The tension between the mathematical and metaphysical strands of Maupertuis' Principle of Least Action

Yannick Van den Abbeel[1]

1. Introduction

In one of the most important twentieth-century textbooks of physics, *Course of Theoretical Physics*, 1962 Nobel prize winner Lev Landau writes that his exposition «makes no use of the historical approach, [and that] from the very beginning it is based on the most general principles: Galileo's principle of relativity, and Hamilton's principle of least action».[2] From these two principles Landau aims to derive the whole of classical mechanics. Without doubt, the principle of least action is a fundamental principle in classical mechanics. In the twentieth century, the intuitions behind the principle of least action were further generalized from classical to quantum mechanics[3] and many other domains in physics. Contemporary physicists, however, consider the PLA as a purely *mathematical principle* – even an axiom which they cannot completely justify. Such an account stands in sharp contrast with the historical meaning of the principle of least action.

When the principle was introduced in the 1740s, by Pierre Louis Maupertuis, its meaning was much more versatile. For Maupertuis the prin-

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1 The author's research is funded by a PhD Fellowship of the Research Foundation – Flanders (FWO).

2 Landau & Lifshitz 1969, *Preface*, vii.

3 Yourgrua, Mandelstam 1960.

ciple of least action signified that nature is thrifty or economical in all its actions, i.e., that nature avoids to do anything unnecessary. Maupertuis understood the principle in teleological terms and even considered the principle as an expression of God's wisdom[4]. It has been correctly pointed out by historians that Maupertuis in his later years moved towards a more speculative and metaphysical approach, whereas his contemporary Euler and later Lagrange, wanted to avoid such theological and metaphysical implications and frame the PLA (in line with contemporary standards) in purely mathematical terms.[5]

Such readings, however, have had the unintended side-effect that they lose out of sight the question how the mathematical and metaphysical aspect of the principle of least action *fit together within Maupertuis' own work*. Investigating *if* and *how* the mathematical and metaphysical aspects of the PLA are compatible within Maupertuis' thought will be the main goal of this paper.

In order to address this question properly it is necessary to first say a few things about the complex and changing relationship between metaphysics and physics during Maupertuis' time. It is often assumed that with the publication of Newton's *Principia* physics immediately took a positivistic and mathematical turn. However, the transition from natural philosophy to modern physics did not occur overnight. The disentanglement of

4 Even though Maupertuis' metaphysical and theological speculations on the PLA were quickly ignored in the eighteenth century, the philosophical implications of the PLA remain open for discussion. Unfortunately, despite its growing importance in physics, philosophers of science have been ignoring this topic for a long time. cf. STÖLTZNER 2003.

5 Euler did not really pursue the theological implications of the principle as such, but was more interested in the mathematical exploration of the principle. Euler's program in mechanics, however, was not free from metaphysics and teleology. It was Lagrange who, in the most radical way, purified mechanics from any metaphysics, theology and teleology, and posited the principle of least action as a mathematical consequence of his axiomatic *principle of virtual velocities*.

metaphysics and physics was a complex and gradual process that started to take shape during the eighteenth century but was only completed in the nineteenth century. The transitional nature of this process also explains Maupertuis' ambivalent attitude towards metaphysics throughout his career.[6] In his early years, he was critical of speculative metaphysics and *a priori* reasoning[7] and favored the mathematical and empirical approach of Newton.[8] An important change in attitude occurred when Maupertuis moved from Paris to Berlin. In 1740 Frederick the Great inherited the Prussian throne and wanted to revitalize the academy of letters and sciences in order to rival those of France and England[9]. Voltaire had endorsed Maupertuis as the new president of the Prussian Academy of Sciences, and in 1746 Maupertuis took up the position. As Mary Terrall (2002) discusses in more detail: before his arrival the institution went through a series of structural changes. Its

6 The above biographical introduction is kept to a minimum. For a complete and detailed overview of Maupertuis' life and his scientific accomplishments, the reader is advised to consult the excellent work of BEESON 1992, and the more recent book of TERRALL 2002. Also the recent edited volume HECHT 1999 contains many relevant articles.

7 It must be pointed out that Maupertuis was not anti-metaphysics *per se*. Rather, his point was that metaphysics should not be pursued independently of empirical research or experience. In his early years he did not introduce a new metaphysical scheme but reflected merely on those of others. For example, in his *Discours sur les différentes figures des astres* (1732), which includes a chapter entitled *Metaphysical discussion upon attraction*, Maupertuis argues that the notion of attraction does not logically contradict other properties of bodies and we can therefore on an *a priori* basis not dismiss it. The empirical success of Newton's gravitational theory was actually a strong argument not to reject it.

8 Understanding the dissemination of Newtonianism in France has been an important topic in the literature. The work of BRUNET 1931 is mainly concerned with Maupertuis' role as an advocate of Newton's theories in France against a Cartesian establishment. BEESON 1992 points out in his study that the dichotomy between Newtonianism and Cartesianism is too simplistic and we must give proper due to the impact of Leibnizeanism as well. A different perspective which downplays the impact of Newton is pursued by SHANK 2004. Also his more recent book SHANK 2008 is of interest and includes a chapter on *The Invention of French Newtonianism: Maupertuis and Voltaire*.

9 Cf. TERRALL 2002, 173-198, 231-269. Also her earlier article TERRALL 1990 discusses the culture of science in Frederick the Great's Berlin. Some other relevant articles which provide context are AARSLEFF 1989 and CALINGER 1968.

members were divided into four classes: experimental philosophy, mathematics, speculative philosophy, and literature. It was the speculative philosophy class which would provide Maupertuis with a new environment and incentive to pursue his own metaphysical mechanics.[10] Maupertuis, however, wanted to find a middle way between the extremes of the Leibnizian-Wolffian dogmatic philosophy and the French disgust of metaphysics.[11] In a letter to Bernoulli he wrote:

> German metaphysics is a strange science, but that is not the fault of metaphysics, but rather of the Germans […] The French are too disgusted with metaphysics; the Germans are too mired down in the mud. Perhaps the Swiss can find a viable middle ground.[12]

Maupertuis wanted to reform metaphysics, and he thought this reform needed to happen both on an institutional, philosophical and methodological level.[13] Terrall in her excellent book has explained in detail the story of Maupertuis' role in the Berlin Academy's political organization. In the next section we will show how Maupertuis' discontent with the metaphysical ap-

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10 Cf. Terrall 2002, 237, 239, 270.

