Leibniz's Theory of Time
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Abstract

I have developed an informational interpretation of Leibniz's metaphysics and dynamics, but in this paper I will concentrate on his theory of time. According to my interpretation, each monad is an *incorporeal automaton* programed by God, and likewise each organized group of monads is a *cellular automaton* (in von Neumann's sense) governed by a single dominant monad (*entelechy*). The activities of these produce phenomena, which must be "coded appearances" of these activities; God determines this coding. A crucially important point here is that we have to distinguish the *phenomena* for a monad from its *states* (perceptions). Both are a kind of representation: a state represents the whole world of monads, and phenomena for a monad "result" from the activities of monads. But the coding for each must be different; *R(W)* for the first, *Ph(W)* for the second, where *W* is a state of the monadic world. The reason for this is that no monadic state is in space and time, but phenomena occur in space and time. Now, the *basis* of the phenomenal time must be in the *timeless realm of monads*. This basis is the *order of state-transition* of each monads. All the changes of these states are given at once by God, and these do *not* presuppose time. The coded appearances (which may well be different for different creatures) of this order occur *in time* (for any finite creatures), and its *metric* must depend on God's coding for phenomena. For humans, in particular, this metric time is derived from spatial distance (metric space) via the laws of dynamics. Thus there may well be an *interrelation* between spatial and temporal metric. This means that the Leibnizian frame allows relativistic metric of space-time. I will show this after outlining Leibniz's scenario.

1. Informational Interpretation of *Monadology*

When I first read Leibniz's *Monadology* (1714) and related papers, I was struck by his characterization of monads as "incorporeal or spiritual automata." A monad is, according to Leibniz, a simple substance with the primitive force, and this force governs its state-transition. What we usually regard as the "world" is phenomena produced by the activities of the monads. Many people may think that this whole idea is crazy, but as John Archibald Wheeler said, "crazy ideas are worth pursuing."

First, I should give a rough idea of what *Monadology* is all about. It is Leibniz's almost final formulation of metaphysics. He forcefully argued that there must be simple *substances* (in reality) which support everything we see and feel in this world, the world of *phenomena*. The spheres of phenomena and of reality must be strictly separated. Physics or dynamics studies motions which are supposed to underlie phenomena, but the laws of dynamics must be grounded on metaphysical principles. In particular, space and time, which are usually assumed as a framework of dynamics must be explained from the metaphysical basis. In short, everything must be explained, ultimately, in terms of the activities of the monads. This is Leibniz's grand vision. And this vision was described in more detail in *Monadology*.

Each monad is a simple substance, a metaphysical point with no shape, no parts, no magnitude. But each monad must be different from each other, and this difference comes form its *internal state*, or more precisely, the whole series of states. But, as Leibniz repeats many times, each monad is a "living mirror" of the whole world, and each of its states is a "representation" of an instantaneous state of the whole world. Here, it is clear that representation needs *coding*, since something can be represented by another thing, only by coding. For example, the color red is represented by the word "red" in English, but by the word "rouge" in French. Obviously, these two representations depend on two different codings. Likewise, when the reality (the world of monad) is somehow represented in *phenomena*, this representation needs another coding, since the two spheres, reality and phenomena, are completely different and their ontological status is radically different. See Figure 1 for a rough image.

These representations are, mathematically speaking, a mapping from one domain to another domain, by means of a function. Thus, we can express the representation in a monad by $R(W)$, the representation in phenomena by $Ph(W)$, where W is either the whole world or its relevant portion.This is my shorthand symbolism; if we are to distinguish different monads we need different subscripts for R and Ph. And what is unique in Leibniz is that W itself can be regarded as the totality of representations, R's of all monads. In other words, this totality is nothing but the *information* of the world. In our modern terms, Leibniz's metaphysics is not only informational, but full of *recursion* of this sort.
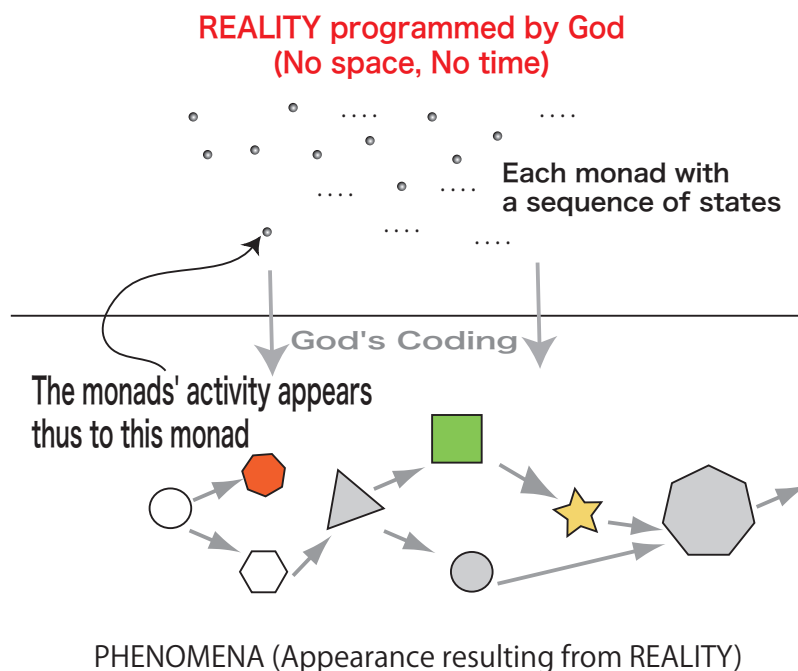

