Compositionality and Modest Inferentialism

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RESUMEN

Este artículo presenta tanto una solución como un problema para la explicación de la composicionalidad en el inferencialismo modesto de Peacocke. El problema inmediato al que se enfrenta la explicación de Peacocke es que hace aparecer la composicionalidad como si sólo pudiera ser entendida al nivel de la semántica, algo que es difícil de reconciliar con el inferencialismo. Aquí, siguiendo una breve sugerencia hecha por Peacocke, presento un armazón formal dentro del cual al nivel de la relación determinante entre inferencia y semántica. Esto proporciona a su vez un "test" para la composicionalidad que, problemáticamente, el armazón de Peacocke de la deducción natural para la lógica clásica no puede superar. Para terminar, bosquejo brevemente un armazón alternativo bilateralista para el inferencialismo modesto para el que vale la composicionalidad.

PALABRAS CLAVE: *composicionalidad; inferencialismo; Peacocke; bilateralismo*.

ABSTRACT

This paper provides both a solution and a problem for the account of compositionality in Christopher Peacocke's modest inferentialism. The immediate issue facing Peacocke's account is that it looks as if compositionality can only be understood at the level of semantics, which is difficult to reconcile with inferentialism. Here, following up a brief suggestion by Peacocke, I provide a formal framework wherein compositionality occurs at the level of the determining relation between inference and semantics. This, in turn provides a "test" for compositionality, which, problematically, Peacocke's natural deduction framework for classical logic cannot meet. To finish, I briefly outline an alternative, bilateralist, framework for modest inferentialism, for which compositionality holds.

KEYWORDS: *Compositionality; Inferentialism; Peacocke; Bilateralism*.

INTRODUCTION

On Peacocke's modest inferentialism, a specified set of natural deduction rules are taken to determine the truth-conditional content of logical constants, where those rules have a substantive connection with ordinary inferential

practices. On Peacocke's [(1987); (1992)] view, this is put in terms of "reading-off" valuational semantics from inference rules that a thinker finds "primitively compelling". An inference is primitively compelling if a thinker finds it compelling, it is underived from other principles, and its correctness is not answerable to anything else [Peacocke (1992)]. Thus, in order for a thinker to have a concept, there are a substantial set of constraints, or acceptance conditions, that a thinker must meet, which also determine the semantic content expressed (ensuring that those inferences are valid) [Peacocke (1986)]. These are a normative set of conditions that express what an agent becomes rationally committed to when they judge a content.[1]

A problem, leveled at modest inferentialism, perhaps most clearly in [Fodor (1991); (1998)], is that it fails to conform to certain intuitively correct constraints upon the compositionality of language. In brief, the claim there is that compositionality requires that the content of complex concepts is fully derivable from the content of their constituent concepts. But, if this is to be built up out of sets of inferences that are encoded within a concept's acceptance conditions, then the complex content may not always be fully derivable from those primitive conditions. In response, Peacocke [(2000)] suggests that an acceptable account of compositionality need only provide a principled means by which the relevant semantics of a complex content can be determined by the relevant semantics of its constituents. Then, the semantic value of a complex content will follow from the determination of the semantic values (by acceptance conditions) of its constituents. For example, Peacocke [(2004)] provides the following definition of compositionality:

> For something to be the complex concept $A \wedge B$ is for there to be some operation $\mathbb{R}$ on semantic values such that the fundamental condition for an entity to be the semantic value $A \wedge B$ is for it to stand in relation $\mathbb{R}$ to the semantic values of the concepts A and B respectively [p. 91].

However, this response is both unclear, and potentially worrying from the point of view of modest inferentialism. First, the determination principle, and the relation $\mathbb{R}$, are never made formally precise, or generalisable, in Peacocke's work (see for example, [Peacocke (1992)]). Second, the response has the air of "giving the game away" from the point of view of modest inferentialism since compositionality is accounted for only at the level of semantic value. Given that part of the motivation for adopting modest inferentialism is that we can distinguish between, for example $\neg(\neg A \vee \neg B)$ and $A \wedge B$, this appears far from satisfactory. Thus, allowing for compositionality only at the level of semantic value is a significant loss for modest inferentialism.

Fortunately, there is a fairly obvious salve, which Peacocke [(2005)] hints at:

We must distinguish sharply. There are

> (a) relations between semantic values, which […] are in general thought-independent things in the world;

> and there are

> (b) relations between the condition for something to be the semantic value of the complex concept and the conditions for things to be the semantic values of its atomic constituents.

> It is the relation between conditions, (b), that I am appealing to in explaining conceptual structure, not the relations (a) between semantic values themselves. […] In the case of a complex concept, as in the case of a simple concept, the concept can be individuated by the fundamental condition for something to be its semantic value [p. 173].

It is this promissory note that I follow up here. In §I, I provide a formal framework for understanding the determination of valuation semantics by an inferentially defined logic. In §II, this is developed providing a definition of Peacocke's relation $\mathbb{R}$ as the "preservation of *V*-validity" over a set of valuations consistent with a rule. I use this to construct a simple "test" for compositionality, which in §III I use to analyse Peacocke's preferred classical natural deduction rules. There, it is shown that both $\lor$ $\neg$ and fail to meet this test. Resultantly, Peacocke's account fails to be compositional. In §IV, I briefly suggest that a more liberal account of inferentialism, bilateralism, can meet this challenge.

I Logic, Semantics and Rules

This section outlines a formal framework for Peacocke's modest inferentialism.[2] First, I define a very general account of a logic *L* (§I.1), which, embellished with a set of rules (§I.2), constrains *L* in a way that is compatible with Peacocke's account. §I.3 gives a generalised account of valuational semantics, before in §I.4, providing a determination theory between the two. Completeness over this determination is outlined in §I.5 in terms of the concept of absoluteness developed in [Hardegree (2005)].

