R are as before;

(ii) Dom is a function mapping each point $s \in S$ to a subset of U (intuitively, the objects that exist according to the precisification s);

(iii) $U = \bigcup \{Dom(s)\}_{s \in S}$;

(iv) for every $n \geqslant 1$, there is some $s \in S$ which is n-*determinate*.

The notion of n-determinacy is defined recursively as follows:

1. s is 1-determinate iff for all s', s'': if sRs' and sRs'', then $Dom(s') = Dom(s'')$

$n+1$. s is $n+1$-determinate iff for all s', s'': if sRs' and sRs'', then s' is n-determinate iff s'' is n-determinate.

As I will discuss in Section 2.3, the property of n-determinacy is crucial, since it makes models based on a variable domain frame $\mathcal{F}^*$ Sider-determinate. If (iv) were dropped from the definition of a frame, the semantics would admit of unintended models, i.e. models that are not Sider-determinate.

Let us now turn to the problem of finding a suitable notion of *local truth*, i.e., truth-at-a-point. In a model based on $\mathcal{F}^*$, a quantifier evaluated at s will range over $Dom(s)$. The nature of the interpretation function σ will depend on the kind of semantics we choose for evaluating formulas with non-referring terms at a specification point. If x exists at point s, what should be the truth-value of '$P(x)$' at a point t where x does not exist? Three options are on the table for dealing with local truth.

Positive local semantics. We could allow an atomic formula like '$P(x)$' to be true at a point where x does not exist. On this view, for any point t, the interpretation of a predicate P is a subset of the frame domain, namely $\sigma(P, t) \subseteq U$, and the value of a free variable an element of U. Accordingly, '$P(x)$' will be either true or false at t depending on whether x is or is not in $Dom(P, t)$.

However, this approach provides a misleading picture of existential vagueness. For, if the same set of objects can be referred to according to all precisifications of the language, existence will be vague only nominally. Indeed, whereas quantifiers are restricted to $Dom(t)$, which varies with t, free variables will constantly range over U. Hence, there could be a precisification in which mereological nihilism is true, and yet it is also true of the sum of this mug and that table that they occupy space in my office. Positive semantics is therefore inadequate for modeling vague absolute quantification.

Neutral local semantics. A more attractive approach is to (i) let a variable at a specification point pick something out of the relevant domain, or nothing at all, and (ii) restrict the interpretation of a predicate in such a way that $\sigma(P,t) \subseteq Dom(t)$. Neutral semantics takes an atomic formula to be indeterminate at all points where some of its terms fail to refer. The picture should then be completed by defining truth conditions for the non-atomic formulas. As it turns out, neutral semantics is not a viable candidate for local truth, either. One of the most attractive features of supervaluationism is that it preserves classical tautologies. The motivating intuition is that vagueness facts should not affect the logic of truth-functors. That nice result breaks down when local truth is defined via neutral semantics. If x does not refer at a specification point s, $x = x$ is indeterminate there and so is $x \neq x$ (under the assumption that local semantics is truth-functional). Thus, $x = x \vee x \neq x$ is indeterminate at s (since, in neutral semantics, a disjunction is indeterminate when both disjuncts are). Hence, some instance of $p \vee \neg p$ is untrue in some model and, therefore, invalid.

To restore the validity of classical tautologies without giving up neutral local semantics, we could tinker with global truth, i.e. truth-in-a-model. Let then ϕ be true in a model $\mathcal{M}^*$ based on $\mathcal{F}^*$ just in case it is true at all points where all of its terms are defined. Since in the evaluation of $x = x$ we rule out every world where x is non-referring, $x = x \vee x \neq x$ must be valid. Out of the frying pan, into the the fire: now $\exists x(x = y)$ is true in a model iff it is true at all worlds where y is defined. But in neutral semantics, y is defined at point s iff it picks a value in $Dom(s)$ iff y exists at s. So, $\exists x(x = y)$ is trivially true in every model and, therefore, valid. However, $\Delta\exists x(x = y)$ is invalid. To see that, consider a model where y is defined at a point s but not at a point t where sRt. Then $\Delta\exists x(x = y)$ is false at s, since t is an accessible point where $\exists x(x = y)$ is untrue. So, $\Delta\exists x(x = y)$ is invalid. It follows that the so-called necessitation rule

(N) If $\models \phi$ then $\models \Delta\phi$

fails under the proposed revision. Since it is a standard desideratum of any semantics for vague language that validity be closed under definiteness, the above theory should be rejected.

One might reply that (N) can be restored by making one simple change: just let $\Delta\phi$ be true at a point s iff, for every point t such that sRt *and* all terms in ϕ are defined at t, ϕ is true at t. This attempt at validating (N) does more harm than good, however, because now $\Delta\exists x(x = y)$ is true—in fact valid—even when there are precisifications of the language according to which there is no y. Since we are looking for a semantics modeling existential vagueness, I take this to be a *reductio* of neutral local semantics.

Negative local semantics. On the third approach for defining truth-at-a-point, value assignments and the interpretation of the language are exactly as in neutral local semantics: (i) a variable at s picks something out of $Dom(s)$, or nothing at all, and (ii) $\sigma(P,t) \subseteq Dom(t)$. These two conditions codify the reasonable assumptions that reference and predication make sense only relative to what exists according to a given precisification. The essential difference is that negative local semantics takes any atomic formula to be false at a point where some of its terms are non-referring. I claim that a *negative supervaluationary semantics*, i.e. a supervaluationary framework based on variable domains and negative local semantics, yields the correct account of vague existence phenomena. To corroborate my claim, I will first provide an exact formulation of the theory, and then tease out its main semantic and logical features.

