

The Ontology of Impossible Worlds

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Abstract The best arguments for possible worlds as states of affairs furnish us with equally good arguments for impossible worlds of the same sort. I argue for a theory of impossible worlds on which the impossible worlds correspond to maximal inconsistent classes of propositions. Three objections are rejected. In the final part of the paper, I present a menu of impossible worlds and explore some of their interesting formal properties.

1 Introduction The notion of impossible worlds has received relatively little attention in the philosophical literature. When they have been mentioned, it has often been in passing, as for example, in ([7], p. 7n.), where Lewis rejects the idea in a footnote. Of those pieces which devote a significant amount of space to the topic, nearly all focus on one or another of the uses to which impossible worlds may be put in, say, the semantics for relevant logic (see Routley [13]) or epistemic logic (see Hintikka [1]). In this paper I would like to address some questions of ontology. Are there such things as impossible worlds? If so, what are they like?

In [16], Yagisawa argues conditionally for a Lewis-style theory of impossible worlds. If the virtues of Lewis's theory of possible worlds give us good reasons to accept it, he says, then these same virtues give us equally good reasons to accept an "extended modal realism" which includes both possible and impossible worlds. The extended theory, like Lewis's, says that 'actual' is an indexical term, that there are nonactual objects, and that worlds are objects like us and our surroundings. In contrast, I will argue (also conditionally) for an actualist, abstractionist theory of impossible worlds. If we have good reasons to believe in the possible worlds of the actualist, abstractionist theory, then we have good reasons to believe in impossible worlds of the same sort. According to this theory, everything which exists, exists actually. Worlds, unlike us and our surroundings, are abstract objects. In particular, worlds are maximal states of affairs. Because of these features the theory to be presented may be regarded as a Plantinga-style theory of impossible worlds.¹

In what follows I assume that entailment is strict implication and that **S5** is the correct logic of "metaphysical" or "absolute" possibility. It is no part of my posi-

Received October 6, 1997; revised November 25, 1997

tion to claim that any impossibility is or might be actual or that any inconsistency is possible. In these respects the theory is quite conservative. Impossible world theory may not require these assumptions, but I leave the adjustment of the theory to contrary assumptions as an exercise for the reader. First, I present the theory; then three objections; then a survey of impossibilities.

2 *Arguing for impossible worlds* I want to argue for the existence of impossible worlds, though I suspect that many or most of those who believe in possible worlds already believe in impossible worlds, whether or not they have thought to call them by that name. On the view of possible worlds that I prefer, there are states of affairs.² Some of these, such as *an apple's being colored if red* and *San Diego's being warm*, are actual; these states of affairs obtain. Others, such as *the Axis powers' having won World War II*, are merely possible; they do not obtain, but it is possible that they obtain. Still others, such as *Paul's having squared the circle* or *the number 9's being a Caesar salad*, are impossible; they do not possibly obtain. Impossible worlds constitute a subclass of this last class of states of affairs, namely, the impossible states of affairs which are maximal (in a sense we shall discuss later).

For philosophers who regard possible worlds as states of affairs, then, very little needs to be said to establish the existence of impossible worlds; it is simply a matter of pointing out which objects are deserving of the title. Nevertheless, it may be worthwhile to review the principal arguments that have been offered for the existence of possible worlds, for in each case there are remarkably similar parallel arguments for impossible worlds. Both of the arguments that I will consider have been put forward by Lewis. I will call them the argument from ways and the argument from utility.

3 *The argument from ways* I believe that, besides the wide variety of possible worlds, there are impossible worlds as well. If an argument is wanted, it is this. It is uncontroversially true that in certain respects things could not have been otherwise than they are. But what does this mean? Ordinary language permits the paraphrase: there are many ways things could not have been. On the face of it, this is an existential quantification. It says that there exist many entities of a certain description, to wit 'ways things could not have been'. Taking this statement at face value, there exist entities that might be called 'ways things could not have been'. In keeping with the terminology used for their possible analogs, I prefer to call them 'impossible worlds'.³

The above argument is an adapted quotation of Lewis's argument for possible worlds in [5], and I think the argument is a good one. However, I take both Lewis's argument and the adaptation to establish something rather different than what Lewis takes them to establish, and in two ways. First, both are arguments for certain kinds of abstract objects, not for concreta that are spatiotemporally unrelated to us. Second, both are arguments for the existence of certain states of affairs, not only for the existence of worlds, which must meet a maximality requirement of some sort.

We *do* speak of ways things couldn't be; told that some object is black and white and red all over, for example, we might say, "It couldn't be *that* way!" Someone will object that ordinary language is ambivalent about this manner of speaking, since we might just as well respond, "There is *no* way things could be like *that*!" But the ambivalence of ordinary language is not a reason to reject the argument. We make the

same sort of remark with respect to logically possible situations: “There is no way I can make it to the church on time” when the only obstacle is my being ten miles away two minutes before the deadline. In both cases, the denial of the existential quantification is implicitly qualified. There is no way I can make it to the church on time; that is, no way compatible with all of my circumstances and the laws of physics, metaphysics, and logic that I can make it to the church on time. There is no way any object can be black and white and red all over; that is, no way compatible with certain necessary truths about color and the laws of logic, no *possible* way. But there is a way things could *not* be, such as a situation in which white is a texture and red is a flavor. Ordinary language permits this way of speaking, just as it permits talk of ways things could be. And if language permits it, we have the prima facie existential quantification we need.

Is there any reason to take the prima facie existential quantification at face value in the argument for possible states of affairs but not in the argument for impossible states of affairs? As Lewis says,

I do not make it an inviolable principle to take seeming existential quantifications in ordinary language at face value. But I do recognize a presumption in favor of taking sentences at their face value, unless (1) taking them at face value is known to lead to trouble, and (2) taking them in some other way is known not to. ([5], p. 84)

So far as I can tell, the thesis that there are impossible worlds does not lead to trouble. The thesis is surprising (or at any rate people are sometimes surprised by it), world-talk having traded exclusively in possible worlds for so long. But none of the objections that one might initially be inclined to array against the thesis—nor any of the others that I have come across—are at all persuasive on a second glance.

Are there other ways of taking our ordinary discourse that are known not to lead to trouble? Earlier in [5] Lewis says this part of our modal discourse may be founded on a confused fantasy (p. 24). But suppose that talk about ways things could be is the literal truth of the matter. Then, I think, rejection of impossible worlds does lead to trouble. Why should one think that talk about ways things couldn’t be is founded on a confused fantasy? Is there a general principle of ontology which would justify our construing these similar parts of our modal language in such dissimilar ways?

There are strong analogies between states of affairs and propositions. Both states of affairs and propositions are representational. States of affairs are ways things could or couldn’t be; propositions describe things as being in a certain way. Now should we exclude representations of the impossible from our ontologies? Necessarily false propositions—impossible propositions, we might say—are not usually regarded as ontologically suspect (at any rate, no more suspect than other propositions). It is no strike against them that they cannot possibly be true. Likewise, there are properties that cannot be exemplified, such as *being the greatest number*, *being green and colorless*, and *nonexistence*. Why should it count against a state of affairs that it cannot possibly obtain? The metaphysical possibility of instantiation seems no more relevant than the nomological possibility of instantiation or the actuality of instantiation.

Lewis’s argument from ways (as I interpret it) hints that the existence of states of affairs may be the sort of thing for which we might not even require an argument. Perhaps this is right: we seem to have a familiarity with states of affairs that makes it

tolerably clear that there are such things. The argument from ways may be viewed as a description of our familiarity and the manner in which our language reflects it. This same familiarity also makes it tolerably clear that some of the examples of states of affairs are of this sort: *9's being even, motherhood's being transitive, something's being identical to something with different properties*. These, of course, are impossible states of affairs. There are also the likes of *there being a private language, something's coming to be from nothing at all, someone's discovering a unicorn, an iron ball's having all the same nonrelational properties as a distinct ball, Peter's freely refraining from an action he strongly desires to do and has no countervailing desires not to do*. These are states of affairs which some philosophers have thought to be impossible, though they are less clearly impossible than the foregoing examples. Such states of affairs help us see that whether or not a state of affairs possibly obtains has no bearing on whether or not there is such a thing. The latter examples are quite obviously examples of states of affairs. We do not need to stop and ask whether it is possible that there be a private language before we can say whether or not there is such a state of affairs as *there being a private language*.

