HOW LOGIC SPEAKS

One of Hilary Putnam's most profound and important essays, "Rethinking Mathematical Necessity", was inspired, he tells us, by a desire to understand an idea of young Wittgenstein's (henceforth YW),

> an intuition that I had never shared. For the early Wittgenstein it was somehow clear that logical truths do not really say anything … (1994: 246)

Whereas it had seemed clear to Putnam that

> sentences of pure logic are statements with content … ; if proved, they are moreover *true* statements, and their negations *false* statements. (Ibid)

The initial problem was to understand what YW could have meant by this. The route Putnam found to an answer went through an idea he found in Kant:

> Logic is not a description of what holds true in "metaphysically possible worlds', to use Kripke's phrase. It is a doctrine of *the form of coherent thought*. Even if I think of what turns out to be a "metaphysically impossible world", my thought would not be a thought at all unless it conforms to logic. (1994: 247)

How to understand this idea? Does it mean that what is, in fact, a law of logic simply *could not* have failed to be no matter how the world were? Kant, as Putnam reads him, tells us:

> The negation of a logical truth is, in a sense, unthinkable; and it is unthinkable precisely *because* it is the negation of a logical truth. Explanation goes no further. (1994: 255)

Trivially, thought, or its logic, could not have turned out to be such that 'blah' where that 'blah' is unthinkable: there is as yet no way it could have turned out. But then, exactly what sort of notion is *unthinkable*? Here, then, is Putnam's concluding way with the 'Kantian' idea:

> My suggestion is not, of course, that we retain this idea of a nature of thought (or judgment, or the ideal language) which metaphysically guarantees the unrevisability of logic. But what I *am* inclined to keep from this story is the idea that logical truths do not have negations that we (presently) understand. It is not that we can say that the theorems of classical logic are "unrevisable"; it is that the question "Are they

> revisable?" is one which we have not yet succeeded in giving a sense. (1994: 256)

This, I will argue, is just the right way with it—as one can see in comparing Frege with YW.

1. *Sätze* and *Gedanke*: In 1919 Frege wrote,

> What is distinctive about my conception of logic is first recognisable by the fact that I place the content of the word 'true' in lead position, and then by the fact that I let thoughts follow immediately as those things by which being true can come into question at all. (1919: 273)

To grasp Frege's conception of logic, then, one must grasp his notion of a thought. Perhaps the most striking thing about that notion is its contrast with something like Russell's (early-20th century) conception of a proposition, inherited YW's of a *Satz*.

Russell, and YW, thought of a proposition (or *Satz* in relevant sense) as something sentence-like in ways a thought in Frege's sense is not. Sentences, or at least sentences of a *language*, have two conspicuous features. First, they have some *perceivable* form (visual, auditory, whatever); a form by which they are recognisable as the sentences they are (without resort to knowledge as to what it is they say). They are the sorts of things suitable for work as vehicles in the expression of our thought; content-bearers. Second, a sentence is something structured—syntactically, and thereby semantically—in some one particular way (bracketing ambiguity). For our purposes (and Russell's and YW's), a sentence can be thought of as generated from a given vocabulary by given syntactic rules—ones by which indefinitely many different sentences are generable from that given vocabulary. It is structured by its derivation in the relevant syntax. Its semantics—what it means, or as such, says—is structured accordingly. The English sentence 'Penguins waddle', for example, speaks of penguins and describes them as waddlers. It is intrinsic to it to do this. A sentence, thus, belongs to a *system* of sentences. That system as a whole is structured by what generates it. The structure of a given sentence locates it in that system. One, if not *the* main task of a syntax is to *identify* each sentence within some given language. To do that is to assign that sentence some structure which is its alone; thus which distinguishes it from every other sentence generated, and, given the complexity of structure here, marks the syntactically (and semantically) relevant similarities between it and given ranges of other sentences which it is marked as resembling.)

It is sometimes unclear whether Russell or YW made that first feature of sentences part of their conceptions of a proposition. Both certainly exploited the second. Every proposition is to be conceived as having some unique representational structure. It is some particular structured way of representing things as some way there is for things to be. So it has (to adumbrate) a unique logical form. Every proposition belongs to a system of propositions, in which its structure positions it, and by which it shares features with given ranges of other propositions. (Wittgenstein espoused this view as late as January 1930. Cf. Waismann, 1979: 89-90.) For both, such relations between propositions are to be the very foundation of logic.

Frege's notion of a thought, however, fits neither of these ideas. It is essential to a thought that it is something invisible (not perceivable by the senses). One reason for this is that a thought is to be *precisely* that by which truth can come into question at all. It is thus to

be identified by just those features, and no more, which identify something on which truth is liable to depend. And whether it is true that penguins waddle does not turn on whether a sentence which so says is written in lower case or capitals. The second idea above—what is most essential to Russell's and YW's, notions of *proposition*—collides, for a start, with Frege's insistence that whole thoughts come first. We do not, he tells us, begin with concepts and put them together so as to construct a thought. Rather concepts *arise* through the decomposition of thoughts. It is a central part of this idea that the *same* thought can be decomposed in many different ways; that semantically, and structurally, differing sentences can all be expressions of the same thought. As Frege also puts it,

> I do not think that for each judgeable content there is just one manner in which it can be decomposed, or that one of the possible manners may always claim an objective priority. (1882: 118)

('Judgeable content' was Frege's first try at the notion 'thought' ('Gedanke') came to stand for.)

