

Vagueness

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What is vagueness?

Standardly, one says that vagueness arises whenever a concept or linguistic expression admits of borderline cases of application. A predicate such as ‘bald’, for example, is vague because there can be situations in which it is indeterminate whether or not it applies to (a name of) a certain object. Some people are clearly bald (Picasso), some are clearly not bald (the count of Montecristo), and some are borderline cases—our concept of baldness and our linguistic practices do not specify any exact number of hairs that marks the boundary between the bald and the non-bald. Similarly, a singular term such as ‘Mount Everest’ is vague because there is no determinate way of tracing the geographical limits of its referent. Some rocks are clearly part of Everest and some are clearly not, but some rocks enjoy a borderline status.

There is, however, dispute concerning this way of characterizing vagueness. For the statement that a term t admits of borderline cases of application—that it is indeterminate whether or not such-and-such objects fall within the boundaries of the entity designated by t —can be given a *de re* reading, as in (1), or a *de dicto* reading, as in (2):

- (1) The term t designates an entity x such that it is indeterminate whether such-and-such objects fall within the boundaries of x .
- (2) It is indeterminate whether the term t designates an entity x such that such-and-such objects fall within the boundaries of x .

On the first reading the indeterminacy is ontological. The predicate ‘bald’ is vague, on this reading, because it stands for a vague set: there is no objective, determinate fact of the matter about whether the borderline cases are included in that set (or about whether they enjoy the corresponding property). Likewise, on this reading ‘Mount Everest’ is vague because it stands for a genuinely vague denizen of reality:

there is no objective, determinate fact of the matter about whether the borderline rocks are part of the mountain. There may also be no determinate fact of the matter about when the mountain itself came into being, for the temporal boundaries of an object may be vague too.

By contrast, the *de dicto* reading corresponds to a purely linguistic (or conceptual) notion of vagueness. On this view the set of bald people is not vague at all. There are exactly 2^n sets of people (where n is the number of all people at the present time), each with its perfectly precise membership function; yet our linguistic stipulations do not fully specify which of those sets can do duty for the extension of the predicate 'bald'. There is, similarly, no vague mountain on this view: instead there are plenty of aggregates of matter, each with its precise spatio-temporal boundary, and when we say 'Mount Everest' we are just being vague as to which such aggregate we are referring to.

The two views are not strictly incompatible, at least insofar as one may be willing to treat some vagueness as ontological and some as linguistic. One could also construe some terms as involving both sorts of vagueness: it would be indeterminate which particular sets or objects those terms designate, and the relevant candidates would include vague specimens along with sharp ones. However, these ways of combining ontological and linguistic vagueness have attracted little attention and current views on vagueness divide rather clearly between one approach and the other. (See Tye 1990 and Lewis 1993, respectively, for two representative position statements, and Evans 1978 for a much debated way of setting up the issue.)

Problems and paradoxes

Insofar as vagueness involves borderline cases, whether *de re* or *de dicto*, it manifests itself semantically in the generation of truth-value gaps. If Jones is a baldish person, then the statement that he is bald appears to lack a definite truth-value; if it is indeterminate whether this rock is part of Everest, then the statement that it is part of Everest is likewise neither true nor false. This has been a natural source of concern for philosophers and logicians since Frege, for the admission of truth-value gaps amounts to a failure of the classical principle of bivalence.

The main source of concern, however, is that vagueness precipitates a deep puzzle. For not only do vague terms involve borderline cases. They also seem to involve borderline borderline cases, or borderline borderline borderline cases. For example, just as there is no sharp line between the bald and the non-bald there does not seem to be any sharp line between the bald and the baldish (or the baldish-ish). In neither case can a single hair make a difference. Intuitively, this means that our notion of baldness satisfies the following principle:

- (3) For every n : if a man with n hairs on his head are bald, then a man with $n + 1$ hairs on his head is also bald.

(Let us suppose that baldness supervenes exclusively on the number of hairs.) However, it is enough to combine this principle with

- (4) A man with no hairs on his head is bald.

to reach the paradoxical conclusion that

- (5) A man with 500,000 hairs on his head is bald.