2.3   Negative supervaluationary semantics

A *negative supervaluationary* (*NS*) model $\mathcal{M}^*$ for $\mathcal{L}_\Delta$ is a pair $\langle \mathcal{F}^*, \sigma^* \rangle$ where $\mathcal{F}^*$ is a variable domain frame and σ^* an interpretation function such that, for every point $t \in S$, (i) $\sigma^*(=,t)$ is the identity relation over $Dom(t)$ and (ii) for P an n-ary predicate, $\sigma^*(P,t) \subseteq Dom(t)^n$.

Let VAR be the set of variables in $\mathcal{L}_\Delta$ and S the set of specification points. A value assignment for VAR over $\mathcal{M}^*$ is a set of partial functions $\{\xi_t\}_{t \in S}$ such that:

1. $\xi_t : VAR \to Dom(t)$

2. $\bigcup \{\xi_t\}_{t \in S}$ is a total function $f : VAR \to U$

3. if $\xi_s(x)$ and $\xi_t(x)$ are both defined, then $\xi_s(x) = \xi_t(x)$

The first condition allows a variable to be undefined at some precisifications, whereas the second condition forces a variable to be defined at some precisification. Consequently, negative supervaluationary semantics is not a framework for definitely non-referring terms, unlike free logic. The third condition guarantees that variable assignments are rigid across specification points.

Truth-at-a-point s (local truth) for ϕ under a variable assignment $\{\xi_t\}_{t \in S}$ in an NS-model $\mathcal{M}^*$, written $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$, is defined recursively thus:

(at) If $\phi = P(x_1, ..., x_n)$, then $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$ iff ξ_s is defined for all $i \in \{1, ..., n\}$ and $\langle \xi_s(x_1), ..., \xi_s(x_n) \rangle \in \sigma^*(P, s)$

($\neg$) If $\phi = \neg\psi$, then $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$ iff $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \not\models_{NS} \psi$

($\wedge$) If $\phi = (\psi \wedge \chi)$, then $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$ iff both $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \psi$ and $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \chi$

($\forall$) If $\phi = \forall x \psi$, then $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$ iff, for every $\{\xi'_s\}_{s \in S}$ such that ξ'_s is defined on x and differs from ξ_s at most on x, $(\mathcal{M}^*, s, \{\xi'_s\}_{s \in S}) \models_{NS} \psi$

(Δ) If $\phi = \Delta\psi$, then $(\mathcal{M}^*, s, \{\xi_s\}_{s \in S}) \models_{NS} \phi$ iff, for every t such that sRt, $(\mathcal{M}^*, t, \{\xi_s\}_{s \in S}) \models_{NS} \psi$

A few more definitions are needed.

A formula ϕ is true in a NS-model $\mathcal{M}^*$ relative to a variable assignment $\{\xi_t\}_{t\in S}$ (i.e., $(\mathcal{M}^*, \{\xi_t\}_{t\in S}) \models_{NS} \phi$) iff, for every $s \in S$, $(\mathcal{M}^*, s, \{\xi_t\}_{t\in S}) \models_{NS} \phi$.

A formula ϕ is true in a NS-model $\mathcal{M}^*$ (i.e., $\mathcal{M}^* \models_{NS} \phi$) iff, for every $\{\xi_t\}_{t\in S}$, $(\mathcal{M}^*, \{\xi_t\}_{t\in S}) \models_{NS} \phi$.

A formula ϕ is a NS-consequence of a set Γ of formulas (i.e., $\Gamma \models_{NS} \phi$) iff, for every NS-model $\mathcal{M}^*$, if $\mathcal{M}^* \models_{NS} \Gamma$ then $\mathcal{M}^* \models_{NS} \phi$

A formula ϕ is NS-valid iff it is a NS-consequence of the empty set.

Let us now turn to the key properties of negative supervaluationary semantics. Standard precisificational theories of vagueness, such as Fine's specification space semantics or Williamson-style constant domain semantics, have the virtue of being classical in a precise sense: given a purely extensional language, classical consequence ($\models_C$) and supervaluationary consequence ($\models_{SV}$) coincide. Namely, for ϕ and Γ formulated in a first-order language $\mathcal{L}$ without definiteness operator,[16]

(Eq$_1$) $\Gamma \models_C \phi$ iff $\Gamma \models_{SV} \phi$

On the other hand, some classical inference rules, which have been regarded as being the source of sorites paradoxes, fail in standard supervaluationary semantics. Existential instantiation does not hold: from 'some number n is the least number such that n grains of sand constitute a heap' we cannot infer the existence of any particular n_0 such that 'n_0 is the least number such that n_0 grains of sand constitute a heap'. The same applies *mutatis mutandis* to universal generalization.[17]

Notice, however, that the result of substituting NS-logical consequence for $\models_{SV}$ in (Eq$_1$) does not hold, due to the fact that local semantics for

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[16]See Keefe [16, pp. 174–81].

[17]For a discussion and defense of this aspect of supervaluationism, see Keefe [16, pp. 181–88].