Grasping Frege's notion of a decomposition is made easier by conceiving a thought in terms of its defining task: to bring truth into question; to make it turn in some determinate way on how things are. Like any task, this one can be broken down into sub-tasks. Serving the drinks can be broken down into serving the martinis, serving the mai tais, serving the margaritas, serving the mojitos, etc. Or it can be broken down into filling cocktail shakers, shaking them, dusting rims of glasses, pouring, etc. Similarly, making truth turn on whether Sid waddles can be broken down into making truth turn (in part) on how Sid is and making it turn (in part) on what waddles. Or it can be broken down into making truth turn on whether the concept *being a waddler* is satisfied by everything which satisfies the concept, *being that very item, Sid*. And, I stress, so on. For a set of subtasks to be a decomposition is just for their joint performance to be (no more nor less than) performing the whole thought's task.

A thought-*element*, on this way of thinking, is, first of all, an element of some decomposition of a thought, and then an element of it *on* that decomposition. For it to be an element on that decomposition is for it to be part of some set of subtasks which, jointly, just are that whole thought. No more is required of a decomposition for it to be a decomposition than just said. Such is what opens the door to multiple decompositions.

Making truth turn in a particular way on how things are is not like serving drinks in *all* respects. In the last case, one can parcel out the tasks: Pia mixes the martinis, Sid serves them. For a thought there is no parallel parcelling out of (proper) subtasks. Making truth turn on what waddles can only be done at all as part of making truth *tout court* turn on how things are. There is no such thing as making truth turn in part on how things are, *punkt*. If Pia fills the shakers and Sid omits to shake, at least we have full shakers. To make truth turn on what waddles and omit to do the rest would be to do nothing at all.

If Sid gets a Lego set for Christmas, he might build a garage. Pia may then decompose it into parts. Or her nephew, Thor, might just oblige. Thor's parts may be very different from what Pia's would have been. None of his would have fit together with any of hers. You can break up a Lego model in many different ways into sub-parts. But—assuming there are no plastic shards about—there is *one* decomposition of the garage that may claim objective priority. It is the decomposition of the garage into those parts in which the Lego set came. *This* is the sort of decomposition one expects of a friend when he returns the Lego set he borrowed. Some philosophers have understood Frege's idea of multiple decomposability on

this Lego model. Such is not Frege's idea.

Between about 1902 and 1904 Frege and Russell debated Frege's notion of *Sinn*. A thought is one sort—the central sort—of *Sinn*. So they were debating, *inter alia*, Frege's notion of a thought. Russell rejected that idea. He (and then YW) preferred their notions of proposition. One notorious point of disagreement concerned whether objects—Sid, or Mont Blanc—could themselves be thought-elements. As Russell seemed to understand this issue, the crucial question was whether the relation between thought-elements which made a thought about an object and the object the thought would thus be of was, or could, be many-one. Russell also seemed to think that Frege's case for *Sinn* was banking on this being many one. Perhaps Frege invited such misunderstanding. Perhaps he was occasionally unclear in his own mind on the point. (See, e.g., 1906.) But this *is* a misunderstanding. The trouble with Mt. Blanc, shared by Sid and the foam on Pia's cappuccino, is that neither these things nor anything to be found in investigating them, can make truth *turn* on anything. Nor is Sid a way for truth to turn on anything. The truth of a thought may well turn, *inter alia*, on how Sid is. But then the relevant element would be one which *made* the thought do this. Such an element would not be Sid. He is the wrong sort for such feats.

Frege did think that the relation in question *is*, in general, many-one. But the existence of *Sinne* depends on no such fact. Rather, many-oneness serves a different end here. Facts as to where there is one thought, where two, need to mesh with facts as to where there is need for proof (and then *what* proof). If the thought that A is the thought that B, then, trivially, anything which is proof of A is proof of B. So there had better be different thoughts wherever proof of B from A would not be immediate, or wherever some proof of A might fail to be proof of B. We might look at arithmetic, for example, to see where proof of *one* thing from another, or of one thing, but not yet of another, would need to be recognised. Such will settle, for arithmetic, just where there are two arithmetical thoughts, where one. For Frege the bare notion of a thought should leave it open for arithmetic to settle such questions. Similarly for other areas of thought. In general, as he puts the point,

> The principles of concepts, and of judgements, serve only as preparation for the theory of *consequence*. (*Kernsatz* 14 (*Nachgelassene Schriften,* p. 190))

Such, I think, is one of Frege's greatest insights.

Compare thoughts and propositions on this point. When are two different decompositions of a thought (each a decomposition of some thought) a decomposition of the same thought? For Frege different decompositions, each of some thought, are to be recognised as decompositions of the same thought where needed so as to represent correctly the facts as to what would be proof of what. That a thought is decomposed in some one way cannot on its own determine what another decomposition of that very thought might be. Nor does the notion of a thought provide us *per se* with any effective condition on thought-identity. Contrast this with the answer to the question when two decompositions of a *proposition* would be decompositions of the same thing. Excluding the Lego model of decomposition, the answer is: only where they decompose it in the same way. A proposition is identified as the proposition that it is by a particular decomposition: that which decomposes it into the structure assigned it by its generation in the system to which it belongs. The structure thus assigned it identifies it as the one it is in the same way that the structure the

syntax of a language assigns a sentence in generating it identifies that sentence as the one it is.

Thus, when YW tells us,

> That the truth of one proposition follows from that of others, we see in the structure of the propositions. (5.13)

He is telling us that, for any stock of questions of truth—ways there are of making truth turn on how things are—there can be no identifying which questions, or ways of turning, these are without *ipso facto* answering the question what, within the stock, would be proof of what. For YW, whatever allowed truth to come into question at all—any actual determinate way for truth to *come* into question—could, *per se*, allow for only one role for what *was* thus in question in the phenomenon of proving and being proved.

It is *just* this idea of a unique way of generating any given proposition which, I hope to show, Putnam cannot accept. In his rejecting it we find his deepest insight about the inevitability (such as it is) of logic.