(This can be shown by 500,000 repeated applications of the rules of universal instantiation and *modus ponens*.) In other words, the intuition that the applicability of ‘bald’ cannot be a matter of a single hair seems to force us to reason from the true premise that Picasso is bald to the false conclusion that the count of Montecristo is also bald. And a corresponding point can be made about the intuition that the applicability of ‘Everest’ is not a matter of millimeters. In both cases it’s hard to come up with a diagnosis, but the clash between logic and intuition is deep.

In its oldest form, this problem is known as the *phalakros* puzzle (from the Greek word for ‘bald’) and is attributed to Eubulides of Miletus, a contemporary of Aristotle. Eubulides is also standardly credited with the formulation of the *sorites* puzzle, which builds in a similar way on the vagueness of ‘heap’ (Greek: *soros*). There are also versions of the puzzle that rely on a different way of expressing the inductive principle (3). For example, already the Stoics considered replacing the embedded conditional ‘if ... then’ with a negated conjunction (it is not the case that a man with n hairs on his head is bald and a man with $n + 1$ hairs is not bald). This makes the paradox even harder, since one cannot just blame the material conditional for the trouble. Another common variant involves replacing (3) with a long chain of conditionals (or negated conjunctions), one for each relevant n . Again this makes the puzzle more robust, since one cannot blame the universal quantifier for the trouble. As it turns out, today one does not make a big deal of these differences and all such puzzles are collectively referred to as sorites puzzles. The problem with vagueness is that it yields such puzzles, in some form or other. And theories of vagueness are naturally compared on the basis of how successful they are in providing a systematic way out.

Theories of vagueness

Broadly speaking, there are two strategies for dealing with the sorites paradox. Focusing on the version exemplified by (3)–(5), one can either (i) reject the argument

as invalid, or (ii) reject it as valid but unsound. (One can also bite the bullet and accept the conclusion, but few would be willing to go that far.)

Strategy (i) comes in two main varieties. On the one hand, one can just insist that logically valid reasoning can only be formulated in a precise language. This was, for example, the response advocated by Russell (1923) in the first full-length paper devoted to the topic of vagueness. Today this is not a popular position because it enforces an intolerable restriction on the scope of logic: since many of the words that we use in ordinary discourse (as well as in much scientific discourse) are vague, logic would be of very little practical use. On the other hand, one can question the validity of the sorites argument by questioning the adequacy of *classical* logic. Here a popular stance is to adopt some kind of many-valued logic in which statements are allowed to take intermediate truth-values and in which the validity of the inference of (5) from (4) decreases as the number of application of *modus ponens* increases.

In fact, because the notion of a borderline case is itself vague, a natural implementation of this strategy allows for a continuum of intermediate truth-values and the result is a fuzzy logic in which sentential connectives, for example, are represented by operations on the real numbers in the interval $[0,1]$ rather than on the two-valued truth set $\{0,1\}$. (See e.g. Machina 1976.) If vagueness is thought of as an ontological phenomenon, this account is naturally combined with a fuzzy semantics in which a predicate, for example, is assigned an extension whose membership function is itself continuum-valued. The closer to 1 the value is, the more the argument is a member of the set (Zadeh 1965). This way of proceeding allows one to resolve the paradox as follows. First, the connectives are characterized so that a conditional of the form

- (6) If a man with n hairs on his head is bald, then a man with $n + 1$ hairs on his head is also bald

is sure to come out true or nearly true. For example, on a popular account the truth-value of a conditional is set equal to 1 minus the surplus of the antecedent over the consequent (if any). Second, there will be values of n such that the truth-value of the antecedent of (6) is slightly higher than that of the consequent. The underlying intuition is that one hair does make some difference after all, albeit a very small and negligible difference. Thus, as long as validity is defined so that the conclusion of a valid argument must be at least as true as each of the premises, the relevant instances of *modus ponens* will be invalid whenever the truth-value of the antecedent is less than or equal to the truth-value of the conditional but (slightly) greater than the truth-value of the consequent. The paradox arises because the error is so small as to be undetectable, and yet it compounds each time a new application of *modus ponens* is invoked.