Figure 1. Reality and Phenomena

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Now, since I wish to concentrate on the problem of time, let me briefly summarize basic features of my Informational Interpretation of *Monadology* (the ground of this interpretation is explained in Uchii 2009, 2014a, b, and c). Some features relevant to Leibniz's theory of time will be discussed in more detail later.

(1) All monads are created by God, as the *ultimate programmer*.

(2) The world of monads is governed by the Pre-established harmony.

(3) The world of monads is without space, and without time.

(4) Each monad changes its state (perception) according to its own distinctive transition function (according to my interpretation), and the whole sequence of its states is given at once (Uchii 2014a, sect. 1).

(5) The unity or individuality of each monad is defined by its sequence of states (in other words, by its initial state and its transition function and both are given by God).

(6) Monads are organized into many groups, each of which is governed by a single dominant monad, called *anima* or *entelechy*. And such groups are again organized into a nested structure, *ad infinitum*. The whole is a single world, ultimately governed by God (Uchii 2014a, sect. 12).

(7) There are many invariant structures in the world of monads. Most important is that the information is conserved, and for each monad, the order of state-transition does not change. As a consequence, the order of world-states does not change either, where a "world-state" is a conjunction of all instantaneous states, aligned by one-to-one correspondence, of monads (Uchii 2014b, sects. 20, 21).

(8) The activities (i.e., state-transitions) of these monads produce phenomena (appearances) for each monad, as was already explained. The genesis of phenomena, which may well be different depending on the grade of monad, depends on God's coding. That is, the same world state W may well appear differently to humans and to angels, for instance.

(9) Further, notice that the *quantitative* features of phenomena, including the magnitude of space and time (in other words, length and duration) must be generated by God's coding of *Ph*, by preserving the invariant structures of the monadic world. According to my interpretation, no other elements of *Monadology* can be responsible for this job (NB: a monad has *no magnitude*).

2. From Information to Energy

Now we begin to see more details. First, feature (7): I understand that all monads are created by God at once, and each monad is given a unique transition function, together with its initial state. This means that the *sequence of states* of each monad is given at once. And this further implies that the information characterizing each monad does not change, unless God annihilate it. Thus it is clear that this information is conserved, and hence the information of the whole world of monads is also conserved.

Now, let us recall that Leibniz repeatedly says that each monad is given the *primitive force* (active and passive), and this is the source of each monad's activities. And the primitive force appears, in the phenomenal world, as the *derivative* force, and this governs the activities of bodies in the phenomena. Further, within Leibniz's dynamics, one of the most important laws is the conservation of *vis viva*, *energy* in modern terms. We can see that this is a rather direct consequence from the conservation of information in the monadic world. For, the primitive force is another name for the principle regulating the state-transition of each monad, and what I called "transition function" of a monad is just a modern expression for saying the same thing. And Leibniz's intention seems clear: the conservation of energy is grounded by the conservation of information in reality, in the world of monads.

