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SORITES PARADOXES AND THE SEMANTICS OF VAGUENESS

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It is sometimes supposed that Sorites Paradoxes are an inevitable consequence of the very nature of vagueness. Take, for example, the term 'bald'. If 'bald' is vague then it lacks precise boundaries. So

- (1) There is a definite number, N , such that a man with N hairs on his head is bald and a man with $N+1$ hairs on his head is not

is false. But intuitively the denial of (1) is equivalent to the assertion of

- (2) For any definite number, N , if a man with N hairs on his head is bald then a man with $N+1$ hairs on his head is also bald.

And (2), together with the obvious truth

- (3) A man with no hairs on his head is bald

entails the obvious falsehood

- (4) A man with a million hairs on his head is bald

via a million applications of modus ponens and universal instantiation. To treat this line of reasoning as a *reductio* of the denial of (1) is to concede that 'bald' is not vague, and hence, in the general case, to concede that no predicates are vague. This conclusion is, of course, itself paradoxical. What, then, has gone wrong?

In this paper I want to present a novel semantics of vagueness which is, I maintain, invulnerable to Sorites Paradoxes such as the one above. I shall begin by briefly sketching and criticizing two well known alternative semantics. The failure of these semantics is, I believe, instructive.

I

It has been suggested by a number of philosophers that a proper understanding of vague discourse requires the admission that truth and set membership come in degrees, rather than being all-or-none.¹ On this approach, real numbers in the interval from 0 to 1 are typically taken to be truth values, with 1 being full-fledged truth and 0 being full-fledged falsity. The same numbers are assigned to the degrees to which objects belong to sets. So, for example, if Herbert is clearly a bald man then the sentence

(5) Herbert is bald

is assigned the value 1, and Herbert is taken to belong to the set of bald men to degree 1. But if Herbert is a borderline bald man then (5) is assigned some number less than 1, 0.6, say, and Herbert's membership in the set of bald men is now taken to be only 0.6 too. In general, a singular sentence '*Fa*' is treated as having the truth value *n*, where $0 \leq n \leq 1$, if, and only if, the referent of '*a*' belongs to the set of *F*s to degree *n*.

A consequence of this approach is that vague predicates do not sharply divide the world into those things to which they apply and those things to which they do not. Rather vague predicates apply to objects to varying degrees. As a man gains hair, for example, he becomes less and less bald; so, the predicate 'bald' applies to him less and less, and the assertion that he is bald diminishes gradually in its degree of truth.

The introduction of degrees of truth permits a relatively straightforward response to the Sorites Paradox with which I began the paper. What the falsity of (4) shows is that (2) is not *wholly* true and hence that (1) is not *wholly* false. It does not follow, then, that 'bald' is really precise. For that would require the stronger claim that (1) is *wholly* true. And, in fact, the value that (1) is given is usually 0.5. This assignment of degree of truth is arrived at as follows: a conjunction is standardly taken to have a degree of truth equal to the minimum value of its conjuncts, and an existentially quantified sentence $(\exists x)Fx$ is standardly taken to have a degree of truth equal to the maximum value of *Fx* for all assignments of objects to *x* in the universe of discourse.² The maximum value, then, for a conjunctive sentence of the form

(6) A man with *N* hairs on his head is bald and a man with *N*+1 hairs on his head is not

is 0.5 (assuming also that the negation of a sentence *P* has a value equal to 1 minus the value of *P*). So, (1) also has the value 0.5 and not the value 0, as the Sorites supposes.³

Notwithstanding this success, the degrees of truth approach sketched above is badly flawed. The problem in its most general form is that vagueness has been replaced by the most refined and incredible precision. Set membership, as viewed

by the degrees of truth theorist, comes in precise degrees, as does predicate application and truth. The result is a commitment to precise dividing lines that is not only unbelievable but also thoroughly contrary to vagueness. The difficulty I am broaching is illustrated in the following sorites:

- (7) A man with zero hairs on his head is bald.
- (8a) If a man with zero hairs on his head is bald then a man with one hair on his head is bald.
- (8b) If a man with one hair on his head is bald then a man with two hairs on his head is bald.
- .
- .
- .
- (8k) If a man with 999,999 hairs on his head is bald then a man with a million hairs on his head is bald.

Therefore, by a million applications of modus ponens, we arrive at the conclusion

- (9) A man with a million hairs on his head is bald.

Since (9) is wholly false, not all of the conditionals, (8a)-(8k), can be wholly true. So, given that every conditional has the value 1 or some lesser value,⁴ there must be a first conditional that is not wholly true. So there must be a first sentence of the form "A man with N hairs on his head is bald" that is not wholly true. But this seems to run directly counter to the idea that 'bald' is vague. Surely it is absurd to suppose that there is some single hair addition that divides the bald from the borderline bald, that changes the degree of truth from 1 to a precise degree less than 1.⁵ Indeed this seems to me no less absurd than supposing that some single hair addition transforms the bald into the non-bald (as would be the case were 'bald' a precise predicate).

It is worth stressing that this problem cannot be overcome simply by supposing that some of the conditionals in the above sequence have an indeterminate truth value somewhere in the interval between 0 and 1.⁶ For even if this is intelligible, it still leaves a sharp dividing line in the sequence between the last conditional that has the value 1 and the first conditional that has a value something other than 1. And this is just not credible. To suppose otherwise is, I suggest, not to take vagueness *seriously*. For it seems to me an essential part of the ordinary vagueness of terms like 'bald', 'young', and 'rich' is that there is no determinate fact of the matter about where truth values change in sequences such as the one, (8a)-(8k). It is this central feature of vagueness which the degrees of truth approach fails to accommodate, regardless of how many truth values it introduces.⁷

The same fact poses a serious difficulty for the supervaluationist approach to vagueness, as we shortly see. On this view, a vague sentence is true if it is true

under all eligible ways of making precise its component vague terms, false if it is false under all eligible ways of making precise those terms, and indefinite or neither true nor false if it is true under some ways and false under others.⁸ Simple sentences about borderline cases are taken to be indefinite in truth-value. However, complex sentences, whose component sentences are indefinite, may be either true or false. Consider, for example, the sentence

(10) Either Herbert is bald or Herbert is not bald

and suppose that Herbert is a borderline bald man. Since any acceptable precisification of 'bald' will guarantee that one of the two disjuncts is true while the other is false, (10) is a logical truth, the component sentences of which are indefinite. So, (10) retains its classical status within the supervaluationist framework.

