Vague Objects

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I believe that there are vague objects. This view apparently is not shared by very many other philosophers. It is often said that the world itself is perfectly precise and that vagueness resides only in language. On the face of it, this is a deeply puzzling position; for common sense has it that the world contains countries, mountains, deserts, and islands, for example, and these items certainly do not seem to be perfectly precise.

Still it cannot be denied that powerful arguments have been levelled against the existence of vague objects. My primary aims in this paper are to clarify the thesis that there are vague objects, as I accept it, and to defend this thesis against the two most influential arguments. I shall also make some criticisms of the opposing thesis that the non-linguistic world is precise.

1. *What can it mean to say that there are vague objects?*

Consider Mount Everest. It seems obvious that there is no line which sharply divides the matter composing Everest from the matter outside it. Everest's boundaries are fuzzy. Some molecules are inside Everest and some molecules outside. But some have an indefinite status: there is no objective, determinate fact of the matter about whether they are inside or outside. Are there any remaining molecules? To suppose that it is true that this is the case is to postulate more categories of molecules than are demanded by our ordinary, everyday conception of Everest and hence to involve ourselves in gratuitous metaphysical complications. It is also to create the need to face a potentially endless series of such questions one after the other as new categories of molecules are admitted. On the other hand, to suppose that it is false that there are any remaining molecules is to admit that every molecule fits cleanly into one of the three categories so that there are sharp partitions between the molecules inside Everest, the molecules on the border, so to speak, and the molecules outside. And intuitively, pretheoretically it is not true that there are any sharp partitions here. What, I think, we should say, then, is that it is objectively indeterminate as to whether there are any remaining molecules. In the ways I have just described, Everest is, I maintain, a vague object.

I propose to generalize from this example. Let us hold that something x is a borderline F just in case x is such that there is no determinate fact of the matter about whether x is an F. Then I shall classify a concrete object o

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as vague (in the ordinary sense in which Everest is vague) if, and only if, (*a*) *o* has borderline spatio-temporal parts and (*b*) there is no determinate fact of the matter about whether there are objects that are neither parts, borderline parts, nor non-parts of *o*.[1]

Some abstract objects are vague too, or so I believe. Consider, for example, the set of tall men. Men who are over 6 feet 6 inches are certainly members of this set and men who are under 5 feet 6 inches are certainly not. Intuitively, however, some men are borderline members. It is, I believe, a mistake to assert that it is true that there are further men who are neither members, borderline members, nor non-members.[2] On the other hand, it seems no less mistaken to assert that it is false that there are such men: intuitively it is not true that the dividing lines between the categories are sharp. I maintain, then, that it is indeterminate. In general, I classify a set *S* as vague, if, and only if, (*a*) it has borderline members and (*b*) there is no determinate fact of the matter about whether there are objects that are neither members, borderline members, nor non-members. This characterization of vague sets may seem to entail that one of the basic axioms of set theory, namely the Axiom of Extensionality, is false. But in reality it does no such thing. I shall elaborate upon this point later.

Like sets, some properties or concepts are vague.[3] The property of baldness, for example, is neither clearly a feature of some people nor clearly not a feature of those people. Baldness, moreover, would have remained vague, even if there had been only very hairy or wholly hairless people. In general I take a property *P* to be vague only if (*a*) it *could* have borderline instances and (*b*) there is no determinate fact of the matter about whether there *could* be objects that are neither instances, borderline instances, nor non-instances. I include clause (*b*) here for essentially the same reasons as those I gave in connection with the earlier (*b*) clauses.

It may be wondered why the conditions I have just stated are not both necessary *and* sufficient for property vagueness. The explanation is as follows. Consider the property of being 2000 feet in height. Intuitively this is a precise property. But it could have a borderline instance, to wit, a vague concrete object whose boundaries are such that it is indeterminate

[1] If there can be non-spatial concrete objects, this account will need to be revised minimally, e.g., by replacing 'spatio-temporal part' with the more neutral 'constituent'. For some comments on the analysis of the ordinary locution 'there is no determinate fact of the matter about whether' and also on its relationship to the predicate 'is indefinite' or 'is neither true nor false', see pp. 11–12 and n. 22 below.

[2] My reasons for making this claim are essentially the same as those presented in the Everest case. I might add that I do not deny that some borderline tall men are closer to being tall than other borderline tall men.

[3] I speak here and elsewhere as if I believe that there are vague properties. This is not exactly my view. What I really believe is that *if* there are such non-linguistic entities as properties then some properties are vague. However, I am inclined to deny that, in the final analysis, the antecedent of this conditional is true. For a discussion of ontological commitment to properties, see my *The Metaphysics of Mind*, New York, Cambridge University Pres, 1989, pp. 43–5.

whether it is 2000 feet in height. So condition (*a*) will be met. It seems to me that condition (*b*) will be satisfied too, if what I say two paragraphs hence about vaguely vague objects is correct.

Still, it is possible, I believe, to construct a defensible equivalence (or rather series of equivalences) for the case of vague properties, given suitable qualifications. Suppose that *P* is a property of concrete objects. Then we may hold that *P* is vague if, and only if, (*a*) *P* could have as a borderline instance a concrete object that does not have borderline spatio-temporal parts, and (*b*) there is no determinate fact of the matter about whether there could be an object of this sort which is neither an instance, a borderline instance, nor a non-instance of *P*. Where *P* is a property of a property of concrete objects (i.e., a second-order property), we may hold that *P* is vague if, and only if, parallel conditions are met in which the relevant instances are now first-order properties that are not vague. And so on at higher levels.[4]

I hope that I have now managed to provide an informal clarification of my use of the term 'vague' in application to concrete objects, sets, and concepts. I do not deny that other kinds of objects may properly be classified as vague but I shall not concern myself with such objects in this paper. Nor do I deny that objects of the three categories I have cited may be vague in other senses of the term 'vague'. For example, some philosophers may be willing to classify a material object as vague if it meets condition (*a*) alone. Nor finally (and relatedly) do I wish to claim that all concrete objects, properties, and sets that are not vague, as I understand 'vague', are precise. To see this, it suffices to realize that no conceptual barrier exists to the admission of properties, for example, which are such that there is no determinate fact of the matter about whether there could be any objects that are borderline instances. Such properties, some of which will concern us later, might be called 'vaguely vague' or 'indefinitely vague'.[5] And what goes here for properties goes *mutatis mutandis* for concreta and sets.

Later on in this paper I shall present a sketch of the logic of vague discourse that is compatible with what I have said above about vague objects. For the moment I want to consider three alternative characterizations about what one is committing oneself to when one accepts the thesis that there are vague objects. The first of these is widely held. To assert

[4] Where *P* is a property of sets or a higher-order property of such a property, the account proceeds in the same way except that the relevant instances at the lowest level are sets that do not have borderline members.

