SUSAN VINEBERG

THE NOTION OF CONSISTENCY FOR PARTIAL BELIEF

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In his famous paper "Truth and Probability", Ramsey introduced the Dutch Book argument to show that degrees of belief should satisfy the probability axioms. The key point in the argument is that degrees of belief that do not satisfy the probability axioms (commonly termed incoherent) are associated with betting quotients that can be exploited by a clever bookie to produce a sure loss. Ramsey held that an agent's degrees of belief can be measured roughly by the bets that she is willing to accept. If they are incoherent, there will be a series of bets, each of which she will be willing to accept, but which are certain to result in a net loss for her. Such a collection of bets is called a Dutch Book, and it is often claimed that it is irrational for someone to have degrees of belief that could lead to having a Dutch Book being made against them.[1]

Numerous objections have been raised against the claim that it is irrational to have degrees of belief (or degrees of confidence) that are incoherent, because they leave a person vulnerable to a Dutch Book. It has been pointed out that incoherence doesn't necessarily involve Dutch Book vulnerability, because there may be nobody who can or will take advantage of the incoherence, and that a Dutch Book can be avoided by refusing to bet. Furthermore, there are cases where such vulnerability does not seem to be irrational.[2] However, it has been suggested recently by Christensen, Howson and Urbach and Skyrms (Christensen, 1991; Howson and Urbach, 1989; Skyrms, 1987), among others, that the Dutch Book argument is misunderstood if it is thought to work by forcing compliance with the probability axioms as a means of avoiding monetary loss. Instead, they claim that, Dutch Book vulnerability should be seen as a symptom of a kind of inconsistency.

For example, Christensen writes

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Dutch Book vulnerability is philosophically significant because it reveals a certain inconsistency in some system of beliefs, an inconsistency which itself constitutes an epistemic defect. (Christensen, 1991, p. 239)

This analysis seems particularly compelling if we focus on the axiom which requires that $p(A \vee B) = p(A) + p(B)$, where A and B are mutually exclusive. To see this, consider a bet on statement S that pays \$m if S is true, and nothing otherwise, for the price of m times the agent's degree of confidence in S. The payoff table for such a bet on S is given below:

S	payoff
T	$m - mDg(S)$
F	$- mDg(S)$

The Dutch Book argument assumes that the agent will evaluate such bets as fair, and accordingly should be willing to buy or sell the bet on S, provided that m is fairly small. Now consider two mutually exclusive statements S_1 and S_2, and suppose that the sum of the agent's degrees of confidence in S_1 and S_2 differ from her confidence in $S_1 \vee S_2$, i.e. $Dg(S_1) + Dg(S_2) \neq Dg(S_1 \vee S_2)$. By assumption, there are separate bets, BS_1 and BS_2, on S_1 and S_2 respectively that the agent evaluates as fair, which taken together are equivalent to a bet on $S_1 \vee S_2$. Since $Dg(S_1) + Dg(S_2) \neq Dg(S_1 \vee S_2)$, there is also a bet $B(S_1 \vee S_2)$ on $S_1 \vee S_2$ that the agent will evaluate as fair that differs from the sum of the bets on S_1 and S_2. It is this difference (inconsistency) that the bookie exploits. Since this inconsistency is tied to the agent's evaluation of the bets constituting a Dutch Book, it appears to reflect an underlying inconsistency in the agent's degrees of belief.

Ramsey himself took Dutch Book vulnerability as arising from an inconsistency in the agent's degrees of belief.

Any definite set of degrees of belief which broke them would be inconsistent in the sense that it violated the laws of preference between options ... If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have book made against him by a cunning bettor and would then stand to lose in any event. (Ramsey, 1926, p. 80)

Ramsey defined an agent's degree of belief in a proposition in terms of her disposition to act on it, and argued that her degrees of belief could be measured in terms of her preferences for bets. On Ramsey's view, degrees of belief are ultimately characterized in terms of preferences among options, and their consistency is understood in terms of rational preference, which amounts to satisfaction of an appropriate set of preference axioms.