I.1 *Logic*

Definition 1. A logic *L* is an ordered pair, $\langle WFF, \vdash_L \rangle$, where *WFF* is the set of well-formed formulas in a language $\mathcal{L}$ (containing an enumerable set of sentences), and $\vdash_L$ is a relation between a subset of formula and a formula $\vdash \subseteq \wp(WFF) \times WFF$.[3]

I will assume throughout that $\vdash$ is reflexive, transitive and monotonic. I also assume that $\vdash$ is finitary, where, for all $\Gamma, \alpha \subseteq WFF$, if $\Gamma \vdash \alpha$, then there is a finite $\Gamma' \subseteq \Gamma$ and $\alpha' \subseteq \alpha$ where $\Gamma' \vdash \alpha'$. Then, given that some form of conjunction exists in $\mathcal{L}$, we have the equivalence: $A_1, ..., A_n \vdash B$ iff $A_1 \wedge ... \wedge A_n \vdash B$.

In its most basic form, then, a logic is simply a pre-order over sets of formulas.[4] This allows the characterisation of a sequent in L as an ordered pair $\langle \Gamma, \alpha \rangle$ where Γ is a set of formulae and α a single formula (where $\Gamma \cup \{\alpha\}$).

I.2 *Rules*

An *n*-premise rule in L is simply an $n + 1$-ary relation on the sequents of L. Where a rule is associated with a connective #, the set of *WFF*'s will be closed under the operation $\#(A_1, ..., A_n)$, such that, when all $(A_1, ..., A_{n+1}) \in \#$, (and $\{A_1, ..., A_n\} \subseteq WFF$), $A_{n+1} \in WFF$. So, for example, typical natural deduction rules are simply a set of ordered pairs consisting of a set of premise sequents and a conclusion sequent (hence, we call this a SET-FMLA framework). Take natural deduction conjunction (written in sequent style):

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \; \wedge\text{-I} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \; \wedge\text{-Ea} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \; \wedge\text{-Eb}$$

The rule, $R^{\wedge}$, is meant to make precise certain acceptance conditions that constrain agent's commitments [Peacocke (1986), p.185ff], in terms of the way in which they are ordered and combined over arguments (the unit of which, to follow Peacocke, I take to be a proposition). For example, an agent accepting that $A, A \vdash A \wedge B$, may be said to be rationally committed to not simultaneously accepting A, B and rejecting $A \wedge B$. Note that this way of putting things is deliberately inequivalent to saying that the agent accepting A, B is thereby rationally committed to accepting $A \wedge B$. This is because, whilst acceptance conditions play a key role in the fixation of beliefs, they neither commit an agent to logical omniscience, nor do they rationally oblige an agent to accept $A \wedge B$ where, for example $A \wedge B$ is unreasonable against the backdrop of the beliefs that agent antecedently holds. These commitments are made perspicuous through the development of proof-systems. So, $R^{\wedge}$, on this view, constrains an agent's commitments by saying that it is rationally prohibitive to reject $A \wedge B$, given acceptance of A, B.

Closure under rules therefore puts constraints upon the pre-order, L, defined above. Call a logic L that is constrained in this way, proof-theoretically defined, when, for each $\Gamma \cup \alpha \in WFF, \Gamma \vdash \alpha$ iff α is provable by the successive application of the rules in L. Then L distributes *WFF* on $\mathcal{L}$ in terms of the collection of sequents that are valid according to L.

Definition 2. Any sequent $\langle \Gamma, \alpha \rangle \in L$ is L-valid, so $\Gamma \vDash_L \alpha$.[5]

I.3 *Semantics*

Definition 3. A semantic structure S is an ordered pair $\langle WFF, V \rangle$, where *WFF* is as above, and V a valuation space where $V \subseteq U$, (U is the set of all possible valuations on $\mathcal{L}$). $\mathcal{V}$ is a set of truth-values, and $\mathcal{D} \subseteq \mathcal{V}$ designated values.[6] A valuation $\mathcal{V}$ is a function on $\mathcal{L}$ assigning a truth-value $\in \mathcal{V}$ to a *WFF* where $\mathcal{V}$: *WFF* $\rightarrow \{\mathcal{V}\}$.

Typically, when we approach logic by way of semantics, we expect V to be induced by the truth-conditional interpretation of each connective defined in L. For example, we might define V_{CPL} as containing those valuations $\mathcal{V}$: WFF $\rightarrow \{1,0\}$ that obey the truth-conditional clauses for the connectives. In many cases, this is equivalent to taking those valuations recursively induced by the defined truth-functions for the connectives.

Definition 4. An n-ary truth-function f^n is any function from $\{1,0\}^n$ to $\{0,1\}$. Call an n-ary connective # of L truth-functional *w.r.t* V if there exists a function f such that, for each $\mathcal{V} \in V$ and for all $\alpha_1 ... \alpha_n \in WFF$, $\mathcal{V}(\#(\alpha_1 ... \alpha_n)) = f^n$ $(\mathcal{V}(\alpha_1)... \mathcal{V}(\alpha_n))$.[8]

I.4 *Determination Theory*

What we want, from the point of view of modest inferentialism, is not to let V be built-up from truth-conditional clauses on connectives, but rather to let V be "carved out" by the set of L-valid sequents defined proof-theoretically. For this, we need a kind of "determination theory" invoked in [Peacocke (1986)], with the general requirement that:

GENERAL REQUIREMENT: The given rules of inference, together with an account of how the contribution to truth-conditions made by a logical constant is determined from those rules of inference, fixes the correct contribution to the truth-conditions of sentences containing the constant [Peacocke (1993), p. 172].