4 The argument from utility Lewis puts the second argument in Quinean terms: improvements in unity and economy of ideology are sometimes worth controversial ontology. The economy and power of set theory gives mathematicians (and the rest of us) good reason to believe in sets. The cost of believing in sets is well worth the benefits for one's total theory. Says Lewis, so it is with possible worlds. The benefits for our understanding of necessity and possibility and for analyses of numerous objects of philosophical inquiry in nearly all subfields of philosophy make the cost of believing in possible worlds worth paying. Weighing the costs of an ontology against its benefits for ideology is a matter of judgment, but in this case, Lewis says, the price is right, even if less obviously so than in the case of sets.

The argument from utility is significantly less impressive than the argument from ways, I think. It is not clear that mathematicians (or anyone else) believe in sets because of the impressive results of set theory and the discovery that all of mathematics can be modeled by set theory plus definitional extensions. Didn't people believe in sets well before these results became accepted early this century, and weren't they right to do so? In both the case of sets and worlds, utility might have relatively little to do with one's reasons for accepting controversial ontology. This is likely to be so in the case of someone who thinks that the argument from ways is a strong argument. The argument stands on its own if it is a strong argument, and it gives us good reason to believe in worlds (or rather, states of affairs) whether or not anyone has demonstrated that they may be used in any enlightening philosophical analysis. The argument obviates considerations of cost, since the conclusion of a sound argument must be accepted whatever the cost.

So I do not think that the argument from utility is a sound argument. However it is not my primary goal here to discredit the argument but to point out that there is again a parallel argument for impossible worlds. Those who are inclined to accept the argument from utility and to think that it provides an important reason for believing in possible worlds will find a very similar argument for impossible worlds, since impossible worlds, like possible worlds, bring the benefits of unity and analytic power

to our total theory.

Unfortunately, I cannot substantiate that claim in this paper. Other articles in this journal may help accomplish the task. In any event, I believe that impossible worlds show great philosophical promise, particularly in the semantics of counterfactual logic, epistemic and doxastic logic, and relevant logic. In some cases impossible worlds enable us to give world-based analyses where no possible worlds analysis could be successful. I hope that those persuaded by the argument from utility will agree.

5 Truth in a state of affairs In everyday discourse, it is common to speak of what is *true in* a given state of affairs or situation ('situation' and various other terms often being synonymous with 'state of affairs'). We may say, "In this case we have three options"—awkwardly paraphrased, "That we have three options is true in this state of affairs." We may say, more naturally, "It is true in most situations that shouting at people will only make them angry." Frequently the mention of truth is suppressed: "In his situation there is no escape." Sometimes the state of affairs is treated grammatically as a place. Thus Kripke: "What do we mean when we say 'In some other possible world I would not have given this lecture today?' We just imagine the situation *where* I didn't decide to give this lecture or decided to give it on some other day" [emphasis mine] ([3], p. 44).

This mode of speech has been incorporated into our more technical possible worlds talk. In his famous "Semantical considerations on modal logic" [4] Kripke appears to assume as a matter of course that certain propositions are true in (and possible in) the various possible worlds, and accordingly he defines a model as a binary function $\varphi(P, \mathbf{H})$ where P is a propositional variable and \mathbf{H} varies over the elements of a model structure which are to be thought intuitively as possible worlds. φ assigns a truth value T or F to each proposition-world pair, and so each model represents each proposition as being true in or false in each possible world.

Others have taken some steps toward explicating this notion of truth in a possible world. Plantinga, for example, says it is part of both what he calls the "Canonical Conception" and the actualist conception of possible worlds that propositions are true in possible worlds, and he offers this analysis of truth in a state of affairs: "A proposition p is true in a state of affairs S if it is not possible that S be actual and p be false . . ." ([11], p. 259). For Plantinga, a proposition is true in a state of affairs just in case the proposition is entailed by that state of affairs, in a naturally extended sense of the term 'entailed'. (We will see a difficulty with this analysis below.) Lewis, in contrast, remarks that the phrase 'at W ' restricts the domains of quantifiers in its scope, and so behaves much like the modifier 'in Australia'. For Lewis, truth at a world is simply a species of truth, a species whose subject matter concerns only the contents of the world in question (in most cases).

These items in the philosophical literature are, I think, attempts to specify or make use of a pretheoretical notion. The pretheory and the language suggest that propositions may stand in a certain relation to states of affairs, a relation of being "true in." How seriously should we take this suggestion? Here the dialectical position is reminiscent of the position in which the argument from ways is given. On the face of it, there is a relation which holds between propositions and states of affairs

and which our “true in” language describes. We needn’t always judge that our usual mode of speech reflects the metaphysical truth of the matter; nonetheless, there is a presumption in favor of that judgment.

I think that our “true in” language springs from an intuitive grasp of states of affairs, propositions, and the relations between them. Two of the things this grasp tells us are the following.

1. There are *many* impossible state of affairs; in particular, there is not only one.
2. States of affairs are to be individuated according to what is true in them, by what we might call *propositional content*.

Thesis 1 is as evident as the thesis that there are many necessarily false propositions. It is quite clear that *Socrates’ being taller than himself* is not the same state of affairs as *addition’s being noncommutative*. Like the propositions *Socrates is taller than himself* and *addition is noncommutative*, the states of affairs are not about the same things. There is an intentional difference between the two.

Note that we may regard (1) as a consequence of the adapted argument from ways. There are many ways things couldn’t be and thus, many impossible states of affairs.

Regarding thesis 2, worlds and other states of affairs, we have been taught, are stipulated, not discovered with powerful telescopes ([3], p. 44). When we do attempt to specify a state of affairs, we try to characterize its content. It is difficult to say just what content is without recourse to metaphors. Content has to do with what a state of affairs contains, what it’s about, what it involves. This is a rough characterization, but it is clear enough that whatever individuates states of affairs must be something in this conceptual neighborhood. Could it be that two different states of affairs have exactly the same content? If two states of affairs share their content, what is left that might distinguish one from the other? (An obvious alternative theory individuates states of affairs according to their logical extension; if, necessarily, state of affairs *A* obtains if and only if state of affairs *B* obtains, then *A* is identical to *B*. But this theory has the consequence that there is only one impossible state of affairs, and so is unacceptable.)

The principal device we have for specifying content is our “true in” language. Indeed, “true in” locutions seem to be designed for that very purpose. Kripke, speaking of how we stipulate possible worlds, says that “A possible world is *given by the descriptive conditions we associate with it*” [Kripke’s emphasis] ([3], p. 44). Here he is speaking of the propositions true in a given world. It seems fitting, then, to call the content of a state of affairs a propositional content. It is a content given by a certain class of propositions, those true in it. If we call the class of propositions true in a state of affairs *S* the *book* on *S* (writing ‘*B_S*’), we may say that states of affairs are to be individuated by their books.

Individuation by propositional content coincides with individuation by extension across possible worlds if it is assumed that the books on states of affairs are closed under entailment. Given both (1) and (2), however, we may conclude that not all books are closed under entailment. For if they were, the book on any impossible state of affairs (any state of affairs such that it is not possible for all the members of its book to be true) would contain every proposition. But then every impossible state of affairs would have the same book, the class of all propositions, and by (2), there

would be only one impossible state of affairs, contrary to thesis 1.

Here, then, we find a swift reply to those who object to the notion of impossible worlds—or to the usefulness of the notion—saying that because a contradiction entails everything, there is at most one impossible world. We may grant that each proposition is entailed by a contradiction. But we deny that all books are closed under entailment. Some books are closed under entailment, for example, the books of possible worlds. These books have closure because they are consistent and maximal. Other books, however, may lack either consistency, as in the case of impossible states of affairs, or maximality, as in the case of states of affairs not complete enough to be worlds.