[5] There also seems to be no immediate conceptual barrier to the admission of properties that are such that (*a*) they could have borderline instances and borderline borderline instances, and (*b*) there is no determinate fact of the matter about whether there could be objects that are neither instances, borderline instances, borderline borderline instances, nor non-instances. These properties might be called 'second-level vague'. Still higher levels of vagueness may be intelligible. However, I can think of no clear common or garden examples of properties exhibiting even second-level vagueness.

that there are vague objects, according to many philosophers, is to assert that there are objects whose identity is indeterminate. I reject this view for two reasons. First, to assert that the identity of an object *o* is indeterminate cannot be to assert that *o* is not determinately identical with *o*. For surely we can all agree that this latter assertion is false, whatever *o* may be. Rather it must be to assert that there is an object *o′* such that it is not determinate whether *o* is identical with *o′*. But this claim does not seem strong enough to guarantee that *o* is vague, if 'vague' is used in the ordinary way. For it could be the case that the vagueness resides in *o′* (Everest, say) and that *o* is a *precise* object which is indeterminately identical with *o′* (for example, a precisification of Everest), assuming *aguendo* that identity is sometimes indeterminate.[6]. So the claim that *o* is vague is not immediately reducible to the claim that the identity of *o* is indeterminate. Second, there is an argument given by Gareth Evans and also independently by Nathan Salmon[7] which I take to show that identity statements are never indefinite in truth-value (so long as the component singular terms are rigid names). But nothing in this argument undermines the *intuitive* claim that Everest, for example, is a vague object. I shall address this issue further in Section IV.

The second view I want to mention is the view that to characterize an object as vague is to characterize it as an object that is capable of being made more precise. This view has a certain plausibility for the case of concrete objects such as Everest. For it is not difficult to imagine circumstances in which Everest is made more precise. Suppose, for example, that extremely powerful bombs are detonated around the base of Everest and that as a result of the explosions Everest's base is much more clearly defined than before. In these circumstances, Everest has fewer indefinite spatio-temporal parts. So Everest is more precise.[8]

In the case of sets, the view is also not without some plausibility. Consider, for example, the set of tall men. How could this set be made more precise? Well, suppose that a significant number of borderline tall men are killed all at once. Then, it might be said, the set of tall men would be significantly more precise than it actually is. As far as properties or concepts are concerned, however, the view under consideration seems to

[6] It is perhaps worth noting here that on supervaluationist approaches to the logic of vague discourse, there clearly can be indefinite identity statements in which the identity sign is flanked by a singular term for a vague object on one side and by a singular term for a precise object on the other. See here my later discussion of supervaluationism, especially p. 23.

[7] See Gareth Evans, 'Can There Be Vague Objects?', *Analysis*, 1978, p. 208; also Nathan Salmon, 'Modal Paradox: Parts and Counterparts, Points and Counterpoints', *Midwest Studies in Philosophy*, 1986, pp. 110–11.

[8] This argument also shows that Everest is not identical with the mereological sum of its parts. For suppose that *t* is one of the chunks of matter Everest would have lost had the bombs been detonated. Then it is true that Everest might have existed without *t*. But the same is not true of the mereological sum of Everest's parts. Hence Everest and the sum differ in a modal property. Of course, I am not denying here that Everest is a material entity.

me rather implausible. I do not deny, of course, that we can sharpen the meanings of vague predicates. But this possibility does not presuppose that the concepts that are, in fact, expressed by these predicates can themselves be made more precise. Rather what is presupposed is that the predicates can be made to express more precise concepts.

I am not convinced, then, that all vague objects are capable of being made more precise. I should also note that, on my account, it is not necessary that all objects that are capable of being made more precise are vague. Take, for example, a material object which has borderline spatio-temporal parts but which is such that the lines dividing those parts from the matter clearly inside the object and the matter clearly outside are sharp. This object can be made more precise by diminishing the number of its borderline parts. But it is not indeterminate enough to count as a vague object, as I use 'vague'.

The third view I reject is one which admits that there are vague concrete objects and sets but which insists that their vagueness is to be understood solely by reference to the vagueness of the appropriate concepts or properties. On this view, the root of non-linguistic vagueness is always conceptual: to say that an entity in the non-linguistic, non-conceptual world is vague is just to say that there is some appropriate concept the entity instantiates that is itself vague.

To see what is wrong with this position, consider again the set of tall men. Call this set S. Whichever vague property of S is chosen, the fact that it is vague does not entail that S is vague. For the vagueness of the property is connected with the fact that it *could* have borderline instances, that is, that there *could* be a set (or some other object) which is such that there is no determinate fact of the matter about whether it is an instance. And from this fact it evidently does *not* follow that there *actually* are any borderline tall men.

The general point, then, is that the vagueness of concepts or properties requires that there could be borderline instances whereas the vagueness of sets requires that there in fact be borderline members. No counterfactual borderline case entails any actual borderline case. It is, therefore, a mistake to claim that non-conceptual vagueness can be analysed in terms of conceptual vagueness in the above manner.[9]

Before I close this section, I want to make a few comments on the vagueness of language. It is sometimes supposed that linguistic vagueness always derives from predicate vagueness. On this view, names are never vague and descriptions are vague only insofar as they contain vague predicates. I deny that linguistic vagueness stems from or is restricted to

[9] It does not help to revise the proposal, I might add, so that an object (e.g., Everest) is counted as vague just in case it is an instance of an appropriate *extensionally* vague sortal concept (e.g., mountain); for there are possible worlds in which Everest is a vague object and in which there are no borderline mountains.

the realm of predicates, just as I denied above that non-linguistic vagueness is rooted in the realm of concepts. On my view, the name 'Everest' is no less vague than the predicate 'is bald', say. In classifying these linguistic items as vague, what I am really claiming is that their meanings are vague. In the case of 'Everest', its meaning is, I believe, its reference. And its reference is the vague object, Everest. In the case of 'bald', its intensional meaning is the property or concept it expresses and its extensional meaning (assuming that meaning can have an extensional sense) is the set of bald men. These, too, are vague. Linguistic vagueness, then, is as widespread as non-linguistic vagueness. Neither, I maintain, is confined to a narrow domain.

2. *Is the world itself precise?*

The claim that vagueness is not to be found in the world is prima facie very perplexing. Consider, for example, the statements 'There are mountains in California' and 'Some mountains are easier to climb than others'. These statements are surely true. Moreover their truth certainly appears to require that there be vague objects, namely mountains and the state of California. Admittedly grammatical form is sometimes misleading. Still it is very hard to see how the above statements can be reconstructed so as to avoid quantification over mountains and, in the former case, reference to California.

There is a proposal (known as supervaluationism) that is often taken to avoid vague objects and that permits vague sentences to retain a logical form closely related to their grammatical form.[10] According to this proposal, a vague sentence P is true if, and only if, P is true under all (eligible) ways of making P completely precise.[11] So, for example, a vague singular sentence is true just in case, under each eligible precisification of its component vague terms, the precise object referred to by its subject has the precise property expressed by its predicate. Similarly, a vague quantified sentence, for example, $(\exists x)Fx$, is true just in case, under each eligible precisification of F, there is at least one precise object which has

[10] For a clear presentation of supervaluationism, see Kit Fine, 'Vagueness, Truth, and Logic', *Synthese*, 1975, pp. 265–300.