FORCE	Primitive (in monad) [informational]	Derivative (in phenomena) [dynamical]
Active (form)	controlled by transition function, and produces the whole sequence of states	motion
Passive (primary matter)		resistance and impenetrability

Table 1. Four Kinds of Force

3. Space and Analysis Situs

But our main concern is time. Time does not exist in the monadic world. Then, how can time be generated in the phenomenal world? Of course, another, exactly similar question arises as regards space too. And my interpretation of Leibniz's theory of time is that he moved from space to time by mediation of motion. So let me briefly touch upon the problem of space and geometry. This problem has been masterfully treated by Vincenzo De Risi (2007). Leibniz's work on *Analysis Situs* (analysis of situations) deals with this question, as De Risi has convincingly argued. In a nutshell, bodies *coexisting* in the phenomenal world are *situated*, or related with

each other, and this situation is the phenomenal expression of the groups of monads corresponding to them. Thus space results, as a totality of such relations. Or more precisely, space in general can be defined by considering all *possible* situations, and the actual space can be defined in terms of the actual situation of the world. Informally speaking, situations are geometrical and metrical relations based on the basic notion of *congruence*, but metric itself is arbitrary in *Analysis Situs*; so that *Analysis Situs* can characterize, in qualitative terms, space and spatial relations.

Now, I have to mention that Leibniz had some difficulty for defining *straight line* in terms of *Analysis Situs*. The source of the difficulty is the gap between "the shortest path" and the "straight line" connecting two points. Leibniz often tries to identify these two. But we, thanks to our hindsight, can see that these two are identical only in Euclidean geometry; the shortest path is now called *geodesic,* and it can be *curved*. As we will see shortly, this difficulty affects Leibniz's dynamics too, because inertial motion is always taken to be straight and uniform, and moreover, straight motion is given a special status in Leibniz's dynamics.

We will ignore these problems for a while, and move on to the problem of time.

4. Motion as a Change of Situation

The crucial question for us is: Given a theory of space, how can we move on to a theory of time? Leibniz is of course well aware of the essential difference between space and time. He frequently puts it this way: "space is the order of coexisting possibles, time is the order of inconsistent possibles" (in a letter to de Volder), or "space is the order of coexistences, time is the order of successions." Here we have to be careful. Space and time are needed only in phenomena; the reality itself has no need for them. But at least some features of reality must be reflected in phenomena so that our mind can know them.

Now, the notion of coexistence presupposes *simultaneity*, and two states of affairs, which are incompatible at the same moment, may well occur in different moments. Thus, many different states of affairs can occur through time. And notice that this feature is similar in reality and in phenomena as well. Thus, Leibniz is saying, in effect, that a change of state is closely connected with the notions of *order* and *time*. First, any monad's activity is nothing but a sequence of states, a sequence of perceptions. Such a sequence has definitely an *order*, and this order is uniquely determined by a transition function. Thus we can clearly see that this is the *basis for time* in the phenomena. Further, in *Initia Rerum Mathematicarum Metaphysica* (1714) [Metaphysical Foundations of Mathematics], he defines motion as a *change of situation*. Here he is talking about phenomena, and hence we need time. And it is

clear that dynamics is concerned with situational changes among the bodies in phenomena. This suggests that spatial geometry can be connected with time by means of motion. But Leibniz begins this paper with a few definitions of temporal concepts, and his definition of motion comes later. So let us see them.

5. Simultaneity and Temporal Order

The temporal concepts defined are "*simultaneous*," "*prior*," and "*posterior*." And the reader may be easily misled to assume that Leibniz is discussing only the temporal concepts in phenomena. But wait a second! The title of the paper refers to the "metaphysical foundations." Then Leibniz must be talking about reality also. Thus I understand that he is trying to point out the connection between these temporal concepts and their counterparts in reality, the world of monads. This conjecture has a strong ground, because Leibniz always assumes *homomorphisms* (partial isomorphism) between reality and phenomena. When he says that certain features of reality is *expressed* in phenomena (e.g., bodies and their motions), he is speaking on this assumption. Likewise, when I am talking about *coding*, I am also speaking on the same assumption. Any coded message must preserve some *essential content or structure* of the original information; otherwise, it should be useless. And if we keep this in mind, Leibniz's definition, and discussion of those temporal concepts should be interpreted in two, interconnected ways, namely, the *basis of time* in reality, and *temporal relations* in phenomena. I named this, *Double Interpretation*.

I will refrain from getting into textual analyses, since it would take much time, and it may be boring to philosophers of science (see Uchii 2014b). However, the following crucial statement must not be ignored, since my interpretation hinges on it.