11 It seems a bit too strong to say that there was absolutely no interest in metaphysical issues in France. Criticism of the notion of force as well as a more general criticism with respect to causes and causal explanations was not uncommon (cf. the work of d'Alembert), but this was hardly a French affair; we can find a similar criticism also elsewhere in Europe. Furthermore, members of the Paris Academy of Sciences were participating in the *vis viva* controversy and the question was even issued as one of the prize questions. Also figures such as Émilie du Châtelet are well-known for promoting a natural philosophy that combines Newtonian physics with Leibnizian metaphysics. It is possible that Maupertuis was exaggerating the French context, because he might have considered his intellectual freedom too limited to pursue further his own (reformed) metaphysical program in Paris.

12 Letter from Maupertuis to Johann II Bernoulli, 18 September 1747, cf. Bernoulli 1747.

13 The new metaphysical method Maupertuis is looking for (contra his contemporaries) is also the topic of Leduc 2015. A somewhat broader story of the faith of metaphysics is given by Clark 1999.

proaches of his contemporaries led him to pursue a new program in metaphysics in which the PLA would play a central role.

2. The Development of the Principle of Least Action

In this part of the paper we will proceed chronologically and discuss the three most important texts related to the development of the PLA. In the first subsection I will focus on *Loi du repos des corps* (1740) which shows Maupertuis' search for general mathematical principles in physics and his application of the calculus to formulate physical problems in terms of minimal conditions. The second subsection discusses the paper *Accord des différentes lois de la nature qui avoient jusqu'ici paru incompatibles* (1744). Not only did Maupertuis in this paper successfully unify the three laws of optics under one higher principle, he also for the first time used teleological terms and even referred to God's wisdom. These speculative elements became more prominent in his next paper, *Les lois du mouvement et du repos, déduites d'un principe de métaphysique* (1746), in which he sees his principle as encompassing all natural phenomena and providing an incontrovertible proof of God's existence. The history of the development of the principle of least action is very rich and broad.[14] In this section, I will limit myself to providing a genealogy which

14 A general overview can be found in the article JOURDAIN 1912 as well in the excellent book of BEESON 1992. Commentators such as PANZA 1999 and PULTE 1987 provide a more detailed and technical account of the mathematics at stake and also take into account Euler who was key for the development of Maupertuis' ideas. Others, such as GERHARDT 1898 and KABITZ 1913, have discussed the controversy with Samuel Koenig, who claimed that Maupertuis had plagiarized Leibniz, the alleged true originator of the principle of least action. Furthermore, as we already pointed out above, TERRALL 2002 is sensitive to the political context and the more general culture of science. BOUDRI 2013, on the other hand, has read the PLA with respect to the changing metaphysical conception of force during the eighteenth century. Finally, FEHER 1988 focusses on the role of metaphor and analogy in the birth of the principle of least action. Providing a *synopsis* of these different perspectives is not required for the purpose of this paper.

highlights both the mathematical and metaphysical aspects of the PLA. Such a developmental story will set the stage for the next section, where we will ask to which extent these two aspects are compatible with each other in Maupertuis' own thinking.

2.1. *Loi du repos des corps*

Maupertuis read to the Paris Academy on the 20th of February 1740 the paper *Loi du repos des corps* in which he presents a new principle in statics. Before turning to a more technical discussion of this principle, he offers a reflection on the nature and types of principles in physics:

> If the sciences are founded on certain simple and clear principles from the first type, on which all the truths that are the object thereof depend, they have yet other principles, less simple truthwise, and often difficult to discover, but which once discovered, are of very great utility. These are in some way the laws which Nature follows in certain combinations of circumstances, and which teach us what she will do on similar occasions. The first principles hardly require any demonstration by their evidence which is obvious to the mind as soon as it examines them. The principles of the second kind, however, do not have a rigorous physical Demonstration, because it is impossible to go through all the cases in which they take place.[15]

Maupertuis introduces a distinction between «clear and simple principles» that «do not require any demonstration» and principles that are neither simple nor generally proven, but that nevertheless, once discovered, can be very useful in specific circumstances.[16] Even though the latter cannot be

15 MAUPERTUIS 1740, 170.

16 Maupertuis twofold division is not unproblematic and one might wonder – given his skepticism and critique of *a priori* rationalism – if he is really convinced that we can have knowledge of principles of the first kind? LEDUC 2015 suggests they are 'mathematical axioms'. BEESON 1992 suggests they are fundamental metaphysical principles (e.g. the principle of non-contradiction). TERRALL 2002 suggests they are physical axioms of

demonstrated rigorously, they have *inductive certainty*.[17] Maupertuis states that physics will never be able to provide an *a priori* proof of these principles, but that perhaps such a proof belongs to «some higher science» (« *quelque science supérieure* »).[18] These remarks are vague, but it is reasonable to align them with his later attempts to provide a metaphysical justification for general principles in physics. In 1740 he does not pursue this further but only points out that the certainty of principles of the second kind is so great that some mathematicians do not hesitate to make them the foundations of their theories. Maupertuis says that these principles are used every day to solve problems and that they function as a «mental shortcut». Indeed, because «our spirit, being a thing of limited scope, often finds that the distance from the first principles to the point it aims at is too great, [it] tires or loses its way».[19] Accordingly, these intermediate principles allow one to dispense with part of the deductive chain and one «often finds that [once applied] the mind has but a little way to go to reach [its] goal».[20] Such shortcuts are particularly useful in statics and dynamics, he says, where «the complicated way that force is

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the mathematical sciences (e.g. Newton's *axiomata sive leges motus*). The last interpretation has some credibility because in the text Maupertuis says that it would be too difficult to solve physical problems starting deductively from (physical?) principles of the first kind. However, one might wonder to which extent first principles such as Newton's laws of motion are 'clear and simple' and 'do not require any demonstration'.

17 « Jamais on n'a donné de Démonstration générale à la rigueur, de ces principes; mais jamais personne, accoûtumé à juger dans les Sciences, et qui connaîtra la force de l'induction, ne doutera de leur vérité. Quand on aura vû que dans mille occasions la Nature agit d'une certaine manière, il n'y a point d'homme de bon sens qui croye que dans la mille-unième elle suivra d'autres loix ». (Maupertuis 1740, 170); cf. also Boudri 2013, 146.

18 « Quant aux Démonstrations a priori de ces sortes de principes, il ne paroît pas que la Physique les puisse donner; elles semblent appartenir à quelque science supérieure ». (Maupertuis 1740, 170)

19 *Ibid.*, 171.