2. Frege's Logical Insights: Frege tells us two or three crucial things as to what logic is. First, he tells us,

> The meaning of the word 'true' is unfolded in the laws of being true. (1918: 59)

Second, he tells us,

> How must I think to reach the goal *truth*? We expect logic to give the answer to this question, but we do not require of it that it delve into the particularities of each area of knowledge and its objects; rather we only assign it the task of setting out the most general things which hold for all areas of thought. … We can thus also say: Logic is the science of the most general laws of being true. (1897: 139)

Third, there is a suggestion in Frege of what sort of generality might be involved in such references to 'the most general.'

First, then, the laws of being true (or laws of truth) are arrived at in unfolding the concept *true* (undefinable, in a sense, but not thereby *altogether* without content). They are laws which hold simply in virtue of what being true is *per se*. Which tells us something as to how to understand Frege's second idea. He frames it in terms of what one might see as logic's universality, or topic-neutrality. Logic, the idea is, tells us how to think insofar as such advice applies to, and holds for, all thinking whatever, no matter what it is about. So its maxims contain no restrictions on the sort of thought to which they apply. Hence logic tells us only that much as to how to reach truth which is contained in what it would be, as such, for something to be true. Since the interest here is in *aiming* at truth, logic would be centrally concerned with relations between the truth of some thoughts and the truth of others; here centrally with truth-preservation. The sort in question would be that ensured simply by truth's

very nature—by what being true is essentially. Such is a way of understanding the idea of topic-neutrality. It is also a clue to what shape, for Frege, logic would take.

What content is there in the bare idea of being true? One strand concerns a certain objectivity. ("Logic begins with the conviction that there is a distinction between truth and falsehood." (*Kernsatz* 12) There *is* a thought—a question of *truth*—just where it can be "grasped as the same" by different thinkers, and, thus, agreed to or disputed. (Cf. 1919: 146) Where there is a thought, it is true, or false, independent of who thinks it or whether it is thought. (Cf., e.g., 1918: 69). Frege's propositional logic is simply a development of this idea. A thought is, or at least aims to be, either true or false. Thus, the set of functions from truth-values, or pairs thereof, to a truth-value will identify all the ways in a thought may be compounded out of others whose truth-values determine its. For each such function there is a logical form formable from any thought, or pair, according as the function takes singletons or pairs as arguments, where what has that form has the value true just where that function maps the values of its elements, so formed, into the value true. Propositional logic just maps truth-preservation across such forms.

Frege's *news* was as to how logic could look inside whole thoughts. There is such a thing as a thought of an object that it is thus and so—a thought which so decomposes—only where there is an object it is thus of. For the thought to be true is then for that object to be as (so decomposed) the thought represents some particular object to be. Suppose we present the form a thought assumes when so decomposed by, say, the symbols F(a) (as Frege notes, as we would typically present the logical form of a (mathematical) function. Then, extracting the form-element F( ) from that whole, we can introduce a new logical constant to combine with that element to form a new logical form. We might write it, e.g., 'ExF(x)'. The basic truth-preserving properties of a thought of that form (insofar as part of what being true is *per se*) would be: such a thought follows from any thought of the corresponding form, F(a) (I here omit details of what 'corresponding' is to mean); if, the role of 'F( )' remaining fixed, G follows from any thought which shares the form 'F(a)' stands in for, then G follows from ExF(x). Now, if you like, introduce a further way of completing 'F( )'—write it, say, 'AxF(x)'—whose basic truth-preserving properties are that it both follows from and is entailed by *not*-Ex-*not*F(x)—letting '*not*' here stand for a standard negation operator. With which we have recognised (at first-order) those logic forms of concern to logic which one discovers by looking inside whole thoughts.

Where logic's laws are expressed in that special way a calculus does this, the relevant properties of those just-indicated constants in logical forms will be made recognisable syntactically. The relevant rules for constructing logical forms for proof, though, are meant to correspond to certain facts of truth-preservation. In the propositional case, e.g., to the facts of when truth would be preserved moving from two thoughts to a compounding of them which took on the value true just when both of them did, and from such a compound to some further thought. In the quantificational case, in the way just indicated.

There is *a* logic—a construction of a special sort. And there is logic—what a logic aims to have represented rightly. Logic—the topic to be represented rightly—has its laws, just as, e.g., mechanics does. For Frege these are the laws of the phenomenon of being true (notably laws of truth-preservation). If there is a topic here, there *might* also be a theory of it. A theory would *mention* the key items that the laws govern—such things as quantifiers, or quantification. It would treat the same phenomenon as a logic does. But it would be answerable to this in a very different way. The above is a somewhat tedious expansion on—if

you like, informal theory of—that to which a logic, or a theory of logic is answerable, each in its own way. The point of the expansion is to begin to point to the different strands which make up the notions logic treats of; notions of that of which its laws are to hold. Multiple core ideas interact here. There are ideas of objectivity. There is the idea of truth's bipolarity. There is an idea of a certain universality and authority to logic—of thought as something *per se* governed by given laws, no matter what, or when, or by whom, the thought is. Such a tangle of threads at least begins to make *truth* a notion of just that sort about which Putnam has had so much to say.

There remains Frege's third idea. Laws of logic, since universal, have a specific sort of generality. They have consequences for the ways *any* thought relates to others. Frege offers a way of understanding the generality of a thought. On this understanding, generality is to be attained through quantification. Where a thought has some specific content—e.g., where it si decomposable into making truth turn on how *Sid* is, and on who smokes—one can move from it to a thought without that content by replacing that element (in relevant decompositions) with a quantifier. If the thought is that Sid waddles, one moves in the right direction by thus moving to a thought that something waddles (or that everything does). One continues in the right direction by moving from there to the thought that something (or everything) does something (or everything). Eventually, the idea is, one reaches a point where there are no more such moves to be made. One would then have attained to a *most* general thought, on this understanding of generality. Frege's idea of universality is often read as the idea that laws of logic (or of truth) belong to the realm of most general thoughts in this sense.