Given this way of understanding consistency for partial beliefs, the question arises as to how consistency for partial beliefs, so understood, is related to the notion of consistency for full beliefs. Full, or simple, beliefs are said to be consistent just in case the propositions believed in are consistent. By tying the notion of consistency for full beliefs to inconsistency for propositions, it is clear that an inconsistent set of full beliefs suffers from an epistemic defect, because it is impossible that all of the propositions believed are true. This is an epistemic defect in the set of beliefs, because having knowledge requires true belief. However, if inconsistency for partial beliefs is taken as being violation of the preference axioms, it is not immediately clear that there is much of an analogy between the notion of consistency for full belief and that for degrees of belief, nor that incoherence involves an epistemic defect.

My objective here is to examine the notion of consistency for partial belief and its connection with the concept for full belief. Since the idea that failure to satisfy the probability axioms involves a type of inconsistency was introduced by Ramsey, in conjunction with the Dutch Book Argument, I shall begin by discussing the argument in more detail, and then consider two proposals for understanding the type of inconsistency involved in being incoherent. After taking up the difficulties with these accounts, I shall turn to another way of understanding the inconsistency involved in violating the probability axioms, which is suggested by non-pragmatic arguments for probabilism.

DUTCH BOOK VULNERABILITY AND INCONSISTENCY

The argument that Dutch Book vulnerability indicates a kind of inconsistency assumes that degrees of belief are associated with particular betting quotients. It is sometimes held further that an agent's degrees of belief just *are* her fair betting quotients. This, in turn, can mean either that the agent is willing to accept bets (at least with small stakes) where her degree of belief matches the betting quotient, or that she will at least evaluate such bets as fair. Suppose for now that degrees of belief are associated with evaluations of bets, at least to the extent that agents who are incoherent will evaluate individual bets where the their degrees of confidence match the betting quotient as fair. In particular, it is to be assumed that an incoherent agent will find the bets offered by the bookie, which lead to a Dutch Book, as at least individually fair.[3] Insofar as degrees of belief, so understood, are tied to preferences, incoherent degrees of belief may be said to involve an inconsistency in the agent's preferences, as suggested by Ramsey. In addition to involving inconsistency of preference, which appears quite removed from the familiar notion for full belief, it has also been maintained that such degrees of belief are inconsistent in a way that is more closely related to inconsistent full beliefs.

An analysis of this kind is suggested by Armendt, who claims that the underlying condition that leads to a Dutch Book is that agents who violate the probability axioms give conflicting evaluations to the same betting options. Armendt terms this divided-mind inconsistency (Armendt, 1992, 1993). If an agent whose degrees of belief are incoherent really does give two different evaluations to the same state of affairs, this would seem to be a kind of epistemic defect, even if it is also a pragmatic liability.[4]

Here again, the analysis looks quite plausible in the case where an agent violates the axiom that requires that $p(A \vee B) = p(A) + p(B)$, where A and B are mutually exclusive. When the agent's degrees of belief violate the additivity axiom for mutually exclusive A and B, [i.e. $Dg(A \vee B) \neq Dg(A) + Dg(B)$, a Dutch Book is made by buying (selling) a bet on the disjunction $A \vee B$ and by buying(selling) bets on A and on B. The agent's degrees of confidence are supposed to result in her evaluating as fair a bet on $A \vee B$ at a differing price from the sum of the prices she takes as fair for bets on A and on B. The bookie will be assured a profit, since a bet on A and on B is equivalent to a bet on A or B (when A and B are mutually exclusive). Thus, Dutch Book vulnerability in this case can apparently be attributed to giving different evaluations to equivalent betting arrangements.