> My earlier state involves a reason for the existence of my later state. And since my prior state, by reason of the connection between all things, involves the prior state of other things as well, it also involves a reason for the later state of these other things and is thus prior to them. (Loemker 1969, 666)

The assertion that everything is connected is one of the distinctive features of Leibniz's philosophy. But, here, this assertion is specifically applied to the problem of simultaneity and temporal order. And we can clearly see that Leibniz is alluding to a homomorphism or correspondence between the state-transition of a monad and the temporal order in the phenomena. Moreover, he is clearly assuming an *alignment* of the states of the world, both monadic and phenomenal. My interpretation is simple: given all monads and their state-transitions, their instantaneous states are aligned, *across* all monads, according to the order of these

states; in other words, these aligned states are connected by this one-to-one correspondence, and this is the *basis* of temporal *simultaneity*.

Notice that the initial states of monads are given by God when He created them; so that there is no difficulty for the alignment of the initial states, nor for the subsequent order and alignment. See Table 2, for a *simplified* toy-model (see also, my explanation below the Table).

MONADS	STATES (second digits are their ORDER)				
monad 1	s(1,**1**)	s(1,**2**)	s(1,**3**)	. . .	. . .
monad 2	s(2,**1**)	s(2,**2**)	s(2,**3**)	. . .	. . .
monad 3	s(3,**1**)	s(3,**2**)	s(3,**3**)	. . .	. . .
. . .	. . .	. . .	. . .	. . .	. . .

Table 2. Monadic States are aligned

I have to emphasize that the notion of *simultaneity* should be understood as a *temporal* concept (for describing phenomena), and the alignment in the Table 2 must not be called a relation of simultaneity. The alignment is a mere one-to-one correspondence with no temporal implication; but it is the *basis* for producing time in phenomena. Thus, any temporal order in the phenomena must preserve the structure of this alignment, although, the temporal order contains *metric*, in addition.

In order to avoid a misunderstanding, I have to add the following important remark: corresponding to the "infinite divisibility of matter and space," any finite time interval should be *infinitely divisible*. This implies that the model of the Table 2 is a *toy-model*. Between s(1,1) and s(1,2), for instance, an infinity of states should recur. Leibniz suggests this in his reply to Clarke (see Fifth paper, sect. 105.) Thus, in order to generalize the idea of Table 2, we have to use a real number (say, in the interval [0,1), the right-hand side open) for signifying the order. For any two order numbers q, r ($q<r$), they show an *order only*, not quantity. (Metric must preserve the order, but order does not imply quantity, which comes from another source.) After all, if a monad is to reflect the whole world in its own way, its sequence of states should be as rich and complex as the whole world!

On this assumption, given all monads and their sequence of states, an alignment of states can have, generally, various possibilities, provided that alignment does not disturb the original order. Such alignments can be *easily* obtained, because one-to-

one correspondences between two or more continua are quite flexible. And one thing is certain. Leibniz has to assume such an alignment; otherwise there should be no *real basis* for time.
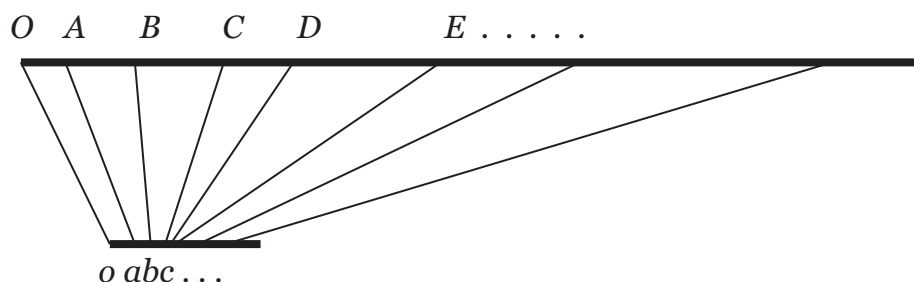

Figure 2. Alignment of States and Order between two Continua

6. How to connect Space and Time?

We now come to the crucial part. Starting from the basis of time in reality, how can Leibniz obtain metric time in the phenomena? We have to employ laws of dynamics, but without getting involved in circular reasoning. Because, laws of dynamics are ultimately grounded on metaphysical basis, and the latter does not presuppose time. I will argue that "timeless order" of state-transition can be projected, so to speak, to the phenomenal world, via coding, and it is this coding that produces metric time. I will also argue that both classical time and relativistic time can be reconstructed, depending on coding. This amazing possibility stems from the timeless nature of reality (monads). Moreover, since coding of phenomena may well be different for higher or lower grades of monads and of organized groups of monads, even *multi-metric dynamics* can be possible.