NS is non-classical. Here is an intuitive example. (A formal countermodel is provided in Appendix B.1.) We know that 'Ted is not a (mereological) simple' classically entails 'Something is not a (mereological) simple'. The same inference, on the other hand, is not NS-valid. Suppose that it is vague whether mereological nihilism or universalism is true. Now, in any precisification that allows the existence of sums, Ted exists and is not a simple; and in any precisification that does not allow the existence of sums, Ted does not exist and *therefore* (local semantics being negative) is not a simple. Hence, Ted is not a simple. But in any precisification that does not allow the existence of sums, it is not the case that something is not a simple. Hence, 'Something is not a (mereological) simple' is untrue, which shows that classical existential generalization is NS-invalid.

Nevertheless, negative supervaluationary semantics can be shown to validate a weaker version of existential generalization, which typically holds in free logic:

$(\exists \mathrm{G}^-) \quad \{\phi(x), \exists y(x = y)\} \models_{NS} \exists x \phi(x).$

In fact, a general result can be proven connecting negative supervaluationary semantics to *negative free logic*: in a purely extensional language, the consequence relation of negative free logic is preserved by negative supervaluationary semantics. Negative free logic—a first-order logic for languages with non-referring terms—is sound and complete with respect to negative semantics, which is the semantics employed here for defining local truth in NS-models.[18] We can think of a model of negative free logic (NF-model) as the degenerate case of a NS-model with a single specification point. A variable assignment over a NF-model is a partial function from free variables to the model domain.[19] In a first-order extensional language, ϕ is

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[18] For the relation between negative free logic and negative semantics see Nolt [22], Burge [7].

[19] Note, however, that a variable assignment over a NS-model with a single specification point is a singleton $\{\xi\}$, where ξ is a *total* function.

a negative-free consequence of Γ ($\Gamma \models_{NF} \phi$) if, for every NF-model and variable assignment, Γ is true only if ϕ is true.

The aforementioned result connecting negative free logic and negative supervaluationary semantics is as follows. For ϕ and Γ formulated in a language $\mathcal{L}$ without 'Δ',

(Eq$_2$)   if $\Gamma \models_{NF} \phi$ then $\Gamma \models_{NS} \phi$.

(For a proof, see Appendix B.2.) On the other hand, the converse of (Eq$_2$) fails, since in negative free logic but not in NS it can be consistently said of something that it doesn't exist (cf. Appendix B.2).

We can tease out a few interesting facts concerning the interaction of existence and identity. First of all, notice that existence is definable via identity in both negative free logic and negative supervaluationary semantics, because each of the two frameworks validates the biconditional $\exists y(x = y) \leftrightarrow x = x$. Since $\neg\exists y(x = y)$ is NF-satisfiable, so is $x \neq x$. On the other hand, in negative supervaluationary semantics nothing is nonexistent, therefore nothing is self-distinct. That is how things should be.

Moreover, the indiscernibility of non-existents

(IN) $\neg\exists z(x = z) \wedge \neg\exists z(y = z) \rightarrow (\phi(x) \rightarrow \phi(y))$

which is valid in negative free logic[20], fails in negative supervaluationary semantics (see Appendix B.3).

Although the converse of (Eq$_2$) fails, a weaker equivalence can be proven. Namely, negative free logic and negative supervaluationary semantics define the same class of valid formulas in a Δ-free language:

(Eq$_3$) $\models_{NF} \phi$ iff $\models_{NS} \phi$.

(For a proof, see Appendix B.4.)

Given a language $\mathcal{L}_\Delta$, negative supervaluationary semantics satisfies the following conditions:

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[20]Nolt [22, p. 1033].

(Tau) $\models_{NS} \phi$, for every tautology ϕ

(K) $\models_{NS} \Delta(\phi \to \psi) \to (\Delta\phi \to \Delta\psi)$

(MP) $\{\phi, \phi \to \psi\} \models_{NS} \psi$

(N)   If $\models_{NS} \phi$ then $\models_{NS} \Delta\phi$

(T) $\models_{NS} \Delta\phi \to \phi$

(B) $\models_{NS} \phi \to \Delta\neg\Delta\neg\phi$

The two conditions (Tau) and (MP) guarantee the validity of classical sentential logic. By the Kripke schema (K), the 'definitely' operator distributes over the material conditional. (T), expressing the facticity of definiteness, holds because the admissibility relation R is assumed to be reflexive. The symmetry of R validates (B). The so-called necessitation rule (N) ensures that validity is closed under definiteness. In fact, a stronger result than (N) is provable in negative supervaluationary semantics (as well as in most supervaluationary frameworks), viz. the definiteness of truth, or Δ-introduction rule:

(N*) $\phi \models_{NS} \Delta\phi$

I now turn to conditions that fail in the present framework. A non-classical aspect of negative supervaluationary semantics, which it inherits from negative free logic, is that validity is not closed under uniform substitution. For P atomic, for instance, $P(x) \to x = x$ is valid whereas $\neg P(x) \to x = x$ is not.

Since negative supervaluationary semantics admits of variable domain models, and in particular it could be that $Dom(s) \subset Dom(t)$ for sRt, the Barcan formula

(BF) $\neg\Delta\neg\exists x\phi \to \exists x\neg\Delta\neg\phi$

has invalid instances, such as $\neg\Delta\neg\exists x(x = y) \rightarrow \exists x\neg\Delta\neg(x = y)$. Think of Ted, for instance. In some sense, he exists—namely, the sense of existence of mereological universalism. But for the nihilist there is no such thing as Ted. Therefore, it is not the case that there is something which, in some sense, is Ted. The failure of the Barcan formula is, of course, not an idiosyncrasy of negative supervaluationary semantics. When '$\neg\Delta\neg$' is substituted with intensional operators of other sorts, it is not hard to find counterexamples to the schema. In the modal case, the sentence

> If Mary could have had a daughter, somebody could have been
> Mary's daughter

is false, barring exotic semantic frameworks.[21] The same occurs (unless we are eternalists) when the possibility operator is replaced with a tense operator, as in:

> If Mary will have a daughter, there exists somebody who will be
> Mary's daughter.

