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Working paper

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Summary

In this paper it is shown that the most rational of all creatures, Homo Oeconomicus, can be enticed to freely engage in an activity that is guaranteed to lead to his ruin. It is furthermore shown that this is possible if and only if utility functions are unbounded. The paper thus develops an argument in favour of bounded utility functions.

Introduction

It is well known that the Devil possesses infinite, though not supreme, power, wealth and knowledge as well as infinite life. It is also known that he derives his pleasures from making other creatures miserable; however, when it comes to creatures endowed with free will, he cannot do anything unless they freely agree to it. The Devil cannot simply force a person to act in accordance with his wishes. This can be very frustrating to the Devil, but in this short paper a strategy is presented whereby the Devil can circumvent this problem and thus, even –or especially– the most rational of all creatures can be brought to misery and ruin none the less.

Homo Oeconomicus is considered the most rational of all creatures. He has perfect insight into his preferences and at all times coolly uses his flawless reason to make the most of his situation: he is the perfect expected utility maximiser. As any standard economic textbook will tell you, his utility function with respect to wealth is strictly concave and non-negative. [1] Usually no assumptions are made with respect to boundedness, and, for now, we shall assume that there is no upper bound.[2] For Homo Oeconomicus's utility function, $U(x)$, where x is his wealth, the following holds: $U(x) \geq 0$, $U'(x) > 0$, $U''(x) < 0$. This is not only known to Homo Oeconomicus, but it is also known to the Devil, whose infinite knowledge of course encompasses the wisdom laid down in standard economic textbooks.

Now assume the Devil offers Homo Oeconomicus the following wager, *R*:
With 99,9% probability the Devil will increase the wealth of Homo Oeconomicus 100 fold, while,

[1] Note that this kind of utility function implies that Homo Oeconomicus is risk averse.
[2] I shall argue that it is in fact better to assume that utility functions *are* bounded. In order to derive this conclusion, I shall show that unboundedness of the utility function leads to problematic results.

with 0,1% probability, Homo Oeconomicus will lose everything. As is well known, the Devil is an evil bastard, but an honourable one who does not cheat,[3] so that Homo Oeconomicus can legitimately assume that the chance experience will be carried out faithfully with the probabilities proposed by the Devil. Suppose that Homo Oeconomicus deduces that the expected utility of accepting this wager is larger than the utility he derives from his present wealth: $E(U(R)) = 0{,}999 \cdot U(100 \cdot x) + 0{,}001 \cdot U(0) > E(U(x)) = U(x)$.[4] Then, being the cool, collected, rational creature he is, he will freely choose to engage in this lottery.

Now assume the Devil offers him this lottery again and again; then at some point, either Homo Oeconomicus will lose or he will decide not to engage in this wager anymore because he has become so wealthy that the 99,9% probability of multiplying his wealth 100 times no longer outweighs the risk of losing it all, even though the probability of the latter is only 0,1%. But suppose that were this to happen, the Devil simply ups the multiplication factor to a level such that Homo Oeconomicus's expected utility of accepting the wager is higher than his present utility. Given the structure of the utility function, this is always possible, and thus the Devil can keep Homo Oeconomicus playing until the time comes that he loses, which will certainly (i.e. with probability 1) happen at some point.

This strategy allows the Devil to ruin Homo Oeconomicus -and thus bring him misery- with certainty, without violating the condition that Homo Oeconomicus must freely choose to participate. In other words, the perfect rational being, Homo Oeconomicus, is willing to play a game in which his optimal strategy is guaranteed to leave him ruined.

Existing Paradoxes

Many paradoxes are known in the literature on decision making. Undoubtedly one of the most famous ones is the St. Petersburg paradox, and this paradox, at first glance, seems closely related to what is happening here. None the less, there are some essential differences. In the St. Petersburg paradox the paradoxical result is obtained by a steady contribution to the expected value of the lottery by each possible outcome.[5] As the probability of a certain outcome is the inverse of the amount of utility that outcome brings, every outcome contributes 1 to the expected value of the lottery. As the number of possible outcomes is infinite, the expected value of the

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[3] Mainly because he does not have to.

[4] If this is not the case, assume that the Devil will not increase his wealth 100 fold, but 1000 or 10000 fold or whatever the amount necessary for this equation to hold. Given the structure of *U(x)*, such a multiplication factor can always be found.

