Theory and Decision (2008) 64:81–101
DOI 10.1007/s11238-007-9026-6 

JAN-WILLEM VAN DER RIJT

# An ALTERNATIVE MODEL OF THE FORMATION OF POLITICAL COALITIONS

ABSTRACT. Most models of the formation of political coalitions use either Euclidean spaces or rely purely on game theory. This limits their applicability. In this article, a single model is presented which is more broadly applicable. In principle any kind of set can be used as a policy space. The model is also able to incorporate different kinds of party motivations: both rent-seeking and idealism. The model uses party preferences and power to identify stable coalitions and predict government policy as well as to indicate which member of the opposition will be able to break up the governing coalition if no stable coalition exists. In the latter case it will also indicate on which issue the government is likely to split. Parties may have preferences over issues such as the composition of cabinet and/or the governing coalition as well as the more traditional issues of government formation. The model also provides a rationale for log-rolling.

KEY WORDS: political coalitions, government formation, coalition formation, log-rolling, government policy.

## 1. INTRODUCTION

Most of the literature on the formation of political coalitions focuses either on Euclidian-spatial analysis (for an overview, see Ordeshook (1997)) or solely on matters of distribution (e.g. Riker (1962)). In models based on Euclidean spaces, the primary motivation for party behaviour in forming coalitions is often idealistic (the goal for parties is to get policy as close to their own preferences as possible). The most famous result of this kind of analysis is Black's median voter theorem (Black (1958)). Many models which focus on positioning of parties prior to elections which use Euclidean spaces are based on purely rent seeking party behaviour (such as the classic models of Hotelling (1929) and Downs (1957)). The behaviour of these parties during the formation of government after the election can still be seen as idealistic as the positions of the parties are fully determined at that time (this argument is also used by Laver and Shepsle (1996)). Other articles using spacial analysis include Baron (1991), McKelvey (1976) and Plott (1967).

Models focusing on distribution place emphasis on rent-seeking (also known as office seeking) as the primary motivation in party behaviour (being in government is equated to power or to the right to distribute a (sizeable) amount of money, and each party intends to get as large a piece of the action as possible for itself and/or its followers). These models tend to rely heavily on game theoretic concepts. A well-known result from this kind of analysis is Riker's theory of minimal-winning coalitions (Riker (1962)). Other works placing emphasis on game-theoretic aspects of government formation are for example Peleg (1981) and Van Deemen (1989).

Most authors acknowledge that both motivations probably occur in real life but argue that either one or the other motivation is of far greater importance and build their model around the motivation of their choosing. Some, however, try to use both motivations, for example Kirchsteiger and Puppe (1997).

Kirchsteiger and Puppe start their analysis with a model based on office seeking behaviour where each party seeks to maximise its relative weight within the governing coalition and propose two different ways of adjusting the model to accommodate policy seeking considerations. In the first approach they assume the largest party in the coalition gets to determine the policy, whereas in the second they base their analysis on a distance measure on the policy space. The first approach is only applicable in situations where the largest party indeed wields excessive influence over policy, which need not always be the case; see for instance the example below. The second approach is problematic in cases where the policy space consists of issues of a different nature, i.e. where some issues can be adequately represented by a Euclidean space, but others require a different kind of set – as this will give rise to difficulties when defining an adequate and realistic distance measure for such policy spaces. Furthermore, they define their distance measure in such a way that parties who are not a member of a winning coalition are indifferent with respect to the policy outcomes, which is not a very realistic assumption.

In this article a model is presented, that is also designed to incorporate both kinds of party motivation and to be as broadly applicable as possible. The main focus of this article is conceptual: next to the presentation of the model, the main emphasis is on the underlying reasons for this adopting this approach. For more technical results the reader is referred to Rusinowska et al. (2005) where such results are deduced for a generalisation of the model presented here. The model also serves as the basis for a number of other papers (see Section 6). In order to illustrate the advantages of applying both rent seeking and policy seeking motivations in a single model, an example will be presented that will show the possibility of the formation of a non-minimum winning coalition [1] (often a problem for models focusing on distribution). Furthermore, the model will be shown to be able to account for the occurrence of log-rolling[2] (the occurrence of which is hard to explain by means of Euclidean-spatial models). It will also be shown that some of the drawbacks of the models proposed by Kirchsteiger and Puppe can be avoided using this model.

## 2. AN ALTERNATIVE MODEL

In this section, an alternative model is constructed that can accommodate a number of the aforementioned objections. Subsequently, the model will be evaluated paying special attention to the capacity of this model to handle issues problematic for standard theory and to objections which may be raised against this new model.

