Platonic Division and the Origins of Aristotelian Logic

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Abstract

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Aristotle's syllogistic theory, as developed in his *Prior Analytics*, is often regarded as the birth of logic in Western philosophy. Over the past century, scholars have tried to identify important precursors to this theory. I argue that Platonic division, a method which aims to give accounts of essences of natural kinds by progressively narrowing down from a genus, influenced Aristotle's logical theory in a number of crucial respects. To see exactly how, I analyze the method of division as it was originally conceived by Plato and received by Aristotle. I argue that, while Plato allowed that some divisions fail to rigorously investigate the essence, he began a program continued by Aristotle (and others in antiquity and the middle ages) of seeking norms for division that would apply in any domain whatsoever. This idea of a rigorous, general method was taken up and developed by Aristotle in his syllogistic. Aristotle also used Plato's conception of predication as parthood in his semantics for syllogistic propositions. As part of my argument, I prove that a semantics based on Platonic divisional structures is sound and complete for the deduction system used in the literature to model Aristotle's syllogistic.

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Chapter 1 Introduction

What is Plato's method of division? Plato himself takes the trouble to show us, at least superficially. In the dialogue *Sophist*, the Eleatic Visitor is trying to explain to his interlocutors how he will pursue his investigation of sophistry. While this is probably not the earliest dialogue in which the method appears,¹ it is the one where we get the most thorough example of someone self-consciously carrying it out in order to show someone how it is to be done:

Visitor: Well then, let's go after the angler from this starting point. Tell me, shall we take him to be an expert at something, or a nonexpert with another sort of capacity?

Theaetetus: He's definitely not a nonexpert.

V: But expertise as a whole falls pretty much into two types.

T: How?

V: There's farming, or any sort of caring for any mortal body; and there's also caring for things that are put together or fabricated, which we call equipment; and there's imitation. The right thing would be to call all those things by a single name.

T: How? What name?

V: When you bring anything into being that wasn't in being before, we say you're a producer and that the thing you've brought into being is produced.

T: That's right.

¹See Lukas (1888) for discussion of division in nine Platonic dialogues. However, he does not discuss the *Euthyphro* (see below, also Meinwald (2002)) or *Cratylus* (Ademollo, 2011). While the method arguably appears frequently, it is only emphasized and thoroughly described in the *Phaedrus, Sophist, Statesman*, and *Philebus*. Although I don't need to argue for it here, the pervasiveness of division throughout the Platonic corpus puts pressure on the idea that there is some sort of development in Plato's thinking from Socratic elenchus to the method of hypothesis to the method of collection and division. Robinson (1953) goes so far as to delimit his subject by the absence of this final stage.

V: And all the things we went through just now have their own capacity for that.

T: Yes.

V: Let's put them under the heading of production.

T: All right.

V: Next, consider the whole type that has to do with learning, recognition, commerce, combat, and hunting. None of these creates anything. They take things that are or have come into being, and they take possession of some of them with words and actions, and they keep other things from being taken possession of. For that reason it would be appropriate to call all the parts of this type acquisition.

T: Yes, that would be appropriate.

V: If every expertise falls under acquisition or production, Theaetetus, which one shall we put angling in?

T: Acquisition, obviously.

V: Aren't there two types of expertise in acquisition?

•••••

V: So now we're in agreement about the angler's expertise, not just as to its name; in addition we've also sufficiently grasped a verbal explanation concerning the thing itself. Within expertise as a whole one half was acquisitive; half of the acquisitive was taking possession; half of possession-taking was hunting; half of hunting was animal hunting; half of animal hunting was aquatic hunting; all of the lower portion of aquatic hunting was fishing; half of fishing was hunting by striking; and half of striking was hooking. And the part of hooking that involves a blow drawing a thing upward from underneath is called by a name that's derived by its similarity to the action itself, that is, it's called draw-fishing or angling—which is what we're searching for.

T: We've got a completely adequate demonstration of that, anyway. (*Sophist* 219a4-d4, -221a7-c4)²

²{ΞΕ.} Φέρε δή, τῆδε ἀρχώμεθα αὐτοῦ. καί μοι λέγε· πότερον ὡς τεχνίτην αὐτὸν ἤ τινα ἄτεχνον, ἄλλην δὲ δύναμιν ἔχοντα θήσομεν; {ΘΕΑΙ.} "Ηκιστά γε ἄτεχνον. {ΞΕ.} Ἀλλὰ μὴν τῶν γε τεχνῶν πασῶν σχεδὸν εἴδη δύο. {ΘΕΑΙ.} Πῶς; {ΞΕ.} Γεωργία μὲν καὶ ὄση περὶ τὸ θνητὸν πᾶν σῶμα θεραπεία, τό τε αῦ περὶ τὸ σύνθετον καὶ πλαστόν, ὃ δὴ σκεῦος ὠνομάκαμεν, ἤ τε μιμητική, σύμπαντα ταῦτα δικαιότατ' ἂν ἑνὶ προσαγορεύοιτ' ἂν ὀνόματι. {ΘΕΑΙ.} Πῶς καὶ τίνι; {ΞΕ.} Πῶν ὅπερ ἂν μὴ πρότερόν τις ὄν ὕστερον εἰς οὐσίαν ἄγῃ, τὸν μὲν ἄγοντα ποιεῖν, τὸ δὲ ἀγόμενον ποιεῖσθαί πού φαμεν. {ΘΕΑΙ.} Όρθῶς. {ΞΕ.} Τὰ δέ γε νυνδὴ <ἂ> διήλθομεν ἄπαντα εἶχεν εἰς τοῦτο τὴν αὐτῶν δύναμιν. {ΘΕΑΙ.} Ἐίχε γὰρ οῦν. {ΞΕ.} Ποιητικὴν τοίνυν αὐτὰ συγκεφαλαιωσάμενοι προςείπωμεν. {ΘΕΑΙ.} Ἔστω. {ΞΕ.} Τὸ δὴ μαθηματικὸν αῦ μετὰ τοῦτο εἶδος ὅλον καὶ τὸ τῆς γνωρίσεως τό τε χρηματιστικὸν καὶ ἀγωνιστικὸν καὶ θηρευτικόν, ἐπειδὴ δημιουργεῖ μὲν οὐδὲν τούτων, τὰ δὲ ὄντα καὶ γεγονότα τὰ μὲν χειροῦται λόγοις καὶ πράξεσι, τὰ δὲ τοῖς χειρουμένοις οὐκ ἐπιτρέπει, μάλιστ' ἅν που διὰ ταῦτα συνάπαντα τὰ μέρη τέχνη τις κτητικὴ λεχθεῖσα ἂν δι-

This passage gives us a model for how to understand division in general, just as it did the interlocutors. A few features are immediately apparent:

- 1. Division in Plato's sense divides a thing into things which can themselves be divided. I will refer to the divisum and dividentia generally as *kinds*. I will generally call the divisum the *genus* and the dividentia *species* in a particular division.
- 2. The dividens is described as a *part* of the divisum. Moreover, parts in this sense seem to be *less general* than their wholes.
- 3. The process of division ends when one has reached the object of investigation. I will call this the *target kind*.
- 4. One can obtain a *definition* of that object of investigation through piecing together the steps of the division.

This helps us get a preliminary grasp on division,³ but does not yet illuminate either the problem that division is meant to solve or show us why anyone would think division is meant to solve it.

I will try to do both of these through a reading of the *Euthyphro*, with some comments about the *Meno*. Both these dialogues (just as many other Socratic dialogues such as they *Laches*, *Charmides*, etc) pursue a *definitional* question, sometimes called a "What is F?" question. In various dialogues, Socrates claims a special status for such questions in the structure of knowledge and inquiry. In the *Euthyphro*, Socrates claims that our questions of what piety and justice are are prior to questions of whether particular actions are lawful (6e), while in the *Meno*, he claims that the question of what virtue is is prior to the question of whether virtue is teachable (71b).

If these claims are right, or even close to right, definitional questions assume enormous importance in Socratic methodology. On the other hand, reading the dialogues shows just how hard it is to find satisfactory definitions. There are so many constraints on a good definition that it is difficult to come up with an account that can simultaneously satisfy them all.

απρέψειεν. {ΘΕΑΙ.} Ναί· πρέποι γὰρ ἄν. {ΞΕ.} Κτητικῆς δὴ καὶ ποιητικῆς συμπασῶν οὐσῶν τῶν τεχνῶν ἐν ποτέρα τὴν ἀσπαλιευτικήν, ὥ Θεαίτητε, τιθῶμεν; {ΘΕΑΙ.} Ἐν κτητικῆ που δῆλον. {ΞΕ.} Κτητικῆς δὲ ἄρ' οὐ δύο εἴδη; [...]{ΞΕ.} Νῦν ἄρα τῆς ἀσπαλιευτικῆς πέρι σύ τε κἀγὼ συνωμολογήκαμεν οὐ μόνον τοὕνομα, ἀλλὰ καὶ τὸν λόγον περὶ αὐτὸ τοῦργον εἰλήφαμεν ἱκανῶς. συμπάσης γὰρ τέχνης τὸ μὲν ῆμισυ μέρος κτητικὸν ῆν, κτητικοῦ δὲ χειρωτικόν, χειρωτικοῦ δὲ θηρευτικόν, τοῦ δὲ θηρευτικοῦ ζωοθηρικόν, ζωοθηρικοῦ δὲ ἐνυγροθηρικοῦ δὲ τὸ κάτωθεν τμῆμα ὅλον ἁλιευτικῆς δὲ ἀγκιστρευτικόν· τούτου δὲ τὸ περὶ τὴν κάτωθεν ἀμασημμέν, ἀλιευτικῆς δὲ ἀρκαιτικῆς δὲ ἀρκουτικοῦ δὲ ἀροθηρικοῦ δὲ ἀνογροθηρικοῦ δὲ τὸ κάτωθεν τμῆμα ὅλον ἀλιευτικόν, ἀλιευτικῆς δὲ πληκτικόν, πληκτικῆς δὲ ἀγκιστρευτικόν· τούτου δὲ τὸ περὶ τὴν κάτωθεν ἄνω πληγὴν ἀνασπωμένην, ἀπ' αὐτῆς τῆς πράξεως ἀφομοιωθὲν τοὕνομα, ἡ νῦν ἀσπαλιευτικὴ ζητηθεῖσα ἐπίκλην γέγονεν. {ΘΕΑΙ.} Παντάπασι μὲν οῦν τοῦτό γε ἱκανῶς δεδήλωται. Text from Burnet (01 7). Translations of Plato are from Cooper (1997) with some alterations, unless otherwise noted.

³This is not the place for detailed defense of these views, which will have to wait for Chapters 2 and 3.

A good example of this is in the *Euthyphro*. In the dialogue, the eponymous character offers three definitions of the pious:

- 1. what I'm doing right now (5d-e)
- 2. what the gods love (7a)
- 3. what all the gods love (9e)

With each definition, Socrates adds a constraint on good definitions:

- 1. The definition must cover all cases of piety. $(6d-e)^4$
- 2. The definition must cover only cases of piety. $(8a-b)^5$
- 3. The definition must be explanatory, stating why cases of piety are so. (10a $ff)^6$

After these refutations, Socrates takes control of the conversation and proposes a new way to approach the question of piety's definition. First he asserts that piety is a part of justice in the way shame is a part of fear and odd a part of number. Shame and odd seem to be parts of fear and number in the sense that the latter kinds are *predicated* of the former. The same seems to be true of piety and justice: everything pious is necessarily just but not the other way around. Socrates seems to think that this is a good starting point:

Socrates: Then consider the next point. If the pious is a part of what's just, we must, it seems, find out what part of the just the pious is. Now if you asked me about one of the things we just mentioned, for example, which part of number is the even—that is to say, what sort of number it is—I'd say that it's any number not indivisible by two, but divisible by it. Or don't you agree?

Euthyphro: Yes, I do.

Socrates: Then you try to teach me in the same fashion what part of the just is pious. $(12d5-e2)^7$

Here we start with an uncontroversial idea:⁸ piety is a kind of justice. Justice,

⁴Euthyphro's mistake here is so egregious that it might even be denied that he answered the question at all. Socrates, however, takes this mistake as an opportunity to explain how he wants something present *in all pious actions*.

⁵The problem with Euthyphro's definitional attempt here is that his definitions of piety and impiety *overlap* when the two are clearly opposites. This implies that one or both of these definitions is too broad.

⁶Here is not the place to give a reading of this complex argument, but see Evans (2012) and Ebrey (forthcoming).

⁷{ΣΩ.} Όρα δὴ τὸ μετὰ τοῦτο. εἰ γὰρ μέρος τὸ ὅσιον τοῦ δικαίου, δεῖ δὴ ἡμᾶς, ὡς ἔοικεν, ἐξευρεῖν τὸ ποῖον μέρος ἂν εἶη τοῦ δικαίου τὸ ὅσιον. εἰ μὲν οὖν σύ με ἡρώτας τι τῶν νυνδή, οἶον ποῖον μέρος ἐστὶν ἀριθμοῦ τὸ ἄρτιον καὶ τίς ῶν τυγχάνει οῦτος ὁ ἀριθμός, εἶπον ἂν ὅτι ὃς ἂν μὴ σκαληνὸς ῇ ἀλλ' ἰσοσκελής- ἢ οὐ δοκεῖ σοι; {ΕΥΘ.} Ἔμοιγε. {ΣΩ.} Πειρῶ δὴ καὶ σὺ ἐμὲ οὕτω διδάξαι τὸ ποῖον μέρος τοῦ δικαίου ὅσιόν ἐστιν.

⁸At least, an idea assumed to be uncontroversial.

whatever *that* is, can and should be understood *independently* of its parts. (*Meno* 79d) Now, Socrates asks, what *else* is there to piety beyond justice? Here Euthyphro gets stuck. His attempted addition to the definition just leads back to the problems faced earlier.

Socrates has asked, in effect, for Euthyphro to use the method of division in the same way that the Eleatic Visitor did for sophistry. The four marks of division all appear there:

- 1. While there is no explicit subdivision, we are clearly here in the realm of *kinds*, as emphasized early on by Socrates' use of the term *idea*.
- 2. The method proceeds beginning with a kind (justice, number, fear) and proceeding down its parts.
- 3. We end when we reach the object of inquiry: piety. In this case, because piety is a very general kind, it does not take so long to get there.
- 4. The stages of division are intended to lead to the definition.

While it is not nearly as elaborate or self-conscious as in the *Sophist*, the very idea of approaching definition by differentiating a kind from other kinds that are also "parts" of the same whole is the core insight of the method of division.

The attempt fails. Euthyphro cannot correctly make out how piety differs from other kinds of justice without collapsing back into his third definition. While Socrates is undeterred and wants to try again to find out what piety is, Euthyphro loses patience and hurries off.

Socrates does not explain in so many words why this method is promising. He just starts doing it, without even giving the kind of demonstration of the method that the Eleatic Visitor does. We can only infer that Socrates thinks this method can produce definitions that satisfy the constraints that emerged in Euthyphro's examination.

And it is natural to think so. If we start from a more general kind and progressively narrow down, we would stop at a part of justice that is coextensive with piety, satisfying the first desideratum. In particular, we would avoid giving too narrow a definition, since we would presumably recognize the correct definition of piety if we arrived at that first. Further, if we clearly state *which* part of justice piety is, our account will not include anything which is not pious or overlap in definition with impiety. Finally, we want our definitions to be explanatory. This requires that they are given in terms of things which are *prior* to the definiendum. There are two main ways that this can fail: one can define a kind in terms of its species⁹ or one can define a kind in terms of affections, Plato's term for derivative or otherwise inessential properties. Division obviously will not lead to the

⁹Indeed, one can make this kind of error in two ways as well. Meno makes both mistakes. One could simply *list* all the species (virtue for a man OR virtue for a horse, etc), or one could define the genus just in terms of one of its species (e.g., by defining virtue in terms of justice).

former kind of error. For the latter, it is plausibly avoided by division because, if one tries to define a kind by an affection, by spelling out the affection, one will make reference to the kind since the account of a derivative property should make reference to its ground.¹⁰ This is exactly what Euthyphro does in the dialogue. My point is not here to show that division will inevitably lead to good definitions, but only that the framework given by division seems to be a promising one for those investigating definitions.

This gives us a positive reason to look towards division as a method for finding definitions. The *Meno* can be read as supporting a similar position *via negativa*.¹¹ Just as in the Euthyphro, the dialogue begins by Socrates canvassing various proposals by Meno about what virtue is, refuting them by citing various constraints on definition that the proposals lack. Eventually, Meno too gets frustrated, and Socrates proposes that they begin to search together. Unlike Euthyphro, however, Meno wheels in an argument to the effect that we cannot inquire at all. One way to think about Meno's motivation for giving this argument, suggested by David Ebrey, is that he is concerned about the stringency of the constraints on giving an adequate definition, in particular the priority of knowledge what (PKW): if one is to know anything about X, one must know what X is. With PKW, investigation directed at finding the definitions of things becomes both *central* and *extremely* mysterious. It is central because it is a principle upon which all other knowledge is based. But it is mysterious because it is a kind of knowledge that seems very difficult to obtain and we as inquirers have no *other* knowledge to use as a guide while inquiring into *it*.

In that dialogue, Socrates uses the example of the slave inquiring into geometry and the theory of recollection to defuse the argument, but is still left with a methodological question. Even if we know that it is possible to inquire, how ought we proceed? This completely reasonable perplexity, it seems to me, is what ultimately drives Meno to bring in his "eristical" argument.

In the *Meno*, Socrates tries to answer the question of methodology by introducing the *method of hypothesis*.¹² This method allows Socrates to inquire into certain questions, such as whether virtue is teachable. Practicing the method effectively reduces this question to a possible definition: virtue is knowledge. That is to say, Socrates aims to show that answering a particular definitional question is both necessary and sufficient for answering the property question. This is an example of a

¹⁰This does not, of course, prevent us from mistaking an affection for an essential property. I will argue in the next chapter that the first definitions of the sophist make precisely this mistake, but it is not obvious on the surface that reference is made to the kind itself. However, I think that, in the end that is exactly what happens. When the interlocutors realize that a sophist is a hunter of young men *because* he is an image maker, they discover that an explanatory account of the hunters of young men requires understanding sophistry itself.

¹¹Here I am following a controversial line of thought inspired by David Ebrey (2013; 2014). However, much of what I say here should be compatible with alternative accounts, such as Fine (2014) or Scott (2006). There is also a similar suggestion in the brief but illuminating Benson (2010).

¹²For an account of the method and its roots in geometrical analysis, see Menn (2002).

successful inquiry. However, it is still unsatisfying because it provides no way to discharge the hypothesis or evaluate its truth on its own.¹³ To use the language of the *Republic*, we have no way to arrive at the *un*hypothetical from the hypothetical.

The method of division promises to be exactly this kind of method. In the *Euthyphro*, it seems like Socrates can use the method consistently with denying that he knows what piety is. Moreover, it does not seem on its face that using the method reduces the definitional question to another question in the way that Socrates reduces the question of the teachability of knowledge to a definitional question. Rather, Theaetetus seems to emphatically say in the *Sophist* that the Eleatic Visitor has just *shown* what sophistry is.

The biggest stumbling block to the method seems to be the very first step. How do we know that piety is a kind of justice, angling a kind of skill, etc? This is important precisely because there are often difficult cases, e.g., whether sophistry is a kind of skill.¹⁴ It is here that, in a very few dialogues (*Phaedrus*, Sophist, and Philebus) a different method becomes important: collection. Somehow, this method is meant to provide a starting point for particular divisions.¹⁵ Unfortunately, Plato does not illustrate or describe this process nearly as thoroughly as he does division. The basic idea seems to be that one grasps the respect in which a group of different things are the same. Theuth, the Egyptian god credited by Plato as inventor of writing, is said to have first looked at the infinite spoken sound and discerned the particular letters that are in common to all sounds.¹⁶ The letter Θ , for example, is something that both Θ exity to c and Θ eódupo c have in common. Plato does not seem to have developed collection as a methodology in anywhere near the same kind of detail as he does division, but we have some evidence for similar procedures in the "similarities" (in his work Tà Oµoía) of Speusippus and the procedure described at the end of Aristotle's Posterior Analytics II.13.¹⁷ This same question still arises in a way at every stage of the division, but it seems to be less of a problem, since there we at least have a determinate plurality of kinds that are already defined under which we can set the target kind.

Despite this and other difficulties, Plato seems to have thought that it was already being usefully deployed in the most successful sciences of his day: harmonics and geometry. In the introductory conversation between Socrates and Theaetetus in the *Theaetetus*, Theaetetus in fact uses the method of division in his generalization of Theodorus' theorems in the numerical application of proportion theory.¹⁸ On the other hand, Plato in the *Philebus* seems to associate the method of

¹³This is not quite right, since one could use the method to refute a definition by showing it entails a contradiction. But that is not enough for the method to determine a particular account.

¹⁴According to Brown (2010), Plato himself denies that sophistry is a skill. Whether or not this is correct, it at least shows how it is a controversial case.

¹⁵Even if they are meant to be used in conjunction (so that a collection precedes a division which in turn precedes a collection, etc), Plato needs the very beginning of this to be a collection.

¹⁶For detailed analysis of the passage, see Menn (1998).

¹⁷I discuss this passage in Ch. 4.

¹⁸Lukas (1888) includes discussion of this passage (pp. 138 ff). There he takes Plato to be dis-

division chiefly with harmonics,¹⁹ which is closely connected to proportion theory. In fact, he seems to be saying there that the discoverers of those sciences actually used the method.

However, it is also important, according to Plato, that this method be practiced in a certain way, *dialectically*, which he contrasts with being *eristic*. Plausibly, one ought to divide dialectically so that one can attain the goal that of division. This means that we have a distinction fundamental to our understanding of division: they can be performed well or badly vis-à-vis knowledge of the definition. We now need to ask two questions where at first we only asked one. First, what is a division? Second, what is a good division?²⁰ I take it that these two questions are related similarly to how syllogisms are related to demonstrations. Demonstrations are the scientific syllogisms, so we ought to first understand what a syllogism is and then what is added to make it scientific.²¹ This is true even if it is a constitutive norm of division that it be scientific, i.e. that nothing is a division unless it is subject to the norm of being scientific. In that case, the questions are still distinct, but part of the answer to to the first will make reference in some way to the second. We have yet to see whether division does have this as a constitutive norm and the example of demonstration shows how it might not be a constitutive norm. For example, dialectical syllogisms are not merely defective syllogisms, they just serve different purposes, for instance, training.

The method of division is Plato's greatest contribution to the dialectical tradition. After both theoretical discussion and practical use of the method in just a few dialogues, it was taken up, debated, refined, and employed for thousands of years.²² In this way, division's influence rivals that of Aristotle's syllogistic theory, finding its way into the methodology of the Platonists (early, middle, and late), Peripatetics, and Stoics, as well as Galen, Boethius, and the medieval tradition. Indeed, thinkers with metaphysical and epistemological positions almost *diametrically opposed* to their interpretation of Plato²³ found it to be important.

²¹Cf *APr* I.1, Crubellier (2008).

tinguishing definitions through examples (as in Theaetetus' first definition) and definition through division. The only other discussion of this is in Sayre (1969). Sayre, however, seems to equate Theaetetus' theorem with the division, which is incorrect. Rather, the division is a precondition for proving the theorem. For discussion of the theorem and its significance, see Heath (1926) on Euclid, *Elements* X.9.

¹⁹As Menn argues, the example from phonetics is used to *illustrate* collection, not division, despite the fact that Theuth also uses division.

²⁰The fact that these are two questions does not mean that Plato has a *theory* of specifically bad divisions, as Aristotle has a theory of eristical syllogisms. Plato might think that all there is to say about such divisions is that they fail to meet the criteria of a good division, or he might have something more robust to say. What I am committed to, however, is that Plato has a theory of division *in general*.

²²A testament to this is that when Jean Buridan, reworking Peter of Spain's *Summule Logicales* in his own *Summulae de Dialectica*, felt Peter's omission of discussion of division to be so conspicuous that he added a whole section on division to the *Summulae*.

²³For instance, the Stoics were materialists and empiricists, Peter Abelard a nominalist.

The aim of this dissertation to argue that the method of division plays a similarly central role in the development of *logic*. Here I am not out to say exactly what logic is. Instead, I take for granted that we know it when we see it. When we look at the method of division, no matter what we make of it, we do not see a logical theory. What I aim to show, however, is that the method of division is crucial for understanding Aristotle's syllogistic, which *is* a recognizably logical theory. This thesis is defended in Chapter 6. This thesis is not original—it has been suggested by many commentators over the last two centuries, but they have not convinced the various detractors largely because they have not identified any very specific aspects of the syllogistic and shown how they were inherited from Platonic division.

To make this claim, however, we need a more nuanced understanding of division as it was originally conceived and received. This is what I do for Plato in Chapters 2-3 and Aristotle's reception of Plato in Chapters 4-5. In the second chapter, I argue for my own conception of the structure of Platonic division, answering the first question that I posed above: what is a division? The answer that I give will also be the key to understanding the puzzle of the *Sophist*. In the third, I give an account of the goal and norms of division as discussed in Plato's dialogues, which will tell us what a good division is. In the fourth chapter, I discuss Aristotle's reception of Platonic division in the work most important for understanding the history of logic, his *Analytics*. In Chapter 5, along with the Appendix, I argue for three precise dimensions of similarity between division and syllogistic and show how it can resolve a puzzle in our interpretation of *Prior Analytics* I.31. In Chapter 6, I argue that the dimensions of similarity identified in the previous chapter strongly suggest that the method of division influenced Aristotle's syllogistic.

In addition to standard philological and philosophical methods of ancient philosophy, at several key junctures I will be employing formal, mathematical tools to support my interpretations of Plato and Aristotle. For instance, I will provide precise formal definitions of structural features of divisions (Chapter 2) and norms of division (Chapters 3 and 4). I will argue that a corollary of a soundness and completeness theorem (proved in the Appendix) motivates a particular interpretation of Aristotelian predication.

Because such heavy use of mathematical and logical tools is not standard in scholarship in ancient philosophy (especially Plato), a brief discussion and defense of this methodology is appropriate.

The advantages of the methodology are clear enough. Precise formal definitions allow one to state views clearly, making it easier to decide when a text is or is not consistent with that view. It also makes comparing the logical relations *between* views much more perspicuous. For instance, if it can be shown that two interpretations can equally well explain a text but that one is logically weaker, this provides *prima facie* evidence in support of the weaker reading. These virtues are not particular to formal methods, but formal methods constitute our most powerful tools for achieving them. Insofar as it brings such clarity, no one would object to these methods. The worry, of course, is that by using these tools, I might be guilty of *anachronism*. Plato and Aristotle didn't have these tools and so may not have been able to make the same connections between views that I did using them. So if we are in the business of figuring out *what they thought*, what good are the fancy formal methods?

This objection poses a serious obstacle to some applications of formal methods in the history of philosophy. However, it does not pose a problem for any of the sorts of applications that I will be discussing. This is because I am not attributing the reasoning *or* knowledge of the connections to Plato and Aristotle. Instead, I am using these to establish, *for us interpreters*, the relationships between the interpretive possibilities. The *actual* relationships are relevant for how we assess evidence for different interpretations, even if Plato and Aristotle themselves could not have established those relationships.

Indeed, the general upshot of my use of formal methods will be to show how interpreters have often imputed views to Plato and Aristotle that are *much stronger* than anything warranted directly by the text. Far from being anachronistic, formal methods can be used effectively to combat anachronism that stems from imprecision. Such "informal" anachronism is especially common when we stand at the other end of a long tradition that we understand very imperfectly, as is the case with both division and syllogistic. When thousands of years of interpretation and misinterpretation, developments and setbacks accrue, we come to the ancient texts with a set of vague assumptions. Just as Frege used formalization as a tool to seek out hidden assumptions in mathematical reasoning and subject them to scrutiny, formalization in the history of philosophy can help to unearth hidden interpretive assumptions.

The true test, of course, will be if these formal methods bear fruit. What follows will hopefully show that it does.

Chapter 2 The Puzzle of the Sophist

Plato's *Sophist* begins brimming with confidence. After Socrates, Theaetetus, and Theodorus ended their discussion aporetically the day before (in the *Theaetetus*), a new character, the Visitor from Elea, enters the scene. Described as a "god of refutation" (*theos elenchtikos*), he promises to distinguish and define sophistry, statesmanship, and philosophy, three crafts that have hitherto been confused with one another. Moreover, he wields a powerful tool of inquiry, the method of division, with which he can, apparently, define anything whatsoever. After showing off his shiny new method with Theaetetus by defining the craft of angling, he proceeds to define sophistry, the first of the controversial trio.

But something goes horribly amiss. The Visitor sure enough obtains a definition of sophistry by division. But then he obtains another, and another, ending up with six in total. The puzzlement voiced by Theaetetus at this point in the dialogue leads to the puzzles concerning falsehood that drive the central section and only gets resolved at the very end, when we finally attain an adequate definition of the sophist (also by division).

My aim is to give an account of this puzzlement. Previous accounts cannot explain the puzzle of the sophist because they rely on what I call the traditional picture of division. In §2.1, I lay out the traditional view, showing that the best account of the puzzlement is inconsistent with the traditional view of division. I will then argue against the traditional view in §2.2 and §2.3 by examining the examples of Platonic division in the *Sophist* and other dialogues. Instead, I will argue that the examples only support a minimal view of Platonic division. I will argue in §2.4 that the minimal view also gives a more satisfactory account of the puzzle of the sophist and suggest that the lesson we are supposed to draw is that division needs norms. So, while division in general does not have very much structure, it can still be useful when those norms are observed.

2.1 The Traditional View of Division and the Puzzle of the Sophist

The traditional view of division is a view about *what division is*. To understand this more clearly, I need to make a few distinctions. The most important is that between *process* and *product* of division. "*Diairesis*", like "division" in English and "*divisio*" in Latin, is ambiguous between referring to the event of a thing being divided and the parts that are the result of the dividing of some whole. For example, "defining by division" would be talking about the process of dividing, while the pseudo-Aristotelian *Divisions* would clearly be about the product.

The traditional view is an answer to the following question: what would Plato consider to be *a division* in the product sense? The restriction to the product sense is important, as it allows us to look not at the mind of the inquirer but instead at the object of her inquiry. An outline account of the process sense would run something like: one first thinks of a general kind that the target falls under. Then one distinguishes various parts of that kind until one arrives at something necessarily coextensive with the target. I don't mean to suggest that it is easy to give this account precisely, but merely what kind of question that would be.

Moreover, I want to focus attention not on the representation created by dividing, but on the relations between things that this mental act is meant to track. Peter Abelard, who is the first person that I'm aware of to *define* division itself, says that it is a sentence (*oratio*) through which something is shown to be divided by other things.¹ Here he clearly has the product sense in mind, but is only defining the linguistic representation, as he does not think that there are any things in the world corresponding to them. The question that I will be discussing abstracts from how one ought to go about dividing and does not consider what are *good* or *scientifically fruitful* divisions. These I admit are important questions, but addressing them requires different considerations and some prior grasp as to what divisions are.²

Even within this question, one might be interested in understanding the metaphysics of division, for instance, whether what is divided are *classes* or Platonic forms.³ One also might be interested in structural features of divisions.⁴ Those who disagree on the precise metaphysics of division can still agree, for instance, on whether divisions are dichotomous or exclusive, or on whether divisions capture relations of essential predication only, or essential and accidental predication. It is primarily these structural features that will occupy us in this chapter.

With these preliminaries aside, I can articulate the traditional view of division. This is encapsulated in the so-called Tree of Porphyry, as described in his *Eisagoge*. There are two components to the traditional view: a conception of the struc-

¹*Dialectica* 535.21-2 de Rijk (1970).

²I will say something about good divisions in §2.4 and much more in Ch. 3.

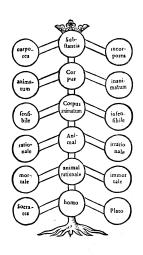
³See the debate between Cohen (1973) and Moravscik (1973).

⁴See Dillon (2003), pp. 28-9 that indicates such a distinction was likely made in the early Academy.

ture of division and a conception of the kind of relation embodied in division.

In Porphyry's Tree, the highest kind substance is divided into corporeal and incorporeal substance. Corporeal substance is in turn divided into animate and inanimate corporeal substance. This procedure of division by exclusive and exhaustive dichotomous division is repeated until one reaches the target kind—in this case, human being. In this picture, all the lower kinds are all species of the higher kinds and this implies that all the relations described are *essential*, since the genus appears in the account of the essence of the species.

According to the traditional view, a division is structured as a *tree*, usually with dichotomous branching.⁵ Crucially, the relation pictured in Porphyry's Tree is supposed to be transitive, exhaustive, and exclusive. This means that any subkind of the highest kind falls under exactly one "line" of division stretching up to the highest kind. Secondly, a division represents a relation of essential predica-



Porphyry's Tree. From Borgnet (1890).

tion between kinds. That is to say, the higher kinds express what the lower kinds *are* in the robust sense that Socrates demands in the Socratic dialogues.

I will argue that proponents of the traditional view have difficulty accounting for the puzzlement expressed in the *Sophist* by the interlocutors themselves. After having given his successful example of the angler, the Visitor gives six accounts of sophistry by division:

- 1. a hunter of rich young men (221c6-223b7),
- 2. a wholesaler of knowledge about virtue (223c1-224d3),
- 3. a retailer of knowledge (224d4-e5),
- 4. a merchant of knowledge who makes his goods (also 224d4-e5),
- 5. a money-making debater (224e6-226a5),
- 6. an educator who uses refutation to purify the soul (226a6-231b9).