It is seemingly less plausible that violation of the other axioms involves divided-mind inconsistency. Consider an agent who violates the axiom requiring that p(T) = 1, where T is a tautology. Let us assume, with Armendt, that degree of belief and probability functions take as arguments propositions.[5] In specifying degree of belief and probability assignments, sentences are used as naming expressions. How then could divided-mind inconsistency occur? Since different tautologous sentences, say '$-p \rightarrow (p \rightarrow q)$' and '$p \vee -p$', name the same tautologous proposition, if a person were to assign a value less than one to the tautology under the first name, but not under the second, then she would in effect be making two different assignments to the same proposition. This, by itself, would certainly constitute a form of inconsistency. If we assume here that degrees of belief are tied to evaluations of bets, such an agent would also possess divided-mind inconsistency, in Armendt's sense. Notice though that an agent could violate the axiom requiring that the probability of the tautologous proposition is one without possessing either sort of inconsistency, provided that she attaches the same confidence level less than one under every name used to associate a level of confidence with T. One could presumably assign a degree of confidence to a tautology by way of the name '$p \rightarrow p$', yet fail to have any other confidence assignments. If this is right, then incoherence does not entail divided-mind inconsistency.[6]

It is clear that Armendt assumes that assigning a degree of confidence to a proposition involves evaluating bets under many different descriptions. To argue that violating the axiom requiring that dg(T) = 1 involves divided mind inconsistency, Armendt says that

> we can point out that a fair betting quotient of other than 1 for a tautology T is an assessment that a bet paying $1 (or 1 utile) if T has value different from a gift of $1 (1 utile), which comes to the same thing. (Armendt, 1993, p. 19)

Assuming, for now, that an agent who assigns a degree of belief to a tautology T has assigned a fair betting quotient to T, must we also accept that she has considered the value of every betting arrangement, which is in fact equivalent to betting on T? Indeed, isn't it possible, if very unlikely, that she thinks that the second description above really does describe the same arrangement as the first, and evaluates it as also being worth less than \$1? Such incoherence would not involve divided-mind inconsistency.

In any case, there is an additional difficulty with the view that the inconsistency that leads to Dutch Book vulnerability involves evaluating the same option in two different ways. Ironically the difficulty emerges from the axiom previously touted as providing motivation for the idea that violation of the probability axioms involves divided-mind inconsistency, namely that for disjoint statements S_1 and S_2, $pr(S_1 \vee S_2) = pr(S_1) + pr(S_2)$. To show that violation of the axiom leads to Dutch Book vulnerability, it is not only assumed that the agent's degree of confidence in S_1 fixes the fair price for a bet on S_1 and similarly for bets on S_2 and on $(S_1 \vee S_2)$, but also that if she finds a bet on S_1 fair and a bet on S_2 fair, she will regard the compound bet on S_1 and on S_2 fair. If she does not evaluate the compound bet on S_1 and on S_2 as fair, then she need not be vulnerable to a Dutch Book, despite the fact that she violates the additivity axiom for probability. Let BS denote a bet on S and let V(BS) stand for the value of that bet. The general assumption needed for the Dutch Book argument is that $V(BS_1) + V(BS_2) = V(BS_1 \vee S_2)$. It is certainly correct that a compound bet on S_1 together with a bet on S_2 is equivalent to a bet on their disjunction, i.e. $V(BS_1 + BS_2) = V(BS_1 \vee S_2)$. What is open to doubt is the further claim that $V(BS_1) + V(BS_2) = V(BS_1 + BS_2)$.[7] If this equality fails to hold, then although the agent may have incoherent degrees of confidence she need not be vulnerable to a Dutch Book. Furthermore such an agent apparently does not evaluate the same betting arrangement in two different ways; since, for her, evaluating the individual bets does not fix an evaluation of the compound bet.

It seems then that violation of the probability axioms in this case need not lead to Dutch Book vulnerability which was the basis for the claim that such violations involve divided-mind inconsistency. It could be argued that measures of utility that can be defined with respect to rational preferences are additive. This would offer a way of filling the hole in the Dutch Book argument. Indeed, if an appropriate set of axioms governing rational preference are assumed, then it can be shown that utility and probability functions p and u exist for which preferences satisfying the axioms maximize utility with respect to p and u. It is such representation theorems which are at the heart of the idea that confidence assignments can be characterized in terms of preferences, and that incoherence reduces to inconsistency of preference.[8] However, this brings us back to the question of how violating the preference axioms is supposed to be analogous to having inconsistent beliefs. Moreover, even if the preference axioms can be justified as constraints on rational beliefs, which I have argued is needed to show that violating the additivity axiom of probability involves divided-mind inconsistency, this still wouldn't show that all forms of incoherence involve divided-mind inconsistency.