One obvious objection to supervaluationism is that the Law of Excluded Middle should fail for some vague sentences. For if (10) is true then either Herbert is bald or he is not bald. If this is the case then the question, "Well, which one is it then?", must surely have an answer. So, if (10) is true then *one* of the disjuncts in (10) must surely be true. But if the second disjunct is true, the first must be false. So, "Herbert is bald" must be either true or false. This runs contrary to the assumption that Herbert is a borderline bald man. So, (10) is not true.

I think that this is a good objection to supervaluationism. But I shall not press it further here.⁹ Instead, I want to focus upon how the view fares with respect to the two sorites arguments presented above. In the case of the first sorites, the supervaluationist claims that the falsity of (4) shows only that (2) is false or indefinite and not that (2) is simply false. However, he concedes that (2) really is false, since a hair splitting N exists for any eligible precisification of 'bald'. So, (1) really is true. But this does not entail that 'bald' is precise. For (1) is true in virtue of there being, for each eligible precisification of 'bald', a *different* number N such that a man with N hairs then satisfies 'bald' whereas a man with $N+1$ hairs does not. So, there cannot be any definite answer to the question "Well, which number of hairs is it that a man must have in order to cease being bald?". And this question must have a definite answer, if the term 'bald' is precise.

I am not entirely persuaded by this response. It seems to me very counter-intuitive to count (1) as true while simultaneously denying that the question just raised is, in principle, answerable. Of course, I do not deny that (1) *would* be true were 'bald' made precise in any acceptable way. What I do deny is that this is a reason to assert that (1) *is* true prior to any eligible precisification of 'bald'.

Turning now to the second sorites, the supervaluationist appears to be in immediate trouble. Since (9) is false, not all of the conditionals, (8a)-(8k), are true. So, given that each of these conditionals is either true or not true, as the supervaluationist supposes, there must be a first conditional that is not true, and

this in turn requires that there be a first sentence of the form “A man with N hairs on his head is bald” that is not true. But, as I noted earlier, to suppose that this is the case is not to take vagueness seriously.

Matters are not quite so simple, however. Although (9)’s falsity does show that not all the relevant conditionals are true and hence that

- (11) There is a pair of adjacent conditionals in the sequence such that the first is true and the second is not

is true, this does not automatically mean that a precise dividing line exists between those conditionals that are true and those that are not. For the predicate ‘true’ may itself be vague. And if this is so then (11) has the same status as (1).

It seems to me that classifying (11) as true is just as counter-intuitive as classifying (1) as true. But even putting this to one side, there remains a further problem. If ‘true’ is vague then it has borderline satisfiers in the sequence.¹⁰ So, some of the conditionals are neither true nor false nor indefinite. Rather they have a fourth truth-value, one we might call ‘indefinitely true’. These lie at the vague border of the true and the indefinite sentences. Now, according to supervaluationism, the Law of Excluded Middle never fails. So, ‘indefinitely true’ itself has borderline satisfiers in the sequence or it does not. If it does not then there is a precise dividing line in the sequence between the true sentences and the indefinitely true ones. On the other hand, if ‘indefinitely true’ has borderline satisfiers in the sequence then some of the sentences are indefinitely indefinitely true. So, the price of avoiding a precise dividing line between the true and the indefinitely true sentences is the admission of sentences that have a fifth truth value. Clearly this argument may be repeated until as many truth-values are introduced as there are conditionals in the sequence so that every single conditional has its own truth-value. The upshot is that there are now precise dividing lines throughout the sequence: the addition of a hair *always* makes a difference. Vagueness has disappeared.

The major lesson, I believe, we should learn from the failure of supervaluationism and the degrees of truth theory, as I have presented them, is that vagueness, if taken at face value, cannot be reconciled with *any* precise dividing lines. It is, I suggest, a central aspect of the vagueness of ordinary terms that it is not true that there are adjacent pairs of conditionals in sorites sequences such as (8a)-(8k) such that the first is true and the second not. For sequences like these, there simply is no determinate fact of the matter about where truth-value changes occur. Any attempt to state truth conditions for vague discourse in precise language will inevitably run afoul of this fact. Vagueness cannot be analyzed away.

I turn next to my own proposal.

II

On the approach I favor, there are three truth-values: true, false, and neither true nor false (or indefinite). The third value here is, strictly speaking, not a truth-value at all but rather a truth-value gap. In my view, there are gaps due to failure of reference or presupposition and gaps due to vagueness.¹¹ Corresponding to the two-valued connectives \neg , \wedge , \vee , \rightarrow , and \leftrightarrow are the three-valued connectives \neg , \wedge , \vee , \rightarrow , and \leftrightarrow . These connectives have the following tables:

P	$\neg P$	P \setminus Q	P \wedge Q			P \vee Q			P \rightarrow Q			P \leftrightarrow Q		
			T	I	F	T	I	F	T	I	F	T	I	F
T	F	T	T	I	F	T	T	T	T	I	F	T	I	F
I	I	I	I	I	F	T	I	I	T	I	I	I	I	I
F	T	F	F	F	F	T	I	F	T	T	T	F	I	T