[11] The following objection to supervaluationism (as stated) might be raised: Consider the sentence 'Smith is bald'. If this sentence is true if, and only if, it is true under all (eligible) ways of making it completely precise, then it is true *only if* it is true under the most extreme (eligible) way of making it completely precise. It follows then that 'Smith is bald' is true only if Smith has no hair whatsoever, i.e., that if 'Smith is bald' is true then Smith is completely hairless (assuming for simplicity that 'bald' is the only vague term in the sentence). This is evidently false. So the above supervaluationist view must be rejected. This objection contains a *non sequitur*. No *eligible* precisification of 'bald' can exclude from its extension any people to whom 'bald' clearly applies prior to precisification. So, 'without any hair whatsoever' is *not* an eligible precisification of 'bald'. So, the claim that 'Smith is bald' is true only if it is true under the most extreme eligible precisification does *not* entail that it is true only if Smith is completely hairless.

the precise property expressed by *F*. All that is required, then, for the truth of a vague sentence, according to most supervaluationists, is that there exist the appropriate precise entities appropriately related. No vague objects are necessary.

There are several difficulties with the above metaphysically reductive version of supervaluationism. To begin with, it should be noted that there is nothing in the initial logical proposal for truth-conditions which *entails* that there are no vague objects. To see this, it suffices to realize that there is no inconsistency in holding that a vague sentence, *Fa*, (in which both *F* and *a* are vague) is true if, and only if, the vague object referred to by *a* has the vague property expressed by *F* and also that this object has this property if, and only if, under every eligible precisification of *a* and *F*, the precise object *then* referred to by *a* has the precise property *then* expressed by *F*.[12] The point here is simply that *a* can refer to something, prior to precisification, which is vague while also referring to something, after precisification, which is precise. And the same goes, *mutatis mutandis*, for *F*.

Now not only is it the case that supervaluationism is consistent with the thesis that there are vague objects but it is also the case, I suggest, that supervaluationism is actually committed to that thesis, at least if it is agreed that there are such entities as sets; for, according to the supervaluationist truth-conditions, any vague term has an associated set of eligible precisifications, each with its own precise meaning. These sets, however, are vague. For example, some precisifications of the term 'bald' are clearly members of the set of eligible precisifications and some are clearly not. But some have a borderline status. Furthermore, it is surely not true that the dividing lines between the three groups are sharp. So vague entities have not been avoided.

A further difficulty arises, once we reflect upon what is involved in, and presupposed by, the process of precisification. Precisification is a process that sharpens the meaning of a sentence or term. This seems to require that there *be* something, namely a meaning, which is capable of being made precise.[13] But nothing that is already precise can be made precise. So again it appears that vague objects or at least objects that are not precise are needed.

Perhaps it will be replied that prior to precisification there is no single object which constitutes the meaning of a vague expression and which is then sharpened. Instead there are indefinitely many different precise objects associated with each vague expression. Precisification then involves selecting a single object from among these as the content of the expression just as, for example, disambiguation involves selecting a single meaning.

[12] In recent conversations on vagueness, Kit Fine has adopted such a view.

[13] Another problem here is that, as I noted earlier, it is not clear that such an entity as a meaning can itself be made more precise anyway (unless the relevant term is a proper name and the meaning is a concrete object such as Everest).

This response faces two objections. First, it seems to require once again that there be vague sets of precise entities associated with vague terms. Secondly, there sems to be no clear way to delineate these sets. Let me explain. Precisification is conceived of above as a process of selection. One precise object is selected from the set of precise objects associated with the given term. Now what determines membership in the relevant set? In the case of 'Everest', for example, it cannot be said that the appropriate set consists of those various objects that result from *making the reference* of 'Everest' completely precise. For this assumes that there is something imprecise that 'Everest' refers to *prior* to precisification. Nor can it be said that the relevant set is the one that consists of all those objects that 'Everest' can refer to *after* it has been made completely precise. For the process of precisification is now opaque: it involves selecting an object from a set comprised of those objects that can be selected via precisification. Unfortunately, I see no other ways of delineating the relevant set. It seems to me, then, that supervaluationism does not successfully eschew vague objects.[14]

Another approach to ridding the world of vagueness is to maintain that mountains and other such concrete objects are not really vague after all.[15] Instead they are to be taken to have precise but unknowable boundaries (unknowable, at any rate, by us). On this approach, the advocate of a precise world need not deny that statements putatively committed to mountains, rivers, corporations, etc. actually have such commitments.

What goes for common or garden concrete objects here is sometimes held to go for common or garden concepts. Just as concrete objects are really precise so too are concepts. On this broader view, not only is vagueness not found in the spatio-temporal world but it is also not found in the conceptual domain either. The claim that entities such as Everest and the property of being bald are really precise is, I think, highly counter-intuitive. It requires us to believe, for example, that the removal of a single hair in a succession of such removals must make a difference to the question of whether it is correct to apply the term 'bald' although we can never know which hair this is, and that the removal of a single molecule in a succession of molecule removals must finally destroy Mount Everest though again we can never know which one will be responsible for Everest's destruction. To hold beliefs like these, we must believe that there is an extraordinary underlying sensitivity in the conditions of application of ordinary names and predicates, a sensitivity which is inaccessible to competent language users. Ordinary non-philosophers do not believe any such thing, of course. And why, on earth, should they believe it?

[14] There are also more general objections to supervaluationism. In particular, I am doubtful that supervaluationism (with or without vague objects) can handle sorites arguments satisfactorily; and, as I show in Section 4, there is a high price to be paid by the supervaluationist response to what I call 'the Argument from Identity'. See also my 'Supervaluationism and the Law of Excluded Middle', *Analysis*, June 1989.

[15] See, e.g., Roy Sorensen, *Blind Spots*, Oxford, Oxford University Press, 1986.

One standard answer to this question is that the admission of vague concreta and concepts, however natural it may seem, encounters insuperable philosophical objections. This too is the view of still other advocates of a precise world, namely those philosophers who maintain both that ordinary statements really are vague and that these statements are false.[16] This position is, of course, even harder to swallow than the last one. In the remainder of the paper, I want to examine the two most important objections to vague entities: the Sorites type of paradox, and the Argument from Identity. What I shall try to show is that these objections are unsound.

3. *The sorites paradoxes*

Sorites paradoxes are generated from vagueness in some central expression. Consider, for example, the following argument which allegedly derives from Eubulides:

(1) A man with no hairs on his head is bald.
(2) For any number, n, if a man with n hairs on his head is bald then a man with $n+1$ hairs on his head is bald.

So,

(3) A man with a million hairs on his head is bald.

The conclusion is derived from the premisses via a million applications of modus ponens and universal instantiation. Now premiss (1) is certainly true and the conclusion, (3), is certainly false. Therefore, premiss (2) is false. Therefore, there is an n such that a man with n hairs on his head is bald and a man with $n+1$ hairs on his head is not bald. Therefore, the predicate 'bald' is precise, contrary to appearances. And what is true in this one case is true by parallel reasoning for any predicate which is ordinarily classified as 'vague'.