20 *Ibid.*

related to matter makes these refuges even more necessary».[21] He mentions the principle of the lowest center of gravity in statics, and the principle of the conservation of living force in dynamics as examples of *unproven but practically useful principles*. The main goal of the *Loi du repos des corps* is to introduce a new principle to the mathematician's toolbox, namely 'the law of rest'. This principle (roughly stated) expresses the conditions of equilibrium for a system of bodies acted upon by any number of central forces which are directly proportional to any integral power *n* of the distance to their centers. Maupertuis writes each central force as $F_i = f_i \cdot z^n$ where the f_i are constants which express the 'intensities' of the respective forces and *z* is the distance to the center. Having explained his terminology we can now turn to the formulation of his principle.

> Consider a system of bodies that weigh, or that are drawn towards centers by forces which act on each separately as a power n with respect to their distances to the centers. In order that all these bodies would remain at rest, the sum of the products of each mass, by the intensity of its force, and by the power n + 1 of its distance to the center of its force (which may be called Sum of the Forces of rest) attains a Maximum or Minimum.[22]

In other words, for a system of bodies of which each is attracted to a center by a force varying as the *n*-th power of the distance from that center, to remain in equilibrium, it is necessary that the quantity

$$\sum m_i \cdot f_i \cdot z_i^{n+1}$$

is a *maximum or a minimum*, where *f* is the intensity of the force which acts on *m*, and *z* is the distance of the mass *m* from its center of force. This condition then reduces to the mathematical equation

21 *Ibid*.
22 *Ibid*.

$$\sum m_i \cdot f_i \cdot z^n = 0$$

Maupertuis proves this principle for two simple mechanical systems.[23] The first is a system with one degree of freedom[24]: a system consisting of a finite number of masses which are rigidly linked to a fixed center and rotate in the same plane. The second case is a system with two degrees of freedom in which the configuration is similar to the first case, except that the rigid connections are replaced with flexible connections in the form of non-elastic cords, and the center can move freely. The mathematical details of Maupertuis' argument will not be outlined here. However, it is worth mentioning that his proof ultimately relies on the *principle of virtual work* of Johann Bernoulli.[25] Even though his previous mentor, who supported him in his mathematical training, used a slightly different terminology and represented the equilibrium conditions in terms of a balance of forces multiplied by elements of distance (infinitesimal displacements), a closer investigation reveals that Maupertuis' law of rest was built on this principle.

Maupertuis' 1740 paper stirred very little interest during his time. From a historical point of view, however, this paper is interesting because it shows Maupertuis' first step towards the explicit formulation of both the term and the concept of the principle of least action. There are some striking similarities with his later thought. Most importantly, the law of rest is expressed as a minimal or maximal condition – a key characteristic of the later mathematical formulation of the principle of least action. Furthermore, Maupertuis'

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23 Cf. Boudri 2013, 148 for a good outline and visualization of the two cases.

24 The notion 'degrees of freedom' belongs to the vocabulary of contemporary mechanics, and refers to the number of independent parameters that determine the configuration of a mechanical system.

25 For a broader discussion, see Hiebert 1962, Lindt 1904, or the more recent discussion in Capecchi 2012, 195-216.

reflection on the nature and usefulness of principles in physics attests to his search for new principles in physics. Finally, despite his critical attitude of metaphysical speculation and *a priori* rationalism, he seems to leave the door open for metaphysics when he says that the demonstration of intermediate principles might be provided by 'some higher science'. We must, however, be careful not to read too much in his earlier work. There is no mention of teleology or God at this stage, nor does Maupertuis have the ambition to provide a very general principle which is applicable across the different sciences (optics, dynamics, and statics). These developments will only appear in the following years, starting with his paper on optics of 1744.

2.2. *Accord des différentes lois de la nature qui avoient jusqu'ici paru incompatibles*

The first major breakthrough in the development of the PLA occurred in the paper *Accord des différentes lois de la nature qui avoient jusqu'ici paru incompatibles* (1744). In this paper, Maupertuis seeks a principle in the field of optics from which the three laws of optics can be deduced. Maupertuis considers the positions of Descartes and Newton[26] to be unsatisfactory and argues that an approach which uses metaphysical principles is preferable. He refers to earlier attempts to explain the laws of optics by using metaphysical principles, which he equates with «those laws to which Nature herself appears to have been subjected by a higher intelligence which, in producing its effects,

26 Maupertuis' view on the historical development of the conflicting theories of refraction is based on Mairan's 1732 study *Suite des recherches physico-mathematiques sur la réflexion des corps*. Another important source of Maupertuis in this paper was Clairaut's 1739 paper *Sur les explications Newtonienne & Cartésienne de la refraction de la lumière* which convinced Maupertuis of the Cartesian position that light moves more quickly in denser media.

causes nature always to act in the most simple way».[27] It soon becomes clear that Maupertuis is referring to *Fermat's principle of least time,* which states that the path taken by a ray of light between two points is the path that can be travelled in the least time. Though Maupertuis was sympathetic to Fermat's effort to subsume the laws of optics under one general principle, he disagreed with Fermat on a theoretical level.

> Fermat himself did not hesitate in believing that light travelled more easily and more quickly in less dense media than in media of higher density […]. Nevertheless, Descartes advanced exactly the opposite, that light moves more quickly in denser media and, although his [mechanical] reasoning was perhaps inadequate, his faults does not stem from his assumption about the speed of light.[28]

Maupertuis endorses the position of Descartes according to which light moves more quickly in denser media. Today we know Fermat was actually right and Descartes was wrong, but this point is not so important for my further discussion. The more important point I want to discuss is how Maupertuis developed an alternative to Fermat's least-time principle:

> After meditating deeply on this matter, I have contemplated whether light, already abandoning the shortest way, which is that of a straight line, when passing from one medium to another, could not also follow that of the shortest time. Indeed, which preference must it have of time over space? Light cannot all at once travel through the shortest way and through that of the shortest time. Why would it rather travel by one of these paths than by the other? So light does not follow either of them, but it takes the path that offers a more real advantage: *the path light takes is that by which the quantity of action is the least*.[29]

Maupertuis points out that there is no reason why light would prefer the shortest time or distance and that the real expense (*dépense*) that nature seeks

