Assertion and grounding: a theory of assertion for constructive type theory

Maria van der Schaar

Abstract   Taking Per Martin-Löf's constructive type theory as a starting-point a theory of assertion is developed, which is able to account for the epistemic aspects of the speech act of assertion, and in which it is shown that assertion is not a wide genus. From a constructivist point of view, one is entitled to assert, for example, that a proposition *A* is true, only if one has constructed a proof object *a* for *A* in an act of demonstration. One thereby has grounded the assertion by an act of demonstration, and a *grounding account of assertion* therefore suits constructive type theory. Because the act of demonstration in which such a proof object is constructed results in knowledge that *A* is true, the constructivist account of assertion has to ward off some of the criticism directed against knowledge accounts of assertion. It is especially the internal relation between a judgement being grounded and its being known that makes it possible to do so. The grounding account of assertion can be considered as a justification account of assertion, but it also differs from justification accounts recently proposed, namely in the treatment of selfless assertions, that is, assertions which are grounded, but are not accompanied by belief.

1 Introduction

Assertions play a role as premises and conclusions in our reasoning. Assertion is therefore an important topic for philosophers. Recent papers on assertion focus on the question: under what condition is one entitled to make an assertion? A well-known account of assertion is given by Williamson (1996), who defends the thesis that one should assert that *S* only if one knows that *S*. Such an account of assertion is called

M. van der Schaar (✉)
Faculty of Philosophy, Leiden University, Leiden, The Netherlands
e-mail: m.v.d.schaar@phil.leidenuniv.nl

a *knowledge account of assertion*. In this paper I give an account of assertion for Per Martin-Löf's constructive type theory. From a constructivist point of view, one is entitled to assert that a proposition *A* is true, only if one has constructed a proof object *a* for *A*, and, in general, an assertion may be made only if it has been grounded by means of an act of demonstration. This account of assertion may therefore be called a *grounding account of assertion*.

A theory of assertion should not merely answer the question under what condition one is entitled to assert. It should also give an account of the relation between assertion and judgement, between the speech act of assertion and the declarative sentence, and it should give an account of the fact that assertions may be correct or incorrect. In constructive type theory the notion of judgement plays an important role, because inference rules are understood as applying to judgements. If it is true that assertion and judgement are correlated notions, the constructivist account of judgement will yield a theory of assertion for constructive type theory. By relating the notion of judgement to the notion of assertion, it is possible to give a pragmatic interpretation of judgemental force.

In order to give a proper evaluation of a constructivist theory of assertion, the question is raised what every theory of assertion has to account for (Sect. 2). From a constructivist point of view, grounding an assertion of the form *A* is true amounts to knowing that *A* is true. It is therefore important to understand to what extent the grounding account of assertion can be understood as a knowledge account of assertion. And, because an assertion that is grounded by an act of demonstration is justified, the grounding account of assertion is also a justification account. The three accounts may all be called epistemic accounts of assertion. In Sect. 3, I focus on the problems that philosophers have put forward for knowledge accounts of assertion. Does this critique apply to all epistemic accounts of assertion? In Sect. 4, the concepts that are essential to a constructivist theory of assertion will be elucidated; these will include the concepts grounding, proof, proposition, knowledge, judgement, judgemental correctness and propositional truth. A full constructivist theory of assertion is developed in Sect. 5. The final Sect. 6 gives an evaluation of the constructivist account of assertion proposed: to what extent does it differ from both knowledge and justification accounts of assertion as they are commonly understood; and is the grounding account of assertion an improvement of these other accounts in at least some aspects?

2 Elements of a theory of assertion

Every theory of assertion has to explain in what sense:

(1)   assertion is a speech act;
(2)   assertions are related to judgements;
(3)   assertions may be correct or incorrect.

And every theory of assertion has to say what type of speech act assertion is by indicating:

(4)   under what condition one is entitled to assert.

(1) *Assertion is a speech act*. All speech acts have certain characteristics in common. Every speech act results in a product. The act of promising results in a promise made;

and the act of pronouncing a judgement upon the accused results in a verdict. The act of promising and the act of judging exist only for a short time, but the promise made and the verdict are still in force after the act of promising or judging has ended. Like other terms for speech acts, *the term 'assertion' shows the act/product ambiguity* (1a): the term may either stand for the act of assertion or for the assertion made. There is an internal relation between act and product: the act of asserting necessarily results in the assertion made. And there is no assertion made without a corresponding act of assertion, although that act may no longer exist when the assertion made is still in force. In reasoning, our assertions function as premises and conclusion (see 3e). These assertions cannot be acts, for these are gone when the conclusion is reached; premises and conclusion are assertions made, the products of acts of assertion.

The act/product distinction is not the same as *the act/proposition distinction* (1b), which holds for most speech acts. The assertion 'John is the father of Mary.' and the question 'Is John the father of Mary?' contain the same proposition *that John is the father of Mary*. The fact that different speech acts may have their proposition in common explains, for example, that the above-mentioned assertion can be considered as an answer to the corresponding question. Propositions are thus essential for relating different speech acts to each other.

In all speech acts we are free either to perform the act or not. Assertion is *up to us* (1c), although we will see below that there is also a sense in which this is not the case.

Each type of speech act has a certain quality that distinguishes it from other types. It is the judgemental or *assertive force* (1d) that makes the act an act of assertion. The assertive force may be indicated by the assertion sign ($\vdash$). The assertion sign shows that the assertion has been made: the assertion sign therefore precedes one's premises and conclusions. The explanation of assertive force is an explanation of what kind of speech act assertion is. And what kind of speech act assertion is, becomes clear when the pragmatic rule of assertion is given, that is, when it is shown under what condition one is entitled to assert (see point 4). Since Russell we use the assertion sign as an undivided unity, but Frege intended it to be composed of the horizontal, called 'content stroke', and the vertical, called 'judgement stroke' ('Urtheilsstrich,' Frege 1879, § 2). The latter is a sign that the sentence is used with judgemental or assertive force. The content stroke is a sign that what follows is a judgeable content. According to Tyler Burge: "the vertical judgement stroke represents judgmental force, and the horizontal alone comes to represent a semantical predicate, such as 'is a fact,' or 'is true'" (Burge 1986, 114). The 'is true' part is thus not a sign that the content is asserted.[1] The 'is true' part rather indicates that the sentence is standardly used with assertive force, in contrast, for example, to a part such as 'Is it true …?' or 'May it be true'. A theory of assertion has to make a distinction between the 'is true' part and a sign of judgemental force. Further, a theory of assertion should answer the question whether there is only one type of assertive force, as Frege has argued for, or whether one has to acknowledge besides assertoric force a special force of denial. It is to be noted that the phrase 'I assert that', like the phrase 'is true', cannot function as sign of assertive force, because we may use this phrase in front of a declarative that is used as

---

[1] Frege himself is ambiguous concerning the phrase 'is a fact' in the *Begriffsschrift*, for he seems to imply that the predicate 'ist eine Tatsache' may function as sign of judgemental force (Frege 1879, § 3).

antecedent of a conditional sentence. To assert, though, *is* a performative verb, because the verb may be used to effect what it signifies. Without indications to the contrary, *in* saying 'I assert that *S*,' I did assert that *S* (Austin 1962, 122); 'to assert' is therefore a performative verb, on Austin's account.

Each speech act is in need of language in order to be performed. In order to be well understood, one may use a special type of sentence for making a certain type of speech act. In the case of assertions, it is the utterance of the declarative sentence that is standardly used for making an assertion. A theory of assertion has to account for the *special connection between assertion and the declarative sentence* (1e). Without indications to the contrary, we understand an utterance of the declarative to make an assertion. It is not to be denied that the occurrence of a declarative sentence may be used for other purposes, too. We often use (the occurrence of) a declarative to ask a question, or to express the antecedent of a conditional. Furthermore, the use of the declarative is not necessary to make an assertion: we sometimes assert by means of the utterance of an interrogative sentence, or by nodding our head. A theory of assertion should also account for the use of the declarative on stage to make a mock assertion (1e′), a "Scheinbehauptung", as Frege put it. Not all philosophers believe that there is a special relation between the speech act of assertion and the declarative sentence (cf. Davidson 1984). But even these philosophers will agree that we cannot assert by means of an isolated utterance of a simple collection of terms, such as 'runs, sits, walks'; neither can we use an isolated utterance of a that-clause or a phrase like 'the death of Caesar' to make an assertion. Apart from the special relation between the declarative and the assertion, one may wonder whether there is *a special form* (1e″), or a special linguistic structure, that is somehow essential or basic to assertion and judgement. Do all assertions and judgements have a subject-copula-predicate (S-is-P) structure, or is it rather the predicative form … *is a fact* that is common to all judgements, as Frege proposed in the *Begriffsschrift*?