3. YW On Logic: The instigation to our present exploration was YW's idea that logical 'truths' (if such they really are) say nothing. YW, in fact, means several different things by this. The most plausible of these turns *au fond* on the notion of representing-as. YW, though, puts it in slightly different terms:

> In tautology the conditions on agreement with the world—the representing relations—cancel each other out, so that it stands in no representing relation to reality. (1922: 4.462)
>
> It is the distinctive mark of a logical proposition that one can recognise in the symbol alone that it is true; and this fact contains the whole philosophy of logic. (1922: 6.113)

Consider the relation of representing-as. One might see this as three-place: in the first place there is a representer—even if, sometimes, only a stand-in for one in the form of something like a thought or a proposition; in the second place, something which is represented *as* something or other—in the main cases so far, either things or *a* thing; in the third place, that which what is in the second place is represented *as* (being)—in those main cases, some way there is for things, or for *a* thing, to be. In a normal case where there is a question of representing truly or falsely, truth value is a cooperative enterprise: the third term in the relation fixes *how* truth is to turn on the second term—what is demanded of this second term if there is to be truth; the second term, how things are, delivers the outcome of such turning. Normally, the various elements in the relevant thought, or proposition, each contribute

substantially to forming some substantial demand on what occupies the second place. But suppose that instead of this, in YW's terms, those would-be partial demands contained in these elements 'cancel each other out', so that really no demand is placed by the third term on the second. So that, in YW's more metaphorical terms, one can recognise in the third term itself that the would-be representing (if either true or false) must be true—or, again, must be false. Then there is no real role for the second term here. It makes no difference at all *how*, or what, it is. It might as well be anything. The result is already determined. Such, if it happened, would be a plausible case for mere *schein*-representing-as; a case where nothing was really represented as anything. Perhaps there was a *schein*-occupier of the second place in the relation. But at best we have only a degenerate case of the obtaining of *this* relation.

Putnam suggests that this effect of cancelling out shows up only in unembedded items of relevant forms, not in embedded ones. One thing this suggests is that cancelling out has to do more with force than simply with content. More generally, though, it suggests that cancelling out is somehow all relevant to context, on some notion of context (yet to be explored); that cancelling out is not something which that which brings truth into question (a thought or a propositions) does as such. This more generally suggestion points in the direction in which we are now headed.

But YW also offers another account of of saying nothing. Such turns more patently on what is peculiar to a proposition as opposed to a thought in Frege's sense, though YW seems to see it as merely continuous with the first idea, above. 6.1222 expresses it as follows:

> Not only must a proposition of logic be incapable of refutation by any possible experience, but it must also be incapable of being confirmed by any such.

If *propositions* were what brought truth into question, these two ideas might fuse. For if what brought truth into question was what was identified, *per se*, by a proposition's structure, then wherever truth was borne on, *eo ipso* what was borne on could not be the *same* proposition as any whose structure cancelled out demands on truth as per above. The ideas separate, though, if what brings truth into question is a thought. Putnam showed us why they must: there is no such legislating of a question of truth what will, what not, matter to its answer.

A law of logic *would* be *absolutely* impervious to worldly bearing on its truth *if* it merely reflected structure which made the thoughts which had it the thoughts they are. There could not fail to be thoughts so structured. The law could not fail to apply to them. YW this idea in 6.341-6.342 by comparison of logic with what I will call a *special system*. Such a system generates a stock of propositions from given vocabulary by given syntax. The structure thus assigned to each such proposition distinguishes it from any other proposition in the system—or from any *tout court*: to be that proposition is to have that structure; to be it is to be generated by that system. (Compare English sentences.) So that structure, with the contrasts it makes with other propositions, identifies the content of that proposition as what it is (identifies the question of truth thus posed). The system need not generate *all* propositions. The structure it assigns *its* may be largely proprietary. It yields a *particular* scheme for describing things; one, perhaps, among many possible. Its concern may be some particular subject matter. Now the crucial idea is this. On the whole, the system generates propositions whose parts do *not* cancel each other out. These are genuine descriptions of the world. But it may also generate, or fix, ones whose parts *do* cancel out in the above-scouted sense. These

merely tell us what system we are dealing with; *what* content the first-mentioned propositions have in relating to each other as the syntax of the system makes them do. The illuminating comparison is to be between such proprietary dicta of a special system and laws of logic.

YW offers two examples. The first is a hypothetical system for describing black and white patterns on a white wall. In the system such are described in terms of a (notional) net. The net consists of labelled cells of a particular size and shape (say, hexagonal). (A label might be a pair of coordinates for row and column.) A description in the system supposes this net placed over the wall in a particular orientation. It is then a conjunction, each conjunct pairing a cell-label with one of the descriptions, 'black', or 'white'. A rule of the system is, say, that a cell is to be paired with 'black' just in case it is at least 50% black; otherwise white. Such fixes when a given description within the system would be true of a given wall. For any given wall, the system might also generate, for each cell, ψ, in the net, the (would-be) proposition, 'ψ is not both black and white'. Or it might generate a generalisation of this, such as 'no cell of this wall is both black and white', or, still more generally, 'there is never a cell of any wall which is both black and white'. But such would-be propositions would say nothing as to how any white wall was *in re* being black-patterned. These pseudo-propositions merely fix how this particular scheme for describing walls works. They help identify the content of a *genuine* proposition such as '… & <<17,39>, black> …'; what it says, insofar as to say this is just what it is to be the proposition in question. Thus, the idea is, the pseudo proposition is in no way liable to proving false.