27 MAUPERTUIS 1744, *Accord de différentes loix de la nature*, 421.
28 *Ibid.*, 422.
29 *Ibid.*, 423.

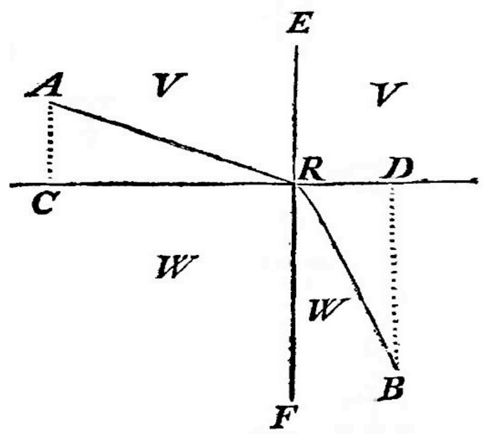
to economize is the 'quantity of action', which he understands as follows: action appears to be a measure of what is required to move a body from *A* to *B* at a particular velocity and along a particular path. More generally, when a particle changes its speed along its path, action becomes proportional «to the sum of the distances, each multiplied by the velocity at which the body passes through [these distances].»[30] In a formula this becomes:

$$Action = \sum v_i \cdot \Delta s_i$$

But how can Maupertuis derive from this principle the three laws of optics? In his paper he only derives the third law of optics. But by way of introduction, I shall illustrate how the first laws of optics follows easily from his action principle. The first law of optics states that *light moves in a straight line in a uniform medium.* So how can we prove this? Since the ray of light does not change its medium its velocity will remain constant. So the above expression becomes: $Action = v \cdot \Delta s$. Since v is constant, minimizing this expression reduces to minimizing the distance Δs. But the shortest distance between any two points (in Euclidean space) is given by a straight line. So we have derived the first law of optics, i.e. light moves in straight lines. It must be pointed out that in a homogeneous medium the action-principle is equivalent to the least-time-principle of Fermat. The situation becomes different for the third law of optics, which deals with refraction due to a change of transmission medium. I will now outline Maupertuis' proof of the third law.[31] Consider the following situation:

30 *Ibid.*

31 Maupertuis' proof is given on p. 424. The above proof is the same in spirit, but adopts a more modern formulation (using the notion of a derivative of a function, using equations instead of proportions). This modern perspective does not distort the argument, but makes it easier for the reader to follow the proof.



where V and W are the respective velocities in the different media, which are separated by the border CD. The principle of least action states that the following quantity should be minimal:

$$V \cdot AR + W \cdot BR = V \cdot \sqrt{AC^2 + CR^2} + W \cdot \sqrt{BD^2 + RD^2}$$

Since $RD=CD-CR$, we have that $RD^2=CD^2-2CD \cdot CR+CR^2$ and the above equation becomes

$$V \cdot \sqrt{AC^2 + CR^2} + W \cdot \sqrt{BD^2 + CD^2 - 2\,CD \cdot CR + CR^2}$$

Since AC, BD and CD are constants, and we can take CR as the variable (since R is the variable place where the light changes the medium), one can look mathematically at the first derivative of the above function with respect to CR. This leads to the equation

$$\frac{V \cdot 2 \cdot CR\, dCR}{\sqrt{AC^2 + CR^2}} + \frac{W \cdot 2 \cdot (CD - CR)\, dCR}{\sqrt{BD^2 + CD^2 - 2\,CD \cdot CR + CR^2}} = 0$$

which after cancellation, and substituting the denominators back, gives

$$\frac{V \cdot CR}{AR} = \frac{W \cdot DR}{BR}$$

or, after reordering:

$$\frac{W}{V} = \frac{CR/AR}{DR/BR} = \frac{\sin(\beta)}{\sin(\alpha)}$$

Where *W/V* is a constant proportion, α is the angle between *AR* and the normal, and β is the angle between *BR* and the normal. This is then Maupertuis' (reversed) version of Snell's law. Maupertuis also provides a teleological interpretation of the PLA in the concluding passages of his paper. Maupertuis understood the PLA to mean that Nature is thrifty or economical in all its actions, i.e. that Nature aims not to do anything unnecessary or needless. This interpretation clearly ascribes some form of teleology to Nature, which for Maupertuis was moreover an expression of God's wisdom. Maupertuis says that

> One cannot doubt that everything is regulated by a supreme Being, while he has imprinted in matter forces which denoted power, has destined it to execute effects that mark his wisdom. And the harmony between these two attributes [i.e. power & wisdom] is so perfect, that undoubtedly all the effects of Nature could be derived from each one taken separately. The first of these ways [i.e. focussing on the properties of material bodies and the causes of their physical effects] is the one most within our reach, but does not take us far. A second type of ways [i.e. based on final causes] may lead us stray, since we do not know enough of the goals of Nature and we can be mistaken about the quantity that must be considered as the true expense of Nature in producing its effects.[32]

Even though Maupertuis says that it is more difficult to attain *knowledge* about final causes, he does affirm that they *are* inherent to the natural world. The basic idea in the above passage sounds very familiar.[33] Leibniz also fam-

32 *Ibid.*, 425-426.

33 HECHT 2001 discusses the relation between Leibniz' concept of possible worlds and the analysis of motion in eighteenth-century physics in more detail. It is important to point out that the comparison between Leibniz and Maupertuis is our own. Maupertuis never explicitly mentions his indebtedness to Leibniz. On the contrary, Maupertuis' knowledge of the Leibnizian positions, as he himself claims, is quite poor until the end of the 1740s.

ously claimed that all existent facts can be explained in two ways – through a kingdom of power or efficient causes and through a kingdom of wisdom or final causes.[34] This double method, which provides a role for final causes in physics, is stated explicitly in Leibniz' controversial text *Tentamen Anagogicum*, where he refers to the two 'kingdoms' which exist even in corporeal nature and says that they

> interpenetrate without confusing or interfering with each other - the realm of power, according to which everything can be explained mechanically by efficient causes when we have sufficiently penetrated into its interior, and the realm of wisdom, according to which everything can be explained architectonically, so to speak, or by final causes when we understand its ways sufficiently.[35]

Maupertuis, in a Leibnizian spirit, likewise saw his principle as affirmation of God's wisdom and the inherent teleology in nature. As TERRALL 2002 summarizes: isolated mechanical interactions might appear 'blind and necessary', but considered in a metaphysical context, they become part of «the designs of the most enlightened and free Intelligence».[36]

Maupertuis stressed the theological aspects more in the following years. On 6 October 1746, he read a paper to the Berlin Academy entitled *Sur les lois du mouvement et du repos déduites des attributs de Dieu*. The title of this paper is somewhat misleading, because it suggests that Maupertuis intended to provide an *a priori* proof of the laws of nature in the same manner as Descartes deduced the laws of motion from the immutability of God. Maupertuis was very critical of such *a priori* arguments. Probably, what he

34 Cf. HIRSCHMANN 1987.
35 LEIBNIZ 1890, Vol. 7, 273.
36 TERRALL 2002, 179.

really meant to say was that *divine wisdom* reveals itself when we take the dimension of *final causality in nature* into account. It is probable that this was one of Maupertuis' reasons to change the title of the published version of his paper into *Les lois du mouvement et du repos, déduites d'un principe métaphysique*.