HOWSON AND URBACH'S VERSION OF THE DUTCH BOOK ARGUMENT

In their book *Scientific Reasoning: The Bayesian Approach*, Howson and Urbach present a version of the Dutch Book argument, which they seem to take as demonstrating the connection between incoherence and the ordinary notion of consistency for full belief. Unlike some versions of the argument, they do not assume that an agent will actually be willing to place bets in accordance with her degrees of belief, but rather take degrees of belief to involve an analysis that certain bets are fair. Moreover, they state quite emphatically that they do not assume that there is any definite connection between degrees of belief and action.[9] For Howson and Urbach degrees of belief are defined in terms of subjectively fair odds, which are those odds that, as far as the agent can tell, confer no positive advantage or disadvantage for either side. Betting quotients are then defined in the usual way in terms of odds, so that if a person's subjectively fair odds on h are q, then her subjectively fair betting quotient for h is q/(q + 1). Howson and Urbach then associate the agent's degrees of belief with her subjectively fair betting quotients. Objectively fair odds are defined as those odds which, in fact, do not confer an advantage to either side.

With these definitions in place, Howson and Urbach appeal to the Dutch Book theorem, which states that

> if a set of betting quotients do not satisfy the probability axioms, there is a series of bets in accordance with those betting quotients that is bound to lose.

They then argue that since a set of betting quotients which is bound to lose must confer an advantage to one side, such betting quotients cannot all be fair. It then follows from the Dutch Book theorem that the betting quotients is a set which does not satisfy the probability axioms *cannot* all be fair. Finally, since an agent's degrees of belief have been associated with the betting quotients she thinks fair, the Dutch Book theorem is taken to show that if those degrees of belief do not satisfy the axioms, then despite the fact the agent thinks they correspond to fair betting quotients, they cannot do so.

Howson has since claimed further that fairness is the analog for truth in assessing degrees of confidence. He states:

> The probability axioms are a sound and complete syntax with respect to the semantic criterion of consistency-coherence. (Howson, 1997, p. 278)

Not only has he drawn an analogy between fairness and truth, and hence between the notion of consistency for degrees of belief and full belief, but the former notion has in effect been reduced to the latter. To have a degree of belief r in proposition p is to believe that r is the fair betting quotient for p, according to Howson and Urbach's definition. Let $r_1, \ldots r_n$ be an agent's degrees of belief in propositions $q_1, \ldots q_n$. Let p_i be the claim that r_i is the fair betting quotient for q_i. For each i, the agent believes that p_i is true. If the agent's degrees of belief are incoherent, the agent then believes that p_i is true, for each i, when the set of propositions p_i is inconsistent.

There are several difficulties with this analysis of the sort of inconsistency involved in violating the probability axioms. The first concerns how advantage, and with it fairness, is to be defined. Howson and Urbach understand the concept of advantage informally, but Maher (Maher, 1997) has recently pointed out that if 'advantage' is understood in the ordinary sense, of benefit, profit or gain, then their version of the Dutch Book argument will not go through. Of course, advantage might be defined in terms of expected utility, but this would just lead back to understanding fairness, and ultimately degrees of confidence, in terms of preference.

Suppose though that somehow the notion of fairness has been well defined, so that it would make good sense to say that an agent's degree of belief in q is her subjectively fair betting quotient for q. The adequacy of the proposed analysis of consistency for partial beliefs then depends on whether having a degree of belief of r in proposition q really is to take r as the fair betting quotient for q. I have suggested that this means that the agent believes that r is the fair betting quotient for q, for this seems the natural interpretation. However, this interpretation is somewhat problematic for Howson and Urbach, since they do not endorse a notion of acceptance or belief, apart from identifying it with high probability. But, understanding belief as high degree of belief would make their definition of degree of belief circular. Howson and Urbach's characterization of degrees of belief, on which their analysis of consistency for degrees of belief depends, thus appears to require an additional notion of belief or acceptance that is not reducible to degrees of belief. A probabilist might reject their analysis of consistency, rather than admit an additional propositional attitude of belief or acceptance, though there are independent reasons for supposing that such a notion is needed.[10]

