A Note on the Transitivity of Parthood

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Abstract. That parthood is a transitive relation is among the most basic principles of classical mereology. Alas, it is also very controversial. In a recent paper, Ingvar Johansson has put forward a novel diagnosis of the problem, along with a corresponding solution. The diagnosis is on the right track, I argue, but the solution is misleading. And once the pieces are properly put together, we end up with a reinforcement of the standard defense of transitivity on behalf of classical mereology.

1. Is parthood transitive? Classical mereology says it is. If x is part of y and y is part of z, then x is part of z—no matter what x, y, and z are. Indeed, on standard models of extensional mereology, parthood is isomorphic to set inclusion, and there is no question that the latter relation is transitive. Nonetheless, an increasing number of authors have been protesting that many legitimate senses of 'part' appear to violate this principle. Typical examples would include:

(1) The handle is part of the door, which in turn is part of a house, but the handle itself is not part of the house.
(2) The nucleus of a cell is not a part of the organ of which that cell is part.
(3) An arm may be part of a musician who is part of an orchestra, yet no arm is part of an orchestra.

(See [11, 7, 9], respectively). Authors who find such examples convincing have offered various accounts of the underlying phenomena. For example, it has been suggested that 'part' is context-sensitive [3], or that transitivity only holds relative to certain conditions [8], or that there really are several meronymic relations, only some of which are transitive [5], or again that each such relation can be meaningfully (and transitively) predicated only relative to entities of a restricted sort, such as individuals, groups, quantities, etc. [10, 14]. One way or the other, the failure of transitivity as a general part-whole principle would appear to have important philosophical ramifica-

tions. Among other things, it would be detrimental to the claim, familiar from the philosophical literature, that the parthood relation modeled by classical mereology is truly governed by formal ontological principles, i.e., principles that are metaphysically neutral and domain-independent and, therefore, realized or exemplified by any entities whatsoever.

There is, however, a standard way of resisting such conclusions on behalf of classical mereology [12, 2]. Broadly speaking, the reply moves from the consideration that the putative counterexamples stem from the implicit narrowness with which we may be inclined to use the relational predicate 'part' in ordinary discourse. What counts as a *functional* part of a door—the handle—may not count as a functional part of the house, but that is not to say that it is not part of the house at all. On the contrary, it has all those features that a genuine part is supposed to have: the handle contributes to the mass and shape of the whole house, it occupies part of the space occupied by the house, it gets annihilated if the house is annihilated, and if it gets annihilated the house itself changes (albeit insignificantly). Similarly, if there is a sense of 'part' in which the musician's arm is not part of the orchestra, or a biological subunit part of the organ to which it belongs, it is a narrow sense: the arm is not *directly* part of the orchestra, though it is directly part of a body that is directly part of the orchestra; biologically the nucleus is not a *distinguished* part of the organ, though it is a distinguished part of the cell and the cell a distinguished part of the organ. It is obvious that if the interpretation of 'part' is narrowed by additional conditions (e.g., by requiring that parts make a functional, direct, or otherwise distinguished contribution to the whole), then transitivity may fail. In general, if x is a ϕ-part of y and y is a ϕ-part of z, x need not be a ϕ-part of z: the predicate modifier 'ϕ' may not distribute over parthood. But that shows the non-transitivity of 'ϕ-part', not of 'part', regardless of whether 'ϕ' is explicitly mentioned in our statements and regardless of whether the corresponding relation can be treated as a significant case of meronymy. (It may also be that different narrowing conditions are operating within the same context: obviously the fact that the heart is a distinguished part of a musician, and the musician a direct part of an orchestra, does not imply that the heart is in any of these senses part of the orchestra.)

