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## Infinity and the Sublime

by

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# Infinity and the Sublime

Karin Verelst \*

*In their recent work, L. Graham and J.-M. Kantor discuss a remarkable connection between diverging conceptions of the mathematical infinite in Russia and France at the beginning of the 20<sup>th</sup> century and the religious convictions of their respective authors. They expand much more on the Russian side of the cultural equation they propose; I do believe, however, that the French (or rather 'West European') side is more complex than it seems, and that digging deeper into it is worthwhile. In this paper I shall therefore broaden the path laid out in Graham and Kantor's work, by connecting two different strands of research concerning the origin of what I loosely call 'formal' ideas: firstly, the rich but complex relation between logic and rhetoric throughout European cultural history, and secondly, the impact of religious convictions on the formation of certain mathematical and scientific ideas during Renaissance and Early Modernity, especially but not exclusively in France.*



Je pense pour mon compte (...) que  
l'important c'est de ne jamais  
introduire que des êtres que  
l'on puisse définir complètement en un  
nombre fini de mots (Henri Poincaré)  
Nommer, c'est avoir individu (Nikolai N. Luzin)

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## 1. Introduction

The beginning of the 20<sup>th</sup> century witnessed a revolution not only in the natural sciences (Einstein's special relativity theory and Planck's quantum of action), but also, less well known, in mathematics. Two decades before the turn of the century an avalanche of discoveries in function theory, topology and number theory followed the invention of set theory and the (re)discovery of the astonishing fact that mathematical infinity comes in kinds, even in infinitely many ones. To these initial groundbreaking steps the name of Georg Cantor, a German mathematician with Russian roots, is first and foremost connected. Many of the subsequent discoveries belong to a field called descriptive set theory and have intuitively to do with what infinite 'divisions' (subsets) are possible of the real numbers line. They often have a counter-intuitive, if not downright paradoxical, flavour.

Names attached to these discoveries include famous Russian and Polish mathematicians, such as Luzin, Suslin, Sierpinski and Kuratowski, but several notorious French mathematicians, like Baire and Borel, made fundamental contributions<sup>1</sup>. However, although working on at first glance similar or in any case related problems, these two groups of men were often moved by radically different viewpoints on their nature and meaning, and consequently developed widely diverging approaches with respect to them. It turns out that, while the Russian mathematicians felt comfortable in the presence of infinite entities, mathematical or otherwise, their French counterparts rather felt profoundly disturbed, and feared that the rational nature of mathematical inquiry *per se* was at stake. Graham and Kantor discuss in detail the relevance of the different intellectual and cultural influences working on the two main groups during their most productive phase, and come to the conclusion that these influences played a decisive rôle in the development of the mathematics of the infinite, and more generally, in the evolution of mathematical ideas as a whole:

<sup>1</sup> A readable presentation of the most important issues can be found in Michael Potter, *Set Theory and its Philosophy* (Oxford: Oxford University Press, 2004). An excellent discussion of the history of set theory and of its philosophical implications is Mary Tiles, *The Philosophy of Set Theory: an Historical Introduction to Cantor's Paradise* (New York: Dover, 2004 [1989]).

We believe that our study of French and Russian developments in set theory and the theory of functions points strongly toward the importance of cultural factors in the process and creation of mathematics—in the French case, Descartes, positivism, and Pascal; in the Russian case, mystical religious beliefs, particularly those of the Name Worshipping movement. As a result the French and the Russians followed different approaches<sup>1</sup>.

They immediately add that such “intellectual causation” can never be proved rigorously, only made plausible—and they are evidently right. But it is equally evident that this holds true for many phenomena investigated in (again loosely speaking) cultural studies, without making them any less relevant. Graham and Kantor also point out that comparative studies add up to more credible evidence, a point with which I agree. Therefore I propose in the present paper an enlargement of the comparative scope of their work into the past. The question is then whether their conclusions, with respect to the profound influence of cultural factors such as religious convictions on scientific or mathematical ideas concerning infinity, do still stand. I shall limit myself deliberately to the ‘French’ part of their comparative equation, because they elaborate the French background only superficially in comparison to the Russian one, probably because they assume it to be sufficiently well known. However, what we are familiar with is the *result* of a profound cultural transformation, while for our purposes it is the *process* leading to it that really matters.

How far in the past do we have to go? The starting point for our investigation is provided by Graham and Kantor when they state that the French intellectual framework, with respect to “rationalism”, has been shaped decisively by Descartes and Pascal. This brings us to the beginnings of the modern period, a time of cultural, religious and scientific turmoil in the wake of the religious wars and the (counter-)reformation that shattered Europe already since more than a century. Of course this crucial period in European history has been studied by many from a variety of perspectives, but I do believe that the topic of the mathematical infinite, when placed in context and pursued, along its ramifications, into other domains of intellectual life, can shed additional light on the deep cultural transformations that shaped the now dominating conceptions concerning creativity, rationality and the nature of the human mind.

<sup>1</sup> Loren Graham and Jean-Michel Kantor, “A comparison of two cultural approaches to mathematics France and Russia, 1890-1930”, *Isis* 97, no. 1 (2006).

Exemplary research over the past decades has shown that during the Early Modern period a debate—really more like an intellectual war—was going on concerning the nature of rhetoric, and more specifically centering on the notion of the *sublime* as a valid source for rhetorical invention. Acceptance or rejection of the sublime in rhetoric is linked to the belief in an absolute or infinite realm to which a great soul can connect, a belief which has far-reaching theological consequences and bears upon precise conceptions of the relation between language, existence and imagination. The sublime paves the way to the acceptance or rejection of the notion of mathematical infinity. This intellectual connection is quite straightforward and can be shown to shape, e.g., certain oppositions between Descartes and Pascal, but I shall show that it remains operative well into the twentieth century, where it explains the different approaches developed to the actual infinite in the foundations of mathematics by the French rational—heir to Descartes and Port Royal—school (Poincaré, Lebesgue, Hadamard, Baire, Borel), and by the Moscow school of mathematics (Egorov, Luzin, Florenskii, Suslin), fueled by a strand of orthodox mysticism called the ‘Name-worshippers’, who shared a central tenet of the defenders of the sublime, i.e. that, at least to a certain extent, ‘to name’ and ‘to exist’ coincide<sup>1</sup>.

## 2. Method, Invention and the Sublime

During Renaissance, medieval dialectic as a part of the scholastic tradition comes under heavy fire, at first from the humanist movement that sweeps through Europe from Italy (in the 14<sup>th</sup> century) to the North (around the end of the 15<sup>th</sup> century). The original criticisms centered mainly on the lack of originality and the triviality of the seemingly pointless logical nitpicking of the Schoolmen, their ‘barbaric’ use of Latin and the overall uselessness of their debates with respect to civil activity and the real world. This rhetorisation of invention and

<sup>1</sup> Along with the paper already cited, see Loren Graham and Jean-Michel Kantor, “Name Worshipers. Russian religious mystics and French rationalists: Mathematics 1900-1930”, *American Academy of Arts and Sciences Bulletin* 58, no. 3 (2005). Much more details in Loren Graham and Jean-Michel Kantor, *Naming Infinity. A True Story of Religious Mysticism and Mathematical Creativity* (Cambridge, Mass.: Belknap Press, 2009).

knowledge, fostered by the rediscovery of numerous lost works by ancient authors, implied a ciceronian shift from certain knowledge back to probable. A second line of attack on the medieval School grew stronger towards the end of Renaissance—one that focused on ways to acquire true, certain knowledge, not anymore based on logic, but on observations of the real world formalised according to a new model for certainty: mathematical quantification. This tendency will ultimately prevail and destroy not only the methods of the scholastic logicians, but also the artistic, rhetorical, even magical approach to reality, so typical for high Renaissance, forcing art out of science, and giving rise to a new debate concerning style and literature apparently completely detached from endeavours to try to understand the world<sup>1</sup>.