The model will, given the outcome of the election and preferences of all parties, be able to answer the following questions:

1. Can a government be formed?
2. If so, can the coalition which will form this government be determined?
3. Is this coalition stable?

The model to be presented here shows strong resemblance to standard *non-transferable utility* games. It cannot be fully described by such standard games, though, as coalitions may generate pay-offs to parties which are not part of the coalition. For more on games with this spillover property (see Thijssen et al. (2002)).

An $M$-dimensionial policy space $B$ is assumed, consisting of $M < \infty$ mutually independent sub-spaces: $B = B_1 \times B_2 \times \cdots \times B_M$. Each of these sub-spaces can be regarded as a separate issue on which government must make a decision. Many such issues can be represented using (a sub-set of) $\mathbb{R}$, many others cannot. In the latter case one could think of issues where the number of alternatives is limited, such as implementing (or abolishing) universal conscription. One either does or does not implement such a policy, no in between alternative is available. $B_j$ can be either a Euclidean space, or (in principle) any other type of set (including multi-dimensional ones).

In addition to the policy space $B$, a set of parties $P$, consisting of $N < \infty$ parties, is assumed. All parties are supposed to have preferences on all issues. For the time being all parties are considered to be purely idealistic in their motivation; later on, rent-seeking behaviour will also be included in the model.

A policy can be represented by a point $x \in B$. Policy on a certain issue $j$ can subsequently be denoted as $x_j \in B_j$. A party $i \in P$ is assumed to give one of three qualifications to a policy on a certain issue $x_j$: desirable, acceptable and unacceptable.

The points party $i$ finds desirable on a certain issue $j$ can be represented by a set $W_j^i \subset B_j$, which will henceforth be referred to as the *desirability-set* of party $i$ on issue $j$. In the same way all points which are deemed to be acceptable to party $i$ can be represented by an *acceptability-set* $A_j^i \subset B_j$. Points considered unacceptable are, consequently, all points in the complement of $A^i_j$, $(A^i_j)^c$. Points deemed desirable are naturally deemed acceptable too: $W^i_j \subset A^i_j$. As the election result is assumed to be given, a party $i$ can be fully described by a multiple $(p^i, A^i, W^i)$, where $A^i$ is defined as $A^i = A^i_1 \times A^i_2 \times \cdots \times A^i_M$, $W^i$ as $W^i = W^i_1 \times W^i_2 \times \cdots \times W^i_M$, and $p^i$ as the fraction of the votes received by party $i$. Hence, the following holds: $\sum_i p^i = 1$.

Parties can get into office by forming a coalition which supports a certain policy, represented by a point $x \in B$. Forming a government is, therefore, equivalent to a mapping of the parties to the policy space with a supporting coalition:

$$(p^i, A^i, W^i)_{i \in P} \longmapsto \begin{cases} x \in B, \\ S \subset P. \end{cases} \tag{1}$$

When starting negotiations, each party seeks to get a desirable result on as many issues as possible without having to put up with policies it deems unacceptable. A party $i$'s behaviour can then be described thus:

$$\max_{x \in A^i} |\{j \in \{1, \ldots, M\} | x_j \in W^i_j\}|, \tag{2}$$

where $x \in A^i$ means that $\forall_{j=1}^{M} [x_j \in A^i_j]$.

### 2.1. *Solutions*

In the following it is assumed that a coalition may form a government if it represents a majority of the voters. In order to accommodate this assumption, the set $P^*$ is introduced as the set of majority coalitions:

$$P^* = \left\{ T \subset P | \sum_{i \in T} p^i > \frac{1}{2} \right\}. \tag{3}$$

In order to determine which government will be formed, the value of such a government to each party must first be defined. While a government is determined by its policy $x \in B$ and a supporting coalition $S \subset P^*$, its value to a party is only dependent on the policy $x$.

### 2.1.1. *Value of policy*

In line with the preceding, the value of a policy $x$ to a party $i$ can be defined as the number of issues on which a party succeeds in getting its desires fulfiled, at least as long as no unacceptable policy is implemented on any other issue: $U^i(x) = |\{j=1,\ldots,M | x_j \in W_j^i\}|$ if $x \in A^i$. If the value of an unacceptable policy is set at $-\infty$, $U^i$ can be fully described as follows:

$$U^i(x) = \begin{cases} |\{j=1,\ldots,M | x_j \in W_j^i\}|, & \text{if } x \in A^i, \\ -\infty, & \text{if } x \notin A^i. \end{cases} \tag{4}$$

$U^i$ is, therefore, a function from the policy space to the set of natural numbers united with $-\infty$ (and 0):

$$U^i : B \longmapsto \mathbb{N} \cup \{0\} \cup \{-\infty\}. \tag{5}$$

The value of a policy $x$ can then be described as $U(x) = (U^i(x))_{i \in P}$. From this $N$-dimensional vector, the revenue of policy $x$ can be read for each party. Since it concerns *non-transferable utility*, this value cannot be described by a number but must remain a vector.