All speech acts are done by an agent. For assertion this means that an assertion is always *made from a certain perspective*: the perspective of the asserter (1f).

Furthermore, all speech acts are social acts. One uses an assertion to convince others, or to defend the truth of a declarative in a dialogue. And, an assertion licenses others to reassert; *entitlements to assert can be conferred upon others* (1g).

Speech acts have special relations with other speech acts. A theory of assertion has to account for the fact that an assertion can always be considered as *answer to a question* (1h). Before we use the occurrence of a declarative sentence *S* to assert that *S*, we often wonder whether *S*, which wonder may be expressed by means of the question 'Is it true that *S*?'. If someone makes the assertion that *S*, we assume that he *apprehends the meaning of S* (1h′). If someone asserts that *S* without apprehending the meaning of *S*, the assertion is somehow unhappy or inappropriate. Some religious thinkers, though, believe that one may properly assert and judge what one does not fully apprehend.[2] The fact that *S* has a meaning may be considered as a presupposition of the assertion that *S*. If the presupposition is false, the assertion is void to such an

---

[2] According to Cardinal Newman, one can genuinely assent to a proposition that one does not fully understand. One needs to understand the predicate, but not the subject of the proposition (Newman 1870, 16, 17).

extent that no assertion is made. A theory of assertion has to give an account of the notion of presupposition, in order to answer the question *whether every assertion has a presupposition* (1h″).

(2) *Assertions have a special relation to judgements*. According to Frege, the act of assertion is the expression or announcement of a judgement [made] (Frege 1918, 62). And according to Dummett, the act of judgement is the interiorization of the external act of assertion (Dummett 1973, 362). Whether one takes the linguistic notion or the mental notion to be prior in the conceptual order, there seems to be an important relation between the two notions. Instead of the term 'judgement', the term 'belief' is often used in modern philosophy. Although one may decide to use these terms synonymously, there is in general an important distinction between the notions judgement and belief. Judgement, like assertion, is primarily an act, and the act of judgement is an all or nothing affair: judgemental or assertive force does not have degrees. Belief is primarily a mental state, and is generally understood to have degrees; belief in this sense is *conviction*, not *judgement*.

Assertion and judgement, though, need not go together (2′). Although an assertion is generally taken to be an announcement of a judgement made, the person who is making the assertion may not have made the relevant judgement. *We are capable of lying* (2′a). A lie is an assertion with the intention to mislead the hearer, in such a way that the hearer is to understand that the speaker believes or judges what he asserts, whereas the speaker believes or judges that the proposition he asserts to be true, is actually false. An assertion may also mislead the hearer in another way: by putting forward a declarative sentence with unqualified assertive force, the asserter presents himself as having a ground for his assertion (see 4). *If the asserter does not have such a ground, his assertion is misleading* (2′b) (independent of the question whether the asserter had the *intention* to mislead or not), although we do not call it a lie when the asserter believes what he asserts. When President George H.W. Bush asserted that there were weapons of mass destruction in Iraq, and used the assertion as a premise in the argument for attacking Iraq, he sincerely believed, I suppose, that there were such weapons; this means that he cannot be accused of lying. Still, the assertion was misleading, because he did not have grounds for his assertion. The only 'ground' that Bush had for his assertion, I suppose, was the strong conviction in the truth of what he asserted, that is, his assertion was the expression of a *prejudice*. Strong convictions do not seem to be the right type of ground for assertions that do not have convictions as their subject. A theory of assertion thus has to say something about what may count as ground for a certain assertion.

(3) One of the most important characteristics of assertion is that *they may be correct or incorrect*. Many philosophers explain the correctness of an assertion in terms of the truth of the proposition contained in the assertion, and it is therefore common to speak of the truth or falsity of an assertion. There are, though, other ways to explain the correctness of an assertion, as will become clear in this paper. In general, one may say that a theory of assertion has to give an account of *the relation between the correctness (or incorrectness) of an assertion and the truth (or falsity) of the proposition asserted to be true in the assertion* (3a). Further, it seems that there are *other speech acts whose product may be correct or incorrect* (3b). One may think of a speech act in which a guess is expressed, or in which a suggestion is done. A theory of assertion has to give

an account of the relation between assertion and these other speech acts: is assertion a *genus* containing the speech acts just mentioned as *species*, or is assertion related in a different way to these other speech acts? The characteristic of being correct or incorrect also holds for the mental counterpart of assertion, that is, for judgement. Judgement, though, is not up to us, precisely because of its being correct or incorrect. When I look out of the window, I cannot help but judge that it is raining. I could have decided not to look out of the window, and in that case I, perhaps, would not have made the same judgement. So there is a sense in which we can indirectly influence our judgements. Given, though, that I have been looking out of the window, I am no longer free to judge that it is not raining. In contrast, I am free to assert or not to assert that it is raining (see 1c). We may express the point thus: as speech act, assertion is up to us; as expression of our judgement, assertion is not completely up to us. If I am allowed, for the moment, to use the metaphor, introduced by Bernard Williams, of aiming at truth, one may say that *insofar as we aim at truth, assertion is not up to us* (3c).

Two last points relating to (3): assertions are related to other assertions, especially if they are made by the same agent. If the asserter realizes that he has asserted that *S*, and that he also has asserted that *it is not the case that S*, he has to withdraw at least one of his assertions, on pain of the accusation of being *inconsistent or irrational* (3d). This is precisely because assertions aim at truth, and because the assertions *S* and *it is not the case that S* cannot both be correct. Further, assertions can be used *to offer a reason* (3e). If someone asks 'How do you know that this bird is a tree-creeper?', the answer might be: 'It has the typical movements of a tree-creeper.' The assertion made that the bird has the typical movements of a tree-creeper may thus function as a reason for the assertion 'It is a tree-creeper'. Assertions function in our reasoning not only as conclusions, but also as premises on which we base our conclusions.

(4) *Under what condition is one entitled to assert*? Or, what is the pragmatic rule for assertion? Philosophers have given four different types of answer to this question. One may say that one is entitled to assert that *S*, if and only if (i) one believes that *S*; (ii) one has a justification for the truth of *S*; (iii) it is true that *S*; or (iv) one knows that *S*. If one defends the thesis that one is entitled to assert that *S*, if and only if one believes that *S*, one takes belief to be a necessary and sufficient condition for entitlement to assert. *Entitlement to assert* is to be understood exclusively in a cognitive sense: one may thus be entitled to assert that *S*, while it is inappropriate to make the assertion because it is contrary to etiquette, or because it is wrong for ethical reasons. If it is true that someone is entitled to assert that *S* if and only if he knows that *S*, one defends a knowledge account of assertion, and knowledge may be called the norm for assertion.[3]

There are some linguistic facts that may help us decide the question under what condition one is entitled to assert. When an assertion is put forward, *an interlocutor has a right to ask 'How do you know?'* (4a). The question presupposes that the asserter knows what he asserts, and the interlocutor seems to ask how the asserter has obtained his knowledge, or what his grounds are for making the assertion. The interlocutor expects that the asserter knows what he asserts, and demands him to give grounds for his assertion. *The Moorean paradoxes* (4b) also show something about the condition

---

[3] "Something is a norm, or a concept is normative, if it involves some form of *evaluation* or appraisal, or some standard of correctness" (Engel 2002, 131).

under which one is entitled to assert. Each of the sentences 'It rains, but I do not believe it.', 'It rains, but I have no evidence for it.', and 'It rains, but I do not know it.' is paradoxical only when uttered with assertive force. This seems to imply that one is entitled to assert only if one believes and knows what one asserts, and if one has evidence for what one asserts.

Points (4a) and (4b) give us reason to think that knowledge is the norm for assertion, and quite a few philosophers defend a knowledge accounts of assertion, although recently important arguments have been raised against such an account. In the introduction I already mentioned that the constructivist account of assertion is a grounding account of assertion. This account has some similarities with knowledge accounts of assertion, and it therefore has to be seen how the grounding account will answer the critique raised against knowledge accounts in general.

3 Arguments against knowledge accounts of assertion

A knowledge account of assertion has it that one is entitled to assert that *S* if and only if one knows that *S*, where entitlement is understood in the exclusive cognitive sense explained above. On a standard account of knowledge this implies that one is entitled to assert that *S*, if and only if (1) one believes that *S* is true, (2) *S* is true, where *S*'s being true need not be accessible to the asserter, and (3) one has a justification for *S*. Some philosophers, though, think that knowledge is too severe a norm for assertion. A knowledge account of assertion can be criticized from three different points of view: a belief account, a truth account, and a justification account of assertion.