The divorce of rhetorical invention from dialectical invention depends, ultimately, on an even more fundamental divorce of rhetoric from dialectic, philosophy, and any of the scientific fields<sup>2</sup>.

We shall see how, dually, different choices with respect to *inventio* appear to determine the whole intellectual outlook of the individual committed to them. Matters of 'style' can act as light-house fires on conceptual vessels searching for the metaphysical home ports in which they were constructed, and by means of which they ought to be construed: "Les questions controversées de rhétorique (...) jouent donc un rôle déterminant dans l'Europe catholique. Elles témoignent de préférences qui vont quelquefois jusqu'à des choix théologiques tranchés"<sup>3</sup>. This holds true not only for the catholic world, and remains so long after the advent of Modernity. I intend to show how conceptions that were rhetorical in their origin, feeding on metaphysical and even theological grounds continued

<sup>1</sup> Studied in an exemplary manner in Sophie Hache, *La langue du ciel. Le sublime en France au XVII<sup>e</sup> siècle* (Paris: Honoré Champion, 2000). The final victory of one of the intellectual parties in the debate thus brought about the destruction of the other approaches and stood godmother to the birth of modern science, in its rationalist outlook advocated by Descartes, Leibniz and Pascal, in its empirist cloak by Newton and his followers. They all have been relevant with respect to the debate on the mathematical infinite, as will be discussed in more detail below.

<sup>2</sup> M. Cogan, "Rudolphus Agricola and the Semantic Revolutions of the History of Invention", *Rhetorica* 2, no. 2 (1984): 163-94 (179).

<sup>3</sup> Marc Fumaroli, *L'école du silence. Le sentiment des images au XVII<sup>e</sup> siècle* (Paris: Flammarion, 1998 [1994]), 16.

to inform recent mathematical thought, in particular in France, in a way that confirms the findings of Graham and Kantor, but sheds a different light on them.

In order to set out the different positions in the debate, I focus on the notion of the *sublime*, a concept central to early modern controversies on the origin of rhetorical creativity. The sublime enlarges the scope of human creativity and surpasses the rule-governed *inventio*, as if it were a divine breath that infuses inspiration into the mind, causing a *furor* or *enthusiasm* in the mind affected by it, and allowing it to surpass common boundaries to originality and rhetorical power. Early Modern writers claimed to have access to such powers: “Je vise à une plus haute sublimité”<sup>1</sup>. It can be retraced to an ancient source, Pseudo-Longinus’s treatise *Peri hypsous* (*On the Sublime*)<sup>2</sup>, the publication of which in Italy in 1555 from a by then virtually unknown Greek manuscript had, so to say, dropped a bombshell<sup>3</sup>.



According to Adrien Baillet, his biographer, Descartes was well aware of the reality and force of this imaginative enthusiasm:

[Descartes] ne croioit pas qu'on dût s'étonner si fort de voir que les Poètes, même ceux qui ne font que niaiser, fussent pleins de sentences plus graves, plus sensées, et mieux

<sup>1</sup> Jean-Louis Guez de Balzac, *Socrate Chrestien* (1652), quoted in Hache, *La langue du ciel*, 54.

<sup>2</sup> Jules Brody, *Boileau and Longinus*, (Genève: Droz, 1958).

<sup>3</sup> Widely known as *De sublimitate*, and originally ascribed—erroneously—to the Greek writer Longinus, it appeared under the title *Dionysiou Longinou Peri ypsous logou. Dionysii Longini De sublimi genere dicendi. In quo cum alia multa praeclare sunt emendata, tum ueterum poetarum uersus, qui confusi commixtique cum oratione soluta, minus intelligentem lectorem fallere poterant, notati atque distincti* (Venetiis: apud Paulum Manutium, Aldi f., 1555). Its author remains unknown, but is customary referred to as ‘Pseudo-Longinus’. The history of its rediscovery is exposed in detail in Marc Fumaroli, “Rhétorique d’école et rhétorique adulte: la réception européenne du *Traité du sublime* au XVI<sup>e</sup> et au XVII<sup>e</sup> siècle”, in *Héros et Orateurs* (Genève: Droz, 1996), 388-93. J. Brody places its first print in 1554, followed by Hache, *La langue du ciel*, 27.



exprimées que celles qui se trouvent dans les écrits des Philosophes. Il attribuit cette merveille à la divinité de l'Enthousiasme, et à la force de l'imagination<sup>1</sup>.

However, this possibility of a direct contact between finite minds and the divine realm was prone to be considered a blasphemy, and it crumbled under the rationalising weight of the Reformation, both protestant and catholic, that sought to eradicate the last traces of 'paganism' in all aspects of European cultural life:

Dès lors que l'enthousiasme, soumis à la critique patiente et systématique de l'humanisme érudit, depuis Jules-César Scaliger jusqu'à Méric Casaubon, n'apparaît plus comme le principe séminal de la connaissance et de l'invention humaines, dès lors que le principe de raison s'impose à sa place, ce sont des pans entiers de la culture humaniste qui s'écroulent, pour faire place à un nouvel édifice de style moderne<sup>2</sup>.

My aim is to show that this decline ultimately had consequences for formal "arts" like mathematics as well. Many authors on the sublime, Longinus in the first place, stress again and again that its acceptance or rejection in rhetoric is connected to the belief in an absolute or infinite realm to which a great soul can connect. It cuts through the frontlines of the notorious *Querelle des Anciens et Modernes*, a fierce debate on style and creativity in relation to the sublime in 17<sup>th</sup> century France, that starts with Guez de Balzac and finds its culmination in Boileau's 1674 translation of Longinus's treatise<sup>3</sup>. The positions defended in those debates bear upon precise conceptions of the relation between language, existence and imagination. This is why the sublime also paves the way to the possible acceptance or rejection of the notion of mathematical infinity.

In the *Peri hypsous* Pseudo-Longinus discusses the ultimate source on which all invention needs to feed in order to get access to truly new, original ideas, the *sublime* (το ὑπέρσος). One can practice to gain access to this source, in spite of its divine nature, by developing the capacity to master the most vehement emotions and express them in an appropriately grand style. This should not be misunderstood as the mere application of a rhetorical technique:

<sup>1</sup> From Baillet's *Vie de Monsieur Des-Cartes* (1691), quoted in Michael H. Keefer, "The Dreamer's Path: Descartes and the Sixteenth Century", *Renaissance Quarterly* 49 (1996), 30-76 (30).

<sup>2</sup> Marc Fumaroli, "Le crépuscule de l'enthousiasme", in *Héros et Orateurs*, 376.

<sup>3</sup> Hache, *La langue du ciel*, 53 ff. and 262 ff.

L'un des aspects majeurs développés par les traités de rhétorique s'intéressant au sublime est la distinction fondamentale établie entre le style sublime, c'est-à-dire le style le plus élevé dans la tripartition héritée de la tradition antique, et le sublime, au sens longinien du terme, qui outrepassa cette hiérarchie et se définit comme *une puissance* de saisissement du discours<sup>1</sup>.

It is as if the orator develops a kind of potential participation in the divine breath into an actual capacity. Longinus calls this exercise explicitly a *method*<sup>2</sup> and he who masters this art has a literally unlimited source of inspiration at his disposal. He will be able to express everything in the appropriate way immediately, like a prophet or a seer. Longinus compares its effect with that of a bolt of lightning<sup>3</sup>.