[5] Here the version of the St. Petersburg paradox is used that is adjusted for concave utility functions (see, for instance, Mas-Colell et. al. (1995)).

lottery is infinite too. This result is regarded as a paradox because it implies that a rational being would be willing to pay *any* amount to be allowed to play this lottery, knowing that this is most likely to lead to huge losses. As the paradox does not occur when utility functions are bounded, the St. Petersburg paradox can be used as an argument in favour of adding the assumption that Homo Oeconomicus's utility function is bounded (cf. Mas-Colell et al. (1995)).

The main result of this paper (to be presented below) gives further support to this argument. Nonetheless, the St. Petersburg lottery differs on some essential points from the game constructed by the Devil.[6] Not only is the contribution to the expected utility of each (finite) outcome 0 (leading to the certainty of total ruin instead of only a very high likelihood of a very big loss), the expected utility of this sequence of wagers need not be infinite either. In fact, it can be shown that it need only be marginally larger than the utility derived from the initial endowment of Homo Oeconomicus (see Appendix A). In other words, the reward in terms of expected utility that Homo Oeconomicus demands for engaging in a lottery that will certainly lead to his ruin is arbitrarily small.

The paradox that seems closest to what is happening in this story is the 'ever-better-wine'-paradox introduced by Pollock (1983). This paradox is derived as follows: suppose a wine lover has a bottle of wine that gets better and better each day it remains unopened. Suppose furthermore that the increase in its quality is such that it outweighs any loss of utility the wine lover may experience by forgoing the present consumption of the wine in favour of consuming it the next day. Then it is rational for the wine lover to forgo drinking the wine today in favour of letting it get even better. This holds, however, for each day that he would think of consuming the wine. As a result he never gets to drink the wine.

There seems to be a similarity between the decision to stop playing in the story of Homo Oeconomicus's encounter with the Devil and the decision to drink the wine in the puzzle by Pollock. However, an essential difference is that in Pollock's story there is no risk of loss, the higher utility that can be achieved in the future is guaranteed, though never attained if one never decides to drink, whereas in the case of Homo Oeconomicus, no such certainty exists. In fact, the only certainty is ruin.

Devil-proofness

Having thus constructed a strategy that allows the Devil to ruin any specimen of the Homo Oeconimicus species he comes across, it is interesting to try to discover if there are

[6] The strategy adopted by the Devil is, in fact, a kind of Martingale system, which he, because of his infinite wealth, can maintain.

augmentations of the Homo Oeconomicus species possible that are not susceptible to this kind of demonic mischief. For this it is useful to keep in mind that the Devil will only engage in lotteries with human beings if he is certain he will win. The utter humiliation that comes with losing to a mortal is something an infinite being simply cannot afford to risk.[7]

There are a number of aspects of Homo Oeconomicus that affect his resistance to the Devil. One assumption that can be dropped is the continuity condition. This will render invalid the conclusion that the expected utility of the wager R^* can be as close to $U(x_0)$ -the utility of Homo Oeconomicus's initial wealth- as we like as we can no longer control the ε_i's in 11. (see Appendix A), but if strict monotonicity is maintained (along with the other assumptions), so will be the guarantee of ruin. As such, it does not provide an adequate defence against the Devil.

An obvious solution to Homo Oeconomicus's plight would be assuming that there is a maximum to the level of utility Homo Oeconomicus can derive from wealth. When this maximum is attained, or approximated closely enough, there is no way the Devil can seduce Homo Oeconomicus into accepting yet another wager. Assuming there is a maximum level of utility that can be attained is sufficient to show that there is a positive probability that the Devil will fail to achieve his objective of ruining Homo Oeconomicus, as it is then certain that this maximum will also be attained (or approximated closely enough) with some positive probability. This follows from the fact that U_t as defined in Appendix A is a divergent sequence.[8]

Assuming that there is a maximum to the level of utility that can be derived from wealth, however, seems a rather overzealous approach to solving the problem. While maintaining the position that human beings –at least at some point- should just be content with what they have, may be a position that seems appealing to some philosophers and moralists, most economists will be very wary of this limitation on human desires. Assuming that there is a maximum to the utility that a person can achieve implies dropping the property of (local) non-satiability[9], a property that is –often implicitly– used in the derivation of many economic theorems and results. Furthermore, unlimited human greed is often regarded as ensuring the continuance of economic development. The insatiability property may very well be worth saving if this is possible - and it is.

Conclusion: boundedness of the utility function

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[7] Note that as long as the game (or the lottery) is in progress, the Devil hasn't lost and that the Devil's infinite powers enable him to postpone the death of any living creature indefinitely.
[8] The possibility that Homo Oeconomicus will stop playing just short of having attained the maximum results from the fact that U_t (as opposed to the utility function $U(x)$) is not continuous but contains jumps.
[9] See Varian (1992).