Another consequence of theses 1 and 2 is that the Plantingean analysis of truth in a state of affairs fails. If a proposition is true in a state of affairs just in case it is, in the relevant sense, entailed by that state of affairs, then every proposition is true in every impossible state of affairs. Then, by (2), there is only one impossible state of affairs, contrary to (1).

If the Plantingean analysis fails, how shall we characterize the “true in” relation? I do not propose to give necessary and sufficient conditions of P 's being true in S . Of course, it is nice to have such necessary and sufficient conditions to attach to our philosophical notions when we can, but we must not let clarity trump accuracy. If the only analyses available are faulty, then it is better to make do with an unanalyzed notion until a correct analysis is found than to endorse a false account as the truth. (Often in philosophical and, especially, scientific inquiry we quite properly make false simplifying assumptions in order to aid our investigation. Such cases are unlike the present case, where the flaw in the proposed analysis is directly relevant to the questions at hand and would yield faulty results rather than harmlessly simplifying our inquiry.)

Fortunately, there are several features of the “true in” relation which enable us to locate the notion nicely. Two of these features are described above. We may also give a *partial* analysis of it. The condition of a state of affairs S 's entailing a proposition P is not a necessary and sufficient condition of P 's being true in S ; however, the condition is a necessary one. Whenever P is true in S , S cannot obtain unless P is true. We may also note that the modal status of a state of affairs is correlated to the status of its book. A state of affairs is possible if and only if the conjunction of propositions true in it is possibly true, actual if and only if that conjunction is true, and necessary if and only if the conjunction is necessarily true.

Given this notion of truth in a state of affairs we may at last define the term ‘world’. A class of propositions is maximal just in case, for every proposition P , either P or its negation $\sim P$ (and perhaps both) belongs to the class. A state of affairs is maximal if and only if the propositions true in it form a maximal class. A maximal state of affairs is called a ‘world’.

6 Restrictions on books? If the above is correct, then the books on many states of affairs contain propositions which contradict each other. May any collection of propositions whatsoever be the book on a state of affairs, or are there restrictions governing which collections may serve as books? Perhaps we should insist that books are nonempty, since some proposition or other is true in every state of affairs. Even

the books of such “null state of affairs” candidates as *nothing’s existing* and *nothing’s being true* contain (at least) the propositions *nothing exists* and *nothing is true*, respectively. Are books restricted in other ways as well?

The two ways of answering this question lead to two types of impossible world theory. A negative answer yields a theory on which every nonempty collection of propositions is the book on some state of affairs or another, even those collections which are nothing more than a haphazard assortment of propositions with no unifying principle at all. A positive answer cannot require that books be consistent collections of propositions, but it may posit closure under some version of relevant implication, or closure under conjunction, or under modus ponens. A theory of this type might also require, for example, that every necessary truth be true in every state of affairs.

I think that the negative answer is to be preferred. We have seen that not everything is true in each impossible state of affairs, and so states of affairs may fail to address certain issues (as Perry puts it), even when the answers to those issues are entailed by what is true in that state of affairs. If states of affairs may omit these entailed propositions from their books, books are not governed by the rules of logic in the way that we might have expected. Then it is very difficult to see why we should think them governed by a rule of closure under conjunction, or by any similar rule. Some may find it intuitively obvious that if propositions P and Q are true in state of affairs S , then so is $P \& Q$, but I suspect that this is generally a disguised form of the thought that all states of affairs are closed under entailment. One might hold that states of affairs are closed under conjunction but not entailment, I suppose, but it is hard to see what might motivate the thought that there is closure under conjunction besides the thought that there is closure under entailment. The obviousness of the entailment from P and Q to $P \& Q$ is indeed striking, but it does not follow that the latter is true in every state of affairs in which the former are true.

One reason that might be proposed for such restrictions goes like this: “To deny that every state of affairs is closed under conjunction is to misunderstand the referent of a phrase like ‘the state of affairs in which Jack is nimble and in which Jack is quick’. When we use such a phrase, we clearly mean to bring to our attention a situation in which the conjunctive proposition *Jack is nimble and Jack is quick* is true, and in which the propositions *Jack is nimble* and *Jack is quick* are true. One would never raise for consideration a state of affairs in which the latter two propositions are true and remain agnostic as to whether the conjunctive proposition would be true in that case, for clearly it would.” Similar arguments could be given for claims that books are subject to some other restriction, as those mentioned above.

What we have here, at best, is an argument for the conclusion that the states of affairs *that we normally bring to mind* are closed under conjunction. And perhaps this much is true. Let J be the state of affairs whose book B_J contains *Jack is nimble* and *Jack is quick* and nothing else. It is questionable whether, in the normal course of events, we ever refer to or even consider such a sparse state of affairs as J . More often, we consider states of affairs which come much closer to being complete. We think of states of affairs which are agnostic regarding, say, the price of rice wine in China, but which do make true the conjunctions of other propositions true in them. But of course the fact, if it is a fact, that one rarely or never considers states of affairs like J does not give us any reason at all to suppose that there are no such states of

affairs. There may yet be states of affairs that are incomplete in such a way as to be agnostic even about conjunctions of its book's members. Such states of affairs are like the author who believes each claim in her book, but, thinking it likely that she has erred at some point or other, does not believe the conjunction of those claims.

Compare the above view of states of affairs to theories which say that propositions are sets of possible worlds: most sets of possible worlds will be rather motley collections, very much unlike any proposition that arises in normal conversation. (However, it is not too hard to see that for every set of possible worlds some proposition is true in exactly those possible worlds. If our set is $\{W_1, W_2, W_3, \dots\}$, one proposition true in those possible worlds is W_1 is actual or W_2 is actual or W_3 is actual....) Likewise we should expect that the states of affairs that normally come to mind comprise only a tiny subclass of the states of affairs, and that they may differ significantly from other states of affairs, such as the emaciated J .

So it is hard to see why books in general should be thought subject to restrictive rules. And in fact there is significant advantage in supposing that they are not. Better than others, a theory of unrestricted books accommodates conflicting intuitions about what we refer to when we speak of a state of affairs of one description or another. One may think that 'Clinton's winning the election' refers to a sparse state of affairs, such as the one whose book is the single-membered $\{Clinton\}$ wins the election}. Or one might think that this same phrase refers to a rich state of affairs—if not a world, then at least one whose book includes such items as *Dole lost the election*, *most eligible Americans voted*, and *Gore became vice-president*. One might think that this rich state of affairs is not closed under the rules of inductive inference (perhaps *Clinton is an American* is true in it, but *Clinton probably voted* is not), though others may insist that the phrase refers to a state of affairs whose book is closed under this rule (and maybe others). The proposed theory both accommodates and explains these views; there are states of affairs of each sort mentioned, and the language we use to refer to them is almost always ambiguous. We refer to a rich state of affairs on one occasion, a sparse one on another, and something in between on a third. We need not choose between them, for each of them exists.

Some contributors to this issue propose that the only impossible worlds are those governed by some paraconsistent logic, though there is disagreement about exactly which logic should shape our ontology on this point. A theory of unrestricted books admits many more worlds than any of these. Whenever it is useful to consider only the worlds governed by one paraconsistent logic or another, the advocate of unrestricted books may consider them as a subclass of all the worlds, just as the impossible worlds theorist may help him- or herself to the applications of a theory which posits only possible worlds. Thus the present theory of impossible worlds has at least as much utility as any other.