One of the main instruments to achieve this goal is by the intensive training of rhetorical memory, heightening it to the level of an almost visionary mental faculty. During Renaissance, these ancient memory techniques made a forceful re-appearance on the public scene, but now in the potentially explosive mixture of neo-Platonic natural magic, Hermetism, and their concomitant urge to expand the powers of the mind. Interest in these mental powers had been resurrected already before, by the translation and publication by Marsilio Ficino of a Greek manuscript containing the main part of the *Corpus Hermeticum*, ordered by Cosimo de' Medici and published in 1464. Thus the essential trait of Renaissance mnemotechnical art was that it became an indissoluble alloy with magico-religious enthusiasm, which caused a huge and increasingly hostile response.

Cette 'conquête mystique', qui avait eu plus tôt son pendant en Italie et en Espagne, fut contemporaine, dans toute l'Europe, d'une véritable obsession de démonologie et de sorcellerie, et dans les pays protestants, d'une prolifération des sectes d'enthousiastes<sup>4</sup>.

A devastating line of attack against this development was launched by Isaac

<sup>1</sup> Hache, *La langue du ciel*, 334. My emphasis.

<sup>2</sup> *De Subl.*, I, 3; *Du sublime*, ed. Henri Lebègue (Paris: Les Belles Lettres, 1952), 3-4). Right from the start of his treatise (I, 2), Pseudo-Longinus stresses the importance of the acquisition of the correct techniques to handle the sublime. He calls one who ventures unprepared into this realm *améethodos*, unqualified; see *Du sublime*, 2.

<sup>3</sup> *De Subl.*, I, 4; *Du sublime*, 9-10.

<sup>4</sup> Fumaroli, "Le crépuscule de l'enthousiasme", 349.

Casaubon, who destroyed the historical basis of Ficino's neo-Platonic natural philosophy, by demonstrating in 1614 on philological grounds that the *Corpus Hermeticum* dated from the Hellenistic period, and not, as Ficino believed, from times immemorial. A further move, in many respects a decisive one, was made by his son Meric Casaubon, a professor at Oxford, in 1655, when he published his *Treatise concerning Enthousiasm*. He dealt a final blow to the credibility of whatever kind of *furor* by relocating it into the realm of the pathological<sup>1</sup>. This line of thought was taken up again by his Cambridge colleague Henry More, a Platonist of the rationalist kind, in his book *A brief discourse of the nature, causes, kinds and cure of Enthousiasm*<sup>2</sup>.

The Renaissance up to the end of the 16<sup>th</sup> century is characterised by an emerging conflict between the humanism of Ciceronian *controversia* and the newer expressions of a platonizing tendency to restore dialectic as the fundamental intellectual discipline. This conflict comes out clearly in the debates between Erasmus and Luther, which pit a view of rhetoric that is basically bilateral and symmetrical and that aims at the *satis probabile*, against a view like Plato's, where the Truth is to be communicated effectively in a unilateral, asymmetrical setting. (...) Eventually, the humanistic controversialist rhetoric gave way, first, to the platonizing Ramists and, afterward (...) to the empiricists' and Cartesians' (...)<sup>3</sup>.

Especially access through memory to the Lullian *dignitates Dei* or names or properties of God<sup>4</sup> under the form of symbolic *images*, created a forceful reac-

<sup>1</sup> He plays therefore a key rôle in the shift from bodily to mental normality, as it has been described by Foucault in his *Histoire de la folie à l'âge classique* (Paris: Plon, 1961).

<sup>2</sup> It is well known that Newton as a young student was an avid reader of his work. See John Henry, "Henry More and Newton's Gravity", *History of Science* 31 (1993), 83-97. It is relevant to our concerns to point out that More discusses how God's *tituli* precisely serve as an indication of the abyssal gap between Him and his creation. This discussion clearly influenced Newton when writing his General scholium to the *Principia*. For an analysis of the textual relationship, see: Rudolf De Smet and Karin Verelst, "Newton's Scholium Generale. The Platonic and Stoic Legacy: Philo, Justus Lipsius and the Cambridge Platonist", *History of Science* 39 (2001), 10-11.

<sup>3</sup> Thomas M. Conley, *Rhetoric in the European Tradition* (Chicago: University of Chicago Press, 1990), 143-44.

<sup>4</sup> Ramón Lull, a contemporary to Thomas Aquinas, developed an astonishing 'Art', the aim of which was to organise all knowledge on the basis of a meditation on the names of God (the *Dignitates Dei*). There are nine fundamental names of God organised in triads (goodness, greatness, eternity...), which form a kind of ladder of being along which the 'artista' ascends and descends. At

tion from calvinist and puritan sides. They called upon Ramus's imageless dialectical theory of memory as an alternative for the mnemotecnical "idolatry". The puritan theologian W. Perkins even called for an absolute prohibition of the use of images as an aid to the mind. But also catholics who stood under the influence of Augustinianism, like the Jansenists, were very critical. Their common concern is the desire to purify Christianity from all remnants of paganism: "C'est sous l'influence d'un augustinisme plus soucieux de trancher entre paganisme et christianisme, entre nature et grâce, que (...) la notion d'inspiration enthousiaste (...) se verra menacée même dans le domaine rhétorique et poétique"<sup>1</sup>.

Antoine Arnauld and Pierre Nicole were two Port-Royal theologians who more or less strongly adhered to Jansenism<sup>2</sup>. Jansenist theology is Augustinian by inspiration and marked out by strong resemblances to several Protestant tendencies. But Arnauld and Nicole are also the authors of the so-called *Port Royal Logic*, the original title of which is: *La Logique, ou l'art de penser*<sup>3</sup>. Already the first page of the first chapter of the *Logique* contains a reference to St. Augustine. It evidently fits in the reform-movement we outlined before, together with other attempts to revolutionise the 'art of thought', like Johann Clauberg's *Logica Vetus et Nova* (1654), which might have been one of its sources of inspiration<sup>4</sup>.



the same time they ground a memory system organised by means of combinatory wheels laying out the interconnections that everything in creation entertains with the attributes of the Creator.

<sup>1</sup> Fumaroli, "Crépuscule", 351.

<sup>2</sup> And so did Pascal. See Jean-Pierre Chantoin, *Le jansénisme* (Paris: Cerf, 2000); Louis Cognet, *Le jansénisme* (Paris: PUF, 1995 [1961]).

<sup>3</sup> Antoine Arnauld and Pierre Nicole, *La Logique ou l'Art de Penser*, ed. Pierre Clair and François Girbal (Paris: Flammarion, 1970 [1662]).

<sup>4</sup> See the introduction to Johann Clauberg, *Logique Ancienne et Nouvelle*, ed. Jacqueline Lagrée and Guillaume Coqui (Paris: Vrin, 2007).

This very influential and widespread book outlines an anti-Aristotelian conception of logic based on the study of natural language and on the nature of mathematical deductions (as opposed to merely syllogistic ones), and it states explicitly that the authors chose their examples in such a way as to introduce the reader to the correct philosophy of nature, as well as to correct methods for judgment in matters moral and religious. Arnauld and Nicole heavily stress the importance of both simplicity and ‘good sense’ in judgment, and lay this down in eight ‘Rules’ destined to govern the process of reasoning. These are exemplified in the *Quatrième partie. De la méthode*, the last part of their book. They furthermore distinguish carefully between *analyse*, the method of invention, and *synthèse*, the method of composition<sup>1</sup>. Finally, they declare explicitly that in this respect they are followers of Descartes in his *Discours de la méthode*. The example was followed by Newton<sup>2</sup>, who starts the third Book of his *Principia*—not accidentally named after another work by Descartes, the *Principia Philosophiae*<sup>3</sup>—with a notorious set of (different!) “Regulae philosophandi”, rules of an empiricist inspiration to which scientific practice should comply in order to lead to trustworthy new results<sup>4</sup>.