Some philosophers say that one is entitled to assert that *S* if and only if one believes that *S*. By making the assertion that *S* one represents oneself as believing that *S*, as Davidson says.[4] A more developed account of assertion in terms of belief can be found in Bernard Williams' *Truth and Truthfulness* (2002, Chap. 4). For Williams, one of the central requirements for assertion is sincerity, which at the most basic level is simply openness: say what you believe.[5] Williams gives an account of assertion in terms of expression of belief. If one understands assertion as expression of belief, one needs to take special account of lying; not every assertion is an expression of belief. The central question that needs to be answered on such an account is: What is belief? Williams does certainly not neglect that question, and I will come back to his account of belief in the final section, as well as to the question whether sincerity or openness is a specific norm for assertion. Williams' account is supported by one of the characteristics of assertion: standardly, assertion is understood to be the announcement of a judgement made or a belief. According to Williams, knowledge is too strong a norm for assertion: the asserter should be in a position to apply the norm for assertion effectively, and if the norm is knowledge, he may not be in a position to do so, because he may reasonably think that he knows that *S* while not knowing that *S*. The asserter is not to be

---

[4] Davidson leaves open the possibility that there is more at stake than belief: "Someone who makes an assertion represents himself as believing what he says, and perhaps as being justified in his belief" (Davidson 1984, 268).

[5] "Sincerity at the most basic level is simply openness, a lack of inhibition" (Williams 2002, 75).

criticized for his assertion, when he reasonably thinks that he knows what he asserts (Williams 2002, 76, 77). It should be noted that for Williams assertion is, perhaps, a broader notion than it is for those who defend a knowledge account of assertion: "merely telling someone that P, without any special entitlement … is surely *already* assertion," he says (Williams 2002, 77).

Other philosophers have defended a truth account of assertion. According to John Searle, "the [illocutionary] point or purpose of the members of the assertive class is to commit the speaker (in varying degrees) to something's being the case, to the truth of the expressed proposition" (Searle 1979, 12). The illocutionary point, according to Searle, is one of the features of illocutionary force, and Frege's assertion sign marks the illocutionary point of all members of the assertive class, that is, of all assertives. Searle's assertive class includes speech acts that are not assertions: *suggesting* and *putting forward as a hypothesis*. Searle understands assertives also as expressions of belief, where belief may reach to degree zero. 'Belief' is thus to be understood as conviction, which comes in degrees. It is difficult to understand why he uses Frege's assertion sign in front of hypotheses; the acknowledgement of *degrees* of assertive force is not compatible with Frege's account of judgement and assertion.

A truth account of assertion is also defended by Matt Weiner. According to Weiner, assertion is a *genus* comprising "species such as reports, predictions, arguments, reminders, and speculations" (Weiner 2005, 229). If 'assertion' is understood in such a broad sense, knowledge is definitely too strong a norm for assertion. One of the questions that needs to be raised in this paper is how broad the concept of assertion is to be understood. How we choose our terminology is not an arbitrary matter. Somehow, we want to be able to blame a person whose assertions are expressions of mere guesses, that is, a norm for assertion seems to be violated in such cases. According to Weiner, the norm for the different species just mentioned is truth, and not knowledge, which point he elucidates by giving two examples. One example concerns a prediction; the other concerns the detective work of Sherlock Holmes. Suppose that Holmes and Doctor Watson are brought to a crime scene. Holmes looks carefully at the scene, and says: "This is the work of Professor Moriarty! It has the mark of his fiendish genius" (Weiner 2005, 231). According to Weiner, Holmes' remark is an assertion, and indeed there do not seem to be signs that the remark is not to be understood as an assertion. When the assertion turns out to be true, Weiner says, there is no way that the assertion can be called improper, although Holmes seemingly did not know that it was the work of Moriarty, for he based his assertion not on evidence, but on 'his sense of what Moriarty's crimes are like'. When asked 'How do you know?', Holmes is not able to give grounds for his assertion. I will give an evaluation of Weiner's example in the last section. For now, it is important to understand that the truth account has something important to say. If someone tells us the truth without having knowledge of this truth and without being able to give a justification, he is generally not blamed for this, and may even be praised for it. Any knowledge or justification account of assertion has to explain this intuition. Besides, both the belief and the truth account of assertion are supported by the fact that entitlements to assert can be conferred upon others. This may be problematic for those who defend a knowledge or justification account of assertion, because the asserter often does not make explicit the ground for his assertion.

Recently, knowledge accounts of assertion have been criticized from a different angle. Jonathan Kvanvig has defended the claim that it is justification rather than knowledge or truth that is the norm for assertion (Kvanvig 2009). He argues that it is important to make a distinction between two ways in which one may have to take back one's assertion: the agent may have to take back his assertion, because its content is shown to be false, in which case he is not to blame for his act of asserting, or he may have to take back his assertion because there was something wrong with the very act of asserting. In the latter case, one is blamed for making the speech act of assertion if it turns out that the asserter did not have any ground for his assertion. The norm for the *act* of assertion, according to Kvanvig, is not knowledge, but epistemic justification. The point of the question 'How do you know?' when an agent has made an assertion, Kvanvig says, is to find out what the reasons are for the assertion, and when the reasons are given, the questioner is satisfied, because his question is answered. Such reasons may not be sufficient, though, for knowledge.

What counts against both a belief and a knowledge account of assertion, Kvanvig says, is that we sometimes properly assert what we don't believe, as in the case of the teacher who is required to teach certain material: "we assert things we don't believe because of a social role we inhabit" (*Idem*, p. 141). Jennifer Lackey has called such assertions *selfless assertions* (Lackey 2007, 598 ff; cf. Douven 2006, 461). As a teacher, one may fully assert as biological facts the central theses of evolutionary theory, although one's personal religion makes one believe otherwise. Essential to selfless assertions is that the subject makes the assertion because it is best supported by the evidence (Lackey 2007, 603), and that he does not believe personally what he asserts "for purely non-epistemic reasons" (Lackey 2007, 599). These assertions are not insincere, and the speaker is not lying, it is said. Kvanvig, Lackey, and Douven (2006, 460) deny that belief is required as a norm for assertion. In selfless assertions, the speaker *is* able to answer the question 'How do you know?', because he can give grounds for his assertion, that is, because it is rational for him to believe what he asserts. And this in no way implies that the speaker *knows* or *believes* what he asserts. It thus seems that justification is the only norm for assertion. These selfless assertions form a tension with characteristic (1f) of assertions, which says that an assertion is always made from the perspective of the asserter. The perspective from which the assertion is made seems not to be a personal one, but rather that of a role to which the asserter has committed himself. A defender of a knowledge account of assertion has to explain this phenomenon of selfless assertions. In order to understand in what sense the grounding account of assertion proposed in this paper differs from justification accounts of assertion as proposed by Kvanvig and Lackey, an account of selfless assertions needs to be developed.

Lackey and Douven argue that defenders of a knowledge or a truth account of assertion have to acknowledge two forms of propriety with respect to assertion. Matt Weiner has indeed defended such a distinction (Weiner 2005, 229, following Keith DeRose in this respect). The primary propriety is determined by whether the act actually conforms to the norm, whether truth or knowledge, while secondary propriety is determined by whether the asserter reasonably believes that the act conforms to the norm. It does not seem, though, that such a distinction has any value, because the agent is fully entitled to assert if it is reasonable for him to believe what he asserts

(Lackey 2007, 608; cf. Douven 2006, 476ff). There is no way to blame him for his act of assertion, it is said, although the content of his assertion may be criticised in the way Kvanvig has argued.

A grounding account of assertion has to show in what sense it agrees with and in what sense it differs from knowledge and justification accounts of assertion as proposed in recent years. In order to be able to make such a comparison I need to explain what the concepts grounding, proof, proposition, truth, judgement, and knowledge amount to from a constructivist point of view.