It has to be noted that a certain coalition does not necessarily represent a unique value, as a coalition can usually support multiple policies.

The number of possible pay-off vectors $U(x)$ is finite (although the number of possible policies generating this pay-off may be infinite), as there is but a finite number of issues, each generating a value of 0, 1, or $-\infty$ for a party. The total number of pay-off vectors is, therefore, limited:

$$|\{U(x) | x \in B\}| \leq (M+2)^N. \tag{6}$$

### 2.1.2. *Possible governments*

A majority coalition $S \subset P^*$ may be formed if a policy $x \in B$ exists which is acceptable to all parties in that coalition, i.e. $x \in A^S$, where $A^S = \{y \in B | \ \forall_{j=1}^M [y_j \in \bigcap_{k \in S} A_j^k]\}$.

A policy $x \in B$ is said to be feasible if $\exists S \in P^*[x \in A^S]$. $B^*$ is defined as the set of all feasible policies.

Parties endeavour to reach as high a level of utility as possible. Party behaviour is, therefore, aimed at finding support for a policy which gives it the highest pay-off. This can be represented mathematically by

$$\max\{U^i(x)|x \in A^S \text{ for some } S \in P^*\}. \tag{7}$$

If no government is formed, each party's utility is put at $-\infty$.

From this, the set of stable policies $C \subset B$ can be defined. In order to do this a number of concepts must be defined first. For $x, y \in B^*$, $y$ is said to defeat (or dominate) $x$, denoted by ${}_yD_x$, if

$$\begin{aligned} {}_yD_x := \exists S \in P^*[y \in A^S \ \& \ \forall i \in S \ [U^i(y) \geq U^i(x)] \& \exists k \in S \\ [U^k(y) > U^k(x)]]. \end{aligned} \tag{8}$$

A policy $x \in B^*$ is subsequently said to be stable if there is no alternative feasible policy $y$ by which it is dominated:

$$x \text{is stable} := \neg \exists y \in B^* \ [{}_yD_x]. \tag{9}$$

This makes it possible to define $C$ as the set of all stable policies, i.e.

$$C = \{x \in B^* | x \text{ is stable}\}.^3 \tag{10}$$

## 3. GOVERNMENT FORMATION

The three questions posed at the beginning of the previous section can now be answered as follows: it is possible to form a government when a feasible policy exists, in other words, if $B^* \neq \emptyset$.

The governments which can come into existence are basically all majority coalitions for which there exists a policy acceptable to all its members:
$\{S \in P^* | A^S \neq \emptyset\}$. Such a coalition can be said to be a feasible coalition.

A majority coalition $S \in P^*$ can only form a stable government if $A^S \cap C \neq \emptyset$. Such a coalition can be called a stable coalition.

A stable government can, therefore, be represented by a pair $(S, x)$ with $S \in P^*$ and $x \in A^S \cap C$. As it is likely that an unstable government will fall, it is not unreasonable to assume a stable government will be formed if one exists.

This gives rise to a number of interesting observations. The model offers an explanation for the fact that sometimes governments are formed which do not fulfil their term, as it is possible that no stable coalition exists (i.e. $C = \emptyset$). From the fact that the range of $U$ is finite it follows that this can only occur when cycles exist. The model, therefore, not only states when no stable government exists; it also provides an opportunity to determine which members of the opposition are capable of bringing down an unstable government. By the same reasoning it should be rather straightforward to determine the issue on which the governing coalition can be split.

The model also explains that sometimes no government can be formed, which occurs when $B^* = \emptyset$. As this would result in a pay-off to all parties of $-\infty$, this condition may also be viewed necessary for no government being formed.

## 4. REMARKS

### 4.1. *Different kinds of policy*

It is possible to analyse many different kinds of issues at the same time using this model. One issue may be represented by a Euclidean space, while another is of a discrete nature. Matters of allocation (like the ones Riker (1962) focuses on) can also be incorporated into the model. To do this, simply define the policy sub-space in question as the set of ordened vectors assigning a piece of the pie to different recipients. If these recipients are parties or party followers, this allows rent-seeking behaviour to be incorporated into the model, as each party can subsequently designate desirable and (un)acceptable allocations in this set with or without paying attention to the shares allocated to parties other than themselves.