2.3. *Les lois du mouvement et du repos, déduites d'un principe de métaphysique*

As was pointed out in the introduction, during his later years Maupertuis adopted a more speculative attitude. The metaphysical and theological consequences of his principle of least action became more important than the actual mathematics involved. This clearly emerges from his 1746 paper, which opens with a critique of the arguments from design or, as he calls it, «the proofs of God's existence drawn from the marvels of nature».[37] Maupertuis's criticisms are quite elaborate and I will not go through all of them.

His first and most serious opponent is Newton, who in the *Opticks*[38] claimed that the uniform motion of the planets reveals an Intelligent Designer. Maupertuis says that even though it is «extremely improbable that the six planets would move as they do»[39] the probability is not zero. The uniformity of planetary motion is not a *necessary proof* of an Intelligent Designer because it might be the result of *pure chance*. Maupertuis adds a second argument and says that if we were able to acquire a better knowledge of the cause of gravity and how it operates, we would no longer need to resort to God. He refers to the Cartesians for whom a certain «fluid transports the planets or at least

37 The full title of the first section of his paper is *Assessment of the Proofs of God's Existence that are Based on the Marvels of Nature*.
38 Newton, *Opticks*, 1717/1718, Third Book, Query 28, p 344-45, Query 31, p 377-378.
39 MAUPERTUIS 1746, 270-271.

regulates their motion»[40] and suggests that if we would have an explanation of this kind, invoking God becomes superfluous. Maupertuis did not actually present a physical causal explanation for gravity, but his point merely seems to be that *lack of knowledge does not warrant us to posit a necessary connection* between the planetary orbits and God's wisdom.

Maupertuis next attacks Newton's design arguments with respect to living beings. Maupertuis rejects the claim that God perfectly designed the organs of animals, and again suggest that *chance* might have «produced a countless number of individual animals, of which a few were constructed so that they could meet their own needs»[41] and a vast number of other individual animals «perished since their parts were not suitable for survival».[42] Nature is, furthermore, full of contradictory purposes.[43] The naturalist might be perplexed by the wonders of divine providence at work in Nature when he observes the development of a fly or an ant (i.e. in the growing-process the egg seals itself first in a chrysalis and then undergoes a metamorphosis). But, Maupertuis sarcastically adds, these wonders only seem to produce an insect that bothers human beings, will be eaten by a bird, or will get caught in a spider's web.[44] Putting aside such contradictory purposes in nature, Maupertuis admits that even though modern authors have acquired much more knowledge about the finer details and marvels of Nature, these results remain a very weak argument for the existence of God.

In the second part of his paper Maupertuis proceeds *positively* and ar-

40 *Ibid.*, 271.
41 *Ibid.*, 271-272.
42 *Ibid.*, 272.
43 Maupertuis was well-versed in these areas and worked himself as naturalist, cf. *Vénus physique* (1745) and the *Système de la nature* (1754).
44 *Ibid.*, 274.

gues that convincing proofs of God's existence must be based on general laws of nature, which are «founded on the attributes of a supreme Intelligence». Maupertuis' point is clear:

> We should not seek the supreme Being in little details, in the parts of the universe of whose connections we know too little of; rather, we should seek Him in universal phenomena that allow no exception and whose simplicity is entirely exposed to our view.[45]

Even though the human mind cannot comprehend the totality of all natural phenomena, mathematics is able to reveal an underlying order that coincidentally also reveals the divine wisdom.[46] The mathematical approach in itself is, of course, not new and many before Maupertuis exploited the power of mathematics. Maupertuis' key insight and innovation was to *understand mathematical extrema in teleological terms*, such that mathematics became a stepping stone to reveal the wisdom of God.

Before deducing the laws of collision (which are confirmed by experience), he briefly discusses the various debates on the *nature and cause of motion*[47] which had led to an impasse. According to Maupertuis «a true philosopher does not engage in vain disputes about the nature of motion; rather, he wishes to know the laws by which it is distributed, conserved or destroyed, knowing that such laws are the basis for all natural philosophy».[48] Maupertuis was not completely satisfied with some of the answers of his contemporaries. Leibniz' claim that living force (*vis viva*) was conserved, for

45 *Ibid.*, 277-278.
46 Cf. TERRALL 2002, 274-275.
47 Maupertuis briefly mentions questions such as whether motion exists at all, if force has a physical reality, whether motion is an essential property of matter or not, and whether we must make motion dependent on some prime mover or God.
48 MAUPERTUIS 1746, 283.

example, led some people to belief that truly inelastic hard bodies do not exist. Maupertuis rejected this position, arguing that the ultimate particles of matter must be infinitely hard and inelastic. Rather than taking sides with Descartes (and Newton) in the *vis viva* controversy, Maupertuis points out the limitations of *both* Descartes' conservation of quantity of motion (*mv*) and Leibniz' conservation of *vis viva* (mv^2), which each apply only in certain situations. Maupertuis claims that he has discovered a new *universal principle* which applies in all situation and is able to overcome the endless (metaphysical) debates in mechanics.[49] This universal principle is, of course, his own principle of least action,

> a principle so wise and so worthy of the supreme Being, and to which Nature appears to be constantly bound; which one observes not only in all changes, but in its constancy it still tends to observe it. In the collision of bodies, motion is distributed in such a way that the quantity of action is as small as possible, under the supposition that the change has occurred. In rest, the bodies that tend toward equilibrium have to be arranged in such a way that if they were to undergo a small movement, the quantity of action would be smallest.[50]

Every change and constancy in nature, Maupertuis explains, has an action associated with it, which can be defined as the product of mass, velocity, and distance. Maupertuis' definition of the PLA is similar to the one given in his optics paper[51], though, in a strange twist, he suggests that we must actually

49 With feigned modesty Maupertuis says «After all the great men who have worked on this matter, I almost dare not say that I have discovered the universal principle on which all these laws are founded; which extends equally to hard bodies and elastic bodies; on which the motion and rest of all corporeal substances depends». (MAUPERTUIS 1746, 286)