An argument along the same lines can be given to show that Mount Everest is not vague. Here is the argument: Suppose that L is a straight path extending from inside Everest through the earth to New York City. Suppose also that Everest is vague, as is ordinarily assumed. Then it is not the case that there is a very tiny, precise chunk of matter which lies on L within Everest's boundaries and which touches another such chunk which lies on L outside. So, for any two very tiny, precise chunks of matter that lie on L and that touch one another, if one is inside Everest then the other is too. So, by an appropriately extended sequence of modus ponens steps, it follows that a very tiny, precise chunk of matter lying within New York City is inside Everest. So Everest is not vague (or alternatively Everest does not exist).

[16] E.g., Peter Unger, 'There Are No Ordinary Things', *Synthèse*, 1979, pp. 117–54; Samuel Wheeler, 'On That Which is Not', *Synthèse*, 1979, pp. 155–73.

I am not persuaded by these arguments. Let me begin my response by sketching out what seems to me to be an attractive and simple logic which may be applied to vague statements. There are three truth-values: true, false, and neither true nor false (or indefinite). The third value here is, strictly speaking, not a truth-value at all but rather a truth-value gap. In my view, there are gaps due to failure of reference or presupposition and gaps due to vagueness.[17] Corresponding to the two-valued connectives $\sim$, &, $\vee$, $\supset$, and $\equiv$ are the three-valued connectives $\ulcorner$, $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$. These connectives have the following tables:

			$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
P	$\ulcorner P$	$P \backslash Q$	$T\ I\ F$	$T\ I\ F$	$T\ I\ F$	$T\ I\ F$
T	F	T	$T\ I\ F$	$T\ T\ T$	$T\ I\ F$	$T\ I\ F$
I	I	I	$I\ I\ F$	$T\ I\ I$	$T\ I\ I$	$I\ I\ I$
F	T	F	$F\ F\ F$	$T\ I\ F$	$T\ T\ T$	$F\ I\ T$

The guiding principles in the construction of these tables are easily explained.[18] (1) the negation of a statement of given truth-value is its opposite in truth-value. (2) A conjunction is true if both its conjuncts are true and false if either conjunct is false. Otherwise it is indefinite. (3) A disjunction is true if either disjunct is true and false if both disjuncts are false. Otherwise it is indefinite. (4) The truth-value of $P \rightarrow Q$ is to be the same as that of $\ulcorner P \vee Q$. (5) The truth-value of $P \leftrightarrow Q$ is to be the same as that of $(P \rightarrow Q) \wedge (Q \rightarrow P)$.

The tables presented above agree with the usual two-valued ones when only *T*s and *F*s are involved. However, there are no three-valued tautologies, since two-valued tautologies can take the value *I* in the three-valued case. For example the Law of Excluded Middle, that is, $p \vee \ulcorner p$, takes on the value *I* when *p* does so. Let us say that a statement form is a quasi-tautology just in case it has no false substitution instances. Then the Law of Excluded Middle and all other two-valued tautologies are quasi-tautologies in the above system.

It may perhaps be charged that the proposed truth-tables yield some implausible truth-value assignments in connection with certain compound sentences having indefinite components. In particular, if *A* is indefinite,

[17] Where a gap is due to vagueness, I maintain that something is said which is neither true nor false. I deny, however, that anything is said in the case where a gap is due to failure of reference. I am inclined to extend the latter view to gaps due to failure of presupposition.

[18] The tables are due to S. C. Kleene (although Kleene himself did not apply them to vague statements). See this *Introduction to Metamathematics*, Amsterdam and Princeton, 1952, pp. 332–40. For an application of Kleene's approach to the Paradox of the Liar, see Saul Kripke, 'Outline of a Theory of Truth', *Journal of Philosophy*, 1975, pp. 690–716.

then $A \rightarrow A$ is indefinite as is $A \wedge \ulcorner A$. I concede that the tables would certainly be mistaken, if they permitted $A \rightarrow A$ to be false and $A \wedge \ulcorner A$ to be true. But they do no such thing. $A \rightarrow A$ is a quasi-tautology and $A \wedge \ulcorner A$ is a quasi-contradiction. So, while the former statement cannot be false and the latter cannot be true, both can be indefinite. This seems to me entirely palatable. After all, it is surely reasonable to require that sentences of the form $P \rightarrow Q$ be equivalent to sentences of the form $\ulcorner P \vee Q$ and also that $P \vee \ulcorner P$ be equivalent to $\ulcorner (P \wedge \ulcorner P)$. It is also surely reasonable to deny that sentences of the form $P \vee \ulcorner P$ must always be true, given the existence of borderline cases.[19] There is, then, good reason to deny that $A \rightarrow A$ must be true and also good reason to deny that $A \wedge \ulcorner A$ must be false.[20]

Turning next to predicates, I suggest that for the purposes of formal semantics the following treatment suffices for any extensionally vague monadic predicate F: given a non-empty domain D, F is assigned an extension S and a counter-extension S'. S is the set of objects of which F is true; S' is the set of objects of which F is false. Since F is vague, S and S' are vague sets. Here the term 'vague set' is to be understood in the way explained in Section 1.

For any individual constant c, let i_c be the object in D assigned to c. Then Fc is true if i_c belongs to S; Fc is false if i_c belongs to S'; and Fc is indefinite if there is no determinate fact of the matter about whether i_c belongs to S (or to S').[21] The generalization to n-place predicates is straightforward.

It may be objected that my use of the locution 'there is no determinate fact of the matter about whether' introduces a vicious circularity into the above truth-conditions. But I deny that this is really the case. The truth conditions state conditions for the application of the *predicates* 'is true', 'is false', and 'is indefinite'. By contrast, the words 'there is no determinate fact of the matter about whether' form a *sentence operator*. This sentence operator cannot be analysed as, nor is it equivalent to, the predicate 'is indefinite' or 'is neither true nor false'. For one thing, a sentence such as 'Everything James says is indefinite' may be true but 'There is no determinate fact of the matter about whether everything James says' is unintelligible. For another, it seems to me no more plausible to classify assertions of the type 'There is no determinate fact of the matter as to whether p' as covertly meta-linguistic than it is to classify assertions of the

[19] See here my 'Supervaluationism and the Law of Excluded Middle', op. cit.

[20] The claim that $A \rightarrow A$ is sometimes indefinite is, of course, compatible with the claim that A is a logical consequence of A, since one sentence will be a logical consequence of another so long as it is not possible for the first to be anything other than true when the latter is true. Within the above framework, then, the deduction theorem no longer holds.

[21] The term 'if', as it is used in these and the later truth-conditions, is to be taken to indicate a relation of entailment. So, e.g., ''Fc' is true if i_c belongs to S' is true on my view, since ''Fc' is true' must be true when 'i_c belongs to S' is true. The fact that 'i_c belongs to S' is sometimes indefinite is irrelevant.

type 'It is not the case that p' in like manner. Finally, and relatedly, on my view, it makes good sense to say that a given singular sentence is indefinite because there is no determinate fact of the matter about whether the appropriate individual belongs to the appropriate set but not to say that the converse is the case.[22] So, I reject the above charge of vicious circularity.

Turning now to the quantifiers, we may introduce $(\exists x)$ and (x) as follows: $(\exists x)Fx$ is to be true if Fx is true for some assignment of an object of D to x; false if Fx is false for all assignments; and indefinite otherwise. $(x)Fx$ is to be true if Fx is true for all assignments of objects of D to x; false if Fx is false for some assignments; and indefinite otherwise.