YW's second example of a special system is Newtonian mechanics. Here the Newtonian laws and definitions are the pseudo-propositions. For example, the (would-be) proposition, 'Momentum is mass times velocity', merely tells us how the terms of the system describe, just as with the would-be proposition, 'No cell of any wall is both black and white.' On the other hand, 'That six-pack is traveling towards that windshield with momentum 200 m/hr/kg'. as generated by the 'Newtonian special system', *is* a genuine proposition. Refer to the Newtonian laws to see *what* it says.

The comparison is thus between laws of logic and the dicta of special systems. What is the comparison to be? One might (not entirely plausibly) think of the laws of logic as generated by, or by-products of, some vocabulary and syntax which generates all special systems. Or, less implausibly, one might think of them as things generated in generating any consistent system of propositions, or perhaps under some suitable closure of it (say, under suitable compoundings of propositions, and operations on sub-propositional parts). In any case, the idea would be that the laws of logic are pseudo-propositions in the same way that pseudo-propositions of special systems are. They are simply part of what fixes how *any* system of propositions is to work; or that structure, or content, of any proposition which is fixed independent of to what special system it belongs. Hence (the thought is) they are as impervious to being *proven* false (or true, for that matter) by vicissitudes of history as are the dicta of special systems.

The comparison, though, founders at at least two points. First (borrowing again from Frege), it misconstrues the nature of the authority laws of logic can claim over our thought. Second, it misconstrues the way in which it is open to a special system to identify those questions of truth towards which we stand. If the first point is not Putnam's in particular, the second certainly is. First point. To conceive of dicta of special systems as immune to worldly bearing, as YW suggests, is to conceive of them as something like stipulations: the descriptions of this system *are to* work thus. *In the system* a cell is to be called black just in

case there is at least as much black in it as white. In another system, perhaps, not. And one could describe patterns on walls in a system that did not so work at all. Similarly YW suggests, one can capture mechanical phenomena in Newtonian mechanics, in terms of its physical quantities, or, if you prefer, within a different system in terms of others. Or, omitting to speak of mechanics, one can, as one cannot for logic, duvk being subject to any mechanical dicta at all.

By contrast, If the wall has a black spot and the six-pack is hurtling towards the windshield, then the six-pack is hurtling, no matter what the special system. Nor can one *stipulate* whether a conjunction is to be taken to entail its conjuncts; nor whether any particular thoughts we think are conjoinable. Nor would such room for stipulation fit with Frege's, and YW's idea that there is no such thing as illogical thought (an empty idea if one can *stipulate* how thoughts are to behave).

Second point. It is simply not true that momentum is mass times velocity. In thinking that *momentum* is *mass* times *velocity*, one cannot be thinking something which could be made true by placing it in some special system. No proposition of any system that would decribe the mechanics of the world could connect momentum, mass and velocity in that way. Which means that to think that momentum is mass times velocity, whatever this might be, could not be simply to think some given proposition, on YW's conception of what a proposition is. Which brings us to what is most central in Putnam's thought.

4. Open Questions: *Thought* and *proposition* are two rival conceptions of a question of truth. *Pro tem*, abstracting from this disagreement, I will speak simply of questions of truth. A question whether the six pack is flying towards the windshield with such-and-such momentum might be one such. The disputed question is: what identifies a question of truth as the question it is.

Suppose there were some special system, with specified vocabulary and syntax, which generated a proposition, Σ, that the six-pack is flying towards the windshield with momentum 200μ. To be *that* proposition would then be, *per se*, to be structured as that system structures Σ. If Σ* is structured differently, then, *ipso facto*, it is not Σ. If Σ identified a question of truth—a given way of making truth turn on how things are—then things could be made to turn in that way on how things are only in representing structured as Σ is. Σ's structure would be essential to representing in *that* way. In applying logic's laws to given discourse we look, in first instance, for something *essentially* structured in the way a proposition is to which to apply those laws. But this, by itself, does not answer the question that presses here: whether the same question of truth might be posed by representing otherwise structured; and if so, what sort of structuring might do this. This question *cannot* be answered by Σ's parent special system itself. There would be no such question if questions of truth were to be counted as (YW's) propositions are. But both Frege and Putnam give reasons why this way of counting such questions cannot be right. I will focus on Putnam's.

Newtonian mechanics *defines* 'momentum' as mass times velocity. Why can it not just be understood as using that term to speak of something of which this is true? Answer: because Newtonian mechanics is, or was, to be understood as thus speaking of a notion to which many strands belong. These individual strands may prove not to hold together. The Newtonian definition is just *one* such strand. It might prove (and *has* proven) the one that has to go. Among other strands in the *Newtonian* understanding are: that momentum is a physical quantity; that a rigid body has some; that momentum has a certain role in explaining

mechanical phenomena. The definition appears not to fit with these others.

More specifically, if relativisitic mechanics is right, there can be no physical quantity fitting the Newtonian definition. On the other hand there *is* a physical quantity of which all of the above would have been being (quite reasonably) supposed before 1905. Now two possibilities: 'momentum' in Newtonian mechanics referred to nothing; or it referred to this last-mentioned physical quantity. On the first there is no such thing as momentum; so no special system ever spoke of such. On the second, there is such a thing. Many of the above strands hold good of it. What Putnam has shown is how the second can be what is *correct* as to what Newtonian mechanics spoke of.

A simple parallel (from another familiar context). I point and say, 'The man behind the Foster-Grants is on his fifth martini.' But those 'Foster Grants' are really Maui Jims. Nor is that 'man' actually a man. If I said something, it was of someone. But if I said something of someone, that person would not fit the description, 'behind the Foster Grants'. In fact, if I did say something of someone, it is clear who it would be. (*Vide* my pointing.) So either I said nothing of anyone, inclusive the person I clearly meant to speak of, or I said something (false if she is wearing Ray Bans) of that person. The reasonable choice: the last.