50 MAUPERTUIS 1746, 286

51 One of the *theoretical aims* in his previous work on optics was to subsume the three laws of optics under one higher *mathematical principle*. In 1746 the PLA is mainly seen as a cosmological-theological principle with *physical-ontological* content. His primary aim is not to unify a multitude of theoretical knowledge, but to reveal a unifying principle 'be-

consider the *distance travelled per unit time*. As Beeson points out, «the adoption of unit time is entirely arbitrary and unjustified by anything in the nature of the problem. Maupertuis resorts to it only because it gives him a convenient way to provide a quantity which he can identify».[52] The mathematical expression of the quantity of action thus becomes

$$Action_i = m_i \Delta v_i \Delta s_i = m_i (\Delta v_i)^2$$

and when there are more bodies involved in nature's change (i.e. in a collision) we need to take the sum of the respective action quantities as the price which nature needs to pay to realize some change. I will not present all the details of Maupertuis' mathematical argument here. The main idea has not changed. Maupertuis still maintains that «when a change occurs in Nature, the quantity of action necessary for that change is as small as possible».[53] In the case of hard body collisions, he easily deduces[54] that

$$m_1 (v_1 - v_f)^2 + m_2 (v_f - v_2)^2 \Rightarrow m_1 v_1 + m_2 v_2 = m_1 v_f + m_2 v_f$$

which corresponds to the conservation of linear momentum. In the case of elastic collisions he used the same technique to deduce the final velocities u_1 and u_2 of the colliding bodies respectively,

$$m_1 (v_1 - u_1)^2 + m_2 (u_2 - v_2)^2 \Rightarrow \quad u_1 = \frac{m_1 v_1 - m_2 v_1 + 2 m_2 v_2}{m_1 + m_2}$$

$$u_2 = \frac{2 m_1 v_1 - m_1 v_2 + 2 m_2 v_2}{m_1 + m_2}$$

which leads after some calculations[55] to the conservation of kinetic energy

hind' the diversity of natural phenomena.

52 Beeson 1992, 273.

53 Maupertuis 1746, 286.

54 The symbol ⇒ signifies «by minimizing this expression we can deduce that»

55 Some of these calculations are presented in a clear way in the Appendix of Beeson 1992,

$$m_1 v_1^2 + m_2 v_2^2 = m_1 u_1^2 + m_2 u_2^2$$

In his third and final application of his principle Maupertuis seeks the point about which two bodies remain in equilibrium. If *L* is the total length of the lever and *z* is the distance of one mass to the fulcrum then the distance for the other mass is *L–z*. Next he observes that when the lever rotates slightly, the two masses describe geometrically similar arcs whose length is proportional to their respective distance from the point of rotation. Because for Maupertuis these arcs also represent their speeds per unit time, he can just apply a similar technique as before to deduce the equilibrium-point:

$$m_1 z_2 + m_2 (L-z)^2 \Rightarrow z = \frac{m_2 L}{m_1 + m_2}$$

With these three examples Maupertuis illustrates the strength and scope of his principle, but he even expected it to have a wider application:

> We may admire the applications of this principle in all phenomena: the movement of animals, the growth of plants, the revolutions of the planets, all are consequences of this principle. The spectacle of the universe seems all the more grand and beautiful and worthy of its Author, when one considers that it is all derived from a small number of laws laid down most wisely [that is, in accordance with the PLA].[56]

At this stage of his career, Maupertuis did not pursue these other domains[57]

p. 273-276.

56 MAUPERTUIS 1746, 286-287.

57 Though he did not further develop his principle in other domains studying physical reality, one might say that Maupertuis was doing something 'similar' in his later moral philosophy where he introduced a *calculus of pleasure and pain*. In order to determine the good life and measure human happiness, Maupertuis proposed to consider the amount of pleasure and pain in terms of their intensity and duration. Taken together as a summation (over a whole life) this quantity should then be optimized in order to attain the good life. Even though his idea lacks precision, we do recognize a vague resemblance to the least-action principle.

(though Euler tried to apply it to the revolutions of the planets). Even though the principle of least action was Maupertuis' greatest accomplishment, it was only known to a limited audience. In order to make his ideas more known, he combined some of his previous material in his *Essai de cosmologie* (1750) which was meant for a broader audience.[58]

3. The compatibility of the mathematical and metaphysical aspects of the PLA

As has become clear from the previous discussion, the principle of least action has a Janus-face. On the one hand Maupertuis calls it a *principe métaphysique*, a fundamental principle directly dependent on the nature of God. On the other hand, he regards the PLA as a *mathematical principle* from which certain laws of nature can be deduced. As pointed out in the beginning of this paper, there is a certain tendency of commentators to prioritize the metaphysical aspect of Maupertuis' principle, and ascribe the proper mathematical formulation of the principle to his contemporary Euler or Lagrange. Such a perspective, however, begs the question as to how the mathematical and metaphysical 'fit together' in Maupertuis' own thought.

BEESON 1992 was the first to point out an important conceptual tension between the metaphysical and mathematical aspect of Maupertuis' account of the PLA. He remarks that in mathematical analysis one simply looks at zeroes of the 'derivative function' which algebraically comes down to the manipulation of symbols and solving an equation. But there is no mathematical reason why this procedure should yield a minimum.[59] Geometrically speaking, the

58 Cf. TERRALL 2012, 279 for a good discussion.

59 Whereas Maupertuis in his early work *Loi du repos des corps* (1740) speaks about minima

points on the curve[60] which satisfy this condition can be minima, maxima or inflexion points – these are all points where a function momentarily stops increasing or decreasing, i.e., where the first derivative is zero. From a purely mathematical point of view there is no intrinsic connection between extremum points and the human concept of economy. As Beeson puts it, in mathematics «nothing is being saved, held back, kept in reserve for some future application» and we should avoid interpreting «the quantities in question in terms of daily human experience».[61]

In addition to the conceptual dissonance, there is also a tension between the *a priori* and *a posteriori* status of the principle and the deductive link Maupertuis unconsciously seems to establish between the two levels. In his 1746 paper *Les lois du mouvement et du repos, déduites d'un principe de métaphysique*, Maupertuis writes that:

> I could have proceeded from the laws [of motion] given by mathematicians and confirmed by experience and looked there for marks of God's wisdom and power. […] I believe it is more certain and more useful to deduce these laws from the attributes of an all-powerful and all-wise being. If those that I find in this way are the same as those observed in the universe, would this not be the strongest proof that such a being exists and that he is the author of those laws?[62]

Maupertuis seems to suggest that the empirical-mathematical laws can be deduced from some *a priori* metaphysical-theological knowledge. However, we see no traces of such a strict deduction in Maupertuis' text[63]. One does find

and maxima (though, not about inflexion points), in his later work he only speaks about minima (which suits his interpretation of the principle of *least* action).