Although Leibniz did not discuss, systematically, how we humans should determine the metric of time, he described at least an outline of the relation of space and time in *Initia Rerum*. De Risi, after discussing how *Analysis Situs* contributed to the philosophy of space and geometry, criticized Leibniz's treatment of time. He argues that Leibniz's did not clarify the foundation of time. I do not think so.

De Risi's fine analysis of Leibniz's view on space and geometry is quite valuable: the metric of space, according to Leibniz, can be determined if we can have a suitable criterion for *congruence* of distance. In modern terms, if we have a rigid rod, and can assume the rod does not change its length through any motions (translation, rotation, etc.), the distance between any two points can be determined, so that the length of rod can determine the spatial metric. Any repeated use of the rod implies counting the number of unit lengths between the two points. However,

since *Analysis Situs* applies only to *coexisting* bodies, the notion of spatial congruence cannot be applied to time, De Risi argues, because a situation cannot hold among *successive* states (or situations) of bodies. He is right, so far.

But Leibniz, in *Initia Rerum*, dwells on the analogy between space and time. And the crucial point emerges when he defines a *path* of a movable thing.

> A *path* [*via*] is the continuous and successive locus of a movable thing. (*Initia Rerum*, Loemker 1969, 668)

Many Leibniz scholars may quickly point out that Leibniz denies the existence of a motion, because, at any instant, a body in motion can have its place (locus), but its *past* states and *future* states cannot coexist with its present state. Yes, indeed Leibniz himself emphasizes this in *Specimen Dynamicum* (1695). But in the definition of path, he is not saying that a motion exists; he is merely saying that a path, a *spatial* entity, can exist. If you draw a line on a blackboard, that line is a path, and all parts of that line exist simultaneously. Although your motion of drawing does not exist anymore, *its path exists* on the blackboard. This is the crucial connection of time and space, *via motion*.

Further, Leibniz adds the following careful remark:

> But we know as coexisting, not merely those things we perceive together, but also those which we *perceive successively*, provided only that, during the transition of from the perception of one to the other, the former is not destroyed and the latter generated. (Loemker 1969, 671, my italics.)

In terms of our example of the path on the blackboard, Leibniz's point is clear. If the line you have drawn is long, we take time for seeing the whole path by successive perceptions. Suppose our perception, beginning with the left end, finishes at the right end of the line. Then, we are sure that the left end *exists now* when we come to the right end, and we are also sure that, when we started perception, the right end, as well as the left end, *existed then*. The *motion* of your chalk may be regarded as, being *represented* in the *line* on the blackboard, although the motion itself does not exist now. Then, it is clear that the metric of time (of your motion) is somehow connected with the length of the line.

7. Inertial Motion

But we still cannot recognize the metric of time by the length of line alone. We need some standard for defining the unit of time. In the case of spatial metric, the notion

of congruence played a crucial role, and this suggests that we need the notion of *congruence for time interval* also. Is there any motion which can be used for this purpose? We may immediately suggest that *inertial* motions can be useful in this context. An inertial motion (in classical physics) is *straight* and *uniform*. Then, if we can measure the length *L* of a finite portion of the path of an inertial motion, this may serve as a criterion of temporal congruence. By comparing *L* with the length of any portion *P* of the path of another motion, we can know the ratio of *P/L*, which can show the *ratio of time* that motion took, compared to the inertial motion. For instance, Galileo's research on a projectile may be recalled; this is a beautiful example of "measuring a change by a steady change."

In the following Figure, inertial motion can be used as a "clock" (a unit length is repeated uniformly), and an accelerated motion of free fall changes its speed. The speed as well as time can be measured in terms of the unit length of inertial motion.
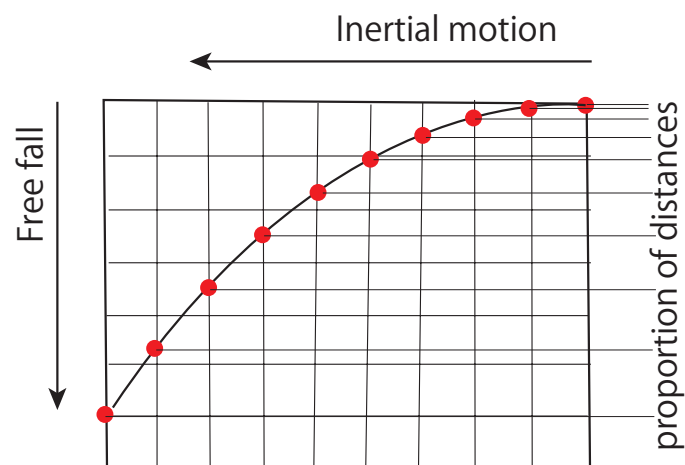

Figure 3. Galileo's Projectile

However, this method for introducing time is question-begging, since the uniformity of inertial motion is presupposed. The "uniformity" is nothing but the uniformity of *speed*, which presupposes the concept of time already.