There are other problems with supposing that an agent's degree of belief in q is identical with her subjectively fair odds for q.[11] First, it is reasonable to attribute degrees of confidence to a person, though perhaps not sharp degrees of confidence, without supposing that the person ever consciously associates their confidences with fair bets, or even has any sort of clear understanding of the concept. Another possibility is that although the person understands her degrees of belief in terms of fair betting quotients, her degree of belief in some proposition h does not equal her fair betting quotient for h, due perhaps to some brain lesion. For example, it seems quite possible that a person might declare that she is highly confident that h is true, and by and large she might act as though she is highly confident of h, but when, and only when, it comes to evaluating explicit bets on h she takes the fair betting quotient for h to be low.

There is another reason to suppose that an agent's degrees of confidence need not be identical with her fair betting quotients. To see this, suppose that the fair betting quotient for A is taken to be r. Where $r < 1$, some bets against A should be taken as fair and this seems to require that $-A$ be taken as possible. Notice that if this is correct, then having degrees of belief that violate the probability axioms because some tautology is assigned a value less than one involves a straightforward inconsistency, since this would involve taking $-T$ as possibly true, when it cannot be. It should be observed that the Dutch Book vulnerability of someone who is less than fully confident in a tautology T turns on the fact that −T is logically impossible. In fact, this suggests a more direct way of arguing that the probability axioms are a consistency constraint on degrees of belief, which does not invoke the pragmatic concept of fairness. The idea here is that degrees of belief could be taken as a sum of the possible ways that propositions could be true. In particular, to have less than full confidence in a proposition would involve taking there to be ways in which it could be false. The problem with this approach is that it is not right to say that having less than full confidence in a proposition A always involves holding that −A is possible. Consider an unproven mathematical claim such as Goldbach's conjecture. It is generally thought that it is either necessarily true or necessarily false. Since it is as yet unproven, it seems quite reasonable to be less than fully confident of its truth; but, the overwhelming numerical evidence seems to warrant high confidence. However, having recognized that it is either necessarily true or necessarily false, one might well not regard Goldbach's conjecture as possibly false. Moreover, in having a high level of confidence r < 1 in Goldbach's conjecture, on the basis of the positive numerical evidence, one need not regard r as its fair betting quotient. Indeed, this could stem from recognizing directly that r cannot be a fair betting quotient.

Where degrees of belief can be understood as fair betting quotients, the probability axioms can be taken as a kind of consistency constraint on them. However, there is an important disanalogy between this notion and that for full beliefs. To believe that p just is to think that p is true, and hence to have inconsistent degrees of belief in a set of propositions is to think that they satisfy a condition that they cannot possibly satisfy. Having degrees of belief that fail to satisfy the probability axioms need not involve taking those degrees of belief to have a property that they cannot possess, though it would if one takes those degrees of belief as fair betting quotients. But, we have seen that degrees of belief need not be taken fair betting quotients. For example, one could have degrees of belief based in some way on the available evidence, and in such a case one might well recognize that those degrees of belief are not fair betting quotients and do not satisfy the probability axioms. How the notion of consistency for degrees of belief, so understood, could be defined remains unclear, but it would not be defined in terms of fairness.

Even assuming that degrees of belief can be taken in a particular context as being fair betting quotients, the analogy between Howson and Urbach's characterization of their consistency and that for full belief breaks down in an additional respect. Howson claims that fairness is a semantic notion, but given that this notion is explicated in terms of advantage, it is not clear that is genuinely semantic. Of course, Howson wishes to refrain from defining advantage in terms of preference, but it is unclear that he can do so and retain his version of the Dutch Book argument for probabilism. If indeed, fairness and advantage are ultimately to be understood in terms of preference, then inconsistency for partial beliefs would be a pragmatic, rather than a semantic concept. Suppose that degrees of belief are taken as fair betting quotients, and that fairness is understood in terms of preference. Howson and Urbach have still shown that there is more of a connection between this understanding of degrees of belief and full belief than is readily apparent from Ramsey's remark that degrees of belief that fail to satisfy the probability axioms are inconsistent in the sense that they violate the laws of preference, for an agent whose degrees of belief fail to satisfy the probability axioms takes her degrees of belief to be fair, when they cannot be.