Given these definitions, we can see why my claim in Section 1 that some sets are vague does not entail that the Axiom of Extensionality is false. What the axiom asserts is this: where S and S' are any sets, S is identical with S' if, and only if, for any object, x, x belongs to S if, and only if, x belongs to S'. Trouble for the axiom lies with the case where S and S' are vague sets which are identical (or which differ only with respect to their borderline members). Here, the statement schema—call it 'A'—that x belongs to S if, and only if, x belong to S' has assignments under which it is not true, since there are objects that are borderline members of S and S'. However, A is not false under these assignments. Rather, by the truth-table for $\leftrightarrow$, it is neither true nor false. So, the universally quantified statement $(x)Ax$ is neither true nor false. So, the statement, $S = S' \leftrightarrow (x)Ax$, has an indefinite right hand side in the above case. So, the Axiom of Extensionality comes out as *indefinite* under the proposed semantics.

Just as the Axiom of Extensionality is not false, on my view, so too are none of the other axioms of set theory. Moreover, it is not merely a contingent fact that the Axiom of Extensionality is not false. Rather it is necessary. My position here with respect to the Axiom of Extensionality is parallel to the one taken above with respect to two-valued tautologies. In the three-valued case, these tautologies become quasi-tautologies. Likewise, the Axiom of Extensionality becomes a quasi-(necessary truth). Of course, if the Axiom is qualified by a clause which restricts S and S' to precise sets then it remains a full-blooded necessary truth.

If, as I am claiming, the Axiom of Extensionality is neither true nor false, it cannot be used to demonstrate that two sets that differ only with respect to their borderline members are not identical. This need not concern us, however. For the sets can be distinguished by means of

[22] The only worthwhile equivalence I can provide for sentences of the type 'There is no determinate fact of the matter about whether p' is 'It is not the case that it is determinate that p and it is not the case that it is determinate that not-p'. This equivalence does not constitute a reductive analysis, since it introduces another comparable sentence operator, namely 'it is determinate that', or, abbreviated, *Det*. Truth-conditions for *Det* may be stated as follows: *Det* p is true if p is true, and false if p is false or indefinite. These truth-conditions do not analyse the meaning of *Det* any more than the truth-conditions for v or $(\exists x)$ analyse the meanings of 'or' or 'some'. See here my comments on truth-conditions on the next page.

Leibniz' Law: one set has a property that the other lacks, namely having such-and-such an object as a borderline member.[23]

At this stage, it might be objected that there is another problem of circularity with my proposal. Vague sets are essentially governed by the logic I have presented. So, the concept of a vague set cannot be understood unless the logic itself is already understood. But the logic makes reference to vague sets. So the concept of a vague set cannot really be understood at all.

The claim that I reject here is the claim that the concept of a vague set cannot be understood unless the logic is already understood. Consider a parallel. The concept of disjunction is essentially governed by the logic of disjunctive sentences. But this logic uses the concept of disjunction: '*p* or *q*' is true just in case '*p*' is true or '*q*' is true. So, understanding the logic cannot be a precondition for understanding the concept. It is, then, a mistake to suppose that the truth-conditions for disjunctive sentences analyse the meaning of the term 'or'. Rather is is because 'or' means what it does that the truth-conditions obtain. One who understands the concept will use it in accordance with the logic but a full grasp of the metalinguistic sentences which utilize the concept in the logic is no part of that understanding.

What is true here for disjunction is true, on my view, for the concept of a vague set. This concept can be explained in an intuitive, pretheoretical way, as it was in Section 1. Grasping this explanation does not itself presuppose a full understanding of the metalinguistic sentences specifying the conditions of application of the truth-value predicates for vague sentences—unless, of course, the operator 'there is no determinate factor of the matter about whether' is to be analysed in terms of the metalinguistic predicate 'is indefinite', a position I have already rejected. So again I deny that there is any troublesome circularity.[24]

With the above semantic framework in place, let us now return to the sorites of the bald man. I have three objections to this sorites. First, since premiss (1) is true and the conclusion, (3), false, what follows is that the premiss (2) is not true, that is, that (2) is either false or indefinite and *not*

[23] I might add that the two occurrences of 'if and only if' in the Axiom of Extensionality do not *have* to be taken as instances of the connective $\leftrightarrow$. If, e.g., they are taken to express a relation of mutual entailment, which is such that *p* if, and only if, *q* is counted as true if *p* and *q* necessarily agree in truth-value and as false otherwise, the Axiom of Extensionality will remain true. In this form the Axiom *will* distinguish vague sets that differ only with respect to their borderline members.

[24] I also deny that the truth-conditions for vague sentences must be stated in a language that is governed by classical logic. Indeed I hold that it is crucial that they not be so stated. The purpose of formal semantics is not to give reductive explanations or analyses of the meanings of various sorts of sentences in classical terms (or in any other terms for that matter). One who lacks the concept of disjunction, for example, will not come to understand it by being shown the truth-conditions for disjunctive sentences. Rather the purpose of a formal statement of truth-conditions is to explain rigorously how the truth-value predicates are to be applied, and to do so in a way that is compatible with our prior, ordinary understanding of the relevant concepts and sentences.

that it is false as the classical reasoning supposes. Secondly, (2) is, in fact, indefinite in truth-value. Let me explain. It is not true that there is any assignment of objects to the statement schema

(4) If a man with n hairs on his head is bald then a man with $n+1$ hairs on his head is bald

under which it is false. However, there are assignments under which both its antecedent and its consequent are indefinite, since there are borderline bald men who would not cease to be borderline by gaining a hair. So, there are assignments under which (4) is indefinite. So, the universally quantified statement, (2), is itself indefinite. Thirdly, if (2) is indefinite then the statement

(5) It is not the case that there is a number n such that a man with n hairs is bald and a man with $n+1$ hairs is not,

which is equivalent to (2), must also be indefinite. So the negation of (5) is indefinite. So it has not been shown that is is true that there is an n such that a man with n hairs is bald and a man with $n+1$ hairs is not. So the argument certainly does not show that 'bald' is precise.

A parallel response is appropriate if the argument is revised so as to proceed indirectly. In this case the derivation of a contradiction from the statement (5) does not demonstrate that the negation of (5) is true and hence that 'bald' is precise. Rather it demonstrates that (5) is not true, that is, that (5) is either false or indefinite. So what is established is that the negation of (5) is either true or indefinite. And, in fact, as I noted above, the negation of (5) is indefinite on my proposal.

A similar approach may be taken to the sorites argument involving Everest. Given that L is a straight path extending from inside Everest through the earth to outside, the claim that Everest is vague entails that the statement

(6) There is a very tiny, precise chunk of matter which lies on L within Everest's boundaries and which touches another such chunk which lies on L outside

is not true but not that it is false. For if Everest is vague then the schema

(7) x is a very tiny, precise chunk of matter $\wedge\, x$ lies within Everest's boundaries $\wedge\, x$ touches another such chunk $y \wedge y$ lies outside Everest

has some assignments under which it comes out indefinite (since there are certainly cases in which the first and third conjuncts are both true whereas the second and fourth are both indefinite), but it is not true that there are any assignments under which it comes out true. So, (6) is indefinite. So, the negation of (6) is indefinite. So,

(8) For any two very tiny, precise chunks of matter that lie on L and that touch one another, if one lies within Everest's boundaries then the other does not lie outside those boundaries

is indefinite too. So the sorites argument fails.