The Barcan formula also fails in fictional contexts. If the operator 'according to fiction S' is construed as the analog of the necessity operator, we can define its dual, 'according to fiction S it might be that'. Works of fiction being typically incomplete, 'according to fiction S it might be that p' is true just in case the fiction does not entail the falsity of p. Thus, consider

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[21]One such framework, involving the use of possibilist quantification, is defended in Linsky and Zalta [20] [21], Williamson [36]. An alternative semantics which validates the Barcan schema is one in which quantifiers range over individual concepts: see Garson [14].

> If according to *Woyzeck* it might be that the Captain has a
> mistress, then there is somebody who according to *Woyzeck*
> might be the Captain's mistress.

Since, according to Büchner's play, the Captain does in fact have a mistress, the antecedent is obviously true. However, the consequent is absurd, unless we buy into realism about fictional characters.[22]

Insofar as there can be specification points s, t such that $Dom(t) \subset Dom(s)$ for sRt, the Converse Barcan formula

(BF) $\exists x \neg \Delta \neg \phi \to \neg \Delta \neg \exists x \phi$

is also invalid.

Further conditions which are invalid in negative supervaluationary semantics are the definiteness of identity

(DI) $x = y \to \Delta x = y$

and the definiteness of distinctness

(DD) $x \neq y \to \Delta x \neq y$.

(Proofs in Appendix B.5.) However, (DI) can never be false in a model, for if it were, that would contradict the fact that $x = y \models_{NS} \Delta x = y$, which is an instance of (N*). Likewise for (DD).

The invalidity of (DI) and (DD) may bring to mind the analogous case of the failure of the necessity of identity and distinctness in some versions of possible-world semantics. However, the analogy is only superficial and, at bottom, misleading. For, in negative supervaluationary semantics, the reason why (DI) and (DD) are invalid is that a term may refer at some but not all precisifications. On the other hand, the run-of-the-mill counterexamples to the necessity of identity and distinctness are formulated in counterpart

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[22]Cf. Sainsbury [24, p. 34]. An alternative option is to go Meinongian and allow quantification over non-existents.

theory or other possible-world semantics relying on non-standard interpretations of *de re* truth.[23] Moreover, in such modal frameworks there are true instances of contingent identity and distinctness, whereas (DI) and (DD) can at most have untrue instances in negative supervaluationary semantics.

Incidentally, since variable assignments over NS-models are rigid, it follows that identity is weakly definite:

(DI⁻) $x = y \rightarrow \Delta(\exists z \exists z'(x = z \wedge y = z') \rightarrow x = y)$

Distinctness, on the other hand, does not satisfy weak definiteness:

(DD⁻) $x \neq y \rightarrow \Delta(\exists z \exists z'(x = z \wedge y = z') \rightarrow x \neq y)$

(See Appendix B.5.)

The importation schema

(IM) $\exists x \Delta \phi \rightarrow \Delta \exists x \phi$

fails, too. (Proof in Appendix B.6.)

We can finally return to the main point, quantifier vagueness. Our goal is to find some NS-model of super-vague existence. First of all, we need to check that the following condition holds:

Sider-determinacy: for all $\mathcal{M}^*$, $\{\xi_s\}_{s \in S}$ and $n \geqslant 1$: $(\mathcal{M}^*, \{\xi_s\}_{s \in S}) \nvDash_{NS}$ $I^n \exists x(x = y)$

for otherwise negative supervaluationary semantics would be inconsistent with Sider's result, or with the higher-order generalization of it. Second, we would have to show

anti-Sider-determinacy: for some $\mathcal{M}^*$ and $\{\xi_s\}_{s \in S}$ and for all $n \geqslant 1$: $(\mathcal{M}^*, \{\xi_s\}_{s \in S}) \nvDash \neg I^n \exists x(x = y)$

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[23]For example, see Lewis [18].

A proof that both conditions hold in negative supervaluationary semantics can be found in Appendix B.7. We can conclude that super-vague existence is NS-satisfiable.

Let's recap. Sider argued that

(V) if vagueness is given a precisificational account and existence is expressed by the unrestricted existential quantifier, then vague existence is incoherent.

According to Sider [26, p. 4], the moral of his alleged *reductio* is that vague existence 'would be radically unlike familiar cases of vagueness. Vague quantifiers may yet be possible, but such vagueness would require an entirely different model from the usual one'. If I am correct, this moral is correct only in a qualified manner. In Section 1 I argued that, although Sider's (generalized) argument proves the impossibility of n-th order vague existence, for all n, the result is *prima facie* consistent with the possibility of super-vague existence. The suggestion has been vindicated in Section 2, where I have developed a generalization of Williamson-style precisificational semantics where Sider-determinacy is satisfied and yet super-vague existence is shown to be a consistent notion. Consequently, Sider's moral holds as long as vagueness is not super-vagueness, *modulo* the present choice of semantic framework.

2.4   Objection from *reductio ad absurdum*

In Section 1.2, I remarked that Sider's argument against vague existence is invalid unless stated in a perfectly precise language, since the argument is a *reductio ad absurdum*, which is a valid inference form only for bivalent languages.

It might be objected that vagueness in the proof's language doesn't suffice to rule out the applicability of *reductio*. For if that language is vague, it would be reasonable to interpret it within a supervaluationist semantics.