Preferences on cabinet formation can also be regarded as a kind of rent-seeking as the distribution of seats in the cabinet provide a party with prestige, benefits and power. To include this into the model, one simply adds a dimension to the policy space. This dimension should be formed by the set of (ordened) vectors assigning particular ministerial positions to particular parties. Within this set, each party can express its preferences again by means of acceptability and desirability sets: $W^i_{M+1} \subset A^i_{M+1} \subset B_{M+1}$. Moreover, for a *fully* rent-seeking party the following will hold: $\forall^M_{j=1}[A^i_j = B_j \ \& \ W^i_j = \emptyset]$.

In much the same way, parties can express their preferences on possible coalition partners. For this, an extra dimension $B_{M+2}$ is to be added 'composition of government coalition' consisting of all possible coalitions. In this, desired (typically, these will tend to be coalitions of which the party is itself a member), but especially unwanted (unacceptable), partners can be denoted by deeming all those coalitions of which an unwanted party is a member unacceptable. That these kinds of considerations may be of importance is also noted by Edelman (1997).

This is especially interesting if one party is denounced by other parties for some reason or other, such as parties whose ideologies are frowned upon by other parties, or parties with whom a party had bad experiences in the past. Such a party could be eligible for government based on their preferences but be prevented from participating by the unwillingness from other parties (this is similar to the concept of quarrelling as mentioned in Brams (1975) and Kilgour (1974)).

### 4.2. *Government negotiations*

Using the acceptability and desirability sets as defined in this article to represent party preferences makes it possible to set absolute boundaries. Certain issues will not be backed by certain parties no matter how many other issues are offered to them in compensation. This is a pleasant feature of the model as it enables one to distinguish pragmatic parties from parties which take a more strict position. If, for example, $W^i \approx A^i$ holds for a certain party, this party can be seen as strict or rigid. The model thus provides an opportunity to examine the interactions between these kinds of parties.

As these boundaries are determined for each issue separately, the possibility also arises to distinguish major issues from minor ones. These can even differ for different parties; an issue can be viewed as being of major importance by one party, while being seen as inconsequential by another. A make-or-break issue can, for instance, be represented by $W^i_j = A^i_j$, whereas an issue which is deemed of no importance whatsoever can be represented by $W^i_j = \emptyset$ and $A^i_j = B_j$. Another way of differentiating between minor and major issues would be to assign different weights to desirable results on different issues in the parties' utility functions $U^i$.

The model provides an opportunity to gain insight into the roles different issues play during government negotiations. In order to do this, a number of possibilities have to be discerned on the joining of two parties ($i$ and $k$) in one coalition. For this, the concept of scoring on issues is introduced. A party $i$ is said to score on issue $j$ if $x_j \in W^i_j$ in the policy $x$ adopted by the government. This leads to the following observations:

1. Parties $i$ and $k$ cannot take part in the same coalition if $\exists j[A^i_j \cap A^k_j = \emptyset]$.
2. Only party $i$ can score on issue $j$ in a coalition with party $k$ if
   $W^i_j \cap A^k_j \neq \emptyset$ and $A^i_j \cap W^k_j = \emptyset$.
3. Neither party $i$, nor party $k$ can score on issue $j$ (in a coalition with the other), but $j$ is not an insurmountable obstacle if
   $W^i_j \cap A^k_j = \emptyset$ and $W^k_j \cap A^i_j = \emptyset$ and $A^i_j \cap A^k_j \neq \emptyset$.
4. Parties $i$ and $k$ can score simultaneously on issue $j$ in a coalition with the other if $W^i_j \cap W^k_j \neq \emptyset$.
5. Issue $j$ is a point of negotiations between parties $i$ and $k$, i.e. both parties can score but not simultaneously, if
   $W^i_j \cap A^k_j \neq \emptyset$ and $W^k_j \cap A^i_j \neq \emptyset$ and $W^i_j \cap W^k_j = \emptyset$.

It is clear from the above that the model allows parties to support a bill which they do not really consider desirable (see 2, 3 and 5) and, hence, explains the occurrence of log-rolling. Consider the case where there are two issues, $j$ and $l$, of type 5, where party $i$ agrees to let party $k$ score on issue $j$, provided party $k$ will do the same for party $i$ on issue $l$. Similar behaviour can occur on issues of types 2 and 3. A party will only agree to support policy on such an issue which it does not find desirable, however, if the resulting coalition generates it enough value. The suggestion that log-rolling makes any statements on the outcome impossible, which is sometimes made, therefore, does not hold in this model, as log-rolling can only occur with respect to issue $j$ if $A^i_j \cap A^k_j \neq \emptyset$.

As this model pays no attention to the voters' preferences, the interesting question of whether log-rolling is welfare enhancing (see, for example, Wittman (1995)) or reducing, cannot be answered here. It is, however, possible to indicate when and on which issues log-rolling may occur, providing a framework in which this question may be addressed.