4 Grounding, knowledge and truth in constructive type theory

In this paper I propose the thesis that a grounding account of assertion suits constructive type theory. In mathematics, a judgement is grounded by an act of demonstration resulting in a theorem, or a judgement is grounded by an act of immediate insight, that is, an act of understanding, resulting in an axiom, such as 0 is a natural number ($0 : N$), A is a proposition (A : prop), or propositions form a type (prop : type). A judgement is thus grounded by a cognitive act that may result in a theorem or axiom; it is the cognitive act that does the grounding, where a cognitive act is either an act of demonstration or an act of immediate insight.[6]

In an act of demonstration for a judgement of the form 'A is true', a proof object a is constructed for a certain proposition A, resulting in the theorem $a : A$. In mathematics, the proof can be named and can itself be treated mathematically, and is therefore a *proof object* (cf. Sundholm 1994, 121). Outside mathematics, one may say, for example, that the (canonical) proof object for the proposition *that the ball is red* is the red-moment of the ball, which is constructed in an act of perception (*cognitive act* is thus becoming a broader notion). On a constructivist account, one is entitled to assert that A is true, only if one has constructed a proof object a for the proposition A, and recognizes a to be a proof for A, thereby demonstrating that A is true. The judgement *A is true* is thus shorthand for the judgement $a : A$. The proof object can be considered as a truth-maker for the proposition A, where A is the truth-bearer (cf. Sundholm 1994, 117ff). What counts as a truth-maker for A is determined by the proposition A, which is understood as a set of proof objects. There is thus an internal relation between the proof object a and A itself. Whether there *exists* a proof object for A is not determined by the proposition A alone, unless A is a tautology. On a constructivist account, one is entitled to assert that there *exists* a proof object or truth-maker for A, only if one has *constructed* a proof object a for A. *Existence* of a truth-maker cannot be identified with the existential quantifier. If one would consider *that there exists a proof object for the proposition A* to be a proposition, the explanation of the truth of propositions would be circular. *There exists a proof object for A* is a judgement, and 'existence' is to be understood as *constructibility. A is true* precisely means that a proof object a for A can be constructed, that is, that there exists a proof object for A. There is thus also an internal relation between the proof object a for the proposition A and the truth of A.

---

[6] I have developed a theory of the cognitive act in my (2010).

On a constructivist account, a judging agent has to understand what the proposition *A* is, in order to recognize an object as a proof object for *A*. The meaning of a proposition *A* is given in terms of its canonical proofs, and in order that a certain object may count as a proof for *A* that object needs to be either a canonical proof for *A* or a non-canonical proof, where a non-canonical proof for *A* consists in a method to obtain a canonical proof for *A*. As Dummett has put it, "the meaning alone determines whether or not something *is* a ground for accepting the sentence" (Dummett 1976, 88).

The difference between a constructivist and a classical logician consists in the different meaning explanations that are given, for example, for negation. What the classical logician accepts as a condition under which ¬*A* is true is not the same as what the constructivist accepts as a condition under which ¬*A* is true. They give thus a different meaning to the proposition ¬*A*, but this is not to imply that we arbitrarily start with a supposition about the meaning of negation. There is a true dispute about meaning, which shows itself in the fact that the intuitionist denies the classical logician to be entitled to make certain assertions. And this means that what the one accepts as a proof system, the other does not accept as such. A proof system is acceptable to me, if its axioms are the result of an act of immediate insight, and the inference(-mode)s are valid, that is, "when a chain of evidence-preserving steps … can be given, which links premises and conclusion" (Sundholm 2004, 455), that is, when each step is justified or grounded by an act of immediate insight. As soon as I understand that the axioms are correct, I understand that the result of the next step has to be correct and justified. For the intuitionist, a proof system that uses elimination of double negation as a rule of inference does not allow for the fact that each step is insight preserving; the proof system is therefore not acceptable to him, and can therefore hardly be called a proof system. Because the axioms are the result of acts of immediate insight, the acts of inference departing from these axioms are insight preserving, and are therefore acts of demonstration. This means that what is shown to be correct within a proof system that is acceptable to me, is justified and known, period, and not merely known, justified, or correct relative to the proof system. It should be noted, though, that from a constructivist point of view justification and knowledge are justification and knowledge from a first person perspective, because knowledge is grounded in a first-person cognitive act (see my 2010). It is precisely for this reason that knowledge is fallible (see below).

What precisely does a grounding account of assertion mean? For common assertions of the from '*A* is true', one may say that one is entitled to assert that *A* is true, only if one has constructed a proof object *a* for *A* in an act of demonstration. The act of demonstration thereby grounds the assertion. A similar account can be given for assertions of the form '*A* is false' and of conditional assertions (for the condition under which one is entitled to assert that A is false, see Sect. 5). There are also assertions, though, that cannot be captured by one of these forms, such as *A is a proposition* (that is, a set), or *sets form a type*. One is entitled to assert that *A is a set*, if and only if one has understood what a set is, and has understood that *A* is such an object. In these cases, one is entitled to assert iff the assertion is grounded by an act of immediate insight, an act of understanding. In a general sense, one may say that one is entitled to assert precisely if the assertion is grounded by a cognitive act (or 'act of knowing', Martin-Löf 1991, 144, 146, 1996, 1). Such a cognitive act is either an act of demonstration or an act of immediate insight, and one may extend the concept of cognitive

act to non-mathematical acts, such as the act of perception. As we will see below, a cognitive act is precisely what makes an assertion justified, and it is for this reason that a grounding account of assertion is a justification account of assertion.

How is the notion of act of demonstration, and that of cognitive act related to the concept of knowledge? According to Martin-Löf, knowledge can be characterized as *justified judgement* (Martin-Löf 1998, 110): knowledge is a judgement that is grounded or justified through an act of demonstration, or an act of immediate insight. A judgement is thus grounded by a cognitive act, whether that act is based on other judgements made, or not. The distinction between act and product, introduced in Sect. 2, applies both to knowledge and judgement. An act of judgement results in a judgement made; in the same way, an act of knowing results in knowledge as product, the justified judgement in the explanation given above. Knowledge as product, or a piece of knowledge ('eine Erkenntnis', Martin-Löf 1996, 20), is an abstract object dependent for its existence on an act of knowing.

Before one makes a judgement, one has to understand the (possible) judgement[7]: "a judgement is defined by laying down what it is that you must know in order to have the right to make it" (Martin-Löf 1998, 108). This means, in case the judgement has the form 'A is true', that one has to understand what kind of proof object has to be constructed for *A*, in order be entitled to make the judgement in question. Judgement is thus a notion that it is explained in epistemic terms. And assertion can be explained on a similar basis, that is, an assertion is defined by laying down what it is that you must *do* in order to have the right to make it: one has to perform a cognitive act, such as an act of demonstration for a judgement of the form 'A is true', in which a proof object *a* for the proposition *A* is constructed, resulting in knowledge that *A* is true. It is in this sense that the grounding account of assertion can be understood as a knowledge account of assertion, although, as we will see below, the constructivist concept of knowledge cannot be identified with the concept of knowledge that is in use in standard knowledge accounts of assertion.

It is now possible to explain the notion of judgemental truth or *correctness*, a notion which will turn out to be relevant for the question what correctness of an assertion means. A judgement is *correct* means that it *can* be grounded or justified ("demonstrated", "made evident", Martin-Löf 1998, 109). A judgement is correct thus means that it is justifiable ("demonstrable", "evidenceable", *Idem*). Correctness of a judgement is not a primitive notion: it is defined by means of the cognitive term 'being grounded'. *Correctness of a judgement* is not to be identified with *truth of a proposition*, but one may say that if and only if a proof object for *A* is constructible, that is, iff *A* is true, then the judgement *A is true* is justifiable, that is, correct.

Although knowledge implies truth in the sense that a judgement that is known is also correct, knowledge does not imply infallible or real truth, according to Martin-Löf: "[A] demonstration purports to make something evident to us, and it is the best guarantee that we have, but it is not infallible" (Martin-Löf 1998, 110). Our acts of demonstration are fallible in the sense that what now counts as a demonstrated judgement, what now counts as a theorem, may in the future turn out to be inconsistent with

---

[7] One may thus speak of a judgement before it is actually judged. I have called this the assertion candidate in my (2007); see also the next section.

other judgements that count as theorems, in which case we have to withdraw one of our knowledge claims (Martin-Löf 1991, 144). And because an act of demonstration, and the act of knowing in general, is fallible, its product, a piece of knowledge, is fallible, too. If we have withdrawn our knowledge claim, we have to admit that what seemed to be an act of demonstration to me then is no longer considered by me to be an act of demonstration for the relevant judgement. On a constructivist account, knowledge does not imply infallible truth, and differs in this sense from standard accounts of knowledge.

The constructivist account of assertion differs from standard knowledge accounts of assertion because the constructivist explains knowledge in a different way. Modern criticism on the explanation of knowledge as justified true belief is directed at the notion justification, whereas the constructivist precisely keeps the 'justification' terminology. On a constructivist account, *justification* is understood in traditional internalist terms, and there is an internal relation between the notions justification and knowledge, and between justification and judgemental correctness.

Further, knowledge is explained here in terms of 'judgement', not in terms of 'belief'. 'Belief' is an ambiguous term. If it is understood as judgement in the way it is explained above, there is no difference between the constructivist and the standard account of knowledge as far as this term is concerned. Generally, though, 'belief' in the explanation of knowledge is understood as a mental *state*, a certain degree of conviction, whereas the judgemental *act* is an all or nothing affair (see Sect. 6).