I am sure that it will now be objected that while my proposal may handle the sorites argument I have discussed it encounters another sorites problem at the meta-linguistic level. For consider the sequence of statements whose members are of the form 'A man with n hairs is bald', where n ranges from 0 to 1,000,000. Call these statements $M_0, M_1, \ldots M_{1000000}$. Surely, it may be said, it can be demonstrated that there is some statement, M_k, such that M_k is true and M_{k+1} is not true. For suppose that there is no such statement. Then it follows that for any statement, M_k, if M_k is true then M_{k+1} is true. And from this, given that M_0 is true, by repeated applications of universal instantiation and modus ponens it may be inferred that $M_{1000000}$ is true. But $M_{1000000}$ is false. So, there is a sharp transition from the true statements in the sequence to the indefinite ones. This claim is no more plausible, however, than the already rejected claim that the addition of a single hair changes a bald man into a man who is not bald.

What this argument shows, in my view, is that the statement

(9) It is not the case that, for some k, there is a statement M_k such that M_k is true and M_{k+1} is not true

is not true. But this does not entail that

(10) For some k, there is a statement M_k such that M_k is true and M_{k+1} is not true

is true, since (9) may be indefinite. And, in fact, in my view, both (9) and (10) are indefinite. My defence of this classification is as follows: in the sequence of statements $M_0, M_1, \ldots M_{1000000}$ there are initially true statements, then later there are indefinite statements, and then finally there are false statements. It seems clear that competent language users will not agree upon precisely where the boundaries are to be drawn in the sequence between the true, the indefinite, and the false statements. Of course, this is not to say that such people will not specify precise points if they are *forced* to assign either 'true' or 'false' or 'neither' to each of the statements $M_0, M_1, \ldots M_{1000000}$ one after another.[25] Still it seems highly unlikely that even one and the same person will pick exactly the same points on different

[25] Indeed, one can imagine people changing their views within the space of a few seconds. Consider, for example, the following imaginary exchange: 'You said that M_{130} is true. Now that you have classified M_{131} as neither true nor false, do you still think that M_{130} is true?' 'Well, I guess not.' 'So, M_{130} is neither true nor false then?' 'I suppose so.' 'What of M_{129}? Is that neither true nor false too or is it true as you held before?' 'Oh, I just don't know what to say.' 'But if you don't know how to classify M_{129}, do you still want to classify M_{130} as neither true nor false?' 'Yes ... No ... I'm befuddled.'

occasions. I grant then, that it is not true that the transitions from true to indefinite statements and from indefinite to false statements are sharp. Consider now the sequence of statements 'M_0 is true', 'M_1 is true', ...'$M_{1000000}$ is true'. Given that it is not true that there is a sharp transition from true to indefinite statements in the object language, I maintain that it is not true that any conjunction of the type 'M_n is true $\wedge$ M_{n+1} is not true' is itself true. So, (10) is not true. But if (10) is false then (9) is true. And (9) is certainly not true. So (10) is indefinite, as is (9).

It may be objected that (10) cannot be indefinite unless some statements of the form 'M_n is true' are themselves indefinite. And, on my account, it is false that some such statements are indefinite. For if any given statement M_i is true then 'M_i is true' is certainly true; and if M_i is either false or indefinite then it is false that M_i is true. Either way, then, 'M_i is true' is not indefinite.

My response to this objection is twofold. First, what my position commits me to is the claim that there is no determinate fact of the matter about whether there are any statements of the form 'M_n is true' that are indefinite and not to the claim that it is false that there are such statements. To see this, suppose that there is a statement 'M_j is true' that is indefinite. Then M_j itself can be neither true nor false nor indefinite. So it is not true that there is a statement 'M_j is true' that is indefinite. So it is either false or indefinite. If it is false then every statement of the type 'M_n is true' is either true of false. And this means that there will be sharp transitions from the true statements of the type 'M_n is true' to the false ones. Intuitively, it is not true that there are such transitions. So it is, I maintain, indeterminate whether there are statements of the type 'M_n is true' that are indefinite.

Secondly, nothing in the earlier semantics requires that an existentially quantified sentence, $(\exists x)Fx$, be indefinite only if Fx comes out as indefinite under some assignments. If there is no determinate fact about whether Fx is indefinite under some assignments then it will not be true that Fx is false under all assignments. So if it is also not true that Fx is true under some asignments, $(\exists x)Fx$ will count as indefinite. So (10) can be indefinite without it being true that some statements of the form 'M_n is true' are indefinite.

It is sometimes supposed that the view that it is not true that there are sharp transitions between the true and the indefinite statements and the indefinite and the false statements in sequences like M_0,M_1,...$M_{1000000}$ is the same as or part and parcel of the view that the predicates 'is true', 'is indefinite', and 'is false' are extensionally vague. This is not correct, however, at least on my account. For if 'is true' is extensionally vague then it follows that the set of true sentences has borderline members. This requires that there be sentences which are such that it is neither true nor false that they are true. And this, in turn, requires that there be sentences that are neither true nor false nor indefinite. I maintain that it is not true that there are such sentences. So 'is true' cannot be classified as extensionally vague. And the same goes *mutatis mutandis* for 'is false' and 'is indefinite'. Of course, in taking this view I am not committing myself to the position that these predicates are precise. Indeed, it is crucial to my account that they *not* be precise. For if they were then every sentence would be either true or false or indefinite, and that would not only generate sorites difficulties of its own (as we shall shortly see) but also run counter to my claim that it is indefinite whether no statement of the form 'M_n is true' is indefinite.[26] Rather my view on the truth-value predicates is that they are vaguely vague: there simply is no determinate fact of the matter about whether the properties they express have or could have any borderline instances. So, it is indefinite whether there are any sentences that are neither true nor false nor indefinite.

Given my position on 'true' and the other truth-value predicates in the first metalanguage, what should be said about the truth-value transitions in the higher metalanguages? The answer must be that in the higher level sequences it is *never* true that such transitions are sharp. Let me explain. Consider again the sequence of statements, 'M_0 is true', 'M_1 is true', ...'$M_{1000000}$ is true'. Suppose that for some n there is a statement of the form 'M_n is true' which is true and which is such that 'M_{n+1} is true' is not true. If any statement of the form 'M_n is true' is true then the corresponding object language statement of the form M_n is true. Also if any statement of the form 'M_{n+1} is true' is not true then the corresponding statement of the form M_{n+1} is not true. For obviously if, for any given n, M_{n+1} is true then it is true that M_{n+1} is true and hence that 'M_{n+1} is true' is true. So if the initial supposition is true, then there is a statement of the form M_n which is true and which is followed by a statement of the form M_{n+1} which is not true. But the consequent here is not true, according to my view earlier. So, it is not true that the initial supposition is true. So, it is not true that the transition from 'true' to 'not true' in the second level sequence is sharp. Clearly, this argument may be generalized to show that it is not true that the transitions from 'true' to 'indefinite' and from 'indefinite' to 'false' are sharp in any of the higher level metalinguistic sequences.