### 4.3. *Drawbacks of the model*

A drawback of this way of modelling is the fact that no differences in preference exist within the sets $W^i$, $A^i$, and $(A^i)^c$. To a degree, this can be circumvented by splitting $W^i$ into multiple sets such as 'slightly desirable', 'desirable' and 'very desirable' and adjusting the utility functions of the parties accordingly, generating higher levels of utility if the policy is in a set of higher desirability. This adjustment of the model has not been pursued in this article in order not to cloud the underlying ideas and reasons for adopting this way of modelling by the additional complexity, but see Rusinowska et al. (2005) for an example of how this adjustment can be implemented in order to derive more precise results.

Some hesitation may arise on the use of infinite negative utility for unacceptable policies. However, I feel this best expresses the meaning of the term unacceptable, and the possibility it provides to incorporate absolute boundaries in the model is very useful to explain (limitations on) log-rolling. For certain issues, no compensation can be given (legalising euthanasia might, for instance, be such an issue for certain religious parties). Any policies a party might be willing to support even though they generate it negative utility in order to achieve greater gains elsewhere must, when it comes down to it, be acceptable to that party. The aforementioned way of further specifying the desirability set can be used also to specify acceptable policies which generate negative utility to a party.

The independence of the separate issues can be debated as it is rather likely that a policy is an integrated whole, rather than just the sum of its parts. What is deemed desirable from a financial point of view is, for example, likely to have an effect on what is considered desirable on the issue of 'expenses on social security'. A number of these problems can be solved by redefining the policy space (the policy subspaces can, in principle, be multi-dimensional themselves). This, however, will not always be possible.

## 5. EXAMPLE

In this example four parties are negotiating the formation of government. The fifth party ($O$) can be regarded as a collection of other (small) parties who, in order to keep the example clear, are supposed not to be eligible for government: $P = \{Q, V, R, D, O\}$. The shares of the votes received by the parties are presented in Table I.

The policy space spans five dimensions ($M = 5$), three of which are sub-sets of $\mathbb{R}$ (e.g. 'health care expenditure', 'amount of cut-backs' and 'level of social security payments' (units may vary over dimensions) and two yes/no decisions (e.g. 'implementing universal conscription' and 'opening civil marriage to couples of the same sex'), $B = \mathbb{R}^3_+ \times \{\text{yes, no}\} \times \{\text{yes, no}\}$. The acceptability and desirability sets are presented in Table II. The majority coalitions are given by: $P^* = \{QVRD, QVR, VRD, QRD, QVD, QV\}$.

TABLE I
Parliamentary balance of power

| Party | Percentage of votes | Party | Percentage of votes |
|---|---|---|---|
| $Q$ | 29 | $D$ | 8 |
| $V$ | 27 | $O$ | 17 |
| $R$ | 19 | | |

TABLE II
Acceptability- and desirability-sets

| Party | Acceptability set ($A^i$) | Desirability set ($W^i$) |
|---|---|---|
| $Q$ | $[7, 20] \times [1, 8] \times [5, 9] \times \{\text{yes, no}\} \times \{\text{yes, no}\}$ | $[10, 14] \times [2, 4] \times [7, 8] \times \{\text{yes}\} \times \{\text{yes}\}$ |
| $V$ | $[2, 7] \times [5, 20] \times [3, 6] \times \{\text{yes, no}\} \times \{\text{yes, no}\}$ | $[4, 7] \times [5, 10] \times [4, 5] \times \{\text{no}\} \times \{\text{yes}\}$ |
| $R$ | $[4, 20] \times [2, 10] \times [5, 8] \times \{\text{yes, no}\} \times \{\text{no}\}$ | $[6, 9] \times [3, 6] \times [6, 7] \times \{\text{no}\} \times \{\text{no}\}$ |
| $D$ | $[1, 14] \times [1, 18] \times [4, 8] \times \{\text{yes}\} \times \{\text{yes}\}$ | $[8, 10] \times [2, 5] \times [5, 6] \times \{\text{yes}\} \times \{\text{yes}\}$ |

Dimension five eliminates every coalition containing '$RD$' as $A_5^R \cap A_5^D = \emptyset$. This reduces the number of feasible majority coalitions to three: $QV$, $QVR$ and $QVD$. These coalitions' options are depicted in Table III.