7 *Lewis's objection* Lewis rejects impossible worlds:

For comparison, suppose travellers told of a place in this world—a marvellous mountain, far away in the bush—where contradictions are true. Allegedly we have truths of the form 'On the mountain both P and not P '. But if 'on the mountain' is a restricting modifier, which works by limiting domains of implicit and explicit quantification to a certain part of all that is, then it has no

effect on the truth-functional connectives. Then the order of modifier and connective makes no difference . . . [T]he alleged truth ‘On the mountain both P and not P ’ is equivalent to the overt contradiction ‘On the mountain P , and not: on the mountain P ’ . . . But there is no subject matter, however marvellous, about which you can tell the truth by contradicting yourself. Therefore there is no mountain where contradictions are true. ([7], p. 7n.)

Lewis goes on to say that ‘at so-and-so world’ is indeed a restricting modifier, unlike ‘in such-and-such story’, since worlds are like the actual world, not like stories.

It is this last point that is of interest here. Lewis’s reasons for rejecting impossible worlds stem from his concretism, that is, his view that worlds are concrete objects much like us and our surroundings. Other worlds differ from the actual world (which he thinks is the same thing as us and our surroundings) in a wide variety of facts, but not in kind.

Worlds, I think, are maximal states of affairs, and states of affairs are not concrete objects but abstract ones. Hence I think that worlds are more like stories than mountains with respect to how the modifier ‘at so-and-so world’ ought to be taken. We noted earlier that states of affairs, like propositions, are in some sense representational. Accurately or otherwise, they represent things as having certain properties and standing in certain relations. This representing of things is a feature which states of affairs and stories have in common, and it is this feature which makes it appropriate to use the modifier ‘at so-and-so world’ much as we use ‘in such-and-such story’. ‘In state of affairs S , $\sim P$ ’ is not equivalent to ‘Not: in state of affairs S , P ’. We might be misled by the special case of possible worlds, since for any possible world W , ‘in W , $\sim P$ ’ is true just in case ‘Not: in W , P ’ is true. But some states of affairs are not maximal; one might be silent about both P and $\sim P$, so that the equivalence fails. Or a state of affairs might represent both P and $\sim P$ as true, and so, again, the equivalence fails. Modifiers like ‘in state of affairs S ’ do operate like ‘in such-and-such story’. And as Lewis himself comments, “If worlds were like stories or story-tellers, there would indeed be room for worlds according to which contradictions are true” ([7], p. 7n.).

Since Lewis’s objection to impossible worlds is aimed only at concretist theories like Yagisawa’s, it is not an objection which my own abstractionist theory needs to refute.⁴ I am quite willing to grant its force against concretist theories of impossible worlds. It will not help the concretist to protest that one only needs to be consistent when telling the truth about the goings on of possible worlds. If a contradiction is true in some world, then some contradiction with completely unrestricted quantifiers is true in every world, the actual world included.⁵ If somewhere in W an object both has property F and does not have property F , then somewhere an object both has F and does not have F . Lewis points out that if a contradiction is true anywhere, then a contradiction is true hereabouts. And a contradiction being true in the actual world is a *reductio* of the concretist theory.

As if this criticism were not powerful enough, I would like to add an objection of my own. If there are impossible worlds, then some world W does not represent itself as concrete. Let us say that none of the propositions which suggest that W is concrete (*W is concrete*, *W has mass*, *W is not an abstract object*...) are true in W , and their negations are. Could such a world be a concrete object? The answer hinges on

how concrete worlds represent propositions as true. If Lewis's usual method applies here and a proposition is true at W just in case it is true when we quantify only over things in W , then W represents itself as concrete if and only if W is concrete (quantifiers restricted to things in W). Here the quantifier restrictions do very little work. If W is concrete (quantifiers restricted to things in W), then W is concrete, and vice versa. By hypothesis, W does not represent itself as concrete, so W is not concrete. If representation does work this way, then any theory according to which all worlds are concrete is inconsistent. (Not all propositions are to be evaluated at a world simply by evaluating the proposition with its quantifiers restricted to things in that world, but W is *abstract*, which concerns only W itself, is not among the exceptions.)

Must concrete worlds represent in the manner described above? For the concretist, the thing which represents Humphrey as waving is a person very much like Humphrey, waving. How might such a thing represent? Lewis examines several possibilities and concludes that the only approach without serious flaws is counterpart theory, according to which an other-worldly person represents Humphrey as waving by being a waving counterpart of Humphrey. Something counts as a counterpart of Humphrey if it is sufficiently similar to him in important respects, whatever these may be.

What counterpart theory explains is *de re* representation, representation of an object, such as Humphrey, as having some property, such as the property of waving. A world represents *de re* of itself that it is concrete just in case the counterpart of that world in that world is concrete. Since the counterpart of a world in that world is just that world itself, a world represents itself as concrete if and only if it is concrete. So counterpart theory gives us the same result as thinking about representation in terms of restriction of quantifiers (as we would expect, since Lewis endorses both).

Unless concrete worlds represent in a manner completely unlike the approaches discussed by Lewis, the above argument against concrete impossible worlds is successful. Barring a successful and hitherto unheard of theory of representation, concrete worlds must represent in a way that is not compatible with the theory that all worlds, possible and impossible, are concrete. The Achilles' heel of a concretist theory of impossible worlds is the fact that there are certain things which concrete worlds cannot represent inaccurately: the concreteness of worlds, for example, and other facts, such as those regarding what occurs at other worlds, or certain truths about whatever transworld objects there would be. In contrast, if worlds are thought to be abstract, there is nothing to prevent inaccurate representation on any topic whatsoever. It might be true in a world W that it is concrete (the proposition W is concrete might belong to its book B_W), despite the fact that W is abstract and not concrete. Put another way, a theory of abstract impossible worlds allows divergence between what is true *about* a world and what is true *in* a world, whereas on a concretist theory these must coincide. In sum, we may agree with Lewis's conclusions: a concretist theory of impossible worlds is not viable, though there is nothing to prevent the abstractionist from recognizing such things.

8 *The analysis of possibility* Another objection to the notion of impossible worlds goes along these lines. "The view that there are possible worlds but not impossible worlds ('PWO' for 'possible worlds only') gives us a nice account of modality: ne-

cessity is truth in *all* worlds and possibility is truth in *any* world. The view that there are possible and impossible worlds ('P&IW') says that necessity is truth in all *possible* worlds. But this view raises a question. What makes the possible ones possible? We can't say without giving some independent account of possibility. All we can say is that possibility is truth in some of a certain collection of worlds; but this criterion is completely uninformative. We are left without any explication of the notion of possibility. So P&IW makes a mystery of modality. If there are only possible worlds, however, the question 'What makes this world possible?' does not arise."⁶

The objection alleges that P&IW raises certain questions about modality which it ought to answer. In particular, it ought to answer the question 'Why are possible worlds possible?' and answer it in an informative way. I think what the objection really means to require of a world theory is an *analysis* of modality. That is, it assumes that a theory of worlds must provide necessary and sufficient conditions for the necessity, possibility, and so forth, of propositions without making any uneliminable use of those notions. It is in this sense that the sentence 'A bachelor is an unmarried man' gives an analysis of the notion of bachelor in terms of the notions of man and of being unmarried. The analysis of modality, it is assumed, is what enables us to give an informative answer to the question of what makes a given world possibly obtain. The answer will take the form 'The satisfaction of condition *C*', where condition *C*, taking its cue from the analysis, will make no use of modal terms.

Lewis's account is an example of a reductive analysis of modality. Possible worlds, he says, are spatiotemporally isolated objects, objects that stand in spatiotemporal relations only with their parts. A proposition's being possibly true is thus to be understood as that proposition's being true in some spatiotemporally isolated object, which in turn is to be understood (in typical cases) as being true when the ranges of the quantifiers involved in the proposition are restricted to some spatiotemporally isolated object. Thus we have an attempt (unsuccessful, in my view) to reduce modality to the notions of spatiotemporal relatedness, truth, object, and so on.