The guiding principles in the construction of these tables are easily explained.¹² (1) The negation of a statement of given truth-value is its opposite in truth-value. (2) A conjunction is true if both its conjuncts are true and false if either conjunct is false. Otherwise it is indefinite. (3) A disjunction is true if either disjunct is true and false if both disjuncts are false. Otherwise it is indefinite. (4) The truth-value of $P \rightarrow Q$ is to be the same as that of $\neg P \vee Q$. (5) The truth-value of $P \leftrightarrow Q$ is to be the same as that of $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

The tables presented above agree with the usual two-valued ones when only Ts and Fs are involved. However, there are no three-valued tautologies, since two-valued tautologies can take the value *I* in the three-valued case. For example the Law of Excluded Middle takes on the value *I* when *p* does so. Let us say that a statement form is a quasi-tautology just in case it has no false substitution instances. Then the Law of Excluded Middle and all other two-valued tautologies are quasi-tautologies in the above system.

It may perhaps be charged that the proposed truth-tables yield some implausible truth-value assignments in connection with certain compound sentences having indefinite components. In particular, if *A* is indefinite, then $A \rightarrow A$ is indefinite as is $A \wedge \neg A$. I concede that the tables would certainly be mistaken, if they permitted $A \rightarrow A$ to be false and $A \wedge \neg A$ to be true. But they do no such thing. $A \rightarrow A$ is a quasi-tautology and $A \wedge \neg A$ is a quasi-contradiction. So, while the former statement cannot be false and the latter cannot be true, both can be indefinite. This seems to me entirely palatable. After all, it is surely reasonable to require that sentences of the form $P \rightarrow Q$ be equivalent to sentences of the form $\neg P \vee Q$ and also that $P \wedge \neg P$ be equivalent to $\neg(P \vee \neg P)$. As I urged earlier, it is also surely reasonable to deny that sentences of the form $P \vee \neg P$ must always be true, given the existence of borderline cases. There is, then, good reason to deny that $A \rightarrow A$ must be true and also good reason to deny that $A \wedge \neg A$ must be false.¹³

Turning next to predicates, I suggest that for the purposes of formal semantics the following treatment suffices for any extensionally vague monadic predicate F : given a non-empty domain D , F is assigned an extension S and a counterextension S' . S is the set of objects of which F is true; S' is the set of objects of which F is false. S and S' are not classical sets, however.¹⁴ Nor even are they sets of the sort countenanced by the degrees of truth theorist (i.e., sets having precise identity conditions and members to precise degrees). Rather it is crucial to a proper understanding of the semantics of vagueness that they be taken to be genuinely vague items. This needs a little explanation.

Consider the set of tall men. Men who are over 6 feet 6 inches are certainly members of this set and men who are under 5 feet 6 inches are certainly not. Intuitively, however, some men are borderline members: there is no determinate, objective fact of the matter about whether they are in the set or outside it.¹⁵ Are there any remaining men? To suppose that it is true that this is the case is to postulate more categories of men than are demanded by our ordinary, non-philosophical conception of the set of tall men and hence to involve ourselves in gratuitous metaphysical complications. It is also to create the need to face a potentially endless series of such questions one after the other as new categories of men are admitted. On the other hand, to suppose that it is false that there are any remaining men is to admit that every single man fits cleanly into one of the three categories so that there are sharp partitions between the men in the set, the men on the border, so to speak, and the men outside. And intuitively, pretheoretically it is not true that there are any sharp partitions here. What, I think, we should say, then, is that it is objectively indeterminate as to whether there are any remaining men. In the ways I have just described, the set of tall men is, I maintain, a vague set.

I propose to generalize from this example. Let us hold that something x is a borderline F just in case x is such that there is no determinate fact of the matter about whether x is an F . Then I classify a set S as vague (in the ordinary robust sense in which the set of tall men is vague) if, and only if, (a) it has borderline members and (b) there is no determinate fact of the matter about whether there are objects that are neither members, borderline members, nor non-members. This characterization of vague sets may seem to entail that one of the basic axioms of set theory, namely the Axiom of Extensionality, is false. But in reality it does no such thing. I shall elaborate upon this point later.

I hope that I have now managed to provide an informal clarification of my use of the term 'vague' in application to sets. I might add here that I do not wish to deny that other kinds of objects may properly be classified as vague. The property of baldness, for example, is, in my view, a vague object. It is neither clearly a feature of some people nor clearly not a feature of those people. Baldness, moreover, would have remained vague, even if there had been only very hairy or wholly hairless people. In general I take a property P to be vague (in the ordinary, robust sense in which baldness is vague) only if (a) it *could*

have borderline instances, and (b) there is no determinate fact of the matter about whether there *could* be objects that are neither instances, borderline instances, nor non-instances.¹⁶ I include clause (b) here for essentially the same reasons as those I gave in connection with the earlier (b) clause for sets.

One further point: once properties are acknowledged that could have borderline instances, no conceptual barrier exists to the admission of properties which are such that there is no determinate fact of the matter about whether they could have borderline instances. Such properties, some of which will concern us later, might be called 'vaguely vague' or 'indefinitely vague'.¹⁷ And what goes here for properties goes *mutatis mutandis* for sets.

Returning now to the formal semantics, with the introduction of vague sets in connection with extensionally vague predicates, we may state truth conditions for vague singular sentences as follows. For any individual constant c , let i_c be the object in D assigned to c . Then Fc is true iff i_c belongs to S ; Fc is false iff i_c belongs to S' ; and Fc is indefinite iff there is no determinate fact of the matter about whether i_c belongs to S (or to S').¹⁸ The generalization to n -place predicates is straightforward.