In order to ensure ruin, it must be possible that the Devil can always entice Homo Oeconomicus to engage in yet another round of play. This implies the following:

- $\exists p \in (0,1)\, s.t. \forall x \exists x': p \cdot U(x') > U(x)$ [10]

This property we shall call demonic susceptibility.[11] It is then possible to prove the following theorem:

Theorem: Homo Oeconomicus suffers from demonic susceptibility if and only if his utility function U is unbounded.

Proof: see Appendix B.

The fact that the certainty of ruin can only be attained if the utility function is unbounded can be used as an argument for adding the assumption of bounded utility functions to the standard version of Homo Oeconomicus.[12] A similar recommendation has also been proposed based on the results from the St. Petersburg Paradox (see, for instance, Mas-Colell et al.(1995)).[13] The argument presented in this paper is stronger, however, as it does not use infinite expected utility as the St. Petersburg Paradox does, and because engaging in an activity that will lead to certain ruin is even more serious then engaging in an activity that will, with a high probability, lead to a very big loss.[14]

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[10] Note that we are assuming neither continuity, nor that x' > x. The latter does seem reasonable, but it is not necessary to achieve the result of certain ruin to assume that U is monotonously increasing. If this is not the case, the Devil may increase utility by offering to take away money from Homo Oeconomicus. Admittedly, however, this would make for very strange utility functions.

[11] It might be argued that the Devil can also ensure that Homo Oeconomicus keeps playing by adjusting the probability of winning. This is true, but ruin would then no longer be guaranteed.
Adjustments of the probability of outcomes also feature in other paradoxes presented by Pollock (1983), in particular in the dare-devil moving ever closer to the edge.

[12] An objection that is often raised against these kinds of paradoxes is that the amount of money, or in this case wealth, in the world is limited. As a result the paradox could not occur. In this light it is worth noticing, however, that for any 'normal' utility function (i.e. those without a vertical asymptote), the assumption of a limited domain on x implies that U is bounded. As such, this remark does therefore not affect the central result of this paper.

[13] Although the assumption of boundedness does not affect many of the main results in economic theory, the assumption may prove problematic in other fields of decision theory (see, for instance, Jeffrey 1983). I thank Richard Bradley for pointing this out to me.

[14] I wish to thank Jan-Willem Romeijn, Boudewijn de Bruin and Barteld Kooi for their helpful comments on earlier versions of this paper.

Appendix A: Deriving the price of certain ruin

In this appendix we shall derive the price of certain ruin - that is, the amount of additional expected utility that has to be dangled in front of Homo Oeconomicus in order to get him to accept the lottery that will certainly lead to his own ruin. To derive this price, we shall first write down this problem in a more precise way.

The following assumptions are made:

1. $\forall x > 0 : U(x) > 0, U'(x) > 0, U''(x) < 0; U(0) = 0$, $U(x)$ is unbounded. (x is Homo Oeconomicus's wealth).
2. The probability of winning p is between naught and one and remains fixed throughout the rounds of play: $p \in (0,1)$.
3. The initial wealth of Homo Oeconomicus is positive: $x_0 > 0$.

Let t denote the round of play, and x_t be the wealth of Homo Oeconomicus at time t. Then the multiplication factor a_t of the wealth in case of winning is determined by:

4. $$\begin{cases} a_0 = 1 \\ a_t \ \ s.t. \, U(x_0 \cdot \prod_{i=0..t-1} a_i) < p \cdot U(x_0 \cdot \prod_{i=0..t} a_i) \quad for\ t \geq 1, \end{cases}$$

A reformulation of this problem as a single compound lottery will be introduced later, but for now let us focus at the repeated offers made by the Devil. For notational ease, we define $U_t = U(x_0 \cdot \prod_{i=0..t} a_i)$, the amount of utility that is achieved after the t'th round of play, conditional on the fact that Homo Oeconomicus has not lost any of the previous rounds.