However, not all possible world accounts purport to provide an analysis of modality. An ontology of possible states of affairs, for example, might make no attempt to explain what a possible state of affairs is without use of the notions of possibility, necessity, or some other modal term. In fact, it is rather commonly thought that any such attempt would be futile because possibility, necessity, and their ilk, form, as it is said, a tight circle of interrelated modal notions, none of which can be properly analyzed without recourse to some element of the circle. If this is so, we may point out useful relations between modal notions (e.g., whatever is possibly true is not necessarily false, and vice versa), but there is no *more* informative analysis of modality to be found.

So is an analysis of modality in nonmodal terms a *sine qua non* for P&IW or not? The objector assumes so, but does not argue for this claim. It is at least plausible that there is no further explication. Analysis must come to an end somewhere, and our failure to produce such an analysis so far (Lewis notwithstanding) gives us some reason to suspect that this is the place. At the very least, some argument for the necessity of a more informative analysis would have to be given before we had a substantial objection here. As it currently stands, the objection merely assumes a premise denied by many modal theorists, and which may well be false.

In any event, a PWO theory which does not actually supply an informative analysis also fails to meet the requirements of the objection. For the question ‘Why is world W possible?’ may also be asked of PWO. If the proposed answer is that W is possible because it exists, we may ask why W exists. One who holds PWO will say that ‘a situation in which Mars is colonized’ succeeds in referring to various entities but that ‘a situation in which Mars is a divisor of 7’ does not. What accounts for this difference? In order to meet the objection, the proponent of PWO must answer the question without any uneliminable use of modal terms. Otherwise she, too, “makes a mystery of modality.”

Conceivably those who say that modal notions form an irreducible circle are mistaken and there exists an answer to the question. But if so, we do not yet know what that answer is. Until it is shown that there is a more informative analysis of modality (and, furthermore, one that cannot be used by the impossible worlds theorist to explain the difference between possible and impossible worlds) the objection puts PWO in a position no better than P&IW. The advocate of impossible worlds, then, has little to fear from this worry.

9 The fine-grainedness objection Yagisawa mentions an interesting objection against his Lewis-style theory of impossible worlds. Part of the motivation for adopting Lewisian (i.e., concrete) possible worlds to begin with, he says, is the resources this gives us for an extensionalist theory of properties. Any theory that identifies a property with the objects that actually instantiate it falls to familiar criticisms. *Having been born with a heart* and *having been born with a kidney* are distinct properties with the same instantiations, so such a theory is too coarse-grained. Concrete possible worlds allow us to distinguish these properties by the possibilities that instantiate them, their extensions in all possible worlds, since it is possible that something be born with a heart but not a kidney.

We naturally hope along similar lines, Yagisawa continues, to use impossible worlds to distinguish between distinct but necessarily coextensive properties such as triangularity and trilaterality. Some impossibilia will have triangularity but not trilaterality, and other impossibilia will have trilaterality but not triangularity, and so the two are distinct. But isn’t such a proposal *too* fine-grained?

It even appears that no property is ever identical with any property whatever, according to the above proposal. For any property P and any property Q , either it is possible for P and Q not to be coextensive or it is impossible. If it is possible, there is a possible world where P and Q are not coextensive. If it is impossible, there is an impossible world where P and Q are not coextensive. Either way, the set of all possibilities and impossibilities having P is different than the set of all possibilities and impossibilities having Q . Therefore, according to the above proposal, P and Q are not the same property. This is true for any P and Q whatsoever, including P and P . So according to the proposal, no property is the same as any property, including itself! This is certainly an unwelcome consequence of any proposal. ([16], p. 195)

Unwelcome indeed! Does this objection weigh against an actualist, abstractionist theory of impossible worlds as well?

First of all, we may note that the objection objects not to impossible worlds per se but to a theory of properties formed in the wider context of a theory of impossible

worlds. Conceivably, it could turn out that the theory of properties fails but that there are impossible worlds nonetheless.

Second, it is no part of the actualist program to promote an extensionalist theory of properties. According to the actualist there are no impossible or merely possible entities that might fill out the extensions of properties as the extensionalist theory requires; nothing belongs to the “set of possibilia and impossibilia having *P*” but what actually has *P*. As it stands, then, the objection does not apply to actualist theories.

But suppose we set aside these points and ask whether the property theories that are most natural for the actualist friend of impossible worlds are susceptible to a similar objection. If so, then perhaps impossible worlds are not quite as useful as we would have thought. (This isn’t very impressive as an objection to impossible worlds, I realize, but discussion of it will be instructive.)

How might the actualist take advantage of impossible worlds when faced with the problem of distinguishing between different but necessarily coextensive properties? The natural approach is to duplicate the identity conditions suggested by the extensionalist view without making reference to nonactual individuals. So one might say that property *P* is identical to property *Q* just in case the following biconditional holds.

- (IC) For every possible or impossible world *W*, an entity *X* has *P* in *W* if and only if *X* has *Q* in *W*.

For the actualist, an entity *X* has property *P* in *W* whenever the proposition *X* has *P* is true in *W*.

The objection could be fitted for this account in this way. For any properties *P* and *Q*, there is an impossible world (if not a possible world) such that some *X* has *P* in *W* but *X* does not have *Q* in *W*. But then IC fails and *P* and *Q*, whatever they are, are distinct—even if *P* is *Q*.

The actualist may reply that IC does hold. For suppose that both ‘*P*’ and ‘*Q*’ are names of redness. Then there is indeed some world in which some *X* has *P* and lacks *Q*, which is to say there is some world in which *X* has redness and lacks redness. Now if *X* both has redness and lacks redness, then *X* has *P* and lacks *P*, and likewise *X* has *Q* and lacks *Q*. But then the biconditional IC does hold, since *X* has *P* in *W* and *X* has *Q* in *W*.

Yagisawa rejects a similar line of argument in the context of his extensionalist theory. The quoted passage continues:

It is a mistake to maintain that the above proposal does not really have this unwelcome consequence, by arguing as follows: “It is not true that the set of all possibilia and impossibilia having *P* is different from the set of all possibilia and impossibilia having *P*. It is certainly true that in some impossible world an object has *P* and does not have *P* at the same time. But such a world is not a world in which *P* and *P* are not coextensive; the extension of *P* contains all objects that have *P*, including those that have *P* and do not have *P* at the same time.” This is a mistake because since it is impossible for *P* not to be coextensive with *P*, it follows, on the extended modal realism, that there is an impossible world in which *P* is not coextensive with *P*; such a world is one in which the extension of *P* is not identical with the extension of *P*. Such a world is more than a world which merely contains something that is and is not *P*; it is a world in

which the law of identity fails for the extension of P . Thus the above, natural proposal cannot be sustained. ([16], p. 195)

According to an actualist theory, too, every proposition is true in some impossible world or another, so it may look as if we have the raw material for a reply to the actualist as well as to the extensionalist. I think, though, that Yagisawa's remarks succeed only within an extensional theory of impossible worlds, whereas the actualist avoids the objection.

Rephrased, the actualist abstractionist's claim is that whenever ' P ' and ' Q ' name the same property, ' X has P ' and ' X has Q ' express the same proposition, and so IC is satisfied. (And whenever ' P ' and ' Q ' name different properties, ' X has P ' and ' X has Q ' express different propositions, and there will be some impossible world such that one of the two is true in it and the other is not.) The supposed difficulty involves worlds in which P is not coextensive with Q , or (in actualist translation) in which the negation of X has P if and only if X has Q is true, even though ' P ' and ' Q ' name the same property. From an abstractionist perspective, however, such a proposition's being true in certain impossible worlds is entirely irrelevant. We would have a problem if $\sim (X$ has P if and only if X has $Q)$'s being true in a world W somehow prevented X has P and X has Q from being true together in W , but it does not. That $\sim (X$ has P if and only if X has $Q)$ is true in W does not change the fact that X has P and X has Q are the same proposition. This is so even if the book on W reports (falsely) that $\sim (X$ has P if and only if X has $Q)$ or reports (falsely) that $\sim (X$ has P in W if and only if X has Q in $W)$ or reports (falsely) that X has P and X has Q are distinct propositions. So the identity criterion IC stands, regardless of whatever propositions turn out to be true in a given impossible world. The abstractionist is successful in holding a position of the sort that Yagisawa rejects.