The principal innovation brought by Arnauld and Nicole is the distinction between a thing and its representation, like a word and the thought that it ex-

<sup>1</sup> Arnauld and Nicole, *Logique*, 368.

<sup>2</sup> Newton had a copy of the latin translation of the Port Royal Logic in his personal library: *Logica, sive Ars cogitandi: in quae praeter vulgares regulas plura nova habentur ad rationem dirigendam utilia. E tertia apud Gallos editioe recognita et aucta in Latinum versa* (Londini: Impensis Richardi Green, 1687). In the Harrison catalogue of Newton’s library, this is item 79. I have reasons to believe it had been given or pointed out to him by his friend Nicolas Fatio de Duillier.

<sup>3</sup> A link evident to his contemporaries, as witnessed by Huygens in a letter to Leibniz of May 29 1694, where he refers to the “raisonnement et experiences de Newton dans ses Principes de Philosophie”; see Letter 2854, in *Oeuvres Complètes de Christian Huygens* (La Haye: M. Nijhoff, 1888 ff.), X, 614. On the relation between Newton and Descartes see, e.g., Andrew Janiak, *Newton as Philosopher* (Cambridge: Cambridge University Press, 2008).

<sup>4</sup> The genesis of Newton’s “rules” is problematic in itself and deserves far more attention than it usually gets. A notable exception is I. Bernard Cohen, “Hypotheses in Newton’s Philosophy”, *Physis* 8 (1966), 163-84. Interesting observations with regard to the different versions of the start of Book III of the *Principia* in Scott Mandelbrote, *Footprints of the Lion: Isaac Newton at Work* (Cambridge: Cambridge University Library, 2001 (the catalogue of an exhibition held there). An interesting study of scope and aim of Newton’s empiricism as voiced in their final version is: Alan E. Shapiro, “Newton’s Experimental Philosophy”, *Early Modern Science and Medicine* 9, no. 3 (2004), 185-217.

presses: “les mots sont des sons distinctes & articulés, dont les hommes ont fait des signes pour marquer ce qui se passe dans leur esprit”<sup>1</sup>. This seemingly innocent statement marks the fundamental breach with the past: even though disposition, style, and delivery are method-governed disciplines, the characteristic proper to inspired discourse is that the word and the thing it represents disappears: “Le sublime (...) abolit la béance entre présence et représentation”<sup>2</sup>. As Foucault points out appropriately<sup>3</sup>, this separation of words and things marks out the transition from the 16<sup>th</sup> to the 17<sup>th</sup> century, and will be part and parcel of any modern linguistic theory: “Ainsi le signe enferme deux idées: l’une de la chose qui représente; l’autre de la chose représentée”<sup>4</sup>. But Arnauld and Nicole operate their distinction in a much larger sense, so as to separate cause and effect in natural phenomena, or even in theology, like the symptoms of an illness, or the stigmata of the suffering Christ. A judgment concerning something is then a proposition consisting of this subject and a praedicate. Starting from this analysis, they defend a new method for invention based on logic to deduce new ideas. The conception of logic they defend is anti-Aristotelian in the sense that it is based on mathematical deductions as opposed to syllogistic ones. They evidently have geometry in mind, which remains after all, as far as proofs are concerned, “an exercise in Logic, classically”<sup>5</sup>. Why then geometry, instead of pure logic? Because of a problem that had been pointed out, among others, already by Descartes, and which has been summed up nicely by J. Henry: “The trouble with ‘reason’, as was clear from the fact that both Calvinists and Papists could claim it for their side, was that it could be made to subserve virtually any cause”<sup>6</sup>. Geometry, by virtue of the fact that it has a proper content related to facts of nature, seemed to be a more reliable guide to certainty than the subtleties of controversial disputation. Arnauld and Nicole then, predictably,

<sup>1</sup> Arnauld and Nicole, *Logique*, 143.

<sup>2</sup> Fumaroli, “Réception”, 383.

<sup>3</sup> See the second chapter of Michel Foucault, *Les mots et les choses. Une archéologie des sciences humaines* (Paris: Gallimard, 1966).

<sup>4</sup> Arnauld and Nicole, *Logique*, 80.

<sup>5</sup> David Finkelstein, “Matter, Space and Logic”, *Boston Studies in the Philosophy of Science*, 5 (1966), 199-215; repr. in *The Logico-Algebraic Approach to Quantum Mechanics II*, ed. Clifford A. Hooker (Dordrecht: Reidel, 1979), 124.

<sup>6</sup> John Henry, “The Scientific Revolution in England”, in *The Scientific Revolution in National Context*, ed. Roy Porter and Mikulas Teich (Cambridge: Cambridge University Press, 1992), 193.

proceed in Part III to a critical discussion of the “lieux”, the traditional *loci* or common places that were in use to structure rhetorical memory.

Arnauld is also co-author of a *Grammaire générale et raisonnée contenant les fondemens de l'art de parler, expliqués d'une manière claire et naturelle*<sup>1</sup> which essentially claims that the basic structures of language are simple, innate and universal<sup>2</sup>. The separation of mentalities can be gauged from the fact that C. Favre de Vaugelas, co-founder of the *Académie* and one of the main editors of its *Dictionnaire*, published only fifteen years before his *Remarques sur la langue française, utiles à ceux qui veulent bien parler et bien écrire*, of which the central tenet was that the best way to master the French language was to copy the ways of speech—“le bon usage”—of the members of the royal court. This, again, is intimately connected to the debate on ‘style’ that ravaged the French *République des lettres* in the same period, where the tendency towards a tamed, christianised version of the sublime found its ally in the growing preference for a ‘natural’, ‘elegant’, ‘simple’ discourse, claimed to constitute the *génie de la langue française*.

For Meric Casaubon, the poetically transgressing ‘inspired’ mind is nothing more than a pathological mind, vulnerable to the ruses of paganism. True judgment requires nothing but ‘bare speech’ to expose ‘the true nature of things’. Words are nothing but representations, the mental face of sensible things. Thus a conceptual line links the in origin Aristotelian conception of words as *symbola rerum* defended by both Scaliger and Casaubon, directly to Locke’s *Essay on Human Understanding* (1690). Indeed, nobody will be surprised to find that it contains a critique of enthusiasm which insists on the lack of proof for the pretenses to certain knowledge that any utterance grounded in it could claim. On the other hand, Henry More successfully ridicules the melancholic temperaments unable to discriminate between a bodily affliction and the workings of God’s grace<sup>3</sup>. While the English Aristotelians and later empirists thus succeeded

<sup>1</sup> Antoine Arnauld and Claude Lancelot, *Grammaire générale et raisonnée* (Paris: Editions Allia, 1997 [1660, 1676<sup>3</sup>]).

<sup>2</sup> The work inspired Chomsky in the development of his ‘generative grammar’, which he himself qualifies as a part of a wider approach to language, ‘Cartesian linguistics’; see Noam Chomsky, *Cartesian Linguistics: a Chapter in the History of Rationalistic Thought* (New York: Harper and Row, 1965).