60 The curve we talk about here is not the trajectory of a moving body, but the curve representing the (equation of the) quantity of action.

61 BEESON 1992, 220, 268.

62 MAUPERTUIS 1746, 305.

63 It is not completely clear if Maupertuis has in mind the contemporary meaning of de-

such an approach in Descartes' *Principia philosophiae* where the laws of motion are deduced in a strict logical manner from God's immutability. One might wonder if Maupertuis really intended to *deduce* the laws from God's two attributes (his power and wisdom) or that he only meant to deduce the laws of motion from the principle of least action which is, in a derivative sense, an expression of God's wisdom. Furthermore, one might wonder to which extent the principle of least action is something we can really know *a priori* (without relying on experience). Even though our understanding might have *a priori* an intuitive grasp of the metaphysical principle of simplicity or efficiency, Maupertuis for sure relied on experience to give the PLA its precise and quantitative formulation. I argue that Maupertuis' claim that we have some kind of *a priori* knowledge of the attributes of God (i.e. his wisdom qua wisdom) or the PLA (i.e. how this wisdom manifests itself in nature), and that from this knowledge we can deduce subsequently the physical laws of nature, is untenable. The mathematical expression depends on the nature of each problem, whereas the metaphysical version is a general statement which lacks determination. The difference between the universal and the particular, between the *a priori* and *a posteriori*, was also noticed by Euler who remarked that:

> we are still very far from that degree of perfection where we are able to assign, for each effect which nature produces the quantity of action which is the smallest, and deduce it from the first principles of our knowledge; and that it will be almost impossible to arrive at it unless we discover, for a great number of different cases, the formulas which become maximal or minimal.[64]

duction. Perhaps he only believes that the PLA can be confirmed in the laws of nature with the help of both mathematics and observation. Admittedly, his terminology is highly problematic.

64 Cf. the Appendix to Euler 1744.

For Euler the fundamental question was how the various mathematical forms of the principle, i.e. the formulas in statics referring to point masses and fluids, the formula for the curvature of a rod, and the formulas in dynamics are related to each other.[65] Stimulated by his correspondence with Maupertuis, Euler wanted to find a 'general mathematical form' behind the manifold of physical problems. Maupertuis did not pursue this mathematical question, and as pointed out above, his cosmological generalizations and theological claims were made in an intuitive *a priori* manner.

A third tension between the mathematical and metaphysical aspect of the PLA is provoked by the following passage of Maupertuis' 1746 paper:

> If it is true that the laws of motion and equilibrium are indeed absolutely necessary consequences of the nature of matter, that proves all the more the perfection of the supreme Being. Everything is so arranged that the blind logic of mathematics executes the will of the most enlightened and free Mind.[66]

One might wonder how the PLA can at the same time be a *logical mathematical necessity* and an expression of the *freedom of God*. Maupertuis seems to make this startling claim casually, but he had thought about the issue earlier in his career. In 1732 Maupertuis meditated in the work *Sur les loix de l'attraction* in a Leibnizian manner on Newton's inverse-square law of attraction. Without actually committing himself metaphysically – but always stating his reasoning conditionally – Maupertuis tried to understand God's possible reasons for choosing the particular form for the mathematical law:

> If God had wished to establish a law of attraction in nature, why would this law

65 Cf. BOUDRI 2013, 164-167 for a brief discussion of this topic.
66 MAUPERTUIS 1746, 303.

> follow the proportion that it seems to follow? […] In the infinity of different relations that seem to have an equal right to being at work in nature, was there some reason to prefer one over another?[67]

Maupertuis, after some reflection, suggests that the geometrical properties of the law can be associated with the criterion of uniformity. He notes that God must have wanted symmetrical macroscopic bodies like spheres «to exhibit the same property of attraction that characterized their smallest particles to attract in the same proportion on all sides».[68] After having established uniformity as a relevant criterion, Maupertuis remarks:

> Once the metaphysical reason for preference was posited, mathematical necessity excluded an infinite number of systems, in which there could not be agreement of the same law in the parts and in the whole.[69]

This passage makes intelligible how the freedom of God is reconcilable with the necessity of mathematics. In a quasi-Leibnizian manner, Maupertuis suggests that God is said to first choose among an infinity of possible worlds,[70] having a certain criterion in mind, namely, uniformity and symmetry of the laws of nature. But once such a world is chosen or actualized, the logic of mathematics determines the necessary form the laws of nature need to have. I argue that Maupertuis could have repeated this same argument in 1750. If the least action principle is indeed the expression of the free will and wisdom of God, the necessary mathematical form embodied in minimizing the quantity

67 MAUPERTUIS 1732, 346–47.
68 Cf. TERRALL 2002, 80-81 for a more elaborate discussion.
69 *Ibid.*, 347.
70 Whether Maupertuis would use the notion of possible worlds in exactly the same sense as Leibniz (as a logically possible collection of interconnected events) is unclear. We do, however, observe an important similarity here.

of action do not contradict the free choice of God but instead realize his choice and wisdom.[71]

In the fourth and last point of this section, we look more closely at the epistemological status of mathematical and metaphysical knowledge with respect to Maupertuis' *skepticism* and *phenomenalism*. It has been pointed out that Maupertuis radicalized the thought of Locke and Berkeley in order to counter the prevailing mechanistic and materialist philosophies.[72] Maupertuis claimed that we cannot have any certain knowledge of external objects – not even what *causes* our sensations – and that all our knowledge is just an ordering of the content of our mind. Maupertuis' skepticism and 'ultra-phenomenalism' (G. Tonelli), however, does not make him a relativist. In his 1756 paper *Examen philosophique de la preuve de l'existence de Dieu employee dans l'Essai de cosmologie* Maupertuis mentions the universal agreement among all men with respect to the propositions of mathematics, and in the first part of this work, *Sur l'evidence & la certitude mathématique*, he further elaborates on the demonstrative certainty of mathematics and its relevance for physics. According to Maupertuis, the certainty of mathematics is grounded in the homogeneity and replicability[73] of the elements with which the mathematician works (e.g.