The general idea is to understand a valuation as providing a counterexample, or not, to the potential validity of a sequent in terms of whether or not truth is preserved when passing from *l.h.s* formulae to right. Then provable sequents determine a set of valuations from the universe of possible valuations over a language $\mathcal{L}$.

Definition 5. Where $\mathcal{V} = \{0, 1\}$ and $\mathcal{D} \subseteq \mathcal{V} =, \{1\}$, a sequent $\langle \Gamma, \alpha \rangle$ (where $\Gamma \cup \{\alpha\} \in WFF$), is refuted by a valuation ν iff, when $\nu(\beta)$ for each $\beta \in \Gamma$, $\nu(\alpha)$ Otherwise ν satisfies the sequent (transitivity ensures extensibility to each *WFF*).

The idea is to construct a valuation-space $V \subseteq U$ from a set of rules defined in a logic, where V contains the set of valuations satisfying each valid sequent in L.

Definition 6. (*V*-validity) A sequent $\langle \Gamma, \alpha \rangle$ is *V*-valid iff, for all $\nu \in V$, ν satisfies $\langle \Gamma, \alpha \rangle$.

We let $V(\langle \Gamma, \alpha \rangle)$ be the set that consists of the valuations in U that satisfy $\langle \Gamma, \alpha \rangle$, and allow a valuation-space $\mathbb{V}(L)$ to be built-up out of these sets over a logic L. Then, let a proof-theoretically defined logic L determine a valuation-space by determining the set of admissible valuations V that are consistent with L.

Definition 7. (*L*-consistency) A valuation $\nu \in U$ is *L*-consistent iff ν satisfies each valid sequent in L. The define the corresponding valuation space: $\mathbb{V}(L) =_{df} \{\nu \in U : \nu$ is *L*-consistent$\}$.

We should note at this point that it is also possible to work the other way around, starting with a valuation-space $V \subseteq U$, and determining the logic or logics that are consistent with it. To do so, we say that a semantic structure, $\langle WFF, V \rangle$ determines a logic L *w.r.t* V (i.e. $\mathbb{L}(V)$) when all the *V*-valid arguments are L-valid.

I.5 *Completeness*

Without placing anything other than the above minimal restrictions on S, L, we have two partially ordered sets P on $\mathcal{L}$ [Hardegree (2005)]:

(P1) The set of all valuation-spaces V on $\mathcal{L}$, ordered by set inclusion;

(P2) The set of all logics L on $\mathcal{L}$, ordered by set-inclusion.

With this, we can define a closure operator *cl* as a function on a poset $\langle P, \leq \rangle$, iff *cl* obeys the following clauses for all *x*, *y* on *P*:

(c1): $x \leq cl(x)$

(c2): $cl(cl(x) \leq cl(\mathrm{x})$

(c3): $x \leq y \to cl(x) \leq cl(y)$

Put succinctly, where *cl* is a closure operator on a poset $\langle P, \leq \rangle$, *x* is an element of *P*, then *x* is closed iff $cl(x) = x$.

This provides a neat formulation of the closure relation between logics and valuation-spaces since the pair <$\mathbb{V}$, $\mathbb{L}$> form an antitone Galois connection between valuation spaces *U* and logics *L* [Dunn and Hardegree (2001); Hardegree (2005); Hjortland (*forthcoming*); Humberstone (2011)].

Fact 8. For each $V \subseteq U$ and $L \subseteq L'$ (for some *WFF*):

(1.1) $L \subseteq \mathbb{L}(\mathbb{V}(L))$

(1.2) $V \subseteq \mathbb{V}(\mathbb{L}(V))$

(1.3) $L \subseteq L' \Rightarrow \mathbb{V}(L') \subseteq \mathbb{L}(V)$

(1.4) $V \subseteq U \Rightarrow \mathbb{L}(U) \subseteq \mathbb{L}(V)$

Proof. Given at length in [Hardegree (2005)].

(1.1) indicates that when we determine $\mathbb{V}(L)$, and then induce a logic $\mathbb{L}$ from the valuation space determined, then $\mathbb{L}$ will contain *L*. Similarly, (1.2) tells us that, when we determine $\mathbb{L}(V)$, and then determine a valuation space $\mathbb{V}$ from the logic determined, that $\mathbb{V}$ will contain *V*.

We may also define a stronger relation since the Galois map between the two form abstract completeness theorems, which, following Dunn and Hardegree [(2001)], I call absoluteness.

Fact 9. [Hardegree (2005)] For any *L*, *V*;

- *L* is absolute iff $L = \mathbb{L}(\mathbb{V}(L))$

- *V* is absolute iff $V = \mathbb{V}(\mathbb{L}(V))$

When absoluteness holds, we have a guarantee that the determining relation-ship between L, V is complete. Absoluteness on V tells us that $V \subseteq U$ is the only valuation-space consistent with $\langle WFF, \vdash \rangle$, and absoluteness on L tells us that L is the only set of sequents that can be associated with $\langle WFF, V \rangle$. So, absoluteness provides a standard by which to analyse the determining re-lationship between a logic and a semantic structure. Our closure operator be-comes an abstract completeness theorem.[9]

Importantly, this ensures that, for each sequent that is derivable by some rule defined in L, it is satisfied by each $\mathcal{V} \in V$ (so long as $\vdash_L$ is closed under reflexivity, monotonicity, transitivity).

Lemma 10. Let Γ be any set of formulas in *WFF*. Define $\mathcal{V}_\Gamma$, as: $\mathcal{V}_\Gamma(\alpha) = 1$ if $\Gamma \vdash \alpha$, and $\mathcal{V}_\Gamma(\alpha) = 1$ otherwise. Then $\mathcal{V}_\Gamma$ is L-consistent and $\mathcal{V}_\Gamma \in \mathbb{V}(L)$.