<sup>3</sup> Fumaroli, “Crépuscule”, 372.

in effectively abolishing any credible remnants of the ‘inspired’ forms of knowledge to the extent that all activity involving possible errants into *l’irrationnel*, *l’inspiration*, *la dictée ‘démonique’*—even philosophy—became suspect, the alliance between *simplicité* and *le sublime* was to shape what René Bray was to call “la doctrine classique”, that gracious philosophical “art de converser” in urban life in France’s 17-18<sup>th</sup> centuries. Even the detractors of rhetorics had to put their cause eloquently! During that period, the debate on the nature of the sublime will resurface, but now within the safe confines of the domain of the arts, in the modern sense of that word. The already mentioned *Querelle des Anciens et Modernes* was to oppose the great French dramatists Racine and Corneille, the writers Perrault and Guez de Balzac<sup>1</sup>. And Boileau, author of a *De l’art poétique*, and, as we have already mentioned, the first translator of the *De Sublimitate* into French, used the latter as a counterpoise to the 16<sup>th</sup> century Aristotelian Julius Scaliger’s pedantic criticisms of any inspired rhetoric<sup>2</sup>. This evacuation of philosophy into the realm of literature went at the cost of its immediate connection to and relevance for reality, whether mundane or sublime: imagination became a sphere of life disconnected from ‘science’ as it was practiced in laboratories or at the theorist’s writing desk:

Car les contraintes psychologiques et même physiques qu’exerça la réforme de l’Eglise—du côté protestant comme du côté catholique—ne furent que de peu inférieures à celles exercées par la Révolution française à son apogée ou—*mutatis mutandis*—par la révolution soviétique. (...) A un moment donné, la censure avait transformé la personnalité : les gens avaient perdu l’habitude d’utiliser activement leur imagination et de penser par ‘qualités’, car cela n’était plus permis. La perte de la faculté d’imagination active entraîna forcément avec elle l’observation rigoureuse du monde matériel et celle-ci se traduisit par une attitude de respect pour toute donnée quantitative et de soupçon envers toute assertion d’ordre ‘qualitatif’<sup>3</sup>.

When the gates of the soul finally closed on the sublime as a road to ultimate reality, the way was free for a strictly rational account of the functioning of the human mind in its relation to the world. Both the empirists and the

<sup>1</sup> As already pointed out, this querelle is studied in detail in Hache, *La langue du ciel*. See also Marc Fumaroli, *La Querelle des Anciens et des Modernes* (Paris: Gallimard, 2001), 7-10.

<sup>2</sup> See Fumaroli, “Crépuscule”, 360.

<sup>3</sup> Ioan P. Couliano, *Eros et Magie à la Renaissance* (Paris: Flammarion, 1984), 240.



Cartesio-Leibnizian rationalists are holding the same line with respect to this, be it armed with very different argumentive weaponry. Style becomes a facet of the first person in the small sense—the birth of the subject, so to speak—and the birth of the idea that debates are a matter of mere literature and personal opinion, as the querelle between the seventeenth century French literary luminaries makes abundantly clear. The question of knowledge belongs from now on to an entirely different realm. The empirists evacuate the question of the sources of knowledge to the outside world, and reduce it to the relation between that world and the sensual impressions we receive from it. This approach will be epitomised in the *Regulae Philosophandi* by which the mature Newton starts Book III of the *Principia*. The clearest example of the nature of the periods' concern with method, however, is Leibniz, who explicitly links the trustworthiness of his *characteristica universalis* to the exhaustive success of his *ars inveniendi*, based as it is on a mechanical procedure to come to new and true propositions. And they all are deeply indebted to Descartes's ideas.

We have by now reached the point where the 'French side' of the story of Graham and Kantor begins. Let me compare this shortly to the description they offer of the mental and cultural background of the Russian mathematicians, and its effect on their dealing with infinity.



### 3. When Existence is in a Name

During the spring of 1913, the Russian navy attacked the monasteries on the peninsula of Athos in order to destroy a Christian-orthodox heresy that had been condemned shortly before by the synod of St. Petersburg. More than 1000 monks were arrested and imprisoned. What was the nature of this heresy, and why did it call for such a violent reaction? Its roots go back to a problem that had haunted theology since centuries: the question whether God can be

known, and if not, how He can be worshipped<sup>1</sup>. In the Western or Latin church, answers to this question had been provided along two different lines: that God is to a certain extent rationally knowable through His creation, but at the same time, as far as His mysterious nature is concerned, He remains out of reach for mortal minds, and one can trust only on faith, as revealed in Scripture and transmitted by the Church. Claims to any more direct access to His nature in mystical experience had been tolerated, but looked upon with suspicion, for they posed a threat to the authority of the Church. The consequences of this ambivalence had stirred Christianity in its foundations already in the thirteenth century, when it developed into the *theory of double truth*<sup>2</sup>, which held that if reason proved something to be true, while its contrary was a matter of faith, then both would retain validity. The initiators of this school of thought were mainly Aristotelian masters at the Faculty of Arts at the University of Paris, known as 'Latin Averroists'. On an opposite stance, Thomas Aquinas developed his magistral *Summa Theologica*. But too sharp a division between reason and faith was at odds also with the neo-Platonic currents feeding Christianity since late Antiquity, a fertile soil on which mystical tendencies using music, poetry, and meditation techniques had flourished throughout the Middle Ages.

During the Reformation period, the chasm between revealed and discovered truth will deepen and it will eventually play an important part in the debates that brought to Galilei's 1633 condemnation<sup>3</sup>. Nevertheless, at both sides of the gap there was agreement on the need to reject and even persecute religious 'enthusiasts' of all stripes, but especially the rapidly spreading millenarian Protestant sects like the 'French Prophets'. They believed they could communicate with the deity directly through possessions, including visions, prophecy, and trances, induced by dancing and music. They belonged to a bigger group of radical Protestants, the *Camisards*, who had been violently oppressed since the revocation of the Edict of Nantes (1685). In 1706 a group of refugees led by

<sup>1</sup> This is in fact a variation on Meno's knowledge paradox, presented by Plato in the dialogue bearing the same name: how can one learn what one does not know, given the fact that one does not know one does not know it?

<sup>2</sup> Étienne Gilson, "La doctrine de la double vérité", in *Études de philosophie médiévale* (Strasbourg: Faculté des Lettres de l'Université de Strasbourg, 1921), 51-69.

<sup>3</sup> Luca Bianchi, *Pour une histoire de la "double vérité"* (Paris: Vrin, 2008). See the *Préface* for an overview.

Elie Marion arrived in London, where they met again with oppressive authorities. Notwithstanding that oppression, they continued to exert lasting influence through their writings. Marion had written a treatise on prophecy, *Les Avertissements prophétiques*<sup>1</sup>, which laid the basis for the later Shaker movement<sup>2</sup>. But in Britain their influence was stamped out by the combined efforts of the Anglican church and the philosophical empiricists, who had been taking over the intellectual lead from the rational Aristotelians of the former generation.

We thus see that, even though inspired movements of all kinds had been present and even dominating throughout large parts of European cultural history, their public influence ceased to exist almost completely after, say, 1750, at both sides of the Channel, save for the literary credibility they retained in France. This had immediate consequences for philosophy in its large sense. Descartes starts his *Regulae* with the remark that people tend to think wrongly that they can conclude validly on the basis of some merely particular similarity (*similitudo*) between things that otherwise differ<sup>3</sup>. We mentioned already that Foucault points out that the correction of this ‘error’ marks the transition from ancient to modern with respect to linguistics. We showed how Arnauld and Nicole (inspired as well by Pascal) elaborate this distinction into a full-fledged theory of representation. In the Cartesian framework, ‘likeness’, as well as any other mental operation that involves a leap of the imagination, has its place only in philosophy or literature, not anymore in science. Furthermore, in the first book of his *Principia philosophiae*, Descartes states clearly that, since we are finite beings, it would be absurd to hope we might ever understand the infinite, and it therefore be better to not think of it at all. He explicitly refers to

<sup>1</sup> Jean-Paul Chabrol, *Elie Marion, le vagabond de Dieu 1678-1713* (Aix-en-Provence: Edisud, 1999).