I want now to consider briefly two further sorites arguments which may seem to present special difficulties for the framework I have sketched. The first of those involves what I take to be vaguely vague predicate, namely the predicate 'is borderline bald'. The argument goes as follows:

(11) Rupert has fewer than one hundred hairs on his head and is borderline bald.

[26] Unless a disquotational approach to the predicate 'is true' is adopted. I reject any such approach.

(12) For any n, if Rupert has n hairs on his head and is borderline bald then he will remain borderline bald if he gains a single hair.

Therefore,

(13) Rupert will remain borderline bald if he gains a million hairs one by one.

(13) follows from (11) and (12). (11) is true, we may suppose. What of (12)? Well, (12) differs in an important respect from the earlier universal statement, (2), involving the vague predicate 'is bald'. For however many hairs Rupert gains through time, it is not true that the schema

(14) If Rupert has n hairs on his head and is borderline bald then he will remain borderline bald if he gains a single hair

has any assignments under which it is indefinite. So, it may seem that my approach compels me to admit that (12), unlike (2), is not indefinite. (12), then, must be false, since (13) is false. So, 'is borderline bald' must be classified as precise, contrary to the initial assumption.

My response to this argument is to insist that (12) *is* indefinite on my semantic analysis. According to what I said earlier, given a non-empty domain D, $(x)Fx$ is true if Fx is true for all assignments of objects of D to x; false if Fx is false for some assignments; and indefinite otherwise. Now, 'is borderline bald' is vaguely vague by hypothesis. So, there is no determinate fact of the matter about whether (14) is indefinite under any assignments. So, it is not true that (14) is true under all assignments. Nor is it true that (14) is false under some assignments. For there is no determinate fact of the matter about whether there are *any* assignments under which (14) has a true antecedent and a false consequent. So, (12) must be classified as indefinite. The point to note, then, is that, on the stated semantics, $(x)Fx$ can be indefinite even if it is not true that Fx has any indefinite assignments. This point parallels the point I made earlier about $(\exists x)Fx$ in response to an objection to my classification of (10) as indefinite.[27]

The final Sorites argument I shall consider takes us back to the case of Mount Everest and other such vague concrete objects. Suppose that $a_0, a_1, \ldots, a_n$ are tiny chunks of matter lying on a straight path L which runs from inside to outside Mount Everest. Suppose also that a_0 touches a_1, that a_1 touches a_2, and so on. Assuming that a_0 is well inside Mount Everest, it is evident that the following conditional is true:

(15) If a_0 touches a_1 and a_0 and a_1 are tiny chunks of matter and a_0 is inside Everest, then a_1 is inside Everest.

[27] Since I take the predicates 'is true', 'is false', and 'is indefinite' to be vaguely vague, I maintain for parallel reasons to those given above in connection with (12) that the generalization, '$(x)(x$ is true $\vee$ x is false $\vee$ x is indefinite$)$,' where the variable 'x' is restricted to sentences, is indefinite.

So, we may infer by modus ponens that a_1 is inside Everest. But if a_1 is inside Everest then surely the following conditional is also true:

(16) If a_1 touches a_2 and a_1 and a_2 are tiny chunks of matter and a_1 is inside Everest then a_2 is inside Everest.

This gives us by modus ponens that a_2 is inside Everest. Repeating this style of argument an appropriate number of times we eventually arrive at the conclusion that a_n is inside Everest which, by hypothesis, it is not. What, then, has gone wrong? According to some philosophers, it is entirely implausible to hold that somewhere in the sequence of conditionals starting with (15) and (16) there is a first conditional that is not true. Our only alternative, then, is to deny that Everest exists and hence to deny that even the assumption that a_0 is inside Everest is true.

My response to this sorites should not be difficult to anticipate: we need not hold either that there is a first conditional in the sequence that is not true or that Everest does not exist. Instead, given the proposed semantics, we should hold that it is neither true nor false that there is a first conditional that is not true. Thus, there are true conditionals initially, and indefinite conditionals later, but it is not true that there is a sharp transition from the former to the latter. That this is the case is evidenced, I suggest, by two facts: (*a*) competent language users will not be prepared to grant that all of the conditionals are true; (*b*) the same language users will think that there is something improper about dissenting from any given conditional C_N in the sequence having just assented to conditional C_{N-1}.

It may seem that there is a difficulty lurking here that I have not fully put to rest. If, as I maintain, the conjunction of conditionals beginning with (15) and (16) is not true then it follows that either (15) is not true or (16) is not true or some later conditional is not true. Surely then at some point in the sequence there must be a pair of adjacent conditionals such that the first is true and the second is not.

There is an unstated assumption in this argument, namely that each conditional in the sequence is either true or not true. Without this assumption, the reasoning is invalid. To see why, consider how the argument must go. (15) is true. So (16) or some later conditional is not true. Suppose (16) is true. Then either the next conditional is not true or the one after that is not true or.... On the other hand, suppose (16) is not true. Then (15) and (16) differ in truth-value and there is a pair of conditionals such that the first is true and the second is not. It is obvious that repeating this style of argument an appropriate number of times will not generate the overall conclusion unless it is assumed that each conditional is either true or not true.

I refuse to accept this assumption. Since, on my view, the truth-value predicates are vaguely vague, I maintain that the claim that every conditional in the sequence is either true or not true (i.e., false or indefinite) is indefinite.[28] So, I am not persuaded that there really is any difficulty.

Before I close this section, I want to address a complication which arises in connection with what I called earlier 'second-level vague' entities.[29] If there can be such entities then the semantics I have sketched will not apply to them. To see this, suppose that Q is a second-level vague property and that a is one of its borderline borderline instances. Then the statement that a has Q will be such that it is neither true nor false that it is neither true nor false. So there will be a statement that cannot be assigned one of the three truth values: true, false, indefinite.

The fact that the semantics I have presented does not cover cases of higher level vagueness is of no great concern to me. I am unconvinced that there are any higher level vague entities. My interest in this paper is primarily with the vague and the vaguely vague, as I have characterized them. Still it seems to me that my semantics can be extended to the higher-level vague without great difficulty. I shall not attempt to show this in the general case. Instead I shall briefly lay out how semantics will go if it is to cover second-level vague objects.

In addition to the values true, false, and indefinite, there is now a fourth truth-value: indefinitely indefinite. This value is such that a statement, P, has it just in case it is neither true nor false that P is neither true nor false.[30] The guiding principles in the construction of truth-tables are as follows: (1) The negation of a statement of given truth-value is its opposite in truth-value.[31] (2) A conjunction is true if both conjuncts are true, false if either conjunct is false, and indefinite if either one conjunct is indefinite and the other true or both conjuncts are indefinite. Otherwise it is indefinitely indefinite. (3) A disjunction is true if either disjunct is true, false if both disjuncts are false, and indefinite if either disjunct is indefinite and neither true. Otherwise it is indefinitely indefinite. (4) The truth-value of $P \rightarrow Q$ is to be the same as that of $\ulcorner P \vee Q$. (5) The truth-value of $P \leftrightarrow Q$ is to be the same as that of $(P \rightarrow Q) \wedge (Q \rightarrow P)$.[32]

Second-level vague predicates are assigned vague extensions and counter-extensions just as first-order vague predicates were earlier. Truth

[28] See here the last footnote. It should be noted that for a finite sequence the claim that each conditional is either true or not true is equivalent to a conjunction the first member of which is that (15) is true or not true. This conjunction is indefinite even though, on my view, it is not true that there is an indefinite conjunct. No problem arises here in connection with the earlier truth-tables, since the principle I stated for the case of conjunction was as follows: a conjunction is true if both (all) conjuncts are true, false if at least one conjunct is false, and indefinite *otherwise*.