What, now, of YW's Newtonian special system? First, if that the six-pack is flying with momentum M is about momentum, then there is no such proposition in that system. For if it is intrinsic to that system to be governed by the Newtonian definition, then it is, so far, a system for speaking of what could not be a physical quantity at all, whereas momentum is one. Second, though, that same question of truth, whether the six-pack has momentum M, *might* have been expressed by a proposition in such a system had Newtonian mechanics proved correct. Hence, that question, while identified with a particular thought in Frege's sense, cannot be identified as such with *any* proposition in YW's. Moreover, if we understand it as intrinsic to YW's system that it is a system of descriptions for mechanical phenomena, then as it turns out there *is* no such system. (The world-dependence of a thought's existence.)

So far, something on which Frege and Putnam agree. A particular expression, or presentation, of a question does not on its own fix what another presentation of the same question would be. But while for both Frege and Putnam what thoughts there *are* is, somehow, a world-involving matter, Frege is unlikely to have anticipated Putnam's take on the idea that so is what would count as a presentation of a *given* thought. Frege had his standards on proper definition. What he may not have anticipated is that whether, e.g., *has momentum M* is well-defined is hostage to how things happen to be. Both agree, though, on this crucial point: identifying different presentations of the *same* question of truth involves extra-logical work—the sort of work involved in settling whether two people are disagreeing (or agreeing) about the *same* thing. For Frege, I suggest, it is the fate of such work which requires *Sinn-Bedeutung* to be a many-one relation.

But for Frege's birth there would have been no thoughts about him. Had Venus had a different history, there might have been no thought about it in which it was presented as the Morning Star. The world does *that* much in deciding what thoughts there are. *A fortiori* there are no thoughts about physical quantities without quantities for them to be about. Putnam's point: here, too, it is for the world to decide just what thoughts there therefore are; and just how any such thought makes truth turn on how things are. The way things are is what, in a thought, we represent as some way or other. What truth thus comes to turn on is also something on which turns just how, in that representing, we made truth turn on it. Such world-involvingness of the identity of thoughts lies at the core of Putnam's response to the

idea that laws of logic say nothing. But before seeing how, there is one more step to take.

5. Logic's Topic: A proposition, for YW, structures elements each of a type to which truth-preservation is sensitive, insofar as such preservation is part of being true as such. So what logic says about, e.g., the relation of a conjunction to its conjuncts applies directly to propositions themselves: what is a conjunct, what its conjuncts, can be read directly off of the structure by which these are to be identified. YW's propositions, however, are not, as we have seen, what identify questions of truth as the ones they are. Frege's thoughts do that. Finding conjunctions and conjuncts within Fregean thoughts, though, involves extra-logical work; *inter alia*, working of seeing in just what ways the *same* thought may be decomposed. Fregean thoughts are not each built in a given way from some given stock of building blocks. A *thought*, as opposed to a proposition, is not something to which logic applies in just one way. Each law of logic, or tautology, as YW sees things, embodies a particular way for elements in a representing to cancel out. Thereby it reflects the structure of some given system or class of systems. A domain of Fregean thoughts, though, does not as such form any such system.

Decompose a thought in some given way and one may arrive at something to which laws of logic apply directly. There then remains a substantial question: how *else* that same thought might be identifiable. Identify a proposition by the structure conferred on it by its derivation in *its* system, and no such substantial question remains. Each element in a decomposition of a thought brings it under a certain generality; presents it as the *same* as some range of thoughts in some given respect. Some such samenesses bring us to that to which logic as such is sensitive. What Frege saw as essential to a question of truth, though, is that for any given such question, there is no *one* right way of doing this. A *decomposition* of a thought cannot on its own provide us with a determinate notion of *same thought*. To which, thanks to Putnam, we may add: for any given candidate way of presenting a thought—even by stipulation—whether this *is* a way of presenting that thought, or any thought, is liable to be a substantial, sometimes world-involving, matter.

How, then, could logic tell us how one must think to reach the goal truth? How could there be such a thing as what follows by *logic* from what? Well, how do we get from a whole thought to a structure to which logic speaks directly? Let the whole thought be that Sid slurps. This decomposes into (roughly) an element which makes the thought one whose truth turns on whether Sid is the ways it specifies, and an element which makes truth turn on whether the object it specifies *slurps*—as it were into a *naming* element, making the thought hostage to how some given object is, and a *predicative* element, making it hostage to which objects are some specified way—here, such as to slurp. Each of these elements is of a given type, instanced in an indefinite range of other thoughts. There is, familiarly, the thought that Sid waddles, etc; and there is the thought that Pia slurps, etc. Where there is such a structuring of elements, there is also a truth-preserving inference to a related thought with a different structure: colloquially, from *Sid slurps* to *someone does*. Such is the sort of thing logic tells us.

On what information does the result depend? Logic takes an interest in the occurrence of predicative elements, and of naming elements, and in their distribution within a given decomposition, or corpus of them. It is *not* interested in whether a thought is about Sid, nor in whether it is about slurping. Whatever is proprietary to such notions is not part of what follows from the notion *being true* as such. It *is* interested in an element's recurrence. It matters that it is the *same* way someone is represented both in *Sid slurps* and in *Someone does*. Whether this happens to be slurping is beside the point.

Let us call what I have just abstracted from that decomposition of the thought that Sid slurps a *logical form*. Logical forms can serve as logic's primary interest. We can think of logic as generating some fixed stock of them. Each form would be a construction of indexed thought-element types, each index marking a distribution of some given element of that type. Such a stock of forms would reflect the most fundamental features of *being true* as such. The rules for such constructions would generate such structures as, e.g., ones in which some concept (more properly what it was a concept of) would be predicated of some object. Laws of logic would then be dicta identifying which transitions from some logical forms to others were truth-preserving. Logic treats the phenomenon *logical form*. A standard logical calculus presents the details (within its scope) in that special way peculiar to such calculi.