This kind of position produces much more difficulty for the extensionalist, because the extensionalist is also a concretist. The concretist regards other worlds as places spatiotemporally unrelated to us, and so regards truth in a world as a species of truth, namely, truth regarding some particular domain. So if it is true in W that the extension of P is not identical with the extension of P , then for the concretist it is true *simpliciter* that the extension of P is not identical with the extension of P . Hence the fact that the extension of P is not identical with itself in some impossible worlds leads to intolerable difficulties for the concretist who identifies properties with their extensions. The abstractionist, though, is never obliged to think that what is true in an impossible world is true, even with respect to some limited domain. Instead, she regards truth in a world as a matter of a proposition characterizing part of the content of the abstract world in question. The propositions that are true in an impossible world may simply be *false* propositions, and so what they say about the extension of P or about the identity criteria of properties or about that world itself is quite beside the point.

We are left with the question of how the extensionalist might deal with the fine-grainedness objection, if not along the lines available to the abstractionist. Yagisawa offers what he calls an "incomplete" solution to the difficulty. His attempt is to identify each property not with its extension in all worlds, but in some smaller group of possible and impossible worlds. His choice is the "analytically familiar worlds," that is, worlds which share all analytic facts with the actual world. It is not an analytic

truth that triangular things are trilateral, so triangularity and trilaterality have different extensions in the analytically familiar worlds. On the other hand, it is analytically true that vixens are female foxes, so in every analytically familiar world, everything that has the property *being a vixen* has the property *being a female fox*, and vice versa. We get the result that these properties are identical.

This solution is not ad hoc, Yagisawa says, since “we naturally expect two synonymous (i.e., analytically connected) predicates to express the same property” ([16], p. 197). Perhaps he means here that if one has already decided to identify a property with its extension in some but not all possible and impossible worlds, then the analytically familiar worlds are a natural choice. It still seems, however, that to identify a property with its extension in any group of worlds other than all the worlds is a significant departure from the extensionalist program. Among the unattractive aspects of Lewis’s structuralist account of properties, Yagisawa lists the “striking feature . . . that it abandons the basic modal extensionalist insistence that a property is to be identified with the set of things which have that property” (p. 193). But Yagisawa, too, it seems, is willing to abandon this “basic modal extensionalist insistence” in favor of a different insistence, identifying each property with something other than the objects which have that property. The reason for this shift is clear enough: the usual extensionalist claim is open to devastating objection, whereas the revised account seems to yield the desired property-identifications. But if this is the reason, then the proposed solution is ad hoc after all; it is motivated only by conditions of adequacy and not by the purported extensionalist insight. It seems an adequate theory of property individuation can be attained only if one gives up the central extensionalist claim, and this gives us reason to give up the central extensionalist claim.

Thus neither the proposal that Yagisawa rejects nor the one he advocates rescues a concretist theory of impossible worlds (coupled with an extensionalist theory of properties) from the fine-grainedness objection, but a version of the former does enable the abstractionist to refute the objection.

A closing remark: we should not think that an impossible-worlds-based theory of property (or proposition) individuation will give us any new information about which properties (or propositions) are identical and which distinct. Like the world-based theory of modality, the purpose of such an account is to explicate certain relationships—in this case between properties and propositions, or between properties and states of affairs—not to arbitrate unclear instances. Our best guide in particular cases will remain conventional usage of the words which refer to the property or properties in question. It is usage that will tell us (or fail to tell us) that *being a vixen* and *being a female fox* are identical, and we may conclude from this that the proposition *Mary is a vixen* is true in a world if and only if the proposition *Mary is a female fox* is true in that world, because “Mary is a vixen” and “Mary is a female fox” express the same proposition.

10 A menu of impossibilities If the theory of impossible worlds given earlier is correct, the content of each state of affairs S is given by some nonempty collection of propositions, its book B_S . Further, book composition is unrestricted, so that every nonempty collection of propositions is the book on some state of affairs or another. We have then, a 1-1 correspondence between nonempty classes of propositions and

states of affairs, as well as a 1-1 correspondence between maximal classes of propositions and worlds. What do impossible worlds look like according to such a theory? The best way to get a feel for the things is to examine a menu of examples. Below, therefore, is a small sampling along with some comments on questions that arise along the way.

One preliminary notion: the books of all impossible worlds (with a single exception) are not closed under entailment. Each impossible world, to some degree or another, compartmentalizes whatever necessary falsehoods or inconsistencies are true in it. To make this idea more precise, let us say that an impossible world W has a *locus of impossibility* L just in case

1. $L \subseteq B_W$,
2. $B_W - L$ is a consistent class of propositions (i.e., possibly the conjunction of all its members is true),
3. no proper subclass L' of L is such that $B_W - L'$ is a consistent class, and
4. no subclass L'' of B_W has cardinality less than that of L and is such that $B_W - L''$ is a consistent class.

(The last two clauses are not redundant, as one of our menu items should make clear.) Informally put, a locus of impossibility is the least that needs to be removed from an impossible world in order to make it possible. The definition allows for the possibility that a given world has more than one locus of impossibility, and it has the consequence that every necessary falsehood true in a given world must belong to each of that world's loci of impossibility.

10.1 λ Every proposition is true in λ . (I borrow the name ' λ ' from Stalnaker's [14] where he uses it to name his absurd world.) Its book is the class of all propositions. λ has no compartmentalization; each consequence of every necessary falsehood (that is, every proposition) is true in it. What are the loci of impossibility of such a world? They are the classes of propositions which contain all propositions but those true in some possible world. Hence λ has as many loci of impossibility as there are possible worlds.

10.2 ω Of course there are other worlds which, due in large measure to multiplicity of inconsistent propositions true in them, are all but impossible to imagine. For example, let ω be the impossible world whose book contains all and only the propositions that are actually false. If A is the class of all propositions and α is the actual world, then $B_\omega = A - B_\alpha$. The world ω is thus a kind of photonegative of actuality. Each of the following propositions is true in ω : *Napoleon was born in 1 A.D.*, *Napoleon was born in 2 A.D.*, *Napoleon was born in 3 A.D.*, and so on, excluding *Napoleon was born in 1769 A.D.* A similar proliferation of propositions about every other topic will be true in ω , so it is clear that ω cannot possibly obtain. Since, for every proposition P , either P or its negation is false, it is also clear that ω meets the maximality requirement for worlds.

It is somewhat tempting to characterize ω as a world with a low degree of compartmentalization, or as being such that all but relatively few propositions are true in it, but this characterization is misleading. The cardinality of B_ω is no greater than the cardinality of B_α , since every proposition in B_ω has a unique negation in B_α .

10.3 δ Consider the impossible worlds whose locus of impossibility has a small, finite cardinality. For example, let δ be the world such that $B_\delta = B_\alpha \cup \{\text{Mars is blue}\}$. In other words, every proposition that is actually true is also true in δ , the additional proposition *Mars is blue* is true in δ , and no other propositions are true in δ . Since every true proposition belongs to B_δ , we are dealing with a maximal state of affairs, and since *Mars is blue* is inconsistent with other propositions in B_δ (e.g., *Mars is not blue*), δ is impossible.

The only locus of impossibility of δ is the single-membered $\{\text{Mars is blue}\}$. It is easy to see that $\{\text{Mars is blue}\} = M$, for short—satisfies conditions 1 and 2 of the above definition. Conditions 3 and 4 are satisfied because M 's only proper subset, and the only subset with a lesser cardinality, is the empty set, and $B_\delta - \emptyset$ is not consistent. δ has no loci of impossibility which are proper supersets of M because of condition 3. δ has no loci of impossibility which are disjoint from M , since such a locus would have to contain all true propositions inconsistent with *Mars is blue*, and each such set is ruled out by condition 4. Hence M is δ 's only locus of impossibility. (Were it not for condition 4, some much larger sets would also be loci of impossibility, so (4) is not a superfluous part of the definition. Condition 3 might then appear to have no function, but it may yet be needed in other cases to rule out infinite sets satisfying (1), (2), and (4) and having proper subsets of the same cardinality which also satisfy (1), (2), and (4).)