<sup>2</sup> This Marion had been a close friend of Fatio de Duillier, himself a close friend of Newton’s at least until the mid 1690s. See Loup Verlet, *La malle de Newton* (Paris: Gallimard, 1993), 194. Newton himself has written extensively on the interpretation of prophecy, and his method with respect to this relates closely to the methods he developed in his scientific works. This has been researched extensively in Maurizio Mamiani, “To Twist the Meaning: Newton’s *Regulae Philosophandi* Revisited”, in *Isaac Newton’s Natural Philosophy*, ed. Jed Z. Buchwald and I. Bernard Cohen (Cambridge, Mass.: MIT Press, 2001), 3-14; Id., “Newton on Prophecy and the Apocalyps”, in *The Cambridge Companion to Newton*, ed. I. Bernard Cohen and George E. Smith (Cambridge: Cambridge University Press, 2002), 387-408.

<sup>3</sup> René Descartes, *Regulae ad directionem ingenii, Texte critique*, ed. G. Crapulli (La Haye: M. Nijhoff, 1966), 1. See also the Adam and Tannery edition of the *Oeuvres de Descartes*: AT X, 359.

mathematical problems, like whether there are infinite lines, and whether their halves are infinite as well; whether there are infinite numbers, and if so, whether they are even or odd, etc. In order to solve such questions, you need an infinite mind. They are incomprehensible to us to the same extent as the mysteries of the Incarnation and of the Trinity, which he discussed in the paragraph before. It is worthwhile to give the full quotation:

Ita nullis unquam fatigabimur disputationibus de infinito. Nam sane, cum simus finiti, absurdum esset nos aliquid de ipso determinare, atque sic illud quasi finire ac comprehendere conari. Non igitur respondere curabimus iis, qui quaerunt an, si daretur linea infinita, ejus media pars esset etiam infinita; vel an numerus infinitus sit par anve impar, et talia: quia de iis nulli videntur debere cogitare, nisi qui mentem suam infinitam esse arbitrantur<sup>1</sup>.

For us, finite minds, when confronted with something of which we can conceive no end, the only viable strategy is to consider it as *indefinite*, i.e., finite, but without a conceivable end. In contemporary terms: all arguments we develop in mathematics are finitary, even if they deal with ‘the infinite’.

Nos autem illa omnia, in quibus sub aliqua consideratione nullum finem poterimus invenire, non quidem affirmabimus esse infinita, sed ut indefinita spectabimus<sup>2</sup>.

He then gives infinite extension and the infinite divisibility of matter as examples of problems which we can only think of in ‘indefinite’ terms. ‘Infinity’ is a name we can apply to God alone: “tum ut nomen infiniti soli Deo reservemus”<sup>3</sup>. We can, however, *know* that there are infinite things— eternity, God—without *understanding* them, because without them, it would not even be conceivable for a finite mind to have the idea of infinity at all. In a certain sense, our finitude points towards the infinity of God, making it accessible to our knowledge, but keeping it far out of reach from our comprehension. There are many other places in Descartes’s work where he varies on this basic idea: “our reason, our powers of comprehension are finite and limited, and since the will’s decisions are determined by reason, we have no, so to speak, immediate access to infinity. We have to content ourselves with recognizing infinity

<sup>1</sup> AT VIII, 14-15; *Principia*, I, 26.

<sup>2</sup> AT VIII, 15.

<sup>3</sup> *Ibid.*; *Principia*, I, 27.

without being able to explain it”<sup>1</sup>. This in fact comes down to the Aristotelian difference in degree of reality between ‘potential’ and ‘actual’ infinity, which are generated ‘stepwise’ and ‘at once’ respectively<sup>2</sup>. The cautious attitude with respect to actual infinity was shared by Huygens and even by Newton in his mature life<sup>3</sup>. Pascal’s introduction of the principle of induction in his *Traité du triangle arithmétique* (1665), seems at first to jeopardise this line of events. But evidently, to establish his induction principle, Pascal uses arguments based on indefiniteness, not infinity, in the sense of Descartes<sup>4</sup>. Moreover, Pascal had warned against epistemological pretenses: even though infinities exist, they remain infinitely far beyond the limits of comprehension of the human mind<sup>5</sup>. So there is much more direct evidence to Descartes’s and Pascal’s stance with regard to infinity than the meager quotation on Descartes’s reductionism Graham and Kantor offer on p. 36 of their book. We also see more clearly by now in what philosophical and religious context they should be placed.

Be that as it may, according to them, this is the intellectual attitude that, almost three centuries later, hampered the French mathematical community in

<sup>1</sup> For this paraphrase and an elaborate discussion in reference to AT III, 293 and II, 138, see A. Drozdek, “Descartes: Mathematics and Sacredness of Infinity”, *Laval théologique et philosophique* 52, no. 1 (1996), 167-78.

<sup>2</sup> Tiles, *The Philosophy of Set Theory*, 26.

<sup>3</sup> Once again, Newton’s attitude is peculiar and ridden with apparent inconsistencies. A case in point is the mathematical methodology of the *Principia*, which is deliberately based on the old geometrical methods, while he had his theory of fluxions already at his disposal. This was part of an ideological agenda to promote ‘ancient wisdom’ as superior over the modern one; see James E. Force, “Newton, the ‘Ancients’, and the ‘Moderns’”, in *Newton and Religion: Context, Nature, and Influence*, ed. James E. Force and Richard H. Popkin (Dordrecht: Kluwer, 1999), 237-57. The implications this had for Newton’s mathematics are brilliantly analysed in Niccolò Guicciardini, *Reading the Principia, The Debate on Newton’s Mathematical Methods for Natural Philosophy* (Cambridge: Cambridge University Press, 2003, [1999]).

<sup>4</sup> Henri Poincaré showed in his 1905 paper “Les mathématiques et la logique”, that complete induction thus construed involves either a *petitio principii*—the set of all natural numbers—, or a circularity—the principle of complete induction itself. The paper has been published in several pieces and variants; I used the annotated reprint in Poincaré, Russell, Zermelo et Peano. *Textes de la discussion (1906-1912) sur les fondements des mathématiques: des antinomies à la prédictivité*, ed. Gerhard Heinzmann (Paris: Blanchard, 1986), 11-54.

<sup>5</sup> Cfr. the famous passage no. 72 on the *ciron* (mite) in the *Pensées*. Arnauld and Nicole elaborate this “problem of infinity” along Pascalian lines in the first chapter of the fourth part (“De la méthode”) of their *Logique*, 359 ff.

its attempts to deal with Cantor's set theory and its concomitant arithmetic of infinity<sup>1</sup>. Cantor had proven in 1873 that the set of natural numbers, or the integers, and the set of real numbers, had different types of infinity of elements, and, even more, that there is an infinite number of types of infinity, which he called 'transfinite' cardinal numbers. He then set out to check whether there are other infinities lying in between the two basic arithmetical infinities, and formulated his famous Continuum Hypothesis (CH), which basically states that this is not the case. Although he never succeeded in proving CH—nobody ever did—he did prove in 1884 the remarkable result that CH holds for all closed subsets of the real line. This kind of investigation had ancient antecedents in the paradoxes of Zeno, and had been problematised before by Bolzano in his *Paradoxien des Unendlichen*. Interestingly, one of the early defenders of Cantor's work in France was the mathematician and historian of mathematics and philosophy Paul Tannery. He introduced in the modern literature the idea that set theory and the paradoxes appearing in it should be related to the work of Zeno<sup>2</sup>. The problem was pinned down accurately in 1905 by yet another prominent French mathematician, Henri Lebesgue (famous for his work on integration theory), when he entered the discussion on another set theoretic source of antinomies, Zermelo's axiom of Choice: "La question revient à celle-ci, peut nouvelle: *peut-on démontrer l'existence d'un être mathématique sans le définir?* (...) définir à tout le temps le sens de: *nommer une propriété du défini*"<sup>3</sup>. This is just another way

<sup>1</sup> The following sketch is based on Graham and Kantor, *Naming Infinity*, 25-40.