This account has a certain *prima facie* appeal but it is open to a number of objections. For one thing, the fuzzy-theoretic machinery appears to replace vagueness with extremely refined precision. To what degree, exactly, is it true that Jones is bald? To degree 0.6? Perhaps to degree 0.59? Or maybe 0.5999? Second, the assumption of a totally ordered set of truth-values is itself problematic. How does the degree to which Jones is bald compare to the degree to which Smith is tall? How does it compare to the degree to which a certain borderline rock is part of Everest? Third, there is the embarrassing presupposition that a point must still exist where one goes from fully fledged truth to partial truth, or from partial truth to fully fledged falsehood. What is the maximum value of n such that a person with n hairs is truly bald, i.e., bald to degree 1? What is the last rock, along a continuous path descending from the peak of Everest, which is definitely part of the mountain? All of these are questions which may not have practical relevance but which appear to undermine the theoretical force of the account.

Turning to strategy (ii) for dealing with the sorites paradox—to accept the validity of the argument but to reject one of its premises as false—one can again distinguish two main approaches. One can either reject the “base step” expressed by (4), or one can reject the “inductive premise” expressed by (3). (In the version of the paradox where (3) is replaced by a chain of conditionals, this amounts to rejecting one of the conditionals. Here we shall not elaborate on this variant.) A rejection of (4) amounts to a radical response to the effect that a vague term such as ‘bald’ is ultimately incoherent. (See e.g. Unger 1979.) Given the pervasiveness of such terms in natural language, this line of response seems to have little advantages over Russell’s version of the first strategy. A rejection of (3), on the other hand, amounts to asserting the existence of a precise number n of hairs separating the bald from the non-bald. This appears to contradict the intuition that ‘bald’ is vague and, by generalization, that there are any vague words at all. Indeed, this conflict is real and ineliminable if vagueness is understood entirely in ontological terms, for then the existence of a relevant cut-off value of n amounts to the existence of a sharp boundary around the relevant set or object. However, if vagueness is understood in linguistic terms there is one popular way of resolving this intuitive conflict. This is what has come to be known as supervaluationism (Fine 1975).

The basic idea underlying supervaluationism is that a vague term is one that admits of various alternative “precisifications”. A vague predicate such as ‘bald’, for instance, could be made precise by deciding that a man is bald if and only if he has at most 10,000 hairs. Or it could be made precise by deciding that a man is bald if and only if he has at most 9,999 hairs. And so on. The predicate is vague precisely because there is indeterminacy between these various ways of picking out a precise cut-off value. Likewise, a vague singular term such as ‘Everest’ could be made pre-

cise by drawing a precise boundary around its referent, but there are many ways of doing this and all of them are compatible with the way we use the name. Given this understanding of vagueness, supervaluationism says that the truth-value of a statement involving vague terms is a function of its truth-values under the various admissible precisifications of those terms. If the statement is true under all such precisifications, then it is true simpliciter: the unmade linguistic stipulations don't matter. In other words, it makes no difference to suppose that the meaning of those expressions could be defined more precisely: what the statement says is true regardless (or "super-true"). Likewise, if the statement comes out false under every precisification then we may regard it as false (or super-false) in spite of its vagueness. This explains, for example, why we can confidently assert (4) and deny (5). On the other hand, when a statement comes out true under some precisifications and false under others the unmade linguistic stipulations become relevant. In such cases, the statement suffers a truth-value gap. This is why, for example, we must suspend judgment when it comes to statements of the form

(7) A man with n hairs on his head is bald

for various intermediate values of n : the truth-value of such statements depends crucially on how we imagine the extension of 'bald' to be precisified.