Now, it is true that *reductio* is invalid in supervaluationism—for instance, we cannot infer from $\phi \wedge \neg\Delta\phi \models \bot$ to $\models \phi \rightarrow \Delta\phi$. However, *reductio* is supervaluationarily valid for Δ-free languages. To see that, let $\models_C$ and $\models_{SV}$ denote the relations of consequence for classical logic and standard supervaluationism, respectively. Now, suppose that $\Gamma, \phi \models_{SV} \bot$. By (Eq$_1$), we know that $\models_C$ and $\models_{SV}$ are equivalent (cf. Section 2.3). So, $\Gamma, \phi \models_C \bot$ and, since *reductio* is classically valid, $\Gamma \models_C \neg\phi$. By (Eq$_1$), it follows that $\Gamma \models_{SV} \neg\phi$. But Sider's argument is formulated in the metalanguage of $\exists x\phi$, which paraphrases away Δ (as well as any other expression defined via Δ) by quantifying over precisifications which are extensional, set-theoretic objects. Thus, the language of Sider's proof being Δ-free, *reductio ad absurdum* appears to be valid after all.

What undermines the above objection is the tacit assumption that, if the semantics for the language of Sider's proof is supervaluationary, then it has to be some kind of *standard* supervaluationism, such as Fine's specification space semantics or Williamson-style constant domain semantics. I claim instead that, if Sider's proof is coached in a vague language, we should model it via *negative supervaluationism*. The reason for this choice is quite straightforward. I argued in Section 1.2 that if Sider's language lacks determinacy, this must be due to vague quantification over precisifications. We need, therefore, a precisificational semantics to deal with vague quantification which, as explained in Sections 2.2 and 2.3, should be negative supervaluationism, with the relevant consequence relation $\models_{NS}$. Now, a feature of negative supervaluationism, which distinguishes it from standard supervaluationism, is that it fails to validate *reductio ad absurdum* even for Δ-free languages. For instance, it is a fact that

Ted doesn't exist$\models_{NS} \bot$

since names and free variables cannot be definitely non-referring in a NS-model. However, it doesn't follow that

$\models_{NS}$Ted exists

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for otherwise existence would always be determinate in negative supervaluationism, which we know not to be the case due to the NS-satisfiability of super-vague existence (Appendix B.7).

It is worth noting that, even though we could give a Sider-style argument against vague existence which doesn't employ *reductio ad absurdum*, the new argument would still have to be formulated in a precise language, in order to be valid. For example, we could give a proof by cases:

i. $\ulcorner \exists x\phi \urcorner$ is either vague or not vague.

ii. Suppose $\ulcorner \exists x\phi \urcorner$ is vague. Therefore, it is true at some precisification, and so true at all precisifications, which means that it is true (cf. steps 3-9 in my reconstruction of Sider's argument, Section 1.1). But if $\ulcorner \exists x\phi \urcorner$ is true, then it is not vague.

iii. Suppose $\ulcorner \exists x\phi \urcorner$ is not vague. Therefore, it is not vague.

iv. Thus, $\ulcorner \exists x\phi \urcorner$ is not vague (*proof by cases*)

The argument has the form:

i. $p \vee \neg p$

ii. $p \models q$

iii. $\neg p \models q$

iv. $\models q$

Now, in standard supervaluationism, proof by cases behaves just like *reductio ad absurdum*: although invalid in general, it is valid for arguments stated in a Δ-free language. But again, if the language in which the above proof is stated is vague due to quantification over precisification, the correct framework is negative supervaluationism, with the relevant consequence relation $\models_{NS}$ which does not validate proof by cases even for Δ-free languages, as the following simple counterexample will show:

i. $a = a \vee a \neq a$

ii. $a = a \models_{NS} \exists x(x = a)$

iii. $a \neq a \models_{NS} \exists x(x = a)$

iv. $\models_{NS} \exists x(x = a)$

Indeed, the three premises are true,[24] whereas the conclusion is untrue.

2.5 Inferentialist objection

I have modeled super-vague existence by means of precisified quantifiers. I will now consider an objection, which is an adaptation of an argument originally formulated by Williamson [34], purporting to show that there cannot be multiple precisifications of the existential quantifier.

Given a language $\mathcal{L}$ with vague $\exists$, define a new language $\mathcal{L}'$ in which $\exists$ is replaced with two precisifications $\exists_1$ and $\exists_2$. For instance, $\exists_1$ could be the ontologically sparse quantifier of the mereological nihilist, whereas $\exists_2$ is the promiscuous quantifier of the universalist. (Likewise, in $\mathcal{L}'$ there will be 'composition$_1$' and 'composition$_2$'). Now, let $\phi(x)$ be a $\mathcal{L}'$ formula. From $\exists_1 x \phi(x)$ we can deduce $\phi(z)$ by existential$_1$ instantiation, where z is chosen so that it does not occur free in $\phi(x)$. By existential$_2$ generalization, $\exists_2 x \phi(x)$ follows from $\phi(z)$. Hence, there exists a deduction of $\exists_2 x \phi(x)$ from $\exists_1 x \phi(x)$. Since we can produce the same kind of argument running in the opposite direction, the two quantifiers are equivalent, which contradicts the initial assumption that $\exists_1$ and $\exists_2$ are *distinct* precisifications.