8. Inertial Systems are interesting enough

The problem seems to boil down to the *foundations* of the law of inertia; thus we have to get into metaphysics. I am now working on a possible reconstruction of the Leibnizian foundations of the law of inertia (generalization, and taking gravity into consideration too), and in this paper, I cannot get into this problem. For, despite the absence of the justification of the law of inertia in Leibniz, we can find several important features of his theory of time, based on the law of inertia. So, let us start

from this law. In modern terminology, we restrict our attention only to inertial systems. After all, the so-called "Newtonian mechanics" can be used for inertial systems, and this circumstance seems to be the same with Leibniz's dynamics.

Thus we take it for granted that any inertial motions have a *uniform speed*. Then, any inertial motion can be used as a "clock" for defining the congruence of time intervals. The reason is quite similar to the case of Galileo's projectile; any finite portion of the path of an inertial motion can give a unit length, and because of the constancy of the uniformity of inertial motion, the chosen length of may be regarded as representing a *unit length of duration*. Further, by comparing inertial motions with different speed, this unit can be divided into any smaller length, so that the situation is exactly similar to the spatial length. And once we establish the criterion of temporal congruence, we can extend it to periodic phenomena, such as the revolution of the earth or the motion of a pendulum, or the vibration of a spring, etc.

So far, you may think there is nothing new. But notice that Leibniz can reduce the congruence of time to that of space. This means he does not need *absolute time*, like Newton. Moreover, *space and time are connected*, as he is saying "everything is connected in the world."

9. From Inertial Systems to Relativity

Since we are concerned with inertial systems, spatial distance can be measured by a straight line, and the geometry of space is Euclidean. But what about time? Here, a novelty appears, because of the connection between space and time. The metric of time is dependent on spatial metric, but there is no need for it to be symmetric with it. Of course, if it is symmetric, Leibniz's dynamics may be, in effect, equivalent with Newtonian mechanics. But, since the *coding for phenomena* comes in for determining temporal metric, some interesting thing can happen. It is possible that the mutual dependence of spatial and temporal metric is such that the resulting metric is the *Lorentz metric* of special relativity. Leibniz knew, in all probability, that the speed of light is finite, because both Newton and Huygens knew Ole Rømer's discovery. Moreover, his insistence on *plenum* may impose a limitation for the propagation of information in the phenomenal world. We know because of our hindsight that relativity together with the constancy of the speed of light lead to the Lorentz metric. I am now saying that Leibniz entertained such possibilities; of course not! I am rather saying that it is worthwhile to explore the potentialities of the Leibnizian dynamics. And I claim that his dynamics had enough flexibilities to

adapt itself to special relativity. This flexibility is out of question for Newtonian mechanics, since it assumed absolute space and time.

Notice that it is perfectly all right, for Leibniz, to have dynamics of special relativity, with *no modifications whatsoever in his metaphysics*. The invariant structure of information is *intact*. Only by changing coding from reality to phenomena, the same invariant structure can produce either classical or relativistic physics.

10. Leibniz's Demon

Let me end this paper with one more amazing example. Do you know *Leibniz's Demon*? It appears in section 61 of *Monadology*:

> every body is affected by everything that happens in the universe, to such an extent that he who sees all can read in each thing what happens everywhere, and even what has happened or what will happen, by observing in the present what is remote in time as well as in space. (Ariew and Garber 1989, 221)

Most modern readers familiar with relativity theories may argue against this, by saying that this demon is against our best theories! On the one hand, Leibniz seems to have a good reason to assert the possibility of this demon. For, in the world of monads (reality), everything is given at once, as timeless entities and series of states. On the other hand, it seems that Leibniz is talking about this possibility in the *phenomenal world*; then this possibility seems to be excluded by relativity. Now I think Leibniz must have had a theory of time that can solve this problem.