As it turns out, this account preserves all theorems of classical logic even though it violates some of its fundamental semantic presuppositions, such as bivalence and truth-functionality. For example, an instance of the law of the excluded middle such as

(8) Either a man with n hairs on his head is bald, or he is not bald.

is sure to come out true even when both disjuncts suffer a truth-value gap. It is precisely this sort of non-standardness that allows supervaluationism to explain away the sorites paradox. Supervaluationally the inductive premise (3) is false because it comes out false on every precisification. However, contrary to the standard semantics for the quantifiers, the falsity of (3) does not imply the existence of a specific n for which the corresponding conditional (6) is false, and this is what allows a supervaluationist to save the intuition that 'bald' is vague. Supervaluationally it is true that there is a number n of hairs that marks the boundary between bald and non-bald, but there is no number n of hairs such that it is true of *it* that it marks the boundary. (The same account applies, *mutatis mutandis*, to a sorites for a singular term such as 'Everest'.)

This account is attractive because it reflects a deep, preanalytical intuition concerning vagueness as it arises in ordinary language: we speak vaguely because in ordinary circumstances the vagueness of our words does not matter. Still, various

objections have been raised. For example, some critics just consider the supervaluational account of the logical operators in (8) and (3) unacceptable. Moreover, supervaluationists have been pressed to provide an account of the phenomenon of higher-order vagueness. This difficulty manifests itself not only in the supposition that there is a clear demarcation between the clear cases and the borderline cases (as in fuzzy logic) but also in the supposition that each vague term comes with a precise set of precisifications. Presumably, if 'bald' could be made precise by deciding that a man is bald if and only if he has at most n hairs, then it could be made precise by deciding that a man is bald if and only if he has at most $n+1$ hairs—and this yields immediately a sorites paradox for the semantic predicate 'could be made precise'. For a supervaluationist this only shows that the metalanguage within which the semantics is formulated is itself vague, but some critics find this line of response unsatisfactory. Lastly, the very assumption that every vague expression can in principle be precisified, or that any number of vague expressions can in principle be simultaneously precisified, has sometimes been regarded with suspicion.

Vagueness and cognitive science

To the extent that vagueness is not entirely a matter of ontology, it falls naturally within the range of interest of the cognitive sciences. Supervaluationism, for example, may be viewed as implementing a certain view about how ordinary speakers manage to communicate and reason even in the absence of a precise language. We speak vaguely because in normal circumstances the vagueness of our words does not matter. In normal circumstances what we say is true under all the admissible interpretations of our words, hence we don't bother being more precise (Lewis 1993).

More generally, the linguistic conception of vagueness has often been associated with the idea that language is but one of many different representation systems. Thoughts and mental images, for some authors, can likewise suffer from the phenomenon of vagueness, and so can every private or publicly accessible representation. Russell himself combined his conservative views on logic with the view that all vagueness is analogous to the vagueness that may exist in a photograph, let alone the figures of an impressionist painting.

It is not clear, however, whether one and the same account can indeed be made to fit all these different cases (Dummett 1975). Compare the vagueness of 'bald' with that of 'looks bald'. If Jones is a borderline case of the latter predicate, a linguistic account would have to say that on certain precisifications Jones will look bald (to me) even though his identical twin, who has just one more hair on his head, will not look bald. Since the two men look exactly alike to me, this seems to contradict

the idea that the predicate ‘looks bald’ is entirely observational, i.e., that it applies only in virtue of appearances. A similar point can be made for observational predicates such as ‘looks square’ (where ‘square’ is non-vague), or for any other predicate expressing properties whose reality, as some like to say, *is* their appearance—e.g., color predicates.

For another example, if there is such a thing as the language of thought, or Mentalese, then it would seem to suffer from a different sort of vagueness than public languages, at least to the extent that the meaning of Mentalese expressions does not depend on their use. A supervaluationary account would therefore seem unjustified in this case. A fuzzy-theoretic semantics would also be inadequate because of the psychologically unrealistic richness of nuances in the underlying space of truth-values. In the case of public languages one may try to ground a fuzzy truth-value assignment on statistical measurements, but Mentalese would defy this way of proceeding. (There is a tradition of psychological studies aimed at measuring the degree to which people are inclined to classify a penguin as a bird, say, but this is irrelevant here: something may fail to be a typical *P* without being a borderline case of *P*, just as a perfectly clear case of prime number may fall short of typicality. See Armstrong et al, 1983.)