It may be objected that my use of the locution 'there is no determinate fact of the matter about whether' introduces a vicious circularity into the above truth-conditions. But I deny that this is really the case. The truth conditions state conditions for the application of the *predicates* 'is true', 'is false', and 'is indefinite'. By contrast, the words 'there is no determinate fact of the matter about whether' form a *sentence operator*. This sentence operator cannot be analyzed as, nor is it equivalent to, the predicate 'is indefinite' or 'is neither true nor false'. For one thing, a sentence such as 'Everything James says is indefinite' may be true but 'There is no determinate fact of the matter about whether everything James says' is unintelligible. For another, it seems to me no more plausible to classify assertions of the type 'There is no determinate fact of the matter as to whether p ' as covertly meta-linguistic than it is to classify assertions of the type 'It is not the case that p ' in like manner. Finally and relatedly, on my view, it makes good sense to say that a given singular sentence is indefinite because there is no determinate fact of the matter about whether the appropriate individual belongs to the appropriate set but not to say that the converse is the case.¹⁹ So, I reject the above charge of vicious circularity.

Turning now to the quantifiers, we may introduce $(\exists x)$ and (x) as follows: $(\exists x)Fx$ is to be true if Fx is true for some assignment of an object of D to x ; false if Fx is false for all assignments; and indefinite otherwise. $(x)Fx$ is to be true if Fx is true for all assignments of objects of D to x ; false if Fx is false for some assignments; and indefinite otherwise.

Given these definitions, we can see why my claim that some sets are vague does not entail that the Axiom of Extensionality is false. What the axiom asserts is this: where S and S' are any sets, S is identical with S' if, and only if, for any object, x , x belongs to S if, and only if, x belongs to S' . Trouble for the

axiom lies with the case where S and S' are vague sets which are identical (or which differ only with respect to their borderline members). Here, the statement schema—call it ' A '—that x belongs to S if, and only if, x belongs to S' has assignments under which it is not true, since there are objects that are borderline members of S and S' . However, A is not false under these assignments. Rather, by the truth-table for \leftrightarrow , it is neither true nor false. So, the universally quantified statement $(x)Ax$ is neither true nor false. So, the statement, $S=S' \leftrightarrow (x)Ax$, has an indefinite right hand side in the above case. So, the Axiom of Extensionality comes out as *indefinite* under the proposed semantics.

Just as the Axiom of Extensionality is not false, on my view, so too are none of the other axioms of classical set theory. Moreover, it is not merely a contingent fact that the Axiom of Extensionality is not false. Rather it is necessary. My position here with respect to the Axiom of Extensionality is parallel to the one taken above with respect to two-valued tautologies. In the three valued case, these tautologies become quasi-tautologies. Likewise, the Axiom of Extensionality becomes a quasi-(necessary truth). Of course, if the Axiom is qualified by a clause which restricts S and S' to precise sets then it remains a full blooded necessary truth.

If, as I am claiming, the Axiom of Extensionality is neither true nor false, it cannot be used to demonstrate that two sets that differ only with respect to their borderline members are not identical. This need not concern us, however. For the sets can be distinguished by means of Leibniz' Law: one set has a property that the other lacks, namely having such-and-such an object as a borderline member.²⁰

At this stage, it might be objected that there is another problem of circularity with my proposal. Vague sets are essentially governed by the logic I have presented. So, the concept of a vague set cannot be understood unless the logic itself is already understood. But the logic makes reference to vague sets. So the concept of a vague set cannot really be understood at all.

The claim that I reject here is the claim that the concept of a vague set cannot be understood unless the logic is already understood. Consider a parallel. The concept of disjunction is essentially governed by the logic of disjunctive sentences. But this logic uses the concept of disjunction: ' p or q ' is true just in case ' p ' is true or ' q ' is true. So, understanding the logic cannot be a precondition for understanding the concept. It is, then, a mistake to suppose that the truth-conditions for disjunctive sentences analyze the meaning of the term 'or'. Rather it is because 'or' means what it does that the truth-conditions obtain. One who understands the concept will use it in accordance with the logic but a full grasp of the metalinguistic sentences which utilize the concept in the logic is no part of that understanding.

What is true here for disjunction is true, on my view, for the concept of a vague set. This concept can be explained in an intuitive, pretheoretical way, as I did earlier. Grasping this explanation does not itself presuppose a full

understanding of the metalinguistic sentences specifying the conditions of application of the truth-value predicates for vague sentences—unless, of course, the operator ‘there is no determinate fact of the matter about whether’ is to be analyzed in terms of the metalinguistic predicate ‘is indefinite’, a position I have already rejected. So again I deny that there is any troublesome circularity.

This brings me to a general point I urged in Section I. It seems to me that if Sorites Paradoxes are to be satisfactorily handled, it is crucial that the truth conditions for vague sentences not be stated in a language that is governed by classical logic. The purpose of formal semantics, in my view, is not to give reductive explanations or analyses of the meanings of various sorts of sentences in vague terms (or in any other terms for that matter). One who lacks the concept *all*, for example, will not come to understand it by being shown the truth conditions for universally quantified sentences. Likewise, one who has no grasp of the concept *possibility* will not be enlightened by being given truth conditions for modal sentences that employ modal primitives.²¹ Rather the purpose of a formal statement of truth conditions is, or should be, to explain rigorously how the truth-value predicates are to be applied, and to do so in a way that is compatible with our prior, ordinary understanding of the relevant concepts and sentences.