According to my Informational Interpretation, space-time metric must be generated by coding for phenomena. And this coding may well be different for different monads, especially if there are different grades of monads, such as humans and angels. Then, because of this difference of coding, the speed of light can be different: much faster for angels, slower for humans (see Figure 4). And again, this is possible *without changing anything in reality*. Leibniz's dynamics has this much of flexibility and potentiality.
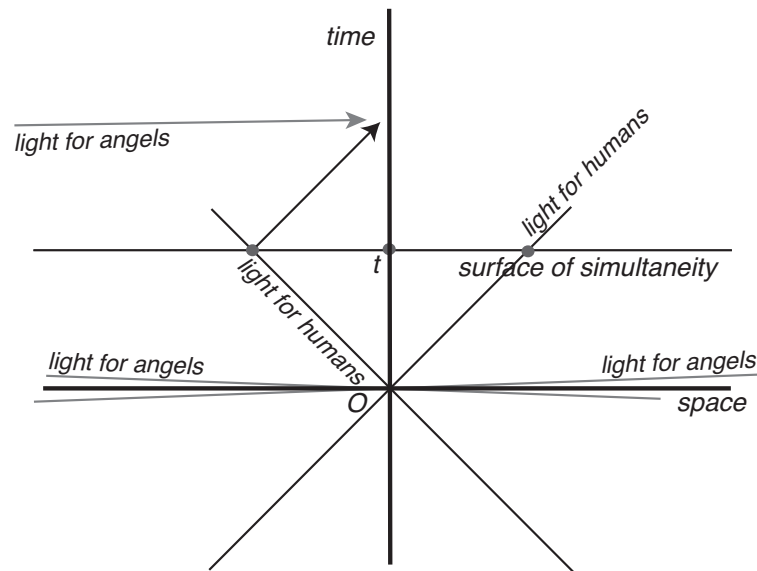
Figure 4. Bi-metric Relativity

References

Adams, R. M. (1994) *Leibniz: Determinist, Theist, Idealist*, Oxford University Press, 1994.

Aiton, E. J. (1985) *Leibniz, a Biography*, Adam Hilger, 1985.

Alexander, H. G., ed. (1956) *The Leibniz-Clarke Correspondence*, Manchester University Press, 1956.

Antognazza, M. R. (2009) *Leibniz, an Intellectual Biography*, Cambridge University Press, 2009.

Ariew, R., and Garber, D., tr. and ed. (1989) *G. W. Leibniz Philosophical Essays*, Hackett, 1989.

Arthur, R. T. W. (1985) "Leibniz's Theory of Time," in K. Okruhlik and Brown, J. R. 1985, 263-313.

———- (2014) Leibniz (Classic Thinkers), Polity Press, 2014.

Barbour, J. B. (1995a) "Mach before Mach," in Barbour and Pfister 1995, 6-8.

———- (1995b) "General Relativity as a Perfectly Machian Theory," in Barbour and Pfister 1995, 214-236.

———- (2000) *The End of Time*, Phoenix, 2000.

———- (2001) *The Discovery of Dynamics*, Oxford University Press, 2001.

Barbour, J. B., and Bertotti, B. (1982) "Mach's Principle and the Structure of Dynamical Theories," *Proceedings of the Royal Society London* 382: 295-306.

Barbour, J. B., and Pfister H., eds. (1995) *Mach's Principle*, Birkhäuser, 1995.

Bertoloni Meli, D. (1993) *Equivalence and Priority: Newton versus Leibniz*, Clarendon Press, Oxford, 1993.

Burks, A. W., ed. (1970) *Essays on Cellular Automata*, University of Illinois Press, 1970.

Cover, J. A. (1997) "Non-Basic Time and Reductive Strategies: Leibniz's Theory of Time," Studies in the History and Philosophy of Science 28 (1997), 289-318.

Davis, M. (2011) *The Universal Computer, Turing Centenary Edition*, CRC Press, 2011.

De Risi, Vincenzo (2007) *Geometry and Monadology: Leibniz's Analysis Situs and Philosophy of Space*, Birkhäuser, 2007.

Earman, J. (1989) *World enough and Space-time*, MIT Press, 1989.

Einstein, A. (1905a) "On the Electrodynamics of Moving Bodies," *Annalen der Physik* 17, reprinted in the *Collected Papers of Albert Einstein* 2, pp. 140-171.

———- (1905b) "Does the Inertia of a Body depend upon its Energy Content?" *Annalen der Physik* 18, reprinted in the *Collected Papers of Albert Einstein* 2, pp. 172-174.

———- (1952) *Relativity, the Special and the General Theory*, Three Rivers Press, 1961. (Original edition in German, 1917, and this translation is 15th ed., 1952)

Garber, D. (1985) "Leibniz and the Foundations of Physics: The Middle Years," in K. Okruhlik and Brown, J. R. 1985, 27-130.