<sup>2</sup> Poincaré was of the same opinion: "il est arrivé qu'on s'est heurté à certains paradoxes, à certaines contradictions apparentes, qui auraient comblé de joie Zénon d'Elée et l'école de Mégare"; Henri Poincaré, *Science et Méthode* (Paris: Kimé, 1998 [1908]), 40. If one renders Zeno's two fundamental paradoxes in what I call their canonical form (as a paradox of plurality: to consist of parts with and parts without magnitude, and a paradox of motion: to count the uncountable), then the family resemblance with the modern set theoretic paradoxes (Burali-Forti, Russell,...), becomes easily visible. See Karin Verelst, "Zeno's Paradoxes. A Cardinal Problem. I. On Zenonian Plurality", in *Paradox: Logical, Cognitive and Communicative Aspects. Proceedings of the First International Symposium of Cognition, Logic and Communication*, ed. Gordana Dodig Crnkovic and Susan Stuart (Riga: University of Latvia Press, 2006). For Tannery's ideas on the subject, see Paul Tannery, "Le concept scientifique du continu. Zénon d'Elée et Georg Cantor", *Revue philosophique de la France et de l'étranger* 20 (1885), 385-410. One will remark that this is a philosophical, not a mathematical journal. It might be worth considering that Tannery is (together with Charles Adam) also the editor of the famous critical edition of Descartes's complete works.

<sup>3</sup> In a letter to Borel published in Jacques Hadamard et al., "Cinq lettres sur la théorie des ensem-

of saying that one has to restrict mathematical notions to those which can be clearly defined in a finite number of words and for which a consistent mental representation is present, requirements which sound familiar in an intellectual environment shaped by Descartes and Port Royal.

Even though French interest in Cantor's results was almost immediate, it was relegated largely and for a long time to the realm of philosophy. When three of the most noted French mathematicians decided (already in 1880) that it would be worthwhile to translate Cantor's work, they appointed the task to a Jesuit priest, because "[Cantor's] philosophical turn of mind will not be an obstacle for a translator who knows Kant"<sup>1</sup>. In 1898, E. Picard, an important member of the French mathematical establishment, nearly dismissed René Baire's doctoral dissertation on discontinuous functions. Baire had, albeit in a very careful way, used concepts of set theory in the theory of functions. Evidently, such a speculative subject witnessed the twists of a philosophical mind: "L'auteur", so Picard, "nous paraît avoir une tournure d'esprit favorable à l'étude de ces questions qui sont à la frontière de la mathématique et de la philosophie"<sup>2</sup>. This was not an innocent remark, nor meant as a compliment.

Even Borel, who invented Borel sets (a way to divide the real line that has a specific algebraic structure) and founded measure theory, and who was as a young mathematician captivated by both the person and the work of Cantor, declared that he had been carried away by Cantor's romanticism<sup>3</sup>, and took some distance; as Jean-Michel Kantor summarizes it, Borel would maintain: "We are serious people; this at least is not philosophy; a disagreement can only be due to a misunderstanding"<sup>4</sup>.

bles", *Bulletin de la S.M.F.* 23 (1905), 261-73 (265-66).

<sup>1</sup> "Leur tournure philosophique ne sera pas un obstacle pour le traducteur qui connaît Kant", Charles Hermite's letter to Mittag-Leffler, quoted in Graham and Kantor, *Naming Infinity*, 30 and 213.

<sup>2</sup> Quoted in Loren Graham and Jean-Michel Kantor, "A Comparison of Two Cultural Approaches to Mathematics: France and Russia, 1890-1930", *Isis* 97, no. 1 (2006), 56-74 (59)

<sup>3</sup> "J'ai été extrêmement séduit, dès l'âge de 20 ans, par la lecture des travaux de Cantor. (...) Georg Cantor a apporté dans l'études des mathématiques cet esprit romantique qui est l'un des côtés les plus séduisants de l'âme allemande"; quoted in Graham and Kantor, "A Comparison", 60.

<sup>4</sup> Loren Graham and Jean-Michel Kantor, "Name Worshippers. Russian Religious Mystics and French Rationalists: Mathematics 1900-1930", *American Academy of Arts and Sciences Bulletin* 58, no. 3 (2005), 17.

Cantor's ideas had indeed flourished in late 19<sup>th</sup> century romantic Germany, but even there Cantor had faced problems throughout his career, because of the staunch opposition to his work by an influential group of mathematicians led by L. Kronecker, who declared in explicitly theological terms, that "God made the integers; all else is the work of man"<sup>1</sup>. Both Cantor and Baire suffered a nervous breakdown as a consequence from the mental strain caused by working on such problems, as well as the burden it put on the development of their respective academical careers. Cantor ended up in a mental asylum, and Baire committed suicide, thus complying to Meric Casaubon's curse in one of those ironical twists history seems so fond of to play on us.

How different the reception of these ideas was in Russia can be gauged from a remark of an initiator of the Russian school of mathematics, which contributed so much to their further elaboration: "Everything seems to be a daydream, playing with symbols, which however, yield great things". This note was not written by an exalted artist, but by one of the greatest Russian mathematicians of the 20<sup>th</sup> century, Nikolai N. Luzin<sup>2</sup>. Graham and Kantor explain this difference by the cultural influence dominant in Russia during the end of the 19<sup>th</sup> and the beginning of the 20<sup>th</sup> century. We are not going to reproduce here in extenso their analysis—I refer to the book and the various papers mentioned in the footnotes—but let us summarize their conclusions, and link them to the bigger historical context we outlined above. In the cultural realm of the Eastern (Greek and Russian) Orthodox churches, nothing comparable to the extinction of religious enthusiasm had happened until much later, not incidentally during the period of Russia's forced modernisation, started by the Czarist regime in the second half of the 19<sup>th</sup> century. The use of exalted symbolism had been common practice in Orthodox liturgy for ages, e.g. in iconography, but especially the use of the controversial 'Jesus Prayer', a ritual in which the worshipper "chants the names of Christ and God over and over again (...) until his whole body reaches a state of religious ecstasy in which even the beating of his heart, in addition to his breathing cycle, is supposedly in tune with the chanted words 'Christ' and 'God'"<sup>3</sup>.

<sup>1</sup> Eric T. Bell, *Men of Mathematics* (New York: Simon and Schuster, 1986), 477.

<sup>2</sup> Quoted in Graham and Kantor, "A Comparison", 71.

<sup>3</sup> Graham and Kantor, "A Comparison", 68.



It had roots in practices going back to fourth century monastic hermits in the Near East, and caused already in the 14<sup>th</sup> century a theological controversy with the more rationalistic Byzantine tradition. Those practicing it were called “Name Worshippers”; those opposing the practice “anti-Name Worshippers”. The conflict came to a climax in a politically highly sensitive period (the impending collapse of the Ottoman Empire and the recent Menchevik Revolution) when in 1907 Ilarion, a monk from a Russian monastery on Mt. Athos, published a very popular book entitled *In the Mountains of the Caucasus*, in which he claimed that the fact that the faithful can reach a state of unity with God by chanting his name proves that the name of God is holy in itself, that the name of God is God. Interestingly enough, he stresses that the process required to learn to do the prayer effectively takes years and has to follow certain methodological steps. With his claims, he provoked a line that no monotheistic theology can afford to be trespassed, and condemnation followed swiftly, which led to the events cited at the beginning of this section<sup>1</sup>.