We are now ready to take up the Sorites Paradoxes. Let us begin with the Paradox from the opening paragraph of the paper. I have three objections to this sorites. First, since premise (3) is true and the conclusion, (4), false, what follows, on my view, is that it is not true that premise (2) is true, and *not* that it is false as the classical reasoning supposes. Secondly, (2) is, in fact, indefinite in truth-value. Let me explain. It is not true that there is any assignment of numbers to the statement schema

- (12) If a man with N hairs on his head is bald then a man with $N + 1$ hairs on his head is bald

under which it is false. However, there are assignments under which both its antecedent and its consequent are indefinite, since there are borderline bald men who would not cease to be borderline bald by gaining a hair. So, there are assignments under which (12) is indefinite. So, the universally quantified statement, (2), is itself indefinite. Thirdly, if (2) is indefinite then the statement

- (13) It is not the case that there is a number N such that a man with N hairs is bald and a man with $N + 1$ hairs is not,

which is equivalent to (2), must also be indefinite. So (1) is indefinite. So it has not been shown that it is true that there is an N such that a man with N hairs is bald and a man with $N + 1$ hairs is not.²² So the argument certainly does not show that ‘bald’ is precise.

Before I turn to the sorites which is based on the sequence of conditionals, (8a)-(8k), I want to discuss another sorites which may seem to create difficulties

for my position at the meta-linguistic level. Consider the list of statements whose members are of the form

(14) A man with N hairs on his head is bald,

where N ranges from 0 to 1,000,000. Call these statements $M_0, M_1, \dots, M_{1000000}$. Surely, it may be said, it can be demonstrated that, on my view, there is some statement, M_k , such that M_k is true and M_{k+1} is not true. For suppose that there is no such statement. Then it follows that for any statement, M_k , if M_k is true then M_{k+1} is true. And from this, given that M_0 is true, by repeated applications of universal instantiation and modus ponens it may be inferred that $M_{1000000}$ is true. But $M_{1000000}$ is false. So, there is a sharp transition from the true statements in the sequence to the indefinite ones. This claim is no more plausible, however, than the already rejected claim that the addition of a single hair changes a bald man into a man who is not bald.

What this argument shows, I maintain, is that the statement

(15) It is not the case that, for some k , there is a statement M_k such that M_k is true and M_{k+1} is not true

is not true. But this does not entail that

(16) For some k , there is a statement M_k such that M_k is true and M_{k+1} is not true

is true, since (15) may be indefinite. And, in fact, in my view, both (15) and (16) are indefinite. My defense of this classification is as follows: in the sequence of statements $M_0, M_1, \dots, M_{1000000}$ there are initially true statements, then later there are indefinite statements, and then finally there are false statements. It seems clear that competent language users will not agree upon precisely where the boundaries are to be drawn in the sequence between the true, the indefinite, and the false statements. Of course, this is not to say that such people will not specify precise points if they are *forced* to assign either 'true' or 'false' or 'neither' to each of the statements $M_0, M_1, \dots, M_{1000000}$ one after another.²³ Still it seems highly unlikely that even one and the same person will pick exactly the same points on different occasions. It is not true, then, that the transitions from true to indefinite statements and from indefinite to false statements are sharp. Consider now the sequence of statements ' M_0 is true', ' M_1 is true', ..., ' $M_{1000000}$ is true'. Given that it is not true that there is a sharp transition from true to indefinite statements in the object language, I maintain that it is not true that any conjunction of the type ' M_n is true $\wedge M_{n+1}$ is not true' is itself true. So, it is not true that (16) is true. Equally, however, it is not true that every conjunct of the above type is false. For that would entail that the millionth statement is true, given that the first is. So (16) must be classified as indefinite, as must (15).

It may be objected that (16) cannot be indefinite unless some statements of the form ' M_n is true' are themselves indefinite. And, on my account, it is false that some such statements are indefinite. For if any given statement M_i is true then ' M_i is true' is certainly true; and if M_i is either false or indefinite then it is false that M_i is true. Either way, then, ' M_i is true' is not indefinite.

My response to this objection is twofold. First, what my position commits me to is the claim that there is no determinate fact of the matter about whether there are any statements of the form ' M_n is true' that are indefinite and not to the claim that it is false that there are such statements. To see this, suppose that there is a statement ' M_j is true' that is indefinite. Then it cannot be true that M_j itself is either true or false or indefinite. So it is not true that there is a statement ' M_j is true' that is indefinite. But neither is it false that there is such a statement. For then every statement of the type ' M_n is true' would be either true or false, with the result that there would be a sharp transition from the true statements of the type ' M_n is true' to the false ones. Intuitively, it is not true that there are such transitions. So it is, I maintain, indeterminate whether there are statements of the type ' M_n is true' that are indefinite.

Secondly, nothing in the earlier semantics requires that an existentially quantified sentence, $(\exists x)Fx$, be indefinite only if Fx comes out as indefinite under some assignments. If there is no determinate fact about whether Fx is indefinite under some assignments then it will not be true that Fx is false under all assignments. So if it is also not true that Fx is true under some assignments, $(\exists x)Fx$ will count as indefinite.²⁴ So (17) can be indefinite without it being true that some statements of the form ' M_n is true' are indefinite.

In claiming that it is not true that there are sharp transitions between the true and the indefinite statements and the indefinite and the false statements in sequences like $M_0, M_1, \dots, M_{1000000}$ I am not thereby claiming that the predicates 'is true', 'is indefinite', and 'is false' are extensionally vague. For if 'is true' is extensionally vague then it follows that the set of true sentences has borderline members. This requires that there be sentences which are such that it is neither true nor false that they are true. And this, in turn, requires that there be sentences that are neither true nor false nor indefinite. I maintain that it is not true that there are such sentences. So I do not accept that 'is true' is extensionally vague. And the same goes *mutatis mutandis* for 'is false' and 'is indefinite'. Of course, in taking this view I am not committing myself to the position that these predicates are precise. Indeed, it is crucial to my account that they *not* be classified as precise. For if they were then every sentence would be either true or false or indefinite, and that would not only generate sorites difficulties of its own (as we shall shortly see) but also run counter to my claim that it is indefinite whether no statement of the form ' M_n is true' is indefinite. Rather my view on the truth-value predicates is that they are vaguely vague: there simply is no determinate fact of the matter about whether the properties they express have or could have any borderline instances. So, it is indefinite

whether there are any sentences that are neither true nor false nor indefinite.