Now, I for one have always found this reply perfectly satisfactory, and I wouldn't feel the need to say anything more in its support. In a recent, informed discussion of the problem, however, Ingvar Johansson [6] raises a

novel challenge. He objects to the anti-transitivity accounts mentioned above, but he also objects to the reply just outlined insofar as it would give rise to an "odd subsumption relation":

> What is true of 'red' is necessarily also true of the 'light red' which it subsumes, what is true of 'running' is necessarily also true of 'running quickly', and what is true of 'x is part of y' ought necessarily [to] be true of 'x is a ϕ-part of y'. Since 'x is part of y' is transitive, 'x is a ϕ-part of y' ought to be so as well. (p. 165; notation adapted)

This objection, I think, misfires, but it is worth explaining why. In addition, it is worth looking into Johansson's own positive account of the matter. For the account itself may seem to offer what everybody is looking for: on the one hand, Johansson reads (1)–(3) and the like as involving truly non-transitive parthood predicates; on the other, he analyses such predicates as involving a departure from the scope of classical mereology:

> There are both intransitive and non-transitive parthood predicates, but […] when examined more closely, these predicates are at least as complex as so-called relative products of other binary relational predicates or as ternary predicates. Only truly *binary* parthood relations are necessarily transitive. (p. 162)

I think this account is flawed, too, though it involves a kernel of truth that deserves some consideration.

2. Here is why the objection misfires. There is, to be sure, something perfectly intuitive in the general subsumption principle that Johansson is assuming:

(4)   When a concept is more specific than some other concept, it inherits the properties of the more general one

(from [1], p. 5). However, it is obvious that a lot depends on how exactly this principle is cashed out, and no reasonable way of doing so justifies the charge that parthood and ϕ-parthood behave oddly.

For one thing, the relevant notion of property must be robust, i.e., (4) cannot be taken to imply that whatever is *true of* a general concept must be true of the more specific one. Surely, if it is true that 'red' stands for a color, then 'light red' must stand for a color, too. But although 'red' can properly

be predicated of the wine in this glass, the same is not true of 'light red'. And although 'red' is more general than 'light red', the same cannot be true of 'light red' itself.

Second, and more to the point, one must distinguish between the properties of a concept as such and the properties of those entities that fall under the concept. Plainly, (4) *must* hold in regard to the second sort of property: if every red object happens to be round, for instance, then *a fortiori* every light-red object is round. It is equally apparent, however, that (4) may fail in regard to the first sort of property: 'red' *subsumes* 'dark red' but 'light red' does not; 'tall' is *vague* but 'one meter tall' is not; 'square' is *consistent* but 'round square' is not. In particular, it is clear that when it comes to relational concepts, greater specificity may come at a cost: a relation may be closed under properties that its subrelations fail to inherit, owing to the lack of suitable relata. Functionality (for example) is necessarily inherited, but reflexivity, symmetry, transitivity, connectedness, density, etc. are not. Thus, either we give up (4) or else we say (correctly, I think, and in agreement with standard practice) that this principle is meant to apply, not to the properties of the concepts themselves, but to those properties that are characteristic of the items in their extensions.

So, now, what about the case at issue? Does the standard line of defense on behalf of mereology violate the subsumption principle (4)? It does not. It says that ϕ-parthood is more specific than parthood, i.e., that 'ϕ' is a predicate modifier with the effect of narrowing the extension of 'part', and pretty clearly any property that is true of each ordered pair in the original extension is going to remain true of those pairs that survive the narrowing. Thus, (4) is trivially satisfied for every such property, regardless of the particular value of 'ϕ'. On the other hand, precisely because of the narrowing, the transitivity of parthood need not be inherited by ϕ-parthood, but we have just seen that this is *not* the sort of property that can plausibly be said to fall within the scope of (4). Hence, no oddness follows from this fact, except of course that it would be a mistake to ignore the narrowing and identify ϕ-parthood with parthood *simpliciter*, or with ψ-parthood for a different condition 'ψ'. Besides, it would be remarkable indeed if things were otherwise. For by the same pattern one could argue that there is an "odd subsumption relation" linking (for example) parthood and *proper* parthood: the former is reflexive, the latter isn't; yet the former is more general than the latter. Surely that would be a poor argument against the thought that the modifier

'proper' in 'proper part' yields a narrower interpretation of the general, re-flexive predicate axiomatized by classical mereology.