Now this strand of exalted theology exerted a direct influence on the mathematical developments in Russia through a former mathematician turned Orthodox priest, P. Florenskii. Two of the founding members of the Moscow school of mathematics, D.F. Egorov and N.N. Luzin, who would become leading mathematicians in the twenties and thirties with major contributions to descriptive set theory and the theory of discontinuous functions, were among Florenskii’s disciples. Florenskii was an adept of the Name Worshippers, and he defended his ideas on several occasions during meetings of a small circle of followers in Egorov’s appartement. He sought to bring together mathematics and religion through the concept of “naming”, as it had been understood by the Name Worshippers: “to name something was to give birth to a new entity. (...) Humans could exercise Free Will and put in perspective mathematics and philosophy”<sup>2</sup>.

Set theory was the field par excellence wherein the mathematical power of naming came to full fruition: discontinuity and infinity became a matter of mental creativity instead of nervous breakdown. Luzin and Egorov were not withheld by the constraints that halted Lebesgue. They fully developed the concept of ‘ef-

<sup>1</sup> A report on the revolt at Mt. Athos and details on the larger historical background can be found in the first chapter of Graham and Kantor, *Naming Infinity*.

<sup>2</sup> Graham and Kantor, “A Comparison”, 70.

fective set' or 'named set', introduced by Lebesgue in 1904 as a tool to avoid the inconveniences of the Axiom of Choice. According to Graham and Kantor, the birth of descriptive set theory coincides with the moment in 1917 on which M. Suslin, a student of Luzin's, entered his office to show a mistake he discovered in a proof by Lebesgue concerning projections of Borel sets on the real line. The famous Polish topologist W. Sierpinski witnessed the scene<sup>1</sup>. Sierpinski's own work would lead later on to another field infested by discontinuity and infinity, the theory of fractals. Luzin was convinced that properly naming a mathematical object was the key to many profound mathematical problems. We can now better understand the motto with which this paper started. Thus, Luzin's work is the fulfilment of the project outlined by Florenskii: "the naming of sets was a mathematical act, just as the naming of God was a religious one, because the point where divine and human energy meet is 'the symbol', which is greater than itself"<sup>2</sup>.

But this leaves us with a seeming historical paradox. In his book on the symbolic revolution in 16-17<sup>th</sup> century European mathematics, M. Serfati has shown beyond a shadow of doubt that the "de-rhetoricalisation" of mathematics and its increasing "symbolisation" go hand in hand<sup>3</sup>. It is nevertheless clear that the Russian use of symbols is kin to that tradition that we have sketched, that favours the use of the imagination as a source of creativity, i.e. the rhetorical tradition. How can these two developments of the use of symbols be matched? We have no intention to go into this question in any detail here, but let us suggest the outlines of a putative answer. Apparently, in the Russian case there is another source of emancipation of the symbol at work, with a very different cultural origin. The emancipation of the symbol is not linked to its being purified from any pre-existing content referring to a "chose"—a *something*—in the sense of the early algebrists, as described brilliantly by Serfati in his book. We suggest that, for the Russians, symbols act like emblems, the existence of which is as much a condition for the existence of what they "symbolise" as it is the other way around. This is confirmed by the way Graham and Kantor un-

<sup>1</sup> Ibid., 72.

<sup>2</sup> Ibid., 70.

<sup>3</sup> M. Serfati, *La révolution symbolique. La constitution de l'écriture symbolique mathématique* (Paris: Pétra, 2005).

derstand the Russian way to resort to the use of symbols: “Symbolism is the use of a perceptible object or activity to represent to the mind the semblance of something which is not shown but realised by association with it”<sup>1</sup>.

Evidently, this is generally *not* the way we construe the operation of mathematical symbols. We do agree, however, that this is exactly what the Russians had in mind: *nommer, c’est avoir individu*. Symbols play a different rôle in the two approaches—the rationalistic and the imaginative one. In the first, symbols are representations of something else, even a ‘no-thing’. In the second, symbols are signs that not simply “re-place” something, but they take place instead of the thing in the place of which they stand, a semiotic rather than a semantic relation, a co-incident rather than an arbitrary union. The messenger in a certain sense *is* the master speaking out directly<sup>2</sup>.

Il faut que ce soit quelque chose de céleste et d’inspiré qui intervienne dans l’éloquence pour exciter les transports et les admirations qu’elle cherche. (...) Il est besoin de quelqu’autre que de l’art, afin que la spéculation se rende sensible, et qu’elle tienne ce qu’elle a promis. Afin que les règles deviennent exemples, afin que la connaissance soit action et que les paroles soient des choses<sup>3</sup>.

## 4. Conclusion

Cultural differences and transformations do not only affect the way we think about the world, but also the way in which we conceive of our mental faculties. The comparative study by Graham and Kantor seems to indicate that a ‘leap of imagination’—in the traditional sense—is needed in order to be able to reach out for certain kinds of formal ideas. We believe the historical research presented in this paper confirms their findings at least to a certain extent. This prominent rôle of, literally, imagination, even when not recognised as such for want of relevant cultural reference points, sheds another light on the tenacity of the

<sup>1</sup> Graham and Kantor, “A Comparison”, 68.

<sup>2</sup> The mechanism of the coincidence between a person preaching on supply and his master has been studied in Aubrey R. Johnson, *The One and the Many in the Israelite Conception of God* (Cardiff: University of Wales Press, 1960), especially with respect to the workings of prophecy.

<sup>3</sup> J.-L. Guez de Balzac, *Paraphrase ou de la grande éloquence*, cited by M. Fumaroli, “Réception”, 395.

infamous “Platonism” which is rumoured to be so popular among practicing mathematicians, to the despair of philosophers of mathematics and logicians alike. It might well be that such type of conviction, no matter how flawed in itself, is a condition of possibility for doing certain types of mathematical inquiry at all. The dual probably also holds, but it remains a dual, not an inverse relationship.

Evidently, this paper is just a first and sketchy attempt to deal with this sort of question. In this specific case, in order to be complete, a detailed study of the philosophical and rhetorical sources of orthodox Christianity would have been needed as well. But I believe nevertheless to have shown that there are some remarkable parallels between the mental universes in which sects like the “Name-Worshippers” and the Early Modern “enthusiasts” operate. This again renders more clear why the destruction of these enthusiast tendencies throughout the European (Counter-) Reformation has had such a marked impact not only on mathematical invention and the place of imagination in philosophy, but on science in general. This line of research widens the scope of existing interpretations of these cultural phenomena: the transition from extatic Renaissance culture into the much more restrained Early Modern civilisation did not only transform forms of knowledge, but also affected the psychology of the knowledge-bearer. It moreover points to some further, potentially interesting research projects, like a re-assessment of the mathematics implied in the diagrammatic representations interspersed throughout the works of Giordano Bruno, or a re-evaluation of the philosophical and theological sources of Cantor’s ideas on infinity, developed as they were in a Germany outliving the Romantic reaction against the excesses of Reformatory Enlightenment, as well as, say, Grothendieck’s reflexions— in the first part of *Récoltes et semailles*—on “Le Reveur”, his source of inspiration, during the period of the beat-generation and the ensuing psychedelic revolution in the West.



*Bible moralisée, Paris, 13<sup>th</sup> century. Österreichische Nationalbibliothek, Cod. Vindob. 2554, f. 1 verso (particular).*