Given my position on 'true' and the other truth-value predicates in the first meta-language, what should be said about the truth-value transitions in the higher meta-languages? The answer must be that in the higher level sequences it is *never* true that such transitions are sharp. Let me explain. Consider again the sequence of statements, ' M_0 is true', ' M_1 is true', ..., ' $M_{1000000}$ is true'. Suppose that for some n there is a statement of the form ' M_n is true' which is true and which is such that ' M_{n+1} is true' is not true. If any statement of the form ' M_n is true' is true then the corresponding object language statement of the form M_n is true. Also if any statement of the form ' M_{n+1} is true' is not true then the corresponding statement of the form M_{n+1} is not true. For obviously if, for any given n , M_{n+1} is true then it is true that M_{n+1} is true and hence that ' M_{n+1} is true' is true. So if the initial supposition is true, then there is a statement of the form M_n which is true and which is followed by a statement of the form M_{n+1} which is not true. But the consequent here is not true, according to my view earlier. So, it is not true that the initial supposition is true. So, it is not true that the transition from 'true' to 'not true' in the second level sequence is sharp. Clearly, this argument may be generalized to show that it is not true that the transitions from 'true' to 'indefinite' and from 'indefinite' to 'false' are sharp in any of the higher level meta-linguistic sequences.

I come finally to the sorites based on the conditionals, (8a)-(8k). My response to this sorites should not be difficult to anticipate: given that (9) is false, we must accept that it is not true that all of (8a)-(8k) are true. But we need not hold that there is a first conditional in the sequence that is not true. Instead, given the proposed semantics, we should hold that it is neither true nor false that there is a first conditional that is not true. Thus, there are true conditionals initially, and indefinite conditionals later, but it is not true that there is a sharp transition from the former to the latter.

It may seem that there is a difficulty lurking here that I have not fully put to rest. Since there are indefinite conditionals in the sequence, (8a)-(8k), not all of the conditionals are true. So, either (8a) is not true or (8b) or some later conditional is not true. Surely then at some point in the sequence there must be a pair of adjacent conditionals such that the first is true and the second is not.

There is an unstated assumption in this argument, namely that each conditional in the sequence is either true or not true. Without this assumption, the reasoning is invalid. To see why, consider how the argument must go. (8a) is true. So (8b) or some later conditional is not true. Suppose (8b) is true. Then either the next conditional is not true or the one after that is not true or On the other hand, suppose (8b) is not true. Then (8a) and (8b) differ in truth-value and there is a pair of conditionals such that the first is true and the second is not. It is obvious that repeating this style of argument an appropriate number of times will not generate the overall conclusion unless we assume

(17) Every conditional in the sequence is either true or not true (i.e., false

or indefinite.)²⁵

I refuse to accept this assumption. On my view, (17) is indefinite. Let me explain.

According to what I said earlier, given a nonempty domain D , $(x)Fx$ is true if Fx is true for all assignments of objects of D to x ; false if Fx is false for some assignments; and indefinite otherwise. Now, the truth value predicates, I claim, are vaguely vague. So, there is no determinate fact of the matter about whether the schema

(18) If x is a conditional in the sequence then x is either true or not true

is indefinite under any assignments. So, it is not true that (18) is true under all assignments. Nor is it true that (18) is false under some assignments. For there is no determinate fact of the matter about whether there are *any* assignments under which (18) has a true antecedent and a false consequent. So, (17) must be classified as indefinite. The point to note, then, is that, on the stated semantics, $(x)Fx$ can be indefinite even if it is not true that Fx has any indefinite assignments. This point parallels the point I made earlier about $(\exists x)Fx$ in response to an objection to my classification of (16) as indefinite.²⁶

I conclude that sorites paradoxes present no real difficulty for my semantics. This is, I maintain, largely because, unlike other prominent semantics, it concedes that the world is, in certain respects, intrinsically, robustly vague; and it avoids, at all levels, a commitment to sharp dividing lines. This position is, I suggest, consonant with both our ordinary, common-sense view of what there is and our pre-theoretical intuitions about vagueness.²⁷

Notes

1. See, e.g., David Sanford, "Borderline Logic," *American Philosophical Quarterly*, 19 (1975), pp. 29-39; George Lakoff, "Hedges: A Study in Meaning Criteria and the Logic of Fuzzy Concepts," in *Contemporary Research in Philosophical Logic and Linguistic Semantics*, ed. by D. Hockney et al. (Dordrecht, Holland: D. Reidel, 1975), pp. 221-272.
2. See, e.g., George Lakoff, *ibid.*, p. 230.
3. (2) likewise has the value 0.5.
4. One degrees of truth theorist who explicitly accepts this assumption is Graeme Forbes. See his *The Metaphysics of Modality* (Oxford: Oxford University Press, 1985), p. 170.
5. It might be replied that a man who has more than zero hairs on his head is not completely bald. So, only (7) has the truth value 1. But, as we ordinarily use 'bald', we certainly take it to apply fully to someone who has a little hair around the rear of the head at the ear level and none above. In any event, as Terry Horgan has noted, this strategy will not work in general for vague terms, since sorites arguments can be constructed that run parallel to the one above and that do not begin with a limit case. Consider, for example, the following: 'A man with 10^6 hairs on his head is hairy', 'If a man with 10^6 hairs on his head is hairy then a man with 10^6-1 hairs on his head is hairy', ..., etc.; or 'A man who is 100 years of age is old', 'If a man who is 100 years of age is old then a man who is 100