To this argument I offer a two-tiered reply. For reasons that will soon become clear, I take the second part of my reply to be the more enlightening one. Firstly, the objection assumes that the precisified quantifiers are classical, in the sense that for each $\exists_n$, the rules of generalization$_n$ and instantiation$_n$ are the classical ones. But this assumption is unwarranted.

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[24] $a \neq a$ is an NS-inconsistency, hence $\exists x(x = a)$ follows from it trivially.

Recall that a precisification of the language is identified with a particular specification point in a model of negative supervaluationary semantics. Local truth, i.e. truth at a specification point, is defined in terms of negative semantics. Moreover, negative free logic is sound and complete with respect to negative semantics. Therefore, the generalization and instantiation rules for a precisified quantifier are the ones of negative free logic:

- from $\phi(z) \wedge \exists_n x(x = z)$ infer $\exists_n x \phi(x)$

- from Γ and $\exists_n x \phi(x)$ infer $\phi(z) \wedge \exists_n x(x = z)$, where z does not occur free in Γ or $\phi(x)$.

With that being said, it is easy to see what goes wrong in the inferentialist objection. Recall that $\exists_1$ is the less promiscuous quantifier, which does not support composition$_1$, whereas $\exists_2$ allows unrestricted composition$_2$. Let $\phi(x)$ be the $\mathcal{L}'$ formula 'x is a mereological compound$_2$', where 'compound$_2$' is the universalist precisification of the vague $\mathcal{L}$-term 'compound'. Since the underlying logic is free, from 'there exists$_2$ the fusion$_2$ of a table and a giraffe' by existential$_2$ instantiation we can infer 'z is the fusion$_2$ of a table and a giraffe and z exists$_2$'. But in order to conclude by existential$_1$ generalization 'there exists$_1$ the fusion$_2$ of a table and a giraffe', we first need to be able to infer 'z is the fusion$_2$ of a table and a giraffe and z exists$_1$'. So, the derivation goes through if 'z exists$_2$' entails 'z exists$_1$', which cannot be assumed without begging the question.[25]

I now turn to the second reply. The inferentialist objection simply assumes that it is possible to define a new language $\mathcal{L}'$ in which the vague quantifier $\exists$ is replaced with the sharp quantifiers $\exists_1$ and $\exists_2$. As it turns out, multiple quantifiers obeying the rules of free logic cannot coexist in the same language. For if there were such $\exists_1$ and $\exists_2$, a new existential quantifier $\hat{\exists}$ could be defined in $\mathcal{L}'$ whose range is the union of the ranges of $\exists_1$ and $\exists_2$:

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[25]Cf. Turner [30, pp. 25–26].

$$\hat{\exists}x(x=z) := \exists_1 x(x=z) \vee \exists_2 x(x=z).$$

Now, if $\exists_1$ is a proper restriction of $\hat{\exists}$, for some z it is true that $\hat{\exists}x(x=z) \wedge \neg\exists_1 x(x=z)$. By definition of $\hat{\exists}$, that is equivalent to

(i) $\quad \neg\exists_1 x(x=z) \wedge \exists_2 x(x=z).$

Since $\exists_1 x(x=z) \leftrightarrow z=z$ is a theorem of negative free logic, from the first conjunct of (i) it follows that

(ii) $\quad z \neq z.$

From (ii) and the second conjunct of (i) we can infer

(iii) $\quad \exists_2 x(x \neq x).$

But

(iv) $\quad \forall_2 x(x=x)$

is a theorem of negative free logic. Hence, the claim (i) that $\exists_1$ is a restriction of $\hat{\exists}$ is inconsistent, provided that the precisified quantifiers obey negative free logic. But according to negative supervaluationary semantics, precisified quantifiers do obey negative free logic. We must conclude that it not possible to define $\hat{\exists}$ and, therefore, that we cannot use both $\exists_1$ and $\exists_2$ within the same language, as the inferentialist objection presupposes.[26]

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[26]It could be objected that a logical constant satisfying some given condition in a language $\mathcal{L}$ need not do so in an expanded language $\mathcal{L}'$. For instance, identity satisfies Leibniz's law if the language is extensional, but not if we add doxastic or epistemic operators. Likewise, the various $\exists_n$ might not satisfy the same inference rules in each precisification $\mathcal{L}_n$ of $\mathcal{L}$ and in the expanded $\mathcal{L}'$. This observation overlooks one important bit of information, namely that the quantifiers $\exists_n$ are all precisifications of the original quantifier $\exists$. I assume the following principle: if c' is a precisification of the the logical constant c, then c' satisfies *at least* the axioms and rules of inference which c satisfies. In fact, I take that condition to partly define what it means for c' to be a precisification of c. It follows in particular that $\exists_1$ and $\exists_2$ must satisfy the axioms and rules of negative free logic.

The moral is that quantifiers behave differently from non-logical predicates in one key respect. If we speak a language where 'bald' is vague, we can define a new language in which the original predicate is replaced with a multiplicity of precise predicates 'bald$_1$', 'bald$_2$' etc. This cannot be done with quantifiers on pain of inconsistency. I hope it is now clear that the deeper reason why the inferentialist objection is unsound is that it assumes that quantifiers governed by negative free logic can coexist in a single language.

3 Conclusion

Whether existence can be vague has consequences both in first-order ontology and in metaontology. In the former case, the possibility of vague existence makes room for vague composition. In the latter, vague existence may be a symptom of the world lacking a unique quantificational structure. Sider has famously submitted a *reductio* of vague existence, on the assumption that vagueness is interpreted precisificationally and existence is absolute.