Worlds like δ immediately enable us to prove certain results about what kinds of impossible worlds there are. For instance, we might wonder whether there are any impossible worlds in which no contradictions are true, or whether there are any impossible worlds in which no necessary falsehoods are true. The answer in each case, perhaps surprisingly, is yes. δ is our example. The proposition *Mars is blue* is not a necessary falsehood, and every other proposition true in δ is true in the actual world. Since no necessary falsehood is true in the actual world, none of the propositions true in δ is necessarily false, and a fortiori none is a contradictory proposition. (Recall that the books of impossible worlds are not closed under entailment, so the fact that both *Mars is blue* and *Mars is not blue* are true in an impossible world does not entail that *Mars is blue and Mars is not blue* is true in that world.) The reason δ is impossible is that some of the propositions true in it contradict each other, and so they cannot possibly be true together. Nonetheless, no necessary falsehood is true in δ . In [5], p. 9, Lewis says,

What is meant by the counterfactual [‘If kangaroos had no tails, they would topple over’] is that, things being pretty much as they are—the scarcity of crutches for kangaroos being pretty much as it actually is, the kangaroos’ inability to use crutches being pretty much as it is, and so on—if kangaroos had no tails they would topple over. We might think it best to confine our attention to worlds where kangaroos have no tails and *everything* else is as it actually is; but there are no such worlds.

δ and its ilk are the worlds whose existence Lewis denies. The world in which kangaroos have no tails and everything else is as it actually is has the book $B_\alpha \cup \{\text{kangaroos have no tails}\}$, or else $(B_\alpha - \{\text{kangaroos have tails}\}) \cup \{\text{kangaroos have no tails}\}$.

The existence of such worlds does not imply that Lewis's semantics give the wrong truth conditions of '*If kangaroos had no tails, they would topple over*'. Worlds with finite loci of impossibility, by the mere fact that they are impossible worlds, are not nearby worlds. They are very dissimilar from the actual world, though some of them have all but identical books. Therefore it remains true that in the worlds most similar to the actual world in which kangaroos have no tails, kangaroos topple over.

The existence of a multiplicity of impossible worlds does, however, present a strong challenge to the Lewisian/Stalnakerian thesis that all counterfactuals with impossible antecedents are vacuously true. An elementary amendment of Lewis's semantics yields a theory on which counterpossibles may be nonvacuously true or false, so a theory which better fits the semantic data. But this is a topic for another paper.

10.4 π Given these results, we might go on to ask whether there are any impossible worlds in which no two propositions contradict each other, that is, whether there is an impossible world such that for any two propositions true in that world, their conjunction is possibly true. Remarkably, there are. Our example may be regarded as an embodiment of the paradox of the preface. If an author says in the preface of her book that some claim made in the book is false, then her beliefs are inconsistent if she believes all the claims made in the book. Nonetheless each individual claim she makes may be consistent with her claim that some part of the book is in error; it is only the conjunction of all the other claims of the book that is inconsistent with the claim of the preface. The world to be presented, π , is structurally similar to this scenario.

The book B_π contains all the propositions that are true in α , the actual world, except the proposition α is actual and any propositions necessarily equivalent to it. Among the excluded propositions will be the conjunction of all truths,⁷ the conjunction of all contingent truths, and the negation of the disjunction of all falsehoods.⁸ Naturally, B_π will contain the negations of α is actual and its necessary equivalents. This specifies all propositions which are true in B_π . π is a world, since for each true proposition that B_π does not contain, its negation does belong to B_π . And π is impossible: each possible world W is such that the proposition W is actual is true in it, but no such proposition is true in π since the only proposition of the sort that is true in α (α is actual) is stipulated not to be true in π .

To see that no two propositions true in π are inconsistent, let T be the proposition α is actual. (' T ' is for 'Truth, the Whole Truth, and Nothing But the Truth'.) Its negation $\sim T$ is itself possibly true, and is of course consistent with any proposition necessarily equivalent to it. Naturally each proposition that is true in α is consistent with every other proposition true in α . Hence there are two inconsistent propositions in π if and only if one of the propositions true in both α and π is inconsistent with $\sim T$.

Now suppose that some proposition P and $\sim T$ are inconsistent; it is not possible that both P and $\sim T$ be true. $\sim T$ is true in every possible world but α , so there is no possible world aside from α in which P is true. Either α is the only possible world in which P is true or P is true in no possible world. If the former, then P is necessarily equivalent to T , so $P \notin B_\pi$. If the latter, then P is necessarily false and $P \notin B_\pi$. In either case $P \notin B_\pi$, so every member of B_π is consistent with $\sim T$. Thus $\sim T$ and its equivalents function as the preface which denies that all of the other propositions of

π have it right, but without contradicting any one of them.

Of course, there is nothing special about our choice of α as the possible world from which π inherits most of its contingent propositions. Take the book on any possible world W , replace W is *actual* and its necessary equivalents by their negations, and you will have the book on an impossible world without inconsistent pairs. The world is W 's *preface world*. This world's preface will be a locus of impossibility.

10.5 ι (?) Is it possible to generalize this result? That is, for what n is it the case that there is an impossible world such that no n propositions true in that world are inconsistent with each other? I am not sure. My guess is that the greatest such n is either 2 or some infinite cardinal. If the former, then every impossible world has some inconsistent triple of propositions true in it. Certainly π and its ilk have this feature: the proposition α is *not actual* and any two propositions (not necessarily equivalent to α is *actual*) whose conjunction is the conjunction of all truths together form an inconsistent triple in B_π .

How might we attempt to construct an impossible world whose book contains no inconsistent triple? Lewis, though no fan of impossible worlds, mentions what he takes to be an example in [6]. Presumably in some possible world, Lewis is exactly seven feet tall. If we are inclined to deal in impossible worlds, says Lewis, we may also say that there is an impossible limit-world ι in which *Lewis is over 7 feet tall* is true, and so are each of *Lewis is less than 7.1 feet tall*, *Lewis is less than 7.01 feet tall*, *Lewis is less than 7.001 feet tall*, and so on. Though it is impossible that all the propositions of this world be true, says Lewis, any finite subset of them is true in some possible world. If he is right, ι not only lacks inconsistent triples, but also inconsistent n -tuples for all finite n .

It is clear enough that any finite subset of $\{\textit{Lewis is over 7 feet tall, Lewis is less than 7.1 feet tall, Lewis is less than 7.01 feet tall, Lewis is less than 7.001 feet tall, \dots}\}$ is a consistent set. The difficulty is that these are not the only propositions true in the world ι , if such a world exists. Can we guarantee that all finite subsets of B_ι are consistent? It is not clear.

To begin we need to see that any impossible world whose book contains no inconsistent pair (and a fortiori any impossible world whose book contains no inconsistent triple) must have at least this much in common with π : no proposition of the form W is *actual* (where W is a possible world) nor any proposition necessarily equivalent to one of these is true in that world. Let us call a world with this property "anonymous." The present claim, then, is that every world whose book contains no inconsistent pair is an anonymous world.