- years -1 day is old', ..., etc. See here Terry Horgan, "Psychologistic Semantics, Robust Vagueness, and the Philosophy of Language," in *Belief Systems in Language: Studies in Linguistic Prototypes*, ed. by S. L. Tsohatzidis (London: Routledge, 1990, pp. 535-557).
6. Nor by claiming that some of the conditionals lack a truth value. In this case, the objection raised below still applies *mutatis mutandis*.
 7. David Sanford has suggested to me that the degrees of truth theorist can handle the above problem by abandoning the principle that every statement has exactly one truth-value. According to Sanford's latest proposal (presented in his "Well-behaved Higher-order Borderline Cases," forthcoming), the phenomena of vagueness require that statements be assigned, in many cases, vague ranges of values instead of unique precise values. This proposal does not overcome the difficulty I am broaching, however. Given that every conditional in the sequence, (8a)-(8k), either has some unique precise value or some range of values, precise or vague, there must still be a first conditional that has some value or range of values other than the value 1 *simpliciter*. And this again, in my view, is just not credible.
 8. See, e.g., Kit Fine, "Vagueness, Truth, and Logic," *Synthese*, 30 (1975), pp. 265-300.
 9. For more on this objection, see my "Supervaluationism and the Law of Excluded Middle," *Analysis*, 49 (1989), pp. 141-143.
 10. So, some sentences in the sequence are such that it is neither true nor false that they are true.
 11. Where a gap is due to vagueness, I maintain that something is said which is neither true nor false. I deny, however, that anything is said in the case where a gap is due to failure of reference. I am inclined to extend the latter view to gaps due to failure of presupposition.
 12. The tables are due to S. C. Kleene (although Kleene himself did not apply them to vague statements). See his *Introduction to Metamathematics* (Amsterdam and Princeton, 1952), pp. 332-340. For an application of Kleene's approach to the Paradox of the Liar, see Saul Kripke, "Outline of a Theory of Truth," *Journal of Philosophy* (1975), pp. 690-716.
 13. The claim that $A \rightarrow A$ is sometimes indefinite is, of course, compatible with the claim that A is a logical consequence of A , since one sentence will be a logical consequence of another so long as it is cannot be true that the first is anything other than true when the latter is true. Within the above framework, then, the deduction theorem no longer holds.
 14. Some philosophers might respond here, "Well, they aren't *sets* at all then." This response is given from a particular theoretical perspective. It forgets that the term 'set' is a perfectly ordinary one and that in its ordinary usage it is *not* reserved for sharp entities. See below for an example of a common or garden vague set.
 15. I might add that I do not deny that some borderline tall men are closer to being tall than other borderline tall men.
 16. For an explanation of why these conditions are not both necessary and sufficient for property vagueness together with a statement of some more complex conditions that are, see my "Vague Objects," *Mind*, 99 (1990), pp. 535-558.
 17. There also seems to be no immediate conceptual barrier to the admission of properties that are such that (a) they could have borderline instances and borderline borderline instances, and (b) there is no determinate fact of the matter about whether there could be objects that are neither instances, borderline instances, borderline borderline instances, nor non-instances. These properties might be called 'second level vague'. Still higher levels of vagueness may be intelligible. However, I can think of no clear common or garden examples of properties exhibiting even second level vagueness.
 18. The term 'iff', as it is used in these truth-conditions, is not to be understood classically. One possible interpretation is in terms of \leftrightarrow . Under this

interpretation, the biconditionals in the text have the status of quasi-(necessary truths). See here page 12. I might add that corresponding points apply to the term 'if', as it is used in the statements of various truth-conditions elsewhere in the text.

19. The only worthwhile equivalence I can provide for sentences of the type "There is no determinate fact of the matter about whether p " is "It is not the case that it is determinate that p and it is not the case that it is determinate that not- p ." This equivalence doesn't constitute a reductive analysis, since it introduces another comparable sentence operator, namely 'it is determinate that' or, abbreviated, *Det*. Truth-conditions for *Det* may be stated as follows: *Det p* is true if p is true, and false if p is false or indefinite. These truth-conditions do not analyze the meaning of *Det* any more than the truth-conditions for \vee or $(\exists x)$ analyze the meanings of 'or' or 'some'. See here my comments on truth-conditions in the sixth and seventh paragraph following the present one in the body of the paper.
20. I might add that the two occurrences of 'if and only if' in the Axiom of Extensionality do not *have* to be taken as instances of the connective \leftrightarrow . Other non-classical interpretations are possible. See here my *Vagueness and Reality*, in preparation.
21. For one such semantics, see Alvin Plantinga, *The Nature of Necessity* (Oxford: Clarendon Press, 1974).
22. On my view, then, what follows from 'bald''s being vague is that (1) is not true and not that (1) is false, as the sorites argument supposes.
23. Indeed, one can imagine people changing their views within the space of a few seconds. Consider, for example, the following imaginary exchange: "You said that M_{130} is true. Now that you've classified M_{131} as neither true nor false, do you still think that M_{130} is true?" "Well, I guess not." "So, M_{130} is neither true nor false then?" "I suppose so." "What of M_{129} ? Is that neither true nor false too or is it true as you held before?" "Oh, I just don't know what to say." "But if you don't know how to classify M_{129} , do you still want to classify M_{130} as neither true nor false?" "Yes...No... I'm befuddled."
24. The understanding expressed here of the 'otherwise' clause in the truth conditions for $(\exists x)Fx$ extends *mutatis mutandis* to the 'otherwise' clauses appearing elsewhere in truth conditions in this paper. This understanding guarantees that it is not true that there are any further alternatives.
25. It should be noted that for a finite sequence the claim that each conditional is either true or not true is equivalent to a conjunction, the first member of which is that (8a) is true or not true. This conjunction is indefinite even though, on my view, it is not true that there is an indefinite conjunct. No problem arises here in connection with the earlier truth-tables, since the principle I stated for the case of conjunction was as follows: a conjunction is true if both (all) conjuncts are true, false if at least one conjunct is false, and indefinite *otherwise*.
26. I want to mention here a complication which arises in connection with what I earlier called (in footnote 17) 'second-level' vague entities. If there can be such entities then the semantics I have sketched will not apply to them. To see this, suppose that Q is a second-level vague property and that a is one of its borderline borderline instances. Then the statement that a has Q will be such that it is neither true nor false that it is neither true nor false. So there will be a statement that cannot be assigned one of the three truth values: true, false, indefinite. For an account of how the semantics I have presented can be extended to the higher-level vague, see my "Vague Objects," op. cit., pp. 554-555.
27. An earlier version of this paper was read at a symposium on vagueness at the American Philosophical Association, Pacific Division, March, 1991. For a full elucidation of the ideas sketched here, see my *Vagueness and Reality*, in preparation.