71 This third point resolves the paradox in the above passage in ontological-modal terms (*how the world could have been or must have been*). The necessity of mathematics can also be addressed from an epistemological perspective (to which extent are mathematical propositions contingent or necessary with respect to the *knowing subject*). This perspective will be taken up in the fourth point above.

72 Cf. the monograph of TONELLI 1987. Also the article of GOSSMAN 1960 makes many interesting points, especially on the role of mathematics in Maupertuis' epistemology.

73 Cf. BEESON 1992 for a brief summary of this topic, i.e. «Mathematics deals with the clearest possible ideas, those of number and extent. These ideas share two properties distinguishing them from all others: one is that of being replicable, by which Maupertuis means that they can take any degree, being multiplicable or divisible any number of times; the other is that they alone are ideas that are supplied by more than one sense (extent is perceived by means of both sight and touch, number by all the senses). Maupertuis proposes that no other property is replicable (that is, quantifiable); those

number and extension).[74] However, mathematics is nothing more than a system of signs, referring to the forms of our perception, and provides no knowledge of (the nature of) external things, nor does it reflect some eternal Platonic truths. The same argument also holds for any physical science which bases itself on purely mathematical principles. Maupertuis says that if the laws of movement could be deduced from the propositions of arithmetic and geometry, they would be necessary laws, but only « du même genre de nécessité » as mathematical propositions themselves.[75] Hence, a purely mathematical reading of the PLA might at best produces a 'subjective necessity' within the human mind, but it cannot produce knowledge about the external physical world. We can, however, ask ourselves to which extent Maupertuis' metaphysical interpretation of the PLA – given his epistemology – is able to grasp physical reality.

In the *Essai de cosmologie* Maupertuis made the controversial claim that «the laws of motion and equilibrium [and by implication also the PLA] are indeed absolutely necessary consequences of the nature of matter».[76] The most vigorous attack against this statement was that of Samuel Reimarus, who in his 1754 book *Die vornehmsten Wahrheiten der natürlichen Religion* argued that the approach taken in Maupertuis' *Essai de cosmologie*, instead of proving the existence and wisdom of God, led to necessitarianism and Spinozism. In the

that seem so in fact possess sub-properties that are reducible to number or extent: for example, variations in the 'blueness' of a colour reflect variations in the quantities of indigo it contains. All this gives mathematics its particular qualities of certainty and self-evidence» (258)

74 « Dans les sciences mathématiques ou les objets, les nombres & l'étendue, sont exactement réplicables, on forme des résultats dont tout le monde convient; parce que c'est sur des sujets qui sont pour tout le monde précisément les mêmes: on est encore plus content de la manière dont soi-même on les conçoit; & c'est en cela que consiste l'évidence & la certitude » (MAUPERTUIS 1746, 399)

75 Cf. GOSSMAN 1960, 319.

76 MAUPERTUIS 1746, 303.

second part of his *Examen philosophique*, which we already mentioned, Maupertuis reconsidered some of his earlier statements and now says that we cannot make any claims about whether or not the laws of motion or the least-action principle are necessary properties of matter. This revised position coincides with his agnosticism of twenty year before. In his earlier work *Discours sur les différentes figures des astres* (1732) Maupertuis also addressed the question whether or not gravity is a necessary property of matter. His answer in 1756, with respect to the laws of motion or the PLA, is the same: we are unable to make such modal claims. Maupertuis, nonetheless, suggests that perhaps more knowledge about the nature of matter might show that least action is a necessary consequence of the essence of matter. But he argues that the same could be said of all known truths.[77] In search for a way out of this agnosticism, he proposes to present a history of the laws of mechanics. He discusses a variety of authors (Leibniz, Newton, Malebranche, Descartes, Huygens) and their respective experimental, metaphysical and mathematical commitments. Maupertuis admired Huygens' approach and following his attitude he started to refer to his least-action principle as an *empirical hypothesis*, «a law of nature supported by observational evidence rather than by abstract mathematical reasoning».[78] Taking these remarks in mind, during the last years of his life, we might wonder to which extent the PLA should still be called a metaphysical principle. As pointed out above, with 'metaphysical' Maupertuis does not mean a (metaphysical) claim about the necessary properties of matter, but perhaps he uses the word in a theological sense (i.e. concerned with the divine attributes). Beeson points out the importance of such a theological reading with respect to Maupertuis'

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77 MAUPERTUIS 1756, part II, article LXXI, 424 ; BEESON 1992, 260, f51
78 Cf. MAUPERTUIS 1756, Article LXIX, 423 ; BEESON 1992, 260.

> man's incapacity to know anything with any certainty by his own powers brings him back into dependence on God. Yet, like the bedrock of certainty Descartes sought under the shifting sands of doubt, so through the dark glass of Maupertuis's phenomenalism can be glimpsed the one incontrovertible source of truth, God.[79]

It is possible that the principle of least action qua metaphysical-theological *principle* was for Maupertuis a rare but proper foundation for our knowledge about the fundamental nature of reality. This *metaphysical knowledge* is different in nature from empirical knowledge (which consists of an ordering of our sensations through language) and mathematical knowledge (which as we stated above only has a subjective necessity in our minds).

4. Conclusion

Whereas commentators either ignore or tend to take for granted the Janus-face of the PLA, I have tried to clarify the intricate relationship between the mathematical and metaphysical aspects of the principle. I have done so by approaching the issue from four different perspectives. First of all, I pointed out that on a purely conceptual level the mathematical and metaphysical interpretation do not collapse, since the mathematics involved not only yields minima, but also includes maxima and inflection points. Secondly, I discussed a tension between the *a priori* and *a posteriori* status of the principle. On the one hand the principle is an *a priori* intuition of the greatest generality but, on the other hand, its mathematical form depends on the nature of each

79 BEESON 1992, 160.

physical problem and thus requires experience. Thirdly, on the level of modality, there *prima facie* seemed to be a tension between the *necessity of mathematics* and the *freedom of God*, but I have suggested a way Maupertuis could have replied to this tension. Finally, by looking more closely to Maupertuis' *epistemology* another difference was revealed. Whereas mathematical knowledge only has a necessity restricted to the human mind, the metaphysical interpretation seemed to have a privileged theological-epistemological status, which for Maupertuis could possibly have been a way to break through the veil of his own skepticism and phenomenalism, and gain knowledge about the fundamental nature of external reality. To which extent Maupertuis was aware of all these tensions, we can only guess, but I hope that the historical perspective I have adopted has shed light on the complex interaction between philosophy and mathematics in eighteenth-century science.

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VRIJE UNIVERSITEIT BRUSSEL

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