Finally, the notion of infallible truth, that is, truth that transcends the individual judger, is not part of the constructivist account of knowledge: knowledge implies judgemental correctness, but not infallible truth.[8]

5 A theory of assertion for constructive type theory

(1) The distinction between act of judgement, judgement made, and possible judgement that was proposed in Sect. 4 can also be made for assertion (1a): there is the act of assertion, the assertion made resulting from such an act, and the possible assertion or assertion candidate. The assertion candidate is an epistemic notion, as it is explained in terms of what one must know in order to be entitled to make the assertion, and is not to be confused with the non-epistemic notion of proposition, which is understood as a set: the former has the form of a declarative (*it rains*, or *that it rains is true*), whereas the proposition has the form of a that-clause (*that it rains*), which form can be used to name an object in contrast to the declarative sentence (1b). The proposition does not have the right form to be asserted; one needs to add the indicative mood, the *is true* part, to the *that* clause in order to obtain an assertible form. The question to what extent assertion is up to the asserter (1c), I answer together with the question to what extent assertion is not up to us (see point 3c below).

When we use an occurrence of a declarative sentence to make an assertion, the (occurrence of the) declarative has assertive force. The assertion sign ⊢ in front of

---

[8] A full comparison between the constructivist account of knowledge and the standard account of knowledge I have given in my (2010).

a declarative sentence shows that the declarative has been used with assertive force (1d). In constructive type theory many judgements have the form *A is true*, but the *is true*-part of these judgements is not to be understood as a sign of assertive force: judgements of the form *A is true* may function as antecedent of a hypothetical judgement, which means that there are contexts in which the judgement *A is true* is not asserted. Here, 'judgement' is to be understood as possible judgement, or what I have called the assertion candidate. Besides, constructive type theory acknowledges judgements of the form *A is false*, and these judgements may have assertive force, too. One is entitled to utter *A is false* with assertive force if and only if one has constructed a refutation of *A*. A proposition *A* is false if there exists a disproof, or refutation, of *A*. And a disproof of *A* is a hypothetical proof of absurdity from *A*, which is a function which takes a proof of *A* into a proof of absurdity. ('Proof' is to be understood in the sense of proof object.) The assertive force that is attached to a judgement of the form *A is false* when the candidate is used to make an assertion, does not differ from the assertive force that is attached to the judgement *A is true* when the latter is used to make an assertion. Although these are different judgemental forms, there is only one type of assertive force. Both the judgements *A is true* and *A is false* are explained by what one has to know in order to be entitled to make it. The assertive force attached to each of these judgements involves a claim that one is entitled to make the relevant judgement. The assertive force is thus the same for each of these judgements, although the sort of proof object that one claims to possess is a different one.

Ranta (1994, 25) has given a different explanation of the assertion sign in CTT: he identifies the assertion sign with the *is true*-part of a judgement, which implies that "assertions will also occur as hypotheses", where a hypothesis functions as antecedent of a hypothetical judgement (Ranta 1994, 26). For Ranta, the assertion sign, which he identifies with the *is true*-part, is a sign of indicative mood. Because the antecedent of a hypothetical judgement has indicative mood, the assertion sign is present in the hypothesis, according to him. My answer to Ranta's proposal is, first, that the *is true*-part does not seem to function as a unique sign of indicative mood; a judgement of the form *A is false* also has indicative mood, but it lacks the *is true*-part; so, Ranta has to acknowledge different signs of indicative mood. Second, one may use the term 'assertion' and 'assertive force' with a new meaning, but then one's logical system is still in need of the notions that are generally called 'assertion' and 'assertive force'. By making a distinction between assertions and assertion candidates one is able to say that not all declaratives have assertive force, although they might all be in the indicative or declarative mood (see my 2007).

Although many judgements have the form *A is true* in CTT, this is not a basic form of judgement (1e″). Apart from the fact that there are also judgements of the form *A is false*, and hypothetical judgements that have the form *B is true (A is true)*, the judgemental form *A is true* is a short-hand for a form that makes the proof object explicit, *a : A*, which is indeed one of the basic judgemental forms in CTT. The important point for the topic of assertion is that there are different types of judgements or assertions, but that they all have the assertive or judgemental force in common, when they are actually judged or asserted.

In CTT, there is an important relation between the notions declarative sentence and assertion (1e). The meaning of the declarative sentence is given in terms of its

assertion condition, that is, in terms of what one has to know in order to be entitled to assert the declarative. This is precisely the way that the notion of assertion candidate or possible judgement is explained, and it can therefore be argued that the assertion candidate is the meaning of the declarative sentence (see my 2007). A presupposition of this semantics for the declarative sentence is that the declarative is standardly used for making an assertion, and that one needs special signs when the declarative is used as the antecedent in a conditional sentence or as an example. In these deviant uses, the declarative is not uttered with assertive force, but it does express an assertion candidate.

Does the assertion sign precede 'assertions' made on stage (1e′)? Dummett has given an affirmative answer to this question (Dummett 1973, 311). It might seem that without such a sign the public will not be able to understand that Desdemona is making an assertion by the utterance of the declarative 'I do love thee.' Othello takes her to be lying, and that presupposes that she is making an assertion. The actress who has the role of Desdemona is not really asserting, though, that she loves the person addressed. The assertion on stage therefore does not seem to be a real assertion, and the assertive force seems to be absent. Is it possible to explain the situation on stage by means of the notion of assertion candidate? When we hear the utterance of a declarative, we apprehend the assertion candidate. Because we know that the utterance is standardly used for making an assertion, the utterance on stage can be understood as an assertion in that context, although the assertive force is absent. This can at most be a partial answer to the problem. For, an utterance of the declarative can be used as assertion on stage, but an occurrence of the same declarative can be used to ask a question on stage, or it can be used as the antecedent of a conditional assertion on stage. The assertion on stage must thus be more than the expression of an assertion candidate. The point, I guess, is rather that everything that happens on stage is bracketed by the stage-setting. The act of killing within a play and the act of asserting on stage are acts modified by the stage setting. The phrase 'asserting on stage' is like the phrase 'painted landscape'. The word 'painted' may be used as a modifying term, in which case it modifies the meaning of the term 'landscape' in such a way that the term no longer refers to a real landscape in this context. The phrase 'on stage' can be understood as a modifying term, too: the term 'killing' in 'killing on stage' does not refer to the activity of murdering, but to murder-on-stage, a mock murder. In an analysis of assertions-on-stage, these assertions are not preceded by a straightforward assertion sign; the most one can say is that they are preceded by the assertion sign-on-stage. This sign is not a special case of the assertion sign, just as a painted lion is not a special kind of lion.

In what sense can we say that the assertion is made from the perspective of the asserter (1f)? The agent is entitled to assert that S, if he has done what the explanation of the assertion candidate demands him to do in order to be entitled to make the assertion. In case the judgement has the form 'A is true', he needs to have obtained the knowledge that A is true, that is, he needs to have constructed a proof object for the proposition A in an act of demonstration, which means that he considers the constructed object as a *proof* for A. The act of demonstration is always an act of *demonstration* from the perspective of the asserter. The act of demonstration or cognitive act purports to make the judgement justified; it is a fallible act as we have seen in the former section, and may not be considered an act of demonstration when looked upon from a new perspective. At a later time, one may no longer call the act an 'act of

demonstration' or a 'cognitive act', and the constructed object is then no longer considered to be a proof for the proposition *A*. The cognitive act is essentially a cognitive act from a first-person perspective (see my 2010).

Can a constructivist account of assertion explain that entitlements to assert can be conferred upon others (1g)? The distinction between canonical and non-canonical proof objects may be of help here (see Sect. 4). On a constructivist account, a proposition is explained in terms of its canonical proof objects. For example, the proposition *A&B* is explained in terms of ordered pairs consisting of a proof object of A and a proof object of B. A non-canonical proof object is a method to obtain a canonical proof object. When the disciples told Thomas that they had seen the Lord after Jesus had died, we may say that he was already entitled to judge that Jesus was among them, because he had obtained a non-canonical proof object for the relevant proposition. Doubting Thomas apparently thought that he was entitled to judge that Jesus was among them, only if he had obtained what is considered to be a *canonical* proof for the proposition, by seeing Jesus with the prints of the nails in his hands. If a proof object, whether canonical or non-canonical, may cross from one person to another, entitlements to assert can be conferred upon others.