Proof. [Hardegree (2005)] If not, there must be an L-valid sequent, $\Delta \vdash \beta$ that is refuted by $\mathcal{V}_\Gamma$, so that $\mathcal{V}_\Gamma \Delta = 1$ and $\mathcal{V}_\Gamma \beta = 0$. $\mathcal{V}_\Gamma$ is defined such that $\Gamma \vdash \Delta$ $\Gamma \vdash \Delta$. Since $\Delta \vdash \beta$ is L-valid, and given that the $\vdash$ associated with L is closed under transitivity, it follows that $\Gamma \vdash \beta$, so by the definition of $\mathcal{V}_\Gamma$, $\mathcal{V}_\Gamma \beta = 1$, so $\mathcal{V}_\Gamma$ does not refute $\Gamma \vdash \beta$.

Now we can show that $L = \mathbb{L}(\mathbb{V}(L))$.

Proof. [Hardegree (2005)] Suppose that some $\langle \Gamma \vdash \beta \rangle \notin L$, to show that $\langle \Gamma \vdash \beta \rangle \notin \mathbb{L}(\mathbb{V}(L))$ (in other words, it is refuted by $\mathbb{V}(L)$). Take the valuation $\mathcal{V}_\Gamma$, which by Lemma 10 is in $\mathbb{V}(L)$. By definition, $\mathcal{V}_\Gamma$ satisfies all derivable sequents of L. Since L is reflexive, each element of Γ is derivable in L, so $\mathcal{V}_\Gamma$ satisfies Γ. But, since $\langle \Gamma \vdash \beta \rangle$ is not L-valid, $\beta \notin \Gamma$, so $\mathcal{V}_\Gamma$ refutes β. Then $\mathcal{V}_\Gamma$ refutes $\langle \Gamma \vdash \beta \rangle$, and so too does $\mathbb{V}(L)$, thus $\langle \Gamma \vdash \beta \rangle \notin \mathbb{L}(\mathbb{V}(L))$.

II. COMPOSITIONALITY

With the above framework in place, we now have a formal structure for analysing Peacocke's claims regarding compositionality. The determination relation between L, and V is the satisfaction relation, and the compositional relation $\mathbb{R}$ that Peacocke alludes to can be recast as the preservation of V-validity, as spelt out below. Defining compositionality in this way provides a means by which the modest inferentialist story can be upheld without "giving up" compositionality to the level of semantic-value, and also allows for the construction of a "test" of compositionality for a rule.

II.1 *Defining compositionality*

For a language $\mathcal{L}$ to be compositional is for each complex formula E of $\mathcal{L}$, consisting of sub-formulas $S_1...S_n$, to be constructible from the application of some rule R such that $E = R[S_1...S_n]$. R is syntactical in the sense that it operates at the level of inference between *WFF's*, with its analogue in the semantic domain in which we are interested (in other words, a valuation-space $V \subseteq U$) will be a function f. There, we require that there be a homomorphism preserving the structure of the application of the rule to some arbitrary set of *WFF's* in the structure of valuations.

Definition 11. Take a complex formula E, where $E(\phi)$ is the immediate sub-formulas of E, $(S_1...S_n)$. Then, an *n*-ary function f^n is a function from $\{0,1\}^n$ to $\{0,1\}$, where for each $\mathcal{V} \in V$ and for all $E(\phi)$, $\mathcal{V}(\#(S_1...S_n)) = f^n(\mathcal{V}(S_1...S_n))$.[10]

The key point here is that the function on the right must be determined by the rule-form *n*-ary connective #. What we want is the stipulated inferential behaviour of the connective to determine a corresponding semantic function preserving the original structure (and so not requiring any sort of ad-hoc manouvering at the level of truth-functions as such like).

With this in mind, we can formalise Peacocke's relation $\mathbb{R}$ in terms of the notion of *V*-validity introduced in Definition 6. There, we defined *V*-validity for sequents, and suggested constructing a set of valuations $\mathcal{V} \in U$ satisfying a sequent, where all $\mathcal{V} \in \mathbb{V} \langle \Gamma, \alpha \rangle$ are called *L*-consistent. This way of going about matters can be extended to a rule defining a connective $R^{\#}$ in terms of the preservation of *V*-validity (since *V*-validity was defined for single sequents) over the total range of $L_{R^{\#}}$-consistent valuations. First, define $\mathbb{V}(L_{R^{\#}})$ to be the total range of $\mathcal{V} \in U$ satisfying every provable sequent of $L_{R^{\#}}$ (i.e. by successive application of $R^{\#}$ over the language $\mathcal{L}$).

Definition 12. (Rule preservation of *V*-validity) A rule $R^{\#}$, consisting of a set of sequent premises and a sequent conclusion $SEQ \rightarrow SEQ$, preserves *V*-validity iff, for every $\mathcal{V} \in \mathbb{V}(L_{R^{\#}})$, and every $\alpha \in WFF$, whenever $\mathcal{V} SEQ = 1$, $\mathcal{V}(SEQ) = 1$.

It is simple to see that, when Peacocke's $\mathbb{R}$ is formalised as the preservation of *V*-validity over *L*-consistent valuations, we have the required operation on underlying valuations (over the total range of valuations consistent with a rule).