Proof: Let W^* be an impossible world such that B_{W^*} contains no inconsistent pair of propositions. Let W be a possible world, and suppose that proposition P is necessarily equivalent to W is *actual* and is a member of B_{W^*} (i.e., suppose that W^* is not anonymous). Then for any proposition Q true in W , Q is true in W^* . (If not, then $\sim Q$ is true in W^* ; but $\sim Q$ is inconsistent with P , so $\sim Q$ and P would form an inconsistent pair in B_{W^*} .) So $B_W \subseteq B_{W^*}$. But then for any proposition R true in W^* but not true in W , its negation $\sim R$ is true in W (since W is maximal), and so the inconsistent pair R and $\sim R$ belongs to B_{W^*} , contrary to supposition. \square

If an anonymous world is to avoid having an inconsistent triple, it needs somehow to get around inconsistent triples of the sort that plague π . One kind of inconsistent triple was mentioned above. Here is another. Each of the propositions *α is not actual*, *Humphrey did not win the election*, and *α is actual or Humphrey won the election* is true in π , and the three form an inconsistent triple. Since any world whose book has no inconsistent triple is anonymous, there is no question of removing the triple by replacing *α is not actual* with *α is actual*. Might we stipulate that *Humphrey did not win the election* (along with propositions necessarily equivalent to it) is not true in our anonymous world, its negation being true there instead? Well, we might, but this does not lead to a promising general strategy. *Every* contingent truth yields a similar inconsistent triple in π , and clearly a world such that each contingent falsehood is true in it will have many inconsistent pairs. Likewise it will not work to replace *α is actual or Humphrey won the election* and its necessary equivalents by their negations. Unfortunately for this strategy, every contingent truth (aside from *α is actual* and its necessary equivalents) is equivalent to the disjunction of *α is actual* with some contingent falsehood. The general strategy of replacing all propositions of this form, then, would force us to replace each contingent truth with its negation, again resulting in many inconsistent pairs.

So if the alleged world ι is meant to be one whose book differs minimally from that of some possible world, it will contain many inconsistent triples like those found in π . If ι is to avoid all these inconsistent triples, its book must be adjusted to differ from B_π (and any similar book) at a great many points, and it is not yet clear whether this can be done without creating new inconsistent triples.

It turns out we can say a bit more about what an impossible world without inconsistent triples would have to be like. Let τ be such a world. As noted above, for every possible world W , the proposition *W is actual* and its equivalents will not be true in τ . So if α and β are both possible worlds, *α is not actual* and *β is not actual* are true in τ . What of the proposition *either α or β is actual*? If it were true in τ , it would form an inconsistent triple with *α is not actual* and *β is not actual*, so it is not, and its negation is. In general, then, if W_1 and W_2 are possible worlds, the proposition *either W_1 or W_2 is actual* and its equivalents are not true in any impossible world without inconsistent triples.

But we may go beyond the two-world case: if γ is a third possible world, then the proposition *either α or β or γ is actual* cannot be true in τ since it is inconsistent with *neither α nor β is actual* and *γ is not actual*. And so on: the general result is that for any finite n , no proposition that is true in exactly n possible worlds is true in an impossible world without inconsistent triples. Let us say that impossible worlds without inconsistent triples are thus “finitely anonymous.” What we have been shown is that the members of a certain class of contingent propositions, viz., those true in only a finite number of possible worlds, cannot be true in impossible worlds without inconsistent triples.

Finally, one last result which may be of interest in the search for an impossible world without inconsistent triples. So far we have seen examples of worlds with inconsistent pairs but no necessary falsehoods, and of worlds with inconsistent triples but no inconsistent pairs. Might there also be worlds with inconsistent quadruples but no inconsistent triples, and so on? Here we may give a firm ‘No’. For any finite

$n > 3$, if a world has an inconsistent n -tuple, then that world also has an inconsistent triple.

Proof: Suppose for reductio that some world W has no inconsistent triple but does have an inconsistent n -tuple (where n is finite and greater than 3). Let $\{P_1, P_2, P_3, \dots, P_n\}$ be one such n -tuple. If B_W did not contain $P_1 \& P_2$, then by maximality it would contain its negation $\sim(P_1 \& P_2)$, which forms an inconsistent triple with P_1 and P_2 . So B_W does contain $P_1 \& P_2$. Then W has an inconsistent $(n - 1)$ -tuple, $\{P_1 \& P_2, P_3, \dots, P_n\}$. We have proved an inductive principle: every world with an inconsistent n -tuple also has an inconsistent $(n - 1)$ -tuple (where $3 < n < \infty$). By repeated applications of this principle, we can prove that W has an inconsistent triple, contrary to supposition. Hence no world has an inconsistent n -tuple ($3 < n < \infty$) and lacks an inconsistent triple. \square

If there are impossible worlds without inconsistent triples to be found, then, they must be worlds with the property that Lewis claims for his impossible limit-worlds: no contradiction can be derived from the propositions true in such a world, since every finite set of propositions true in it which might serve as premises is a consistent set. As it stands, we have neither proof that all impossible worlds have inconsistent triples, nor a construction of a world which lacks them. The only hope for the latter would seem to be a world which differs from π not by systematic removal of its contingent truths, but by removal of some and preservation of others. It is hard to see how such a construction would avoid pairs whose conjunction is equivalent to T , but this difficulty falls short of settling the matter. To the best of my knowledge, the question remains open.

II Conclusion I would like to point out that the worlds surveyed above are rather exotic ones—even for impossible worlds! They are quite unlike the worlds that we most often imagine. If we can be said to imagine entire worlds at all, then the impossible worlds we imagine, whether we are spurred on by inconsistent fiction or by our own musings of what it would be like to trisect the angle, will tend to hide their inconsistencies. The loci of impossibility of these worlds will often be detectable only in our peripheral vision. Our imaginative energies will be focused on consistent subsections of the state of affairs. Of course, there will be exceptions, as when impossible fiction explicitly affirms some contradiction or other necessary falsehood.

Though I have said little about them, it is the more “natural” impossible worlds that are most important in the chief applications of impossible worlds. The more easily imagined worlds will tend to be the impossible worlds most similar to the actual world under the similarity relations operative in the evaluation of counterfactuals. They will also tend to be the worlds not ruled out by (if incompatible with) a given individual’s beliefs. However, the worlds discussed above do, I hope, illustrate something of the breadth and interest of the impossible world theory of this paper.

Acknowledgments I would like to thank Brian Leftow, Alvin Plantinga, Michael Rea, and especially Michael Thrusch for their helpful remarks on drafts of this paper.

NOTES

1. Cf. the theory of possible worlds in [11] and [10].
2. I think these are the same as the situations referred to, for example, by Perry in [9].
3. After writing this passage I learned that Naylor gives the same argument in [8].
4. I do not take up the issues that divide concretists and abstractionists, since I have little to add to what others have said. See, for example, van Inwagen [15] and Plantinga [12].
5. I assume here that a proposition with completely unrestricted quantifiers may be said to be true in the actual world. The nature of the proposition prevents us from assessing its truth value at the actual world by asking whether it is true when we restrict the scope of our quantifiers to the denizens of the actual world. This procedure would only give us the truth value of a different proposition, one without completely unrestricted quantifiers. If the proposition in question is not true in the actual world, then the actual world is not maximal, and thus not a world after all.

This proposition, along with propositions about how many worlds there are, what is happening at other worlds, and others, must be evaluated at a given world in some way other than Lewis's usual one of restricting quantifiers to the given world—presumably just by leaving the quantifiers unrestricted. Lewis allows for exceptions to the usual method (see [7], p. 6).
6. Many thanks to Brian Leftow for helpful articulation and discussion of this objection.
7. If indeed there is such a proposition. I do not know of any good reason for thinking that there are not infinite conjunctive propositions. (Cf. Kim's remark about properties: "such operations as infinite conjunctions and infinite disjunctions would be highly questionable for predicates, but not necessarily for properties—any more than infinite unions and intersections are for classes" ([2], p. 73)—though Kim is defending infinite conjunctive properties against a charge of complexity and artificiality, not of nonexistence.) But even if we are comfortable with infinite conjunctions in general, we may have special reservations about a conjunction of all truths. Is this proposition one of its own conjuncts? The conjunction of all truths is true, so it would seem that it must be (assuming now that conjunctive propositions have conjuncts, contra theories according to which sentences but not propositions exhibit the relevant sort of structure). An unusual proposition! Still, there is nothing wrong with being unusual, and I am hard pressed to find any other charge to bring against it.
8. If indeed there is such a proposition distinct from the conjunction of all truths.

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