The question whether apprehension of the meaning of a declarative sentence *S* by a speaker is assumed when we hear him asserting that *S* is to be answered affirmatively (1h′). It is true for all assertions that in order to make the assertion that *S* one is at least expected to have apprehended the assertion candidate *S*. It is in this sense that all assertions can be considered as answers to a certain question (1h), namely the question 'Is the assertion candidate true (correct)?' The question whether every assertion or judgement has a presupposition is to be answered negatively, though (1h″). In CTT, the judgement *A: proposition* has to be correct in order that a meaning explanation can be given for the judgement *a : A*, that is, the judgement *A : proposition* is a *presupposition* for the judgement *a : A*, and thus for the judgement *A is true*. Propositions or sets are to be considered as types themselves, which means that a further presupposition is needed for judgements of the form *A : proposition*, namely that propositions (or sets) form a type, that is, *prop : type* or *set : type*. In type theory, the judgement *set : type* does not have a presupposition. The judgement cannot be made without understanding what a type is, but the definition of what a type is, is not given in a judgement, and therefore cannot be considered as a presupposition.[9] To use a phrase introduced by Collingwood, we may call the judgement *set : type* an *absolute presupposition*.

(2) On what ground can one defend a judgement/assertion parallel? It is not a sufficient answer that assertion is the announcement of a judgement made, or that judgement is the interiorization of assertion, for we need an explanation why each of these notions can be understood in terms of the other. One of the presuppositions of the parallel is that judgement, like assertion, is understood as linguistically structured. Both the assertion and the judgement that snow is white standardly use an occurrence of the declarative 'Snow is white', either aloud or in silence. Because the meaning of the declarative is explained as what one has to *know* in order to be entitled to assert the declarative, an utterance of the declarative in silence with judgemental force amounts

---

[9] A type is defined by what it means to be an object of that type (the criterion of application), and by what it means for two objects of that type to be identical (the identity criterion).

to the same act as an assertion, except that it is uttered in silence. As Plato says in the *Theaetetus* (190a), judgement is an assertion which is not addressed to another person or spoken aloud, but silently addressed to oneself.[10] The dialogical aspect of assertion is internalized to a dialogue with oneself. This explains that a silent judgement is in need of a ground as much as the overt assertion. It is better to speak of the assertion/judgement parallel than of the assertion/belief parallel: in the case of belief, a ground for one's belief is not demanded. The most important reason, though, to speak of the assertion/judgement parallel, rather than the assertion/belief parallel, is that both assertion and judgement are primarily *acts*: a speech act in the former case, a mental act in the latter. Although the assertion/judgement parallel may at first sight seem to confirm a belief account of assertion, this need not be the case if the notions judgement and belief are held apart, and the (possible) judgement is explained in cognitive terms.

Conceptual space is needed between the notions assertion and judgement because we can lie, and because one can assert without having a justification for one's assertion (2′). If someone lies, he asserts that *S* while judging that it is not the case that *S*, with the intention to mislead the hearer to make him think that the asserter does judge that *S* (2′a). If one asserts without having a ground for one's assertion, the asserter need not lie (2′b). If the hearer finds out that the asserter did not have a ground for his assertion, the asserter's trustworthiness will be diminished, though. The next time the US government puts forward a *casus belli* in an assertion, we will definitely make use of our right to ask 'How do you know?'.

(3) What does it mean to say that an assertion is correct? In the last section we have seen that a judgement is correct if and only if it can be grounded or justified. From a constructivist point of view, if a judgement or assertion is grounded, the assertion is correct. The distinction between an assertion being correct and it being grounded becomes fruitful when applied to the assertion of a third person, which includes assertions made by oneself in the past. Because I myself have grounded the assertion *A is true*, I can call the assertion *A is true* made by another agent correct, independent of the question whether he has a justification for his assertion. In the same way a declarative sentence, expressing an assertion candidate, can be called correct, although it is not actually used to make an assertion. An assertion is thus correct, if and only if it *can* be justified.

What is the relation between the correctness of an assertion of the form *A is true* and the truth of the proposition *A* (3a)? An assertion of the form *A is true* is correct, if and only if it can be justified, that is, if and only if it is possible to construct a proof object for *A*, which means that there exists a proof of *A*, that is, that the proposition *A* is true.

Is it true that there is a *group* of speech acts whose products are correct or incorrect (3b)? Can we speak of a correct guess and a correct suggestion in the sense of judgemental correctness? Because the declarative by means of which a suggestion is done or a guess is made expresses an assertion candidate, the suggestion or guess may be called correct, insofar as the assertion candidate is correct. There is thus a group of

---

[10] I have used Myles Burnyeat's revision of Levett's translation, apart from the fact that I have used 'assertion', where the Levett–Burnyeat translation has 'statement' (Burnyeat 1990, 323).

speech acts that have the characteristic of being correct or incorrect. The crucial question is now: How do these speech acts relate to the speech act of assertion? Is it true that there is a *genus* of assertives with different species such as assertion, expressing a guess, doing a suggestion, and putting forward a hypothesis, as Searle has suggested? If the assertives form a *genus* they ought to have something in common, most likely, a claim to correctness. Against this it may be said that putting forward a hypothesis does not involve a claim to correctness. The members of the class of assertives do have something in common, though: in each case an utterance of the declarative is standardly used to carry out the relevant speech act. Each of these acts thus expresses an assertion candidate, which is explained in terms of what one has to know in order to be entitled to *assert* it. The assertion candidate is explained in terms of the speech act of assertion. The speech act of assertion is thus a notion prior in the order of explanation to the other speech acts in this group. The relation between the speech act of assertion and the other speech acts in this group is not a genus–species relation, for these speech acts are not explained in terms of the speech act of assertion, the *genus*, together with a specific difference. These other speech acts are rather to be understood as etiolations or modifications of the speech act of assertion. When someone expresses a guess, his speech act has some similarities with the speech act of assertion, but the speaker does not claim to have a ground for the assertion candidate expressed by the declarative that is used to make the guess. A sign is needed that the declarative is used in a deviant sense; one may, for example, add 'I guess' as a parenthesis.

Insofar as there is a parallel between assertion and judgement, we are not free to assert whatever we like (3c). In order to be entitled to judge, one needs to have a ground for one's judgement, and what counts as a ground for the judgement is not up to us. What may count as a ground is determined by the explanation of the judgement (see Sect. 4). The act of assertion is in one aspect more free than the act of judgement (1e): we can lie to others in a way we cannot lie to ourselves.

If the asserter has asserted that *A is true*, and at some time later asserts that *A is false*, which is equivalent with the assertion ¬*A is true*, he has to withdraw one of his assertions (3d). Presupposing that *A* is a proposition, if the judgements *A is true* and *A is false* were both knowable, so would be the judgement ⊥ *is true*, because *A is false* means that there exists a hypothetical proof of absurdity from *A*. ⊥, like any proposition, is defined by its introduction rules, which are none, that is, for absurdity there is no canonical proof, which means that there cannot be a non-canonical proof for absurdity either. Therefore, absurdity cannot be known to be true (which is one of the *laws of knowability*, cf. Martin-Löf 1995, 194). And this means that the judgements *A is true* and *A is false* cannot both be knowable. Therefore, as soon as an asserter realizes that he has asserted both that *A is true* and that *A is false*, he will withdraw at least one of his assertions.

When an assertion is offered as a reason, it has the function of a premise in our reasoning (3e). The assertion made rather than the act of assertion functions as a *reason*, because premises are assertions made; the acts have only temporal existence. This notion of reason or ground is not identical with the notion of justification as it is introduced in Sect. 4. A judgement is grounded or justified if it is grounded by a cognitive act, which may be based upon other judgements made, but it need not be, for the cognitive act may be an act of immediate insight, which results in an axiom.

In a more restricted sense of ground or *reason*, the judgements made on which an act of demonstration is based, may be called the *reasons* for the conclusion in the sense of 3e. These reasons are not to be confused with the proof objects for propositions.

(4) We have seen in Sect. 4 that within CTT the judgement is explained by what one has to know in order to be entitled to make the judgement in question. A knowledge account of assertion thus suits CTT, although on a constructivist account this means nothing more than that one's assertion is grounded.[11] Because knowledge is explained as justified judgement, it may also be said that one is entitled to make a judgement or assertion, if and only if one has justified it. An interlocutor has a right to ask 'How do you know?', when someone has made an assertion (4a). The questioner thus presupposes that the asserter knows what he asserts, and if the asserter gives a ground for his assertion, the 'How do you know'-question is answered; nothing further is asked for. The point shows that there is an internal relation between knowledge and justification, and this corresponds to the way knowledge and justification are explained in Sect. 4.

A Moorean paradox arises if the sentence 'It rains, but I do not know it' is uttered with assertive force (4b). Instead of saying that one is entitled to assert if and only if one knows what one asserts, one can also say that by making an assertion one claims to know what one asserts. If this knowledge claim is made explicit, one obtains '(I know that) it rains, but I do not know that it rains.', and the paradox becomes visible (cf. Sundholm 2004, 460, note 15). The same holds for an assertive utterance of the sentences 'It rains, but I do not have evidence for it,' and 'It rains, but I do not believe it.' Making the knowledge claim explicit, one obtains in the latter case '(I know that) it rains, but I do not believe it.' This creates a paradox for standard cases of knowledge, where knowledge involves a certain degree of conviction (belief). In standard cases, the justification for one's judgement results in a certain degree of conviction. There are exceptions, though, for example, when someone asserts 'I have my driver's licence; I can't believe it', just having obtained the licence after a long struggle.