TABLE III
Feasible coalitions

| Coalition | Feasible policies ($A^S$) |
|---|---|
| $QV$ | $\{7\} \times [5, 8] \times [5, 6] \times \{\text{yes, no}\} \times \{\text{yes, no}\}$ |
| $QVR$ | $\{7\} \times [5, 8] \times [5, 6] \times \{\text{yes, no}\} \times \{\text{no}\}$ |
| $QVD$ | $\{7\} \times [5, 8] \times [5, 6] \times \{\text{yes,}\} \times \{\text{yes}\}$ |

It is important to note that both party $R$ and party $V$ will always score on issue 1, whereas $Q$ and $D$ never will. $V$ will, furthermore, always score on the second issue as $A_2^{QV} = A_2^{QVR} = A_2^{QVD} \subset W_2^V$. Also noteworthy is the fact that $Q$ and $V$ can never be faced with unacceptable results. This follows from the fact that they are part of every feasible coalition.

Where the stakes of the game are a matter of maximising their gain for $Q$ and $V$, the game is much more conclusive for $R$ and $D$ as they could be facing unacceptable results at the end of it. An overview of all attainable pay-off vectors is presented in Table IV.

It is striking that the party who emerged strongest from the elections ($Q$) scores consistently less than the other party/parties with whom they form a coalition. In itself this is not particularly relevant as each party only strives to the highest pay-off to itself without paying any attention to the consequences to other parties. On the other hand this also means that a considerable part of the electorate indirectly supports a policy which it may not necessarily want. In any case, it does not hold that government policy coincides largely with the wishes of the largest party, as is, for instance, assumed in Downs (1957) and in one of the models suggested by Kirchsteiger and Puppe (1997). It is furthermore true that no stable governments containing $R$ exist; whereas a stable coalition containing $D$ exists, even though $Q$ and $V$ are also able to form a stable government on their own.

In this example, the set of stable policies is given by

$$C = \{(7; [5, 8]; 5; \text{no;yes}), (7; 5; 5; \text{yes;yes})\}.$$

All stable governments are presented in Table V.

This example shows that it is possible for a party which is not needed for the formation of a majority coalition to still partake in government ($D$).

### 5.1. *Expansion: rent-seeking*

This example will now be expanded by assuming that parties have, next to the aforementioned preferences, ideas about the composition of cabinet.

TABLE IV
Attainable pay-offs

| $x$ | Pay-off $(Q, V, R, D)$ | Coalitions | $x$ | Pay-off $(Q, V, R, D)$ | Coalitions |
|---|---|---|---|---|---|
| $\{7\} \times \{5\} \times \{5\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 3, 3, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (5, 6] \times (5, 6) \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ |
| $\{7\} \times \{5\} \times \{5\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 4, -\infty, 4)$ | $QV,\ QVD$ | $\{7\} \times (5, 6] \times (5, 6) \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 4, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times [5, 8] \times \{5\} \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 5, -\infty, -\infty)$ | $QV$ | $\{7\} \times (5, 6] \times \{6\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 4, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times \{5\} \times \{5\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 4, 4, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (5, 6] \times \{6\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 3)$ | $QV,\ QVD$ |
| $\{7\} \times \{5\} \times (5, 6) \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 3, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (5, 6] \times \{6\} \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ |
| $\{7\} \times \{5\} \times (5, 6) \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 4)$ | $QV,\ QVD$ | $\{7\} \times (5, 6] \times \{6\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 5, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times \{5\} \times (5, 6) \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ | $\{7\} \times (6, 8] \times \{5\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 3, 2, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times \{5\} \times (5, 6) \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 4, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times \{5\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 4, -\infty, 3)$ | $QV,\ QVD$ |

TABLE IV

| $x$ | Pay-off $(Q, V, R, D)$ | Coalitions | $x$ | Pay-off $(Q, V, R, D)$ | Coalitions |
|---|---|---|---|---|---|
| $\{7\} \times \{5\} \times \{6\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 4, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times \{5\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 4, 3, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times \{5\} \times \{6\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 4)$ | $QV,\ QVD$ | $\{7\} \times (6, 8] \times (5, 6) \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 2, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times \{5\} \times \{6\} \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ | $\{7\} \times (6, 8] \times (5, 6) \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 3)$ | $QV,\ QVD$ |
| $\{7\} \times \{5\} \times \{6\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 5, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times (5, 6) \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ |
| $\{7\} \times (5, 6] \times \{5\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 3, 3, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times (5, 6) \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 3, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times (5, 6] \times \{5\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 4, -\infty, 3)$ | $QV,\ QVD$ | $\{7\} \times (6, 8] \times \{6\} \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 3, -\infty)$ | $QV,\ QVR$ |
| $\{7\} \times (5, 6] \times \{5\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 4, 4, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times \{6\} \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 3)$ | $QV,\ QVD$ |
| $\{7\} \times (5, 6] \times (5, 6) \times \{\text{yes}\} \times \{\text{no}\}$ | $(1, 2, 3, -\infty)$ | $QV,\ QVR$ | $\{7\} \times (6, 8] \times \{6\} \times \{\text{no}\} \times \{\text{yes}\}$ | $(1, 4, -\infty, -\infty)$ | $QV$ |
| $\{7\} \times (5, 6] \times (5, 6) \times \{\text{yes}\} \times \{\text{yes}\}$ | $(2, 3, -\infty, 3)$ | $QV,\ QVD$ | $\{7\} \times (6, 8] \times \{6\} \times \{\text{no}\} \times \{\text{no}\}$ | $(0, 3, 4, -\infty)$ | $QV,\ QVR$ |

TABLE V
Stable governments

| | |
|---|---|
| $(QV, (7; [5, 8]; 5;$ no;yes$))$ | $(QV, (7; 5; 5;$ yes;yes$))$ |
| $(QVD, (7; 5; 5;$ yes;yes$))$ | |

Let the cabinet consist of 14 ministerial positions. Parties are assumed to provide candidates for ministerial positions only in governments they support. The parties have the following preferences:

*Q:* Demands

– No other party may occupy half or more than half of the seats in government.
– At least as many ministerial positions as any other party.

Desires

– More ministerial positions than any other party.

*V:* Demands

– Ministerial positions are to be divided amongst coalition partners according to their relative power in parliament (or more are to be given to *V*).

Desires

– six or more ministerial positions (in addition to the above).

*D:* Demands

– At least two ministerial positions.

Desires

– Three or more ministerial positions.

*R:* Demands = Desires

– Divide ministerial positions amongst coalition members according to their relative power in parliament.

It is, of course, possible to write down all possible distributions of ministerial positions over parties and subsequently designate desirability and acceptability sets for all parties using ordened vectors. This somewhat tedious labour is not necessary, however, and is, therefore, omitted.

From this, $A_6^{QV} = \emptyset$ follows, as $V$ demands seven ministerial positions in the coalition $QV$, which is half of the total number of seats in the cabinet, which is unacceptable to $Q$. Remarkably though, $A_6^{QVD} = \{(6,6,2)\} \neq \emptyset$, and $A_6^{QVR} = \{(5,5,4)\} \neq \emptyset$.

If the composition of the cabinet is incorporated in this way, only one stable coalition results: $QVD$. This shows that it is possible that two parties which together occupy more than half of the seats in parliament are not able to form a government on their own, while they can if a third party is included.

Using this way of modelling party preferences over coalitions is preferable over the more standard approach of assuming that each party will attempt to simply maximise the number of seats in government (see, for instance, the way Kirchsteiger and Puppe (1997) implement office seeking considerations). Not only might a party have a special interest in particular ministerial positions (Green parties might for instance be particularly interested in securing the position of 'minister of environmental issues'), it also — and more importantly — allows parties to take the distribution of power into account in a more refined way. As the literature on powerindices shows (see, for instance Laurelle and Valenciano (2005)), it is not only a party's (own) size that determines its power.

## 6. CONCLUDING REMARKS

The model presented in this paper is shown to be widely applicable: accommodating many different kinds of issues, either discrete or continuous in nature; allowing for both rentseeking and idealistic party motivations; for strict and pragmatic parties; discerning major from minor issues; as well as allowing for more refined (and realistic) concerns about the distribution of power. It accounts for the occurrence of issues which often are problematic for standard theories and approaches, the most important being log-rolling and non-minimally winning governments.

The model can be refined in several ways in order to approximate real life situations more closely. One such refinement has been explored in Rusinowska et al. (2005) and software exists that can be used to derive specific information on party preferences for such extentions, see for instance Roubens et al. The concept of stability for these kinds of models is explored extensively in Rusinowska et al. (2005), providing a bases for further analysis, both theoretical and applied, of the government negotiations should multiple stable coalitions exist. (see for instance articles by Berghammer et al., Eklund et al. and Rusinowska and De Swart).

When the outcome of such processes is sufficiently researched the model could, theoretically at least, also be used to provide voters with information on the ultimate results of the outcomes of parliamentary elections, enabling them to use this information when casting their votes. This in turn could be used to analyse party positioning prior to elections. The possibilities this model provides for incorporating distinctions between minor and major issues (which may be different for different parties) as well as the difference between strict and more flexible parties may proof to be very useful in this respect, as this may be particularly relevant to voters.

ACKNOWLEDGEMENTS

I thank Harrie de Swart, Agnieszka Rusinowska, Thijs ten Raa, Peter Borm and Ruud Hendrickx for their many helpful comments and advice.

## NOTES

1. Minimal winning coalitions were introduced by Riker (1962). The main underlying idea is that parties having to divide a fixed sum of money will tend to minimise the number of recipients in order to lay claim to as large an amount as possible. Non-minimum winning coalitions have been known to occur, for example after the 1998 Dutch parliamentary elections, the parties PvdA and VVD had enough seats to form a government, yet the coalition PvdA, VVD and D66 was formed. For empirical research into the occurrence of certain types of government coalitions see De Swaan (1973) and Taylor and Laver (1973).
2. It is well known that parties sometimes support bills they do not prefer in order to gain support for another bill which they deem of great importance. (see for instance Dunleavy (1991)). This kind of vote-trading behaviour is known as log-rolling and is sometimes said to make any predictions on government policy impossible.
3. In Rusinowska et al. (2005) a comprehensive analysis of different definitions for stability for an extended version of this model is provided, including the derivation of necessary and sufficient conditions.

## REFERENCES

Baron, D. (1991), A spatial bargaining theory of government formation in parliamentary systems, *American Political Science Review.* 8(1), 137–164.

Berghammer, R., Rusinowska, A. and De Swart, H. (forthcoming). Applying relational algebra and RelView to coalition formation, *European Journal of Operational Research*.

Black, D. (1958), *The Theory of Committees and Elections*, Cambridge: Cambridge University Press.

Brams, S. (1975), *Game Theory and Politics*, The Free Press: New York.

De Swaan, A. (1973), *Coalition Theories and Cabinet Formation*, Amsterdam: Elsevier.

Downs, A. (1957), *An Economic Theory of Democracy*, New York: Harper & Row Publishers.

Dunleavy, P. (1991), *Democracy, Bureaucracy and Public Choice: Economic Explanations in Political Science*, Harvester Wheatsheaf, Hertfordshire.

Edelman, P. (1997), A note on voting, *Mathematical Social Sciences* 34, 37–50.

Eklund, P., Rusinowska, A. and De Swart, H. *A Consensus Model of Political Decision-making*, Umea University/Tilburg University, Mimeo.

Hotelling, H. (1929), Stability in competition, *The Economic Journal* 39, 41–57.

Kilgour, D. (1974), A shapley value for cooperative games with quarreling. in A. Rapoport (ed.), *Game Theory as a Theory of Conflict Resolution*, Boston: Reidel.

Kirchsteiger, G. and Puppe, C. (1997), On the formation of political coalitions, *Journal of Institutional and Theoretical Economics* 153, 293–319.

Laurelle, A. and Valenciano, F. (2005). Assessing success and decisiveness in voting situations. *Social Choice and Welfare 24*, 171–197.

Laver, M. and Shepsle, A. (1996), *Making and Breaking Governments,* Cambridge: Cambridge University Press.

McKelvey, R. (1976), Intransitivities in multidimensional voting models and some implications for agenda control, *Journal of Economic Theory* 12, 472–482.

Ordeshook, P. (1997), The spatial analysis of elections and committees: four decades of research, in D. Mueller (ed.), *Perspectives on Public Choice: A Handbook,* Cambridge: Cambridge University Press.

Peleg, B. (1981), Coalition formation in simple games with dominant players, *International Journal of Game Theory* 10, 11–33.

Plott, C. (1967), A notion of equilibrium and its possibility under majority rule, *American Economic Review* 57, 787–806.

Riker, W. (1962), *The Theory of Political Coalitions,* New Haven: Yale University Press.

Roubens, M., Rusinowska, A. and De Swart, H. (forthcoming). Using MACBETH to determine utilities of governments to parties in coalition formation, *European Journal of Operational Research.*

Rusinowska, A., De Swart, H. and Van der Rijt, J. (2005), A new model of coalition formation, *Social Choice and Welfare* 24, 129–154.

Rusinowska, A. and De Swart, H. *Negotiating a Stable Government – an Application of Bargaining Theory to a Coalition Formation model*, Tilburg University, Mimeo.

Taylor, M. and Laver, M. (1973), Government coalitions in western Europe, *European Journal of Political Research* 1, 205–248.

Thijssen, J., Hendrickx, R. and Borm, P. (2002), Spillovers and strategic cooperative behaviour, *CentER discussion paper* No. 2002-70. Tilburg University.

Van Deemen, A. (1989), Dominant players and minimum size coalitions, *European Journal of Political Research* 17, 313–332.

Wittman, D. (1995), *The Myth of Democratic Failure, Why Political Institutions Are Efficient*, Chicago: American Politics and Political Economy, University of Chicago Press.

*Address for correspondence:* Jan-Willem van der Rijt, Faculty of Philosophy, University of Groningen, Oude Boteringestraat 52, 9712 GL Groningen, The Netherlands. E-mail: J.W.van.der.Rijt@rug.nl