——— (1995) "Leibniz: Physics and Philosophy," in Jolley (1995), pp. 270-352.

——— (2009) *Leibniz: Body, Substance, Monad*, Oxford University Press, 2009.

Gerhardt, C. I., ed. [abbreviated as Gerhardt M] *Leibnizens mathematische Schriften*, 7 vols., A. Asher, 1849-1863.

Gerhardt, C. I., ed. [abbreviated as Gerhardt] *Die Philosophischen Schriften von Gottfried Wilhelm Leibniz*, 7 vols., Weidmann, 1875-1890.

Hacking, I. (1985) "Why Motion is Only a Well-Founded Phenomenon," in K. Okruhlik and Brown, J. R. 1985, 131-150.

Jolley, N. ed. (1995) *The Cambridge Companion to Leibniz*, Cambridge University Press, 1995.

Lynden-Bell, D. (1995) "A Relative Newtonian Mechanics," in Barbour and Pfister 1995, 172-178.

Loemker, L. E., tr. and ed. (1969) *Gottfried Wilhelm Leibniz, Philosophical Papers and Letters*, 2nd ed., Kluwer Academic Publishers, 1969.

Mach, E. (1960) *The Science of Mechanics*, Open Court, 1960.

Mates, B. (1986) *The Philosophy of Leibniz: Metaphysics and Language*, Oxford University Press, 1986.

Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973) *Gravitation*, W. H. Freeman, 1973.

Newton, I., ed. by F. Cajori (1962) *Sir Isaac Newton's Mathematical Principles*, 2 vols., University of California Press, 1962.

Okruhlik, K., and Brown, J. R., eds. (1985) *The Natural Philosophy of Leibniz*, The University of Western Ontario Series in Philosophy of Science, Reidel, 1985.

Rescher, N. (1991) *G. W. Leibniz's Monadology*, University of Pittsburgh Press, 1991.

Rutherford, D. (1995a) "Metaphysics: The Late Period," in Jolley (1995), pp. 124-175.

——— (1995b) *Leibniz and the Rational Order of Nature*, Cambridge University Press, 1995.

Shimomura, T., et al., eds. (1988-1999) *Leibniz Opera Omnia* (in Japanese), 10 vols., Kosakusha, 1988-1999.

Smith, J. E. H. (2011) *Divine Machines: Leibniz and the Sciences of Life*, Princeton University Press, 2011.

Smith, J. E. H., and Nachtomy, O., eds. (2011) *Machines of Nature and Corporeal Substances in Leibniz*, Springer, 2011.

Taylor, E. F., and Wheeler, J. A. (1992) *Spacetime Physics: Introduction to Special Relativity*, 2nd edition, Freeman, 1992.

Taylor, E. F., and Wheeler, J. A. (2000) *Exploring Black Holes*, Addison Wesley Longman, 2000.

Thatcher, J. W. (1970) "Self-Describing Turing Machines and Self-Reproducing Cellular Automa," in Burks (1970), 103-131.

Turing, A. (1936) "On Computable Numbers, with an Application to the Entsheidungsproblem," *Proceedings of the London Mathematical Society*, ser. 2, 42 (1936), 230-267.

Uchii, S. (1989) *Truth, Proof, and Computation: Logic and Machine* (in Japanese), Kyoto: Minerva, 1989.

——— (2004) *Tracing Einstein's Ideas—Introduction to the Philosophy of Space and Time* (in Japanese), Kyoto: Minerva, 2004.

——— (2006) *The Riddle of Space and Time* (in Japanese), Chuko Shinsho, 2006.

——— (2009) "An Informational Interpretation of Monadology," in *Logic, Methodology and Philosophy of Science, Proceedings of the 13th International Congress* (ed. by Clark Glymour, Wang Wei, and Dag Westerståhl), College Publications, 2009, pp. 344-353.

——— (2014a) "Monadology, Information, and Physics, Part 1: Metaphysics and Dynamics," PhilSci-Archive 10599.

——— (2014b) "Monadology, Information, and Physics, Part 2 (revised): Space and Time," PhilSci-Archive 11088.

——— (2014c) "Monadology, Information, and Physics, Part 3 (revised): Inertia and Gravity," PhilSci-Archive 11125.

Vailati, E. (1997) *Leibniz and Clarke*, Oxford University Press, 1997.

Wang, H. (1957) "A Variant to Turing's Theory of Computing Machines," *Journal of the Association for Computing Machinery* 4, 1957.

[Uchii 2014a, b, c will be revised soon.]