Since multiplications of 0 remain 0, we can denote the expected utility of each strategy of Homo Oeconomicus by the number of the round in which he stops. This will be denoted as S_t. For each strategy, the expected utility is then given by:

5. $$E(U(S_t)) = p^t \cdot U_t .$$

By construction we know:

6. $E(U(S_t)) < E(U(S_{t+1}))\ \forall t.$

This implies that the strategy of never stopping (S_∞) is indeed the dominant strategy. The expected utility of this strategy is:

7. $$E(U(S_\infty)) = \lim_{t \to \infty} p^t \cdot U_t = \lim_{t \to \infty} p^t \cdot U(x_0 \cdot \prod_{i=0..t} a_i) .$$

Analogously to the story behind the St. Petersburg paradox, we can construct a single compounded lottery. Let us assume we toss a not necessarily balanced coin, so that the chance it comes up heads is somewhere between 0 and 1 (and is known). If it comes up heads, all of Homo

Oeconomicus's wealth is eliminated. If it comes up tails, his wealth is multiplied by a_i and the coin is tossed again. This lottery -let us denote it by R^*- then has the same expected utility as S_∞, which by definition is larger than the expected utility Homo Oeconomicus derives from his initial endowment, $U(x_0)$. It is therefore rational for Homo Oeconomicus to accept this lottery, even though it leads to certain ruin.

It may be argued that this is not that surprising, as it is not necessarily irrational to engage in a lottery with infinite expected utility even if the chance of winning is infinitesimally small. While it is indeed clearly possible to construct a_t's in such a way that the expected utility of this lottery/strategy will equal infinity, this need not be the case. It is possible to generate a_t's in such a way that this expected utility is finite. In fact, we can reduce it to only marginally more than the initial utility of Homo Oeconomicus: $U(x_0)$.

In order to do this, let us return to 4. From this we know that:

8. $p \cdot U_{t+1} > U_t \Leftrightarrow p \cdot U_{t+1} - U_t = \varepsilon_t \quad \textit{for some}\, \varepsilon_t > 0$.

We furthermore know that we can give $\varepsilon_t > 0$ any value we like by choosing a_{t+1} accordingly. So let $\varepsilon_t > 0$. Then we can also express 8. by:

9. $p \cdot U_{t+1} - U_t = \varepsilon_t \Leftrightarrow p \cdot U_{t+1} = U_t + \varepsilon_t \Leftrightarrow U_{t+1} = \dfrac{U_t}{p} + \dfrac{\varepsilon_t}{p}$.

From this we can, after some elementary calculations, deduce 10.:

10. $$p^t \cdot U_t = \begin{cases} U_0 & t = 0 \\ U_0 + \sum\limits_{i=1..t} (\varepsilon_{i-1} \cdot p^{i-1}) & t > 0 \end{cases}$$

and then 7. can be rewritten as:

11. $$\begin{aligned} E(U(S_\infty)) &= \lim_{t\to\infty} p^t \cdot U_t = \lim_{t\to\infty} p^t \cdot U(x_0 \cdot \prod_{i=0..t} a_i) = \lim_{t\to\infty}(U_0 + \sum_{i=1..t} \varepsilon_{i-1} \cdot p^{i-1}) \\ &= U_0 + \sum_{i=1}^{\infty} (\varepsilon_{i-1} \cdot p^{i-1}) \end{aligned}$$

For sufficiently small ε_i's, this summation will converge. Furthermore, as we can choose the ε_i's as small as we like, we can also reduce this summation ad libidinem.

Appendix B: Proof of the theorem.

$D.S. \Rightarrow UB$ Suppose: Homo Oeconomicus suffers from demonic susceptibility, i.e.

$\exists p \in (0,1)\, s.t. \forall x \exists y : p \cdot U(y) > U(x)$

Let *p'* be that *p*.

Suppose: *U* is bounded.

Then: $\exists M \ s.t. \forall x : U(x) \leq M$.

Let *M* be the smallest upper bound of *U*.

Let *x* be such that $U(x) \geq p' \cdot M$. This must be possible, or *M* would not be the smallest upper bound.

Then, according to the first premise, Homo Oeconomicus's demonic susceptibility:

$\exists y\, s.t.\, p' \cdot U(y) > U(x) \geq p' \cdot M$

which implies: $U(y) > M$

Which implies a contradiction, as *M* is an upper bound of *U*.

$UB \Rightarrow D.S.$ Suppose: *U* is unbounded.

Then: $\forall M\ \exists y\, s.t. U(y) > M$ (2)

We will show that it is possible to keep constructing new wagers that are agreeable to Homo Oeconomicus for every $p \in (0,1)$.

So let $p \in (0,1)$

Let $x \in \Re^+$

Take $M = U(x)/p$

Take $y\ s.t. U(y) > M$, which is possible because of the unboundedness of *U*.

Then $p \cdot U(y) > p \cdot M = U(x)$

So $\exists p \in (0,1)\, s.t. \forall x \exists y : p \cdot U(y) > U(x)$

Q.e.d.

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