Example 13. (Conjunction) Let $\mathbb{V}(L_{R^{\wedge}})$ be the total range of $\mathcal{V} \in U$: $\mathcal{V}$ is $L_{R^{\wedge}}$-consistent. The conjunction rules defined above ($\wedge$-I, $\wedge$-Ea, $\wedge$-Eb) pre-

serve *V*-validity over $\mathbb{V}(L_{R^\wedge})$. Then, we know that for each $\mathcal{V} \in \mathbb{V}(L_{R^\wedge})$, (from $\wedge$-I) that when $\mathcal{V}(A) = 1$ and $\mathcal{V}(B) = 1$ then $v(A \wedge B) = 1$; (from $\wedge$-Ea) that when $\mathcal{V}(B) = 0$, then $\mathcal{V}(A \wedge B) = 1$; (from $\wedge$-Eb) that when $\mathcal{V}(B) = 0$, then $\mathcal{V}(A \wedge B) = 0$. In other words, by preservation of *V*-validity by $R^\wedge$ we have compositionality over formulas such that $\mathcal{V} \wedge S_1, ..., S_n = f^\wedge$ $(\mathcal{V}(S_1)...\mathcal{V}(S_n))$. Indeed, the function on the right $f^\wedge$ is the classical truth-function for conjunction:

$$f^\wedge (x, y) = \begin{array}{ll} 1 & \textit{if } x{=}1 \textit{ and } y{=}1 \\ 0 & \textit{otherwise} \end{array}$$

Importantly, of course, the function on valuations follows from the determination relation on $R^\wedge$, rather than assuming antecedent knowledge of the classical function.

With this in place, we can construct a simple "test" for the compositionality of formulas in the context of a rule $R^\#$.

Definition 14. (Valuation agreement) Say that for some $\mathbb{V}(L_{R^\#}) \subseteq U$ consistent with a rule $R^\#$, a pair of valuations $\mathcal{V}_1, \mathcal{V}_2 \in \mathbb{V}(L_{R^\#})$ agree on $\mathbb{V}(L_{R^\#})$ when, for $\forall \alpha \in L_{R^\#}$, then $\mathcal{V}_1(\alpha) = \mathcal{V}_2(\alpha)$. [11]

It is obvious that compositionality for the *WFF's* $\in L_{R^\#}$ requires (at least) that, if an arbitrary pair of valuations $\mathcal{V}_1, \mathcal{V}_2 \in \mathbb{V}(L_{R^\#})$ agree on the subformulas of a complex formula (formed using $R^\#$), they must agree on the formula itself.

Conjecture 15. (Compositionality Test)

$$\forall \mathcal{V}_1 \mathcal{V}_2 \in \mathbb{V}(L_{R^\#}) \{ \forall S \in E(\phi) [\mathcal{V}_1(S) = \mathcal{V}_2(S)] \rightarrow \mathcal{V}_1(E) = \mathcal{V}_2(E) \}.$$

This expresses the basic thought that, if an arbitrary pair of valuations agree on all $E(\phi)$ of E, then they agree on E. [12]

III. PROBLEMATIC COMPOSITIONALITY

With this, however, we are far from home and dry for Peacocke's account, since a number of the natural deduction rules for classical connectives fail the compositionality test. Take $\vee$ as example. Sticking with the natural deduction form for a moment, we schematise $R^\vee$ as follows:

$$\frac{A}{A \lor B} \ \lor I^A \qquad\qquad\qquad \frac{B}{A \lor B} \ \lor I^B$$

$$\frac{A \lor B \qquad \overset{[A^u]}{C} \qquad \overset{[B^v]}{C}}{C} \ \lor E^{u,v}$$

The issue arises for cases in which $\mathcal{V}(A) = \mathcal{V}(B) = 0$ for E^{uv}. In this case, we can not ensure that $\mathcal{V}(A \lor B) = 0$ since we are able only to conditionally infer C from $A \lor B$, given independent sub-derivations to C (which will not figure in the immediate sub-formulas of complex formulas involving $\lor$). We don't have in schematic form all of the relevant information encoded within the immediate sub-formulas involved in the derivation. Given that we are concerned here only with formally valid reasoning, what E^{uv} tells us is just that, if there are proofs available from A to C and B to C, then we have proofs of $A \supset C$, and $B \supset C$. With these, and the disjunction elimination rule we can then show only that V-validity will include $(A \lor B),(A \supset C),(B \supset C) \vdash C$.

With this in mind, we may rewrite the rules in sequent form (again, call this $R^\wedge$):

$$\frac{\Gamma \vdash A \lor B \qquad \Gamma, A \vdash C \qquad \Gamma, B \vdash C}{\Gamma \vdash C} \ \lor\text{-E} \qquad\qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \lor B} \ \lor\text{-Ia}$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \lor B} \ \lor\text{-Ib}$$

However, this won't fix matters, since, again, we have a situation in which there are valuations agreeing on the immediate sub-formulas, A, B, but not on the formula $A \lor B$ itself. Again, the issues arise with the elimination rule, which only gives us something along the lines of: if we can infer C from A, and we can infer C from B, and $\mathcal{V}(A \lor B) = 1$, then $\mathcal{V}(C) = 1$. If we consider the derivations of C from A, B in terms of V-validity, however, we require for $\Gamma, A \vdash C$ only that $\mathcal{V}(C) \neq 0$ when $\mathcal{V}(A) = 1$. Then, for $\Gamma, A \vdash C$ (and equally, $\Gamma, B \vdash C$), we need either that $\mathcal{V}(A) = 0 \ (= \mathcal{V}(B))$, or $\mathcal{V}(C) = 1$. This provides a counterexample to compositionality for $R^\wedge$ since we need only find a pair of valuations $\mathcal{V}_1$, $\mathcal{V}_2$ for which $\mathcal{V}_1(A, B) = \mathcal{V}_2(A, B)$ but $\mathcal{V}_1(A \lor B) \neq \mathcal{V}_2(A \lor B)$. Set $\mathcal{V}_1(A) = \mathcal{V}_2(B) = 0 = \mathcal{V}_2(A) = \mathcal{V}_2(B)$. The first

case has $\mathcal{V}_1(C) = 0$, and the second $\mathcal{V}_2(C) = 1$. As is obvious, this gives us $\mathcal{V}_1(A \vee B) = 1$, but $\mathcal{V}_2(A \vee B) = 0$. For example, if $A = C$, then both of the conditional premises are satisfied by $\mathcal{V}_1$. So, in order for $\mathcal{V}_1$ to satisfy the rule, it must be the case that the major premise is satisfied, and so $\mathcal{V}_2(A \vee B) = 1$. Hence, there are valuations that are equivalent on the subformulas but that do not agree on $\mathcal{V}(A \vee B)$, and so $R^{\wedge}$ is not compositional.