[29] See n. 5.

[30] Any statement has the value, indefinitely indefinite, just in case it is either indefinitely true or indefinitely false. Those sentences that lie on the border between the true sentences and the indefinite ones are indefinitely true. Those sentences that lie on the border between the indefinite sentences and the false ones are indefinitely false. For ease of exposition, in what follows I ignore the fact that the indefinitely indefinite sentences can be divided into two sub-classes.

[31] The opposites of 'indefinite' and 'indefinitely indefinite' are taken to be themselves.

[32] The connectives $\rightarrow$, $\wedge$, $\vee$, $\ulcorner$ *are now four-valued.*

conditions are also stated in like manner for second-level vague singular sentences except that a fourth case is now added: where F is a second-level vague predicate and c an individual constant for object i_c in D, Fc is indefinitely indefinite if there is no determinate fact of the matter about whether there is no determinate fact of the matter about whether i_c belongs to S (the vague set assigned to F as its extension).

The quantifier $(\exists x)$ is introduced as follows in connection with second-level vague predicates: $(\exists x)Fx$ is to be true if Fx is true for some assignment of an object of D to x; false if Fx is false for all assignments of objects of D to x; indefinite if Fx is indefinite for some assignment of an object of D to x and true for no assignment; and indefinitely indefinite otherwise. $(x)Fx$ is to be true if Fx is true for all assignments of objects of D to x; false if Fx is false for some assignments; indefinite if Fx is indefinite for some assignments and false for none; and indefinitely indefinite otherwise.

Given the above semantics, sorites arguments against the existence of second-level vague objects may be countered along parallel lines to those adopted in my defence of (first-level) vague objects. And what is true at the second-level is, I believe, true *mutatis mutandis* at higher levels.

So much for sorites arguments. In the final section, I examine another major objection to vague objects: The Argument from Identity.

4. *The Argument from Identity*

This argument, in its original form, is due to Gareth Evans.[33] According to Evans, the thesis that the world contains vague objects or that the world might contain such objects rests in part upon the thesis that identity statements are sometimes vague. Evans maintains that this latter thesis is false. For suppose that 'a' and 'b' are singular terms and that '$a=b$' is indefinite in truth-value. Then, if we let '∇' symbolize 'indefinitely', the following is true:

(17) $\nabla(a=b)$.

(20) ascribes to b the property '$\hat{x}\ \nabla\ (x=a)$', so that

(18) $\hat{x}\ [\nabla\ (x=a)]\ b$

is also true. Now surely we have

(19) $\ulcorner\ \nabla\ (a=a)$

and hence

(20) $\ulcorner\ \hat{x}\ [\nabla\ (x=a)]\ a$.

[33] See his 'Can There Be Vague Objects?' op. cit.

By Leibniz' Law, from (18) and (20),

(21) $\ulcorner (a=b)$

follows. And (21) contradicts the initial assumption that '$a=b$' is indefinite in truth-value.

This argument is a little rough and ready, as it stands. For one thing, it is not clear that Evans is entitled to claim that (18) ascribes to b the property '$\hat{x} \nabla (x=a)$' and that (20) ascribes to a the property '$\ulcorner \hat{x} \nabla (x=a)$', unless he assumes that the terms 'a' and 'b' are rigid names. So his conclusion is best taken to concern only those identity statements in which the identity sign is flanked by the appropriate designators. For another, (21) does not *directly* contradict the initial assumption that '$a=b$' is indefinite in truth-value. Given the logic sketched earlier, however, it is a trivial matter to expose a contradiction.

One common response to Evans's argument by advocates of vague objects is to attempt to restrict the applicability of Leibniz' Law so that (21) cannot be inferred from (18) and (20). This response strikes me as *ad hoc*. Moreover it is unnecessary. For one can grant that the argument demonstrates that identity statements (in which the identity sign is flanked by rigid names) cannot be indefinite in truth-value without admitting that such statements cannot be vague. To say that an identity statement is vague, on my view, is to say that it has a vague meaning. This will be the case, I maintain, if either of the singular terms flanking the identity sign is vague. But the vagueness of 'a' or 'b' in '$a=b$' does not require that '$a=b$' might be indefinite in truth-value. Take, for example, the name 'Everest' and suppose that 'm' is a name for a more precise mountain that differs from Everest only in that it lacks certain chunks of matter that are indefinite constituents of Everest. In my view, the statement '$m=$Everest' is vague, since 'Everest' is vague (as also is 'm' unless it names an object that is *completely* precise). But '$m=$Everest' is not indefinite in truth-value. Rather it is false. This is shown by the Evans's argument. And it may also be shown by the following simple argument: suppose that 't' names a chunk of matter which is an indefinite constituent of Everest and which is definitely not a constituent of m. Then, where 'C' abbreviates 'is a constituent of', we have

(22) $\ulcorner \hat{x} [\nabla Ctx]m$.

But if we abbreviate 'Everest' by 'e', we also have

(23) $\hat{x} [\nabla Ctx]e$.

So, by Leibniz' Law, we may conclude

(24) $\ulcorner (m=e)$.[34]

[34] I might note that (24) is compatible with the earlier claim that Everest is *capable* of being made more precise. The latter will be true if Everest has the property of *possibly* having chunk of matter t (or some such other indefinite constituent of Everest) as a definite constituent. What demonstrates that (24) is true is the fact that Everest, unlike m, lacks the property of actually having t as a definite constituent.

It is worth noting here that if one adopts a supervaluational approach to vagueness, one will not be able to respond to Evans's argument in the manner I have suggested. According to supervaluationists, the identity statement '$a=b$' is true (false) just in case it is true (false) under all ways of making the terms 'a' and 'b' completely precise, and '$a=b$' is indefinite just in case it is true under some ways of making the relevant terms completely precise but false under other ways. Thus, on this account, if 'a' is a vague name and 'b' names one of the objects that results from making the referent of 'a' completely precise, '$a=b$' will be classified as indefinite. So, (17) will be true. So, supervaluationists must challenge the applicability of Leibniz' Law in the argument. And they must do likewise with the argument from (22) and (23) to (24). This, I think, is a strong reason for preferring the truth-value gap approach I have proposed.

I conclude that neither sorites arguments nor the Argument from Identity present any real difficulties for the thesis that there are vague objects. This result is, I suggest, unsurprising. Philosophical arguments rarely, if ever, demonstrate that fundamental common-sense beliefs are mistaken or incoherent.[35]

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