CONSISTENCY WITHOUT FAIRNESS

Insofar as coherence can be viewed as an epistemic notion, it should be possible to characterize the inconsistency involved in violating coherence without employing non-epistemic terms. One avenue makes use of Van Fraassen's observation that incoherence precludes vindication, in the sense that incoherent degrees of belief cannot be perfectly calibrated (van Fraassen, 1989). To explain the concept of calibration suppose that you turn to the Weather Channel and the forecaster announces that there is a 60% chance of snow showers for the New York area. The forecaster is said to be perfectly calibrated just in case,

> The proportion of days with snow showers among those when the forecaster predicts a 60% chance of snow showers is 60%.

The concept of calibration provides a way of making sense of the correctness of probability judgments in cases where probabilities can also be interpreted as relative frequencies. It is generally supposed that in cases where predictions are made about an event of a particular type, for which there is data about the relative frequency of events of that type occurring, it is reasonable, if not required, to have a personal probability judgment equal to the observed relative frequency. However, calibration is only appropriate as a measure of the correctness of personal probability judgments about A in cases where A describes an event that belongs to a class of essentially similar events for which frequencies can be obtained. For most propositions, such as the claim that there are leptoquarks, there will be no appropriate reference class, which will allow us to make sense of the rightness of such judgments in terms of the concept of calibration.

The possibility of perfect calibration would provide a characterization of consistency for partial belief that is closer to that for full belief than is obtained by invoking the concept of fairness, because, like truth, calibration is a measure of the accuracy for beliefs. However, calibration is not the only measure of accuracy for partial beliefs. Recently, Joyce has introduced what he calls a measure of gradational accuracy for degrees of belief that differs from calibration (Joyce, 1998). The highest level of gradational accuracy accrues to someone who believes all truths to the highest degree and attaches the lowest degree to each falsehood. Since actual agents will almost certainly have beliefs that fail to attain the highest level of gradational accuracy, they must be concerned with the various ways of being wrong. The precise details are unimportant here, but on Joyce's evaluation scheme, the worst way to be wrong is to be fully confident in a false proposition and to be minimally confident in a true one. This makes it reasonable to be less than fully confident in at least most contingent propositions. After providing an exact definition of gradational accuracy, Joyce then proves that if a set of degrees of belief is incoherent, there must be another set of degrees of belief that has greater gradational accuracy in his sense, regardless of which propositions turn out to be true. Joyce uses his result together with the following norm of gradational accuracy to argue for probabilism.

> Norm of Gradational Accuracy (NGA): An epistemically rational agent must evaluate partial beliefs on the basis of their gradational accuracy, and she must strive to hold a system of partial beliefs that, in her best judgment, is likely to have an overall level of gradational accuracy at least as high as that of any alternative system she might adopt. (Joyce, 1998, p. 579)

Suppose a person's degrees of belief are understood as those that she takes as likely to have an overall level of gradational accuracy that is at least as high as that of any alternative. If her degrees of belief are incoherent, then they cannot be likely to have a level of gradational accuracy at least as high as that of any alternative set of degrees of belief. Thus, the concept of gradational accuracy leads to yet another way of understanding consistency for partial belief, and it is one which has the virtue of not appealing to any pragmatic notions such as fairness or preference.

It is a reasonable epistemic goal for an agent to try to maximize the level of gradational accuracy of her degrees of belief. However, it is not the only reasonable goal. On Joyce's definition of gradational accuracy, assigning any necessary truth N less than probability one will result in having degrees of belief that are less gradationally accurate than they could be, regardless of the facts. This is because the necessary truth N will be true in every possible world, and so a set of degrees of belief S in which N has a degree of belief of less than one must be less gradationally accurate than one which is just like S except that N is assigned one. There certainly appear to be circumstances in which it would be reasonable to be less than fully confident in a necessary truth. For example, it seems quite reasonable to be less than fully confident in unproven mathematical propositions, such as Goldbach's Conjecture. As noted previously, where degrees of belief are taken as reflecting the amount of evidence, or lack of evidence, for a proposition, there would be circumstances in which necessary truths would receive a value less than one. Of course, Joyce says only that one should strive for gradational accuracy, so he could agree that a person would not be epistemically irrational in violating the probability axioms by being less than fully confident in some necessary truths. However, he clearly intends the norm of gradational accuracy to carry force in compelling satisfaction of the probability axioms, in a way that the goal having true beliefs is not supposed to work as a sanction against having less than full confidence in all true propositions now.