This becomes particularly problematic when we consider formulas involving negation such as $A \vee \neg A$. First, consider the valuation $\mathcal{V}^*$ [Belnap (1990)], which is defined for each formula $\alpha \in WFF$ in relation to *CSL*-admissible valuations (where a *CSL*-admissible valuation is one which satisfies the typical truth-functional conditions of classical sentential logic). Then let;

$\mathcal{V}^*(\alpha) = 1$ iff each *CSL*-admissible valuation verifies α.

$\mathcal{V}^*(\alpha) = 0$ otherwise.

Lemma 16. $\mathcal{V}^*$ is consistent with *CSL*, whilst $\mathcal{V}^*$ is not *CSL*-admissible.

Proof. [Hardegree (2005)] First, check that $\mathcal{V}^*$ is *CSL*-consistent. Suppose otherwise, in which case $\mathcal{V}^* \notin V_{CSL}$. Then, $\mathcal{V}^*$ must refute at least one sequent, $\Gamma \vdash \alpha$, in $\mathbb{L}(V_{CSL})$. In other words, we must have $\mathcal{V}^*(\Gamma) = 1$ whilst $\mathcal{V}^*(\alpha) = 0$. Let β be some element of Γ. Then $\mathcal{V}^*(\beta) = 1$, and, by the definition of $\mathcal{V}^*$, every *CSL*-valuation verifies β, so β is *CSL*-valid. We know, therefore, that every $\beta \in \Gamma$ is *CSL*-valid. Since $\Gamma \vdash \alpha$ is *CSL*-valid, α must be *CSL*-valid. Then, every *CSL*-valuation verifies α, so, by the definition of $\mathcal{V}^*$, $\mathcal{V}^*(\alpha) = 1$, which contradicts our hypothesis that $\mathcal{V}^*$ refutes $\Gamma \vdash \alpha$.

To see that $\mathcal{V}^*$ is not *CSL*-admissible, it is enough to see that $\mathcal{V}^*(A) = \mathcal{V}^*(\neg A) = 0$, which, of course, violates the typical truth-functional constraints on V_{CSL}.

It is simple to see from that the fact that $\mathcal{V}^*$ is consistent with *CSL*, that compositionality will not hold for derivations involving $A \vee \neg A$. Set $\mathcal{V}_1$ to be a "typical" valuation, and let $\mathcal{V}_1(A) = \mathcal{V}_1(\neg A) = 0 = \mathcal{V}^*(A) = \mathcal{V}^*(\neg A)$. Then, whilst $\mathcal{V}_1$ and $\mathcal{V}^*$ are equivalent on relevant sub-formulas, $\mathcal{V}_1(A \vee \neg A) = 0$, whilst $\mathcal{V}^*(A \vee \neg A) = 1$. So, again, we have a case in which there are valuations consistent with $R^{\vee}$ that are equivalent on sub-formulas, yet disagree on the formula itself.

Resultantly, then, Peacocke's preferred natural deduction framework fails to be compositional. In effect, whilst we can now avail ourselves of a possible means of compositionality which is respectable from the point of view of modest inferentialism, we have simply formalised the Fodorian objection with which we started. The valuation of a complex formula is underde-

termined by the valuations of its immediate sub-formulas, where those valuations are determined by the inferential rules for negation and disjunction.

IV. BILATERALISM

Whilst Peacocke's formulation of modest inferentialism fails to be compositional, this is not the case for alternative frameworks, particularly those allowing for multiple succedents. Here, I briefly outline the bilateralist account developed by Greg Restall [(2005); (2009)], before showing that it passes the compositionality test.

IV.1 *The Bilateralist Framework*

Bilateralism is a form of inferentialism developed in [Restall (2005)] , which takes the view that the validity of inferences concerns not only acceptance conditions, but both acceptance and rejection conditions, and the connections between the two.[13] The inference rules defining logical connectives are then understood in terms of coherence constraints over how an agent should treat the acceptance and rejection of formulas of a language. Crucially, both acceptance and rejection are taken to be primitive in the sense that both the structural and inferential rules of a proof-theory are explained in terms of these prior notions. Hence, rejection is not understood as simply the acceptance of a negation, since the rules for negation are themselves built out of constraints over acceptance and rejection. One obvious motivation for bilateralism lies in its ability to secure an inferentialist account of first-order classical logic. As such, we can see the bilateralist account in relation to the problems diagnosed above regarding Peacocke's attempt to do the same.

Restall's [(2005)] suggestion is to think of logic as governing positions involving accepted and rejected sets of formulas.

Definition 17. (Position) A position $\Gamma : \Delta$ is a pair of sets of formulae where Γ is the set of accepted formulas, and Δ the set of rejected formulas.

A position expressed in a language may be used to represent an agent's rational commitments in terms of the coherence between acceptance and rejection. Where $\Gamma : \Delta$ is a position, we allow that $\Gamma, A : \Delta, B$ is the state adding the formula A to the left set Γ, and B to Δ. Think of the above coherence constraints over rational commitment as saying that, a position $\Gamma : \Delta$ is incoherent if it contains some formula in both the left set and the right set, so that $\Gamma \cap \Delta \notin \varnothing$. Thinking of this in terms of an agent, such a position indicates that some statement is both accepted and rejected, and so incoherent. Incoherence allows us to characterise sequent provability.