3. So I think that Johansson's objection misfires. Nonetheless, I agree that more could (and should) be said about the sort of "narrowing" that is at issue in the putative counterexamples. Standardly, a predicate modifier is just an-other predicate. A male parent is someone who is both a parent and male; a blue shirt is something that is both a shirt and blue. It is well known, however, that not every predicate modifier can be analyzed along these stan-dard lines: a fake diamond is not a diamond, an alleged thief need not be a thief, and a good pianist need only be good *qua* pianist. (See [4] for an overview.) In fact, even the examples used by Johansson seem to run afoul of the standard cases: a light red object may be darker than a dark yellow one, and someone may be running quickly while journeying slowly. So how exactly is the relevant predicate modifier to be handled when it comes to the various cases of ϕ-parthood exhibited by the putative counterexamples to transitivity?

Clearly, 'ϕ' cannot be an irregular modifier, like 'fake' or 'alleged', otherwise ϕ-parthood would not qualify as a *specification* of parthood. But 'ϕ' cannot be just a regular modifier either, like 'male' or 'blue'. For then the standard conjunctive analysis would go through, and on such an analysis transitivity would have to hold. If 'x is a ϕ-part of y' meant 'x is part of y and x is ϕ', then the relevant transitivity statement would have the form:

(5) $(Pxy \mathbin{\&} \phi x) \mathbin{\&} (Pyz \mathbin{\&} \phi y) \rightarrow (Pxz \mathbin{\&} \phi x)$,

and surely this statement is a logical consequence of the transitivity of 'P', parthood:

(6) $Pxy \mathbin{\&} Pyz \rightarrow Pxz$.

So what sort of modification do we have in cases such as (1)–(3) and the like? Here is where Johansson's positive account comes into the picture.

With reference to the example in (1), Johansson offers the following analysis. When we assert that a handle, x, is a functional part of a door, y, we mean to assert that x is part of y *and* that there is a z (the door's panel) such that acting on x makes something happen to z that is relevant for x's function in relation to y (such as opening and closing y). Modulo minor details, this strikes me as right on target. The modifier 'functional' does not attribute a

5

monadic property, like 'male' or 'blue'; it attributes a relational property. And this relational property, F, need not itself be transitive: x may be F-related to y and y to z, but there is no guarantee that x is F-related to z.

Of course this is just the beginning. The "minor details" I mentioned concern questions such as the following:

(i)   Should this account require that y be a functional unity or an integral object of some kind? (Johansson thinks so, for otherwise we would get the odd result that the handle is a functional part of the whole solar system, but I am not sure this is right: on the one hand, the handle does not have any function in relation *to* the solar system; on the other, it seems to me that the solar system *is* an integral or functional whole anyhow.)

(ii)   Should we require that x be always in contact with z? (Johansson says so, but I am inclined to disagree: presumably my mouse is a functional part of my computer, even though it is wireless and, therefore, spatially disconnected from the rest.)

(iii)   Should we require that z be always a third entity, distinct from x and y? (My understanding is that Johansson would think so, and surely this seems right in the door handle case. But what about the handle of a walking cane, for example: isn't that a case in which $z = y$? Or consider the door panel: arguably, that is a functional part of the door, too. And doesn't pushing the panel—at least in some cases, as with ordinary grocery-store doors—make something happen to the panel itself that is relevant to its function in relation to the door, i.e., isn't that a case where $z = x$?)

These are not easy questions to answer, especially in the absence of a substantive theory of what functionality amounts to (by itself a challenging task; see for instance [13]). But never mind that. At least as a first approximation, it certainly seems to me that the account is on the right track: in cases such as (1), the implicit predicate modifier does not attribute a monadic property to the part; it attributes a relational property to the part-whole pair. And this should suffice to dispel once and for all the confusion surrounding the transitivity issue—as a clarification of the standard account, not as an alternative.