We began from a conception of logic on which the distinctive feature of laws of logic, aside from their truth, was their maximal generality, in a sense in which such generality is achieved through maximal quantification. On this conception logic speaks to thoughts through a reverse process, instantiation—what, on our present conception, would lead to a decomposition of a thought; *one* such among, perhaps, many. So conceiving laws of logic in this way is one way to try to capture logic's topic-neutrality; its having no special subject matter.

But we now have another way of conceiving the matter. Such leaves logic both universal and topic-neutral. But it does not achieve this by quantifying away from all subject-matter. Rather, it achieves this by virtue of the special subject-matter it makes logic's topic. That special subject-matter is the domain of logical forms. Logic unfolds the notion *being true* in, and by, unfolding the notion *logical form*.

This changed conception of logic is occasion once again to consider the idea that *being true* (the notion logic unfolds), like the notion *momentum*, is made up of many strands. Logic's concern is meant to be something intrinsic to being true itself, hence to any thought. Part of that something is now that such-and-such *are* the logical forms. So, first, any thought is decomposable into some of these. Second any decomposition of any thought is of some one of these. Third, perhaps, for any form whose constituents are place-holders for thoughts or predicative elements thereof, any thoughts or predicative elements (as appropriate) may replace those place-holders to form a compound thought. Thus, any move from thoughts of a given forms to something *of* a form which follows from these by logic's laws is a move *to* a thought, moreover one true if these first ones are. Only some quantifying, e.g., may preserve truth; but all of it preserve being truth-valued. Logic may never lead from thoughts to non-thoughts.

Logic does not tell us what thoughts have which forms. It is thus far insulated from confirmation or refutation by the way things happen to be. But it unfolds a notion with enough independent threads to justify Putnam's refusal to endorse "metaphysical guarantees of the unrevisability of logic." Logic may fix 'the form of coherent thought' as such; in which case there is no such thing as thinking something so, not subject to its dictates. Such is what it is for logic to be universal. Universality does not shield the question *what* laws have such scope from proving, like laws of momentum, hostage to how things are.

6. Truth's Threads Revisited: Having pointed out that laws of logic were not laws of *holding* true (when thoughts would be *held* true), hence not psychological laws, Frege continued,

> It is because of this that they have authority for our thought if it would attain to truth. (1893: xvi)

But, as MW later wrote, neither, equally, is when to count something as being red a matter of when it would be held red. (Cf. *Zettel*: §§429-432) The facts of what would count as red (or green, etc.) hold a certain authority over all thought about colour (or those colours). But it is not that special authority logic holds over all thought. Such, then, must have another source.

Objectivity is *per se* authority external to us. But one can omit attitudes towards colour, or at least towards being red or green. There is no such opting out of logic's laws, or none we currently understand. Also, while we do understand how the world may reveal what momentum really is, we as yet have no idea what it would be for it to reveal to us what (the) logical forms really are. Logic holds a *special* authority over us. Such cannot derive just from the fact that its laws are not psychological.

Frege's idea was: logic is not just universal, but also ineluctable. There is only one thing it could have been, no matter what. Logic (for Frege) unfolds what belongs to *being true* as such. To be a thought, for Frege, is just to fix (or be) some given question of truth. ("I place the content of the word 'true in lead position …and I let *thoughts* follow immediately as that by which truth can come into question at all." (1919: 273) Logic thus could not but be universal. Further, if to think is (*inter alia*) to think thoughts, then logic thus governs all thinkers. These points do not depend on what logic's dicta *are*, or on just what they dictate.

Still, being true is a notion made up of separable threads; as are concomitant notions such as *logical form*. We have seen how such a feature mattered to what momentum is. How might it matter here? In search of an answer one might first look more closely at the notion *ineluctable*. When Frege tries to imagine not being bound by our familiar laws of logic (see 1893, xvi), what he thinks of is thinkers who *flout* these laws. Another idea would be: the thoughts they think (or some) are—despite what *seems* to be just intrinsic to *being true* as such—simply not articulable into those logical forms in terms of which our familiar laws are defined. So our familiar laws would be, for part of their thinking, at least, not flouted, but inapplicable. Here is a new understanding of being ineluctable. Those laws we know may well be inescapable for thought insofar as it articulates into those forms in which the logic we know trades, and yet not of force where (if anywhere) thoughts did not assume those logical forms for which the laws were designed.

Such an idea fits with the way in which we have seen YW's two different ideas of logic's laws as saying nothing come apart. One idea is that these laws say nothing in that their elements cancel each other out. We might thus think of logic's dicta as differing from other thoughts in that, since their role is to govern any though whatever, no provision has been made for them to be hostage in any way to how things turn out to be. In other cases, to put it in YW's terms, lack of provision is within some special system. That system of Newtonian descriptions of mechanical phenomena provides no way for the Newtonian definition of momentum to prove either false or senseless. It stands *within* the system as a fixed point. But the life of the thought that momentum is mass times velocity is not confined to its place in any given special system. Logic is a different story. The universality attaching to its role means that in the case of its dicta cancelling out is not just within any special system.

YW's other idea of saying nothing (6.1222) is: being neither supported, nor called into question, by anything experience may show as to how things are. But saying nothing on the first idea of doing so does not seem to entail saying nothing on this second. Laws which, *if*

laws, hold of all thoughts are plausibly ones whose holding is not hostage in any determinate way to what, given them, is liable to be true or not. They are precisely what would hold independent of the truth-value of such things. YW's idea of cancelling out is a not implausible way of thinking of this sort of insulation from worldly vicissitudes. Such laws, as Putnam's Kant has it, merely limn the form of thought as such (for Frege, of being true as such). But what would be so of laws which *held* cannot, it would seem, by itself select which laws *do* hold. Or not unless the ways in which the world is *liable* to bear on truth and falsity are identified, uniquely, by some structure intrinsic to the domain of thoughts as such—an idea which Putnam has certainly given us reason to reject.