Definition 18. (Sequent provability) If $A:B$ is incoherent, then $A \vdash B$.

This is because, if a position consisting of accepting A and rejecting B is incoherent, then $A \vdash B$, and an agent who accepts A and rejects B, as said above, has made a mistake.

The definition generalises in cases involving sets of accepted and rejected propositions. In a multiple-conclusion (SET-SET) framework, $A \vdash B$ may be read in terms of the underlying atomic formulae $\alpha_1, ...\alpha_n \vdash \beta_1, ...\beta_n$, which is (classically) equivalent to $\alpha_1 \wedge \alpha_2 ... \wedge \alpha_n \rightarrow \beta_1 \vee \beta_2 ... \vee \beta_n$.

Definition 19. (Sequent provability generalised) If $\Gamma : \Delta$ is incoherent, then $\Gamma \vdash \Delta$.

By incoherence, we mean any position $\Gamma : \Delta$ for which an agent who accepts each member of Γ, and rejects each member of Δ is incoherent. Then, $\Gamma \vdash \Delta$, and an agent is mistaken to accept all $\alpha \in \Gamma$ and reject all $\beta \in \Delta$.

The general idea is to construct a sequent calculus out of these constraints over acceptance and rejection. For example, since both accepting and rejecting the same formula is incoherent, from $\Gamma, A : \Delta, \mathrm{A}$ and Definition 19, we have the usual identity axiom for all atomic formulas. We also have weakening, since, if a position is incoherent, the addition of accepted and rejected formulas will not bring it back to a coherent position. Contrapositively, if $\Gamma : \Delta$ is coherent, and $\Gamma' \subseteq \Gamma$ and $\Delta' \subseteq \Delta$, then $\Gamma' : \Delta'$ will be coherent. Restall also suggests that we construct cut by thinking of extensibility constraints on accepted and rejected formulas. For a position $\Gamma : \Delta$, if the positions $\Gamma, A : \Delta$ or $\Gamma : \Delta, A$ are incoherent, then the original position $\Gamma : \Delta$ must already be incoherent. In other words, if a position is coherent, it should be extensible by a formula A to a coherent position where A is either accepted or A is rejected. So, where $\Gamma : \Delta$ is coherent, either $\Gamma, A : \Delta$ or $\Gamma : \Delta, A$ is coherent.

More importantly, operational rules for the connectives can also be constructed out of positions. For example, if the position $\Gamma : \Delta, A \wedge B$ is coherent, then, $\Gamma : \Delta, A$, $\Gamma : \Delta, B$ or both, are coherent. Contra-positively, if $\Gamma : \Delta, A$ and $\Gamma : \Delta, B$ are incoherent, then so too is $\Gamma : \Delta, A \wedge B$. In this case, we know that $\Gamma \vdash \Delta, A$, and $\Gamma \vdash \Delta, B$, so that $\Gamma \vdash \Delta, A \wedge B$. This gives us:

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge\text{-L} \qquad \frac{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B} \wedge\text{-R}$$

We construct the rules for classical negation by taking a negation $\neg A$ to be acceptable when A is rejectable, and vice-versa. So, if $\Gamma : \Delta, A$ is incoherent, then so too is $\Gamma, \neg A : \Delta$. This gives us Gentzen's classical negation rules:

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \ (\neg\text{-L}) \qquad\qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \ (\neg\text{-R})$$

Analogous accounts can be provided for all of the classical sequent rules [Restall (2005)]. This gives us a construction of the classical sequent rules in multiple-conclusion form, which is built out of a simple and plausible account of agents' rational commitments.[14]

IV.2 *Semantics for Bilateralism*

Before we even consider a determination theory for bilateralism, it should be fairly clear that, unlike Peacocke's system, we can derive identity ($A \vdash A$) for all complex formulas, given only the assumption that we have identity for atomic formulas. That is, because we are building up the accounts of sequent rules from both acceptance and rejection, we already have compositionality for complex formulas involving inferential rules. This carries over to the valuational semantics, which we can see on the above determination theory with just a few tweaks.

Definition 20. For a set of formulae *WFF* in a language $\mathcal{L}$, a multiple-conclusion sequent is an ordered pair, Γ, Δ (where $\Gamma \cup \Delta \in WFF$, and where Γ, Δ are sets of formulae of *WFF*). A multiple-conclusion logic L is an ordered pair $\langle WFF, L \rangle$, where L is the set of binary relations $\vdash_L$ between finite subsets of *WFF* and finite subsets of *WFF*. We call the set of provable sequent in L, L-valid, such that $\Gamma \vdash \Delta =_{df} \{\langle \Gamma, \Delta \rangle$ is L-valid$\}$.

Definition 21. A sequent $\Gamma \vdash \Delta$ is *satisfied* by a valuation $\mathcal{V}$ just in case $\mathcal{V}(\alpha) = 0$ for some $\alpha \in \Gamma$, or $\mathcal{V}(\beta) = 1$ for some $\alpha \in \Delta$, otherwise $\mathcal{V}$ *refutes* the argument.

Then, let L determine a valuation-space by determining the set of admissible valuations V that are consistent with L as before: $\mathbb{V}(L) =_{df} \{\mathcal{V} \in U : \mathcal{V}$ es L-consistent$\}$.

IV.3 *Compositionality for Bilateralism*

Again, we formulate Peacocke's relation $\mathbb{R}$ as the preservation of V-validity by a rule $R^{\#}$ (over $\mathbb{V}(L_{R^{\#}})$).