In Section 1, I argued that a precisificational framework per se does not allow us to disprove vague existence, i.e., to prove that it is definitely not vague. At most it can be proven that existence is not definitely vague. The same applies to a disproof of higher-order vague existence. The upshot of the discussion turned out to be that Sider's argument is compatible with existence being neither definitely vague nor definitely precise, at every order. I named this specific phenomenon *super-vague existence*. In Section 2, I provided a precisificational model theory, dubbed *negative supervaluationary semantics*, with the aim of modeling super-vague existence and its logic. Moreover, an objection from *reductio ad absurdum* and an inferentialist objection have been taken care of.

If existence is super-vague, we ought to accept that composition might be super-vague (i.e., vague at all orders) and that the world may lack a

unique quantificational structure.

Appendices

A   Finean supervaluationism and existence

In Fine's supervaluationary semantics it can be proved that, as long as there definitely are finitely many objects, there is no vague existence at any order (cf. Section 2.1).

For the proof, it suffices to show that (i) existence is definite, i.e., $\exists x(x = y) \to \Delta \exists x(x = y)$, and that (ii) definite statements cannot be indefinitely definite, i.e., $\Delta \phi \to \Delta \Delta \phi$. Let n be the cardinality of the domain of the largest complete specification point in a space. (If the cardinalities had no upper bound, it would not be the case that there definitely are finitely many objects.) Notice that the domain of the base point @ is a subset of the cardinality of any complete specification point, since if $\exists y(x = y)$ is true at @ (given an assignment for x), then it is true at every accessible point. So, there is some $m \leqslant n$ which corresponds to the cardinality of the domain of @. Let $\forall y(y = x_1 \vee y = x_2 \vee ... \vee y = x_m)$ express which things exactly exist at @. By the construction of a specification space, that sentence must be true at all complete specification points, which must therefore have constant domain. So, $\exists y(x = y)$ is true at a complete specification point s only if it is

true at all complete specification points. Therefore, at every s it is true that $\exists x(x = y) \rightarrow \Delta\exists x(x = y)$. We can conclude that existence is not vague. Moreover, Fine's theory does not admit of cases of higher-order vagueness to be expressed in the object-language: what is true/false/indeterminate is definitely true/false/indeterminate. Which is to say, both (a) $\Delta\phi \rightarrow \Delta\Delta\phi$ and (b) $\neg\Delta\phi \rightarrow \Delta\neg\Delta\phi$ are supervaluationarily valid in Fine's model. As to (a), suppose that $\Delta\phi$ is true at a complete specification point s. So, ϕ is true at all complete specification points. It trivially follows that it is true at s that every complete specification point is such that ϕ is true at all complete specification points. Which is to say, $\Delta\Delta\phi$ is true at s. *Mutatis mutandis* for (b). As a consequence, since existence is precise, it must be precise at all orders. Q.E.D.

B   Elements of negative supervaluationary semantics

B.1 NS and existential generalization

We want to show that, for ϕ and Γ formulated in a first-order language $\mathcal{L}$ without Δ, it is not the case that

$$\Gamma \models_C \phi \text{ iff } \Gamma \models_{NS} \phi$$

where $\models_C$ ($\models_{NS}$) indicates the classical (NS-) consequence relation. To see that, let $\mathcal{M}^*$ be a NS-model with only two specification points s and t, where $Dom(s) = \{a\}$ and $Dom(t) = \{a, b\}$. Also, suppose that $\sigma^*(P, s) = \sigma^*(P, t) = \{a\}$. Consider a value assignment mapping x to b at t and leaving it undefined at s. Since $P(x)$ is false at both s and t, $\neg P(x)$ is true in the model. However, $\exists x \neg P(x)$ is true at s and false at t, therefore indeterminate in the model. Thus, existential generalization does not hold in general: $\phi(x) \nvDash_{NS} \exists x \phi(x)$. The same reasoning applies *mutatis mutandis* to universal instantiation.

B.2 NS and negative free logic I

For ϕ and Γ formulated in a language $\mathcal{L}$ without 'Δ',

(Eq$_2$)   if $\Gamma \models_{NF} \phi$ then $\Gamma \models_{NS} \phi$.

To see that, let Γ be true in the NS-model $\mathcal{M}^*$ given a variable assignment $\{\xi_t\}_{t \in S}$. Then, for every ξ_t, Γ is locally true at t. Since local truth for a Δ-free language in a NS-model is tantamount to truth in a NF-model, and since ϕ is an NF-consequence of Γ, it follows that ϕ is locally true at t under ξ_t. Hence, ϕ is true at $\mathcal{M}^*$ under $\{\xi_t\}_{t \in S}$.

However, the converse does not hold. Because a variable assignment over a NS-model maps each variable to an object at some specification point, $\neg \exists y (x = y)$ can only be false or indeterminate in a model. Hence, it must be that $\neg \exists y (x = y) \models_{NS} \bot$. On the other hand, $\neg \exists y (x = y)$ is NF-satisfiable.

B.3 NS and the indiscernibility of non-existents

The indiscernibility of non-existents

(IN) $\neg \exists z (x = z) \wedge \neg \exists z (y = z) \rightarrow (\phi(x) \rightarrow \phi(y))$

fails in negative supervaluationary semantics. In order to see that, consider a NF-model with two specification points s, t such that tRs and $\sigma^*(P, s) = \{b\}$. Now, assume that $\xi_s(x) = a$, $\xi_s(y) = b$, $a \neq b$ whereas both $\xi_t(x)$ and $\xi_t(y)$ are undefined. Then, the instance of (IN) obtained by substituting $\Delta \neg P$ for ϕ is false at t under $\{\xi_s\}_{s \in S}$ and therefore untrue in the model.