Sorensen (1991) has suggested that cases such as these favor an epistemic account whereby vagueness is a kind of ignorance. On this account, the indeterminacy associated with a vague expression stems primarily from our inability to determine its exact reference (extension). More generally, the epistemic account has been proposed as an alternative to all the theories mentioned in the previous section, also because it provides a straightforward way out of all sorts of sorites paradoxes. If the vagueness of ‘bald’ is a matter of ignorance, then a critical cut-off value of *n* does exist which separates the bald from the non-bald, except that it is unknown to us. Moreover, the relevant value cannot be known by us and this would explain our inclination to regard a statement such as (3) as true when it is, in fact, false. In this sense, epistemicism can be viewed as an alternative to supervaluationism in providing an implementation of strategy (ii) above. Both validate all theorems of classical logic; unlike supervaluationism, however, epistemicism also validates all classical semantic presuppositions, including the principle of bivalence.

The epistemic account of vagueness is generally met with astonishment. How can there be a sharp boundary demarcating the extension of ‘bald’ if nobody ever bothered making the necessary semantic stipulations? What could possibly be the explanation of the relevant ignorance? One response, articulated in some detail by Williamson (1992), is that the boundaries associated with vague terms are unknowable because they violate a general principle that characterizes reliable knowledge. Briefly, this is a principle to the effect that our beliefs are reliable only if we leave a

margin for error. For example, the belief that a general condition obtains in a particular case can be reliably true only if that condition obtains in every similar case (the relevant notion of similarity depending on context and cognitive capacities). In the case at issue this would mean that we cannot *know* that a certain person is bald if people with just one more hair on their head *are not* bald. The vagueness of a predicate such as ‘bald’ would then be captured, intuitively, not by a principle such as (3) but rather by a margin-of-error principle such as (9):

- (9) For every n : if a man with n hairs on his head is known to be bald, then a man with $n + 1$ hairs on his head is bald.

And this principle does not combine with (4) and (5) to generate a paradox even if classical logic is retained *holus bolus*.

Some support for the epistemic conception of vagueness seems to derive from recent experimental data. Notably, Bonini et al (1999) have found that ordinary speakers react to questions about vague predicates as if they were not sure about their boundaries, which leads to the hypothesis that vague predicates are mentally represented like sharp predicates with crisp true/false boundaries of whose location one is uncertain. On the other hand, such findings seem compatible also with the view that vagueness is at bottom a phenomenon that reflects the fluid judgmental spreadings involved in human categorization. According to Raffman (1994), ordinary subjects are always likely to break the slippery slope of a sorites series precisely because a sharp category shift is likely to occur at some point on each run of judgments. The point of shift varies with the judgments of different speakers and those judgments in turn vary with the contexts in which they are made. Rather than explaining this phenomenon in epistemic terms, however, Raffman conjectures that the point of shift is determined by a constellation of psychological factors, such as the strength of the judgmental inertia induced by the anchoring heuristics employed by the subjects as they proceed along the series. (One will categorize a greater number of people as bald if one begins from the hairless side of a corresponding sorites series than if one begins from the hairy side: see Tversky and Kahneman 1974.) In other words, the relevant category shifts are not to be viewed as boundary crossings but as Gestalt-like changes of perspective. If this is right, then it is also plausible to suppose that a subject’s judgments may vary depending on whether the items in a sorites series are considered individually or pairwise. The basic premise and the conclusion of a sorites argument derive their plausibility from individual judgments. But only the second, pairwise type of judgment satisfies the inductive premise of the sorites paradox. This means that (3) would have to be rewritten as

- (10) For every n : if a man with n hairs on his head is bald then a man with $n + 1$ hairs on his head is also bald, insofar as the two men are judged pairwise.

And with (3) replaced by (10) the paradoxicality of the argument would dissolve into a fallacy of equivocation.

It is indeed regretful that the available experimental data are still too scarce to throw light on these conjectures. A psychologically plausible account can hardly fail to include some hypothesis about the mental representations that underlie our usage of vague words. Still, few theorists seem inclined to believe that the paradox can succumb to empirical considerations, just as few theorists are willing to accept a purely epistemic account. For a vast majority, the paradox is a genuine one. And for many vagueness remains a deep and bewildering philosophical conundrum.

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