Definition 22. (Rule preservation of V-validity) A rule $R^{\#}$, consisting of a set of sequent premises and a set of sequent conclusions $SEQ_P \rightarrow SEQ_C$, preserves V-validity iff, for every $\mathcal{V} \in \mathbb{V}(L_{R^{\#}})$, and every $\alpha \in WFF$, whenever $\mathcal{V}(SEQ_P) = 1$.

Bilateralism gives us a compositional theory for all $WFF's \in L_{R^{\#}}$. The reason is that, unlike the $SEQ \rightarrow SEQ$ formulation, the characteristic function on $\mathbb{V}(L_{R^{\#}})$ allows us to carve up valuations in $\mathbb{V}(L_{R^{\#}})$ into two sets by assigning 1 to every element of $\mathbb{V}(L_{R^{\#}})$, and 0 to every element of $U \in \mathbb{V}(L_{R^{\#}})$. As a result, we no longer have to worry about valuations such as $\mathcal{V}^{*}$.

Example 23. (Disjunction) In this setting, we schematise $R^{\vee}$ as follows:

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \text{ } \vee\text{-L} \qquad\qquad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \text{ } \vee\text{-R}$$

Let $\mathbb{V}(L_{R^{\vee}})$ be the total range of $\mathcal{V} \in U : \mathcal{V}$ is $L_{R^{\vee}}$-consistent. $R^{\vee}$ preserves V-validity over $\mathbb{V}(L_{R^{\vee}})$. Now, if we take any arbitrary pair of valuations $\mathcal{V}_1, \mathcal{V}_2 \in \mathbb{V}(L_{R^{\vee}})$, then, where $\mathcal{V}_1, \mathcal{V}_2$ agree on sub-formulas A, B, they will agree on the complex formula $A \vee B$. The problematic case identified above occurred when $\mathcal{V}(A) = 0 = \mathcal{V}(B)$. Here, we know (from $\vee$-L) that, for each $\mathcal{V} \in \mathbb{V}(L_{R^{\vee}})$, when $\mathcal{V}(A) = 0$ and $\mathcal{V}(B) = 0$, then $\mathcal{V}(A \vee B) = 0$. In other words, in the bilateralist framework, preservation of V-validity by $R^{\vee}$ gives us compositionality over formulas such that $\mathcal{V}(\vee(S_1,\ldots,S_n)) = f^{\vee}(\mathcal{V}(S_1)\ldots\mathcal{V}(S_n))$.

Example 24. (Negation) Define $R^{\neg}$ as above (§IV.1). Let $\mathbb{V}(L_{R^{\neg\vee}})$ be the total range of $\mathcal{V} \in U : \mathcal{V}$ is $L_{R\neg\vee}$-consistent. $R^{\neg}$ and $R^{\vee}$ preserve V-validity over $\mathbb{V}(L_{R^{\neg\vee}})$. Unlike before, we no longer have it that $\mathcal{V}^{*}$ is $L_{R\neg\vee}$-consistent, and it is also the case that $R^{\vee}$ is compositional by preservation of V-validity (example 23). It follows, straightforwardly, that for an arbitrary pair of valuations $\mathcal{V}_1, \mathcal{V}_2 \in \mathbb{V}(L_{R^{\neg\vee}})$, when $\mathcal{V}_1(A) = \mathcal{V}_1(\neg A) = 0 = \mathcal{V}_2(A) = \mathcal{V}_2(A)$, $\mathcal{V}_1(A \vee \neg A) = 0 = \mathcal{V}_2(A \vee \neg A)$.

CONCLUSION

This paper has presented both a solution and a problem for the account of compositionality in Peacocke's modest inferentialism. In providing a formal framework for the determination of a valuational semantics by an inferentially defined logic, I suggested reformulating the "compositional" relation,

$\mathbb{R}$, alluded to by Peacocke, as the preservation of *V*-validity by a rule. This provides a "test" for the compositionality of a logic, which, problematically, the classical natural deduction framework cannot meet. To finish, I briefly outlined an alternative account of modest inferentialism, bilateralism, that is capable of meeting this challenge.[15]

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NOTES

[1] I stick with Peacocke here in talking of the cognitive states of acceptance and rejection, rather than speech acts of assertion and denial. However, I take it that for the purposes of this paper at least, since assertion and denial at least typically express acceptance and rejection, respectively, both are equally rationally constrained in the ways discussed here.

[2] Further details can be found in [Trafford (*under submission-1*)].

[3] Assuming the standard formulation SET-FMLA. We will have cause to discuss other formulations later.

[4] See Strassburger (2007) for further details.

[5] Hereafter, I drop $\vdash_L$ for $\vdash$, assuming the definition is relative to a logic throughout.

[6] There are a number of plausible candidates for $\mathcal{V}, \mathcal{D}$, depending upon the logic in question, but here I will be interested only in cases where $\mathcal{V} = \{1,0\}$, and D = $\{1\}$.

[7] See also Hardegree (2005).

[8] On this, and for further detail, see Humberstone (2011), §3.

[9] Though I won't go into it here, it should be said that absoluteness for $V = \mathbb{V}(\mathbb{L}(V))$ does not hold for classical propositional logic (in SET-FMLA). For details, see Hardegree (2005); Hjortland (*forthcoming*).

[10] On this, and for further detail, see Humberstone (2011), §3.

[11] A similar notion can be found in Humberstone (2011).

[12] It should be noted that this is a fairly basic account of compositionality which does not directly deal with issues regarding intersentential compositionality regarding inferences that exploit the particular constituent structure of premises and conclusion. Nonetheless, I am confident that the account here can be extended to deal with these issues since the valuations of compound sentences formed with connectives of will be compositionally determined from their components. I leave the details for further work. Thanks to an anonymous referee for drawing my attention to this matter.

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