After completing the paper, I received a copy of Terry Horgan's contribution to this volume ("Robust Vagueness and the Forced-March Sorites"). Since Horgan's essay presents a direct challenge to both the metaphysical position I

have elaborated and the associated logic, I want now to take up Horgan's central argument and to explain briefly where, in my view, he goes wrong.

Consider a sorites sequence of baldness statements, $B(0)$ - $B(10^7)$, beginning with $B(0)$ and its right hand neighbour, $B(1)$. Either $B(1)$ has the same truth-value as $B(0)$, or else $B(1)$ and $B(0)$ differ in truth-value. But if $B(1)$ and $B(0)$ differ in truth-value, then there is a sharp boundary (contrary to my view). So, given that $B(0)$ is true, $B(1)$ must be true. Consider next $B(1)$ and $B(2)$. A structurally identical argument for $B(1)$ and $B(2)$ establishes that $B(2)$ must be true. Ten million such arguments eventually establish that $B(10^7)$ must be true. But $B(10^7)$ is not true. So, Horgan concludes, there can be no sorites sequence with true statements at the beginning and false statements at the end, but no sharp transitions anywhere in between.

My initial reaction to this sorites is the one Horgan himself foresees. According to my logic, the statements

- (1*) For any n , either $B(n)$ and $B(n+1)$ have the same truth-value or they differ in truth-value

and

(2*) For any n , sub-argument $A(n)$ (concerning $B(n)$ and $B(n+1)$) is sound are indefinite. This does not require, of course, that there be some particular sub-argument which is such that there is no determinate fact of the matter about whether it is sound, or relatedly that there be some particular pair of statements, $B(i)$ and $B(i+1)$, for which it is indeterminate whether its members have the same or different truth-values. It suffices that there be no determinate fact of the matter about whether there is a sub-argument which is not sound and whether there is a pair of statements that have neither the same nor different truth-values. But it is true that the overall sorites argument is sound only if it is true that each and every sub-argument is sound. So, it is not true that the overall argument is sound.

Horgan claims that this reply, which is dictated by the logic I have sketched, can be shown to be incoherent. Take some single pair of statements, $B(i)$ and $B(i+1)$ in the sorites sequence. Horgan argues that if forced to confront the question, "Are $B(i)$ and $B(i+1)$ alike in truth-value?," a defender of the logic has no option but to reply "Yes". So, then, I must affirm

- (3*) Either $B(i)$ and $B(i+1)$ have the same truth-value or else they differ in truth-value.

This line of reasoning may be repeated for each and every adjacent pair of statements in the sorites sequence, taken one pair at a time. So, eventually, according to Horgan, I will be forced to affirm every instance of the following open sentence:

- (4*) Either $B(n)$ and $B(n+1)$ have the same truth-value or else they differ in truth-value,

where $0 \leq n \leq 10^7$. But it is logically incoherent to affirm every instance of (4*) while also refusing to affirm (1*).

I am entirely unpersuaded by this argument, ingenious though it is. Horgan assumes that it is legitimate for him to force the defender of the logic to face a question of the form "Are $B(n)$ and $B(n+1)$ alike in truth-value?" for *every* pair of adjacent statements in the sorites sequence, one after the other. But this can be legitimate only if *each* such question has an answer. And, in my view, it is simply not true that each question has an answer. As before, *and in parallel ways*, this assertion is indefinite. This follows directly from the logic I have elaborated. (To suppose that each question has an answer is effectively to suppose either that all the statements in the sequence are alike in truth-value or that there are sharp boundaries.) So, the question and answer game Horgan describes is one which an astute defender of the logic must, by the very

principles of the logic, refuse to play.

This is not to say, of course, that if I am asked only whether $B(0)$ and $B(1)$, say, have the same truth-value, it is incorrect for me to say "Yes". Nor is it to claim that this reply is not to be taken at face value. The point is rather that if I am a cogent defender of the logic, I should not allow myself to be drawn into answering questions about *every* member of a sorites sequence. For if I am so drawn in, I am no longer a cogent defender of the logic, and hence my answers are irrelevant. What they reveal is merely that I have not understood the logical underpinnings of my own position or the way in which vagueness intrudes at all levels. There is, then, no logical incoherence of the sort Horgan claims to find.

Of course, Horgan may insist that it does make sense to raise questions individually with respect to *all* the adjacent pairs in the sequence. Perhaps so, from his own theoretical perspective. But to the extent that my logic successfully captures our ordinary reasoning about vagueness, he should realize that he is no longer working within the commonsense framework. And look at the metaphysical consequences of this insistence: There are no such properties as baldness, and no such objects as mountains, tables, and people. Horgan accepts these apparently lunatic consequences, or so he says. But I cannot myself take them seriously.