Unfortunately, Johansson does not think so. He regards the analysis just outlined as involving a major deviation from the standard account. For he maintains that the analysis brings out an important feature of functional parthood that has hitherto been neglected, and according to Johansson it is precisely the failure to appreciate *that* feature that explains the confusion:

> When we claim that a handle (x) is a functional part of a door (y), we seem to be using a binary relational predicate. In fact, however, we are using a predicate that contains a relative product and that, therefore, involves at least three relata (x, y, and z). And the same applies to the door-to-house case, too. […] The sentences 'The handle is a functional part of the door' and 'The door is a functional part of the house' fall outside mereology as the theory of the binary parthood relation. (p. 173)

This strikes me as both incorrect (in the diagnosis) and independently confusing (in the verdict).

First of all, standardly a relative product of two relations R and S (also known as composition) is defined as follows:

(7) $R/Sxy =_{df} \exists z(Rxz \ \& \ Szy)$.

In other words, the tie between the two relata is guaranteed by a third, middle term. The relational predicate 'aunt' is in this sense a relative product of the predicates 'sister' and 'parent'. But surely 'functional part' does not, on the proposed analysis, have this form. Let '$Mxyz$' abbreviate the predicate 'x makes something happen to z that is relevant for x's function in relation to y', and define

(8) $Fxy =_{df} \exists z(Mxyz)$.

Then the analysis of 'x is a functional part of y' amounts to the following:

(9) $FPxy =_{df} Pxy \ \& \ Fxy$.

And this is not a relative product, not even a "qualified" relative product (i.e., a product involving further qualifications on the middle term). It is just a conjunctive predicate. Johansson is right in pointing out that 'F' involves "a hidden and indefinite reference to a third relatum" (p. 171), namely, the door panel. But *pace* Johansson, this third relatum does not act as a middle term; it is just an item that needs to be mentioned in the explicit definition of 'F'. Moreover, the relatum in question is bound by a quantifier, so obviously its showing up in (8) does not affect the number of arguments of the definiendum, which is perfectly binary. This is true also of binary products, of course: the definiens in (7) involves three variables; but the third variable, z, is bound, so we end up with a relation of the same arity as R and S.

Accordingly—and this is the confusing bit—Johansson cannot go on to say that 'functional part' picks out a qualified relative product which, as

such, is akin to a ternary relation (p. 172). Whether or not we think of it as a product, 'F' is just as binary as 'P', hence we cannot conclude that the putative counterexample to transitivity exhibited by (1) founders because of the peculiar nature of the relevant relation—a relation that involves "at least three relata … not just two, as in the parthood relation of mereology" (p. 170). It founders because the logical form of (1) does not correspond to the transitivity axiom, (6), but to the conditional

(10) (Pxy & Fxy) & (Pyz & Fyz) $\rightarrow$ (Pxz & Fxz).

And this conditional, unlike (5), is logically independent of (6) and of the other axioms of mereology.

4. My conclusion is therefore twofold. Not only does Johansson's objection to the standard account misfire. His own account is incorrect, too—and misleading. It is incorrect and misleading in spite of the correct and fitting intuition on which it rests. Of course, I have only illustrated these claims in connection with (1), but I would argue that similar considerations apply to Johansson's treatment of the other putative counterexamples to transitivity. In his paper he offers a thorough analysis of such predicates as 'direct part', 'distinguished part', etc., each of which involves its own subtleties. I leave the details to the interested reader. For what it's worth, my assessment is the same in each case. The basic analysis is correct, but the ensuing picture is defective. And once the picture is fixed along the lines illustrated above, we do not end up with an alternative account of the matter; we just have a more careful, accurate formulation of the standard account on behalf of classical mereology. Parthood is transitive; ϕ-parthood—for many values of 'ϕ'—is not.

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