6 An evaluation of the constructivist account of assertion

How does a grounding account of assertion deal with the problems for a knowledge account of assertion described in Sect. 3? We have seen that, according to Bernard Williams, knowledge is too strong a norm for assertion, because the asserter is not able to determine whether his assertion is made in accordance with the norm. The standard account of knowledge understands knowledge to imply truth, where truth is understood as transcending the individual judger, what has been called infallible or real truth in Sect. 4. If knowledge is the norm for assertion, transcendent or infallible truth is thereby a norm for assertion, too, on the standard account. The problem with

---

[11] In 1988 Göran Sundholm has already pointed out that Martin-Löf's constructive type theory can account for the fact that the announcement of a judgement made by means of an assertion involves a knowledge claim (Sundholm 1988, 17; cf. Sundholm 1999, 2004). In a letter from 10 September 2007, Martin-Löf wrote to me that "the judgemental force … is defined by the pragmatic rule, To have the right to make a judgement, you must have justified (grounded) it." The two explanations, one in terms of knowledge, the other in terms of justification, do not differ on a constructivist account, because knowledge is precisely the justified or grounded judgement.

knowledge accounts of assertion as they are recently proposed, is that we are never able to determine whether our judgement is true in this sense. And this criticism thus applies to a truth account of assertion, as well, if truth is understood in a non-epistemic, transcendent sense. Within CTT, knowledge is not explained in terms of a transcendent notion of truth; knowledge is understood as justified judgement, where the justification does not transcend the asserter, as we have seen in Sect. 4. The asserter is in a position to determine whether his judgement is justified or grounded, because the cognitive act through which the judgement is justified is an internal, first-person act, and is thus immanent to the asserter.

Williams himself gives an account of assertion in terms of belief: "A utters a sentence "S," where "S" means that P, in doing which either he expresses his belief that P, or he intends the person addressed to take it that he believes that P" (Williams 2002, 74). Because not all assertions are expressions of belief—there are lies, too, Williams has to take account of assertions that are not expressions of belief in the account of assertion itself. According to Williams, it is an important characteristic of assertions that they can be used insincerely, that is, that they can be used to lie, and this characteristic, he says, should be accounted for in the explanation of assertion. It is perfectly possible, though, to understand what assertion is without understanding that it can be used for lying; lying is a parasitic phenomenon. Giving an explanation in terms of *entitlement* to assert leaves open the possibility of insincere assertions, without distinguishing between two types of assertion in the account of assertion, as Williams has to do. To explain assertion in terms of belief is also problematic, because the notion of assertion is clearer than the notion of belief: assertions, in contrast to beliefs, can directly be perceived and are non-dispositional. Besides, belief is a highly ambiguous notion: it may mean *a disposition to judge*, where judging is an all or nothing affair and a normative or rational notion; *conviction*, which has degrees; *opinion*, which is opposed to knowledge; or (religious) *faith*. (This analysis will turn out to be of importance for the treatment of selfless assertions at the end of the section.) Related to these ambiguities, the notion is sometimes used in a naturalistic context, in which case the central question is how we come to have the beliefs we have ('belief' in the sense of *conviction*). At other times, it is used in a normative context, where truth is the norm for belief ('belief' in the sense of *disposition to judge*), and the central question is what reasons we have for our judgements. Unravelling the ambiguities of the term 'belief' will show that Williams is not able to give a naturalist foundation for normative notions such as assertion and for epistemic virtues such as sincerity, but this has to be done in a new paper. Besides, sincerity is not the only demand for assertion. In the case of the *casus belli* with respect to Iraq, the assertion that there were mass destruction weapons was, most likely, sincere, but there was definitely something wrong with it. Williams can invoke his notion of accuracy here, but the problem with these demands of sincerity and accuracy is that they are not exclusive for assertion. The norm of sincerity is not typical of the speech act of assertion: one should not make insincere promises, or advise someone insincerely; and the same holds for accuracy.

Searle and Weiner combine a truth account of assertion with the thesis that there is a broad class of assertives, including expressions of guesses and speculations, because for these acts truth also is the norm, they say. According to Weiner, these speech acts are acts of assertion. This implies, according to Weiner, that the asserter may be fully

entitled to make his assertion, because what he asserts is true, although he is not able
to answer the question 'How do you know?' If one takes the class of assertives in this
general sense, one may still wonder under what condition one is entitled to assert in the
more specific sense; such an entitlement seems to demand more than the entitlement
for guesses and speculations demands. Holmes' remark 'This is the work of Professor
Moriarty,' is an assertion, according to Weiner. Although Holmes is not able to answer
the 'how do you know'-question, he is entitled to make the assertion, Weiner says,
because what Holmes says is true. A defender of an epistemic account of assertion
may answer Weiner in either of two ways. One may say that Holmes did have a ground
for his assertion—we are told that Holmes had knowledge of the *modus operandi* of
Moriarty; the ground may simply be difficult to explain to an interlocutor. Or, one may
say that Holmes uses the utterance of the declarative 'This is the work of Professor
Moriarty' not to make an assertion in the strict sense, but rather to express a proposal
for further investigation. The quality of Holmes as a detective consists in putting for-
ward the most relevant candidate for assertion. Certainly, we do praise people when
they have put forward a hypothesis that turns out to be correct, but we praise them not
because of their entitlement to assert, but because their hypothesis is valuable.

According to Kvanvig, the norm for assertion is justification, and not truth or knowl-
edge. The grounding account of assertion can be considered as a justification account
of assertion, as we have seen in Sect. 4, and it is therefore important to understand in
what sense the constructivist account of assertion differs from justification accounts of
assertion defended by Kvanvig, Douven, and Lackey. These philosophers have argued
against knowledge and truth accounts of assertion that two forms of propriety are
needed on such accounts to explain that someone's assertion may have a secondary
propriety, because he reasonably believes that he is entitled to assert, whereas his
assertion does not have primary propriety, because he is not really entitled to assert on
such an account. From the constructivist point of view, no such distinction is needed:
entitlements to assert are based upon internal justification, and it is this justification that
determines exclusively whether the asserter is entitled to assert. There is thus only one
type of propriety, as it is on the justification account defended by Kvanvig and others.

There are also some important differences, though, between the constructivist
account and justification accounts as generally understood. According to Kvanvig,
the norm for the content of assertion is not to be confused with the norm for the act of
assertion. Whereas truth and knowledge are the norm for the content of the assertion,
justification is the norm for the act of assertion, he says. From a constructivist point
of view, there is not a similar way to separate these norms. That one has obtained
the knowledge demanded by the assertion candidate *A is true*, because one has con-
structed a proof object for *A*, precisely gives one the entitlement to make the assertion.
Further, both Kvanvig and the constructivist can explain that an interlocutor has a right
to ask for grounds, when an assertion is made. Kvanvig cannot explain, though, the
presupposition of the question 'How do you know?', namely that the asserter knows
what he asserts.

According to Douven, Kvanvig and Lackey, a central argument against both a belief
and a knowledge account is the possibility of *selfless assertions* (Lackey 2007, 598).
A teacher seems to be fully entitled to assert that modern day *Homo sapiens* evolved
from *Homo erectus*, although she does not believe the proposition to be true, because

she has "a belief in the truth of creationism, and, accordingly, a belief in the falsity of evolutionary theory" (*Idem*, 599). "[S]he readily admits that she is not basing her own commitment to creationism on evidence at all but, rather, on the personal faith that she has in an all-powerful Creator. … She regards her duty as a teacher to include presenting material that is best supported by the available evidence, which clearly includes the truth of evolutionary theory" (*Idem*). If assertions are essentially made from the perspective of the asserter, how can the agent assert selflessly?

Does the constructivist defend the thesis that belief that *S* is a necessary condition for being entitled to assert that *S*? If 'belief' means a disposition to judge resulting from a cognitive act the answer is 'yes'. If 'belief' means conviction, the answer is not a straightforward 'yes', because one needs to acknowledge those cases where one possesses a proof object for *A*, and recognizes it as such, although one has not obtained the degree of conviction that standardly accompanies such a possession, as we have seen in the case of the driver's licence. In the example of the biology teacher, 'belief' in the sense of *faith* plays an important role, which is generally accompanied by the syntactic form 'to believe *in*'. Insofar as religious faith and judgement do not belong to the same category the biology teacher may assert and judge that *Homo sapiens* evolved from *Homo erectus* on the basis of evidence, and also have faith in the God of the Bible. The case of selfless assertions is not restricted, though, to cases where belief means *faith*. There are more cases imaginable where we assert things we do not believe because of a social role we inhabit.