Since it appears that one could be epistemically rational in having degrees of belief that do not satisfy the probability axioms, we should not suppose that having a set of partial beliefs is to think that those beliefs are likely to maximize gradational accuracy. So having degrees of belief that do not satisfy the probability axioms is not to think that they are likely to have the property of maximizing gradational accuracy, when they do not. Again, there is a difference with having full beliefs, which involves thinking that the propositions believed are true, and so having inconsistent beliefs means that the propositions believed cannot have the property that one attributes to them by the act of believing in them. The disanalogy reflects the fact that we have failed to locate a univocal concept of partial belief. Nevertheless, there is a central concept of partial belief, such that agents aim to have degrees of belief that are likely to maximize gradational accuracy. The notion of consistency here is analogous in important respects to that for full belief. For instance, as believing that a necessarily false proposition involves an inconsistency, because it cannot be true, so having less than full confidence in a necessary truth involves having degrees of confidence that could not be optimal with respect to gradational accuracy. More importantly, it is those degrees of belief that one takes as optimizing gradational accuracy that it makes sense to take as fair betting quotients, so it is unsurprising that the notion of consistency for degrees of belief understood in both senses coincides with satisfaction of the probability axioms.

NOTES

[1] For details on how a Dutch Book can be constructed against someone whose degrees of belief violate the axioms, see Skyrms' *Choice and Chance* (Skyrms, 1975). For the argument that it is irrational to have degrees of belief that do not

satisfy the axioms see Jackson and Pargetter's, "A modified Dutch Book Argument" (Jackson and Pargetter, 1976).

[2] See for example Adams and Rosenkrantz, 1980; Kennedy and Chihara, 1979; Seidenfeld et al., 1990; Maher 1993; Foley 1992.

[3] Insofar as coherence is thought to be a norm of rationality, it is a substantial assumption that incoherent agents would evaluate bets in this way.

[4] Skyrms also suggests that incoherence is tied to the familiar notion in pointing out that "Ramsey ... has provided a way in which the fundamental laws of probability can be viewed as pragmatic consistency conditions: conditions for the consistent evaluation of betting arrangements no matter how described" (Skyrms, 1980).

[5] The majority of philosophers take propositions or statements as the arguments of probability functions, although some take probabilities to apply to sentences. The subject of whether probabilities attach to propositions, statements or sentences is mainly an issue within the philosophy of logic. For most discussions of scientific reasoning, it matters little which sort of object is taken as the appropriate argument for a probability function. A notable exception involves Garber's solution to the problem of old evidence, which seems to require that probabilities attach to sentences (Garber, 1983).

[6] A related point was made by Titiev in (Titiev, 1993).

[7] There are many examples where $V(BS_1) + V(BS_2) = V(BS_1 + BS_2)$ is apparently violated. For instance, I might be willing to pay $1 to bet on the toss of a fair coin, where I will win $2 if it comes up heads, and also willing to bet $1 on the toss of a fair die, where I will win $2 if the number is even, but be unwilling to take both bets, because I need at least $1 for bus fare home.

[8] For a presentation of representation theorems and their use in arguing for probabilism, see (Maher, 1993).

[9] In this, they differ with Armendt, who assumes that degrees of belief are guides to action, though with Howson and Urbach, Armendt does not assume that having degrees of belief means that one will take bets that are fair or advantageous.

[10] See, Kaplan, 1996; Maher, 1993.

[11] Christensen has recently made similar complaints about the idea that degrees of belief are, or reduce to, fair betting quotients (Christensen, 1996).

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Department of Philosophy
Wayne State University
Detroit, MI 48202, USA
E-mail: susan.vineberg@wayne.edu