B.4 NS and negative free logic II

Negative free logic and negative supervaluationary semantics define the same class of valid formulas in a language without 'Δ':

(Eq$_3$) $\models_{NF} \phi$ iff $\models_{NS} \phi$.

The left-to-right direction is an immediate consequence of (Eq$_2$). As to the converse, consider a NF-model and a partial function ζ mapping the free variables of ϕ to the domain. Truth in that model is tantamount to local truth at a specification point t of some NS-model $\mathcal{M}^*$ under $\xi_t = \zeta$. Since ϕ is NS-valid, it is true at $\mathcal{M}^*$ under $\{\xi_s\}_{s \in S}$ and therefore locally true at t under ξ_t. Consequently, ϕ is true in the original NF-model under the variable assignment ζ.

B.5 NS and identity

A condition which fails in negative supervaluationary semantics is the definiteness of identity

(DI) $x = y \rightarrow \Delta x = y$

To see that, just consider a NS-model and a variable assignment in which x and y co-refer to a at point s, whereas t is a point such that sRt and $a \notin Dom(t)$. Since $x = y$ is true at s and false at t, (DI) is false at s and therefore untrue in the model. A symmetrical scenario yields a counterexample to the definiteness of distinctness

(DD) $x \neq y \rightarrow \Delta x \neq y.$

However, (DI) can never be false in a model, for if it were, that would contradict the fact that $x = y \models_{NS} \Delta x = y$, which is an instance of (N*). Likewise for (DD).

Since variable assignments over NS-models are rigid, it follows that identity is weakly definite:

(DI$^-$) $x = y \rightarrow \Delta(\exists z \exists z'(x = z \wedge y = z') \rightarrow x = y)$

Distinctness, on the other hand, does not satisfy weak definiteness:

(DD$^-$) $x \neq y \rightarrow \Delta(\exists z \exists z'(x = z \wedge y = z') \rightarrow x \neq y)$

For suppose a variable assignment maps x and y to the same object a at point t, and let sRt, where $a \notin Dom(s)$. Since the assignment is rigid, x and y will fail to refer at s, and so the antecedent of (DD$^-$) must be true at that point. The consequent, on the other hand, is false at s.

B.6 NS and the importation schema (IM)

The importation schema

(IM) $\exists x \Delta \phi \rightarrow \Delta \exists x \phi$

is not NS-valid. To construct a counterexample, let $\mathcal{M}^*$ be a NS-model with specification points s and t where sRt, $Dom(s) = \{a\}$ and $Dom(t) = \{b\}$. Suppose that $\sigma^*(P, s) = \emptyset$ and $\sigma^*(P, t) = \{b\}$. Then, it is true at s that $\exists x \Delta \neg P(x)$, since $\neg P(x)$ is true at s under $\xi_s = \{\langle x, a \rangle\}$ and true at t under $\xi_t = \emptyset$. But $\exists x \neg P(x)$ is false at t and therefore $\Delta \exists x \neg P(x)$ is false at s. So, $\exists x \Delta \neg P(x) \rightarrow \Delta \exists x \neg P(x)$ is untrue in $\mathcal{M}^*$.

B.7 NS and super-vague existence

In order to show that negative supervaluationary semantics can model super-vague existence, it suffices to show that the following conditions hold:

Sider-determinacy: for all $\mathcal{M}^*$, $\{\xi_s\}_{s \in S}$ and $n \geqslant 1$: $(\mathcal{M}^*, \{\xi_s\}_{s \in S}) \nVDash_{NS}$
$I^n \exists x (x = y)$

anti-Sider-determinacy: for some $\mathcal{M}^*$ and $\{\xi_s\}_{s \in S}$ and for all $n \geqslant 1$:
$(\mathcal{M}^*, \{\xi_s\}_{s \in S}) \nVDash_{NS} \neg I^n \exists x (x = y)$

The first part, Sider-determinacy, follows immediately from the fact that every variable domain frame $\mathcal{F}^*$ of a NS-model is n-determinate, for every $n \geqslant 1$ (cf. Section 2.2).

An NS-model satisfying anti-Sider-determinacy can be constructed as follows. Let $\mathcal{M}^* = \langle S, U, R, Dom, \sigma \rangle$ be a model for a $\mathcal{L}_\Delta$ language without non-logical constants, where

- $S = \{s_0, s_1, s_2, ..., s_n, ...\}$ (the specification points)

- $U = \{a, b\}$ (the individuals)

- R is the reflexive and symmetric closure of $\{\langle s_n, s_{n+1} \rangle\}_{n \geqslant 0}$

- $Dom(s_0) = \{a, b\}$; $Dom(s_{n+1}) = \{a\}$

- $\sigma(=, s_n)$ is the identity relation over s_n

Now, let $\{\xi_s\}_{s \in S}$ be an assignment over $\mathcal{M}^*$ such that $\xi_{s_0}(y) = b$. It is easy to show that, for all $n \geqslant 1$, $(\mathcal{M}^*, s_n, \{\xi_s\}_{s \in S}) \models_{NS} I^n \exists x (x = y)$. Hence, $(\mathcal{M}^*, \{\xi_s\}_{s \in S}) \nvDash_{NS} \neg I^n \exists x (x = y)$, for all $n \geqslant 1$.

We can conclude that super-vague existence is possible in negative supervaluationary semantics.[27]

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