The possibility of selfless assertions supports Jonathan Cohen's thesis that assertions are announcements of what we accept rather than of what we believe, where acceptance is an act of choice based on evidential or prudential reasons that need not be accompanied by belief (Cohen 1992). Acceptance as Cohen explains it is always relative to a certain context: one may thus accept the truths of evolutionary theory in one context, while not accepting them in another. I have argued, in my (2009), that Cohen's distinction between belief and acceptance is not a satisfying one: the same notion of acceptance takes in both evidential reasons and prudential or practical reasons. I have argued that accepting for evidential reasons is not context relative, and should be replaced by the notion of judgement, whereas acceptance for purely prudential or practical reasons is a notion we are in need of, and is indeed context relative. Constructivism does not say anything about acceptance for prudential or practical reasons, but it can certainly allow for it. This gives a solution to the point that we may accept ('assert') things we do not believe because of a social role we inhabit, although I would prefer not to use the term 'assertion' here. We have seen that there are a lot of speech acts that have some agreement with assertion, such as making a guess, or putting forward a hypothesis, where the declarative is the standard linguistic form to perform the speech act, and acceptance may be included among them. We may accept that *S* for prudential reasons, while not judging that *S* because of evidential reasons, and the other way round. In accordance with this, I understand the example of the biology teacher in a different way: the teacher judges that evolutionary theory is true, because of the evidence available, and is thus entitled to assert, while she does not accept the evolutionary theory for religious reasons. I thus understand the religious reasons to be prudential reasons. This solution allows also for cases in which non-religious, purely prudential reasons play a role.

We can make the example of the biology teacher a bit more difficult to explain, though. In a footnote, Lackey gives a quotation from an interview with Marcus R. Ross, a creationist who just finished a Ph.D. in paleontology: "the methods and theories of paleontology are one 'paradigm' for studying the past, and Scripture is another … I am separating the different paradigms" (Lackey 2007, 620, note 12). It seems that Ross is thus willing to assert both that the evolutionary theory is true, and, for example, that *Homo sapiens* evolved from *Homo erectus*, *and* that creationism is true, that evolutionary theory is false, and that *Homo sapiens* did not evolve from *Homo erectus*, although he is not willing to make those assertions within the same paradigm.

Those who defend a justification account of assertion may say that Ross is entitled to assert that the evolutionary theory is true, because he has some evidence for this assertion, and that he also is entitled to assert that evolutionary theory is false, because he has some evidence for that assertion, too. From a constructivist point of view, one cannot be entitled to assert that *A is true* and that *A is false*, because these judgement candidates are not both knowable (see Sect. 5, point 3d); they cannot both be grounded. For a constructivist 'being entitled to assert that *A* is true with respect to an admissible paradigm' means that one is entitled to assert *A is true*, period. If Ross were to assert both that the evolutionary theory is true and that the evolutionary theory is false, he is inconsistent, and is thus logically to be blamed, if he realizes what he is doing, and does not withdraw one of his assertions, *or* Ross is a relativist with respect to truth ('true', for him, means *true relative to a paradigm*).

7 Conclusion

The paper has developed a full theory of assertion for constructive type theory, in which it is shown that the theory is able to account for many of the epistemic aspects of the speech act of assertion, and that it is supported by the thesis developed here that assertions do not form a wide genus. The paper has shown that a grounding account of assertion suits constructive type theory. Because grounding the assertion that *S* by means of an act of demonstration amounts to knowing that *S*, the question to what extent the constructivist account of assertion differs from knowledge accounts of assertion generally proposed had to be answered. It is especially the internal relation between the proof for a proposition and its truth, and the internal relation between a judgement being grounded and its being known that distinguishes the constructivist account of assertion from standard knowledge accounts of assertion. Because a constructivist explains knowledge as grounded judgement, it also had to be shown in what sense the constructivist account of assertion differs from justification accounts of assertion recently proposed. It is especially the treatment of selfless assertions that makes the constructivist account of assertion different from these justification accounts.

References

Austin, J. L. (1962). *How to do things with words*. Oxford, New York: Oxford University Press. 1984.
Burge, T. (Ed.). (1986). Frege on truth. In *Truth, though, reason: Essays on Frege* (pp. 83–132). Oxford: Clarendon Press. 2005.
Burnyeat, M. (1990). *The Theaetetus of Plato* (with a translation of Plato's Theaetetus). Indianapolis, Cambridge: Hackett Publishing Company.
Cohen, L. J. (1992). *An essay on belief and acceptance*. Oxford: Clarendon Press.
Davidson, D. (Ed.). (1984). Communication and convention. In *Inquiries into truth and interpretation* (pp. 265–280). Oxford: Clarendon Press.
Douven, I. (2006). Assertion, knowledge and rational credibility. *Philosophical Review, 115*, 449–485.
Dummett, M. (Ed.). (1973). Assertion. In *Frege: Philosophy of language* (Chapter 10, pp. 295–363). London: Duckworth. Second edition 1992.
Dummett, M. (Ed.). (1976). What is a theory of meaning (II). In *The seas of language* (pp. 34–93). Oxford: Clarendon Press. Originally in G. Evans & J. McDowell (Eds.), *Truth and meaning*. Oxford: Clarendon Press, 1993.
Engel, P. (2002). *Truth*. Chesham: Acumen.
Frege, G. (1879). *Begriffsschrift*. In I. Angelelli (Ed.), *Begriffsschrift und andere Aufsätze*. Hildesheim, New York: Georg Olms. 1971.
Frege, G. (1918). Der Gedanke. *Beiträge Zur Philosophie Des Deutschen Idealismus, 1*, 58–77.
Kvanvig, J. (2009). Assertion, knowledge, and lotteries. In P. Greenough & D. Pritchard (Eds.), *Williamson on knowledge* (pp. 140–160). Oxford: Oxford University Press.
Lackey, J (2007). Norms of assertion. *Nous, 41*, 594–626.
Martin-Löf, P. (1991). A path from logic to metaphysics. In *Atti del Congresso Nuovi problemi della logica e della filosofia della scienza* (Vol. II, pp. 141–149). Bologna: CLUEB.
Martin-Löf, P. (1995). Verificationism then and now. In W. Schlimanovich, E. de Pauli, & F. Stadler (Eds.), *The foundational debate* (pp. 187–196). Dordrecht: Kluwer.
Martin-Löf, P. (1996). On the meanings of the logical constants and the justification of the logical laws. *Nordic Journal of Philosophical Logic, 1*, 11–61. (Originally presented in 1983.)
Martin-Löf, P. (1998). Truth and knowability: On the principles C and K of Michael Dummett. In H. G. Dales & G. Oliveri (Eds.), *Truth in mathematics* (pp. 105–114). Oxford: Clarendon Press.
Newman, J. H. (1870). *An essay in aid of a grammar of assent*. Oxford: Clarendon Press. (I used the edition of 1985.)
Ranta, A. (1994). *Type-theoretical grammar*. Oxford: Clarendon Press.
Searle, J. R. (1979). *Expression and meaning*. Cambridge: Cambridge University Press.
Sundholm, G. (1988). *Oordeel en gevolgtrekking; bedreigde species?* Inaugural lecture. Leiden: Leiden University.
Sundholm, G. (1994). Existence, proof and truth-making: A perspective on the intuitionistic conception of truth. *Topoi, 13*, 117–126.
Sundholm, G. (1999). MacColl on judgement and inference. *Nordic Journal of Philosophical Logic, 3*, 119–132.
Sundholm, G. (2004). Antirealism and the roles of truth. In I. Niiniluoto, M. Sintonen, & J. Wolenski (Eds.), *Handbook of epistemology* (pp. 437–466). Dordrecht: Kluwer.
van der Schaar, M. (2007). The assertion-candidate and the meaning of mood. *Synthese, 159*, 61–82.
van der Schaar, M. (2009). Judgement, belief and accepance. In G. Primiero & S. Rahman (Eds.), *Acts of knowledge: History, philosophy and logic; essays dedicated to Göran Sundholm* (pp. 267–286). London: College Publications.
van der Schaar, M. (2010). The cognitive act and the first-person perspective; an epistemology for constructive type theory. *Synthese*. doi:10.1007/s11229-009-9708-4.
Weiner, M. (2005). Must we know what we say?. *Philosophical Review, 114*, 227–251.
Williams, B. (1973). Deciding to believe. In *Problems of the self* (pp. 136–151). Cambridge: Cambridge University Press.
Williams, B. (2002). *Truth and truthfulness*. Princeton: Princeton University Press.
Williamson, T. (1996). Knowing and asserting. *Philosophical Review, 105*, 489–523.