Such abstract rumination begins to gain content with our shifted understanding of what logic is about. Suppose that logic's laws have an identifiable subject matter: logical forms—not what forms given thoughts assume, but what forms there are for thoughts to assume, and how these relate to one another. Here there seems, at first blush, something for a would-be law to get right or wrong. Would not any claim as to what forms there are (in Frege's phrase) expose itself to risk of error—a risk, of course, which some such claims would escape?

But when we look more closely, perhaps such dealing in abstraction is mere word play, a flight of fancy. For what *do* laws of logic say? What features of being true do they unfold? Well, for example, what is truth-valued is either true or false; where which of these it is is independent of what we think. There are thus an easily surveyable variety of ways for the truth-value of a thought to be fixed by the truth-values of others. Logic provides us with logical forms corresponding to these ways; and then rules determining how truth is preserved in moves from some forms to others. So, for example, logic speaks of a way for a thought to be formed from, *A&B*. It tells us that a move from such a thought to the corresponding thought, A, is truth-preserving. Call a thought which assumes this form relative to some thought, A, and some thought, B, a *conjunction* of A and B. Which thoughts are conjunctions of which others? Such is not for logic to say. What it does say is when a thought would be a conjunction of two others: for a start, when that first thought is true *just* where those two others are. Now look at that fact about truth-preservation. How minimal can a fact get and still be one? Such, I think, exemplifies the sort of thing laws of logic (viewed one way) say.

But there is more. Logic tells us what logical forms there are: *any* thought must assume some of these. These then identify *the* ways for thoughts to relate *logically* to one another. Moreover, moves which logic tells us preserve truth it also tells us preserve thought-hood. So, too, with all ways of compounding thoughts (or making thoughts of predicative elements) always preserve thought-hood. So, for example, though identifying thoughts is extra-logical work, for any two thoughts, A and B, there is a third, A&B, and a fourth, and etc. Any two thoughts can be compounded or disjoined, disjunction distributing over conjunction and vice-versa. On the other hand, for a given sort of thought—say, of given water, at a time, that it is boiling—there is, on the one hand, the way a thought of that type would represent things as being, and there is what it so represents: some particular case, that water's being as it then is. To be able to think water to be that way is to be able to recognise, of particular cases (of suitable sort) when they would, when not, be cases of things being the way in question. Such exemplifies abilities we have to recognise the obtaining of a relation to which logic does not speak: a relation between something governed by its laws, and something—bits of history—not. Such an ability can at least be exercised as a control on what logic tells us here. Is it really so that wherever there is a conjunction of a thought with a disjunction there is a disjunction of the conjunction of that thought with each disjunct? Perhaps. But might there not be

something here for logic to be hostage to?

We can now understand how Putnam stands in relation to both YW and MW. Putnam can accept the universality of logic. Universal is just what it is designed to be. Logic, as Frege put it, unfolds the content of the notion *being true*. A thought is precisely 'that by which truth can come into question at all'. So logic applies to all thoughts. Concern with any thought is just its business. There is no pressure for things to be otherwise.

Nor need Putnam dissent from YW's first notion of saying nothing. Logic's concerns (truth-preserving) touch one side of the representing-as relation: ways to represent things; not (directly) things so to represent. Truth belongs to representation. Normally it is a joint product of both sides of that relation: one side *makes* truth turn in a certain way on the other; the other then yields a product of so turning. What represents a law of logic to hold, though, cancels out work of that other side; makes truth turn in the null way on it. Or better: it provides no way for truth thus to turn. Putnam need not balk at this either.

Putnam must, though, with MW, resist YW's second account of saying nothing. A logical truth, like any thought, represents things as a certain way there is for things to be. There are things to be understood as to just what way that is. But it is a world-involving matter what ways for things to be there are (and what each is). So that what is to be understood as to what any given one is is relative to a perspective on it and on the world. What is (now) to be understood as a law of logic would be might, from a more revealing perspective on the world, be to be understood differently, or, perhaps, prove simply ununderstandable, as the world has proven to be. Which rules out YW's idea of *absolute* immunity to worldly bearing. It may belong to a law of logic that *for it to hold* is for the world to be denied a role in deciding that it does. Whether there *is* such law, or such a thing to hold or not, is another matter.

I have here championed one idea of Frege's at the expense of another. Frege was right, and Russell and YW not, about what questions of truth there are, hence about what the truths are across which truth is to be preserved. Thoughts are distinguished from propositions in the room a decomposition leaves for different applications of the notion *same thought*. Just this leaves the space Putnam has charted for the world's role in fixing what ways there are for things to be and how any given one in fact makes truth turn on how things are.

Frege's proving right on this point, though, brings with it a conception of logic decidedly not his. A law, so logic's, *holds*. It is neither true nor false. So it is not a thought. But it *could* relate to what it governs as a quantification to its instantiations. For the project *Begriffsschrift* addressed it was important for logic's laws to do so. On the present view, they do not. They govern relations between thoughts. But in first instance they speak of forms abstracted from thoughts. How they apply to given thoughts is thus not fixed until extra-logical work is done. (Compare the way mechanic's laws apply to given rigid bodies.)

This idea of application-at-a-distance is sometimes said just to be our current conception of logic; a conception reflected in that peculiar form of expression of what logic says, a calculus. If this is right, though, the current conception is one current philsoophers all too often forget.

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