The presence of all these accounts has puzzled commentators just as they puzzled the interlocutors. Theaetetus gives voice to this puzzlement:

Theaetetus: Let's say that. But the sophist has appeared in lots of different ways. So I'm confused about what expression or assertion could convey the truth about what he really is. $(231b-c)^6$

⁵For a description of the traditional view, see Barnes (2003). The attribution of the traditional view to Plato is widespread, but is explicit in a wide range of commentators: Crivelli (2012); Miller (2016); Rickless (2010).

⁶{ΘΕΑΙ.} Λεγέσθω μέν· ἀπορῶ δὲ ἔγωγε ἤδη διὰ τὸ πολλὰ πεφάνθαι, τί χρή ποτε ὡς ἀληθῆ λέγοντα καὶ διισχυριζόμενον εἰπεῖν ὄντως εἶναι τὸν σοφιστήν.

There are three plausible interpretations of the reason that Theaetetus is giving for his confusion:⁷

- 1. His confusion is **logical**. He is confused because he does not think all of those accounts are *logically consistent*. Nothing could satisfy all of these accounts.⁸
- 2. His confusion is **epistemological**. He is confused because the plurality of accounts, especially the sixth account of sophistry that seems to apply to philosophers, makes him unsure which of these accounts is true.⁹
- 3. His confusion is **metaphysical**. He is confused because, even though all these accounts might be true of sophistry, he still does not know what sophistry is. Finding out the nature of sophistry will disclose which of the other accounts are merely true descriptions, but not accounts of the essence of sophistry or of what sophistry really is.¹⁰

It is important here to be precise about what account (*logos*) here means. It is not here restricted to definitions, although that is can certainly a possible use of *logos* in Plato. Everyone besides Moravcsik (1973) in this debate agrees that there are not seven correct definitions of the sophist. Rather, I mean here the sense of account which is broader, to include any true predication. This is the sense of *logos* present, for instance, in the *Euthyphro*, when Socrates agrees that something might be pious just in case the gods love it, but does not take it to be a definition, only a quality. Aristotle in the *Topics* makes essentially the same move, distinguishing definitions from the predications of genera, differentiae, propria, and accidents. It is possible for something to be said ($\lambda \epsilon \gamma \epsilon \tau \alpha t$) of something even if it is not in the essence.¹¹

Paying attention to Theaetetus's language suggests that 1 is not the source of the confusion. He is not saying that the multiplicity of appearances implies that at most one of them can be true. Instead, he is saying that the multiplicity shows that we do not yet know what the sophist is. What the Visitor does next seems to disconfirm 1, as he then focuses on a feature of sophistry that can account for the multiplicity of appearances:

⁷Not all views in the literature about what it is puzzling about the sophist count as views about what puzzles the *interlocutors*. So, for instance, according to Brown (2010) all of the accounts are false because there is no kind "sophistry" at all. If true, this fact evidently eludes the Visitor and Theaetetus. Similarly, Moravcsik (1973) takes all the accounts to be definitions and denies that there must be a unique one. This too would seem to elude the interlocutors, since they have an evident preference for the final account (268d) over all the others that is unexplained on this view. This isn't to rule out such interpretations of what *we* should take away from the many accounts of the sophist. But, insofar as we can give interesting and plausible accounts of the interlocutors' puzzlement, such esoteric views are undermined.

⁸Cornford (1957); Rickless (2010).

⁹Notomi (1999). Note that he thinks that another source of confusion is metaphysical.

¹⁰There are quite a few variations on this view, held by Notomi (1999); Sayre (2006); Gill (2010). For my purposes, it is not important whether the first definitions express *kinds* of sophistry, *aspects* of the sophistic activity, or even particular sophists.

¹¹Aristotle uses λόγος to describe true accidental predications, e.g., in *Topics* 102b32.

Visitor: Well then, suppose people apply the name of a single sort of expertise to someone, but he appears to have expert knowledge of lots of things. In a case like that don't you notice that something is not sound with the way he appears? Isn't it obvious that if somebody takes him to be an expert at many things, then that observer can't be seeing clearly what it is in his expertise that all those pieces of learning focus on—which is why he calls him by many names instead of one?

Theaetetus: That definitely does seem to be the nature of the case.

Visitor: So let's not let laziness make that happen to us. First let's take up one of the things we said about the sophist before, which seemed to me to exhibit him especially clearly. (232a-b)¹²

Although there was something "not sound" about the appearances of the sophist, the direction that the Visitor wants to go is not to assess the truth of any of the previous accounts, but rather to find "what it is in his expertise that all those pieces of learning focus on" ($\epsilon l \zeta \delta \pi \alpha \nu \tau \alpha \tau \alpha \mu \alpha \vartheta \eta \mu \alpha \tau \alpha \tau \alpha \delta \lambda \epsilon \pi \epsilon \iota$) and the reason ($\delta \iota \delta$) that he is called by all these names. If 1 were true, presumably the Visitor would try to figure out which of the accounts he had before was false and then try to say what he is in truth. Rather, what is "not sound" is precisely that he does not yet appear as he *really* is ($\delta \nu \tau \omega \zeta \epsilon \epsilon \nu \alpha \iota$). Given that the interlocutors share the goal of discovering the nature of sophistry, any account that fails to be a definition will be "unsound." So we need not think that the appearances are false, but only that they fail to capture the essence of sophistry.

The defender of 1, however, would reply that what is being explained are the *appearances*, not the accounts themselves. While the Visitor does describe it that way too, he does not need to be using the language of appearance to suggest falsity, but instead how the sophist has "shown up" in their inquiry. The final account of the sophist as an appearance maker explains *not only* how he can appear as, say, a hunter of rich young men, but also how he actually is able to hunt them.

This also suggests that, unlike reading 2, the Visitor is not worried that the method he used up to now was unreliable in the sense that some of these accounts are false, since he makes use of them in the search for the final definition. Their being false would, I think, actually undermine the purpose that the Visitor puts them to. If they are false, then why would we try to explain these names by appeal to something else which the sophist is?¹³ If the Visitor were trying to use the correct

¹²{ΞΕ.} ^{*}Αρ' οῦν ἐννοεῖς, ὅταν ἐπιστήμων τις πολλῶν φαίνηται, μιᾶς δὲ τέχνης ὀνόματι προσαγορεύηται, τὸ φάντασμα τοῦτο ὡς οὐκ ἔσθ' ὑγιές, ἀλλὰ δῆλον ὡς ὁ πάσχων αὐτὸ πρός τινα τέχνην οὐ δύναται κατιδεῖν ἐκεῖνο αὐτῆς εἰς ὁ πάντα τὰ μαθήματα ταῦτα βλέπει, διὸ καὶ πολλοῖς ὀνόμασιν ἀνθ' ἑνὸς τὸν ἔχοντα αὐτὰ προσαγορεύει; {ΘΕΑΙ.} Κινδυνεύει τοῦτο ταύτη πῃ μάλιστα πεφυκέναι. {ΞΕ.} Μὴ τοίνυν ἡμεῖς γε αὐτὸ ἐν τῆ ζητήσει δι' ἀργίαν πάσχωμεν, ἀλλ' ἀναλάβωμεν <ἕν> πρῶτον τῶν περὶ τὸν σοφιστὴν εἰρημένων. ἑν φάρ τί μοι μάλιστα κατεφάνη αὐτὸν μηνῦον.

¹³This language leads many to follow a suggestion in Cornford (1957) that the first definitions were really a disguised collection, to think that there is here a "collection" of definitions, from which the correct definition is derived. The idea behind this tradition (which includes Notomi

account of sophistry to explain why the sophist falsely appears to be a hunter, that would be one thing. But that does not seem to be the aim. Rather, the Visitor is trying to explain *how it is* that the sophist *can* hunt those men. If the sophist could not do that, i.e., if that were a false appearance, then the explanatory enterprise would be very different. Reading 3, on the other hand, has no difficulty with this passage, since the appearances are true but not revealing of what sophistry is.¹⁴ This should make us prefer it.

The final definition, it seems, succeeds at explaining that towards which all the other pieces of learning look precisely by making the sophist a kind of appearancemaker. Importantly, however, this does not undermine the truth of what was said previously about the sophist. Rather, it seems to me to *vindicate* the other claims made about him and explain why they are true, even though they do not express the sophist's nature. For instance, it explains how the sophist could be a hunter of rich young men *because* he is able to give off the appearance of knowledge. All of the activities described in the earlier accounts: debating, selling knowledge, even purifying the city of its ignorance, are possible when someone has the skill in making appearances. The final definition constitutes a significant improvement because it describes the one kind of skill that can ground the possession of all of the others.

We see this clearly in the way that the dialogue ends. When the Visitor returns to the divisions with an account of false speech, belief, and imitations in hand, he says:

Visitor: Then let's try again to take the kind we've posited and cut it in two. Let's go ahead and always follow the righthand part of what we've cut, and hold onto things that the sophist is associated with until we strip away everything that he has in common with other things. Then when we've left his own peculiar nature, let's display it, especially to ourselves but also to people to whom this sort of procedure is naturally congenial.

Theaetetus: All right.

Visitor: Didn't we begin by dividing expertise into productive and acquisitive?

Theaetetus: Yes.

Visitor: And under the acquisitive part the sophist appeared in hunting, combat, wholesaling, and types of that sort.

Theaetetus: Of course.

¹⁴Thanks to Katja Vogt for pushing me away from an overly Aristotelian reading of these texts.

^{(1999);} Sayre (2006); Gill (2010); Crivelli (2012)) is that the previous definitions have some use or tell us something right about the sophist. Usually these accounts accept something along the lines of readings 1 or 2, and so have to be careful about the extent to which the first definitions get anything right.

Visitor: But now, since he's included among the experts in imitation, first we obviously have to divide productive expertise in two. $(264d-265a)^{15}$

The idea here is that they are now in a position to say what the peculiar nature $(\tau\dot{\eta}\nu)$ oixe($\alpha\nu$, $\phi\dot{\upsilon}\sigma\nu$) of sophistry is. Again, there does not seem to be a rejection of his previous appearances once again. One who wants to find such a rejection could read the "But now" (N $\ddot{\nu}\nu$ $\delta\dot{\epsilon}$) adversatively and take the appearance language to indicate distancing from these previous accounts. I don't think there is a knockdown argument against this reading, however, I find the fact that he still uses the language of seeming to describe their attitude toward the last definition (268d) to suggest that, at least in this regard, there is not a significant difference.

If we do not take the adversative reading, the rest of the passage is straightforward. We get a final definition of sophistry as a kind of production. Once this is established, Theaetetus says: "And now at last I see that we have to call him the person who is really and truly a sophist" (268c).¹⁶ Here Theaetetus is echoing what he said in 232b-c, where he expressed puzzlement about what the sophist *really* was. He apparently feels that his question is answered. Similarly, when the Visitor closes the dialogue, he says "Anyone who says the sophist really is of this "blood and family" will be saying, it seems, the complete truth" (268d).¹⁷ Again, we are told that the sophist *really* is this kind of person. The repetition of ὄντως seems to be telling the reader that this account says something important about what sophistry is, something that the other accounts were missing.

I have argued that neither 1 nor 2 attribute a very reasonable confusion to Theaetetus, for why would it be surprising that we can say many different true things about the sophist?¹⁸ The only motivation, I think, for either reading of the confusion comes from holding on to the traditional view of division, as Rickless explicitly does. With the traditional view, the higher kind is divided into lower kinds that exclude one another and the higher kind is essentially predicated of those lower kinds. With either of those two assumptions in place, along with the

¹⁵{ΞΕ.} Πάλιν τοίνυν ἐπιχειρῶμεν, σχίζοντες διχῆ τὸ προτεθὲν γένος, πορεύεσθαι κατὰ τοὐπὶ δεξιὰ ἀεὶ μέρος τοῦ τμηθέντος, ἐχόμενοι τῆς τοῦ σοφιστοῦ κοινωνίας, ἕως ἂν αὐτοῦ τὰ κοινὰ πάντα περιελόντες, τὴν οἰκείαν λιπόντες φύσιν ἐπιδείξωμεν μάλιστα μὲν ἡμῖν αὐτοῖς, ἔπειτα καὶ τοῖς ἐγγυτάτω γένει τῆς τοιαύτης μεθόδου πεφυκόσιν. {ΘΕΑΙ.} Όρθῶς. {ΞΕ.} Οὐκοῦν τότε μὲν ἡρχόμεθα ποιητικὴν καὶ κτητικὴν τέχνην διαιρούμενοι; {ΘΕΑΙ.} Ναί. {ΞΕ.} Καὶ τῆς κτητικῆς ἐν θηρευτικῆ καὶ ἀγωνία καὶ ἐμπορικῆ καί τισιν ἐν τοιούτοις εἴδεσιν ἐφαντάζεθ' ἡμῖν; {ΘΕΑΙ.} Πάνυ μὲν οῦν. {ΞΕ.} Νῦν δέ γ' ἐπειδὴ μιμητικὴ περιείληφεν αὐτὸν τέχνη, δῆλον ὡς αὐτὴν τὴν ποιητικὴν δίχα διαιρετέον πρώτην.

¹⁶καὶ σχεδὸν ἤδη μεμάθηκα ὅτι τοῦτον δεῖ προσειπεῖν ἀληθῶς αὐτὸν ἐκεῖνον τὸν παντάπασιν ὄντως σοφιστήν.

¹⁷ "ταύτης τῆς γενεᾶς τε καὶ αἴματος" ὃς ἂν φῆ τὸν ὄντως σοφιστὴν εἶναι, τἀληθέστατα, ὡς ἔοικεν, ἐρεῖ.

¹⁸Note that these readings do not say that one of these accounts must be false *qua* definition. There is no word or phrase in any of the six accounts like " $\delta\nu\tau\omega\varsigma$ " to mark such a restriction. Rather, they are presented as just bare propositions. On readings 1 and 2, the confusion must lie in the truth or falsity of the bare propositions, not in the question of whether they are definitions. To say that they are false *qua* definition is really just a terminological variant of reading 3.

background assumption that one thing has at most one essence, the different accounts must be mutually inconsistent.

To see this, recall that for any two putative accounts of sophistry, there is a node in the divisional tree at which they diverge. For simplicity, just consider the final definition and the first. The final definition places sophistry under production, while the first puts it under acquisition. If divisions are exclusive, then nothing can belong to both branches of the tree, so nothing can be both a productive craft and an acquisitive one. *A fortiori*, sophistry cannot be both a subkind of the productive and of the acquisitive crafts. The same will be true if divisions only capture essential relations. In that case, the two accounts will claim, respectively, that sophistry is *essentially a particular sort of acquisition* and that it is *essentially a certain sort of production*. But these essences are different. Since one craft only has one essence, sophistry cannot be both.¹⁹

So here we are. The account of the puzzle that I have offered is in conflict with the Traditional View of Platonic division. In what follows, I will argue that the Traditional View is false and, instead, argue for a Minimal View that is compatible with the most natural reading of the puzzle of the sophist. Recall that, according to the Traditional View, in a Platonic division, the relation that holds between higher and lower kinds is one of essential predication and is transitive, dichotomous, and exclusive. According to the Minimal View, by contrast, the relation is one of predication and transitive, but not necessarily essential, dichotomous, or exclusive.²⁰

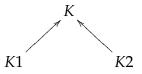
In what follows, I will argue for the Minimal View, but before doing so, I want to address the question: why would someone care at all about division if it were correct? This is a significant worry, since it seems that, on the Minimal View, division is useless for pursuing essences, which was the whole point of introducing the method! Fortunately, the Minimal View does not have this consequence. Just because *some* divisions are useless does not mean that *all* are, just as the fact that, according to Aristotle, some syllogisms are useless in the pursuit of demonstrative knowledge does not show that all are useless. In particular, if one could divide in such a way as to secure essential connections between kinds, that would be *scientifically preferable*. So we will then have a distinction between divisions that are deficient from the point of view of science and those that are not. Then, if we could somehow make sure that our divisions were only of the latter sort, it could really be quite useful. I will pursue this point in greater detail after I have argued for the Minimal View.

¹⁹These points are made with great force by Rickless (2010).

²⁰In arguing for the Minimal View, I am not denying that there could be other properties of Platonic divisions.

2.2 Minimal View: Predication and Transitivity

Since at least the early Middle Ages and probably earlier, divisions have been pictured in a very obvious way:



It has not been traditional to write arrows on the ends of lines. The idea of directionality, however, was implicit in the medieval representations, which, instead of using arrows, used relative height on the page. Because the more general kind was pictured above, I put the arrows facing "up".) I prefer the arrows because they are both a more general way of depicting binary relations and easier to read when the divisions get complicated. Now we can transform our question of the structure divisions into the question: what are the properties of the binary relation represented by this line? Note that at this point, I am not ruling out what some might consider to be crazy structures for division, e.g.:



In order to ask whether this structure is a possible division, we need to know something about how to interpret the lines. Here I will run with the idea that the lines are meant to represent *predication*, so that $K \to K'$ only if K' is predicated of K.²¹ Otherwise, it seems unclear how this could be giving us a *logos* of the objects of something like the Sophist or Statesman.²² It might seem to follow from these considerations alone that, for instance, divisional lines are transitive or that they need not be dichotomous. I will not, however, use this to infer directly any properties of the lines from this interpretation for two reasons. Firstly, this interpretation is not obvious. If we find some similarities between the formal structure of the arrows and the predication relation, I want to take this as evidence for the interpreting the lines as the predication relation without any kind of vicious circularity. Secondly, the idea that these lines represent predication may not be the whole story. In particular, I cannot dismiss the possibility that the lines represent a *special kind* of predication, as the Traditional View holds.²³ In that case, there is no guarantee

²¹Note that this view is not in competition with one saying that this relation is fundamentally grounded in something more metaphysically demanding.

²²Those who think that there is a distinction between essential and inessential predication in Plato (e.g., Meinwald (1992)) would naturally see these as the former, which Meinwald calls "Tree Predication".

²³Meinwald (1992) and Moravcsik (1973) are explicit about this.

that the structure of a division will have much in common with the structure of predication without knowing more about what kind of predication it is. This is just because, if relation R is a sub-relation of R', we cannot infer, for instance, from the transitivity of R' to the transitivity of R.

Is the binary relation that we are considering transitive? This would mean that if $K \to K' \to K''$, then $K \to K''$. Or, if you like, you would still be dividing a genus into species if you divided it into the subspecies of those species. There are three reasons to think that Plato considered his divisions to be structured by transitive relations. I will then consider two problem cases for this view and show why they are not problems.

First of all, the language of parts and wholes pervades talk of division. Plato often thinks of what is being divided as a kind of whole and its species as parts, for instance:

This kind of hunting [of underwater things] might be divided into two main parts. (220b)

So we'll call this part of hunting enclosure-hunting or something like that. (220c)

The relation of parts and wholes is generally thought to be transitive when it comes to ordinary material objects, so it would be natural for Plato's reappropriation of the language for kinds to have this same structure. Indeed, that the classes are parts is presupposed by the Visitor when he tells Young Socrates in the *Statesman* that he should divide by kinds, and not into *mere* parts.²⁴ This reason is, of course, highly defeasible. Plato might, for all we have said at this point, think that the relation between kinds is altogether different from the ordinary one and is not in general transitive.

Secondly, without transitivity, how could the divider be sure that, having reached the object of definition, she is entitled to say that anything before the penultimate step is still true of the *definiendum*? Recall that we are working with the idea that the relations between genus and species are relations of *predication*. A failure of transitivity would jeopardize the whole idea of a *logos* being all of the kinds one meets along the way from the genus to the object of definition. The Eleatic Visitor himself mentions this sort of reason:

Visitor: And then from the controlling sort, we took one that was set over inanimate products, and one set over living creatures; and it's by splitting things up in just this way that we have been progressing all the time to the point where we are now. We haven't forgotten that it's knowledge, but as for what sort of knowledge it is, we're not yet able to give a sufficiently accurate answer. (*Statesman* 292b-c)²⁵

²⁴Statesman 263a ff.

²⁵{ΞΕ.} Κάκ τῆς ἐπιστατικῆς τὴν μὲν ἐπ' ἀψύχοις ἔργοις, τὴν δ' ἐπὶ ζώοις· καὶ κατὰ τοῦτον δὴ τὸν τρόπον μερίζοντες δεῦρ' ἀεὶ προεληλύθαμεν, ἐπιστήμης οὐκ ἐπιλανθανόμενοι, τὸ δ' ἥτις οὐχ ἱκανῶς πω δυνάμενοι διακριβώσασθαι.

If transitivity did not hold, then it cannot be guaranteed that this controlling sort is still a sort of knowledge.

Finally, transitivity makes very good sense of the longer and shorter paths in *Statesman* 265 ff. The idea there is that there are two ways to go from collectively rearing in herds to collectively rearing human beings. The shorter path divides into land and aquatic, divides land into two- and four-footed, and the last divides biped into winged and bare. The longer way, by contrast, divides into land and aquatic, land into winged and footed, footed into horned and hornless, hornless into non-interbreeding and interbreeding, and finally non-interbreeding into four-footed and two-footed, which is identified as humankind. Important here is the relationship between the herds which are on dry land and humankind. In the longer road, there were three intermediate kinds, while in the shorter there was only one. Transitivity would show us why, in general, if the longer way is a division, the shorter way is as well.

Grams (2012) has recently argued that transitivity does not hold as a general rule in Platonic divisions, giving two examples from the *Sophist*. The first is at 225-6, where skill (*techne*) has been divided into the art of disputation and forensics (225b). The Sophist is put into the former category and then it is divided into, on the one hand, "random and unskillful" and on the other, "artful and eristic". Grams argues that this random and unskillful kind of disputation cannot be a part of the genus *skill*, so, by dividing *skill*, we have moved outside of the genus.

To see whether Grams is right, let us see how the Visitor actually makes this division with a relatively neutral translation:

One part of the art of contradiction involves controversy about contracts (συμβόλαια) and is managed in a random and inexpert way (εἰκῆ καὶ ἀτέχνως). We should set it down as a type since the account can distinguish (διέγνωκεν) what makes it different. Nevertheless, it has not been a name given by earlier people and it is unworthy of getting one from us. [...] And on the other hand, the expert part, which debates about just and unjust matters themselves and concerning other things generally, aren't we used to also calling it "eristic"? (225b-c)²⁶

Interestingly, in this passage there seems to be two ways in which one could distinguish the kinds. The first, favored by Grams, is between the kind of art of contradiction which is skillful as opposed to that which is unskillful. However, there is clearly here another principle by which one can distinguish the subkinds: the art of controversy concerning contracts as opposed to concerning matters of justice and injustice. This is quite unlike most of the other divisions, which only give us one such principle. As these two principles will evidently lead to very differ-

²⁶Τοῦ δὲ ἀντιλογικοῦ τὸ μὲν ὅσον περὶ τὰ συμβόλαια ἀμφισβητεῖται μέν, εἰκῆ δὲ καὶ ἀτέχνως περὶ αὐτὸ πράττεται. ταῦτα θετέον μὲν εἶδος, ἐπείπερ αὐτὸ διέγνωκεν ὡς ἔτερον ὄν ὁ λόγος, ἀτὰρ ἐπωνυμίας οὕθ΄ ὑπὸ τῶν ἔμπροσθεν ἔτυχεν οὕτε νῦν ὑφ΄ ἡμῶν τυχεῖν ἄζιον. [...] Τὸ δέ γε ἔντεχνον, καὶ περὶ δικαίων αὐτῶν καὶ ἀδίκων καὶ περὶ τῶν ἄλλων ὅλως ἀμφισβητοῦν, ἄρ΄ οὐκ ἐριστικὸν αὐ λέγειν εἰθίσμεθα;

ent divisions,²⁷ we as interpreters are left with a choice. Based on more general Platonic considerations,²⁸ it seems like the better choice is that concerned with the objects of the craft as opposed to how people currently practice it.²⁹ This reading allows us to say that the visitor's division is based on a perfectly good distinction, which does not bring us outside of the genus, but that the only important sense in which this lacks expertise is that *the current practitioners happen to be inexpert*. This is further suggested by the way that these adverbs are introduced, namely as modifiers of $\pi \rho \alpha \tau \tau \epsilon \tau \alpha t$. Thus, it seems to me that we can have a perfectly good understanding of this division that does not require giving up transitivity at all. Given its antecedent plausibility, this reading is preferable to Grams's.

Grams's second example is from the first definition of sophistry. First, the Visitor takes up the distinction between productive and acquisitive arts from the division of the angler (219a-d) and then divides the acquisitive into the kind that does so by exchange and that which does so by conquest. Eventually, however, dividing the conquering part of acquisition, the Visitor makes a division of *privately hunting young men by persuasion* into those that give the persuasion as a gift and those that get payment. Grams rightly notes that getting payment is exactly what characterizes acquisition by exchange. So we have strayed outside of the kind *acquisition*. This is only the case, however, if acquisition and production were mutually exclusive kinds. Since I will deny that divisions must be mutually exclusive (and that the Sophist is a paradigmatic example of this) then there is no longer any pressure to give up transitivity.

2.3 Against the Traditional View

I have argued for the two positive structural features of the Minimal View: transitivity and predication. Now I will show that we should reject the extra structure of the Traditional View: dichotomy, exclusivity, and essential predication. As I said above, this means that: not every division, according to Plato, must have these properties. I am not denying that these are important properties for divisions to have from the point of view of science.

I will proceed by looking at the *examples* of predication in the *Sophist* as well as other dialogues. This methodology might strike some as backwards: would it not be better to see what Plato considers to be the important features of division in his theoretical discussions of the method, and only then decide whether the examples fit the bill? Would this not prevent us from coming to the conclusion that Plato might be making intentional mistakes in the examples in order to provoke his audience? I think that the way that I go has two advantages. First, the more

²⁷Surely it is possible that a kind of controversy is unskillful without being about contracts! ²⁸For instance, in the argument against the lovers of sights and sounds in *Republic* V.

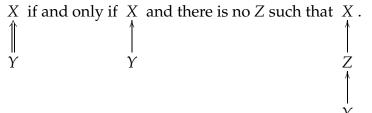
²⁹Schleiermacher seems to imply this in his translation: "...wird durcheinander und kunstlos". The idea here is that the contract-controversialist *becomes* random and non-expert, but that is not what he *is*. Thanks to Katja Vogt for the reference.

theoretical discussions are so condensed and obscure that may not actually give us enough information to decide many of the questions with which we are concerned, so one would be forced to look to examples anyway. Secondly, the role of examples here is similar to how we treat Aristotle's in the *Analytics*. What I take to be important is what Plato considers to be *admissible structures*. One could always debate about whether his own substantive views conflict with that particular example, but so long as those views don't have to do with structural features of division, there could very well be a different division that has the same structure but is no longer in conflict with the substantive views.

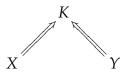
2.3.1 Dichotomy

To get an idea of the sorts of a condition on divisions that Plato would not accept before moving on to the more controversial case of exclusivity, I want to discuss dichotomy. Although most commentators do not take Platonic division to be necessarily dichotomous, there is a not insignificant minority who do.³⁰ Most with this view of Plato are coming from an examination of Aristotle's arguments against dichotomy in the *Parts of Animals* and assume that the only plausible target is Plato. Even if it is now widely accepted in the literature that Platonic division need not be dichotomous, my aim in arguing for it here is to get a sense of the kind of claim that I am arguing for as I set up for a much more controversial claim later.

To state the thesis that all divisions are dichotomous, we need a notion of a *proximate dividens*,³¹ which can be nicely defined as:



The claim that all divisions are dichotomous, then, is to say that, for any *K* to be divided, there are exactly two *proximate* dividentia, so, there are exactly two *X*, *Y* such that



Notably, most of the divisions in the *Sophist* and *Statesman* are dichotomous in this sense. Is it, however, a requirement on divisions? I follow most commentators in

³⁰See Shorey (1903); Stenzel (1940); von Fragstein (1980); Pellegrin (1986) and, perhaps less explicitly, Hambruch (1904). Balme (1987) thinks Plato is a dichotomizer in a different sense. For discussion of his notion of exclusivity, see n. 49.

³¹Cf Boethius De Divisione 877c12-d6 Magee (1998); Abelard Dialectica 538.35 de Rijk (1970).

thinking that it is not,³² because of the presence of the following kind of example:

Visitor: We recognize monarchy, don't we, as one of the varieties of rule in cities?

Young Socrates: Yes.

Visitor: After monarchy one would, I think, list the holding of power by the few.

Young Socrates: Of course.

Visitor: And isn't a third type of constitution rule by the mass of the people, called by the name of "democracy"?

Young Socrates: Most certainly. (Statesman 291d)³³

From context, it is clear that the Visitor considers this to be a division of rule in cities. He is directly dividing it into three kinds: monarchy, oligarchy, and democracy—a division familiar to both *Republic* and Aristotle's *Politics*. In fact, one may even want to count as many as six divisions, depending on how one understands the three perverted kinds of rule corresponding to the three above. From these extra-textual considerations, it seems clear that Plato endorses a division along these lines and, more importantly, that this is never thought of in terms of dichotomous divisions. One could try to defend dichotomy by adding some kind, say "ruling by more than one" to reduce the proximate kinds of ruling to two. This, however, simply does not seem to be Plato's practice, which does not postulate such a kind. Because of this kind of example, we are entitled to say that Plato did not think it was a requirement on divisions that they are dichotomous.³⁴

Also, the description of the god-given method in the *Philebus* explicitly mentions the possibility of division into three or more parts: "And once we have grasped it [the one], we must look for two, as the case would have it, or if not, three or some other number." (16d) From context, it is clear that Plato is talking about division and the number of kinds into which one must divide. Together, these indicate that the phenomenon of non-dichotomous divisions, although certainly prevalent in the *Statesman*, is found in a wider range of Platonic texts.³⁵

³²Cornford (1957) thinks that dichotomy is a feature of definitory divisions but not of divisions used to classify a genus.

³³{ΞΕ.} ᾿Αρ΄ οὐ μοναρχία τῶν πολιτικῶν ἡμῖν ἀρχῶν ἐστι μία; {ΝΕ. ΣΩ.} Ναί. {ΞΕ.} Καὶ μετὰ μοναρχίαν εἴποι τις ἂν οἶμαι τὴν ὑπὸ τῶν ὀλίγων δυναστείαν. {ΝΕ. ΣΩ.} Πῶς δ΄ οὔ; {ΞΕ.} Τρίτον δὲ σχῆμα πολιτείας οὐχ ἡ τοῦ πλήθους ἀρχή, δημοκρατία τοὕνομα κληθεῖσα; {ΝΕ. ΣΩ.} Καὶ πάνυ γε.

³⁴For the full range of examples, see Lukas (1888).

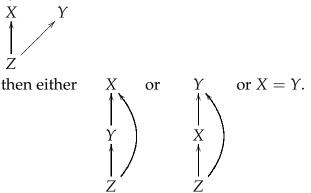
³⁵One could, of course, respond that the *Philebus*, like the *Statesman*, is later than the *Sophist* (and *Phaedrus*). Aside from being otherwise unmotivated chronology, it cannot handle the internal evidence of the *Sophist*. The stylometric evidence groups all of the dialogues other than the *Phaedrus* very close together, so it seems unlikely that there were major doctrinal shifts. Further, *Phaedrus* has a number of non-dichotomous divisions as well, so the developmental story would have Plato begin with non-dichotomous division, change to dichotomous division for one dialogue, and then revert to the original view.

Some might hesitate to say that this shows anything about the *Sophist* in particular. Perhaps Plato uses only dichotomous division in the *Sophist* to make a particular point, at odds with his general point. For instance, according to Gill (2010), Plato uses the failure of dichotomy in the *Sophist* and beginning of the *Statesman* as an argument for a new kind of method in the latter part of the *Statesman*. However, there is relatively explicit use of non-dichotomous division in the *Sophist*. The first plausible case comes from the introduction of discrimination in addition to acquisition and production as kinds of art (226b ff). Rickless (2010) tries to ameliorate this problem by putting discrimination in the genus of production. However, as he recognizes, this is not directly in the text. Nevertheless, we get another example from the length- and breadth-wise divisions to be discussed in more detail below (266a ff), where we have four sub-kinds of production: human, divine, copy-making, and reality-making. *Pace* Henry (2011), these are not dichotomous, since all of these kinds are distinct from one another and proximate. So, even the *Sophist* has some non-dichotomous divisions.

I have argued that Platonic divisions need not be dichotomous in general or in the *Sophist* in particular. This does not imply that dichotomous divisions are not *special* in some way in Plato. In fact, in the next chapter, I will argue that Plato endorses a norm with the implication that dichotomous divisions, when available, are to be preferred. To be clear: what I am saying now is not in conflict with that normative claim. I am only arguing that it is possible for something to be a division and not into two species.

2.3.2 Exclusivity

I began with dichotomy primarily as an exercise in looking at texts to see how we can look at examples of Platonic divisions to judge a given formal condition on divisions. The more important condition for the puzzle of the sophist is exclusivity. The kind of exclusivity that I have in mind holds just in case, when there exist X, Y, and Z such that:



This sort of exclusivity has to do with kinds, as opposed to individuals. Rather than saying that two kinds are mutually exclusive just in case no individuals fall under both, this condition says that any two different kinds in which one is not subordinated to the other will have no third kind which falls under both. It turns out that this condition is of the utmost importance for understanding how the many divisions of the *Sophist* relate to one another. If it holds, then it seems like we interpreters need to make sense of why what seems to be *one* kind, namely sophistry, can end up subsumed under so many mutually exclusive kinds. In what follows, I will argue that Plato does not require this kind of exclusivity. This is compatible with him requiring a weaker form of exclusivity, or understanding exclusivity, like dichotomy, to be a feature of *good* divisions, if not all.³⁶

There are two important reasons one might take this condition seriously as a condition for Platonic divisions. The first comes from how Plato talks about division. When we consider the butcher analogy of the *Phaedrus* or *Statesman*, for instance, it seems like the pieces that are the product of division are mutually exclusive in the above sense. If I cut up a chicken into thighs, legs, breasts, and wings, there will not be any part of the wing that will also be the part of the thigh. This motivates the idea of exclusivity above since, for whenever a small piece Z of the chicken is generated by cutting up X and Y, which are also pieces of the same chicken, it seems plausible that X and Y are the same piece, or that one is a part of another. If we carry over analogy to Plato's method, it seems unlikely that we would find ourselves making a division of a kind into two (or more) kinds which would later be found out to overlap. Secondly, there is a long tradition of taxonomy following Plato which does seem to employ some kind of exclusivity assumption. While we may be hesitant to attribute the kind of rich taxonomic structures of Linneas or even Aristotle to Plato, it is not totally implausible to think that they were drawing on features which they took to be at the very least implicit in Plato's conception.

Here I will argue that, despite appearances, we should not accept this condition on divisions.

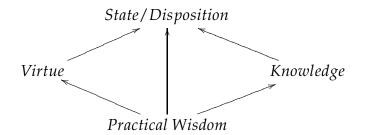
Before giving positive reasons against the condition in Plato, I want to undermine the idea that the early tradition gives us much reason to think that Platonic division is exclusive. In particular, there is a passage in Aristotle's *Topics* that seems to allow for exactly the structure on divisions that I was considering:

Look and see, also, if there is any other genus of the given species which neither embraces the given genus nor falls under it, e.g. suppose any one were to lay down that knowledge is the genus of justice. For virtue is its genus as well, and neither of these genera embraces the remaining one, so that knowledge could not be the genus of justice; for it seems that whenever one species falls under two genera, the one is embraced by the other. Yet a principle of this kind gives rise to a difficulty in some cases. For some people hold that practical wisdom is both virtue and knowledge, and that neither of its genera is embraced by the other— although certainly not everybody admits that practical wisdom is knowledge. If, however, any one were to admit the truth of this assertion, yet

³⁶I discuss one sort of weak exclusivity in n. 52.

it would still be thought to be necessary that the genera of the same object must at any rate be subordinate either the one to the other or both to the same thing as actually is the case with virtue and knowledge. For both fall under the same genus; for each of them is a state and a disposition. You should look, therefore, and see whether neither of these things is true of the given genus; for if the genera are subordinate neither the one to the other nor both to the same thing, then what is given could not be the genus. (121b24-2a2)³⁷³⁸

It is not important to me whether Aristotle himself endorses this final possibility.³⁹ What is important is that some people around him, presumably in the Academy, are debating this sort of structure:



This *topos* strongly suggests that there was some sort of debate surrounding exclusivity. Those who held practical wisdom to be both a kind of virtue and a kind of knowledge probably didn't arrive at that view by doing division, but the surrounding text strongly suggests that it was asked whether this sort of definition

³⁸Cf. 144a11 and Hambruch (1904), pp. 17-18. He seems to both say that Aristotle is agreeing with "akademischer Schuldoktrin" and that Xenocrates is the source of the claim about practical wisdom, which would require a *modification* of the traditional doctrine.

 $^{^{37}}$ Σκοπεῖν δὲ καὶ εἴ τι ἄλλο γένος ἐστὶ τοῦ ἀποδοθέντος εἴδους, ὃ μήτε περιέχει τὸ ἀποδοθὲν γένος μήθ' ὑπ' ἐκεῖνό ἐστιν. οἴον εἴ τις τῆς δικαιοσύνης τὴν ἐπιστήμην θείη γένος· ἔστι γὰρ καὶ ἡ ἀρετὴ γένος, καὶ οὐδέτερον τῶν γενῶν τὸ λοιπὸν περιέχει. ὥστ' οὐκ ἂν εἴη ἡ ἐπιστήμη γένος τῆς δικαιοσύνης· δοκεῖ γἀρ, ὅταν ἐν είδος ὑπὸ δύο γένη ῆ, τὸ ἔτερον ὑπὸ τοῦ ἑτέρου περιέχεσθαι. ἔχει δ' ἀπορίαν ἐπ' ἐνίων τὸ τοιοῦτο· δοκεῖ γὰρ ἑνίοις ἡ φρόνησις ἀρετή τε καὶ ἐπιστήμη εἴναι καὶ οὐδέτερον τῶν γενῶν ὑπ' οὐδετέρου περιέχεσθαι. ἐχει δ' ἀπορίαν ἐπ' ἐνίων τὸ τοιοῦτο· δοκεῖ γὰρ ἑνίοις ἡ φρόνησις ἀρετή τε καὶ ἐπιστήμη εἴναι καὶ οὐδέτερον τῶν γενῶν ὑπ' οὐδετέρου περιέχεσθαι. οὐ μὴν ὑπὸ πάντων γε συγχωρεῖται τὴν φρόνησιν ἐπιστήμην εἴναι. εἰ δ' οὕν τις συγχωροίη τὸ λεγόμενον ἀληθὲς εἶναι, ἀλλὰ τό γε ὑπ' ἄλληλα ἢ ὑπὸ ταὐτὸ ἄμφω γίγνεσθαι τὰ τοῦ αὐτοῦ γένη τῶν ἀναγκαίων δόξειεν ἂν εῖναι, καθάπερ καὶ ἐπὶ τῆς ἀρετῆς καὶ τῆς ἑπιστήμης συμβαίνει· ἄμφω γὰρ ὑπὸ τὸ αὐτὸ γένος ἐστίν· ἑκάτερον γὰρ αὐτῶν ἕξις καὶ διάθεσίς ἐστιν. σκεπτέον οῦν εἰ μηδέτερον ὑπάρχει τῷ ἀποδοθέντι γένει. εἰ γὰρ μήθ' ὑπ' ἄλληλά ἐστι τὰ γένη μήθ' ὑπὸ ταὐτὸν ἄμφω, οὐκ ἂν εῖη τὸ ἀποδοθὲν γένος. Τhe texts of Aristotle used are Ross (1953, 1958, 1964). Translations of Aristotle are from Barnes (1984) with some alterations, unless otherwise noted. Thanks are due to Laura Castelli for alerting me to this passage and its importance.

³⁹Some might worry, on Aristotelian grounds, whether this is a good example. Practical wisdom might be some kind of *composite* of virtue and knowledge, and thus not a species at all of virtue and knowledge. First of all, it is Aristotle himself who is offering the example as one where practical wisdom is a species of virtue and knowledge, and, although it might not be his own view, he does not think that it is unintelligible or incoherent. The *topos* shows that *even he* is flexible enough to make sense of these kinds of overlapping genera. Another, perhaps Socratic reply, would resolve the tension by simply identifying virtue and knowledge.

came into conflict with structural features of division and, in particular, exclusivity as I defined it above. Surprisingly, Aristotle does not *immediately* reject the possibility of such a structure as absurd, even if his complete theory of division would rule it out. Instead, he suggests, perhaps on someone else's behalf, that some such structures could be permissible and that exclusivity could be weakened. This kind of move would be unnatural if division were *introduced* by Plato *as having this property.*⁴⁰ In that case, such an account of practical wisdom would be dead in the water. A more satisfying story would be one in which Plato introduced division without taking a stand on exclusivity, which was a later innovation that seems to have been endorsed by people like Aristotle.

We see even more evidence for the possibility for non-exclusive divisions in Boethius's *De Divisione*, which specifically countenances multiple divisions that would produce this diamond structure:

One and the same genus is divisible in more than one way just as all corporeal entities and all things of determinate magnitude are. For just as we distribute a circle into semicircles and into what the Greeks call *tomeis* (we can say "divisions"), or a square into triangles (by drawing a diagonal through the angle), in another into parallelograms, in another into squares, so too we distribute a genus. For example, we say, "Of numbers, some are even, others odd;" or alternatively, "some are prime, others not-prime." (*De Divisione* 885b-c)⁴¹

These multiple divisions will produce kinds such as even, odd, prime, and composite, which cut across one another so that there will be kinds (such as smallest even number) which are both prime and even, despite the fact that neither of these contains the other.⁴² Moreover, this text, *De Divisione*, is known to have drawn extensively on Porphyry's commentary on the *Sophist*. One could speculate that multiple divisibility played a role in Porphyry's interpretation of the many divisions of sophistry. These two examples show that the argument from tradition that I appealed to earlier is not as well-supported as one might initially think.

There is also, however, evidence that Plato himself did not regard divisions as necessarily being exclusive. The first reason for this is that one would expect Plato to actually tell us why, in particularly unclear cases, the kinds into which he

⁴⁰This is especially true of those views such as Rickless (2010) and Miller (2016), who think that it is an essential property of division that it is exclusive.

⁴¹Fit autem generis eiusdem multipliciter diuisio, ut omnium corporum et quaecumque alicuius sunt magnitudinis. Sicut enim circulum in semicirculos et in eos quos Graeci tomeas uocant (nos diuisiones possumus dicere) distribuimus, et tetragonum alias ductor per angulum diametro in triangula, alias in parallelogrammata, alias in tetragona separamus, ita quoque genus, ut cum dicimus "numerorum alii sunt pares, alii impares" et rursus "alii primi, alii non primi". Translation from Magee (1998).

⁴²This example is also interesting since it apparently violates some kind of rule in the Academy that there are no genera of ordered series (cf *Metaphysics* B.3, 999a6-16, *Nichomachean Ethics* I.6, 1096a17-23).

divides are exclusive. For instance, why can't there be an art that is both acquisitive and productive? Let us see how he makes the division:

Visitor: But expertise as a whole falls pretty much into two types.

Theaetetus: How?

Visitor: There's farming, or any sort of caring for any mortal body; and there's also caring for things that are put together or fabricated, which we call equipment, and there's imitation. The right thing would be to call all those things by a single name.

Theaetetus: How? What name?

Visitor: When you bring anything into being that wasn't in being before, we say you're a producer that the thing you've brought into being is produced.

Theaetetus: That's right.

Visitor: And all the things we went through just now have their own capacity for that.

Theaetetus: Yes.

Visitor: Let's put them under the heading of production.

Theaetetus: All right.

Visitor: Next, consider the whole type that has to do with learning, recognition, commerce, combat, and hunting. None of these creates anything. They take things that are or have come into being, and they take possession of some of them with words and actions, and they keep other things from being taken possession of. For that reason it would be appropriate to call all the parts of this type acquisition.

Theaetetus: Yes, that would be appropriate.

Visitor: If every expertise falls under acquisition or production, Theaetetus, which one shall we put angling in? (219a-d)⁴³

⁴³{ΞΕ.} Άλλὰ μὴν τῶν γε τεχνῶν πασῶν σχεδὸν εἴδη δύο. {ΘΕΑΙ.} Πῶς; {ΞΕ.} Γεωργία μὲν καὶ ὅση περὶ τὸ ϑνητὸν πᾶν σῶμα ϑεραπεία, τό τε αὕ περὶ τὸ σύνϑετον καὶ πλαστόν, ὃ δὴ σκεῦος ἀνομάκαμεν, ἤ τε μιμητική, σύμπαντα ταῦτα δικαιότατ' ἂν ἑνὶ προσαγορεύοιτ' ἂν ὀνόματι. {ΘΕΑΙ.} Πῶς καὶ τίνι; {ΞΕ.} Πᾶν ὅπερ ἂν μὴ πρότερόν τις ὂν ὕστερον εἰς οὐσίαν ἄγῃ, τὸν μὲν ἄγοντα ποιεῖν, τὸ δὲ ἀγόμενον ποιεῖσθαί πού φαμεν. {ΘΕΑΙ.} Όρϑῶς. {ΞΕ.} Τὰ δἑ γε νυνδὴ <δ> διήλϑομεν ἅπαντα εἴχεν εἰς τοῦτο τὴν αὐτῶν δύναμιν. {ΘΕΑΙ.} Όρϑῶς. {ΞΕ.} Τὰ δἑ γε νυνδὴ <δ> διήλϑομεν ἅπαντα εἴχεν εἰς τοῦτο τὴν αὐτῶν δύναμιν. {ΘΕΑΙ.} Εἴχε γὰρ οῦν. {ΞΕ.} Ποιητικὴν τοίνυν αὐτὰ συγκεφαλαιωσάμενοι προςείπωμεν. {ΘΕΑΙ.} Ἔστω. {ΞΕ.} Τὸ δὴ μαϑηματικὸν αῦ μετὰ τοῦτο είδος ὅλον καὶ τὸ τῆς γνωρίσεως τό τε χρηματιστικὸν καὶ ἀγωνιστικὸν καὶ ϑηρευτικόν, ἐπειδὴ δημιουργεῖ μὲν οὐδὲν τούτων, τὰ δὲ ὄντα καὶ γεγονότα τὰ μὲν χειροῦται λόγοις καὶ πράξεσι, τὰ δὲ τοῖς χειρουμένοις οὐχ ἐπιτρέπει, μάλιστ' ἅν που διὰ ταῦτα συνάπαντα τὰ μέρη τέχνη τις κτητικὴ λεχϑεῖσα ἂν διαπρέψειεν. {ΘΕΑΙ.} Ναί· πρέποι γὰρ ἄν. {ΞΕ.} Κτητικῆς δὴ καὶ ποιητικῆς συμπασῶν οὐσῶν τῶν τεχνῶν ἐν ποτέρα τὴν ἀσπαλιευτικήν, ῷ Θεαίτητε, τιθῶμεν;

Here it might initially seem that the Visitor is giving a reason for us to think that production and acquisition are mutually exclusive when he says that "None of these creates anything."44 This, however, is not right. Rather, the Visitor is talking about all of the examples acquisition that he had just discussed—this is why he uses the plural τούτων. The kind *acquisition*, however, presumably includes more than just these examples and the Visitor has not given us any reason to think that one of these could not also be productive. This consideration does not show that Plato did not *want* his division here to be exclusive, only that the considerations he gave *could not establish* exclusivity. If we can make good sense of his procedure without having to postulate exclusivity, however, then this would be preferable than attributing to him both exclusivity and bad arguments. Note too that this is a very frequent feature of Platonic divisions. For instance, when fishing is divided into enclosure-hunting and strike-hunting, we get baskets, nets, slipknots, and creels as examples of the former kind and hooks and tridents for the latter. Instead of stating principles which would guarantee exclusivity, the divider finds a bunch of examples, thinks of something that they all have in common, and then establishes a kind on that basis.⁴⁵

It cannot be denied that exclusivity can serve some useful purposes. Indeed, I will argue in Ch. 4 that Aristotle brings exclusivity to the fore to do some epistemological heavy-lifting. But again, we must keep in mind the distinction between a division and a *good* division. I have nothing to say now about the normative status of exclusivity, but instead intend to argue that it is not a structural feature of every Platonic division.

The second text comes from Grams:

Exchanging (μεταβλητικός)—which the Visitor specifies is accomplished through gifts (δωρεά), wage-earning (μίσθωσις), and selling (ἀγόρασις)—is initially marked off as a kind of "mutually agreed" acquiring, in opposition to conquering (χειρωτικός) that takes by force (219d5).

However, both wage-earning ($\mu \iota \sigma \vartheta \alpha \rho \nu \eta \tau \iota \varkappa \delta \varsigma$) and gift-giving ($\delta \omega \rho o \rho \rho \rho \iota \varkappa \delta \varsigma$) kinds appear again as subsequent divisions of conquering (222d). (Grams, 2012)

As Grams notes, there are two ways of accepting this division as legitimate: either division is not transitive (and we have left the kind of conquering) or it is not ex-

⁴⁴This is the view of Rickless (2010).

⁴⁵It is here that we see an important role, even in definitory divisions, for collection. As described in the *Phaedrus*, collection is the procedure for finding a single thing scattered among many. This seems to be precisely how the Visitor is using examples here: he lists a number of classes and then says that there is a single kind encompassing all of them on account of sharing some feature. Importantly, this feature in common has not yet found a special place in the theory of division and dialectic as the *differentia specifica*, something we find clearly already in Aristotle. Nevertheless, Plato does clearly talk about this feature in his discussions of division. See Meinwald (2002).

clusive (and the relevant kinds are overlapping in these cases).⁴⁶ The way that the Visitor describes this distinction, it is much more plausible to think Plato would prefer the latter method. This is because the kinds of wage-earning and gift-giving that we are dealing with in 222d are ways of *hunting tame animals by persuasion in private*. Hunting is, according to the Visitor, conquering secretly. Exchange, on the other hand, is distinguished from taking possession by being mutually willing. Because of the secrecy of hunting, it seems plausible that one could have an art of acquiring that is both hunting and mutually willing. In such a case, the art would hunt the victim partially by making that person think that they are just exchanging, and therefore enter into the relationship willingly. Consider, for instance, a pyramid scheme. The people being bamboozled think that they are entering into a fair agreement and so willingly give their money to the head of the scheme. Thus, these kinds would plausibly be taken to be both cases of exchange and conquering.

The third text, I think, is a much more direct problem for the exclusivity assumption as defined above.⁴⁷ Here, at the very end of the dialogue, the Visitor returns to the art of production, and makes two divisions of it, on the one hand into divine/human and on the other into production of originals/images:

Visitor: It's as if you'd already cut production all the way along its width, and now you'll cut it along its length.

Theaetetus: All right.

Visitor: That way there are four parts of it all together, two human ones related to us and two divine ones related to the gods.

Theaetetus: Yes.

Visitor: Then if we take the division we made the first way, one part of each of those parts is the production of originals. Just about the best thing to call the two parts that are left might be "copy-making." That way, production is divided in two again. (266a)⁴⁸

The point of this passage is to say that we are actually making two distinct divisions of *production*. This is not, as some commentators think, a case of dividing the production into human/divine and then subdividing human production into

⁴⁶Of course, one could also say that Plato was just snoozing here and made a mistake by his own lights. There is something to be said for this reading and it would give little reason to jettison exclusivity or transitivity in the absence of independent evidence. However, with the corroborating evidence of the other passages, the interpretation provided by Grams becomes more plausible.

⁴⁷However, unlike the previous two texts, this is not a counterexample to the weaker form of exclusivity discussed in n. 52.

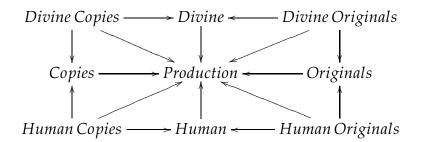
⁴⁸{ΞΕ.} Οἶον τότε μὲν κατὰ πλάτος τέμνων τὴν ποιητικὴν πᾶσαν, νῦν δὲ αῦ κατὰ μῆκος. {ΘΕΑΙ.} Τετμήσθω. {ΞΕ.} Τέτταρα μὴν αὐτῆς οὕτω τὰ πάντα μέρη γίγνεται, δύο μὲν τὰ πρὸς ἡμῶν, ἀνθρώπεια, δύο δ' αῦ τὰ πρὸς θεῶν, θεῖα. {ΘΕΑΙ.} Ναί. {ΞΕ.} Τὰ δέ γ' ὡς ἑτέρως αῦ διηρημένα, μέρος μὲν ἐν ἀφ΄ ἑκατέρας τῆς μερίδος αὐτοποιητικόν, τὼ δ' ὑπολοίπω σχεδὸν μάλιστ' ἂν λεγοίσθην εἰδωλοποιικώ· καὶ κατὰ ταῦτα δὴ πάλιν ἡ ποιητικὴ διχῆ διαιρεῖται.

copy-making and original-making (and repeating the process for divine production). If that were what the Visitor was describing, then we would not have cut *one thing*, production, first along its width and then along its length.

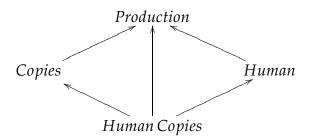
So we should think here of a total of nine kinds, pictured below. They are organized in three levels:

- 1. Production
- 2. Parts of production: Human, Divine, Copy-Making, Original-Making
- 3. Parts of parts of production: Human-Copy-Making, Divine-Copy-Making, Human-Original-Making, Divine-Original-Making

The levels here are organized by their generality, every level 2 kind is contained in the level 1 kind and every level 3 kind is contained in two level 2 kinds. However, no level 1 kind is contained in a level 2 kind, nor is a level 2 kind contained in a level 3 kind. The problem that this passage poses for the exclusivity assumption is that we have the following kind of structure:



and if we just look at a small part of it and rotate, we see that:



This is precisely the kind of thing that exclusivity does not allow, since we get direct argumentation for the claim that there are kinds of copy-making that are not human (dreams) and kinds of human production that are not copy-making (carpentry).⁴⁹ A defender of exclusivity might want to resist this line of reasoning

⁴⁹Note that this is also a counterexample to Balme (1987), who claims that Plato only divides by one difference at a time. So in that respect, Plato and Aristotle in the *Parts of Animals* are not so different. However, Aristotle's account of these sorts of cases will differ from Plato's in other respects because, it seems, he unlike Plato holds on to a stronger form of exclusivity.

by saying that there seem to be, by Plato's own lights, *two different divisions* of production. While there might be two different "cuts", it seems quite important for Plato's purposes that he has one division, in some sense. For he wants to give a single divisional structure that encapsulates all the features necessary for defining the sophist and both of the features described above are necessary. So we are forced to put the two divisions together. And in the resulting structure, there are two coordinate kinds that each contain another kind and exclusivity, as I defined it, claimed that this structure did not allow for a kind to fall under two kinds unless one contained the other. So exclusivity is violated.

What is important about the exclusivity assumption is that it puts strong constraints on how groups of divisions must hang together—in particular, requiring that no two divisions cut across the distinctions of the other. And it is this very consequence of exclusivity that is shown problematic in the text. Furthermore, there seems to me to be little point in claiming that these are not "real" or "natural" kinds for Plato, as the distinctions between human and divine and real and image are indisputably central to much of his thinking and are often made independently of one another. Finally, this does not seem to me to be a mere slip on Plato's part; he spends more time explaining this division than practically any other in the dialogue, so we should take it that he is trying to tell us something important in it.

Non-exclusive divisional structures are also extremely common in science, so it would make Plato's view more plausible if he allows for them. To give an example close to Plato's own in the *Philebus*, the generic kind *Turkish vowel* is divided in three different ways, one might say, cut lengthwise, breadthwise, and depthwise:

- 1. whether the tongue is in either the front or the back part of the mouth,
- 2. whether the lips are either rounded or unrounded,
- 3. whether the tongue is either high or low in the mouth.

This series of distinctions produce a yet more complicated version of the structure described in the *Sophist*. Every combination of these three distinctions results in exactly one vowel, e.g., /i/ = front, unrounded, high. These are phonetic kinds. If the Traditional View's insistence on exclusivity were correct, then Plato's requirements on division rules out these structures *tout court*, something that does not sound very plausible.⁵⁰ This feature, however, is not restricted to the "special" sciences. Even the Standard Model of particle physics distinguishes 24 kinds of elementary fermions, divided by color charge, electric charge, weak isospin, and mass. These four distinctions are all fundamental to the theory, as they determine how the particle acts under the fundamental forces.

It does not help to say here that these are merely "dialectical" divisions, since exclusivity was taken to be a conditions on *all* divisions, dialectical or not. Perhaps, as I said above, all scientifically valuable divisions are in fact exclusive. All

⁵⁰Thanks to John MacFarlane for the suggestion.

that I need for now is the claim that there are divisions that do not satisfy such requirements. That is what the Traditional View claims. I have argued that Plato's texts and the practice of science are evidence against it.

2.3.3 Essential Predication

In this section, I will argue against the second part of the traditional view of division. Again, this requirement of divisions is quite natural. First of all, since division is supposed to produce a *logos*, it makes sense that the relations are ones of what Aristotle would call predication. Moreover, since the whole point is to get at what something *really is*, the relationship should only include relations of essential predication. Otherwise, when you identify the target kind at the bottom of a division, you won't get a definition, which was what was originally sought.

While it seems to be right that all divisions involve predication, what I will here be disputing is whether they need be essential predications. From the mere fact that divisions are *intended* to result in definitions, it does not follow that divisions which fail to achieve that end are not divisions. All that can be concluded is that the *methodologically useful* divisions include essential predications, not that every division does.

Not only does the argument fail, however. There are two pieces of positive textual evidence that support the claim that some divisions capture *accidental* predication relations. The first comes from looking at the arguments that support a division. Let's return to the angler (here omitting Theaetetus's replies):

But expertise as a whole falls pretty much into two types. [...] There's farming, or any sort of caring for any mortal body; and there's also caring for things that are put together or fabricated, which we call equipment, and there's imitation. The right thing would be to call all those things by a single name. [...] When you bring anything into being that wasn't in being before, we say you're a producer and that the thing you've brought into being is produced. [...] And all the things we went through just now have their own capacity for that. [...] Let's put them under the heading of production. [...] Next, consider the whole type that has to do with learning, recognition, commerce, combat, and hunting. None of these creates anything. They take things that are or have come into being, and they take possession of some of them with words and actions, and they keep other things from being taken possession of. For that reason it would be appropriate to call all the parts of this type acquisition. [...] If every expertise falls under acquisition or production, Theaetetus, which one shall we put angling in? (219a-d)

If the Traditional View is true and all divisions involve essential predication, when the Visitor divides expertise into production and acquisition, he should try to show two things. First, he should show that production and acquisition are essentially kinds of expertise. What if, instead of this division, the Visitor divided expertise into those practiced indoors and outdoors? If we proceeded in this way, part of the definition of angling would be that it is practiced outdoors. But this is not plausibly part of the essence of angling. What is important to note is that here, in the very first division, where the Visitor is going at great lengths to explain what he is doing, he does not argue that acquisition or production are essentially kinds of expertise or even flag the assumption, but merely says that every expertise falls into one or the other. The same, of course, would be true of the indoor/outdoor division.

The second thing that the Visitor should show, on the Traditional View, is that each of these expertises is essentially either productive or acquisitive. But again, he does nothing of the sort. Instead, he just says that each *is* an expertise that produces something or acquires something. But that doesn't say anything about what kind of expertise it is essentially. Again, it is true that angling is practiced outdoors, but not essential to it. So it seems that, on the Traditional View, Plato is guilty of some pretty sloppy argumentation. And the definition of angling is supposed to be the easy definition that is got unproblematically. This suggests that the Traditional View is just mistaken. It would be more charitable to attribute good arguments and a weaker view to Plato than a stronger view that is not well-supported.

The *Sophist* thus suggests that charity considerations tell against the Traditional View. Once we look outside of that dialogue, however, the textual evidence is much more direct. In the *Statesman*, Young Socrates divided herd-rearing too quickly into rearing of human herds and beast herds. The Eleatic Visitor explains the problem as ahving "rushed the account" (*Statesman* 262a-b). To illustrate the mistake, the Visitor gives two other divisions that reach the same end point but do so more quickly and more slowly and in which the fast division leads to a mistake. The Traditional View has a very difficult time accounting for this passage, since Young Socrates succeeded in making a division but evidently did not mark out essential features of animals. It seems like the whole point of the distinction between the two kinds of divisions is that, while the Socrates's merely got "parts", a proper division should aim at "ideas" or "kinds". The Eleatic Visitor is quite emphatic about this distinction:

That whenever there is a kind of something, it is necessarily also a part of whatever thing it is said to be a kind of, but it is not at all necessary that a part is a kind. You must always assert, Socrates, that this is what I say rather than the other way around. (*Statesman* 263b7-10)⁵¹

This is a distinction between two kinds of things one can divide into. On the one hand, one can divide into "mere parts", such as animals into humans and beasts. On the other hand, one could divide into genuine kinds. The first kind seems to be

⁵¹ Ως είδος μὲν ὅταν ἦ του, καὶ μέρος αὐτὸ ἀναγκαῖον είναι τοῦ πράγματος ὅτουπερ ἂν είδος λέγηται· μέρος δὲ είδος οὐδεμία ἀνάγκη. ταύτη με ἢ 'κείνη μᾶλλον, ὥ Σώκρατες, ἀεὶ φάθι λέγειν.

problematic *precisely because* birds are not essentially beasts. A complete and accurate account of the essence of birds would not, according to the Visitor, mention the kind "beast". We can see this from the Visitor's own parallel case: dividing people into Greeks and barbarians. Because the term "barbarian" picks out a group that has no language or practices in common, it does not belong to the essence of any particular person or group of people that they are barbarians.

Instead, an account of the essence will only include *ideas*, which seem to be genuine unities. Thus it seems like Plato here is explicitly making room, in the *Statesman*, for genuine divisions that do not mark out essential relations of predication. This corroborates the interpretation suggested by charity considerations in the *Sophist*. Here they are clearly described both as divisions and as failing to capture the essence. In sum, we have good reason to doubt that Platonic divisions mark out relations of essential predication.

Just like the structural features of division, the points made in this section suggest that the Traditional View's requirement that divisions only mark out relations of essential predication, is false. Of course the interlocutors are *trying* to mark out essential predications, since they are *trying* to define the sophist and statesman. And of course the division would be successful if it did those things. All that I am arguing is that the interlocutors can succeed at dividing without accomplishing this goal.

2.4 The Puzzle Again

If we discard the Traditional View of division in favor of the Minimal View, we can now easily dissolve problems for the straightforward interpretation of the puzzle of the sophist. Let's see how discarding exclusivity and essential predication can help.

Consider exclusivity. We can say that every division in the *Sophist* is really a division, with the one kind *Sophistry* simply being an example of a kind that is a species of several different, coordinate genera. Both in the divisions of sophistry and the length-wise/breadth-wise division we are concerned with putting together multiple divisions. Without exclusivity, we are no longer forced to say that these accounts are incoherent because jointly inconsistent. Instead, we *can* say that (even if only one of them expresses the essence of sophistry) they all are true together. Of course, one might still have qualms about the truth of particular accounts (e.g. the Noble Sophist), but these worries would arise independently of there being other divisions. Importantly, the doubts expressed by both the Visitor and Theaetetus in the passage about the Noble sophist have to do with whether this account is true of sophists *at all*, so would have been an issue even if it were the very first division.⁵² Now consider essential predication. We can again say

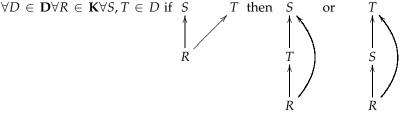
⁵²Someone might object here that the length-wise/breadth-wise divisions do not provide sufficient support for this claim. Just because *Human-Copy-Making* falls under two distinct coordinate

that every division in the *Sophist* really is a division, with the one kind *Sophistry* being an example of a kind that is accidentally acquisitive but essentially productive. Thus all of the Visitor's argumentation while making the divisions is correct, as far as it goes. It is just not enough to get what he wanted, until he reached the last.

I have argued that the Traditional View of division must be discarded if we are to give the correct interpretation of the puzzle of the sophist. In its place, I have elaborated and defended a Minimal View of division, on which it still has some structure, in particular, transitivity, and does mark out relations of predication, but is not necessarily dichotomous or exclusive, and can fail to mark out relations of essential predication. So, while there is much less structure than the Traditional View originally claimed, there still remains some significant structure.

The question remains, however, what Plato was trying to use the puzzle to show us. The discussion above gives us some hint: not all divisions get at relations of essential predication. But this was exactly what we were after by doing division. The program going forward, then, is clear. What does one dividing need to do to rule out the problematic divisions? What norms should she obey? Progress on this program would be articulating norms that rule out problematic divisions. This is exactly what happens in the *Statesman*. The Eleatic Visitor there argues for a norm to divide slowly (261e-264b) and to divide into the smallest number possible (287b-c). While the *Sophist* investigates division generally, without regard to whether the division is useful for the purposes of science, while the *Statesman* starts to articulate norms for scientifically sound divisions.⁵³

kinds does not mean that exclusivity is not a requirement on each *dimension* of a division. On this view, what's important is that exclusivity hold between *human* and *divine* production, since these were divided at the same time. One way to spell this out: instead of a pair $\langle \mathbf{K}, \uparrow \rangle$, where **K** is a set of kinds and \uparrow the set of lines connecting them, we will need a triple $\langle \mathbf{K}, \uparrow, \mathbf{D} \rangle$, where **D** is a set of subsets of **K** of kinds belonging to the same dimension. Plausibly, **D** will be closed downwards under \uparrow , i.e., if $S \in D \in \mathbf{D}$ and $R \to S$, then $R \in D$. This basically means that subkinds are parts of the same dimension as the higher kinds. Also, we would want to require that $\bigcup \mathbf{D} = \mathbf{K}$, i.e., that every kind belongs to some dimension. We can then state exclusivity along a dimension thus: $\forall D \in \mathbf{D} \forall R \in \mathbf{K} \forall S, T \in D$ if S and T then S or T or S = T. Clearly, this view



can make sense of the length-wise and breadth-wise divisions, since there we are obviously talking about different dimensions. This is true because allows that exclusivity as I defined above does not hold in some cases. However, the other problems for the stronger exclusivity assumption detailed above reemerge for this view. It is still the case, for example, that the Visitor's argumentation in the productive/acquisitive division does not suffice for even this kind of exclusivity. For, although he is clearly talking about one dimension, his collection does not pretend to be exhaustive, so he cannot guarantee that there would not be one kind falling under both the acquisitive and productive arts.

⁵³As I will argue in the next chapter, the norms articulated in the *Statesman* and alluded to in the

On the picture being sketched, the point of introducing the puzzle is to motivate the search for norms that goes on in the *Statesman*. While division is necessary for scientific inquiry into essence, the divider must be careful. The best way to see this is to see how, if one is not careful, the method can run wild.

Philebus are not jointly sufficient to prevent the mistakes being made in the *Sophist*. There I will argue that this indicates that Plato took his project in the *Statesman* to be incomplete.

Chapter 3

The Goal and Norms of Platonic Division

After the successful definition of the *Sophist*, the Eleatic Visitor continues his project of defining the controversial trio of sophist, statesman, and philosopher in the *Statesman*. Here he switches interlocutors, but not (apparently) his method. At the very least, he continues to use division in order to try to obtain a definition of statesmanship.

There are a number of important differences, despite the evident continuities:the same starting point is taken, division is used to introduce puzzles in the "core" of the dialogue, the method aims at definition but only achieves it after some initial failures. First, the Visitor lets Socrates try his hand at making divisions and corrects him. Second, he gives examples of alternative ways of dividing, asking Young Socrates (and, indirectly, us_ to work out which of these ways is preferable.

In this chapter, I will argue that Plato proposes two norms of division in the *Statesman*. I argued in the last chapter that the puzzle of the sophist was introduced to show us the *need* for norms. In the *Statesman*, Plato is beginning to articulate such norms. Not only does he lay down two norms, but he also gives a general recipe for justifying them, akin to the use of soundness proofs in modern logic. One has the goal of only moving from truths to truths in one's arguments. Observing the rules of logic *guarantees* this to happen. However, it is perfectly possible that one will move from one truth to another while making an *invalid* inference.

The Eleatic Visitor tries to justifies norms of division by showing how they can guarantee, or at least make it much more probable, that the divider reaches the goal. Just as an invalid argument might *sometimes* move from a true premise to a true conclusion, in division one might accidentally make a correct cut, isolating the theoretically important kind, but your procedure may not have guaranteed that. Plato himself seems conscious of this distinction, making it in the *Statesman*:

Visitor: ... it is most fine to separate off the object of inquiry from the others straightaway, should you do it correctly, just as a moment ago,

when you thought that you had the division, you rushed the account, seeing that it was headed towards humans. But my friend, it is not safe to do such fine work... This makes all the difference in investigations. (*Statesman* 262a-c)¹

The structure of the chapter is as follows: first, I will articulate the goal of division. Next, I will look at the two norms of division found in the *Statesman*, which both have parallels in the *Philebus*. Finally, I will speculate about the completeness of the Platonic theory.

3.1 Goal

Put most vaguely, the goal of the method of division in the *Sophist* and *Statesman* is clear enough. The interlocutors are trying to get definitions—accounts of what sophistry, statesmanship, and philosophy *really are*. At several points in the dialogues we get some help about what that involves, points familiar from the Socratic definitional dialogues. First, division seeks essential and not merely accidental features of the kinds to be defined. Second, the definition needs to contain *all* the essential features. It seems clear that, if it could accomplish these two things, then there would be nothing further for it to do.²

So, while more can be said about what an essential feature *is* and what it is for an account to contain *all* the essential features, this is enough to give us a provisional account of the goal of division:

Goal Do not leave anything essential out of the account.

While this is very plausible as the goal of division in the *Sophist* and *Statesman* and the way that Aristotle thinks about his predecessors in the *Analytics*,³ there are some texts that give a different impression. In the *Phaedrus* and *Philebus*, the very dialogues in which division is given its most important role in scientific inquiry, the purpose of division is not to define target kinds but to model knowledge of an entire kind of thing. This has been called the *classificatory* use of division by Cornford (1957).

¹{ΞΕ.} ... κάλλιστον μὲν γὰρ ἀπὸ τῶν ἄλλων εὐθὺς διαχωρίζειν τὸ ζητούμενον, ἂν ὀρθῶς ἔχῃ, καθάπερ ὀλίγον σὺ πρότερον οἰηθεὶς ἔχειν τὴν διαίρεσιν ἐπέσπευσας τὸν λόγον, ... ἀλλὰ γάρ, ῷ φίλε, λεπτουργεῖν οὐκ ἀσφαλές... τοῦτο δὲ διαφέρει τὸ πᾶν πρὸς τὰς ζητήσεις.

²This is an interesting point of contrast with Aristotle. As I will argue in the next chapter, Aristotle does not think that division can do these things, but he thinks that division can help us discover something else about the essence that he thinks is also necessary: the order of the essential features.

³Aristotle says of "all those who made use of it" (τοὺς χρωμένους αὐτῇ πάντας) that "they tried to convince us that it is possible for a demonstration concerning essence, or what something is, to come about." (πείθειν ἐπεχείρουν ὡς ὄντος δυνατοῦ περὶ οὐσίας ἀπόδειξιν γενέσθαι καὶ τοῦ τί ἐστιν.) Since Plato clearly is intended to be one of those "all" *pace* Cherniss (1944) and Striker (2009), Aristotle interprets Plato as trying to use division to demonstrate the what it is, or essence, of the target kind.

Is this incompatible with the goal of division being to give a complete account? Only if what is important or useful about the classification is *not* that it gives the scientist accounts of all the things she is interested in in a given domain. But it seems that this is just what division does in these situations as well. Most importantly, Plato seems to freely move between these two goals in the *Statesman*. On Cornford's account, he should only be interested in division's use in giving definitions, but when he pauses to discuss the importance of the method practiced correctly, he seems to advert to a different notion:

For it is indeed the case, in a certain way, that all the products of the various sorts of expertise share in measurement. But because of their not being accustomed to carrying on their investigations by dividing according to kinds, the people in question throw these things together at once, despite the degree of difference between them, thinking them alike—and then again they also do the opposite of this by dividing other things not according to parts, when the rule is that when one perceives first the community between the members of a group of many things, one should not desist until one sees in it all those differences that are located in classes, and conversely, with the various unlikenesses, when they are seen in multitudes, one should be incapable of pulling a face and stopping before one has penned all the related things within one likeness and actually surrounded them in some kind. (285a-b)⁴

It seems to me, then, that the classificatory use of division is not at all in tension with the definitional use. In fact, they come together in a very important way. Definitions of the genus/differentia sort aim to show what something is by saying how it relates to other things. Classifications are useful precisely when they allow one to identify a kind in relation to others.

Secondly, I fail to see what is philosophically interesting about a *merely* classificatory use of division, one on which the main purpose is to say how many subkinds there are of a given kind. This point is made with great force by Zabarella in *De Methodiis* III.9. It does not seem like such an enumeration is enlightening, unless we know, what each of these subkinds *are*. But *that* was the job of the definitional use of division. Even worse, Zabarella points out, it seems like the enumeration presupposes an identification of the subkinds, and hence, knowledge of the essence anyway.

This does not mean that Cornford's distinction has no basis. There is *something* different in the two groups of texts, just not quite the one that he recognized.

⁴μετρήσεως μὲν γὰρ δή τινα τρόπον πάνθ' ὁπόσα ἔντεχνα μετείληφεν· διὰ δὲ τὸ μὴ κατ' εἴδη συνειθίσθαι σκοπεῖν διαιρουμένους ταῦτά τε τοσοῦτον διαφέροντα συμβάλλουσιν εὐθὺς εἰς ταὐτὸν ὄμοια νομίσαντες, καὶ τοὐναντίον αὕ τούτου δρῶσιν ἕτερα οὐ κατὰ μέρη διαιροῦντες, δέον, ὅταν μὲν τὴν τῶν πολλῶν τις πρότερον αἴσθηται κοινωνίαν, μὴ προαφίστασθαι πρὶν ἂν ἐν αὐτῆ τὰς διαφορὰς ἴδῃ πάσας ὁπόσαιπερ ἐν εἴδεσι κεῖνται, τὰς δὲ αῦ παντοδαπὰς ἀνομοιότητας, ὅταν ἐν πλήθεσιν ὀφθῶσιν, μὴ δυνατὸν εἶναι δυσωπούμενον παύεσθαι πρὶν ἂν σύμπαντα τὰ οἰκεῖα ἐντὸς μιᾶς ὁμοιότητος ἕρξας γένους τινὸς οὐσία περιβάληται.

Rather, it is that in the *Sophist/Statesman* divisions, there is one thing in particular that is being targeted for a definition, whereas the *Phaedrus/Philebus* divisions simply try to define *everything* under the kind.

At this point, I have argued that the fundamental goal of Platonic division, not only in the *Sophist* and *Statesman*, but even in these superficially quite different dialogues, is to give a complete account of the essence of a target kind. This can be done one kind at a time or all at once, but in both cases the insight is the same: we arrive at an account of, say, the essence of sophistry by systematically comparing it to other kinds, asking how it is the same or different. A complete account of that will tell us what the thing is *in itself*.

But how ought one go about making such comparisons? Surely not every comparison is equal. One particularly striking example is, of course, the *Sophist*. In the last chapter, I suggested that the lesson Plato wants us to draw from the *Sophist* was precisely this. But we are now in a quandary: how do we make the right distinctions to achieve the goal?

3.2 Two Norms

I will now show how Plato made a start at answering this last question in the *Statesman*. As I noted earlier, here the Visitor departs from the procedure in the *Sophist* by giving Young Socrates the reins and correcting his division, as well as being more open about why he makes the various choices that he does. At these points, I argue, he introduces two norms of division. These norms are intended to be applied when inquiring into anything at all. We can be fairly sure of this because 1) there is no qualification in the statement of the norms, and 2) the Visitor explicitly says that he is doing all this for the sake of making Young Socrates and Theaetetus better dialecticians *about all things*. (285d) In this section, I will analyze these norms, show how they relate to one another, and describe Plato's method of justifying them.

3.2.1 The Tortoise Norm

The first norm introduced in the *Statesman* comes when Young Socrates divides the knowledge of collective herd rearing:

Visitor: ... Do you see how by showing the collective rearing of herds to be twin in form one will make what is now being sought in double the field then be sought in half of that?

Young Socrates: I shall try my hardest. It seems to me that there is a different sort of rearing of human beings, and in turn another sort where beasts are concerned. (*Statesman* 261e-2a)⁵

 $^{^{5}{\{\}Xi E.\}}$ την δὲ ἀγελαιοτροφικην ἅρ' ἐννοεῖς πῆ τις δίδυμον ἀποφήνας τὸ ζητούμενον ἐν διπλασίοισι τὰ

The Visitor thinks that there is something right about what Young Socrates did, but also a big mistake:

Visitor: Ah, yes. You've made a very zealous and courageous division! However, we should try not to have this happen ever again!...We should not carve off one small part from many big ones nor apart from a kind. Instead, let the part at the same time be a form. For it is most fine to separate off the object of inquiry from the others straightaway, should you do it correctly, just as a moment ago, when you thought that you had the division, you rushed the account, seeing that it was headed towards humans. But my friend, it is not safe to do such fine work. Instead, cutting through the intermediate stages⁶ is safer and one would more encounter ideas. This makes all the difference in investigations. (*Statesman* 262a-c)⁷

What the Visitor is saying here is that, while Young Socrates happened to get the right answer here by doing what he did, he did not do so "safely". Presumably this means that, if he were to always go on the way did just then, he would be prone to making errors. But what exactly *was* the problem with how he went on? The Visitor explains by showing what would happen in another case if one were to divide in the same way:

Visitor: It's as if someone tried to divide the human race into two and made the cut in the way that most people here carve things up, taking the Greek race away as one, separate from all the rest, and to all the others together, which are unlimited in number, which don't mix with one another, and don't share the same language—calling this collection by the single appellation 'barbarian'. Because of this single appellation, they expect it to be a single family or class too. Another example would be if someone thought that he was dividing number into two real classes by cutting off the number ten-thousand from all the rest, separating it off as a single class, and in positing a single name for all

νῦν ἐν τοῖς ἡμίσεσιν εἰς τότε ποιήσει ζητεῖσθαι;

[{]NE. ΣΩ.} Προθυμήσομαι. καί μοι δοκεῖ τῶν μὲν ἀνθρώπων ἑτέρα τις εἶναι, τῶν δ' αὖ θηρίων ἄλλη τροφή.

⁶Most translations render this as the "middle of things" or "in halves", but it seems pretty clear that this phrase is closely connected with the gift of the gods passage in the *Philebus* (see below). There Plato discusses "μέσα" as intermediate stages between the one and the infinite that one should not overlook. Here exactly the same point is made, so we should render "διὰ μέσων" similarly. Cf *Prior Analytics* I.31 46a31, "ἡ διὰ τῶν γενῶν διαίρεσις".

⁷{ΞΕ.} Παντάπασί γε προθυμότατα καὶ ἀνδρειότατα διήρησαι· μὴ μέντοι τοῦτό γε εἰς αὕθις κατὰ δύναμιν πάσχωμεν. ...Μὴ σμικρὸν μόριον ἐν πρὸς μεγάλα καὶ πολλὰ ἀφαιρῶμεν, μηδὲ εἴδους χωρίς· ἀλλὰ τὸ μέρος ἄμα εἴδος ἐχέτω. κάλλιστον μὲν γὰρ ἀπὸ τῶν ἄλλων εὐθὺς διαχωρίζειν τὸ ζητούμενον, ἂν ὀρθῶς ἔχῃ, καθάπερ ὀλίγον σὺ πρότερον οἰηθεἰς ἔχειν τὴν διαίρεσιν ἐπέσπευσας τὸν λόγον, ἰδὼν ἐπ' ἀνθρώπους πορευόμενον· ἀλλὰ γάρ, ῶ φίλε, λεπτουργεῖν οὐκ ἀσφαλές, διὰ μέσων δὲ ἀσφαλέστερον ἰέναι τέμνοντας, καὶ μᾶλλον ἰδέαις ἄν τις προστυγχάνοι. τοῦτο δὲ διαφέρει τὸ πῶν πρὸς τὰς ζητήσεις.

the rest supposed here too that through getting the name this class too came into existence, a second singe one apart from the other. But I imagine the division would be done better, more by real classes and more into two, if one cut number by means of even and odd, and the human race in its turn by means of male and female, and only split off Lydians or Phrygians or anyone else and ranged them against all the rest when one was at a loss as to how to split in such a way that each of the halves split off was simultaneously a real class and a part.

...Visitor: I think it was pretty much the point at which you were asked how to divide herd-rearing, and you said with great keenness that there were two classes of living creatures, one human, and a second single one consisting of all the rest—the beasts—together.

...Visitor: And to me you appeared then to think that in taking away a part you had left behind the rest as in its turn a single class, consisting of all of them, because you had the same name, 'beasts, to apply to them all.

...Visitor: And yet, my courageous friend, maybe, if by chance there is some other animal which is rational, as for example, the crane seems to be, or some other such creature, and which perhaps distributes names on the same principles as you, it might oppose cranes as one class to all other living creatures and give itself airs, taking all the rest together with human beings and putting them into the same category, which it would call by no other name except—perhaps—'beasts'. So let's try to be very wary of everything of this sort.

...Visitor: Well then, let's not divide in the way we did then, looking at everything, or in a hurry, just in order to get quickly to statesmanship. $(262b-264c)^8$

⁸Τοιόνδε, οἴον εἴ τις τἀνθρώπινον ἐπιχειρήσας δίχα διελέσθαι γένος διαιροῖ καθάπερ οἱ πολλοὶ τῶν ἐνθάδε διανέμουσι, τὸ μὲν Ἑλληνικὸν ὡς ἕν ἀπὸ πάντων ἀφαιροῦντες χωρίς, σύμπασι δὲ τοῖς ἄλλοις γένεσιν, ἀπείροις οῦσι καὶ ἀμείκτοις καὶ ἀσυμφώνοις πρὸς ἄλληλα, βάρβαρον μιῷ κλήσει προσειπόντες αὐτὸ διὰ ταύτην τὴν μίαν κλῆσιν καὶ γένος ἐν αὐτὸ εἶναι προσδοκῶσιν· ἢ τὸν ἀριθμόν τις αῦ νομίζοι κατ' εἴδη δία ἰαριφώνοις πρὸς ἄλληλα, βάρβαρον μιῷ κλήσει προσειπόντες αὐτὸ διὰ ταύτην τὴν μίαν κλῆσιν καὶ γένος ἐν αὐτὸ εἶναι προσδοκῶσιν· ἢ τὸν ἀριθμόν τις αῦ νομίζοι κατ' εἴδη δία ἰαριφώριος ἀπὸ πάντων, ὡς ἐν εἴδος ἀποχωρίζων, καὶ τῷ λοιπῷ δὴ παντὶ θέμενος ἐν οῦο διαιρεῖν μυριάδα ἀποτεμνόμενος ἀπὸ πάντων, ὡς ἐν εἴδος ἀποχωρίζων, καὶ τῷ λοιπῷ δὴ παντὶ θέμενος ἐν οῦνομα διὰ τὴν κλῆσιν αῦ καὶ τοῦτ' ἀξιοῖ γένος ἐκείνου χωρὶς ἕτερον ἐν γίγνεσθαι. κάλλιον δέ που καὶ μᾶλλον κατ' εἴδη καὶ δίχα διαιροῖτ' ἄν, εἰ τὸν μὲν ἀριθμὸν ἀρτίῳ καὶ περιττῷ τις τέμνοι, τὸ δὲ αῦ τῶν ἀνθρώπων γένος ἄρρενι καὶ θήλει, Λυδοὺς δὲ ἢ Φρύγας ἤ τινας ἑτέρους πρὸς ἄπαντας τάττων ἀποσχίζοι τότε, ἡνίκα ἀποροῖ γένος ἄμα καὶ μέρος εὑρίσκειν ἑκάτερον τῶν σχισθέντων. …

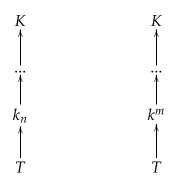
[{]ΞΕ.} οἶμαι μὲν γὰρ μάλιστα, ὅθεν ἐρωτηθεὶς σὺ τὴν ἀγελαιοτροφίαν ὅπη διαιρετέον εἶπες μάλα προθύμως δύ΄ εἶναι ζώων γένη, τὸ μὲν ἀνθρώπινον, ἕτερον δὲ τῶν ἄλλων συμπάντων θηρίων ἕν. ...

[{]ΞΕ.} Καὶ ἔμοιγε δὴ τότ' ἐφάνης μέρος ἀφαιρῶν ἡγεῖσθαι καταλιπεῖν τὸ λοιπὸν αῦ πάντων γένος ἕν, ὅτι πᾶσι ταὐτὸν ἐπονομάζειν ἔσχες ὄνομα, θηρία καλέσας. πειραθῶμεν οῦν ἡμεῖς ἐξευλαβεῖσθαι πάνθ' ὁπόσα τοιαῦτα. ...

[{]ΞΕ.} Τὸ δέ γε, ὥ πάντων ἀνδρειότατε, τάχ' ἄν, εἴ που φρόνιμόν ἐστί τι ζῷον ἕτερον, οἶον δοχεῖ τὸ τῶν γεράνων, ἤ τι τοιοῦτον ἄλλο, ὃ χατὰ ταὐτὰ ἴσως διονομάζει χαθάπερ χαὶ σύ, γεράνους μὲν ἓν γένος ἀντιτιθὲν τοῖς ἄλλοις ζώοις χαὶ σεμνῦνον αὐτὸ ἑαυτό, τὰ δὲ ἄλλα μετὰ τῶν ἀνθρώπων συλλαβὸν εἰς ταὐτὸ

Young Socrates's methodology was problematic, it seems, because it proceeded *too quickly* to humans. He got away with it because he knew antecedently that statesmanship would not be a rearing of any other kind of creature. But in doing so, he grouped together all of the non-human animals as "beasts", when these have as little in common as a whole group as "barbarians". Such a group he calls a "part" instead of a "form", with the implication that the latter, but not the former, should be placed in accounts of the essence.

Helpfully, the Visitor also gives some guidance for hitting on forms rather than parts. He advises Socrates to "divide through intermediate stages"—which seems to be a way to more precisely capture the idea that one should go slowly, since the more intermediate stages one has to go through, the longer it will take to get to the end. This would be formalized by saying, of these two divisions:



one should prefer the former to the latter if n > m. Interestingly, Plato's Socrates endorses the same norm in the *Philebus*:

While the gods, just like I said, handed down to us this way of inquiring, learning, and teaching each other, the contemporary wise guys **make a One in a chance way and a Many faster and slower than they ought.** After the one they go directly to the unlimited and **the intermediates escape them**, which are what determines whether we make our accounts for one another dialectically or eristically. (*Philebus* 16e-17a, emphases mine)⁹

Here again is the point that hitting the intermediate stages makes one's divisions dialectical rather than eristic.¹⁰

ούδεν άλλο πλην ίσως θηρία προσείποι. πειραθῶμεν οὖν ήμεῖς ἐξευλαβεῖσθαι πάνθ' ὑπόσα τοιαῦτα.

[{]ΞΕ.} Μὴ τοίνυν διαιρώμεθα ὥσπερ τότε πρὸς ἄπαντα ἀποβλέψαντες, μηδὲ σπεύσαντες, ἵνα δὴ ταχὺ γενώμεθα πρὸς τῆ πολιτικῆ.

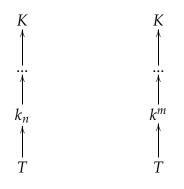
⁹οί μὲν οὖν θεοί, ὅπερ εἶπον, οὕτως ἡμῖν παρέδοσαν σχοπεῖν καὶ μανθάνειν καὶ διδάσκειν ἀλλήλους· οἱ δὲ νῦν τῶν ἀνθρώπων σοφοὶ ἐν μέν, ὅπως ἂν τύχωσι, καὶ πολλὰ θᾶττον καὶ βραδύτερον ποιοῦσι τοῦ δέοντος, μετὰ δὲ τὸ ἐν ἄπειρα εὐθύς, τὰ δὲ μέσα αὐτοὺς ἐκφεύγει – οἶς διακεχώρισται τό τε διαλεκτικῶς πάλιν καὶ τὸ ἐριστικῶς ἡμᾶς ποιεῖσθαι πρὸς ἀλλήλους τοὺς λόγους.

¹⁰It might be objected that making the accounts is what is either dialectical or eristic, not the division. Even if this is so, we want to make the kinds of division that lead to dialectical accounts and so the difference in accounts leads to a difference in the kinds of division as well. On the other

On my reconstruction, the emphasis in both of these passages is exclusively on going slowly. This seems in tension with the way that it is described in the *Philebus* passage. Why isn't the norm instead to divide *at the correct* speed?¹¹ To respond, I want to both show that this is not how Plato understood the norm and that it is better that he did not.

First, the interpretive issue. In both passages above, Plato formulates the same norm in two ways. One formulation is in terms of speed, where he says both not to hurry (in the *Statesman*) and not to go too fast or slow (in the *Philebus*). The other way is in terms of "intermediate stages". When he puts it in this way, the idea is that the mistake was to *skip steps*. The Visitor treats this as equivalent to the other formulation, but this is only so if the only kind of "speed" mistake that really matters is going too fast. You can't skip any steps by going too slow! Otherwise, in addition to the problem of skipping steps, a divider could also err by including *too many* intermediate stages.

Moreover, the mistakes that Plato diagnoses with this norm are always of the "too fast" variety. Clearly this is the case with Young Socrates, but it is equally true in the *Philebus*, where the contemporary wise guys err by omitting the intermediates and, in Socrates's words, going "directly" to the unlimited from the one. So it seems clear that Plato understands the norm in the way that I have suggested: of two divisions:



one should prefer the former to the latter if n > m.

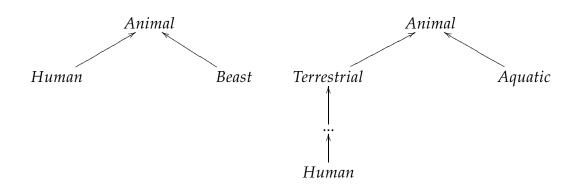
But was Plato *right* to formulate the norm this way?¹² I believe that Plato had good reason to hold the view that he does.

First, it can be justified as a necessary means of reliably reaching the goal of division. Consider a slightly adapted version of the case in the *Statesman*:

hand, it is plausible that a dialectician making a division *is itself* a giving of an account in a broad sense, so the question may be ill-posed just because Plato is not here distinguishing the division and the account that is produced. Thanks to Sean Kelsey for pushing me on this point.

¹¹This objection could also be made with respect to the *Statesman* by bringing in the discussion of giving "due measure" in one's accounts. (283d-e)

¹²Thanks to John Wynne for pressing me on this issue.

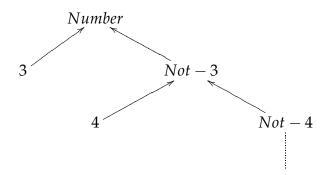


With the latter division we have made much more progress in understanding what humans are, especially since our only grip on "Beast" is as non-human animal. In this way, going too fast leads to the possibility of missing common elements of the essence of humanity. This brings me to a more formal justification, in the same vein as soundness proofs. This sort of argument is, I think, suggested, but not supplied by the passage quoted above, where the Visitor says that going slower is safer.

Argument from Goal to Tortoise: Suppose that you violate Tortoise. Then there could be a kind more general than the kind that you divided into that was passed over. However, this will not be part of the account, as you will already be dealing with smaller classes. Thus you will not meet your goal of having a complete account of the essence.

Second, purported cases of going *too* slow are really violations of other norms. There are two kinds worries about going too slow, as far as I can tell.

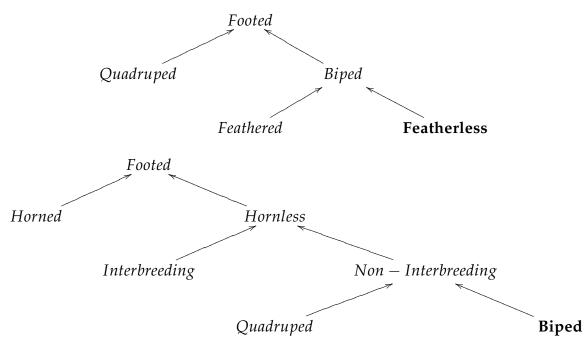
The first objection just comes from looking at the consequences of dividing slowly. Suppose one wants to define the number 2. One could try to divide:



Something like this seems to be the slowest way to get to 2, but this does not give us a complete account, since it will be infinitely long! But this is only a slow division with respect to *one* kind. While in the context of the *Sophist* and *Statesman*, we are primarily concerned with defining a single kind, if the norm is expected to be applied to all kinds simultaneously, we see that this is in fact a very fast division to 3. This doesn't suggest a problem with the norm itself, but does show that a divider will, in general, have to balance the need to go slow to one kind with the need to

move slowly to another. This doesn't undermine the utility of the norm, since some divisions go too quickly with respect to *all* kinds (e.g., dividing *Number* directly into all the individual numbers) and the fact that it cannot uniquely determine the correct division is not a problem so long as it is not the only norm.

The second objection is that shorter divisions are better. Consider what happens just after Young Socrates makes his mistake. The Eleatic Visitor goes on to give two divisions, one longer and one shorter, to illustrate the mistake that Young Socrates made. They are (beginning from the place they diverge):



The worry, expressed by Gill (2012), is that *both* of these err in being too long and Young Socrates was right in emphasizing the fundamental difference between the projects of rearing human herds and rearing herds of other animals. The Visitor is suggesting that the relevant point of difference between the kinds of rearing is biological, whereas it is really psychological. While this is a possible objection to the division (and a recommendation for something like Porphyry's Tree), I see no suggestion of this view in Plato or even Aristotle when discussing division of animals. For both of them, e.g., in the *Timaeus* and in *Parts of Animals* I, humans are correctly classed as a variety of terrestrial animal more similar to pigs and cows than to birds and fish. Indeed, in the *Statesman* itself, one of the primary duties of the statesman is to direct the breeding of humans, so the fact that they are not interbreeding could turn out to be quite important. (310b ff) Only in the longest route is the kind of breeding that humans do mentioned, so it seems to me that this feature of the herds is actually very important to the kind of rearing one is to do.

Even if Gill is right and Plato *did* want us to pick Young Socrates's division in the end, his reason should not have been because the other was too slow. It would be because it made some other sort of error. To return to Aristotle, the longer routes

could have erred either because it includes subdivision of privations like *Hornless* (*PA* I.3), or because it does not divide by a difference of a difference (*Met.* Z.12), since being horned is not a way of being footed and having feathers is not a way of being biped. These, I think, are better explanations of what could possibly be going wrong in the Visitor's divisions than just the fact that they go too slowly.

Finally, the norm of not going too slow is superior to the other candidate norm: go at the right speed. The real problem with this norm is that it is of no help to an inquirer actually doing a division. When presented with two divisions, one shorter than the other, which should the inquirer choose? It is of no help to say: the one that is the right length. That is to say, the other candidate interpretation gives us a norm that is not action guiding at all. On the other hand, the "go slow" norm *is* action guiding, since it is actually rules out some divisions in favor of others. It may not uniquely identify the right division. Going slow down one path has to be balanced with going slow down another, but at least it rules out some divisions in favor of others.

Recapping: I have defended the Tortoise norm, both as an interpretation and as a philosophically defensible position. It is justifiable as a necessary condition on reliably achieving the goal of division.

3.2.2 The Minimization Norm

The Eleatic Visitor is not content with this one norm. At the next crucial juncture in the dialogue, he suggests that they must try something new. They reached an account of statesmanship as knowledge of rearing a certain sort of herd animals (humans) by dichotomous division, but this account fails to distinguishing statesmanship from generals, doctors, and other caretakers of human herds. At this point, the Visitor says he must stop using dichotomous division:

Visitor: Well then, the king has been separated off from the many sorts of expertise that share his field—or rather from all of them concerned with herds; there remain, we are saying, those sorts of expertise in the city itself that are contributory causes and those that are causes, which we must first divide from each other....

Visitor: So do you recognize that it is difficult to cut them into two? The cause, I think, will become more evident if we proceed....

Visitor: Then let's divide them limb by limb, like a sacrificial animal, since we can't do it into two. For we must always cut into the nearest number so far as we can. $(Statesman 287b-c)^{13}$

¹³{ΞΕ.} Οὐκοῦν ἀπό γε τῶν πολλῶν ὁ βασιλεὺς ὅσαι σύννομοι, μᾶλλον δὲ ἀπὸ πασῶν τῶν περὶ τὰς ἀγέλας διακεχώρισται· λοιπαὶ δέ, φαμέν, αἱ κατὰ πόλιν αὐτὴν τῶν τε συναιτίων καὶ τῶν αἰτίων, ἂς πρώτας ἀπ' ἀλλήλων διαιρετέον. ...

[{]ΞΕ.} Οἶσθ' οὖν ὅτι χαλεπὸν αὐτὰς τεμεῖν δίχα; τὸ δ' αἴτιον, ὡς οἶμαι, προϊοῦσιν οὐχ ἦττον ἔσται καταφανές. ...

The last sentence of this passage suggests that, although he does not think that all good divisions need to be dichotomous, he stills holds on to a weaker norm that, if one is presented with two divisions of some kind *K*:



the former should be preferred to the later if n < m. Like the Tortoise norm, there is also a passage in the *Philebus* expressing the same norm:

Since this is the structure of things, we have to assume that there is in each case always one form for every one of them, and we must search for it, as we will indeed find it there. And once we have grasped it, **we must look for two, as the case would have it, or if not, for three or some other number**. And we must treat every one of those further unities in the same way, until it is not only established of the original unit that it is one, many and unlimited, but also how many kinds it is. (*Philebus* 16d)¹⁴

Just like the Tortoise norm, the Minimization norm can be justified as a means of reliably obtaining the goal of division. To see how this is an effective norm, let's look at an example. Suppose that, instead of dividing Knowledge into Practical and Theoretical (258e), one divided it into Practical, Directive, and Judgment-Making (260a-b). It seems like this latter division misses something which is captured by the former. If you divide into small numbers, you will not split up a kind so as to miss something common between a set of its subkinds. Although the Visitor does not explicitly assert that this norm is subject to the same kind of justification as the Tortoise norm, it is easy to see how such a justification would go:

Argument from Goal to Minimization: Suppose that you violate Minimization. Then there could be a kind which belongs to the essence of the target kind and is the genus of several of the kinds divided into. However, this will not be included, as you divided directly into its subkinds. Thus you will not meet your goal of having a complete account.¹⁵

In both of these passages, dichotomy emerges as a special case: it is the smallest possible number into which one can divide. So, according to this norm, it should be

[{]ΞΕ.} Κατὰ μέλη τοίνυν αὐτὰς οἶον ἱερεῖον διαιρώμεθα, ἐπειδὴ δίχα ἀδυνατοῦμεν. δεῖ γὰρ εἰς τὸν ἐγγύτατα ὅτι μάλιστα τέμνειν ἀριθμὸν ἀεί.

¹⁴δεῖν οὖν ἡμᾶς τούτων οὕτω διακεκοσμημένων ἀεὶ μίαν ἰδέαν περὶ παντὸς ἑκάστοτε θεμένους ζητεῖν – εὑρήσειν γὰρ ἐνοῦσαν – ἐἀν οὖν μεταλάβωμεν, μετὰ μίαν δύο, εἴ πως εἰσί, σκοπεῖν, εἰ δὲ μή, τρεῖς ἤ τινα ἄλλον ἀριθμόν, καὶ τῶν ἐν ἐκείνων ἔκαστον πάλιν ὡσαύτως, μέχριπερ ἂν τὸ κατ' ἀρχὰς ἕν μὴ ὅτι ἕν καὶ πολλὰ καὶ ἄπειρά ἐστι μόνον ἴδη τις, ἀλλὰ καὶ ὁπόσα·

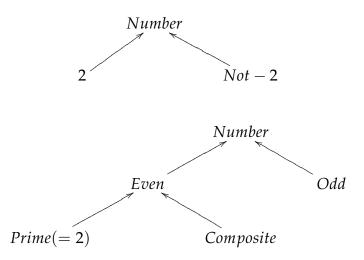
¹⁵Aristotle gives a different justification for the same norm in *Topics* 109b13-29. In this passage, he argues that arguments based on such divisions proceed more quickly, since they deal with a small number of large groups instead of a large number of small groups.

preferred whenever available. Evidently, Plato does not think it is *always* available, since otherwise it would be vacuous and, in any case, he goes on to divide into many more right after articulating this norm.¹⁶

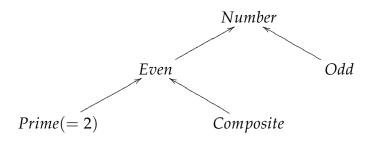
One should always first try a dichotomous division and then expand if that fails. Indeed, this is frequently what happens in Platonic divisions and explains why so many are dichotomous. For example, in the *Timaeus*, the speech begins with a division between forms and sensibles (27d), but later a third kind is added, the receptacle (48e). Similarly in the *Sophist* with the separative art (226b-c) and the *Philebus* with the fourfold division of being (23c ff).

How does this norm relate to the Tortoise norm? First, I will show how neither norm implies the other locally, so that they give genuinely different advice to the inquirer. Then I will argue that they are also compatible norms.

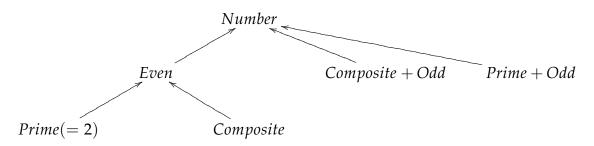
Take 2 to be the target kind and Number to be the starting point.



Both of these divisions use only dichotomous division and hence are equal by the lights of the Minimization norm, but the latter is preferable to the former according to the Tortoise norm. So the Tortoise norm does not imply the Minimization norm. Now consider:



¹⁶This strongly suggests that Plato is not among the "dichotomizers" of Parts of Animals I.



The two divisions get to 2 with the same number of intermediates, hence are equal by the lights of the Tortoise norm, but the former is preferable to the former according to the Minimization norm. So the Minimization norm and the Tortoise norm are independent.

The difference between them is apparent *locally*, with respect to a single target kind, but if we ask whether they are compatible, we should look at them *globally*, that is, as entire classifications of a genus. How do we measure whether one division is *globally* slower, or *globally* divided into fewer? For now, let's hold the genus and the infimae species fixed. At least in this case, it is relatively straightforward to measure them. A division is globally slower when it has more kinds that are neither the genus nor the infimae species. A division is globally divided into fewer subkinds, one takes proportionally more divisions to classify the same number of infimae species.

If we accept these kinds of global measurement, there is a surprising result. Again, holding fixed the genus and infimae species, a division *A* is globally slower than *B* if and only if *A* is globally divided into fewer than *B*. This is because the number of intermediates is just one less than the number of vertices. Unwittingly, we have found that, not only are the two norms compatible, but, at least in this circumstance, come to the same thing.

This might make it seem as if there is no difference between them, but we saw above that from the local perspective, which is the perspective that matters to the divider, they differ significantly.

3.3 Conclusion: Plato's Program

Plato articulated two norms of division in the *Statesman* and briefly alluded each in the *Philebus*. These norms have an abstract, formal characteristic and are intended to provide guidance in any sort of inquiry one might engage in. They are, more-over, the only norms that Plato (or any of his characters) ever explicitly endorses.¹⁷

¹⁷Some might argue that in the *Phaedrus* 263 there is a norm as well, namely, to divide "according to the natural joints". However, while this is a norm in a sense, given the goal of division, it really seems to boil down to the claim that one should divide in the objectively correct way. Even if this is not a triviality, it is of little help to someone who is ignorant of the relevant facts and using division to discover them. In fact, context strongly suggests that this is *not* meant to be particularly helpful, since Socrates professes a certain amount of ignorance concerning this technical field and admiration for those who can accomplishment successfully. Rachel Barney (personal communication) has

However, it is clear that these norms do not, by themselves, pin down a unique correct answer in every investigation. What are we to make of this?

As far as I can see, there are two possible responses. One is taken by Henry (2011) and Moravscik (2004). On this view, the formal norms give way at the end of the day to insight. We shouldn't take the Tortoise norm or Minimization norm to be rules but more like training wheels for young dialecticians like Theaetetus and Young Socrates. The Visitor uses these norms to help them gain the ability to spot the right divisions intuitively, without the use of rules.

The other option is suggested by Galen:

And whereas both Aristotle and Plato thought it such a great and difficult a task to divide genera into their appropriate differentiae, and although after them Theophrastus and the other philosophers tried to work out the method because it had not yet been correctly done even in their schools... (*Method of Medicine* 26K)¹⁸

On Galen's view, it seems that Plato *began* the project of elaborating norms of division, but did not complete that project. According to Galen, this was a project common to Aristotle, Theophrastus, and even the Stoics.

In favor of Henry's proposal, one might say first that certain kinds of "substantive" mistakes occur in the Platonic dialogues that could not be plausibly fixed by adding more formal norms. How could more rules have helped the Visitor and Theaetetus place sophistry under production instead of acquisition? In the *Statesman*, the Visitor uses observations from the Myth of Cronus to correct earlier errors in his division, but it is unclear how more rules would have helped him. Secondly, Plato gives no hint about the existence of any *further* norms.

I believe that Galen's view can respond to these challenge. The defense I will present decisively shows this, but in general, presents a Plato with a more interesting and provocative view. So, since I think that the text only weakly favors Galen's position, charity considerations strongly support it.

In reply to the first objection, it is not clear to me that Plato distinguishes between substantive and formal error, just as Robinson (1953) showed that Plato (in the Socratic dialogues) did not distinguish between different kinds of *elenchus*. Without that distinction (one that, incidentally, Aristotle made in the *Analytics*), it is hard to know how Plato could have ever arrived at the conclusion that some errors could not be avoided by any norm at all.

suggested that the *Phaedrus* norm might be thought of as a kind of meta-norm. This would mean that, although it does not itself give any useful instruction to the divider, it does help us sort out what the norms themselves *are*. This would be in line with the project of justifying the norms that I pursue in this chapter.

¹⁸εἴτ[°] Ἀριστοτέλους τε καὶ Πλάτωνος οὕτω μέγα καὶ χαλεπὸν εἶναι νομιζόντων εἰς τὰς οἰκείας διαφορὰς ἀκριβῶς τὰ γένη τέμνειν καὶ μετ[°] αὐτοὺς Θεοφράστου τε καὶ τῶν ἄλλων φιλοσόφων ἐξεργάζεσθαι πειρωμένων τὸν τρόπον, ὡς οὕπω κατωρθωμένον οὐδὲ παρ[°] ἐκείνοις... Text is from Johnston and Horsley (2011). Thanks to David Sedley for help with the translation.

The most popular way to maintain this objection is to argue that Plato invoked another method to correct the substantive errors. In the *Statesman*, the Visitor invokes the notion of models to help Young Socrates understand how the statesman stands to the other crafts that help to maintain the city. This leads to the final steps of the definition of the statesman. Several recent commentators have suggested that the use of models is indispensible for rigorous definition by division.¹⁹ If Plato introduced models to play this methodological role, then it seems like he recognized that norms of division are not enough to correct for substantive errors and that another method was needed.

However, it seems to me hard to reconcile this with the idealized descriptions of dialectic in any of the late dialogues, such as *Sophist* 253d ff and *Philebus* 57e ff. In all of these, collection and division figure prominently, but there is no hint at all of the need of anything resembling models. Instead, it seems that models (and not collection and division) are the pedagogical tools. Like the correction to the division that was illustrated by myth of Cronus earlier in the dialogue (which was explicitly introduced as a bit of play), they help Young Socrates see a point that could have been made without the myth. For instance, it helps us to see the importance and interest of limb by limb division to use the model of the weaver, but the model seems to me to be much more dispensable than the division.

In reply to the second objection, it seems like there is an indirect suggestion of the need for more norms. This comes from two observations. 1) The norms in the *Statesman* are not enough to solve the problem of the *Sophist* (as the objection said): the divisions are both very slow and almost always into two. 2) The Eleatic Visitor suggests that a dialogue *Philosopher* will follow the *Statesman*. It doesn't seem to me to be a stretch to think that this dialogue would contain the complete description of the skill of the philosopher: dialectic. It would be very misleading for Plato to suggest that there is such a dialogue but not think that it contained something new about division, the most prominent dialectical method in the *Sophist* and *Statesman*.

The main positive reason for rejecting Henry's view, however, is that it does violence to Plato's conception of dialectic as the supreme craft or science. Craft and science, according to Plato, are able to *say* what they are doing. When Plato talks about the relationship between dialectic and the other crafts and sciences, he consistently claims that it is more precise, its accounts more stable, even that it has more of a claim to be called knowledge.

On the Henry interpretation, however, I fail to see how this is true. Philosophy turns out to look much more like rhetoric in the *Gorgias*, a kind of experience that is fundamentally *stochastic*, but which possesses no account of what it does. The kind of training that Theaetetus and Young Socrates are receiving does not seem to me to give them anything more. Unless some magical change happens, they will, at best, become only more accurate guessers about what the right divisions are. But they will have no clue how they can do it, since that would amount to precisely the kind of general specification that would eliminate the need for insight.

¹⁹In different ways, see Gill (2012); Rickless (2010).

Going with Galen, however, we can make sense of this. There is a complete story to be told, one that would be brilliant in its precision. But the dialogues have made only the most preliminary progress.

After Plato, accordingly, we see significant refinement of the method of division, as well as reflections on its power and limitations. The first, and in many ways most important reflections on division come in Aristotle's *Analytics*.

Chapter 4

The (Ab)use of Division in the *Analytics*

In Chapters 2-3, I argued that Plato considered the method of division as a tool to discover essences. Not every division can discover the essence of a target kind, but if one adheres to certain norms, one will more reliably hit on the essence.

It is generally thought that Aristotle *rejects* the Platonic picture on account of two polemical chapters in the *Analytics*: *Prior Analytics* I.31 and *Posterior Analytics* II.5. At the same time, in *Posterior Analytics* II.13, Aristotle seems to assign some importance to it. Strikingly, through a series of back references, these seem intended to be understood as a single, sustained inquiry into the method of division. How are we to reconcile this tension?

Most scholars ease the tension by interpreting Aristotle as giving division a very different purpose than the one that Plato assigned to it.¹ It is not useful as a method for discovering essences, but could have some other point. Cherniss (1944) and Ross (1949) take division to have an entirely dispensable, merely heuristic role. Goldin (1996) takes division to play a merely "pedagogical role" (p. 88). According Detel (1998) to says that it is not a method, but gives *criteria of adequacy* for a good definition. Bronstein (2016), by contrast, has argued that Aristotle, like Plato, considered division to be indispensable in the discovery of essences by discovering the differentiae in a definition and ordering the elements in the definition.

Through a close analysis of all of the relevant texts, I will argue that Bronstein is mostly correct. Aristotle thought the Platonic method could be used for discovering essences. However, Aristotle is not committed to it being the only method and he only thinks that division can do part of what Plato and Bronstein's Aristotle take it to.

In what follows, I will argue first examine the critical passages (*APr* I.31, *APo* II.5), arguing primarily against the first camp. In these texts, Aristotle does not reject division *as such*, but only as a syllogism or a demonstration of the essence.

¹Zabarella (1578), by assigning division to order as opposed to method (*De Methodiis* III) could also be read in this way.

Then I will argue that in *APo* II.13, Aristotle allows a useful role for division in the hunt for essences of some kinds. However, he is much clearer here than Plato ever was about the limitations of division and what knowledge must be presupposed by the one dividing. So while Aristotle does allow for an important role for division, he thinks that it can achieve comparatively less than what his predecessors thought.

4.1 *APr* I.31: Division does not syllogize

The first discussion of division occurs as part of a coda with I.30. In I.30, Aristotle makes some remarks concerning the scope of the "route" (*hodos*) that he had just discussed, emphasizing its generality and use in science, dialectic, or any art whatsoever.

Following this discussion, Aristotle claims that division of genera (*diairesis dia ton genon*) is only a small part of the method he had just discussed:²

It is easy to see that division of genera is a small part of the aforementioned method. For division is a kind of weak syllogism. For, on the one hand, it asks for what it ought to show and on the other, it deduces something higher up.

And at first this eluded all those using division and they tried to persuade $\langle us \rangle$ in what way it is possible for a demonstration concerning essence and the what it is to come about, with the result that they neither understood what particular thing³ it is possible that those dividing⁴ deduce, nor that it was possible only as we said.

In demonstrations, on the one hand, whenever it should be necessary to deduce that something belongs, it is necessary that the middle term through which the syllogism comes about is always less than and not a universal of the first of the extremes.⁵ But division wants the opposite. For it takes the universal as a middle term.

²There has been controversy since antiquity concerning the scope of the back reference. Is it only to 27-30 or the entire book up to that point? See Alexander *In APr* 333, 10-20 Wallies (1883), where he gives reasons for each possible interpretation without explicitly settling on one. Crubellier (ms); Duncombe (ms); Ferejohn (1991); Wiener (2015) all take him to be restricting himself to the *ars inveniendi* of 27-9. Alexander, however, seems to make the decisive point in favor of the broader interpretation that Aristotle's claim that division is a weak syllogism is irrelevant for whether division can provide premises for deduction or demonstration. Moreover, Aristotle makes more use of results from I.23 and 26 than anything in 27-9 for his argument that division cannot syllogize its goal. Finally, the back reference to I.31 in *APo* II.5 says that it is " ϵ_{V} $\tau_{\tilde{\eta}}$ $\dot{\alpha}v\alpha\lambda$ úσει $\tau_{\tilde{\eta}}$ $\pi\epsilon\rho$ $\tau\dot{\alpha}$ $\sigma_{X}\eta\mu\alpha\tau\alpha''$ which could not plausibly be restricted to 27-31.

³Reading ő τι instead of őτι.

 $^{^{4}}$ Reading διαιρουμένους with Waitz over διαιρουμένοις (Ross) διαιρουμένοι (Cherniss).

⁵This is shown in APr I.26.

For: let A be on animal, B on mortal, Γ on immortal, and Δ on man, whose account it is necessary to understand. Then it assumes all animal is either mortal or immortal: All this (what would be A) is either B or Γ . Again the one dividing always places man to be an animal, so that he assumes A belongs to Δ . On the one hand, there is a syllogism that all Δ will be either B or Γ , so that necessarily man is mortal or immortal, but it is not necessary that it is a mortal animal, but asked. But this was what was necessary to deduce.

And again, having set A to be mortal animal, B on footed, Γ on footless and Δ on man, in the same way he takes A to be either in B or in Γ (for every animal is either mortal or immortal), and A of Δ (for he assumes man is a mortal animal). It results, on the one hand, that necessarily man is a footed or footless animal, but it is not necessary that it is footed, but assumed. And this, *again*, is what it was necessary to show.

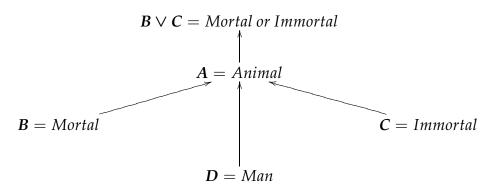
And in this manner it always happens for the one dividing to take the universal to be the middle term of what it is necessary to show and the differences to be the extreme. And the goal, that this is man (or whatever should be the object of inquiry) they do not say anything clear so that it is necessary. For they make the make the other route [=division] everything, not realizing that there are possible solutions. $(46a31-46b25)^6$

Aristotle makes two claims against division: 1) that "it asks for what it ought to be proving and always deduces something higher up"(46a33-4) and 2) that "they tried to convince us that it is possible for a demonstration concerning essence, or what something is, to come about." (46a35-7) This passage only argues for the first

⁶Ότι δ' ή διὰ τῶν γενῶν διαίρεσις μιχρόν τι μόριόν ἐστι τῆς εἰρημένης μεθόδου, ῥάδιον ἰδεῖν· ἔστι γὰρ ή διαίρεσις οἶον ἀσθενής συλλογισμός· ὃ μὲν γὰρ δεῖ δεῖξαι αἰτεῖται, συλλογίζεται δ' ἀεί τι τῶν ἄνωθεν. πρῶτον δ' αὐτὸ τοῦτο ἐλελήθει τοὺς χρωμένους αὐτῆ πάντας, καὶ πείθειν ἐπεχείρουν ὡς ὄντος δυνατοῦ περί οὐσίας ἀπόδειξιν γενέσθαι καὶ τοῦ τί ἐστιν. ὥστ' οὕτε ὅ τι ἐνδέχεται συλλογίσασθαι διαιρουμένους ξυνίεσαν, οὔτε ὄτι οὕτως ἑνεδέχετο ὥσπερ εἰρήχαμεν. ἐν μὲν οῦν ταῖς ἀποδείξεσιν, ὅταν δέῃ τι συλλογίσασθαι ὑπάρχειν, δεῖ τὸ μέσον, δι΄ οῦ γίνεται ὁ συλλογισμός, καὶ ἦττον ἀεὶ εἶναι καὶ μὴ καθόλου τοῦ πρώτου τῶν ἄχρων ἡ δὲ διαίρεσις τοὐναντίον βούλεται· τὸ γὰρ χαθόλου λαμβάνει μέσον. ἔστω γὰρ ζῷον ἐφ' οὖ Α, τὸ δὲ ϑνητὸν ἐφ' οῦ Β, καὶ ἀϑάνατον ἐφ' οῦ Γ, ὁ δ' ἄνϑρωπος, οῦ τὸν λόγον δεῖ λαβεῖν, ἐφ' οῦ τὸ Δ. ἄπαν δὴ ζῷον λαμβάνει ἢ ϑνητὸν ἢ ἀϑάνατον· τοῦτο δ΄ ἐστίν, ὃ ἂν ἢ Α, ἄπαν εἶναι ἢ Β ἢ Γ. πάλιν τὸν ἄνϑρωπον ἀεὶ διαιρούμενος τίθεται ζῷον εἶναι, ὥστε κατὰ τοῦ Δ τὸ ${
m A}$ λαμβάνει ὑπάρχειν. ὁ μὲν οὕν συλλογισμός έστιν ὅτι τὸ Δ ἢ B ἢ Γ ἄπαν ἔσται, ὥστε τὸν ἄνθρωπον ἢ θνητὸν μὲν ἢ ἀθάνατον ἀναγχαῖον εἶναι, ζῷον θνητὸν δὲ οὐχ ἀναγχαῖον, ἀλλ΄ αἰτεῖται· τοῦτο δ΄ ἦν ὃ ἔδει συλλογίσασθαι. χαὶ πάλιν θέμενος τὸ μὲν Α ζῷον θνητόν, ἐφ' οὕ δὲ τὸ Β ὑπόπουν, ἐφ' οῦ δὲ τὸ Γ ἄπουν, τὸν δ' ἄνθρωπον τὸ Δ , ὡσαύτως λαμβάνει τὸ μὲν Α ἦτοι ἐν τῷ Β ἢ ἐν τῷ Γ εἶναι (ἄπαν γὰρ ζῷον ϑνητὸν ἢ ὑπόπουν ἢ ἄπουν ἐστί), κατὰ δὲ τοῦ Δ τὸ Α (τὸν γὰρ ἄνθρωπον ζῷον θνητὸν εἶναι ἔλαβεν)· ὥσθ' ὑπόπουν μὲν ἢ ἄπουν εἶναι ζῷον ἀνάγκη τὸν άνθρωπον, ὑπόπουν δ' οὐχ ἀνάγχη, ἀλλὰ λαμβάνει· τοῦτο δ' ῆν ὃ ἔδει πάλιν δεῖξαι. χαὶ τοῦτον δὴ τὸν τρόπον ἀεὶ διαιρουμένοις τὸ μὲν καθόλου συμβαίνει αὐτοῖς μέσον λαμβάνειν, καθ' οὕ δ' ἔδει δεῖξαι καὶ τὰς διαφορὰς ἄχρα. τέλος δέ, ὅτι τοῦτ' ἔστιν ἄνθρωπος ἢ ὅ τι ποτ' ἂν ἢ τὸ ζητούμενον, οὐδὲν λέγουσι σαφές ὥστ' ἀναγχαῖον εἶναι· καὶ γὰρ τὴν ἄλλην ὁδὸν ποιοῦνται πᾶσαν, οὐδὲ τὰς ἐνδεχομένας εὐπορίας ὑπολαμβάνοντες ὑπάρχειν.

of these claims, which removes the Platonist motivation for thinking that demonstration of the essence is possible.⁷

In this chapter, I will not be asking whether the comparison with the Platonist is *fair*—this will be discussed at length in Chapter 5. Instead, I will consider what his criticisms show about his attitude towards division. The argument for the claim that division asks for what it ought to be proving proceeds by means of an example. Here are what Aristotle takes to be the *assumptions* of a typical case of division:



In words:

- 1. All Animal is Mortal or Immortal.
- 2. All Mortal is Animal.⁸
- 3. All Immortal is Animal.
- 4. All Man is Animal.

This seems to be plausible reconstruction on Aristotle's part of how Plato normally conducts divisions, for instance in the *Sophist*:

Visitor: But expertise as a whole falls pretty much into two types.

Theaetetus: How?

V: There's farming, or any sort of caring for any mortal body; and there's also caring for things that are put together or fabricated, which we call equipment; and there's imitation. The right thing would be to call all those things by a single name.

⁷One could take the $\pi\rho\omega\tau\sigma\nu$ to take scope up to 37 and the second point to be introduced by the $\omega\sigma\tau\epsilon$ While I don't think it makes much of a difference for the structure of the passage as a whole, it seems less probable to me syntactically, since the xal following $\pi\rho\omega\tau\sigma\nu$ naturally introduces the second idea.

⁸This seems to be an implicit assumption that one might think is part of the claim that all animal is mortal or immortal. I think this is plausibly based on the formulas used for introducing divisions in both Plato and Aristotle, especially *men/de* clauses and part/whole language. When someone says "tou A to men B, to de C", they are committed to there being some B which is A.

T: How? What name?

V: When you bring anything into being that wasn't in being before, we say you're a producer and that the thing you've brought into being is produced.

T: That's right.

V: And all the things we went through just now have their own capacity for that.

T: Yes.

V: Let's put them under the heading of production.

T: All right.

V: Next, consider the whole type that has to do with learning, recognition, commerce, combat, and hunting. None of these creates anything. They take things that are or have come into being, and they take possession of some of them with words and actions, and they keep other things from being taken possession of. For that reason it would be appropriate to call all the parts of this type acquisition. T: Yes, that would be appropriate.

V: If every expertise falls under acquisition or production, Theaetetus, which one shall we put angling in?

T: Acquisition, obviously. (Sophist 219a-d)

Now, we want to conclude that all man is mortal. If we were to try to get a syllogism of the goal out of the assumptions at our disposal, it would have to have the terms Animal, Mortal, and Man and to make a second figure syllogism of the form:

- 1. All Man is Animal.
- 2. All Mortal is Animal.
- 3. So, all Man is Mortal.

This must be what someone trying to make a syllogism out of a division would have to do,⁹ since 1) he is going for that conclusion and 2) the only term in the assumptions which is common to both Man and Mortal is Animal, so that *must* be the middle term in any syllogism. As Aristotle points out, however, the conclusion that all man is mortal does not follow.¹⁰ This is something that the divider "wants" but is not entitled to.

⁹The extent to which this is unfair on Aristotle's part is discussed in the following chapter.

¹⁰The counterexamples to this second figure arrangement are given A=substance, B=animal, C=man and A=substance, B=animal, C=number, *APr* 27a19-20.

There is a deduction in this diagram, but not a very interesting one, namely that all *D* is $B \lor C$, since "division...takes the universal as the middle term" (46b2-3) and "in demonstrations, the middle term must always be less than and not a universal of the first of the terms." (46a39-b2) This gives the sense in which division deduces something "higher up".¹¹ Striker (2009) points out that there is a problem here: *mortal or immortal* may not be strictly more general than animal (if we take god to be alive), since the latter terms convert, so in what sense can animal then be "less than" mortal or immortal?¹² The problem, I take it, is that we want *less than* to be a strict ordering because it is irreflexive. Nothing is less itself. The natural way to resolve the issue, it seems to me, is to think of it rather as a weak ordering, like "less than or equal to".¹³ Then we would understand Aristotle as claiming that one can deduce nothing lower than the divided genus, such as that all man is mortal. What is important is that the disjunctive term is predicated universally of the genus divided. It is this universal predication that grounds the Barbara syllogism, regardless of whether the converse universal predication is true as well.

So when Aristotle says:

It is necessary for a man to be either mortal or immortal. For a man to be a mortal animal, however, is not necessary, but rather is asked for: yet this was what needed to be deduced. (46b10-13)

he means that there is a syllogism, but not the intended one. Indeed, he complains that the divider "does not explain at all clearly how the goal [i.e., the definition that they want to deduce] is necessary." (46b22-3) The necessity here, then, is not the kind that we find in modalized propositions, but the kind of necessity present in all syllogisms, namely that the conclusion comes about by necessity *given the premises*. (24b19-22, cf 91b14-6) The only way, then, for the divider to get to the goal is by asking for something else, namely the goal itself.

How much does Aristotle's argument here establish? Not very much. He has only shown that one division does not give us any of the desired syllogisms, not that many repeated ones, for instance, could not produce them, nor that if the divider applied a special procedure (such as dividing by means of privations), there could not come to be the desired syllogisms. These are serious concerns because Plato, in his discussions of division, both emphasizes the importance of many stages of division and gives norms for division. In the previous chapter, for instance, I showed how he endorses the Tortoise and Minimization norms.

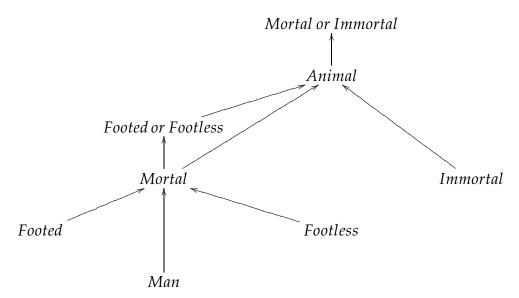
While the example that follows (46b7ff) could just be a repetition of the argu-

¹¹Cf APo 82a23-4 for the definitions of ano and kato.

¹²Perhaps it is that mortal or immortal is higher than *human*. But 1) this is just as problematic a claim (there can be a Barbara syllogism even when all three terms convert) and 2) it does not fit with the argument, which appeals to a conclusion of I.26, saying that the middle term is less than the first extreme. In this case, it means that animal must be less than mortal or immortal.

¹³It is best interpreted, I think, as the preorder used in Malink (2013) and in my own completeness proof for the assertoric syllogistic in the Appendix.

ment above,¹⁴ if it is brought together with the first example, it can address some of the worries above.¹⁵ Aristotle's argument here relies on the claim that the only valid deduction that can be made on the basis of these assumptions is that all *D* is $B \lor C$ and that this remains true no matter how many times one iterates the division:



On the basis of the second division, it is now clear that the only things which can be deduced on the basis of these further divisions is at least as general than the kind *mortal*. In fact, this is a necessary consequence of any kind of division, which always proceeds from more general to more specific kinds. Already having assumed that mortals are animals does not add anything that is relevant for deducing that man is footed.

There is one possible exception to this. Having placed man under mortal, could one not infer that man is not immortal?¹⁶ Since immortal is neither above nor below mortal, it would seem to be a problem for Aristotle's claim that division *only* deduces what is higher up, i.e., propositions whose predicate is at least as general

¹⁴"*Palin*" frequently plays this role in Aristotle, but in his discussions of division (e.g. *APo* II.5), Aristotle can also use it to indicate performing another division.

¹⁵While this interpretation has not been defended, it seems to be the standard one. See also Barnes (1975); Ferejohn (1991).

¹⁶This is a suggestion from Cavini (1995), who analyzes the deductive conclusion of division as always an instance of *modus ponendo tollens* or *modus tollendo ponens*. This leads him to conclude that Aristotle's analysis is gravely mistaken, primarily because it confuses "all A is B or C" with "either all A is B or all A is C", the latter of which is more important for Plato. Unfortunately, he fails to provide a single instance of Plato actually making the inference that he takes to be all important and Aristotle does not ever seem to commit the fallacy that he is accused of. The earliest mentions of this kind of inference that I could find in connection to division are Alexander *In APr* ad loc and Philoponus(?) *In APo* 350.7-20 Wallies (1909)), although there could be some connection with the Peripatetic hypothetical syllogistic, where a disjunction is called *diairetike* (e.g., Galen *Institutio Logica* 3.3-5 Kalbfleisch (1896), Theophrastus fr. 111D Huby (1992)). See Castelli (2015) for discussion of Alexander's interpretation of the passage.

as the middle term. There seem to be several options here for saving Aristotle's argument:

- 1. Reject the assumption that I started with in the second example, which took it as an elaboration of the first. Then we can say that Aristotle was restricting his claim to the one-off division and was not considering cases of iterated division when he showed that division only deduces what is higher up. In this case, what is really important to Aristotle is that division does not succeed in deducing what it really wants to deduce, namely the essence. Thus, while he is wrong to say that the only consequence in a many-stage division is a disjunctive claim, it is much less important.
- 2. Reject the claim that the assumptions pictured in the diagram above actually entail that man is not immortal. This is because Aristotle does not assume here that every division is exclusive. Instead, the divider can only assume that "All man is mortal or immortal" if the "or" is read inclusively. In this way, we can hold on to the generality of the argument.
- 3. The conclusion of the inference here is not a case of "something else coming about" and therefore not a syllogism at all. In favor of this suggestion is the clear informal equivalence between "All man is mortal" and "No man is immortal". However, when Aristotle uses negative terms in counterexamples¹⁷ he does not seem to assume that they are equivalent to the relevant proposition, but rather that they are terms in their own right. This of course is merely negative evidence. On the other hand, there seems little in the way of support for the claim either.

There is nothing decisive in this text to support 1 or 2. 1, however, has the disadvantage of making Aristotle say something false about division in general. When Plato advised the use of division in the *Philebus*, for example, he clearly envisaged the inquirer repeatedly dividing. This is borne out in his practice of division in the *Sophist* and *Statesman*, which contain numerous examples of many-staged division. Indeed, Aristotle indirectly refers to this in *APo* II.5 91b21-2 when he says that it makes no difference how many divisions one makes. Furthermore, I have argued at length in the Chapter 2 that Plato did not require divisions to be exclusive. Therefore, the second interpretation both gives Aristotle a better interpretation of Plato and a better argument against him, so I take charity considerations to push us toward the second line of interpretation. On this reading of the argument, Aristotle does not assume that when one has divided a genus into species, there does not exist another species which is subordinate to both, even though Aristotle does believe that divisions *ought* to be exclusive in *APo* II.13, discussed in §4.3.

¹⁷In the assertoric syllogistic, he assumes all the following to be true: "All ignorance is a condition", "All snow is inanimate", "Some inanimate is white", "No ignorance is good", "No man is inanimate", "No swan is inanimate", "No animal is inanimate", "Not all inanimate is white". See Malink (2013), Appendix C for the full list.

The last part of the text concerns other limitations of the method of division:

It is clear that with this method it is not possible to refute, nor to deduce concerning an accident or property or genus, nor in those cases when it is unknown whether this or that holds, such as whether the diagonal is commensurable or incommensurable. For if one assumes that every length is commensurable or incommensurable and the diagonal is a length, it has been deduced that the diagonal is commensurable or incommensurable, but if it has been assumed to be incommensurable, what was needed to be deduced was assumed. For the route is the same, but it is not through this. (A on incommensurable or commensurable, B on length, Γ on diagonal.)

Then it is clear that this manner of inquiry does not fit every inquiry, nor is it useful even in those for which it seems most appropriate. $(46b26-37)^{18}$

Let us examine them in order:

Division cannot refute. As Aristotle takes refutation to be a kind of syllogism (66b4-17), this follows straightforwardly from the fact that it does not produce a non-trivial syllogism. The kinds of refutations that are at issue here are not of the disjunctive claims but rather when a dialectical opponent affirms some definition of the target kind and one is in the role of arguing against that definition. It does not come from the fact that it can only produce conclusions which are universal affirmatives (as Alexander *In APr* 338.6-7), since "Not all man is mortal" would be refuted by an argument whose conclusion is "All man is mortal".

Division cannot deduce accidents, properties, or genera. This too is a direct consequence of the claim that divisions are not deductions of anything. Here, how-ever, one might raise the worry that it does seemingly deduce one kind of accident such as "Man is mortal or immortal". Perhaps since "mortal or immortal" is not *one thing*, this is not a problem. Surely, the distinctions between definitions, accidents, etc. in the *Topics* does not seem to have complex predicates in mind.

Division presupposes knowledge of which choice to make in the division. This idea goes hand-in-hand with the major claim that division does not deduce its goal, although it is not exactly the same—one could be able to know the proper choice by non-deductive means. Knowing that a diagonal is commensurable or incommensurable gets you nowhere in trying to find out which of those it is just because the goal simply does not follow. As a more general worry about the epistemology of division, however, this isn't ever really answered by Aristotle. Rather,

¹⁸Φανερὸν δ' ὅτι οὕτ' ἀνασκευάσαι ταύτῃ τῇ μεθόδῳ ἔστιν, οὕτε περὶ συμβεβηκότος ἢ ἰδίου συλλογίσασθαι, οὕτε περὶ γένους, οὕτ' ἐν οῖς ἀγνοεῖται τὸ πότερον ὡδὶ ἢ ὡδὶ ἔχει, οἴον ἄρ' ἡ διάμετρος ἀσύμμετρος ἢ σύμμετρος. ἐὰν γὰρ λάβῃ ὅτι ἄπαν μῆκος ἢ σύμμετρον ἢ ἀσύμμετρον, ἡ δὲ διάμετρος μῆκος, συλλελόγισται ὅτι ἀσύμμετρος ἢ σύμμετρος ἡ διάμετρος. εἰ δὲ λήψεται ἀσύμμετρον, ὃ ἔδει συλλογίσασθαι λήψεται. οὐκ ἄρα ἔστι δεῖξαι· ἡ μὲν γὰρ ὁδὸς αὕτῃ, διὰ ταύτης δ' οὐκ ἔστιν. τὸ ἀσύμμετρον ἢ σύμμετρον ἐφ' οῦ Α, μῆκος Β, διάμετρος Γ. φανερὸν οῦν ὅτι οὕτε πρὸς πᾶσαν σκέψιν ἁρμόζει τῆς ζητήσεως ὁ τρόπος, οὕτ' ἐν οἶς μάλιστα δοκεῖ πρέπειν, ἐν τούτοις ἐστὶ χρήσιμος.

he seems to, at best, avoid the problem in his positive theory. As we will see in §4.3, he gives division a different role in the inquiry into essence. It can be illuminating, he thinks, but not for these sorts of questions but instead of questions about how to order the elements in the essence.

In conclusion, Aristotle criticizes division on several counts here: it does not deduce the thing it aims to, it is not applicable to many different kinds of problems, and that it presupposes significant knowledge. None of this adds up to a wholesale rejection of its use in science. His style of argument *couldn't establish that* without the dubious further assumption that scientific methods are all deductive.¹⁹

4.2 APo II.5: Division and demonstration

The second critical discussion of division occurs amid a series of *aporiai* intending to show how demonstrating a definition is problematic. This is importantly related to division, since Aristotle portrays Platonists as interested in division precisely because they think it will allow them to demonstrate the essence. The chapter divides neatly into two sections, one giving a series of criticisms of division and the second trying to resolve them. First:

Yet truly the route through divisions does not deduce, just as was said in the analysis concerning the figures. For nowhere does that matter come to be that necessarily, these things being the case, and instead it does not demonstrate just as the one performing an induction. For it is not necessary to ask the conclusion, nor is it necessary for it to be given, but it is necessary given those things, even if the answerer does not affirm it. Is man animal or inanimate? Then he took animal, but has not deduced. Again, every animal is either terrestrial or aquatic. He took terrestrial. And for man to be the whole, terrestrial animal, is not necessary from the things said, but he assumes this also. And it makes no difference to go about in this way many times or a few. For it is the same. (In truth, non-deductive use is made by those proceeding in this way even of what can be deduced.) For what prevents this being true: that the whole is <said> of man without it making clear what it is or what it is to be? Further, what prevents something being added, taken away, or skipping over the essence? $(91b12-27)^{20}$

¹⁹It is notable that Zabarella, when he argues against division, *does* defend that claim.

²⁰'Αλλά μὴν οὐδ' ἡ διὰ τῶν διαιρέσεων ὁδὸς συλλογίζεται, καθάπερ ἐν τῆ ἀναλύσει τῆ περὶ τὰ σχήματα εἴρηται. οὐδαμοῦ γὰρ ἀνάγκη γίνεται τὸ πρᾶγμα ἐκεῖνο εἶναι τωνδὶ ὄντων, ἀλλ' ὥσπερ οὐδ' ὁ ἐπάγων ἀποδείκνυσιν. οὐ γὰρ δεῖ τὸ συμπέρασμα ἐρωτᾶν, οὐδὲ τῷ δοῦναι εἶναι, ἀλλ' ἀνάγκη εἶναι ἐκείνων ὄντων, κἂν μὴ φῆ ὁ ἀποκρινόμενος. ἄρ' ὁ ἄνθρωπος ζῷον ἢ ἄψυχον; εἶτ' ἔλαβε ζῷον, οὐ συλλελόγισται. πάλιν ἄπαν ζῷον ἢ ἔνυδρον· ἔλαβε πεζόν. καὶ τὸ είναι τὸν ἄνθρωπον τὸ ὅλον, ζῷον πεζόν, οὐκ ἀνάγκη ἐκ τῶν εἰρημένων, ἀλλὰ λαμβάνει καὶ τοῦτο. διαφέρει δ' οὐδὲν ἐπὶ πολλῶν ἢ ὀλίγων οὕτω ποιεῖν· τὸ αὐτὸ γάρ ἐστιν. (ἀσυλλόγιστος μὲν οῦν καὶ ἡ χρῆσις γίνεται τοῖς οὕτω μετιοῦσι καὶ τῶν ἐνδεχομένων συλλογισθῆναι.) τί γὰρ κωλύει τοῦτο ἀληθὲς μὲν τὸ πᾶν είναι κατὰ τοῦ ἀνθρώπου, μὴ μέντοι τὸ τί ἐστι

The structure of this passage is fairly clear.²¹ It begins by a brief recapitulation of the main thrust of *APr* I.31: division does not deduce the goal since does not follow necessarily from the assumptions of division, repeating the point that at each step the divider begs the question. This entails that, like induction, division is not demonstrative. Interestingly, he does add the point that the number of cuts does not make a difference, seemingly in response to Plato's insistence that one must divide slowly in the *Philebus* and *Statesman*. After this, however, there is a problem: the new point at b23 does not seem at all to be supported by what follows with the "gar" clause. Barnes, following Ross, sensibly places this new point in brackets to indicate that what we have here are two different criticisms of division, but only the second one is an argument against division demonstrating the definition.

The first new criticism is that the divider assumes what she could have proven had she gone by a different method (91b23-4). What are the things that could have been proven, but were not? Barnes (1975, p. 201) thinks that this is a reference to the trivial deductions to conclusions like "All man is mortal or immortal" discussed in I.31. However, it is more plausible if we think instead of things like "All man is animal" as being demonstrated from (e.g.) "All man is mortal." and "All mortal is animal".²² The reason for this is that Aristotle does seem to think these propositions can be demonstrated in the traditional *APo* I-style demonstrations. On the other hand, in *APr* I.31, Aristotle actually *claimed* that "division always syllogizes" the disjunctive conclusions put forward by Barnes, so it would be bizarre if now, in a passage directly drawing on the results in the earlier chapter, he was saying that division assumes and does not demonstrate them. Rather we should look for actual examples from the previous text of what Aristotle says division assumes.

The second criticism is that, even granting the truth of all those predicates established by means of division, we may not yet have secured a definition (91b25-8). This, then, is another argument for the claim that division cannot deduce definitions, but coming from the angle of definitions. Aristotle then lists all those things which distinguish definitions from merely true statements: one can "not make clear what it is" (e.g., by using obscure language²³), "posit something additional" (by including "grammatical" in the definition of human, which is true of all and only humans but only accidentally), "abstract something" (by stopping, for instance, at "footed animal"), or "skip over something in the essence" (by, for instance, defining man as "two-footed animal" instead of "two-footed terrestrial animal").²⁴

μηδὲ τὸ τί ἦν εἶναι δηλοῦν; ἔτι τί xωλύει ἢ προσθεῖναί τι ἢ ἀφελεῖν ἢ ὑπερβεβηxέναι τῆς οὐσίας; ²¹See Barnes (1975) for a similar take.

²²See Philoponus (?) *In APo* 350.7-20 Wallies (1909). Although the text is corrupt, he is clearly looking to *APo* I-style demonstrations as well.

²³*Topics* 139b19ff.

²⁴Cf *Topics* 143a15-28. The basic difference between abstracting and skipping seems to be that in the latter case, one has the final differentia correct but omits the proximate genus. In the former, by

Aristotle then claims that one can resolve these puzzles:

While these things are left aside, it is possible to solve them by taking all things in the what it is, and making the order by division, the first <term> being asked for, and leaving nothing out. And this is necessary if everything falls into the division and nothing < from the things in the what it is> is left, for²⁵ it must at this point be atomic.²⁶ But nevertheless it is not a deduction and if so, it makes it known in another way, and this is nothing absurd, for the one performing induction equally does not demonstrate, but still might make something clear. But the one stating a definition from division does not state a deduction. For just as in the conclusions without middle terms, if someone should state given those things, that this is the case, it would be possible to ask why, in this way too with the divisional definitions. What is a man? An animal that is mortal, footed, biped, and featherless. Why, for each addition? For, he will say and will show in the division, as he thinks, that everything is mortal or immortal. But every such account is not a definition, so that even if he demonstrates by division, the definition does not come to be a deduction.²⁷ (91b28-92a5)

The idea, it seems, is that division's role in inquiry should not be thought of as *discovering* the elements of an essence (genus, differentiae, or whatever else might be in there).²⁸ Rather, the divider can make sure that she arrives at a correct definition by division if she assumes all of these at the outset and then uses division (in the appropriate way, yet to be discussed) to put the pieces into a coherent whole. Aristotle's reason for this will be further elaborated in II.13, but here he gives a sketch

contrast, one fails to give a definition because one has not given the final differentia.

²⁵Following Waitz and all the older MSS AB*n*D in eliminating the second τοῦτο δ' ἀναγκαῖον, which Ross brackets. This looks to have crept into Eustratius through Philoponus' paraphrase of the section and is not part of Philoponus' citation, a point completely obscured by Ross's apparatus.

²⁶Barnes, who rightly notes that the various emendations proposed here are not needed, still excises this sentence on the grounds that it is only intelligible in virtue of II.13. This whole passage, it seems to me, is pointing towards that discussion, so I don't see why one would only excise that one part. Admittedly the text is somewhat obscure, but not unintelligible as it stands.

²⁷Ταῦτα μὲν οὖν παρίεται μέν, ἐνδέχεται δὲ λῦσαι τῷ λαμβάνειν ἐν τῷ τί ἐστι πάντα, καὶ τὸ ἐφεξῆς τῆ διαιρέσει ποιεῖν, αἰτούμενον τὸ πρῶτον, καὶ μηδὲν παραλείπειν. τοῦτο δ΄ ἀναγκαῖον, εἰ ἄπαν εἰς τὴν διαίρεσιν ἐμπίπτει καὶ μηδὲν ἐλλείπει· [τοῦτο δ΄ ἀναγκαῖον], ἄτομον γὰρ ἤδη δεῖ εἶναι. ἀλλὰ συλλογισμὸς ὄμως οὐκ ἔστι, ἀλλ' εἴπερ, ἄλλον τρόπον γνωρίζειν ποιεῖ. καὶ τοῦτο μὲν οὐδὲν ἄτοπον· οὐδὲ γὰρ ὁ ἐπάγων ἴσως ἀποδείκνυσιν, ἀλλ' εἴπερ, ἄλλον τρόπον γνωρίζειν ποιεῖ. καὶ τοῦτο μὲν οὐδὲν ἄτοπον· οὐδὲ γὰρ ὁ ἐπάγων ἴσως ἀποδείκνυσιν, ἀλλ' čἰμως δηλοῖ τι. συλλογισμὸν δ΄ οὐ λέγει ὁ ἐκ τῆς διαιρέσεως λέγων τὸν ὁρισμόν. ὥσπερ γὰρ ἐν τοῖς συμπεράσμασι τοῖς ἄνευ τῶν μέσων, ἐάν τις εἴπη ὅτι τούτων ὄντων ἀνάγκη τοδὶ εἶναι, ἐνδέχεται ἐρωτῆσαι διὰ τί, οὕτως καὶ ἐν τοῖς διαιρετικοῖς ὅροις. τί ἐστιν ἄνθρωπος; ζῷον ϑνητόν, ὑπόπουν, δίπουν, ἄπτερον. διὰ τί, παρ' ἑκάστην πρόσθεσιν; ἐρεῖ γάρ, καὶ δείξει τῆ διαιρέσει, ὡς οἴεται, ὅτι πῶν ἢ ϑνητὸν ἢ ἀθάνατον. ὁ δὲ τοιοῦτος λόγος ἅπας οὐκ ἕστιν ὁρισμός, ὥστ' εἰ καὶ ἀπεδείκνυτο τῆ διαιρέσει, ἀλλογισμος ζύνος ἀλος ἕστι ἀ ἀλλος τοῦτος οὐ συλλογισμὸν δ΄ οὐ λέγει οἰς διαιρετικοῖς ὅροις. τί ἐστιν ἀνθρωπος μαφαλοίται, ὑπόπουν, ὑπόπουν, ὑποίπουν, ὅποτον, ὑιὰ τί, σῦτως καὶ ἐν τοῖς διαιρετικοῖς ὅροις. τί ἐστιν ἀνθρωπος τοῦς σύκρισμος μας τι πῶν ἢ ϑνητὸν ἢ ἀθάνατον. ὁ δὲ τοιοῦτος λόγος ἅπας οὐκ ἕστιν ὑρισμός, ὥστ' εἰ καὶ ἀπεδείκνυτο τῆ διαιρέσει, ἀλλ' ὅ γ' ὁρισμὸς οὐ συλλογισμὸς γίνεται.

²⁸From this, it does not follow that division isn't involved in inquiry for Aristotle. As we will see in II.13, he thinks that division plays an important role in the discovery the definition from its raw materials.

of a way to divide that guarantees one obtains a definition. In order to do this, he thinks that we must divide exhaustively until one has used up all the elements of the essence, at which point one will reach something indivisible.

Even granting this solution, Aristotle argues that division cannot deduce (and hence not demonstrate). Instead, he claims it might only be able to "make us familiar" with the definition of the thing. The reason that they have not deduced, on Aristotle's view, is that one can still ask for the reason why each assumption was made in the division. Even if one tried to answer this kind of question by means of the division, the definition that one gave will not be a deduction (and hence not a demonstration). Instead, the account one gives either elucidates or explains the definition.

Even more clearly than in *APr* I.31, this passage clearly does not show Aristotle rejecting Platonic division as a method *tout court*, as Cherniss, Goldin, and Detel claim. Rather the last passage indicates just how aware Aristotle is that there remain options open, since he describes one.²⁹ Indeed, this option is taken up and developed in *APo* II.13.

4.3 APo II.13: Defining with division

Between this chapter and the last discussion of division, Aristotle has resolved the *aporiai* concerning definition. The topic now is how to hunt for what is predicated in the what it is, the *ti esti*. In the *Analytics*, this clearly refers to in the first meaning of "*per se*" in *APo* I.4. There Aristotle speaks of things "in the *ti esti*" including, for instance, the line in the *ti esti* of the triangle (because the latter is a figure bounded by three *lines*). Here what we are looking at a more narrow conception of what is in the *ti esti*, since a triangle is not a line. This actually seems to be guaranteed by Aristotle's phrasing, since although line belongs *per se* 1 to triangle, it is not a *predicate* of triangle. Indeed, Aristotle uses the phrase "predicated in the what it is" to explicitly describe genera as well as definitions, since both are proper answers to the question "what is it?" (*Topics* 102a31ff).

As I will argue, Aristotle will say, in this chapter, how to hunt for each of these, the definition and the genus. As this chapter is significantly more complex than the first two, I will first give an outline of my reading and then, beginning with a translation, try to say something about each of the parts except for the last, which contains miscellaneous advice about constructing definitions.

 96a24-b14: Here Aristotle, in preparation for his discussion of how to find things predicated in the essence, says how the essence of things like "three" is "contains" predicates which are independently necessary and insufficient,

²⁹It will not do to say that this is some *different* method, as Ferejohn (1991) does, since Aristotle evidently thinks that it has precisely the same shortcomings with respect to deductions as the Platonic method, as well as the same goal.

but which jointly are necessary and sufficient. This tells us about the relationship between the two kinds of things predicated in the *ti esti*: the genus and essence.

- 96b15-25: This passage argues for a particular order of inquiry into essences: first one discovers the genus, then the essence of the species through division, and finally the essence of attributes through demonstration.
- 96b25-97b6: Dividing by means of the differences is useful for the task of finding the essence of species (which are not the *summum genus*³⁰ of the science). Some advice on how to divide and then how to use the divisions to form definitions, responding to two objections to the method.
- 97b7-25: How to investigate with similarities. This is how one ought to investigate the second kind of thing predicated one needs to begin division.
- 97b25-39: Miscellaneous remarks on finding and properly phrasing definitions.

I offer this account of the structure as a hypothesis for now, since one of the most controversial aspects of this passage since antiquity has been the structure or even overall coherence of the chapter. To the extent that, within this framework, I can give convincing readings of the various passages, I take this structure to be confirmed. Thus we must await final judgment until we have reached the end of this section.

If this general structure is on the right track, then we see that Aristotle here is giving division a prominent place in his theory of the discovery of definitions. Indeed, one way of seeing this would be to notice that, on this view *every species of the summum genus of a science can be defined by means of division*. So, for instance, *substance* would not be defined by division, but *animal, terrestrial animal, line, three,* and *virtue* all would be. Depending on whether one takes division to include the division of what would later be called "integral wholes" (for instance, of a house into roof, walls, and floor)³¹, one might even want to say that the definition of every kind other than the *summa genera* will get their definition by means of division. Most treatments implicitly do not even take there to be an overarching structure to the chapter, since they discuss various parts of it in completely different order.³² On the other hand, Barnes (1975) and Detel (1998) more or less divide the chapter in three by putting together 96b15-25 and 96b25-97b6.³³ Because the chapter is not

³⁰A *summum genus* is the most kind general kind which is the object of scientific scrutiny, which includes everything under the purview of a science. For example, *number* is the genus relevant for arithmetic, *living being* for biology, etc. Contra McKirahan (1992), I take this to be the same as the genus which is divided in the process of division, for reasons that will become clear only once we have a complete view of II.13.

³¹As it was already in antiquity: Alcinous *Didaskalikos* Whittaker (1990), Sextus Empiricus *PH* II.215-8 Mutschmann and Mau (1914-61), Boethius *De Divisione* Magee (1998).

³²Bronstein (2016); Charles (2000); Ferejohn (1991); McKirahan (1992).

³³However, see Goldin (2004) for reasons to separate them.

very well sign-posted, we can only get a sense of the internal structure with a firm grip on the content of each of the sections.

4.3.1 Essence and Genus

Of the things always belonging to something, some extend more broadly but not outside of the genus. By "belonging more broadly", I mean whatever things belong to something universally, but nevertheless also belongs to something else, just as there is something which belongs to every three, but also to a non-three, so that being belongs to three and also non-number. But odd belongs both to every three and belongs more broadly for it also belongs to five, but not outside the genus. For five is a number, but nothing is odd other than a number.

One should take those kinds of things until the first point in which each thing taken belongs more broadly, but all together do not belong more broadly. For necessarily this is the essence (*ousia*) of the subject matter. For example, number belongs to every three, as do odd and prime (both in the sense of not being measured by number but also not composed of numbers). Now then, *this* is what three is: a number, odd, prime, and prime in the other sense. For some of these belong to every odd number, while the final belongs to two as well, but all together belong to nothing else. But since it was made clear by us in the above discussions that the predicates in the "what it is" are necessary³⁴ (and the universals necessary), and the things taken are in the "what it is" for three (and for whatever else is taken), thus necessarily three is these things.

That it is the essence, is clear from the following. For necessarily, if this should not be the being for three, it would be some genus, either named or nameless. Now, it will belong more broadly than to three. For let a genus be supposed to be such a thing so as to possibly belong more broadly. Now if it belongs to nothing other than the indivisible threes, this would be three's being. (For let this also be supposed, the essence of each thing is the last such predication over the indivisibles). Thus for anything else shown in this way, the being for that thing will be similar. (96a24-b14)³⁵

³⁴Reading *anagkaion* with MSS, Barnes. Ross emends to *katholou*.

³⁵Τῶν δὴ ὑπαρχόντων ἀεὶ ἐκάστῷ ἔνια ἐπεκτείνει ἐπὶ πλέον, οὐ μέντοι ἔξω τοῦ γένους. λέγω δὲ ἐπὶ πλέον ὑπάρχειν ὅσα ὑπάρχει μὲν ἑκάστῷ καθόλου, οὐ μὴν ἀλλὰ καὶ ἄλλῷ. οἴον ἔστι τι ὃ πάσῃ τριάδι ὑπάρχει, ἀλλὰ καὶ μὴ τριάδι, ὥσπερ τὸ ὄν ὑπάρχει τῆ τριάδι, ἀλλὰ καὶ μὴ ἀριθμῷ, ἀλλὰ καὶ τὸ περιττὸν ὑπάρχει τε πάσῃ τριάδι καὶ ἐπὶ πλέον ὑπάρχει (καὶ γὰρ τῆ πεντάδι ὑπάρχει), ἀλλ' οὐκ ἔξω τοῦ γένους· ἡ μὲν γὰρ πεντὰς ἀριθμός, οὐδὲν δὲ ἔξω ἀριθμοῦ περιττόν. τὰ δὴ τοιαῦτα ληπτέον μέχρι τούτου, ἕως τοσαῦτα ληφθῆ πρῶτον ὥν ἕκαστον μὲν ἐπὶ πλέον ὑπάρξει, ἄπαντα δὲ μὴ ἐπὶ πλέον· ταύτην γὰρ ἀνάγκῃ οὐσίαν εἶναι τοῦ πράγματος. οἴον τριάδι ὑπάρχει πάσῃ ἀριθμός, τὸ περιττόν, τὸ πρῶτον ἀμφοτέρως, (καὶ ὡς μὴ μετρεῖσθαι ἀριθμῷ καὶ ὡς μὴ συγκεῖσθαι ἐξ ἀριθμῶν). τοῦτο τοίνυν ἤδῃ ἐστὶν ἡ τριάς, ἀριθμὸς περιττὸς

Here I will give a reconstruction that slightly reorganizes the material in this section in order to get at what is interesting. I'll begin with three definitions:

X belongs more broadly than Y: 1) X universally belongs to Y³⁶ 2) but also to something else.

X is a genus of **Y**: X possibly belongs more broadly than Y.³⁷

X is the essence of **Y**: X is the last thing predicated over the indivisible Ys.

Aristotle begins with the assumption that "among those things which always belong to something, there are some things which 1) belong more broadly and 2) do not stretch outside the genus." He then describes the following procedure: Concatenate predicates which individually belong more broadly until you get one which no longer belongs more broadly.³⁸ He then claims that the result is the essence. The two initial options are that it is a genus or the essence. Aristotle didn't say why, but presumably, if B and C are genera of A in the sense defined above (which they must be, since they belong more broadly), then BC either extends over more atoms or just the atoms of A. But it is not the genus, since the genus potentially belongs more broadly but the term which is the result of the concatenation does not.

Interestingly, this procedure requires that there are at least two such predicates. Bronstein (ms.) and McKirahan (1992) have suggested that this is inconsistent with other Aristotelian commitments. I will discuss this later in the chapter, when we have occasion to see the allegedly inconsistent passage. One consequence of this is that the procedure will not work for the highest species of a genus, which only have one differentia. Is this a problem? I take it that here Aristotle is primarily concerned with definitions involving several predicates and the way that they relate

³⁶This seems to be a reference to the sense of of "universal" in *APo* I.4. In this sense, X universally belongs to Y if it belongs per se and as a whole. However, there it is commonly thought that the subject and predicate must be coextensive. However, we might take this passage as evidence for the view of Barnes (1975), pp. 247-8 that Aristotle nowhere commits himself to this understanding of universal.

³⁷Couldn't the same definition hold of differentiae? Aristotle may be doing as he did in the *Topics* I.4 101b17-19, where he freely treats genera and differentiae in the same way. Note, however, that this does not prevent him from distinguishing differentiae from genera in *Topics* IV.6 128a20ff.

³⁸The concatenation BC of two terms is rather like the intersection of B and C. However, just as there have been powerful objections by, for instance, Martin (1997) and Malink (2009) to interpreting Aristotle's terms extensionally, the same should apply in this case. The concatenation, on Martin's view, should be thought of as the strongest term such that B belongs to it and C belongs to it. Following him, I will denote it BC. Aristotle here speaks of "such predicates" in the plural, but the notion of concatenation clarifies the distinction between "each" and "all" at a33-4.

πρῶτος καὶ ὑδὶ πρῶτος. τούτων γὰρ ἕκαστον, τὰ μὲν καὶ τοῖς περιττοῖς πᾶσιν ὑπάρχει, τὸ δὲ τελευταῖον καὶ τῆ δυάδι, πάντα δὲ οὐδενί. ἐπεὶ δὲ δεδήλωται ἡμῖν ἐν τοῖς ἄνω ὅτι ἀναγκαῖον μέν ἐστι τὰ ἐν τῷ τί ἐστι κατηγορούμενα (τὰ καθόλου δὲ ἀναγκαῖα), τῆ δὲ τριάδι, καὶ ἐφ' οὕ ἄλλου οὕτω λαμβάνεται, ἐν τῷ τί ἐστι τὰ λαμβανόμενα, οὕτως ἐξ ἀνάγκης μὲν ἂν εἴη τριὰς ταῦτα. ὅτι δ' οὐσία, ἐκ τῶνδε δῆλον. ἀνάγκη γάρ, εἰ μὴ τοῦτο ῆν τριάδι εἶναι, οἶον γένος τι εἶναι τοῦτο, ἢ ἀνομασμένον ἢ ἀνώνυμον. ἔσται τοίνυν ἐπὶ πλέον. ἐἰ τῆ τριάδι ὑπάρχον. ὑποκείσθω γὰρ τοιοῦτον εἶναι τὸ γένος ὥστε ὑπάρχειν κατὰ δύναμιν ἐπὶ πλέον. εἰ τοίνυν μηδενὶ ὑπάρχει ǎλλῷ ἢ ταῖς ἀτόμοις τριάσι, τοῦτ' ἂν εἴη τὸ τριάδι εἶναι (ὑποκείσθω γὰρ καὶ τοῦτο, ἡ οὐσία ἡ ἑκάστου εἶναι ἡ ἐπὶ τοῖς ἀτόμοις ἔσχατος τοιαὐτη κατηγορία)· ὥστε ὁμοίως καὶ ἄλλῷ ὁτῷοῦν τῶν οὕτω δειχθέντων τὸ αὐτῷ ἔσται.

to the essence. If we just have one difference, then this relationship is rather clearer and a definer could probably figure out how to construct the definition without an algorithm. In this passage, Aristotle never actually asserts that everything which is definable is definable in this way. He is only talking about one kind of case, in which the things predicated in the essence extend more broadly.

Another problem, discussed in detail by McKirahan (1992), ch. 9, is that Aristotle does not seem to distinguish here between the immediate and demonstrated necessary predications, only the former of which should be included in the definition. This would be a serious problem. The whole point of definitions is that they can serve as an explanatory bedrock. Including the *explananda* in the definition would remove the possibility of it being able to explain it. However, it is not at all clear that Aristotle is thinking that everything which is predicated universally will make an appearance in the definition. Rather, he is making a distinction between two kinds of predicates which belong universally and using it to explain the relationship between the things predicated in the essence and the essence itself. While he is saying that we should take the predicates in one of these groups, he does not thereby say that we should take *all* of them, although there is no indication in this passage how exactly those predicates are to be chosen.

After describing the procedure, he gives an example of this construction: *three*.³⁹ First he describes choosing the predicates. Being belongs to three, but it also belongs to non-number–so it is not in the genus. Therefore it will not belong in the definition. Odd, however, belongs to three, belongs more broadly (because it includes five), and does not extend outside the genus (since five is a number and no odds are non-numbers). Three predicates (in addition to the genus) are discussed that fit the bill: odd, prime, and prime' (not composed out of numbers).⁴⁰ So this is what the essence of three is: number odd prime prime'.

The weird thing about this example is that everything which is prime' (just two and three) is prime, so the third item in the definition is not needed. I can see several possible explanations here. The first is that Aristotle just picked a bad example. I find this hard to swallow, since Aristotle could have just used that very same example and only discussed the more narrow sense of prime without running into a problem.⁴¹ The second is that the procedure is not meant to exclude these kinds of superfluous elements. This is not completely implausible, especially since the construction that Aristotle discusses does not put any kind of ordering

³⁹*Three* seems to be an example of an infima species. This might seem odd to us, since it might be more plausibly thought to be a particular. His use of the quantifier "every" at 96a35 further shows that he has in mind something general. For this reason, some have thought that he is talking rather about triples and doubles as opposed to three and two. On the other hand, the Greeks generally thought of numbers as pluralities of units (cf Euclid Book VII Def 2), so the idea that it is like a species, also a collection, may not be strange.

⁴⁰In Greek mathematical thinking, all numbers other than two and three are composed out of two and three by summation (one is not a number). This is in fact the meaning of "composite" in this very passage.

⁴¹Or he could have chosen to define two using either notion of prime.

on the predicates which are concatenated. In fact, one might follow Bronstein in seeing Aristotle as assuming that there is a proper ordering in this passage and then giving the rules for obtaining such an ordering later on. Finally, one could say that Aristotle is not actually mentioning both notions of prime in the definition, instead either reading the *kai* epexegetically to specify only the latter meaning of prime⁴² or reading it as a compound predicate.⁴³ This would give Aristotle a nicer definition, even if only by accident.

There seems, to my mind, little to choose between the second and third readings. Both of them have the benefit of rendering Aristotle's doctrine consistent with the example, which the first reading cannot do. The important issue, which both readings should agree about, is that Aristotle's definitory procedure here is too loose to guarantee the kind of definition that he wants, for instance, in *Topics* VI and VII, where no part of the definition is superfluous in the stronger sense that the definition without that term would not give necessary and sufficient conditions. If this were the last word on hunting for the definitions in the chapter, this would certainly be something to worry about. However, the more specific accounts that he will give in what follows seem to circumvent this problem.

4.3.2 The order of inquiry

But it is necessary, whenever one is dealing with some whole, to divide the genus into the indivisibles in species, which are the primary things⁴⁴ (such as number into three and two), then in this way try to get definitions of *them* (for instance straight line, circle, and right angle). And after this, having determined what the genus is⁴⁵ (for instance whether it is among quantities or qualities), [one must] contemplate the proper attributes through the common⁴⁶ primaries. For in the things put together from the indivisibles, the characteristics will be obvious from the definitions [of the simples], on account of the definition and the simple being the principle of all things, and those characteristics belonging to the simples alone on account of themselves, while they belong to everything else on account of the simples. (96b15-25)⁴⁷

⁴²This suggestion is due to Alan Code and Henry Mendell.

⁴³As in Charles (2000).

 $^{^{44}\}mathrm{As}$ Goldin (2004) points out, this could also read "the primary things in species, which are indivisible."

⁴⁵Note that, I take the aorist participle to indicate that the activity has occurred *before* what he then tells us to do. The relative priority of determining the genus and dividing the species I take to be unspecified, allowing us to maintain the more traditional Aristotelian thesis that the genus is prior in definition to the species. To get the grammar to fit with this claim, we should interpret the "after this" rather unnaturally as not taking scope over the participle. Thanks to Marko Malink for pressing me on this point.

⁴⁶Cf *APo* I.9.

⁴⁷Χρὴ δέ, ὅταν ὅλον τι πραγματεύηταί τις, διελεῖν τὸ γένος εἰς τὰ ἄτομα τῷ εἴδει τὰ πρῶτα, οἶον ἀριθμὸν εἰς τριάδα καὶ δυάδα, εῖθ΄ οὕτως ἐκείνων ὁρισμοὺς πειρᾶσθαι λαμβάνειν, οἶον εὐθείας γραμμῆς καὶ κύκλου,

This passage is incredibly contested. From Goldin (2004), there are something like 32 interpretive possibilities, many of which have actually been held from the time of the ancient and Renaissance commentators. Without engaging with all these viewpoints, I will try to give an interpretation that works both internally and in context fitting in with the main lines of recent interpretation.⁴⁸

Here we are hunting the parts of the essence of the derivative thing. In contrast to most of the literature,⁴⁹ I take the phrase "some whole" ($\delta\lambda$ ov τ t) with which one is dealing to be any kind at all and *not* the genus specifically which is said to be divided in the following line. One might think that it is mandated by the verb translated "is dealing with" ($\pi\rho\alpha\gamma\mu\alpha\tau\epsilon\eta\tau\alpha$ t) that the $\delta\lambda\sigma\nu\tau$ t refers to the genus, since Aristotle has the view that the subject of a science is delimited by the genus that one is working with. However, one might take that point without saying that the object of the scientist's concern is, at every moment, the genus and not something relating to that genus in a particular way. The genus interpretation also makes it difficult to make sense of the appearance of genus ($\gamma\epsilon\nu\sigma\varsigma$) in the next clause, which looks to be introducing a new idea. On the genus interpretation we would have instead expected a pronoun referring to it. Not much really hangs on this word, although on the interpretation that I'm proposing, the transition would be much smoother. Furthermore, this would be an unprecedented use of $\delta\lambda\sigma\nu\tau$ t in Aristotle.

Rather, the primary use of the phrase in the *Analytics* can be seen in *APo* I, where 7 of the total 18 occurrences lay. Here I will just quote one instance, which gives the flavor of its general meaning: "Now when either A or B is in some whole or both are, it is not possible for A to belong to B primitively" (79a37 ff). This sense is defined in *Metaphysics* V.26 as "that which is said of something as a whole" and thereby said to be "some whole" (1023b29-30). Here that to which A and/or B belong is just some predicate, not necessarily the summum genus. This is especially clear in 79b6-12, which explicitly discuss chains of predications.

The interpretation of $\delta\lambda o\nu \tau \iota$ that I am proposing is sufficiently broad that it can capture even otherwise strange uses, such as that of *Physics* I.1:

At first the mixed up things are more obvious and clear to us, while later from these the elements and principles come to be knowable to those dividing them [the mixed up things]. Thus one must proceed from the entirety to the things taken one by one. For the whole is more knowable by perception, and the entirety is some whole (*holon ti*). For the entirety encompasses many things as parts. $(184a21-26)^{50}$

καὶ ὀρθῆς γωνίας, μετὰ δὲ τοῦτο λαβόντα τί τὸ γένος, οἶον πότερον τῶν ποσῶν ἢ τῶν ποιῶν, τὰ ἶδια πάθη θεωρεῖν διὰ τῶν κοινῶν πρώτων. τοῖς γὰρ συντιθεμένοις ἐκ τῶν ἀτόμων τὰ συμβαίνοντα ἐκ τῶν ὁρισμῶν ἔσται δῆλα, διὰ τὸ ἀρχὴν εἶναι πάντων τὸν ὁρισμὸν καὶ τὸ ἁπλοῦν καὶ τοῖς ἁπλοῖς καθ' αὑτὰ ὑπάρχειν τὰ συμβαίνοντα μόνοις, τοῖς δ' ἄλλοις κατ' ἐκεῖνα.

⁴⁸In particular, Bronstein (2016); Charles (2000); Goldin (2004).

⁴⁹Starting from Philoponus(?) *In APo* II 400,30 Wallies (1909).

⁵⁰ἔστι δ΄ ἡμῖν τὸ πρῶτον δῆλα καὶ σαφῆ τὰ συγκεχυμένα μᾶλλον· ὕστερον δ΄ ἐκ τούτων γίγνεται γνώριμα

Here he seems to think of composites being "some whole". Even though I lack a definition of this kind of composite, I take it to include at least the following kinds of things: five, square, the snub, pale man, etc. In these sorts of cases, the kind or the particular is not sufficiently unified to receive the same type of unified definition that one might give a more fundamental natural kind. For example, Aristotle understands five as a complex of two and three. There is, as it were, no more to defining five than understanding it as a complex of two and three and knowing what two and three are. On the other hand, the more basic kinds are not so decomposable (even though Aristotle does think they too have definitions). So while the genus interpretation would have to give an alternative meaning to this use of the phrase "some whole", the more expansive account fits much better with the text.⁵¹

Here in *APo* II.13, Aristotle is presenting a very condensed summary of how one is to conduct inquiry into the essence of anything at all. However, his goal seems to be, in particular, to say how one finds the essence of composites, since he only briefly discusses finding primary subject kinds, but without mentioning how one is to do so (this will be the topic of the rest of II.13). Rather, the end goal seems to be deriving the definitions of complexes and demonstrating characteristics which, following Goldin, could well have been identified by Aristotle with complexes (or at the very least, closely associated with them).⁵²

In order to define a complex, Aristotle says that one must have already done quite a lot, including having divided the genus into the primitive atoms, defined the primitive atoms, and grasped the genus. The basic idea, then, is that once we have the definitions of the basic elements of the science (the species and genus), the things predicated in the essence of the composites will be obvious. Unlike the species and genus, Aristotle does not give us a recipe for discovering these definitions. One reason for this, if one looks at the examples that I gave above, is their sheer variety. Aristotle might think that some of these need to be defined in very different ways. For example, for the snub and other natural beings (as in *Physics* II.2), the complexity comes from the presence of form and matter. This is surely not true of five. So we might expect that definitions which include both form and matter would be very different in structure from those that do not. At the very abstract and general level of II.13, however, Aristotle is merely saying that these composites are defined *after* the subject genus and its species because it

τὰ στοιχεῖα καὶ αἱ ἀρχαὶ διαιροῦσι ταῦτα. διὸ ἐκ τῶν καθόλου ἐπὶ τὰ καθ' ἕκαστα δεῖ προϊέναι· τὸ γὰρ ὅλον κατὰ τὴν αἴσϑησιν γνωριμώτερον, τὸ δὲ καθόλου ὅλον τί ἐστι· πολλὰ γὰρ περιλαμβάνει ὡς μέρη τὸ καθόλου.

⁵¹It also occurs in *Metaphysics* 1084b30 in this sense. In some texts, Aristotle uses ὅλον τι to mean "everything", for instance *De Caelo* 294b32, *De Gen et Corr* 232b17, *Physics* 253a34, *Metaphysics* 993b6. At *Metaphysics* 1069a19, it is used as part of a metaphor, 1075a9 as paraphrastic for ὅλφ τινι χρόνφ.

⁵²Bronstein (2016); Goldin (2004); McKirahan (1992) have wondered to what extent Aristotle could really believe that we can read the properties of complexes off of the simples. Here I follow Bronstein in taking Aristotle to be making a relatively weak claim here and restricting himself to properties had by both the simple and complex. For example, assuming that 2 and 3 are the basic numbers, the composite 5 is going to be a number *because* 2 and 3 are numbers.

is defined *in terms* of them.

4.3.3 Defining the simple species

This passage is the most important for my purposes, so I will take it section by section. The basic strategy of this passage is to:

- 1. Raise an objection about the use of division and respond to it, and in the process describe how to divide in such a way as to avoid that objection.
- 2. Raise another objection about omniscience and respond to it by introducing another norm of division.
- 3. Show how to construct a definition by means of the divisions.

As a whole, this section shows how Aristotle, despite his insistence on the limitations of division, is working squarely within the Platonic program. As we saw in Ch. 3, Plato introduced norms division that are necessary if a divider is to reliably achieve her goal. In what follows, we see that Aristotle is committed to the very same program, even though the particular norms that he proposes are different.

4.3.3.1 Objection to using divisions

And the divisions according to the differences are useful for going about in this way. However, how they prove was discussed earlier.⁵³ But they might be useful towards syllogizing the "what it is" in this way alone. And yet it might be thought to be of no use, but to take everything straightaway, as would be the case if someone were to take [everything] from the beginning without the division. But the order in which the predications are predicated makes a difference, such as to say animal-tame-biped or biped-animal-tame. For if all is from two and animal-tame is a unity, and again man (or whatever unit comes to be) is made up of this and the difference, it is necessary, having divided to ask for it. (96b25-35)⁵⁴

The objection seems to be a generalization of points made by Aristotle himself in other parts of the *Analytics*. If division does not syllogize (*APr* I.31) or prove (*APo*

 $^{^{53}}APo$ II.5.

⁵⁴αί δὲ διαιρέσεις αἰ κατὰ τὰς διαφορὰς χρήσιμοί εἰσιν εἰς τὸ οὕτω μετιέναι· ὡς μέντοι δεικνύουσιν, εἴρηται ἐν τοῖς πρότερον. χρήσιμοι δ' ἂν εἴεν ὥδε μόνον πρὸς τὸ συλλογίζεσθαι τὸ τί ἐστιν. καίτοι δόξειέν γ' ἂν οὐδέν, ἀλλ' εὐθὺς λαμβάνειν ἄπαντα, ὥσπερ ἂν εἰ ἐξ ἀρχῆς ἐλάμβανέ τις ἄνευ τῆς διαιρέσεως. διαφέρει δέ τι τὸ πρῶτον καὶ ὕστερον τῶν κατηγορουμένων κατηγορεῖσθαι, οἶον εἰπεῖν ζῷον ἤμερον δίπουν ἢ δίπουν ζῷον ἤμερον. εἰ γὰρ ἅπαν ἐκ δύο ἐστί, καὶ ἕν τι τὸ ζῷον ἤμερον, καὶ πάλιν ἐκ τούτου καὶ τῆς διαφορᾶς ὁ ἄνθρωπος ἢ ὅ τι δήποτ' ἐστὶ τὸ ἐν γινόμενον, ἀναγκαῖον διελόμενον αἰτεῖσθαι.

II.5), then isn't it completely useless?⁵⁵ How could this help one in the search for what is in the essence?

First, we must understand what he means by the notion of syllogizing the what it is.⁵⁶ There are two texts that bear on the issue: *Topics* VII.3 and *APo* II.8-10. I will start by just quoting the passages. It is important to resolve this question, since the conception of a syllogism of the essence is quite different in these two passages and thus the purpose of division will be quite different depending on which sort of syllogizing it is supposed to do. Let us start with the *Topics* passage:

At present it concerns us only so far as is required for our present purpose, and accordingly we need only make the bare statement that it is possible for there to be a deduction of a thing's definition and essence. For if a definition is an account signifying the essence of the thing and the predicates contained therein ought also to be the only ones which are predicated of the thing in what it is, and genera and differentiae are predicated in the "what it is," it is obvious that if one were to get an admission that these are the only attributes predicated in what it is, the account containing them will of necessity be a definition; for it is impossible that anything else should be a definition, seeing that there is not anything else predicated of the object in what it is. That a definition may thus be reached by a process of deduction is obvious. The means whereby it should be established have been more precisely defined elsewhere, but for the present purposes of the inquiry now before us the same commonplace rules serve. (*Topics* 153a12-26)⁵⁷

Here, the syllogism of the essence is supposed to take the following form:

- 1. G is the genus of S.
- 2. D₁, ..., D_n are differentiae of S.
- 3. There is nothing in the definition of S other than $G, D_1, ..., D_n$.
- 4. Therefore, the definition of S is $GD_1...D_n$.

⁵⁵Cherniss (1944), it seems, takes Aristotle to agree in spirit with this objection, primarily as a result of not reading *APo* II.5 as a polemic, not an aporia. I take this passage to be decisive evidence against such a view.

⁵⁶Thanks to Robert Bolton for pressing me on this issue.

⁵⁷νῦν δ' ὄσον ἱχανὸν πρὸς τὴν παροῦσαν χρείαν, ὥστε τοσοῦτον μόνον λεχτέον ὅτι δυνατὸν γενέσθαι ὑρισμοῦ καὶ τοῦ τί ῆν εἶναι συλλογισμόν. εἰ γάρ ἐστιν ὅρος λόγος ὁ τὸ τί ῆν εἶναι τῷ πράγματι δηλῶν, καὶ δεῖ τὰ ἐν τῷ ὅρῷ κατηγορούμενα ἐν τῷ τί ἐστι τοῦ πράγματος μόνα κατηγορεῖσθαι, κατηγορεῖται δ΄ ἐν τῷ τί ἐστι τὰ γένη καὶ αἱ διαφοραί, φανερὸν ὡς εἴ τις λάβοι ταῦτα ἂ μόνα ἐν τῷ τί ἐστι τοῦ πράγματος κατηγορεῖται, ὅτι ὁ ταῦτα ἔχων λόγος ὅρος ἐξ ἀνάγκης ἂν εἴη· οὐ γὰρ ἐνδέχεται ἕτερον εἶναι ὅρον, ἐπειδὴ οὐδὲν ἕτερον ἐν τῷ τί ἐστι τοῦ πράγματος κατηγορεῖται. Ὅτι μὲν οὖν ἐγχωρεῖ συλλογισμὸν ὅρου γενέσθαι, φανερόν. ἐκ τίνων δὲ δεῖ κατασκευάζειν, διώρισται μὲν ἐν ἑτέροις ἀχριβέστερον, πρὸς δὲ τὴν προχειμένην μέθοδον οἱ αὐτοὶ τόποι χρήσιμοι.

It is clear that, by contemporary standards, this is a valid argument: there is no way in which the premises could be true and conclusion false.⁵⁸ However, this is evidently not a syllogism that could be brought into the three figures of the *Prior Analytics*.⁵⁹ This is because Aristotle's logic has no way to move from a plurality of terms to a compound term within an argument. He can perfectly well account for the argument:

- 1. All A is BC.
- 2. All BC is DEF.
- 3. Therefore, all A is DEF.

However, he has no way to infer from this, for instance, that all A is D. The argument above requires the converse sort of move, from "All A is B" and "All A is C" to "All A is BC". Again, whether because he thought that this was not a syllogism or he simply did not have the logical resources to deal with it, the theory of deduction in the *Analytics* cannot account for the argument in the *Topics*, which is well known to have have a much broader view about what counts as a deduction.⁶⁰ Moreover, it seems probable that the end of the *Topics* passage refers to a more definitive treatment of the problem, which would presumably be the one in *APo* II.⁶¹ Therefore, it seems unlikely (although not impossible) that Aristotle had this sort of syllogism in mind.

The second passage might then seem promising:

When it is clear that A belongs to C, then to seek why it belongs is to seek what B is—whether screening or rotation of the moon or extinction. And this is the account of the one extreme, i.e. in this case of A. For an eclipse is a screening by the earth.⁶²

⁵⁸Of course, casting it into modern notation would be a tricky business. That, however, is really just due to the difficulty of setting out the relations of essence/genus/differentiae/definition into the logical vocabulary. Adequate syntax would surely lead to this being a valid argument.

⁵⁹McKirahan (1992), pp. 149-159 argues on the basis of *APr* I.23 that Aristotle was not concerned with actually giving the syllogistic arguments because he thought he had proved a theorem showing that all deductions can be recast in the figures. Therefore, when he gets to the application of his logical apparatus, he does not feel the need to actually give the syllogism. Nevertheless, it is generally clear in *APo* how to recast his arguments into the syllogistic form, since he normally alerts his reader about, for instance, the relevant middle terms. In the case of the argument above, not even this seems to be true. He could give a syllogism like "S is $GD_1...D_n$ " meets conditions XYZ, anything that meets XYZ is a definition, so "S is $GD_1...D_n$ " is a definition, but that is not a syllogism of the *content* of the definition, which is what Aristotle seems to think is at issue both in the *Topics* and the *Analytics*.

⁶⁰See Malink (2015) for discussion about the differences between syllogisms in the *Topics* and *Analytics*.

⁶¹Although there is a difficulty here since he takes the existence of a demonstration of the what it is here to be quite unproblematic, whereas he objects to this very procedure in *APo* II.6. Thanks to Michel Crubellier for this point, which is also made in Maier (1896) II.2, p. 80 n. 3. See Allen (2011) for discussion of the relationship of these sections.

⁶²NB: Here, as elsewhere in *APo*, Aristotle is only referring to a lunar eclipse.

What is thunder? Extinction of fire in the cloud. Why does it thunder? Because the fire in the cloud is extinguished. Cloud C, thunder A, extinction of fire B. Thus B belongs to C, the cloud (for the fire is extinguished in it); and A, noise, to this; and B is indeed an account of A, the first extreme. And if again there is another middle term for this, it will be from among the remaining accounts. We have said, then, how what a thing is is grasped and becomes familiar, hence no deduction and no demonstration of what a thing is comes about—yet it is clear through deduction and through demonstration. Hence without a demonstration you cannot become aware of what a thing is (in cases where the explanation is something else), yet there is no demonstration of it (as we said when we went through the puzzles). (*APo* 93b3-20)⁶³

Here Aristotle is much closer to the syllogistic model of APr, which is later on called a syllogism of the what it is (94a12):⁶⁴

- 1. Extinction of fire (B) belongs to cloud (C).
- 2. Thunder (=a certain kind of noise= A^{65}) belongs to B.
- 3. Therefore, A belongs to C.

There are still some differences with the *APr* model. For example, all the propositions here would be classified as "indeterminate" (meaning that the quantifier being used is not made explicit) and should have quantifiers added on to them before they can be real syllogisms. On the other hand, the sort of recasting that this argument might need is much less than in the *Topics* passage. Indeed, it seems like the natural way to go here would be to think of this as a syllogism in Darii, as the first premise is obviously false with a universal quantifier.⁶⁶ Then we would have:

⁶³δήλου δ΄ ὄντος ὅτι τὸ Α τῷ Γ ὑπάρχει, ἀλλὰ διὰ τί ὑπάρχει, τὸ ζητεῖν τὸ Β τί ἐστι, πότερον ἀντίφραξις η̈ στροφὴ τῆς σελήνης ἢ ἀπόσβεσις. τοῦτο δ΄ ἐστιν ὁ λόγος τοῦ ἑτέρου ἄχρου, οἴον ἐν τούτοις τοῦ Α· ἔστι γὰρ ἡ ἔχλειψις ἀντίφραξις ὑπὸ γῆς. τί ἐστι βροντή; πυρὸς ἀπόσβεσις ἐν νέφει. διὰ τί βροντᾶ; διὰ τὸ ἀποσβέννυσθαι τὸ πῦρ ἐν τῷ νέφει. νέφος Γ, βροντὴ Α, ἀπόσβεσις πυρὸς τὸ Β. τῷ δὴ Γ τῷ νέφει ὑπὰρχει τὸ Β (ἀποσβέννυσθαι τὸ πῦρ ἐν τῷ νέφει. νέφος Γ, βροντὴ Α, ἀπόσβεσις πυρὸς τὸ Β. τῷ δὴ Γ τῷ νέφει ὑπὰρχει τὸ Β (ἀποσβέννυσθαι τὸ πῦρ ἐν αὐτῷ τὸ πῦρ), τούτῳ δὲ τὸ Α, ψόφος· καὶ ἔστι γε λόγος τὸ Β τοῦ Α τοῦ πρώτου ἀχρου. ἀν δὲ πάλιν τούτου ἄλλο μέσον ῆ, ἐχ τῶν παραλοίπων ἔσται λόγων. Ὁς μὲν τοίνυν λαμβάνεται τὸ τί ἐστι καὶ γίνεται γνώριμον, εἴρηται, ὥστε συλλογισμὸς μὲν τοῦ τί ἐστιν οὐ γίνεται οὐδ΄ ἀπόδειξις, δῆλον μέντοι διὰ συλλογισμοῦ καὶ δι' ἀποδείξεως· ὥστ' οὕτ' ἄνευ ἀποδείξεως ἔστι γνῶναι τὸ τί ἐστιν, οῦ ἔστιν αἴτιον ἄλλο, οὕτ' ἔστιν ἀπόδειξις αὐτοῦ, ὥσπερ καὶ ἐν τοῖς διαπορήμασιν εἴπομεν.

⁶⁴In 93b38-94a14, Aristotle articulates three kinds of definitions, one of which is indemonstrable, one which is a demonstration and one which is a conclusion of a demonstration and thus clearly demonstrable.

⁶⁵Aristotle seems here to be assuming that A both stands for thunder and what might be called the preliminary account of thunder as such-and-such a noise or the nominal definition of thunder, which is what the term "thunder" signifies.

⁶⁶What modality would it be in? This depends on whether the relations between noise, extinction of fire, and cloud are necessary. Since Aristotle seems convinced of this as a result of reflections on the various per se relations in *APo* I.4, it is pretty plausible to say that it would be an NNN

- 1. Extinction of fire belongs to some cloud.⁶⁷
- 2. Noise belongs to all extinction of fire.
- 3. Noise belongs to some cloud.

As Aristotle refers to II.8-10 in the beginning of II.13 and comes only a few pages later, this gives us an additional reason to take him to be referring to the second kind of syllogism. Aristotle would not be telling us that division is useful for syllogizing the "what it is" for any old thing, but only those things whose definitions are given in this quasi-demonstrative format. As Aristotle makes clear in *APo* II.9, we are here dealing with definitions of those things whose explanations are *something different* from the thing defined. For example, the explanation of eclipse is the earth's interposition and for thunder, fire being extinguished. We should not assume, then, that everything has this kind of quasi-demonstrative definition. Other things (which we might presume to be subject kinds) have immediate and indemonstrable essences. The point of the claim, then, would be that divisions (done properly) can be useful in syllogizing the essences of those things which are composed out of the subject kinds. This fits in nicely with the interpretation of §4.2, which also emphasized the definitional priority of the simples and the way in which their characteristics belong *because of themselves* (*kath auta*).

Summarizing, I take Aristotle to be referring in II.13 to demonstrations of the essence in II.8-10 for three reasons. First, the candidate syllogism is much closer to what we would expect in the *Analytics* than what we find in the *Topics*. Second, II.13 already contains one obvious back reference to these chapters, so we should already be prepared to see it in this context.⁶⁸ Finally, a plausible interpretation of 96b15-25 already makes a similar claim. This may not have a direct bearing on his view of *the nature of division* in this chapter, however, it is important because we new have a clearer sense of its purpose. Aristotle is trying to show a way in which division is useful for the project of II.8-10.

Now, returning to the uselessness objection, how is division supposed to contribute to construction of these quasi-demonstrative definitions? Aristotle's official response to this reasonable worry is to say that order matters for giving definitions of basic kinds, which is necessary for quasi-demonstrative definitions. Division, on his view, ensures that we give the definition in such an order that we will not give a definition of a simple subject like "biped-animal-tame". Aristotle does not explain why order makes any difference at all. Aren't the two candidate definitions equivalent logically, indeed, in any possibly important sense ?

syllogism, although he also takes Darii NXN to be valid. If these propositions are necessary, this would pose a problem for some accounts of Aristotle's apodeictic syllogistic such as Malink (2013), which require that, since cloud is a substance, anything predicated of it is also a substance, which would implausibly also include extinction of fire.

⁶⁷Even as a particular premise, this is a strange thing to say, and the corresponding claim for the eclipse demonstration: the earth coming between belongs to some moon, is even stranger. See *APr* 48a40 ff.

⁶⁸For more on the relationship between these parts of *APo* II, see Charles (2000), pp. 221 ff.

Aristotle claims that the improperly ordered definition does not have or does not display the proper unity. Ordering gives the definition the proper structure for us to say that it is one thing.⁶⁹ While this is motivated by the text, it does not yet answer the question. Why should we care about this sort of unity of the definition at all? How does the correct order aid in the project of the *Posterior Analytics*?

It seems to me that the language of unity and insistence of the verbal ordering should be read as a convention for the reader to understand the real order in which the definitional elements are applied. The point of the convention is that it allows one to reconstruct the entire genus/species structure in the right way simply from the definitions of all the infimae species. This can be done by reversing a procedure that will be described later in the chapter: essentially pulling off the last differentia in the definition one at a time to get to the next highest genus. If we understand the genus/species relationships to have explanatory power, then encoding it into the definition will give more information to someone possessing the definition about how to use it in explanations. In fact, in the procedure for finding demonstrations (or dealing with what he here calls "problems"), this sort of structure is exactly what is used.⁷⁰

4.3.3.2 How to divide according to the differences

Further, only in this way is it possible to leave nothing out in the "what it is" (*ti esti*). For whenever the first should be taken as genus, were someone to take one of the lower divisions, not everything would fall into it, for instance, not every animal is either whole-winged or splitwinged, but every *winged* animal is. For this is a difference of that [winged animal]. Rather, the first difference of animal is that into which every animal falls. Similarly also for the differences of each of the others, both the difference of the outer genera and those below it, for example the difference of bird <is that> into which every bird falls and the difference of fish <is that> into which every bird falls and the way for those proceeding to know that nothing has been left out. But otherwise necessarily one will leave something out and not know. (96b35-97a6)⁷¹

Recall that the question Aristotle must answer, following on the previous section: *what is the order in which to take the differences in order not to leave anything out?* It

⁶⁹This point is rightly emphasized by Bronstein (2016), although he does not sufficiently address the significance of unity in Aristotle's discussion.

⁷⁰See Ferejohn (1991); Lennox (1987).

⁷¹ Έτι πρός τὸ μηδὲν παραλιπεῖν ἐν τῷ τί ἐστιν οὕτω μόνως ἐνδέχεται. ὅταν γὰρ τὸ πρῶτον ληφθῆ γένος, ἂν μὲν τῶν κάτωθέν τινα διαιρέσεων λαμβάνη, οὐκ ἑμπεσεῖται ἄπαν εἰς τοῦτο, οἴον οὐ πᾶν ζῷον ἢ ὅλόπτερον ἢ σχιζόπτερον, ἀλλὰ πτηνὸν ζῷον ἅπαν· τούτου γὰρ διαφορὰ αὕτη. πρώτη δὲ διαφορά ἐστι ζῷου εἰς ἢν ἅπαν ζῷον ἐμπίπτει. ὁμοίως δὲ καὶ τῶν ἄλλων ἑκάστου, καὶ τῶν ἔξω γενῶν καὶ τῶν ὑπ' αὐτό, οἴον ὄρνιθος, εἰς ἢν ἅπας ὄρνις, καὶ ἰχθύος, εἰς ἢν ἅπας ἰχθύς. οὕτω μὲν οῦν βαδίζοντι ἔστιν εἰδέναι ὅτι οὐδὲν παραλέλειπται· ἄλλως δὲ καὶ παραλιπεῖν ἀναγκαῖον καὶ μὴ εἰδέναι.

seems like what is described is a kind of depth-first procedure:

- 1. Start with a genus.
- 2. Take a difference such that the whole genus falls into the difference. (E.g. winged or wingless)
- 3. Choose a part of the difference. (E.g. winged)
- 4. Repeat 2, 3 with this part as the new whole until the parts are indivisible.

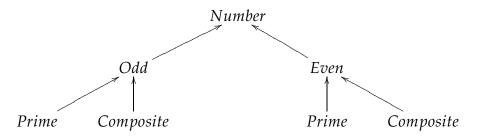
The idea, presumably, is to repeat this procedure for each of the differences. Step 2 adds a norm for dividing correctly (I call it the Covering Norm) regarding the relationship between a genus and the difference chosen: the difference has to *cover* the genus. By this, Aristotle apparently has in mind the idea that the disjunction of species should (necessarily, per se, etc) be true of the genus. This might seem a norm obviously embedded in divisional practice, but Aristotle thinks that observing it shows what division adds to the mere assumption of essential predicates.

There are two basic questions for the procedure as elaborated here:

- 1. What are we trying not to leave out? Why shouldn't we leave them out?
- 2. The procedure described above clearly generates an ordering of differences. Is this ordering *unique*? If not, is this a problem for the account?

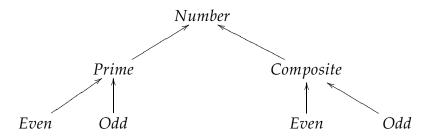
First, it should be clear from the context that we are not to leave any of the *things predicated in the essence* out. As we saw in 96a24ff, to get the essence of something from the things predicated in it, we must use all of those things. Furthermore, there is no worry of leaving too much in, since we assumed at the outset (96b28-30) that these are in the essence, not demonstrable per se attributes.

To elaborate the second worry, let us consider the genus *number* from above. Aristotle there gave examples of two pairs of differentiae which each cover the entirety of the genus: prime/composite and odd/even.⁷² He evidently takes the division of number to be, where the labels represent the additional differentia added (so that the bottom left should be taken to represent number-odd-prime):



⁷²Indeed, he introduces a second notion of "prime" which could equally be used in this case, but the point being made needs only the two distinctions.

Clearly, this division satisfies Aristotle's requirement. However, so does the division:



So, while the Covering Norm can rule out some divisions, it does not seem like enough, even in this artificially simple arithmetical case, to uniquely specify the divisional tree. Is this a problem? Aristotle seemed to think so, but this points not to some gross inadequacy of his account, but rather an incompleteness. Aristotle's worry here (cf. *APo* II.6, *Met* Z.12) is that there can only be one definition of one thing. If there is more than one scientifically sound division, then one would have the materials for making several different definitions. The literature has proposed two different kinds of strategies to remedy this incompleteness.

Perhaps Aristotle thought that he could add more general kinds of norms to guarantee that odd would be divided before prime. Charles (2000) might be read as endorsing such a view, noting that Aristotle frequently associates definitions with algorithms and that there was a method of sorting primes out of odd numbers (Eratosthenes' sieve),⁷³ but none which describe the opposite procedure.⁷⁴ Then we could imagine a pretty general norm: divide according to differences in which there is an algorithm or method for sorting members of the divisum into species. There are two challenges for this account. First, could it be applied as generally as it needs to be? For instance, one could come up with exactly the same problem by looking at the division of animals into blooded/not blooded and into terrestrial/aerial/aquatic. In this particular case, the reply does not help, however, since there is a simple algorithm for finding odd primes from primes: remove 2! Charles' defender could respond perhaps that this algorithm is of no mathematical significance, but in this case we would really be appealing to the second reply.

On the other hand, he could have thought norms of inquiry from particular sciences would be sufficient for sorting out these issues.⁷⁵ Then it would be particularly *arithmetical* considerations that would lead us to divide number first into

⁷³Incidentally, Eratosthenes lived in the third century, so Aristotle himself may not have known this method. However, in Nicomachus' *Introduction* I.13 Hoche (1866), he only attributes the *naming of the method* to him, so it may be have been discovered earlier.

⁷⁴This norm could be justified either because such algorithms could make it easier *for us* to figure out what falls into the kinds or because the algorithms are themselves indicative of some fundamental structure in reality. Thanks to Julian Jonker and Dylan Murray for pointing me in this direction.

⁷⁵This is the view of Bronstein (2016).

odd/even. What could these arithmetical considerations possibly be?⁷⁶ Would arithmetical practice be at all altered by the ordering of these cuts? Bronstein has no suggestions in this regard. The only refuge I can see for such a view is again to look at Eratosthenes' sieve as implicitly committing arithmetic to the priority of the odd/even distinction.

Interestingly, the tradition of division has not, in the whole, followed Aristotle's obsession for uniqueness in this respect. Boethius is quite explicit on this point, saying "One and the same genus is divisible in more than one way just as all the corporeal entities and all things of determinate magnitude are." (*De Divisione* 885b)⁷⁷ His examples for this include the very one under discussion: odd/even and prime/composite. We have already seen that Plato seems content to divide the same kind (productive art) lengthwise and breadthwise without thinking that one division comes first.

In looking at a case like that, it seems to me that one would only seek an ordering if one were guided by deep theoretical considerations. Here we see a bit of what Aristotle's considerations could have been. He thought that the set of definitions of infimae species and the division tree needed to carry all the same information and so, if there had to be a single correct definition, then that would force us to order the division tree in a particular way. However, this just leads us back to the question: why is it that the definition number-odd-prime any worse than number-prime-odd? Aristotle could have had the view that these are mere syntactic variations of the very same definition. I don't know for sure why he did not have this view, but we can at least begin to make sense of the view that he did have, which has two components: the strong division/definition isomorphism and the strong uniqueness of definition. Together these entail an equally strong uniqueness requirement on division.

4.3.3.3 The Omniscience Objection and Response

But it is not at all necessary that one defining and dividing knows all the things that are. And yet, some people say that it is impossible to know the differences with respect to each thing while not knowing each thing, and one does not know each thing without the differences. For a thing is the same as that from which it does not differ, but something other than that from which it differs. (97a6-11)⁷⁸

The conclusion of objection here is quite simple: division presupposes omniscience. The first premise says that necessarily, if you know the differences with

⁷⁶Perhaps the conceptual simplicity of odd/even, which, unlike prime/composite only makes reference to division by one number while the latter makes reference to divisibility in general.

⁷⁷Fit autem generis eiusdem multipliciter diuisio, ut omnium corporum et quaecumque alicuius sunt magnitudinis.

⁷⁸οὐδὲν δὲ δεῖ τὸν ὁριζόμενον καὶ διαιρούμενον ἄπαντα εἰδέναι τὰ ὄντα. καίτοι ἀδύνατόν φασί τινες εἶναι τὰς διαφορὰς εἰδέναι τὰς πρὸς ἕκαστον μὴ εἰδότα ἕκαστον· ἄνευ δὲ τῶν διαφορῶν οὐκ εἶναι ἕκαστον εἰδέναι· οῦ γὰρ μὴ διαφέρει, ταὐτὸν εἶναι τούτῳ, οὕ δὲ διαφέρει, ἕτερον τούτου.

respect to each thing (or, what differentiates X from each), you know each of these other things. This is at least initially plausible (at least, on a very weak reading), since knowing A is different from B does presuppose having a clue about what B is. And the second premise says that necessarily, if you know those things, then you know their differences. The reasoning behind both premises seems to be that since X is the same as Y iff Y is not different from X, being able to distinguish two things presupposes you can identify them, but that just requires that you know how to distinguish *them* from everything else. Aristotle's response goes in two steps:

First off, this is false. For something is not other because of *every difference*. For many differences belong to things which are the same in species, but neither in respect of their essence or in respect of themselves. Next, whenever one takes opposites and the difference and that everything falls either here or there, and one takes the object of inquiry to be in the one, and one knows this, then it makes no difference to know or not to know upon how many things the differences of the others are predicated. For it is clear that if, proceeding in this way, one arrives at those things of which there are no longer differences, one will have the account of the essence. And it is not a postulate that everything will fall into the division, if they are opposites of which there is nothing between, for necessarily everything is in one of them if it will be⁷⁹ that thing's difference. (97a11-22)⁸⁰

First, Aristotle denies the reasoning in support of both premises (and along with it Premise 2), saying that, since only some differences make a thing different, we only need to know some differences. In particular, things of the same species are different from one another, but one does not have to have that kind of knowledge in order to know what something is. To give a concrete example, one can know what a dog is without knowing what distinguishes Fido from Lassie nor what distinguishes Aristotle from Plato. Instead, knowing what a dog is seems to involve being able to distinguish it from, for instance, things without souls in general, or biped animals in general, without having to know anything in particular about rocks or humans. This may not be much comfort to Speusippus (or whoever came up with the argument⁸¹) without assurances that knowing one thing does not in-

⁷⁹Reading *estai* with *ABn*, Ross reads *esti* with *d*.

⁸⁰πρῶτον μὲν οῦν τοῦτο ψεῦδος· οὐ γὰρ κατὰ πᾶσαν διαφορὰν ἕτερον· πολλαὶ γὰρ διαφοραὶ ὑπάρχουσι τοῖς αὐτοῖς τῷ εἴδει, ἀλλ' οὐ κατ' οὐσίαν οὐδὲ καθ' αὐτά. εἶτα ὅταν λάβῃ τἀντικείμενα καὶ τὴν διαφορὰν καὶ ὅτι πᾶν ἐμπίπτει ἐνταῦθα ἢ ἐνταῦθα, καὶ λάβῃ ἐν θατέρῳ τὸ ζητούμενον εἶναι, καὶ τοῦτο γινώσκῃ, οὐδὲν διαφέρει εἰδέναι ἢ μὴ εἰδέναι ἐφ' ὅσων κατηγοροῦνται ἄλλων αἱ διαφοραί. φανερὸν γὰρ ὅτι ἂν οὕτω βαδίζων ἔλθῃ εἰς ταῦτα ῶν μῃκέτι ἔστι διαφορά, ἔξει τὸν λόγον τῆς οὐσίας. τὸ δ' ἅπαν ἐμπίπτειν εἰς τὴν διαίρεσιν, ἂν ἢ ἀντικείμενα ῶν μὴ ἔστι μεταξύ, οὐκ αἴτημα· ἀνάγκῃ γὰρ ἅπαν ἐν θατέρῳ αὐτῶν εἶναι, εἴπερ ἐκείνου διαφορά ἔσται.

⁸¹See Falcon (2000) for strong reasons to accept the identification of Speusippus with the author of the argument.

volve also knowing an extremely large number of things.⁸² Aristotle solves this by saying that one needs to know only the differences at every point of a division. If one divides a kind into two species and places the definiendum into one of them, he says that there is no need to know what falls or does not fall under the other. One knows that everything must go somewhere, one has shown the definiendum's essential similarities and differences to everything else implicitly, without having to take each thing one by one. To return to the dog example, we would need to know, perhaps, how dogs differ from non-blooded animals *as a whole*, or birds *as a whole*. But that does not imply that one needs to know anything about beetles or penguins in particular.

Here we also see the introduction of opposites being used to generate the divisions.⁸³ The opposites under discussion here are usually *contraries*, but not contradictories—they are properties that cannot be jointly satisfied, but it is possible that they are jointly *un*satisfied. This is, I take it, the first explicit mention of the notion in discussions of division. While Plato clearly discusses opposites even in his early dialogues (e.g., *Protagoras* 332a-3b), he does not lay any real weight on it. Sometimes he mentions (e.g., *Sophist* 220e8, *Statesman* 289d7) that he is dividing into opposites, but this is not applied consistently throughout the dialogues. Even in that dialogue, for instance, production and acquisition, the greatest kinds into which art is divided are clearly not opposites.

Moreover, if one is dividing by means of differentiae, the requirement that differentiae be opposites is equivalent to the requirement that divisions ought to be exclusive. To see this, suppose that differentiae are opposites. Then nothing can fall under both. Likewise, if divisions ought to be exclusive and are generated by differentiae, then the only way to distinguish species with differentiae so that there is no species subordinate to both is to use differentiae which are opposites. Note that for Plato these questions may not come to the same because he does not make the explicit distinction between species and differentiae.

If I am right that the introduction of exclusivity as a condition on divisions is here, then we see that its introduction was not an Aristotelian (mis)understanding of Plato's own method,⁸⁴ but rather a proposed *refinement* of the method in response to a certain kind of epistemological worry. Exclusivity of divisions allows one to ignore great swaths of the divisional structure in one's defining practices and construct a definition without complete knowledge of the entire field. If Plato did not feel the force of the epistemological objection, or thought that it could be avoided in a different way, then there is no reason to suppose that he came up with the condition.

⁸²Bronstein (2016) helpfully rephrases the objection in response to this in terms of "essential differences." The objection then says that we need to know all the essences in order to know one, since we have to know everything that essentially distinguishes something from everything different in kind from it.

⁸³The notion of opposites also has a prominent place in *PA* I.2-3, but this is not likely to have been written before *APo*.

⁸⁴As Moravscik (1973) has it.

This will be more clear if we think about how exclusivity alleviates the worry. Suppose that, after dividing A into B and C and placing the target kind K into B, K could still also be in C (i.e., we have a failure of exclusivity). Then one had better divide both B and C, just in case K is also a species of C (in which case, one would have missed a part of the definition of K, namely C). Since this same issue could be raised at any point on the divisional tree, the threat of radical holism looms.

If one requires exclusive divisions by contrast, then one's knowledge can be much more compartmentalized. This is what Aristotle means when he says that, given that one knows that the genus divides into such and such kinds and that the definiendum falls into one, it doesn't matter what is predicated of the others. In that case, one really need not know anything outside the genus⁸⁵ of inquiry and one can discard the parts of each division which the target kind does not fall into.

This restriction, however, comes with a significant cost. While the assumption of exclusivity has epistemological benefits, it is, in general, not easy to establish when one is restricted to essential predicates. One needs to have a reason for saying that every subkind of the genus to be divided falls into exactly one of the divisions and *that* seems to require knowing at least a bit about every species of that genus. So while it might seem that Aristotle has made a significant advance over Plato, he has really just shifted the epistemological burden to establishing exclusivity.

On the other hand, sometimes establishing exclusivity does not actually involve looking at species one by one and checking that they all fall into exactly one genus. For example, if one were to divide lines into straight and curved lines, then one knows at the outset that no kind of line will be both. And knowing this clearly does not require knowing every species of line or particular line. This is because something is curved just in case it is not straight, in some sense, by the definition of those two notions. There is no need for these sorts of cases just to involve two species. This will come back in the *Parts of Animals* and was already discussed in the *Topics*, where Aristotle calls it division by negation.⁸⁶

To see Aristotle's view more clearly, it might be useful to compare it with one understanding of Plato's view. From his discussion of Theuth in the *Philebus* (in particular, his insistence that one must completely count out all the kinds in the genus), one might get the impression that, in order to know any one of the letters, one would have to know how it fits in with all the other letters: in effect, one could not know anything about an object in the relevant genus (here, letter) without knowing everything about that genus. This position is intermediate between Aristotle's (which only requires knowledge of some of the elements of the genus)

⁸⁵Bracketing for now those common axioms one must know in order to know anything at all. (*APo* I.2) Given Aristotle's examples, however, there does not seem to be an inordinate number of these.

⁸⁶Based on his characterization in *Metaphysics* I.8 of non-dichotomous divisions as having a *metaxu* that negates both extremes, he might even think of extending this model to divisions into multiple species.

and the conclusion of the Speusippean argument.⁸⁷

Before moving on, I want to close with a possible objection to this interpretation based on our knowledge of Speusippus.⁸⁸ Many assume that the kind of division popular in the Academy (and associated with Plato's successor in particular) was primarily a kind of division that satisfies exclusivity because it always divides a genus into two species, where one difference is the negation of the other. Why would Speusippus have embraced the omniscience objection if his own method of division did not succumb to it?

There are a number of things to say. First, the evidence that we have about Speusippus only tells us that he came up with the argument. However, that does not mean that he embraced the conclusion, just as most interpreters would not take Aristotle's reference to Meno's Paradox in *APo* I.1 to mean that Plato thought inquiry was impossible. Rather, Speusippus might have given the argument because he thought there is something interesting to be said about how it goes wrong (just like Aristotle). In fact, this is exactly the view of Falcon, who accuses Ps-Philoponus of misunderstanding Speusippus' intention. On the other hand, Aristotle does seem to think that some people actually believe the conclusion since he begins the argument saying "some say".⁸⁹

Secondly, there is only minimal evidence that Speusippus had any method of division at all, much less the particular method described above.⁹⁰ While the book lists in Diogenes Laertius and a fragment of the comic poet Epicrates preserved by Athenaeus do give us reason to think that Speusippus was part of the group which debated about and probably used division,⁹¹ is he the one referenced by *Topics* VI.6 or *PA* I.2? Our extant sources fail to tell us for sure. Even if Speusippus did embrace the conclusion and used exclusive dichotomous division, he may not have seen the connection between the two views that Aristotle is claiming here. It is certainly not an obvious move for Aristotle to say that one can completely ignore the sides of a division into which one does not place the definiendum. Perhaps Speusippus simply thought that one's knowledge of horse would be somehow degenerate if one did not possess knowledge of partridges.

⁸⁷If Plato held, however, that there is just one genus of being (as Aristotle reports), then his position is in danger of collapsing into the more radical one.

⁸⁸The attribution of this objection to Speusippus is universal among the commentators: Philoponus(?) *In APo* 405,28, Themistius *In APo* 58,4 Wallies (1900), Eustratius *In APo* 202,17 Hayduck (1907), Anonymous *in APo* 584,17 Wallies (1909). Anonymous gives Eudemus as the authority. See Falcon (2000) for a reconstruction primarily on the basis of the anonymous commentary.

⁸⁹This point is not decisive. Speusippus may not have been convinced by the argument but others were.

⁹⁰Falcon has made this point especially forcefully.

⁹¹See Dillon (2003).

4.3.3.4 How to define with divisions

And it is necessary when constructing a definition by means of divisions, one must aim at three things: 1) taking the predicates in the "what it is", 2) arranging these according to which is first or second, and 3) that these are *all the predicates*.

The first one of these is through being able (as with respect to the accident to syllogize that it holds) to construct [the definition] by means of the genus. And the arranging will be as it ought to be if one should take the first predicate. And this will be if the predicate is taken such that 1) it follows all the [other predicates] and 2) all the [other] predicates do not follow it. For necessarily there is such a thing. Having taken this, for the lower predicates it is the same procedure. For the first of them will be the second and the first of those kept is the third. For, after having separated off the topmost, the next predicate will be first among the others. And similarly for the others. That these are all the predicates, is clear from taking with respect to the first term in the division, that every animal is this or that, and this one belongs < to what is being defined>, and then taking the difference of *this* whole, and that there is no longer a difference of the final whole, or with just this last difference, this no longer differs from its whole in species. For it is clear that neither has more been added on (for everything was taken from the things in the what it is) nor is anything left out. For either it would be 1) a genus or 2) a difference. But 1) both the first thing and it [the first one] taken with the differences are the genera. And 2) all the differences are contained <in the definition>, for there are no subsequent ones, for the last would have differed in species, but it was said not to differ. $(97a23-b6)^{92}$

Now Aristotle describes how to put the method of division to use in the construction of the definition. The procedure must satisfy three requirements:

1. The predicates must all be in the "what it is".

- 2. They must be arranged in the right order.
- 3. These must be all the predicates in the "what it is".

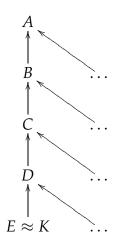
The procedure:

- Base case: Start with all of the predicates. The first item in the definition is the one that follows everything but nothing follows it.
- Inductive step: Assume that you have arranged *n* out of *k* predicates. The *n*+1 item in the definition will be the one of the remaining that follows everything but none of the remaining predicates follow it.

This procedure assumes that the relation of "following" well-orders the predicates in the essence. A well-ordering is a relation that is transitive, reflexive, anti-symmetric and has the least upper bound property, basically assuring us that there will always be at most one thing "next" in the sequence. For otherwise, we would not be guaranteed the existence of a unique term that comes next in the process. Is the assumption justifiable? As we have seen before, Aristotle has set himself the project for formulating norms for division (covering and exclusivity) that are meant to ensure the attainment of the uniquely correct divisional tree. Although there are difficulties for him in this regard, if he does manage to specify such norms, I take it that his job in what follows is complete. This is because any such division will have the following properties:

- 1. There will be a unique path from the genus, through all the subordinate species, to the target kind.
- 2. That path will be finite and hence a well-ordering.⁹³

The procedure, then, can make use of *that very same well-ordering*. The idea is to begin with a tree going down to our target kind *K* (ignoring the other branches):



⁹³Otherwise the division would never have been completed.

According to the procedure, we can now get a definition for *K* in terms of *ABCDE*. Much, however, depends on how we construe those letters and the relation of following (which Bronstein calls "subsumption" and McKirahan says "specify increasingly narrow subclasses") between the elements of the definition. In the context, I take it that the most natural, and weakest, notion of subsumption will be satisfied trivially: if a kind is divided into two sub-kinds, the kind will "follow" both of the sub-kinds. Bronstein and McKirahan instead take this to be a claim about the differentiae, the properties used to divide these sub-kinds, claiming that *they* must stand in a subsumptive relationship.⁹⁴ The genus-species relation being subsumptive is weaker because it is implied by the differentiae being subsumptive,⁹⁵ but not vice versa.

To see why the weaker reading should be preferred to the McKirahan-Bronstein reading, we must look at a passage in the *Sophistical Refutations* where Aristotle discusses the relationship between the species and differentiae.

In the *Sophistical Refutations*, Aristotle considers arguments of the following sort:

- 1. Odd is a number containing a middle.
- 2. There is an odd number.
- 3. So, there is a number number containing a middle. (SE 173b8-10)

Finding premises obviously true and the conclusion unintelligible, Aristotle proposes a general solution to these problems (in *SE* 13 and 31), saying that it was wrong to assume that "odd" has the same meaning in isolation and when put together with its genus. It is either the case that in isolation it has no meaning at all or that its meaning is different from when it is in context with its genus.⁹⁶ This is tantamount to saying that the "odd" in the second premise cannot be substituted with the definition of "odd" in the first. The idea, then, is that by using "differentia" terms we can either indicate something that the genus follows (the species) *or* something that it does not.⁹⁷ It is key to understanding the argument that follows that each successive "taking" in the procedure that Aristotle described above moves one step down the divisional tree (hence why he said he claimed to be describing a process of establishing a definition *through division*).

⁹⁴As they seem to in, for instance, *Metaphysics* Z.12 or *PA* I. Even there, however, I am not sure that Bronstein and McKirahan are right.

⁹⁵Suppose the differentiae are subsumptive, then if A is a species of B and B of C, the differentia of A will be subsumed in both B and C, guaranteeing that A is also subsumed.

⁹⁶In 13, Aristotle has three different kinds of examples: double (in the context of the half), snub (in the context of nose), and odd. Although he does not repeat all of these examples in 31 (where he gives the general form of solution), he gives no indication that there will be different solutions for the case of odd.

⁹⁷This point actually needs to be further refined. In this passage, as well as in *Topics* 144b12-144b30, Aristotle says that all the differentiae have to imply the summum genus of a science, but may not follow lower genera. But the point still stands: when differentia terms are used in the context of a definition, they do not, in general, imply the genus.

Each thing taken, then, seems to be taken first on its own and then attached onto the longer definitional phrase. So, when we evaluate following, we will be taking, say, *B* on its own and asking whether *A* follows it. Since *B* is a species of *A*, this trivially holds. The idea, then, is to add *B* onto the end of *A* so that we have *AB*. This, given the doctrine of *SE*, means that *B* no longer has the same meaning taken alone that it had before. But the difference is harmless, now that *AB* now clearly has the meaning that *B* ambiguously had. This means that we can understand his procedure as only requiring a weak form of subsumption, one trivially satisfied by any division.

Since we are clear on the procedure, we can understand Aristotle's argument that this is the definition. Suppose first that we have a division done according to the covering and exclusivity norms which has K as a final⁹⁸ node and in which all and only elements of K's essence have been used in a cut.⁹⁹ Then consider the definition of K as *ABCDE*. First of all, it clearly includes all and only the predicates in the essence, thus satisfying desiderata 1 and 3. Secondly, it is easy to check that it will be in the right order, at least insofar as Aristotle has given us guidance on the proper ordering.

Now we are in a position to evaluate Bronstein's and McKirahan's charge that Aristotle's conception of a divisional definition here is inconsistent with what he said earlier (96a24-b14) about the relationship between the things predicated in the essence. For instance, here is McKirahan (1992):

The method described here [96a24ff] is different from that for reaching type A definitions [definitions by genus and differentiae], where we take successively narrower differentiae until we reach one that exactly covers the desired class; here we find a set of predicates, each of which is wider than the desired class. (Hence the two approaches are actually incompatible.) (pp. 114-115)

The worry works in the following way: suppose that the subsumption requirement held not for species (as I had argued) but for *differentiae*. Then the final differentia will be co-extensive with the target kind—otherwise we would not have a complete definition and will have instead defined a genus. However, one of the requirements for the things predicated in the essence from above was that each of the terms *extends more broadly* than the target kind. Therefore the final differentia cannot be co-extensive with the target kind. Contradiction.

The solution to this problem is already laid out in the interpretation above. The subsumption requirement has to hold of the definitional predicates qua species of the genus. In this way, then, the final predicate *E* when taken on its own will be co-extensive with the essence. On the other hand, qua differentiae, all the elements of

⁹⁸The assumption that it is the final one is unnecessary in what follows, but seems to be what Aristotle has in mind.

⁹⁹Note that there is nothing to prevent several parts of the essence from being used in the same cut.

the essence must extend more broadly than the target kind. And we should actually expect this from how Aristotle discusses them. In the first passage, Aristotle is looking at a definition like *ABCDE* and asking what role each of those parts plays. In that context, it is natural to think that each part should be doing some work and hence no part should just be co-extensive with the target kind. In the present passage, the very same kind of term is being used to name something into which the genus is divided, which would be one of its species.

4.3.4 The account of the summum genus

And it is necessary that the one observing things that are similar and without differences investigates first what same thing they [the similar things] all have, and then again [investigate] *other* things that are 1) in the same genus as them [the similar things], but 2) are the same in species as themselves and 3) are different in species from them [the similar things]. And when one has taken for these that same thing which they all have, and similarly for the different things, then for the remaining things it is necessary to investigate if it has the same thing until one comes to *one account*. For this will be the definition of the subject. But if one should not proceed to one account, but rather to two or many, it is clear that the object of inquiry would not be some one thing, but many.

I mean, for instance, if we were to inquire what is pride, we ought to look at some proud people whom we know, what one thing they all have as such. For instance, if Alcibiades is proud as well as Achilles and Ajax, what one thing do they all have? Not being tolerant when insulted: for one made war, the other was wrathful, the last killed himself. Then for others, such as Lysander or Socrates. If they are indifferent to good or bad fortune, taking these two things I investigate what is the same thing that both insensitivity to fortune and intolerance of being dishonored have. If nothing, then these would be two species of pride. (97b7-25)¹⁰⁰

Here Aristotle describes a procedure for investigating similarities for a definition of a subject:

¹⁰⁰Ζητεϊν δὲ δεῖ ἐπιβλέποντα ἐπὶ τὰ ὄμοια καὶ ἀδιάφορα, πρῶτον τί ἄπαντα ταὐτὸν ἔχουσιν, εἶτα πάλιν ἐφ΄ ἑτέροις, ἂ ἐν ταὐτῷ μὲν γένει ἐκείνοις, εἰσὶ δὲ αὐτοῖς μὲν ταὐτὰ τῷ εἴδει, ἐκείνων δ΄ ἔτερα. ὅταν δ' ἐπὶ τοῦτων ληφθῆ τί πάντα ταὐτόν, καὶ ἐπὶ τῶν ἄλλων ὁμοίως, ἐπὶ τῶν εἰλημμένων πάλιν σκοπεῖν εἰ ταὐτόν, ἔως ἂν εἰς ἕνα ἔλθη λόγον· οὕτος γὰρ ἔσται τοῦ πράγματος ὁρισμός. ἐἀν δὲ μὴ βαδίζη εἰς ἕνα ἀλλ' εἰς δύο ἢ πλείους, δῆλον ὅτι οὐκ ἂν εἴη ἕν τι εῖναι τὸ ζητούμενον, ἀλλὰ πλείω. οἴον λέγω, εἰ τί ἐστι μεγαλοψυχία ζητοῖμεν, σκεπτέον ἐπί τινων μεγαλοψύχων, οῦς ἴσμεν, τί ἔχουσιν ἐν πάντες ἢ τοιοῦτοι. οἴον εἰ Ἀλκιβιάδης μεγαλόψυχος ἢ ὁ Ἀχιλλεὺς καὶ ὁ Αἴας, τί ἐν ἅπαντες; τὸ μὴ ἀνέχεσθαι ὑβριζόμενοι· ὁ μὲν γὰρ ἐπολέμησεν, ὁ δ' ἐμήνισεν, ὁ δ΄ ἀπέκτεινεν ἑαυτόν. πάλιν ἐφ΄ ἑτέρων, οἴον Λυσάνδρου ἢ Σωκράτους. εἰ δὴ τὸ ἀδιάφοροι εἶναι εὐτυχοῦντες καὶ ἀτυχοῦντες, ταῦτα δύο λαβών σκοπῶ τί τὸ αὐτὸ ἔχουσιν ἢ τε ἀπάθεια ἡ περὶ τὰς τύχας καὶ ἡ μὴ ὑπομονὴ ἀτιμαζομένων. εἰ δὲ μηδέν, δύο εἴδη ἂν εἴη τῆς μεγαλοψυχίας.

- 1. For every group *g* of similar things within a genus, figure out what one thing everything in *g* has. Call each of those a similarity.
- 2. Repeat 1, now treating each looking at groups of similarities, until one thing is reached which everything has in common.¹⁰¹

If this unifies all the groups, then one has given the definition of the subject at hand. Otherwise, one has realized that there is not *one* subject at all.

Aristotle here is giving us a procedure which seems intended to define something quite *general*, not an infima species, since he is starting with individuals or possibly infimae species and then attempting to reach something which they have in common. It is natural to think that he is giving us an idea of how one could come to understand the genus according to the 96b15-25 procedure. This would be expected from the beginning of the chapter, since the things predicated in the what it is include both definitions and genera, and claiming that one of the preliminary things one must do before defining composites is to discover the genus. The previous method he discussed—that of division according to differentiae—cannot accomplish this, since the genus is always going to be at the top of the hierarchy. In order to ensure us of the possibility of finding definitions of all the kinds relevant to the science, he needs to make sure that the genus is covered. However, there is no reason to think that it could not also be used for definitions of other kinds. (More on this below.)

While the overall way of going is relatively clear, there are some interesting questions about how this relates to procedures described in Plato and Aristotle. Is it, as Bronstein claims, the use of induction to arrive at (certain) principles? Or is it more like an application of Platonic collection? I find it quite plausible as an instance of the latter, as we are clearly in an Academic context (with the discussions of division and definition) and Plato's discussion in the *Phaedrus* and *Philebus* of collection seems to have this very same feel.¹⁰² Additionally, Aristotle nowhere discusses *sunagoge* with the Platonic meaning, and as this seems like an integral part of Platonic dialectic, it would be natural to seek *some engagement* with it by Aristotle in which he does not mention it by name. This, indeed, is not as incompatible with Bronstein's approach as it might initially seem. There is considerable unclarity concerning Aristotle's notion of induction and it does not seem unlikely that he would have thought it closely connected to what Plato called collection.

4.4 Conclusion

From these texts, we can clearly see the role that Aristotle sees for division in science. Like his predecessors, Aristotle takes division's only use to be in the realm of

¹⁰¹It is necessary that these similarities be somehow explanatorily relevant, something not mentioned by Aristotle in this passage.

¹⁰²In contrast, he primarily associates induction with Socrates (*Metaphysics* M.4).

inquiry into the essence. Moreover, even in this domain, division is primarily used in finding out the essence of kinds which are neither composites like snub-nose nor summa genera¹⁰³ and can only do this with the *assumption* of all the things predicated in the essence. This does not render division completely impotent, on Aristotle's view, since it can still play the important role of properly ordering the parts of the definition, as well as show exactly how much a definer of a kind X needs to know about those things which are not X.

This should be enough to show that Cherniss, Ross, Detel, and Goldin, who attribute a merely heuristic (in the sense of a method of guess and check) or pedagogical role to division in the *Analytics* are simply not giving it its due.

However, need we go so far as Bronstein? Recall that, on his reading, Aristotle is saying division is *the only way* to arrive at definitions of the relevant kinds of objects and that, in addition to discovering the order, it discovers the differentiae.

Take the second issue first. Aristotle, when describing correct division at several points in the critical sections, Aristotle clearly argues that division requires the assumption of "everything in the essence" (II.5 91b27 ff), a requirement repeated as a premise in the Omniscience Objection that Aristotle does not dispute. This would include the differentiae. Knowing the differentiae of a target kind in advance is supposed to allow the divider to make the right divisions, so it should not be something that the inquirer discovers by making the divisions. Further, he claims in *APr* I.31 that division is useless when the inquirer is ignorant of which differentia applies to a target kind. This point seemed right. It doesn't seem like doing division helps one make the choice of which differentia to place the target kind under. So the differentiae should be presupposed, just like he says in *APo* II.5 and II.13. So, on the whole, it seems that Aristotle held that division needed to assume the differentiae (and thus could not discover them) and was only useful in generating the order. Moreover, given the kinds of criticisms he himself gives of division, this seems like the right view.

Nor is it the case that Aristotle thinks division is unique. First of all, Aristotle has set himself a how-to question at the beginning of II.13 and generally successful answers to such questions do not presuppose that they are the *only* answer to that question. For instance, if I answer the question "How does one get to the freeway?", I am not saying that the route I have described is the only such way or even the best way. So, although Aristotle is here clearly saying that division plays an important role in our discovering certain kinds of definitions, why shouldn't there be other ways? Indeed, Aristotle might even be thought to give an example of this.¹⁰⁴ For we see, in his discussion of defining the genus, that he could be thought of as defining kinds in between infimae species and genus. It is not inconsistent with any important doctrines that this collection (or, if Bronstein is correct

¹⁰³On this point, of course, Platonists agree and postulate collection to fill that role.

¹⁰⁴In *De Anima* I.1, Aristotle explicitly raises the question of whether there is one way to discover the *ousia* of a thing and seems to give a negative answer. It is not decisive, however, as there it might be the case that by *ousia* he means substance, not essence.

about understanding §4.3.4, induction) could not be equally effective in defining such an intermediate kind as division. On Bronstein's view, this should be a cause for alarm.¹⁰⁵ However, on a more relaxed conception, it is not troubling that Aristotle has two methods for covering the same kinds, so long as 1) they produce the same definition and 2) every type of scientific object capable of definition is in fact covered by the methods. On my interpretation, these hold even though Aristotle could have thought multiple methods could arrive at the definition.

Secondly, the only evidence Bronstein (indeed, following Barnes and Goldin) has for Aristotle holding the stronger view is that he translates, for instance, "Zŋtɛĩv $\delta \epsilon \delta \epsilon \tilde{\epsilon} t \epsilon \mu \beta \lambda \epsilon \pi \sigma v \tau \alpha \epsilon \pi \tilde{\epsilon} \tau \alpha \delta \mu \sigma \alpha x \alpha \tilde{\epsilon} \delta \epsilon \delta \epsilon \sigma \sigma ought to look to$ $ward the similar and undifferentiated things, and then investigate in such-and-such ways. However, the participle "<math>\epsilon \pi \mu \beta \lambda \epsilon \pi \sigma v \tau \alpha$ " should not be translated as another infinitive, but instead as restricting the scope of the " $\delta \epsilon \tilde{\epsilon}$ " to those who are doing the looking and being neutral on whether one should look toward those things and only giving instruction to those who in fact are looking towards these things. If, instead of the participle, there was a noun like "dog" occupying the same grammatical role in the sentence, we would not be tempted to say that Aristotle thinks it is necessary that everyone be a dog and inquire, but rather that *dogs* should inquire. The conclusion, then, is not that Aristotle tells everyone to inquire in a certain way, but rather that, if they are doing it in a certain way already, then observing certain rules will allow them to reach their goal. This latter claim is significantly weaker in not ruling out alternative procedures of inquiry and is all that the text warrants.¹⁰⁶

Perhaps the most troubling section for this more permissive view is 96b15, when he says that in dealing with any kind one *must* ($\chi\rho\dot{\eta}$) do various things, such as divide a genus and define it. However, there he does not specify the way in which one is to divide the genus to guarantee a definition (this is put off for 10 lines) and when he does, he calls divisions according to differentiae "useful" ($\chi\rho\dot{\eta}\sigma\mu\sigma\varsigma$, 96b25). This does not mean that *some* kind of division isn't necessary for any definitional inquiry, only that the use of differentiae is a particularly good method.¹⁰⁷ The use of differentiae in this passage allows Aristotle to skirt various objections to the use of division and was not clearly distinguished in Plato who divided into and according to kinds. It is reasonable, then, that Aristotle's most general way of discussing divisions (see, e.g., the opening of *APr* I.31) does not

¹⁰⁵Bronstein argues that the method in §4.3.4 could not do this because it may not get the order of the differentiae right. However, the "greatest" kinds of a genus could still be defined in both ways.

¹⁰⁶One might worry that Aristotle's empiricism and emphasis on observation tells us independently that he is committed to the necessity of looking toward the similar things. However, it is not obvious to me that he has in mind here something as general as observation. It seems like there are many ways that one could make observations, for instance, looking at how things are different (or how they are both different and similar), in order to accomplish the same task. Modern observational sciences like astronomy do not follow this procedure but nevertheless are empirical.

 $^{^{107}}$ Ferejohn tries to get a stronger reading out of χρήσιμος on analogy with χρή in b15, but this just does not seem to be how Aristotle uses that word. The opposite of χρήσιμος does not mean unnecessary, but *useless*.

make references to differentiae.

Finally, this weaker reading allows us to see that the passages here are not actually in conflict with Aristotle's aporematic methodology in, for instance, the *Physics* or *De Anima*. There he uses what are apparently very different methodologies in coming to the first principles of science. On my view, this is not surprising, as Aristotle's discussion in the *Analytics* is noncommittal as concerns other methods of finding the principles.

On the other hand, it seems that Aristotle at least sometimes does proceed by this method in his scientific practice. Here I will briefly mention what I take to be the clearest example of this. In *Categories* 14, Aristotle uncontroversially divides change into generation, destruction, increase, diminution, alteration, and change of place. When he goes on in *Physics* III.1-3 to give his scientific account of change, he starts in III.1 by giving and then defending a particular definition of change as a whole: change is the actuality of the potential being as such.¹⁰⁸ When summarizing what he's done, he makes the following observation:

It was said both what change is both generally and part by part. For it is quite obvious how each of its species will be defined. For alteration is the actuality of the alterable as such. And still more scientifically, the actuality of what potentially acts and is affected as such, both in general and again for each [species], whether housebuilding or healing. And one will speak in the same way concerning each of the other [kinds of] change. (*Physics* 202b23-29)¹⁰⁹

Here we therefore get a move from the first stage of inquiry, the definition of the genus *change*, to the definition of its species. Aristotle here thinks that the hard part about this particular case was getting the correct definition of the genus. We might even see a move to the final stage of definition if we take his later discussion of, for instance, time, to be defining a complex. Time, which he says is the number of change with respect to earlier and later (*Physics* 219b1-2), is plausibly such a complex because the two main components of its definition (number and change with respect to before and after) are understandable quite independently of one another and properly studied in different sciences. They closely resemble "snubness", which is one of the prime examples of such complexes. So, we have some evidence in Aristotle's own scientific practice that he made use of division in the search for definitions in the very same way in which *APo* says that it should.

We can also see the role of norms in this discussion. Like Plato, Aristotle thinks that we must observe certain norms in order for division to be practiced correctly.

¹⁰⁸The precise meaning of this definition is subject to seemingly endless controversy. Fortunately, none of it concerns us here. For a useful overview and the difficulties with current proposals, see Anagnostopoulos (2010).

¹⁰⁹τί μέν οὖν ἐστιν κίνησις εἴρηται καὶ καθόλου καὶ κατὰ μέρος· οὐ γὰρ ἄδηλον πῶς ὑρισθήσεται τῶν εἰδῶν ἕκαστον αὐτῆς· ἀλλοίωσις μέν γὰρ ἡ τοῦ ἀλλοιωτοῦ, ἤ ἀλλοιωτόν, ἐντελέχεια. ἔτι δὲ γνωριμώτερον, ἡ τοῦ δυνάμει ποιητικοῦ καὶ παθητικοῦ, ἤ τοιοῦτον, ἁπλῶς τε καὶ πάλιν καθ' ἕκαστον, ἢ οἰκοδόμησις ἢ ἰάτρευσις. τὸν αὐτὸν δὲ λεχθήσεται τρόπον καὶ περὶ τῶν ἄλλων κινήσεων ἑκάστης.

In addition, we can see that Aristotle's introduction of norms follows a very specific pattern: they are always introduced as responses to objections to division. Thus we should not expect in these passages anything like a *complete* theory of divisional norms unless we also think that Aristotle has expressed every objection he could think of. Indeed, because of further objections such as concerning the unity of the definition in *Metaphysics* Z.12, it is more plausible to think that Aristotle only formulates norms to deal with objections that are relevant to his current discussion.

Chapter 5

Division, Syllogistic Method, and Science in *Prior Analytics* I.31

In the first book of the *Prior Analytics*, Aristotle sets out the first logic. It consists of a deductive system (I.4-22), meta-logical results (I.23-26), and a method for finding and giving deductions (I.27-29) that can apply in "any art or science whatsoever" (I.30). After this, Aristotle compares this method with Plato's method of division, a procedure designed to find essences of natural kinds through systematic classification.

This critical comparison in *APr* I.31 raises an interpretive puzzle: how can Aristotle reasonably juxtapose two methods that differ so much in their aims and approach? What can be gained by doing so? Previous interpreters have failed to show how this comparison is legitimate or what important point Aristotle is making.

The goal of this chapter is to resolve the puzzle. Aristotle's comparison makes sense once we view both division and the syllogistic method as general, rigorous scientific methods aimed at investigating part-whole relations between kinds. The point of the criticisms is to show that his syllogistic method, unlike Platonic division, allows the scientist to produce valid arguments, which are of crucial importance to the scientific enterprise. With the methods situated in this wider context, Aristotle's critical comparison doesn't just make sense, it serves a crucial function within his overall project: the comparison with division highlights how the valid arguments produced by the syllogistic method are valuable in science. These valid arguments could not be produced by the method of division. Far from being an anomalous chapter in the treatise, *APr* I.31 highlights the broader philosophical and scientific interest of Aristotle's logical work.

The plan for this chapter is, first, to introduce the puzzle of Aristotle's comparison and show how previous attempts to resolve it are inadequate (§5.1). Then, I will argue that Aristotle's comparison is legitimate because both methods are intended to contribute towards a methodology for science that 1) applies in any domain and 2) is rigorous (§5.2). Not only are they similar as methods, the sorts of claims that each method yields are part-whole (mereological) relations among kinds (§5.3). