Spontaneous symmetry breaking in the Higgs mechanism

August 2012

Abstract

The Higgs mechanism is very powerful: it furnishes a description of the electroweak theory in the Standard Model which has a convincing experimental verification. But although the Higgs mechanism had been applied successfully, the conceptual background is not clear. The Higgs mechanism is often presented as spontaneous breaking of a local gauge symmetry. But a local gauge symmetry is rooted in redundancy of description: gauge transformations connect states that cannot be physically distinguished. A gauge symmetry is therefore not a symmetry of nature, but of our description of nature. The spontaneous breaking of such a symmetry cannot be expected to have physical effects since asymmetries are not reflected in the physics. If spontaneous gauge symmetry breaking cannot have physical effects, this causes conceptual problems for the Higgs mechanism, if taken to be described as spontaneous gauge symmetry breaking.

In a gauge invariant theory, gauge fixing is necessary to retrieve the physics from the theory. This means that also in a theory with spontaneous gauge symmetry breaking, a gauge should be fixed. But gauge fixing itself breaks the gauge symmetry, and thereby obscures the spontaneous breaking of the symmetry. It suggests that spontaneous gauge symmetry breaking is not part of the physics, but an unphysical artifact of the redundancy in description.

However, the Higgs mechanism can be formulated in a gauge independent way, without spontaneous symmetry breaking. The same outcome as in the account with spontaneous symmetry breaking is obtained. It is concluded that even though spontaneous gauge symmetry breaking cannot have physical consequences, the Higgs mechanism is not in conceptual danger. The mechanism relies on the non-zero ground state value of the Higgs field, rather than on spontaneous symmetry breaking.

Contents

1 Introduction

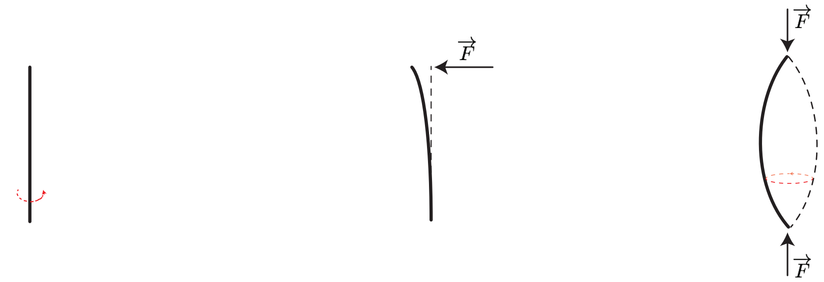
The Higgs mechanism is an important part of the Standard Model of particle physics. It provides the theory with mass for the gauge bosons of the weak interaction and for the fermions. The electroweak theory in which it plays a prominent role had a convincing experimental verification. The Higgs mechanism is generally described as a case of spontaneous symmetry breaking, such as in Peskin and Schroeder [36, Section 20.1] and Weinberg [54, chapter 21].

The notion of spontaneous symmetry breaking (SSB) will be at the basis of this essay. To place the abstract ideas of SSB that will be discussed in a concrete context, a very simple example of SSB will be described here, that can also be found in Ryder [38, pp. 282, 283]. The example emphasizes that spontaneous symmetry breaking is not the same as explicit breaking of a symmetry.
Explicit symmetry breaking is caused by an external force that actively breaks the symmetry. It appears as a symmetry breaking term in the Lagrangian or in another formalism from which the equations of motion can be derived. Spontaneous symmetry breaking however does not require such an external force; this is where the term 'spontaneous' comes from. The equations of motion and the Lagrangian then still obey the spontaneously broken symmetry. The symmetry considered in this example is the rotational symmetry of a rod around its axis as in figure 1(a). The symmetry can be broken by applying an external force to one end of the rod: the rod will bend and lose its rotational symmetry as in figure 1(b). This is explicit symmetry breaking: it is caused by an external force that enters the equations of motion. The case of spontaneous symmetry breaking is encountered if a force is applied in the longitudinal direction of the rod.[1] The rod will now also bend and is no longer rotationally invariant (figure 1(c)). It is worth noting two properties of this spontaneously broken state. The first is that the direction in which the rod bends is arbitrary - there are several 'ground states' that could all equally well be occupied: the ground states are degenerate. The second thing to notice is that the different directions of bending (the different ground states) are related by the original rotational symmetry. The symmetry has thus not disappeared but is 'hidden' in the relation between the ground states. This reflects the fact that the equations of motion still obey the symmetry, although the ground states do not. For this reason spontaneous symmetry breaking is sometimes called a 'hidden symmetry'.

The usual account of the Higgs mechanism describes it as a case of spontaneous symmetry breaking. The spontaneously broken symmetry is however not a physical symmetry as in the example above. The rotational symmetry in the example relates observably different states: the ground states of the rod are physically different. In the Higgs mechanism the symmetry that is spontaneously broken is not a physical symmetry, but a gauge symmetry. A gauge symmetry relates states that are physically the same, but differ in their mathematical description. It is not a symmetry of the physical states; it is a symmetry of the description of a physical state. A more comprehensive study of the nature of gauge symmetries is presented in section 2.

Since gauge transformations relate different descriptions of the same state, spontaneous gauge symmetry breaking is different from the spontaneous symmetry breaking of the physical symmetry above. The degenerate ground states of the system are not physically different in the case of gauge symmetry breaking.

[1] Although an external force is also applied here, this case can be justified as being spontaneous symmetry breaking by considering that the applied force works perpendicular to the bending direction of the rod.



(a) Unbroken symmetry: the rod in its original state is rotationally invariant

(b) Explicitly broken symmetry: the rod bends due to an external force and loses rotational invariance

(c) Spontaneously broken symmetry: the rod bends in an arbitrary direction and loses rotational invariance

Figure 1: Spontaneous symmetry breaking as seen in the bending of a rod

It therefore does not matter which ground state the system 'picks': all states describe the same physics. The asymmetry of the ground states has no physical results. Since a gauge symmetry is not a physical symmetry, spontaneous breaking of this symmetry cannot be expected to have physical consequences. But the Higgs mechanism does have physical consequences: it gives mass to the particles in the standard model. This causes a conceptual problem about the Higgs mechanism: if gauge symmetry breaking cannot have physical consequences, how can the Higgs mechanism which is a case of gauge symmetry breaking, have any?

This conceptual problem has concerned philosophers of physics since Earman [9] in 2002 called for the investigation of this subject. In Brading and Castellani [6, part III] (2003) many aspects of symmetry breaking are addressed. Smeenk [43] gives a good overview of the role of gauge symmetry breaking in the Higgs mechanism. A debate on the ontology and explanatory power of the Higgs mechanism is found in three subsequent papers by Lyre [31], Wüthrich [56] and again Lyre [32]. Kosso [29] investigates the epistemology of broken symmetries and questions the analogy between electromagnetism and the weak theory as regards their basis in gauge symmetry. Indeed, the conceptual discussion about the Higgs mechanism is not only due to philosophers of physics. Articles by Witten [55] and 't Hooft [47, pp. 65 - 95] also contribute to the debate.

Different approaches to spontaneous symmetry breaking are followed in the literature. For example Earman in Brading and Castellani [6, pp. 335-346] follows an algebraic approach to symmetry breaking. This approach is different from the approach that will be followed in this essay. The algebraic approach is axiomatic: it is based on abstract algebras, and their state spaces. Spontaneous symmetry breaking is then studied through unitarily inequivalent representations. The heuristic approach in this essay follows the historical line of development and is the approach adopted by most physicists: spontaneous symmetry breaking in the Standard Model is developed in the classical case, and from there extended to quantum field theory. Throughout this essay symmetry breaking will be considered in the classical domain. It is possible to stay in the classical domain because the presence of spontaneous symmetry breaking is already manifest before quantization, and the conceptual problems discussed in this essay thus already appear in the classical case.

In recent papers by Struyve [44], Ilderton et al. [26], Chernodub et al. [7], Faddeev [12] and Masson and Wallet [33], it has been suggested that the Higgs mechanism can be described without spontaneous symmetry breaking. These papers discuss gauge invariant approaches to the Higgs mechanism. A gauge invariant approach to the Higgs mechanism is however not a completely new discovery, as pointed out by Struyve [44]. Higgs himself [25] (1966) already mentioned a gauge invariant approach, and it is also discussed by Rubakov [37, chapter 6] (1999). In this essay the gauge invariant approach is discussed in section 6. It will be argued that although concerns about spontaneous gauge symmetry breaking are legitimate, the content of the Higgs mechanism is not threatened. The gauge invariant approach shows that the Higgs mechanism is based not on spontaneous breaking of a gauge symmetry, but rather on the non-zero vacuum expectation value of the Higgs field.

In section 2 of this essay the concept of symmetry will be introduced: the distinction between global and local symmetries, and their relation to gauge symmetry is discussed. Section 3 will place the Higgs mechanism in its physical and historical context. It will be seen why the Higgs mechanism plays such an important role in the Standard Model. After that, spontaneous symmetry breaking will be investigated. In section 4 spontaneous breaking of a global symmetry is discussed, and in section 5 the discussion proceeds to local gauge symmetries. This is the case of spontaneous symmetry breaking that is usually attributed to the Higgs mechanism. In section 6, it is shown that the Higgs mechanism can be described without relying on spontaneous symmetry breaking.

2 Global, local and gauge symmetries

At the classical level, a symmetry is a transformation that preserves relevant physical properties. More precisely, a symmetry transformation of a system (such as the positions of particles or the value of a field at each point in space-time) leaves the action of each history (i.e. time-sequence of states) invariant. Since the equations of motion are satisfied by a history whose action is an extremum, the invariance of the action under a symmetry implies that the transformed history also obeys the equations of motion. The action is built from the Lagrangian, so a symmetry can also be described as a transformation that leaves the Lagrangian invariant. An example of a symmetry is seen in the Introduction; the rod in figure 1(a) is symmetric under rotation around its axis. In the context of field theory, symmetries are transformations of fields that leave the action or Lagrangian invariant. Symmetries can be classified in terms of their properties. In this section the distinction between global and local symmetries is described, and light is shed on their relation to gauge symmetry.

A *global symmetry* has a parameter that is constant throughout space-time. The symmetry transformation is thus executed by the same "amount" at every point. The rotational symmetry of the rod in the introduction is a global symmetry: the rod is symmetric under a rotation that is the same at every point in space, and throughout time. More generally a rotation can be implemented by a constant rotation matrix R_{ij}. (R_{ij} is rotation matrix if it is in the group SO(N), thus if it is an orthogonal matrix with unit determinant. i and j run over $1, ..., N$. For a rotation in three dimensions $N = 3$.) A rotation applied to fields ϕ_j is then $\phi'_i = R_{ij}\phi_j$. The fields that this rotation relates are physically distinct; the orientation of a field is a measurable quantity. Another example of a global symmetry is a phase transformation of a field, where the phase transforms by the same amount at every point in space-time. A phase transformation is described by a global U(1) group of transformations. The U(1) transformation will be considered throughout this essay since it is the building block of a simple but adequate model of spontaneous symmetry breaking. A global U(1) transformation of a field is thus

$$\phi(x) \longrightarrow e^{i\alpha}\phi(x), \tag{1}$$

where α is a global (constant) parameter. A phase transformation like this is different from a rotation, because unlike the rotational symmetry, phase transformations of a field are not observable. The complex phase of a field does not appear in measurable quantities, that are in general built from pairs of $\phi^*(x)$, the complex conjugate field, and $\phi(x)$. There are two ways to interpret the unobservable nature of the complex phases.[2] In one interpretation it is concluded that states described by a different complex phase are physically the same and differ only in mathematical description. Another way to look at it is that the states are observationally indistinguishable, but are nevertheless physically different. In the first of these two cases, the global symmetry is labeled as a gauge symmetry, while in the second case it is not. A gauge symmetry is a symmetry that relates states that are physically the same. The states it connects are then only different in their mathematical structure. The presence of a gauge symmetry thus reveals a redundancy in description: there is more description, more mathematical structure, than is strictly necessary to describe the physical reality.

When it is believed that the complex phase transformation relates physically equivalent states, the global U(1) symmetry is a gauge symmetry. The term gauge symmetry is however more often used in the context of local symmetries. In the next part of this section the origin of this will be disclosed: a local symmetry is necessarily a gauge symmetry if it is to be part of a deterministic theory.

A *local* transformation has a parameter that depends on space-time. In the case of rotation, the angle of the rotation can change for different locations and times. For U(1) it means that the parameter α in equation (1) is a space-time dependent function $\alpha(x)$. A local transformation is fundamentally different from a global transformation. The difference lies in their consequences for the determinism or indeterminism of the theories concerned. There are several ways to describe determinism of a theory. For example an approach through the Hamiltonian formalism is possible, but here the question of determinism will be approached through the principle of least action.

Determinism as understood in terms of the action principle is discussed by David Wallace in Brading and Castellani [6, pp. 163 - 173]. His approach gives a very good insight into the relation between symmetries and determinism and especially the connection between local and gauge theories, and will therefore be the backbone of the discussion here.

David Wallace approaches determinism of a theory through the least action principle. In classical mechanics, to describe the motion of a particle, an action is ascribed to every possible path the particle can take between two fixed points. The action is a function of the path of the particle, such that the path that extremizes the action obeys the equations of motion of the system. Through extremizing the action it can thus be determined which path will be realized between two fixed points. The principle of least action can be extended to field theory, where rather than the position of a particle the values of a field are considered. The term 'path' is now not accurate, since the field has values at every point in space, and will therefore be replaced by the term 'history' (following David Wallace). The history between two fixed field configurations, for which the action is an extremum will

[2] The distinction made here follows Struyve [44].

be the realized history. The theory is deterministic if this history is unique, i.e. if there is only one history that extremizes the action between two fixed configurations. So let us now ask: How is this idea of determinism related to global and local symmetries?

Recall that a symmetry is defined as a transformation that keeps the action invariant. So if a transformation is applied to a history that extremizes the action, the transformed history will thus also extremize the action. There are now two histories that extremize the action, and thus are realized. Will this necessarily violate determinism, which, recall, says that there should be a unique history between two endpoints? No, this is not always the case. If the symmetry also changes the endpoint of the history, the transformed history minimizes the action for a *changed* situation. So determinism is not violated. But if the endpoints stay the same under the symmetry transformation, while a part of the history does change, then the theory will be indeterministic. There is then more than one realizable history for a given initial and final configuration of the field; knowledge of the initial and final configuration is not enough to uniquely determine the history of the field.

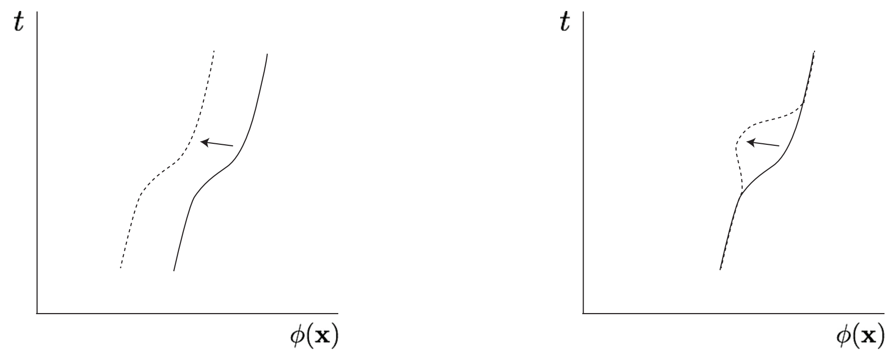
The presence of a global symmetry cannot make a theory indeterministic in such a way: the global parameter ensures that transformations of the field are the same throughout time. It ensures that a non-trivial transformation of the history at any time implies that the initial and final configurations transform non-trivially as well. This is illustrated in figure 2(a). The initial and final field configurations can never be fixed while the rest of the history is transformed, and so the theory remains deterministic.

The only exception to this would occurs if the initial and final field configurations are invariant under the transformation. By the definition of determinism given above, the theory would then be indeterministic. But this would only happen for a certain theory and for certain endpoints and certain transformations, and so would not be a general feature of the theory. This indeterminism for the global symmetry does not occur if we slightly reformulate determinism. Thus determinism can also be defined as the requirement that knowledge of an arbitrarily short segment of the history uniquely determines the rest of the history. Adopting *this* definition, a global symmetry can never make a theory indeterministic.

For a local symmetry, however, this is not the case. The time dependence of a transformation for a local symmetry will ensure that a transformation can leave the initial and final configurations invariant, while transforming the history between those two points. Thus for the local U(1) symmetry in equation (1) with $\alpha = \alpha(t, \mathbf{x})$, this is achieved if the parameter $\alpha(x)$ vanishes at the initial and final times, t_i and t_f, for every $\mathbf{x}$, while it has an arbitrary non-zero value at at least one point $\mathbf{x}$ for a time t, for which $t_i < t < t_f$. Since such a transformation is a symmetry of the theory, the transformed history will extremize the action for the same initial and final configuration. A theory with a local symmetry will thus always be indeterministic. This is illustrated in figure 2(b).[3]

A local symmetry is connected to indeterminism because of the time dependence of its transformation parameter. Since it is desirable that a theory is deterministic, it will here be investigated under what circumstances this can still be the case for a local symmetry. To retain determinism field configurations connected by a local transformation must be physically identified. Histories that are related by a local transformation are then physically the same, and the theory is deterministic

[3]The alternative definition of determinism that was mentioned in the last paragraph will naturally by the same reasoning also lead to indeterminism in the case of theory with a local symmetry.



(a) A global symmetry must have the same parameter at each time. The theory remains deterministic.

(b) A local symmetry has a time-dependent parameter. The theory is indeterministic.

Figure 2: In contrast to a global symmetry, a local symmetry is necessarily indeterministic.

since the initial conditions specify a unique history. Since the local symmetry now relates physically identical states, it is a gauge symmetry. Thus, in a deterministic theory a local symmetry must thus necessarily be a gauge symmetry.

To describe the physics of the system by the equations of motion, every physical state must make an unambiguous appearance in these equations: that is, a single representative needs to be picked from the descriptions that are identified as the same state. This procedure is called 'gauge fixing'. A gauge can be fixed by imposing a gauge condition on the field descriptions, that picks out a representative for each physical field. This gauge condition becomes part of the theory. When a gauge is fixed, a theory loses its gauge symmetry, for a gauge transformation would transform the fields in such a way that they no longer satisfy the gauge condition. The transformed fields do not fit into the theory and therefore a gauge transformation is not a symmetry of the theory.

Aspects of global, local and gauge symmetries have now been discussed. It has been seen that a local symmetry is connected to indeterminism. If the states that are related by the local transformation are identified, determinism is retained; the local symmetry is then a gauge symmetry. A gauge needs to be fixed to be able to describe the physics that the theory represents. Upon gauge fixing the theory loses its invariance under the local gauge transformation. It remains to be discussed how global and local symmetries are implemented in the Lagrangian of a theory.

A general Lagrangian for a single complex scalar field ϕ has the form

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - V(\phi). \tag{2}$$

The example of a U(1) transformation as in equation (1) will be used here to demonstrate global and local symmetries of the Lagrangian. Since the kinetic term of the Lagrangian is invariant under the global U(1) transformation, U(1) is a symmetry of the Lagrangian if the potential $V(\phi)$ is also invariant under the transformation. For example in complex ϕ^4 theory, for which the potential is $V(\phi) = (\phi\phi^*)^2$, the theory is invariant under a global U(1) transformation.

For a local U(1) symmetry the situation is however more complicated, since the kinetic term in equation (2) is not invariant under a local gauge transformation. The derivatives in the kinetic term will also act on the space-time dependent parameter $\alpha(x)$, so that extra terms appear in the Lagrangian upon a local transformation. The Lagrangian of equation (2) is thus not gauge invariant.

It is nevertheless possible to construct a Lagrangian of a different form that obeys the local U(1) symmetry. In a naive account the above Lagrangian is altered such that it is gauge invariant, thereby imposing the existence of a new field: the gauge field. This gauge field (A_μ) has the following gauge transformation:

$$A_\mu \longrightarrow A_\mu + \frac{1}{q}\partial_\mu \alpha(x), \tag{3}$$

where q is the charge of the scalar field. It interacts with the gauge fields in such a way that the interaction compensates for the non-invariance of the kinetic term. As such it appears in the 'covariant derivative': $D_\mu = \partial_\mu - iqA_\mu$, that takes the place of the normal derivative ∂_μ in the Lagrangian. The gauge field also has its own (gauge invariant) dynamical term: $F_{\mu\nu}F^{\mu\nu}$. $F_{\mu\nu}$ is the field strength of the gauge field:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{4}$$

The complete U(1) gauge invariant Lagrangian then has the form

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi D^\mu\phi^* - V(\phi). \tag{5}$$

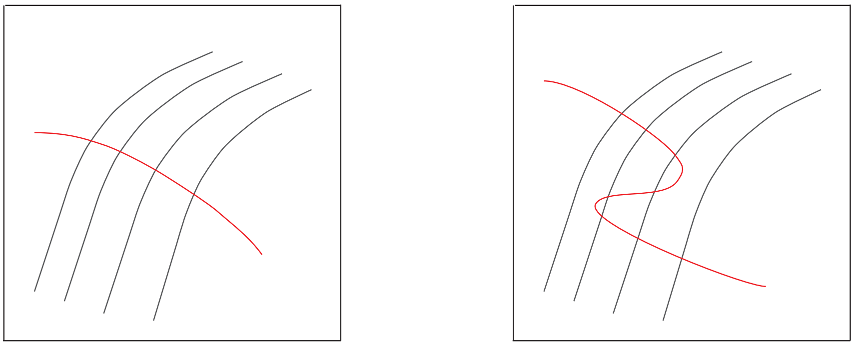
This is the Lagrangian that describes quantum electrodynamics. It is thus the basis of an important theory in physics, whose predictions are experimentally confirmed with high precision.

U(1) is an Abelian gauge symmetry, which means that all elements of the group commute. There are also symmetries for which the elements do not commute, such as for the SU(2) symmetry that occurs in the electroweak theory of the Standard Model. The elements in the Lagrangian of this non-Abelian symmetry are a bit different. The gauge symmetric theory for SU(2) was developed by Yang and Mills [57] in 1954, and in honor of these two physicists the theory is known as Yang-Mills theory. Under a local SU(2) transformation the field transforms as

$$\phi \longrightarrow \exp(i\alpha^i(x)\frac{\sigma^i}{2})\phi, \tag{6}$$

where σ^i are the Pauli matrices: the three non-commuting generators of the SU(2) transformations. Since SU(2) has three generators there are also three gauge fields A^i_μ ($i = 1, 2, 3$). The covariant derivative for a SU(2) gauge invariant Lagrangian is $D_\mu = \partial_\mu - iqA^i_\mu \frac{\sigma^i}{2}$. Due to the non-commutativity of the generators of this symmetry, the field strength $F_{\mu\nu}$ has an extra term: $F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + q\epsilon^{ijk}A^j_\mu A^i_\nu$. The altered covariant derivative and field strength for the SU(2) symmetry describe the Yang-Mills theory when inserted in the Lagrangian in equation 5.

The examples in this essay will contain the Abelian U(1) symmetry, but can be extended to non-Abelian SU(2) and to SU(2)×U(1) that describes the full electroweak theory of the Standard Model. The extension from an Abelian to a non-Abelian symmetry does not influence the conceptual difficulties discussed in this essay. Nevertheless one complication for non-Abelian gauge symmetries should be mentioned. For a non-Abelian gauge symmetry it is not possible to completely fix the gauge. Fields that are connected by a gauge transformation are said to lay on the same orbit - a 'line' can be drawn that connects these states. A gauge fixing condition crosses each orbit once and thereby selects a representative for each physical state. In the case of a non-Abelian gauge theory no gauge fixing condition can be found that picks out a single representative for each state. Every condition crosses the orbits on which the fields lie more than once, as is shown in figure 3(b). This is known as the Gribov ambiguity, named after Vladimir Gribov who presented the ambiguity in 1978 [19] for the Coulomb gauge. Singer [42] proved not much later that for topological reasons the ambiguity also applies for any other gauge.[4] The ambiguity only arises for non-perturbative situations, when the magnitude of the fields becomes large, as proved by Daniel and Viallet [8], and also emphasized by Jackiw et al. [27] and Esposito et al. [11]. Since the Higgs mechanism operates in the regime of perturbation theory, the Gribov ambiguities do not directly affect most of the discussions in this essay. Nevertheless in section 6 for the gauge invariant approach to the Higgs mechanism, Gribov's ambiguity does play a role, that will be discussed.



(a) A gauge fixing condition must cross each orbit once.

(b) For a non-Abelian gauge symmetry any condition crosses some orbits more than once.

Figure 3: Due to the Gribov ambiguity, no gauge fixing is possible for a non-Abelian gauge theory.

It has now been seen how gauge invariant Lagrangians for U(1) and SU(2) can be constructed, but the method through which the Lagrangian in equation (5) is obtained above is naive. There seems to be no ground for imposing local invariance on the Lagrangian. Although in many treatments the gauge invariant Lagrangian is constructed through this naive account, there is also a more convincing route to obtain this Lagrangian. A more natural approach is adopted by Weinberg [53, section 5.9].[5] He shows that a massless spin 1 vector field (A_μ) cannot be a four-vector under Lorentz transformations. He does this by considering the action of the creation and annihilation operators for massless spin 1 fields.[6] The transformation of $A_\mu(x)$ under a Lorentz transformation Λ is

$$U(\Lambda)A_\mu(x)U^{-1}(\Lambda) = \Lambda^\nu{}_\mu A_\nu(\Lambda x) + \partial_\mu \Omega(x, \Lambda), \tag{7}$$

[4]The ambiguity causes problems when quantizing a gauge field, since the functional integration (in the path integral approach) over-counts the duplicated fields. A solution to this is to restrict integration to fields in a region where the duplication ambiguity does not occur. This region is known as the Gribov region.

[5]Martin in Brading and Castellani [6, pp. 29 - 60] mentions this approach.

[6]Weinberg here looks at the quantized field. The approach in this essay has been classical up till here, but for this argument the quantum nature of the field is used.

where $\Omega(x, \Lambda)$ is a function of the creation and annihilation operators. To implement the vector field in a Lorentz invariant theory it must not only be invariant under the Lorentz transformation ($A_\mu \longrightarrow \Lambda^\nu{}_\mu A_\nu$) but also under the transformation $A_\mu \longrightarrow A_\mu + \partial_\mu \Omega$. This last transformation is recognized as the gauge transformation (3). The gauge invariance of the Lagrangian follows then as a more natural result of the Lorentz transformation of the massless spin 1 vector field A_μ.

In this section global, local and gauge symmetries have been discussed. Global symmetries have a global parameter, whereas the parameter of local transformations is space-time dependent. Local symmetries are therefore connected with indeterminism, but can be part of a deterministic theory if they are gauge symmetries; i.e. if they connect physically equivalent states. To extract physics (like the equations of motion) from a theory with a local gauge symmetry a gauge needs to be fixed. The gauge-fixed theory is no longer gauge symmetric.

A gauge invariant Lagrangian has the form of equation (5). The interaction with the gauge field A_μ is necessary for gauge invariance.

In the next section a short historical overview is given of how these gauge invariant theories have become the backbone of the Standard Model of physics, and the essential role that the Higgs mechanism plays in it.

3 Mass and renormalization

The Higgs mechanism is an essential part of the Standard Model. It describes how the gauge field A_μ in Lagrangian in equation (5) can be massive. In this section it will be explored why the Higgs mechanism is so important. Why is a gauge invariant Lagrangian used to describe the electroweak theory, and why is the Higgs mechanism necessary for mass generation?

In the 1940s quantum electrodynamics was finalized and its predictions were experimentally verified to high precision. Due to the work of Bethe [5], Tomonaga [49][50], Schwinger [39] and Feynman [14] between 1946 and 1949, infinities arising in the theory could be dealt with by renormalization. Counterterms were introduced in the Lagrangian that cancel the infinities in the integrals. The counterterms can be absorbed in the 'bare' coupling constants, bare mass and bare fields of the theory. Schwinger [40] in 1948 calculated the value for the anomalous magnetic moment of the electron to second order (one-loop). The predictions by Schwinger were in wonderful agreement with the measurements by Kusch and Foley [30] (1948). And at present, the predicted and experimental values of the anomalous magnetic moment of the electron are of such high precision and in such agreement that QED is the most precisely tested theory of physics.[36]

Thus at the beginning of the 1950s QED had taken up the position of a well established theory. The weak interactions were however not yet described in a satisfying way. Fermi [13] in 1933 described weak beta decay as a contact interaction, the interaction occurring at one point. His theory was accurate, but it was generally believed that for higher energies a description is required where the interaction is mediated by particles, analogous to the photon in QED.[20]. The particles that mediate the weak force must have properties different from the photon: the weak interaction is short-ranged, whereas electromagnetism is a long range interaction. This manifests itself in the mass of the particles. In 1935 Yukawa [58] found a relation between the mass of the intermediate particle and the range of the interaction. This is expressed in the Yukawa potential:[7]

$$U(r) = \frac{g^2}{4\pi r} e^{-mr}. \tag{8}$$

Here r is the range of the interaction and m is the mass of the intermediate particle. The potential decays exponentially for $m \neq 0$. The decay rate is proportional to m, so the decay is faster and the interaction range shorter for a heavy intermediate particle. For a massless intermediate particle like the photon, the exponential term reduces to $e^0 = 1$ and the potential in equation (8) becomes the long range Coulomb potential of QED. The weak interaction is known to be of very short range (which is the underlying feature that makes the contact model of Fermi such a good approximate description), and must therefore be described by massive intermediate particles.

The success of QED from the 1940s inspired physicists to develop a model for the weak interactions analogous to QED. The intermediate particles should be described as gauge bosons in a gauge invariant Lagrangian. In 1954 Yang and Mills [57] constructed a non-Abelian gauge theory based on the SU(2) symmetry, in the hope that this would fit the weak interactions. Glashow [16] in 1960 extended this idea to the group SU(2)×U(1) that should be in the theory underlying the combined electromagnetic and the weak interactions.

The main reason to seek a gauge invariant model for the weak interactions analogous to QED is the renormalizability of QED. Although it was not yet proven for the general case, it was at that time generally believed that the gauge invariance of the Lagrangian is crucial to ensure renormalizability (a general proof for this would not be established until the 1970s by 't Hooft). The massive intermediate particle of the weak interaction however did not fit nicely in a gauge invariant theory. The gauge invariant Lagrangian of QED is linked to masslessness of the intermediate particle. In the previous section it has been shown how Weinberg [53, section 5.9] justifies the gauge invariant Lagrangian from the Lorentz transformation (equation (7)) of a *massless* spin 1 vector field A_μ. The masslessness of the vector field is explicitly necessary for the derivation of this Lorentz transform, and thus for the natural appearance of the gauge field in the gauge invariant Lagrangian. The masslessness of the gauge field in a gauge invariant theory also follows from the fact that a mass term of the form $m^2 A_\mu A^\mu$ is not gauge invariant. Including such a term in the Lagrangian of equation (5) would destroy the gauge symmetry of the Lagrangian.

The mass term in the Lagrangian not only destroys the gauge symmetry, but also the hope of renormalizing the theory. The renormalization of a massive gauge field would require an expression that is not compatible with unitarity of the scattering matrix S. That is: an expression of the massive gauge field that respects unitarity would make the theory non-renormalizable. A clear description of this can be found in an overview of renormalization and gauge invariance, as presented by 't Hooft [48] in Kyoto in 2006.

The mass of the weak interaction gauge bosons thus seems to be incompatible with a renormalizable theory. A different approach to the masses of the gauge bosons is needed. The solution to the problem comes from an analogy with a process in condensed matter: superconductivity.[8]

[7] For a historical discussion of the application of the potential to the strong and weak nuclear forces, see Weinberg [53, pp. 29, 30]. A physical derivation is found in Peskin and Schroeder [36], Tong [51] or the original paper by Yukawa [58].

[8] Discussions of the analogy between spontaneous symmetry breaking in particle physics and super-

Superconductivity was in 1950 described by Ginzburg and Landau as spontaneous breaking of the gauge symmetry of electromagnetism. This description of superconductivity is macroscopic. In 1957 Bardeen, Cooper and Schrieffer [2, 3] developed a microscopic theory in terms of the interactions between electrons and phonons. The theory became known as the BCS theory. As Gor'kov [18] in 1959 showed, the macroscopic Ginzburg-Landau model can be deduced from the microscopic BCS theory.

Nambu and Jona-Lasinio [34] applied the Ginzburg-Landau model of spontaneous symmetry breaking to field theory. But the application of this model to field theory seemed to be incorrect when in 1962 Goldstone, Salam and Weinberg [17] prove 'the Goldstone theorem': whenever a symmetry is spontaneously broken a massless particle must be present. Particles that could serve as those 'Goldstone bosons' are not known to exist, so an application of spontaneous symmetry breaking to field theory seems impossible. But in the same year Schwinger [41] wrote an article in which he emphasized the possibility of combining mass with gauge fields. This article inspired Anderson [1], who described an exception to the Goldstone theorem for a superconductor: he noted that below a certain 'plasma frequency' electromagnetic waves do not propagate in a charged free-electron gas - the photon can under these circumstances be interpreted as massive.[9] Anderson concluded his article with the suggestion that a similar process in particle physics could be responsible for giving mass to Yang-Mills gauge bosons. This is the hint that led to the development of the Higgs mechanism.

Little more than a year after the 1962 article of Anderson, Higgs [23, 24, 25], Englert and Brout [10] and Guralnik, Hagen and Kibble [21] each independently found a mechanism to incorporate spontaneous gauge symmetry breaking in a gauge invariant field theory. This mechanism, that became known as 'the Higgs mechanism', gives the gauge bosons a mass. Weinberg [52] in 1967 and Salam independently applied the mechanism to SU(2)×U(1) gauge theory. The SU(2)×U(1) symmetry is partly broken to a U(1) subgroup, so that of the four gauge bosons associated with SU(2)×U(1), three (the W^+, W^- and Z bosons) become massive, while the photon remains massless. In this model not only do the gauge bosons become massive through the Higgs mechanism; fermions get their mass in a similar way.

The motivation to look for an alternative method of giving mass to gauge bosons, was to renormalize the weak interactions. Recall that an explicit mass term in the gauge invariant Lagrangian (equation (5)) would make the theory non-renormalizable. The Higgs mechanism provided massive gauge bosons, without such a term. But renormalization of a theory with the Higgs mechanism was not yet proved. Thus the application of the Higgs mechanism to SU(2)×U(1) as by Weinberg was not an acknowledged description of the weak interactions, as long as it was unknown if the theory could be renormalized. In 1971 Gerard 't Hooft [46], in close collaboration with his PhD supervisor Veltman, showed that theories with massive gauge bosons due to spontaneous gauge symmetry breaking are renormalizable. 't Hooft and Veltman established this through a procedure called dimensional regularization. 't Hooft explicitly showed how the model of leptons developed by Weinberg in 1967 can be renormalized.

After the renormalizability of Weinberg's model had been shown, the model is recognized as an accurate description of the unification of the weak and electro-

conductivity can be found in Weinberg [54, section 21.6] and Witten [55].

[9] Anderson argues that the longitudinal degree of freedom that is usually attributed to the plasmon can be interpreted as the longitudinal component of a massive photon.

magnetic interactions. The theoretical description was followed by experimental confirmations: the predicted neutral current[10] was experimentally found at CERN in 1973 by Hasert et al. [22]; the W and Z bosons were found in 1983, also at CERN. In the summer of 2012, scientists at CERN announced the discovery of a particle consistent with the Higgs boson: this implies that the Higgs mechanism truly is responsible for mass generation.

Would it be possible to describe the electroweak theory without the Higgs mechanism? Other theories have been developed, but none of them has achieved the generally accepted status that the Higgs mechanism has. One of these theories is technicolor. Technicolor is intended as the analog of the BCS theory of superconductivity: it should give a 'microscopic' explanation for the mass generation in the Standard Model. In the simplest technicolor model mass is generated by a bound state of two 'techniquarks'. The drawback of the model is that technicolor does not automatically create fermion masses. 'Extended technicolor' models have been developed that take care of this, but they add complications to the theory, that make technicolor unsatisfactory as model for mass generation. To sum up: there is no satisfactory analog to the BCS theory in particle physics; and the Higgs mechanism is currently the best explanation for mass generation.

The great achievement of the model of the electroweak theory is that it contains massive gauge bosons, but is nevertheless renormalizable. The Higgs mechanism plays an important role in combining these two requirements. The role of the Higgs mechanism is so important, that in the context of the electroweak theory it would be foolish to question its validity.

Nevertheless the Higgs mechanism should be the subject of a conceptual discussion. The Higgs mechanism describes how gauge bosons become massive through spontaneous breaking of a local symmetry. Why is this problematic? Recall the simple example of spontaneous symmetry breaking in the Introduction. In the spontaneously broken case, the ground states of the rod are degenerate and asymmetric, as in figure 1(c). But in the Higgs mechanism the broken symmetry is a local symmetry. In section 2 it was discussed that a local symmetry is necessarily a gauge symmetry: in a deterministic theory it must relate physically indistinguishable states. The physical indistinguishability also applies to the ground states of the system in the broken case. These states are thus not degenerate, and not physically asymmetric as in the example of the rod, since all orientations correspond to the same physical state. Thus the spontaneously broken symmetry in the Higgs mechanism is a gauge symmetry, that does not apply to physical states, but to the *description* of a physical state. Breaking this symmetry cannot have physical consequences. But nevertheless the Higgs mechanism is ascribed physical consequences: the gauge bosons acquire a mass.

In this essay it will be argued that this discrepancy is not a serious problem for the Higgs mechanism. Even though spontaneous gauge symmetry breaking as such cannot have physical consequences, this does not threaten the Higgs mechanism. The Higgs mechanism can be described without relying on gauge symmetry breaking, as will be discussed in section 6. That discussion will be preceded by a section on global symmetry breaking (section 4) and a discussion of the Higgs mechanism as spontaneous gauge symmetry breaking (section 5).

[10]The neutral current supports interactions that involve the exchange of a neutral Z boson

4 Spontaneous breaking of global symmetry

The idea of gauge symmetry breaking is designed in analogy with global symmetry breaking, but has a different outcome and a different conceptual content. To form a clear concept of spontaneous gauge symmetry breaking it is therefore essential to start by forming an idea of spontaneous breaking of a global symmetry. Part of the discussion will be the description of the Goldstone theorem. This theorem predicts the existence of a massless particle for each broken symmetry. Since these particles are not present in the case of spontaneous gauge symmetry breaking, the breakdown of the Goldstone theorem marks a difference between global and local symmetry breaking.

In section 5 the ideas of global symmetry breaking will be used to describe the Higgs mechanism as spontaneous breaking of a local gauge symmetry. It will be studied how the Goldstone theorem breaks down for gauge symmetry breaking. After that, light will be shed on a different account of the Higgs mechanism: an account that does not need gauge symmetry breaking.

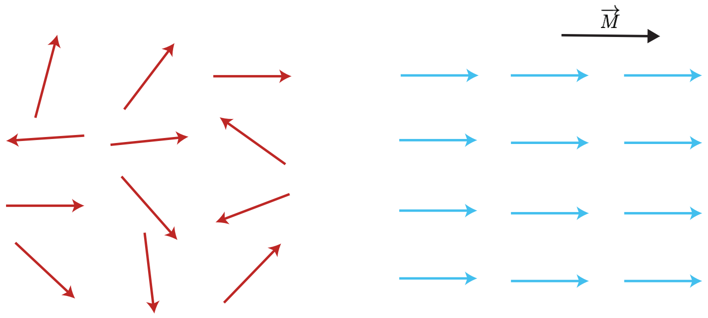
The discussion of spontaneous global symmetry breaking will be started off with the example of a ferromagnet. After that spontaneous symmetry breaking in the global U(1) model is described, thereby using the language of classical field theory. At the end of this section the Goldstone theorem will be discussed: it states that for every broken continuous symmetry there must be a massless particle. The general proof of the Goldstone theorem that will be presented there applies to the domain of quantum mechanics.

4.1 Spontaneous symmetry breaking in a ferromagnet

The purpose of this example is to qualitatively explain spontaneous symmetry breaking. The example of the ferromagnet is widely used in literature.[11] A ferromagnet can be modeled as spins on a grid. At high temperature the spins point in arbitrary directions, as in figure 4(a). Macroscopically (globally) the ferromagnet is invariant under rotations due to the disordered formation of the spins. The expectation value of macroscopic quantities such as the magnetization vanish in this situation. When the temperature is lowered below a critical temperature, the Curie temperature, the spins start to align. The magnetization becomes non-zero and points in some specific direction. Thus the alignment of the spins breaks the invariance of the ferromagnet under global rotations. In this situation the same two comments can be made as were made for the example of the rod in the Introduction: the direction in which the spins align is arbitrary, and the different directions are related by the original symmetry of the ferromagnet: rotations. The symmetry is thus again hidden.

The description of a ferromagnet bears a close analogy to field theories. The ferromagnet can be seen as a statistical system in which the free energy is minimized. Below the Curie temperature, the free energy is minimized if the magnetization has a non-zero value. In the Landau-Ginzburg theory the free energy is expressed in terms of the magnetization, in a form that is analogous to (Euclidean) field theory where the Hamiltonian is expressed in terms of the fields [36, Section 11.3]. The description of spontaneous symmetry breaking in a ferromagnet is thus very analogous to spontaneous symmetry breaking in field theory. Sponaneous breaking of a global symmetry in field theory is the subject of the next subsection.

[11] See for example Ryder [38, section 8.1] and Zee [59, pp.199, 200].



(a) The ferromagnet for $T > T_c$. The global rotational symmetry is unbroken.

(b) The ferromagnet for $T < T_c$. The global rotational symmetry is spontaneously broken.

Figure 4: Two dimensional model of spontaneous breaking of global rotational symmetry in a ferromagnet.

4.2 Spontaneous breaking of global U(1) symmetry

The discussion of global symmetry breaking will be centered around a simple model: ϕ^4 theory with a U(1) symmetry. This simple model will provide insight into the analogies with local gauge symmetry breaking, that will also be considered for the U(1) model. U(1) is an internal symmetry, that describes for example conservation of electric charge.

The U(1) transformation of a complex field ϕ that will be considered here looks like

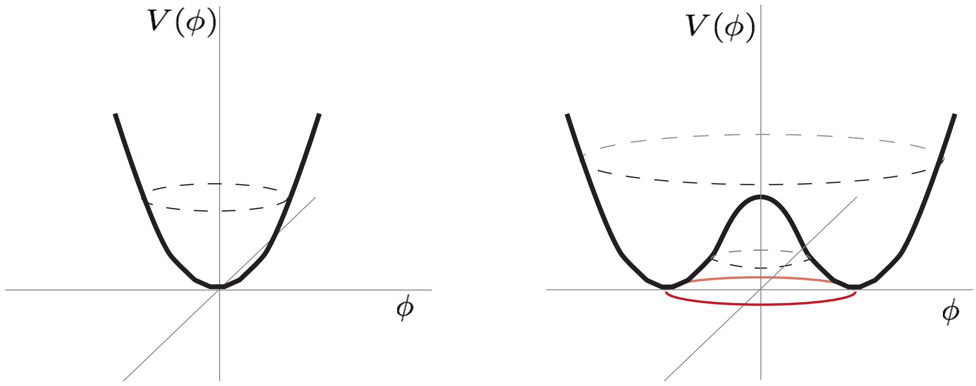
$$\phi(x) \longrightarrow e^{i\alpha}\phi(x), \tag{9}$$

where α is a constant and hence a global parameter, independent of space-time. Such a global parameter defines a global symmetry, as discussed in section 2. The Lagrangian of complex ϕ^4 theory is invariant under this global U(1) symmetry. It is

$$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi^* - \mu^2\phi\phi^* - \lambda(\phi\phi^*)^2, \tag{10}$$

In a theory without symmetry breaking the second term is the mass term, with $\mu^2 > 0$ and the potential $V = \mu^2\phi\phi^* + \lambda(\phi\phi^*)^2$ has the form of a parabola. This is sketched in figure 5(a). There is a unique vacuum that is symmetric: perturbations around the ground state are symmetric under the U(1) phase rotation. For spontaneous symmetry breaking the sign of the mass term is however chosen negative instead of positive: $\mu^2 < 0$. The mass can be seen as imaginary; the field is tachyonic. A tachyonic field is however unstable and will condense into a stable state. The potential has the shape of a Mexican hat as sketched in figure 5(b). The local maximum of the potential at $\phi(x) = 0$ is an unstable point associated with the tachyonic field. The field will condense into a stable ground state. From the figure it is clear that there is not a unique ground state for this system. There is a set of degenerate vacuum states, lying on a ring in the complex plane (as indicated by the red ring in the figure).

The value of the fields in the ground states can be found in a very standard procedure by setting the derivative of the potential to zero. For the case $\mu^2 > 0$ the minimum is found to be at $\phi = 0$, as is expected on inspection of the sketch of the parabola-shaped potential. Since in the unbroken case the ground state is thus at $\phi = 0$, perturbations around this ground state are expressed in small



(a) For $\mu^2 > 0$ there is a unique ground state at $\phi = 0$, sharing the U(1) symmetry of the Lagrangian.

(b) For $\mu^2 < 0$ the ground state is degenerate. Each ground state is asymmetric under the U(1) symmetry of the Lagrangian.

Figure 5: The potential corresponding to the Lagrangian in equation (10) for two different values of μ^2.

values of the field ϕ. The situation is however different if the symmetry is broken in the ground state. When $\mu^2 < 0$ the field $\phi = 0$ describes a local maximum, rather than a minimum. The minima are found by evaluating

$$0 = \frac{\partial^2 V(\phi)}{\partial \phi^2} = \mu^2 + 2\lambda\phi\phi^*. \tag{11}$$

From this it follows that $\phi\phi^* = -\frac{\mu^2}{2\lambda}$ and hence that the ground states all have the same absolute value $|\phi_0|$:

$$v := |\phi_0| = \sqrt{\phi_0\phi_0^*} = \sqrt{\frac{-\mu^2}{2\lambda}} \tag{12}$$

The minima of this system are thus degenerate; there are multiple states with the same vacuum energy. The different orientations in the complex plane define different states. The orientation of these states is comparable to the direction of alignment of the spins in the ferromagnet that was described in section 4.1. These ground states are asymmetric under U(1): applying the U(1) transformation to any of the vacuum states will rotate it to a different orientation that describes a different physical state. For the spontaneous symmetry breaking in the rod and in the ferromagnet two comments were made that apply equally to this case: each of the ground states has an equal chance to be the ground state of the physical system, and the ground states are related to each other by the U(1) symmetry of the Lagrangian. Relating this situation to quantum mechanics might provoke suspicion: through tunneling the real ground state could surely be a superposition of the individual ground states, which would not be degenerate. But although such a situation could occur in ordinary quantum mechanics, it does not apply to quantum field theory. The fields live in an infinite volume and thus have infinitely many degrees of freedom. Tunneling cannot happen in this case. All vacuum states are orthogonal. This is extensively discussed by, for example, Weinberg [54, pp. 163-167].

After identifying the vacua the next step is now to analyze the content of this theory, by taking a closer look at the Lagrangian of equation (10). To analyze this

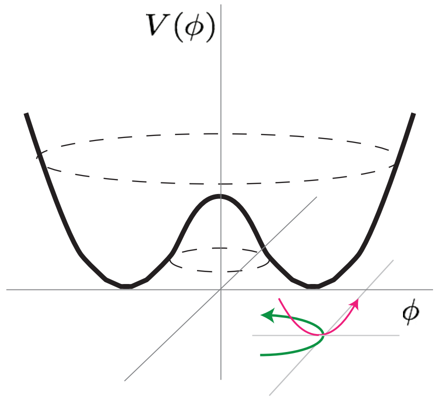


Figure 6: The Mexican hat potential of spontaneously broken U(1) symmetry, with excitations of the Goldstone field indicated by the orange arrow.

Lagrangian it is necessary to set up a perturbation expansion around the ground state. Since the ground state of ϕ is non-zero, it is not possible to perturb around this field. ϕ must be expressed in terms of a constant part that reflects the non-zero value of the ground state and a variable field that is small for perturbations around the ground state. That can be achieved by writing ϕ as

$$\phi(x) = (v + \eta(x))e^{i\xi(x)}. \tag{13}$$

$\eta(x)$ and $\xi(x)$ are real fields while $\phi(x)$ is a complex field, so writing $\phi(x)$ in terms of two real fields is valid. If this expression is plugged into the Lagrangian, the particle content of the theory should be more straightforwardly visible:

$$\begin{aligned}\mathcal{L} &= \partial_\mu\eta\partial^\mu\eta + (v+\eta)^2\partial_\mu\xi\partial^\mu\xi - \mu^2(v+\eta)^2 - \lambda(v+\eta)^4 \\ &= \partial_\mu\eta\partial^\mu\eta + (v+\eta)^2\partial_\mu\xi\partial^\mu\xi - 4\lambda v^2\eta^2 + \lambda v^4 - 4\lambda v\eta^3 - 4\lambda\eta^4. \end{aligned} \tag{14}$$

Here $\mu^2 = -2\lambda v^2$ is used for the elimination of μ^2 out of the expression, after which some terms cancel. The Lagrangian now shows that there is a mass term for the η field, with the mass given by $m_\eta^2 = 4\lambda v^2$. The coefficient of this mass term is positive. There is no mass term for the ξ field. The massless field is the Goldstone boson, that always shows up for spontaneous breaking of a global symmetry. The occurrence of this massless particle can be inferred from the Mexican hat shape of the potential. In figure 6 the freedom of the Goldstone field is marked with a green arrow - it is the angular displacement. A perturbation in this direction does not face any resistance since the energy in the adjacent state is the same. This field is therefore massless.

The presence of a massless boson for each broken symmetry is formulated in the Goldstone theorem. The general proof for this is due to Goldstone, Salam and Weinberg in 1962 [17].[12] A slightly altered proof built from the same principles, is given by Gilbert [15]. This second proof gives a clear insight into the assumptions made and will be discussed in section 4.3. This will be useful in section 5 when considering why the Goldstone theorem does not hold for gauge symmetries.

[12]The proof can also be found in Weinberg [54, section 19.2] and in Osborn [35, section 1.3]. A proof has also been presented by Dr Wingate in the lectures on the Standard Model in part III of the Cambridge Mathematical Tripos.

4.3 The Goldstone theorem

The Goldstone theorem proves the existence of massless bosons in a theory with a spontaneously broken (global) symmetry. The most general proof of the Goldstone theorem is formulated in the language of quantum mechanics. In this section the classical approach will be departed from temporarily. This will change some things: for example the field and charge will now be operators. In this section the details of the proof of the Goldstone theorem will follow a discussion by Gilbert [15].[13]

In the proof it will be shown that states must exist, that have zero mass and spin 0 - the Goldstone bosons. Before proceeding to the details of the proof some basic ideas are stated and an outline of the proof will be given.

The Goldstone theorem uses the notion that according to Noether's first theorem, for every global symmetry there is a conserved current j_μ, such that $\partial^\mu j_\mu = 0$. From this conserved current a charge can be formed. For a global symmetry this charge is time-independent. The charge (which is now an operator) is given by

$$Q = \int d^3x j^0(x), \tag{15}$$

and it generates the transformation of the fields ϕ^i, where i labels the different fields in the theory. This is seen from the commutation relation between the charge and the field:

$$[Q(t), \phi^i(x)] = \tau^{ij}\phi^j(x), \tag{16}$$

where a summation runs over the repeated indices j.[14] The non-zero vacuum expectation value as a result of spontaneous symmetry breaking can then be expressed as

$$\langle 0| [Q, \phi^i(x)] |0\rangle = \tau^{ij} \langle 0| \phi^j(x) |0\rangle \neq 0. \tag{17}$$

The proof of the Goldstone theorem centers around the Fourier transform of the vacuum expectation value of the commutator of the current $J_\mu(x)$ and field $\phi(0)$:

$$M^i_\mu(k) := \int d^4x \langle 0| [J_\mu(x), \phi^i(0)] |0\rangle e^{ikx}. \tag{18}$$

This quantity is the starting point of the Goldstone theorem. The evaluation of this quantity itself is however not the goal of the proof; the objects of interest are the intermediate states that will be inserted in this relation. States can be inserted in equation (18) between $J_\mu(x)$ and $\phi^i(0)$ as long as they form a complete set of states that satisfy the completeness relation $\sum_n |n\rangle \langle n| = 1$. The eigenstates of the momentum operator $\hat{P}$, that satisfy $\hat{P}|n\rangle = p_n |n\rangle$ form such a set of states. The Goldstone theorem will prove that one of these states must necessarily be a spin zero, massless particle: the Goldstone boson.

The set of states $\sum_n |n\rangle \langle n| = 1$ will now first inserted into equation (18). The form that the quantity $M^i_\mu(k)$ then takes is used to present the outline of the rest of the proof of the Goldstone theorem.

[13] This form of the proof can also be found in Bernstein [4, section II].

[14] This proof can be generalized to multiple charges Q, each arising from a different symmetry. These symmetries can all independently be broken. For each broken symmetry the existence of a Goldstone boson can then separately be proven. The generalization does not give additional insight into the Goldstone theorem and its breakdown in the case of spontaneous breaking of a local symmetry. It will therefore be omitted here.

Inserting the intermediate states $\sum_n |n\rangle \langle n| = 1$ in equation (18) gives:

$$M_\mu^i(k) := \int d^4x \, \langle 0| \left[J_\mu(x), \phi^i(0) \right] |0\rangle \, e^{ikx} \tag{19}$$
$$= \sum_n \int d^4x \left(\langle 0| J_\mu(x) |n\rangle \langle n| \phi^i(0) |0\rangle - \langle 0| \phi^i(0) |n\rangle \langle n| J_\mu(x) |0\rangle \right) e^{ikx}.$$

The x-dependence of the current can be expressed as $J_\mu(x) = e^{-i\hat{P}x} J_\mu(0) e^{i\hat{P}x}$, where $\hat{P}$ is the momentum operator. The momenta of the states $|n\rangle$ are p_n. Using this, the expression above can be evaluated to give:

$$M_\mu^i(k) = \sum_n \int d^4x \left[e^{ip_nx} \langle 0| J_\mu(0) |n\rangle \langle n| \phi^i(0) |0\rangle - e^{-ip_nx} \langle 0| \phi^i(0) |n\rangle \langle n| J_\mu(0) |0\rangle \right] e^{ikx}$$
$$= \sum_n (2\pi)^4 \left[\delta(p_n + k) \langle 0| J_\mu(0) |n\rangle \langle n| \phi^i(0) |0\rangle - \delta(p_n - k) \langle 0| \phi^i(0) |n\rangle \langle n| J_\mu(0) |0\rangle \right]. \tag{20}$$

From this relation a first observation can already be made about the intermediate states. Any intermediate state $|n\rangle$ that contributes non-trivially to $M_\mu^i(k)$ must give a non-zero value to the matrix element $\langle n| \phi^i(0) |0\rangle$. For this matrix element to be non-zero, the states $\phi^i(0)|0\rangle$ and $|n\rangle$ must not be orthogonal. The state $|n\rangle$ must thus have the same quantum numbers as the field ϕ^i. Since ϕ^i is a scalar field it has no spin. To make a non-zero contribution to $M_\mu^i(k)$, the state $|n\rangle$ must thus have spin zero as well. It can furthermore not be the vacuum state, since ϕ^i acting on the vacuum state creates a one-particle state. If $|n\rangle$ is not orthogonal to $\phi^i(0)|0\rangle$ it must thus be a one-particle state with spin zero. The goal of the Goldstone theorem is now to show that $M_\mu^i(k)$ is non-zero, such that the spin zero particle must necessarily exist. Because $M_\mu^i(k)$ will turn out to be proportional to $c_1\delta(k^2)$. c_1 is shown to be non-zero, so that the spin zero particle must exist, while the delta function in k^2 ensures that the particle is massless. The existence of Goldstone bosons is then proven.

The next step in the proof is to write equation (20) in a general form. In equation (20) $M_\mu^i(k)$ is written in terms of the intermediate states. The goal is to find that some of these states must be massless bosons. This will be deduced from a general expression for $M_\mu^i(k)$ that is constructed from the Lorentz structure. Since J_μ is a four-vector the matrix elements $\langle 0| J_\mu(0) |n\rangle$ must be proportional to a four-vector. The four-vector present in the theory is $p_{n,\mu}$. Following Gilbert [15] the matrix element is expressed in terms of this four-vector in order to maintain manifest Lorentz covariance:

$$\langle 0| J_\mu(0) |n\rangle = a(p_n^2) p_{n,\mu}, \tag{21}$$

where $a(p_n^2)$ is an arbitrary function of p_n^2, the only Lorentz scalar that can be formed. This expression is manifestly Lorentz covariant. If manifest Lorentz covariance were not required, a term proportional to an arbitrary four-vector η_μ could be added to the general expression in equation (21).

Here we note that Higgs [23] pointed out that this is the case for a gauge symmetry, thereby destroying the argument of the Goldstone theorem. Inspired by the possibility of spontaneous breaking of gauge symmetries Higgs developed the Higgs mechanism in a subsequent paper.[15] The breakdown of the Goldstone theorem will be discussed in the next section. For now manifest Lorentz covariance

[15]Guralnik et al. [21] and Englert and Brout [10] independently developed descriptions of the same mechanism.

is required and the general form in equation (21) is used.[16]

The matrix element $\langle n| J_\mu(0) |0\rangle$ is similarly expressed in a general form as $\langle n| J_\mu(0) |0\rangle = b(p_n^2)p_{n,\mu}$. Using the general forms for these matrix elements in equation (20) gives

$$\begin{aligned} M^i_\mu(k) &= (2\pi)^4 \sum_n \left[\delta^4(p_n+k)a(p_n^2)p_{n,\mu} \langle n| \phi^i(0) |0\rangle - \delta^4(p_n-k) \langle 0| \phi^i(0) |n\rangle b(p_n^2)p_{n,\mu}\right] \\ &= (2\pi)^4 \left(-a(k^2)k_\mu \langle n| \phi^i(0) |0\rangle \theta(-k_0) - b(k^2)k_\mu \langle 0| \phi^i(0) |n\rangle \theta(k_0)\right). \end{aligned} \tag{22}$$

The step functions $\theta(k_0)$ and $\theta(-k_0)$ arise because the states $|n\rangle$ are required to be physical states and thus have $p_0 = E_n > 0$. This leads to $\delta(E_n + k_0) = 0$ if $k_0 > 0$ and $\delta(E_n - k_0) = 0$ if $k_0 < 0$. The expression is now rewritten in a convenient form, replacing the step functions by a function $\epsilon(k_0)$ that is given by

$$\epsilon(k_0) = \begin{cases} -1 & \text{if } k_0 < 0, \\ 1 & \text{if } k_0 \geq 0. \end{cases} \tag{23}$$

The general form that M^i_μ now takes is

$$M^i_\mu = \epsilon(k_0)\rho^i_1(k^2)k_\mu + \rho^i_2(k^2)k_\mu, \tag{24}$$

where ρ^i_1 and ρ^i_2 are functions of $a(k^2)$, $b(k^2)$ and the matrix elements $\langle n| \phi^i(0) |0\rangle$ and $\langle 0| \phi^i(0) |n\rangle$.[17]

Now $M^i_\mu(k)$ is written in this general form, it is possible to show that ρ^i_1 and ρ^i_2 are proportional to delta functions. This can be done by considering the conservation of the current $\partial_\mu J^\mu = 0$. This implies that $\partial^\mu M^i_\mu(k) = ik^\mu M^i_\mu(k) = 0$, where the derivative is evaluated for $M^i_\mu(k)$ as in equations (18) and (20) respectively. Applying this to the general expression in equation (24) gives

$$\epsilon(k_0)\rho^i_1(k^2)k^2 + \rho^i_2(k^2)k^2 = 0, \tag{25}$$

which results in two constraining equations; one for $k_0 < 0$ and one for $k_0 > 0$:

$$\begin{aligned} \rho^i_1(k^2)k^2 + \rho^i_2(k^2)k^2 = 0; \\ -\rho^i_1(k^2)k^2 + \rho^i_2(k^2)k^2 = 0. \end{aligned} \tag{26}$$

The only solution to these equations is $\rho^i_1(k^2) = c^i_1\delta(k^2)$ and $\rho^i_2(k^2) = c^i_2\delta(k^2)$, where c^i_1 and c^i_2 are undetermined parameters, that could at present still be zero.

It should be remembered that the i indices label different fields in the theory. To prove the presence of Goldstone bosons, at least one of the parameters for a field should be non-zero. If both parameters were zero, ρ^i_1 and ρ^i_2 and consequently M^i_μ would vanish, so that no conclusions can be drawn about the intermediate states. But if one of the parameters is non-zero the delta function $\delta(k^2)$ ensures that there are intermediate states that have $k^2 = 0$, and hence must be massless. The last part of the proof of the Goldstone theorem is thus to show that $c^i_1 \neq 0$

[16] The explicitness of this assumption in the proof by Gilbert is the reason that here this proof is discussed, rather than that of Goldstone, Weinberg and Salam.

[17] More precisely, the functions are:
$\rho^i_1(k^2) = \frac{(2\pi)^4}{2}\left[-a(k^2)\langle n| \phi^i(0) |0\rangle + b(k^2)\langle 0| \phi^i(0) |n\rangle\right]$
$\rho^i_2(k^2) = \frac{(2\pi)^4}{2}\left[-a(k^2)\langle n| \phi^i(0) |0\rangle - b(k^2)\langle 0| \phi^i(0) |n\rangle\right]$

or $c_2^i \neq 0$.

To do this, the non-zero vacuum expectation value of ϕ (equation (17)) that has so far not been used, is put into play. It will be shown that c_1 is proportional to the non-zero vacuum expectation value. Therefore the zeroth component of $M_\mu^i(k)$ needs to be considered:

$$M_0^i = \int d^4x \left\langle 0\right| \left[J_0(x), \phi^i(0)\right] \left|0\right\rangle e^{ikx} \tag{27}$$

$$= \epsilon(k_0)k_0\rho_1^i(k^2) + k_0\rho_2^i(k^2), \tag{28}$$

where respectively the original definition of M_μ^i (equation (18)) and its general form (equation (24)) are used. Now the existence of Goldstone bosons can be proved by considering the integral of M_0^i over k_0, setting $\mathbf{k} = 0$. The results of doing this for M_0^i as in equations (27) and (28) are then compared. For M_0^i as in equation (27) the integral becomes:[18]

$$\begin{aligned}
\int dk_0 (M_0^i)_{\mathbf{k}=0} &= \int d^4x \int dk_0 \left\langle 0\right| \left[J_0(x), \phi^i(0)\right] \left|0\right\rangle e^{ik_o x_0} \\
&= \int d^4x\, 2\pi\delta(x_0) \left\langle 0\right| \left[J_0(x), \phi^i(0)\right] \left|0\right\rangle \\
&= 2\pi \int d^3x \left\langle 0\right| \left[J_0(\mathbf{x}, 0), \phi^i(0)\right] \left|0\right\rangle \\
&= 2\pi \left\langle 0\right| \left[Q, \phi^i(0)\right] \left|0\right\rangle \\
&= 2\pi\tau^{ij} \left\langle 0\right| \phi^j(0) \left|0\right\rangle \neq 0.
\end{aligned} \tag{29}$$

In the last lines the assumption of spontaneous symmetry breaking as in equation (17) is used:

$$\langle 0| [Q, \phi^i(x)] |0\rangle = \tau^{ij} \langle 0| \phi^j(x) |0\rangle \neq 0.$$

The integral over k_0 is then again evaluated, for the general expression of M_0^i as in equation (28). To evaluate this expression the following identity is used:

$$\delta(k^2) = \frac{\delta(|\mathbf{k}| - k_0)}{2|\mathbf{k}|} + \frac{\delta(|\mathbf{k}| + k_0)}{2|\mathbf{k}|}. \tag{30}$$

The integral can be evaluated to give

$$\begin{aligned}
\int dk_0 (M_0^i)_{\mathbf{k}=0} &= \int_{-\infty}^{\infty} dk_0 \left[\epsilon(k_0)k_0 c_1^i + k_0 c_2^i\right] \delta(k^2) \\
&= \int_{-\infty}^{0} dk_0 (-k_0 c_1^i + k_0 c_2^i) \frac{\delta(|\mathbf{k}| + k_0)}{2|\mathbf{k}|} \\
&\quad + \int_0^{\infty} dk_0 (k_0 c_1^i + k_0 c_2^i) \frac{\delta(|\mathbf{k}| - k_0)}{2|\mathbf{k}|} \\
&= \frac{|\mathbf{k}|}{2|\mathbf{k}|}(c_1^i - c_2^i) + \frac{|\mathbf{k}|}{2|\mathbf{k}|}(c_1^i + c_2^i) = c_1^i.
\end{aligned} \tag{31}$$

Now equations (29) and (31) are combined to find $c_1^i \neq 0$. Since c_1 is non-zero, ρ_1^i and thus $M_\mu^i(k)$ are non-vanishing.[19] There must thus be matrix elements $\left\langle n\right| \phi^i(0) \left|0\right\rangle$ for some states $|n\rangle$ that are non-zero. For these matrix elements to

[18] The metric + - - - is used.

[19] The presence of the $\epsilon(k_0)$ function prohibits the cancellation of a non-zero value of c_1 by any value of c_2. It can also be proved that $c_1 = \left\langle 0\right| \phi^j(0) \left|0\right\rangle / 2\pi$, but the proof of this will not be of extra value for the present purposes.

be non-zero $|n\rangle$ must have the quantum numbers of $\phi^i(0)$: it must have spin zero. The proportionality to $\delta(k^2)$ implies the masslessness of this state. Furthermore it cannot be the vacuum state, since it is not orthogonal to the one-particle state that results from $\phi^i(0)$ acting on the vacuum ($\phi^i(0)\,|0\rangle$). This thus establishes the proof of the existence of a massless particle with spin 0: the Goldstone boson.

In this section spontaneous symmetry breaking for a global symmetry has been discussed. It is shown how a massless boson appears for the U(1) symmetry, and the appearance of Goldstone bosons has in general been proven by the Goldstone theorem. In the next section it will be seen that for spontaneous breaking of a gauge symmetry this massless particle will not be present. Rather, the component of the field that would become the Goldstone boson disappears after gauge fixing. The degree of freedom left over is picked up by the gauge field that becomes massive. This is the context of the Higgs Mechanism which will be discussed in the next section.

5 The Higgs mechanism as spontaneous gauge symmetry breaking

In the previous section, spontaneous breaking of a global symmetry has been discussed. In this section the Higgs mechanism will be presented as the spontaneous breaking of a gauge symmetry. The Goldstone bosons that appeared upon the breaking of a global symmetry in the last section will not be found for spontaneously broken gauge symmetry: the Goldstone theorem breaks down. In this section the Higgs mechanism is described as spontaneous breaking of a simple U(1) gauge symmetry. After that it will be shown why the Goldstone theorem breaks down. Thereby it will be seen that the possibility to fix a gauge plays an important role in the difference between global and gauge symmetry breaking.

5.1 The Higgs mechanism

The simple Abelian U(1) gauge theory that will be discussed here describes electromagnetism. The gauge boson that in this model become massive through spontaneous symmetry breaking is the photon. In nature photons are generally massless particles, since electromagnetism is a long range force. The U(1) symmetry is then unbroken. A situation in which photons are massive is in a superconductor where the U(1) gauge symmetry is spontaneously broken. This was briefly described in section 3. The Higgs mechanism explains how a field with an (asymmetric) non-zero ground state can be the source of massive gauge bosons.

The Higgs mechanism can also be applied to non-Abelian gauge theories such as the SU(2)×U(1) gauge symmetry that describes the electroweak interaction. It is in this model that the Higgs mechanism found its most important application: the mechanism allows the W and Z bosons to be massive. The non-Abelian gauge symmetry has an extra difficulty rising from the Gribov ambiguity. This theorem states that a gauge cannot be completely fixed for a non-Abelian gauge theory. Instead of picking one representative for states that are related by a gauge transformation, more than one representative state is picked. As was discussed in section 2, the Gribov ambiguity is not a problem for perturbation theory - the regime in which we will operate in this section. In section 6 the Gribov ambiguity must be considered, when the gauge invariant approach to the Higgs mechanism is discussed.

The cornerstone in the Higgs mechanism is the assumption of a new field, the Higgs field, whose non-zero vacuum expectation value breaks the U(1) gauge symmetry of the Lagrangian. This feature is implemented in the Lagrangian in a similar fashion as for the case of global symmetry breaking. The field is given a negative parameter for its mass term ($\mu^2 < 0$); thus becoming tachyonic.

The Lagrangian for ϕ^4 theory with a U(1) gauge symmetry, has the form

$$\mathcal{L} = -F^{\mu\nu}F_{\mu\nu} + D^{\mu}\phi D_{\mu}\phi^* - \mu^2\phi\phi^* - \lambda(\phi\phi^*)^2. \tag{32}$$

The gauge field A_μ interacts with the field ϕ in such a way that the Lagrangian is invariant under the gauge transformations of ϕ and A_μ, as seen in equations (1) and (3). As discussed before, a mass term would break the gauge invariance of the Lagrangian.

In the Higgs mechanism $\phi(x)$ is the Higgs field and $\mu^2 < 0$, so that the potential has the Mexican hat shape as in figure 5(b). To analyze the theory in a perturbation expansion, it is as in the global case necessary to expand around a variable field that is small for perturbations around the ground state. Therefore the field is again written out in a polar decomposition, similar to the global symmetry case:

$$\phi(x) = (\rho(x) + v)e^{i\xi(x)} \tag{33}$$

where again $v = \sqrt{\frac{-\mu^2}{2\lambda}}$, and $\rho(x)$ and $\xi(x)$ are real scalar fields. Plugging this into the Lagrangian equation (32) gives:

$$\begin{aligned}
\mathcal{L} &= -F^{\mu\nu}F_{\mu\nu} + (\partial_m u + iqA_\mu)(\rho+v)e^{i\xi}(\partial^\mu - iqA^\mu)(\rho+v)e^{-i\xi} \\
&\quad -\mu^2(\rho+v)^2 - \lambda(\rho+v)^4 \\
&= -F^{\mu\nu}F_{\mu\nu} + \partial_\mu\rho\partial^\mu\rho + (\rho+v)^2\partial_\mu\xi\partial^\mu\xi + q^2(\rho+v)^2A_\mu^2 \\
&\quad -2q(\rho+v)^2\partial_\mu\xi(x)A^\mu - 4\lambda v^2\rho^2 - \lambda\rho^4 - 4\lambda\rho^3 v + \lambda v^4 \\
&= -F^{\mu\nu}F_{\mu\nu} + \partial_\mu\rho\partial^\mu\rho + v^2\partial_\mu\xi\partial^\mu\xi - 4\lambda v^2\rho^2 + q^2v^2A_\mu^2 - 2qv^2\partial_\mu\xi(x)A^\mu \\
&\quad + \text{cubic and quartic terms.}
\end{aligned} \tag{34}$$

To see the particle content of this theory, only the quadratic terms in this equation are interesting. The term $q^2v^2A_\mu^2$ shows that the gauge field A_μ has become massive, due to its interaction with the constant part of the Higgs field. One of the components of the Higgs field, $\rho(x)$, is massive due to the term $4\lambda v^2\rho^2$. The other component $\xi(x)$ seems to be a massless Goldstone mode, like in the global case. $\xi(x)$ is however not a physical particle: it is a function that results from the freedom to pick a gauge. The freedom to pick a gauge can be used to set $\phi(x) = (\rho(x) + v)$ (so that thus $\xi(x) = 0$). This choice of gauge is called the 'unitary gauge'. It shows the particle content of the theory clearly, and is therefore used here. With another gauge, such as the Lorenz or Coulomb gauge the outcome is the same but the physics is less straightforwardly visible.

To see that $\phi(x) = \rho(x) + v$ defines a gauge, it must be noted that this can be achieved by a gauge transformation with $\alpha(x) = -\xi(x)$:

$$\begin{aligned}
\phi(x) &\longrightarrow e^{-i\xi(x)}\phi(x); \\
A_\mu &\longrightarrow A_\mu - \frac{1}{q}\partial_\mu\xi(x).
\end{aligned} \tag{35}$$

With the gauge fixing, the gauge symmetry is removed from the Lagrangian. The fields no longer have the freedom to transform under gauge transformations, because the one representation $\phi(x) = \rho(x)+v$ is fixed. In this gauge the Lagrangian then looks like

$$\mathcal{L} = -F^{\mu\nu}F_{\mu\nu} + \partial_\mu\rho\partial^\mu\rho - 4\lambda v^2\rho^2 + q^2v^2A_\mu^2 + \text{ cubic and quartic terms.} \quad (36)$$

This Lagrangian now clearly shows that there is a massive gauge field and that the Higgs field has one massive component. There are no massless particles in this theory, and the field $\xi(x)$ has completely disappeared from the Lagrangian.

Looking at the outcome of the Higgs mechanism as compared to the global case, it is hard to believe that both result from a similar mechanism. For the gauge symmetry, instead of ending up with a massless Goldstone boson, there is a massive gauge field. Why did the local gauge invariance of our theory change the mechanism so severely? Part of the answer lies in the inapplicability of the Goldstone theorem to gauge symmetries. This will be considered below. In section 6, a gauge independent formulation of the Higgs mechanism will be described, that provides more insight to what makes the Higgs mechanism so different.

5.2 Breakdown of the Goldstone theorem

The way in which the Goldstone theorem breaks down depends on the gauge in which the theory is evaluated. The result of course is the same for every choice of gauge: no massless bosons. The role of gauge fixing in the breakdown of the Goldstone theorem can give insight into the position of gauge symmetry breaking with respect to global symmetry breaking. Accordingly, the breakdown of the Goldstone theorem will be considered here for two choices of gauge: the unitary gauge and the Coulomb gauge. These choices cover two interesting cases. In the unitary gauge the Goldstone theorem itself does not break down, but the 'Goldstone bosons' can nevertheless be rotated away. In the Coulomb gauge the Goldstone theorem does break down: some of the assumptions made become invalid. The discussion in this section applies to the case of Abelian gauge theory. In the non-Abelian case gauge fixing is not always unambiguously possible due to the Gribov ambiguity. But this section nevertheless gives insight in the role of gauge fixing in the Higgs mechanism.

The unitary gauge was used for spontaneous gauge symmetry breaking in section 5.1, equation (35). In the Higgs mechanism as described above, the unitary gauge is used to reveal the particle content of the Lagrangian. Choosing the unitary gauge, the Goldstone theorem is not violated, but the Goldstone bosons (ξ) are 'rotated' away as is described by Weinberg [54, pp.295-296]. It is possible to make a rotation such that all the fields are orthogonal to the Goldstone fields. The rotation is explicitly only possible if the Lagrangian is symmetric under a local transformation. The rotation of the fields that rotates away the Goldstone bosons is

$$\tilde{\phi}_i(x) = \sum_k \gamma_{ij}^{-1}(x)\phi_j(x), \quad (37)$$

where $\gamma(x)$ is a suitable rotation of the fields and $\tilde{\phi}_i(x)$ are the rotated fields, that do not include the Goldstone bosons. The specific rotation (with a fixed parameter $\gamma(x)$ as in equation (37)) that makes the fields $\tilde{\phi}$ orthogonal to the Goldstone bosons is in fact equivalent to choosing the unitary gauge. The Goldstone theorem does not break down for this gauge choice, but the 'Goldstone bosons' are not physical particles.

The Coulomb gauge shows a different effect. The gauge fixing constraint is given by $\nabla \cdot \mathbf{A} = 0$.[20] This gauge preserves Lorentz covariance, although this is not manifest in the description. This will violate the Goldstone theorem, as Higgs [23] was the first to point out.[21] In the Goldstone theorem the quantity $M_\mu^i(k)$ is written in a general form. This general form was chosen to show manifest Lorentz covariance, and therefore be proportional to k_μ, as in equations (21) and (24). Thereby proportionality to an arbitrary four-vector η_μ was disregarded through the argument of preservation of manifest Lorentz covariance. Since in the Coulomb gauge this argument does not hold, the vector η_μ should now not be disregarded. For gauge theories a general form must first be found for $M_\mu^i(k)_A$, the quantity equivalent to M_μ^i from equation (18) but with A_μ taking the place of J_μ. The general form of this quantity then also contains proportionality to the arbitrary vector η_μ:

$$\begin{aligned} M_\mu^i(k)_A &= \int d^4x \, \langle 0| \left[A_\mu(x), \phi^i(0)\right] |0\rangle \, e^{ikx} \\ &= a^i(k^2, \eta k) k_\mu + b^i(k^2, \eta k)\eta_\mu + c^i \eta_\mu \delta^4(k). \end{aligned} \tag{38}$$

η_μ is a four-vector in a given Lorentz frame, that can be chosen to be $\eta = (1, 0, 0, 0)$. The two Lorentz scalars that can occur in the considered functions are k^2 and ηk. Now this general expression and the relation between current J_μ and gauge fields A_μ[22] can be used to construct a general expression for $M_\mu^i(k)_J$, the quantity under consideration in the Goldstone theorem (equation (18)). It turns out that the general expression then is

$$M_\mu^i(k)_J = b^i(k^2, \eta k) \left[k^2 \eta_\mu - k_\mu(\eta k)\right]. \tag{39}$$

It is not possible to prove the existence of Goldstone bosons from this expression. The procedure in the Goldstone theorem was to use conservation of the current to show $k^\mu M_\mu^i(k) = 0$, and from that find that the terms in $M_\mu^i(k)$ must be proportional to delta functions. But for the general structure as in equation (39) this procedure fails, since $k^\mu M_\mu^i(k)_J$ is trivially zero. No conclusions can thus be drawn from the general structure of $M_\mu^i(k)$ and the Goldstone theorem breaks down.

The mathematical descriptions of what happens to the Goldstone bosons in the case of gauge symmetry thus depend on the gauge in which the theory is analyzed. From the discussion in this section it is important to notice the essential role that gauge fixing plays. This puts the asymmetry of the ground state in a different light: the gauge fixing procedure itself breaks this symmetry. What is the status of an asymmetric vacuum, if gauge fixing is essential to see the physical content of a theory? To investigate this and the role that gauge fixing plays, we will discuss in section 6 accounts of the Higgs mechanism where gauge fixing is not employed.

6 The Higgs mechanism without spontaneous symmetry breaking

The procedure that was used in section 5 to describe the Higgs mechanism, uses explicit gauge fixing by requiring that $\phi(x) = \rho(x) + v$ (thereby setting the imaginary part of $\phi(x)$ to zero). But this gauge fixing gets rid of the gauge symmetry,

[20] In momentum-space this constraint takes the form $(kA(k)) + (\eta k)(\eta A(k)) = 0$, where η is a four-vector that, by choosing a suitable Lorentz frame, can be set to $\eta = (1, 0, 0, 0)$.

[21] Bernstein [4, section III] discusses the details of this.

[22] Namely the relation $j_\mu(x) = \frac{\partial}{\partial x_\nu}\left(\frac{\partial A_\nu(x)}{\partial x^\mu} - \frac{\partial A_\mu(x)}{\partial x^\nu}\right)$.

and thus obscures the meaning of gauge symmetry breaking. Happily, it is possible to describe the Higgs mechanism in a gauge invariant way. This was already done by Higgs, in a third paper on the mass of gauge bosons in 1966. Also, Kibble [28] described this for non-Abelian gauge theories. In some recent papers these gauge independent accounts of the Higgs mechanism are also discussed, either referring to the old accounts or re-inventing them.[23]

The same result as in the discussion above is obtained by making convenient field transformations. The only fields that remain in the theory are gauge invariant. The fields that do transform under a gauge transformation are factored out from the theory. These fields are what could be seen as the Goldstone bosons. This procedure without gauge fixing is comparable to applying the field transformations corresponding to the fixing of the unitary gauge (equation (35)), but without considering $\xi(x)$ as a fixed function, so that the procedure is not a gauge fixing. Rather $\xi(x)$ is still a variable field, and will disappear from the Lagrangian in a 'natural' way. It can be shown that the fields that then still occur in the Lagrangian are invariant under the U(1) transformation. The new fields in terms of which the Lagrangian is now written have a unique non-zero vacuum expectation value. Through this non-zero value the same results are obtained as seen earlier for the Higgs mechanism interpreted as spontaneous gauge symmetry breaking.

The procedure will be worked out in more detail below for U(1) gauge symmetry breaking. With a similar treatment it would be possible to do the same for SU(2)×U(1), leaving a U(1) symmetry in the Lagrangian. This can be found in Masson and Wallet [33]. But for the conceptual understanding of the role of gauge symmetry the U(1) model is sufficient, so the case for SU(2)×U(1) is not worked out here. It should be noted that the Gribov ambiguity cannot be ignored for the non-Abelian case. The Gribov ambiguity indeed seems to appear for the SU(2)×U(1) symmetry; but, as shown by Ilderton et al. [26] this can be circumvented for the case in which the scalar Higgs field has a non-zero vacuum value. Treatments of the gauge invariant approach to the Higgs mechanism for SU(2)×U(1) are given by Chernodub et al. [7], Faddeev [12] and Masson and Wallet [33]. After describing the Higgs Mechanism in this gauge-independent way, a discussion will follow on what this means for the understanding of spontaneous gauge symmetry breaking.

For the description of the Higgs mechanism without symmetry breaking the first important step is to write the theory in terms of gauge invariant variables. This will make the theory independent of the gauge group under consideration. The starting point is again the U(1) gauge invariant Lagrangian for ϕ^4 theory as in equation (32). Through field transformations the set of fields (A_μ, ϕ) is transformed to a set of new fields (B_μ, ρ, ξ). The old and new fields are related by

$$\begin{aligned} B_\mu &= A_\mu - \frac{1}{q}\partial_\mu \xi(x); \\ \phi(x) &= \rho(x) e^{i\xi(x)}, \end{aligned} \tag{40}$$

where B_μ(x) is a vector field and ξ(x) and ρ(x) are real scalar fields. ρ(x) can be chosen positive.

These transformations look a lot like the transformations that determined the gauge fixing in equation (35). They are however not the same. The difference

[23] Recent papers are Struyve [44], Masson and Wallet [33], Chernodub et al. [7] and Faddeev [12]. The gauge independent account is also discussed in Rubakov [37, section 6.1].

between the two transformations is that in the gauge fixing ξ(x) was a fixed field, but now ξ(x) is one of the variable fields. While earlier after the gauge fixing, the gauge transformation could no longer be applied to the fields, now the gauge is not fixed and the gauge transformations can be applied to the fields. Doing so, it can be seen what the effect of the U(1) transformations of $\phi(x)$ and $A_\mu(x)$ (equations (1) and (3)) on the fields is:

$$\begin{aligned} &B'_\mu(x) = A'_\mu(x) - \frac{1}{q}\partial_\mu \xi'(x) = A_\mu(x) + \frac{1}{q}\partial_\mu \alpha(x) - \frac{1}{q}\partial_\mu \xi'(x); \\ &e^{i\alpha(x)}\phi(x) = \phi'(x) = \rho'(x)e^{i\xi'(x)}. \end{aligned} \tag{41}$$

These transformations imply that $B_\mu(x)$ and $\rho(x)$ are invariant under U(1), while $\xi'(x) = \xi(x) + \alpha(x)$. This is clear if equation (41) is worked out further:

$$\begin{aligned} &B'_\mu(x) = A_\mu(x) - \frac{1}{q}\partial_\mu\left(\xi'(x) - \alpha(x)\right) = A_\mu - \frac{1}{q}\partial_\mu \xi(x) = B_\mu(x); \\ &\phi(x) = \rho(x)e^{i\xi(x)} = \rho'(x)e^{i\left(\xi'(x)-\alpha(x)\right)}. \end{aligned} \tag{42}$$

From the transformation $\xi'(x) = \xi(x) + \alpha(x)$ it follows that the field $\xi(x)$ is a pure gauge variable. It is the only one in the set of new fields that is not gauge invariant. The Lagrangian in equation (32) can be rewritten in terms of the new fields to give

$$\mathcal{L} = -B^{\mu\nu}B_{\mu\nu} + (\partial_\mu + iqB_\mu)\rho(\partial_\mu - iqB_\mu)\rho - m^2\rho^2 - \lambda\rho^4, \tag{43}$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The field $\xi(x)$ does not appear in this description; it has been factored out. All fields in this theory are now invariant under the U(1) gauge transformation; the U(1) symmetry has no grip on this theory.[24] Now the U(1) symmetry has been factored out, the ground state of the system is no longer degenerate. There is a unique ground state with $B_{\mu,0} = 0$ and $\rho_0 = v$. To describe perturbations around this ground state, the field $\rho(x)$ is rewritten as $\rho(x) = v + \eta(x)$, with $\eta(x)$ a variable field and $v = \sqrt{\frac{-\mu^2}{2\lambda}}$ as before. The Lagrangian that describes perturbations is now

$$\mathcal{L} = -B^{\mu\nu}B_{\mu\nu} + \partial_\mu\eta\partial^\mu\eta + \mu^2\eta^2 + q^2v^2B_\mu B^\mu + \text{ cubic and quartic terms,} \tag{44}$$

where only the quadratic part is shown explicitly. This Lagrangian is the same as in equation (36), section 5, obtained for the Higgs mechanism with spontaneous symmetry breaking. The Lagrangian contains a massive gauge boson $B_\mu(x)$ and one massive component of the Higgs field $\eta(x)$. The model is renormalizable; and this can be shown by the same method as renormalizability for the equivalent Lagrangian in equation (36) has been shown by 't Hooft.

It has been shown how a description of the Higgs mechanism can be given where gauge symmetry breaking plays no role. There are two reasons that this treatment seems to be so different from the treatment involving gauge symmetry breaking in section 5.1. The first one of course is that no gauge has been fixed, because the procedure works with a gauge independent Lagrangian. The second difference is the order in which the two steps in the treatment of the previous section are executed. The gauge symmetry seems to be spontaneously broken if first the vacuum expectation value of the field is analyzed, and after that the gauge is fixed as in section 5.1. These two steps could be done the other way around. If

[24] It might look like a discrete symmetry $\rho(x) \longrightarrow -\rho(x)$ still remains from the gauge symmetry, but this is not so. $\rho(x)$ is chosen to be positive in the field transformations equation (40), and thus does not allow for this transformation.

the gauge were fixed first, there would be a unique vacuum state and there would be no spontaneous symmetry breaking either.

Spontaneous breaking of the gauge symmetry is thus not a necessary step in the Higgs mechanism. Is the possibility of describing the Higgs mechanism as gauge symmetry breaking then still problematic? No, spontaneous gauge symmetry breaking is not connected to the description of physics, since the meaning of the symmetry breaking disappears once the physics comes into play and a gauge is fixed. The possibility of giving a gauge independent description of the Higgs mechanism underlines that.

Then if it is not spontaneous gauge symmetry breaking, what is the underlying feature of the Higgs mechanism? The approaches discussed have one crucial element in common: the non-zero vacuum expectation value of the Higgs field. A further conceptual study of the Higgs mechanism should focus on this. What does it mean for a field to have non-zero expectation value? Morrison [6, pp. 347 - 363] discusses this. What the Higgs mechanism misses is an analog of the BCS theory of superconductivity: there is no currently accepted "deep" explanation for the non-zero vacuum value of the Higgs field.

7 Conclusions

Local symmetries in a deterministic theory are necessarily gauge symmetries - they relate physically identical states. The spontaneous breaking of such a symmetry cannot have any physical consequences. Nevertheless, the Higgs mechanism is generally described as the spontaneous breaking of a local gauge symmetry. And the Higgs mechanism furnishes the mass generation of the W and Z gauge bosons in the electroweak theory: important physical consequences. This gives rise to an apparent conceptual problem.

To study this, spontaneous breaking of a local symmetry was compared to breaking of a global symmetry. It was seen that through the fixing of a gauge the Goldstone theorem of the global case breaks down. The possibility to fix a gauge by a local transformation marked the difference between global symmetry breaking and spontaneous symmetry breaking of a gauge symmetry as in the Higgs mechanism. That gauge fixing makes spontaneous gauge symmetry breaking so different from global symmetry breaking is not surprising: the gauge fixing breaks the symmetry that should be broken in spontaneous symmetry breaking.

By considering a gauge independent approach to the Higgs mechanism, it has been seen that gauge symmetry breaking is not an essential ingredient of the Higgs mechanism. Gauge symmetry breaking is a mathematical artifact from the redundancy in description related to a gauge symmetry, but is not directly responsible for any of the physics of the Higgs mechanism. It has been shown that the Higgs mechanism does not rely on spontaneous gauge symmetry breaking and so is not threatened by the absence of physical implications of the spontaneous breaking of a gauge symmetry. It is the non-zero vacuum expectation value of the Higgs field that has physical implications. The fundamental question that the Higgs mechanism does not answer, and which we leave open, is why and how the Higgs field has a non-zero vacuum expectation value.

Acknowledgements

I would like to thank Nazim Bouatta and Jeremy Butterfield for their valuable and detailed comments on this essay. I am also grateful for meetings with Nazim, that inspired the focus of this essay.

References

[1] P. W. Anderson. Plasmons, gauge invariance, and mass. *Phys. Rev.*, 130: 439–442, Apr 1963. doi: 10.1103/PhysRev.130.439.

[2] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Microscopic theory of superconductivity. *Phys. Rev.*, 106:162–164, Apr 1957. doi: 10.1103/PhysRev.106.162.

[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. Theory of superconductivity. *Phys. Rev.*, 108:1175–1204, Dec 1957. doi: 10.1103/PhysRev.108.1175.

[4] Jeremy Bernstein. Spontaneous symmetry breaking, gauge theories, the Higgs mechanism and all that. *Rev. Mod. Phys.*, 46:7–48, Jan 1974. doi: 10.1103/RevModPhys.46.7.

[5] H. A. Bethe. The electromagnetic shift of energy levels. *Phys. Rev.*, 72: 339–341, Aug 1947. doi: 10.1103/PhysRev.72.339.

[6] Katherine Brading and Elena Castellani, editors. *Symmetries in Physics; Philosophical reflections*. Cambridge University Press, 2003.

[7] M.N. Chernodub, Ludvig Faddeev, and Antti J. Niemi. Non-abelian supercurrents and de Sitter ground state in electroweak theory. *Journal of High Energy Physics*, 2008(12):014, 2008.

[8] M. Daniel and Claude-M. Viallet. The gauge fixing problem around classical solutions of the Yang-Mills theory. *Physics Letters B*, 76(4):458 – 460, 1978. ISSN 0370-2693. doi: 10.1016/0370-2693(78)90905-X.

[9] John Earman. Laws, symmetry, and symmetry breaking; invariance, conservation principles, and objectivity, November 2002.

[10] F. Englert and R. Brout. Broken symmetry and the mass of gauge vector mesons. *Phys. Rev. Lett.*, 13:321–323, Aug 1964. doi: 10.1103/PhysRevLett.13.321.

[11] Giampiero Esposito, Diego N. Pelliccia, and Francesco Zaccaria. Gribov problem for gauge theories: a pedagogical introduction. *Int.J.Geom.Meth.Mod.Phys.*, 1:423, 2004.

[12] L. Faddeev. An alternative interpretation of the Weinberg-Salam model. arXiv:0811.3311v2 [hep-th], 2008.

[13] E. Fermi. Versuch einer Theorie der β-Strahlen. i. *Zeitschrift f ur Physik A Hadrons and Nuclei*, 88:161–177, 1934. ISSN 0939-7922. 10.1007/BF01351864.

[14] R. P. Feynman. Space-time approach to quantum electrodynamics. *Phys. Rev.*, 76:769–789, Sep 1949. doi: 10.1103/PhysRev.76.769.

[15] Walter Gilbert. Broken symmetries and massless particles. *Phys. Rev. Lett.*, 12:713–714, Jun 1964. doi: 10.1103/PhysRevLett.12.713.

[16] Sheldon L. Glashow. Partial-symmetries of weak interactions. *Nuclear Physics*, 22(4):579 – 588, 1961. ISSN 0029-5582. doi: 10.1016/0029-5582(61)90469-2.

[17] Jeffrey Goldstone, Abdus Salam, and Steven Weinberg. Broken symmetries. *Phys. Rev.*, 127:965–970, Aug 1962. doi: 10.1103/PhysRev.127.965.

[18] L.P. Gor'kov. Microscopic derivation of the Ginzburg-Landau equations in the theory of superconducivity. *Soviet Physics JETP*, 36(9):1364–1367, Dec 1959.

[19] V.N. Gribov. Quantization of non-Abelian gauge theories. *Nuclear Physics B*, 139:1 – 19, 1978. ISSN 0550-3213. doi: 10.1016/0550-3213(78)90175-X.

[20] David Griffiths. *Introduction to Elementary Particles.* Wiley-VCH, 2008.

[21] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble. Global conservation laws and massless particles. *Phys. Rev. Lett.*, 13:585–587, Nov 1964. doi: 10.1103/PhysRevLett.13.585.

[22] F.J. Hasert et al. Observation of Neutrino Like Interactions Without Muon Or Electron in the Gargamelle Neutrino Experiment. *Phys.Lett.*, B46:138–140, 1973. doi: 10.1016/0370-2693(73)90499-1.

[23] Peter W. Higgs. Broken symmetries, massless particles and gauge fields. *Physics Letters*, 12(2):132 – 133, 1964. ISSN 0031-9163. doi: 10.1016/0031-9163(64)91136-9.

[24] Peter W. Higgs. Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.*, 13:508–509, Oct 1964. doi: 10.1103/PhysRevLett.13.508.

[25] Peter W. Higgs. Spontaneous symmetry breakdown without massless bosons. *Phys. Rev.*, 145:1156–1163, May 1966. doi: 10.1103/PhysRev.145.1156.

[26] Anton Ilderton, Martin Lavelle, and David McMullan. Symmetry breaking, conformal geometry and gauge invariance. *Journal of Physics A: Mathematical and Theoretical*, 43(31):312002, 2010.

[27] R. Jackiw, I. Muzinich, and C. Rebbi. Coulomb gauge description of large Yang-Mills fields. *Phys. Rev. D*, 17:1576–1582, Mar 1978. doi: 10.1103/PhysRevD.17.1576.

[28] T. W. B. Kibble. Symmetry breaking in non-abelian gauge theories. *Phys. Rev.*, 155:1554–1561, Mar 1967. doi: 10.1103/PhysRev.155.1554.

[29] Peter Kosso. The epistemology of spontaneously broken symmetries. *Synthese*, 122(3):pp. 359–376, 2000.

[30] P. Kusch and H. M. Foley. The magnetic moment of the electron. *Phys. Rev.*, 74:250–263, Aug 1948. doi: 10.1103/PhysRev.74.250.

[31] Holger Lyre. Does the Higgs mechanism exist?, June 2008. Penultimate version, forthcoming in "International Studies in the Philosophy of Science".

[32] Holger Lyre. The just-so Higgs story: a response to Adrian Wüthrich, 2010.

[33] Thierry Masson and Jean-Christophe Wallet. A remark on the spontaneous symmetry breaking mechanism in the standard model. arXiv:1001.1176v1 [hep-th], 2010.

[34] Y. Nambu and G. Jona-Lasinio. Dynamical model of elementary particles based on an analogy with superconductivity. i. *Phys. Rev.*, 122:345–358, Apr 1961. doi: 10.1103/PhysRev.122.345.

[35] H. Osborn. Part III lecture notes on the standard model, 2011.

[36] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to Quantum Field Theory*. Westview Press, 1995.

[37] Valery Rubakov. *Classical Theory of Gauge Fields*. Princeton University Press, 1999.

[38] Lewis H. Ryder. *Quantum Field Theory*. Cambridge University Press, 2 edition, 1996.

[39] Julian Schwinger. Quantum electrodynamics. I. A covariant formulation. *Phys. Rev.*, 74:1439–1461, Nov 1948. doi: 10.1103/PhysRev.74.1439.

[40] Julian Schwinger. On quantum-electrodynamics and the magnetic moment of the electron. *Phys. Rev.*, 73:416–417, Feb 1948. doi: 10.1103/PhysRev.73.416.

[41] Julian Schwinger. Gauge invariance and mass. *Phys. Rev.*, 125:397–398, Jan 1962. doi: 10.1103/PhysRev.125.397.

[42] I. M. Singer. Some remarks on the Gribov ambiguity. *Communications in Mathematical Physics*, 60:7–12, 1978. ISSN 0010-3616. 10.1007/BF01609471.

[43] Chris Smeenk. The elusive Higgs mechanism. *Philosophy of Science*, 73(5): 487–499, 2006.

[44] Ward Struyve. Gauge invariant accounts of the higgs mechanism. *Studies in History and Philosophy of Science Part B*, 42(4):226–236, 2011.

[45] G. 't Hooft. Renormalization of massless Yang-Mills fields. *Nuclear Physics B*, 33(1):173 – 199, 1971. ISSN 0550-3213. doi: 10.1016/0550-3213(71)90395-6.

[46] G. 't Hooft. Renormalizable lagrangians for massive Yang-Mills fields. *Nuclear Physics B*, 35(1):167 – 188, 1971. ISSN 0550-3213. doi: 10.1016/0550-3213(71)90139-8.

[47] G. 't Hooft, editor. *50 years of Yang-Mills theory*. World Scientific, 2005.

[48] G. 't Hooft. Renormalization and gauge invariance, Dec 2009. URL `http://www.staff.science.uu.nl/~hooft101/gthpub/GtH_Yukawa_06.pdf`.

[49] S. Tomonaga. On a relativistically invariant formulation of the quantum theory of wave fields. *Progress of Theoretical Physics*, 1(2):27–42, 1946. doi: 10.1143/PTP.1.27.

[50] S. Tomonaga and J. R. Oppenheimer. On infinite field reactions in quantum field theory. *Phys. Rev.*, 74:224–225, Jul 1948. doi: 10.1103/PhysRev.74.224.

[51] D. Tong. Quantum field theory, 2007. Lecture notes for University of Cambridge, Part III of the Mathematical Tripos.

[52] Steven Weinberg. A model of leptons. *Phys. Rev. Lett.*, 19:1264–1266, Nov 1967. doi: 10.1103/PhysRevLett.19.1264.

[53] Steven Weinberg. *The Quantum Theory of Fields I, Foundations*. Cambridge University Press, 1995.

[54] Steven Weinberg. *The Quantum Theory of Fields II, Modern Applications*. Cambridge University Press, 1996.

[55] Edward Witten. From superconductors and four-manifolds to weak interactions. *Bull. Amer. Math. Soc.*, 44:361–391, Jul 2007. doi: 10.1090/S0273-0979-07-01167-6.

[56] Adrian Wüthrich. Eating Goldstone bosons in a phase transition: A critical review of Lyre's analysis of the Higgs mechanism, January 2010.

[57] C. N. Yang and R. L. Mills. Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, 96:191–195, Oct 1954. doi: 10.1103/PhysRev.96.191.

[58] H. Yukawa. On the interaction of elementary particles. *Proc. Phys.-Math. Soc.*, 17:48–57, 1935.

[59] A. Zee. *Quantum Field Theory in a Nutshell*. Princeton University Press, 2003.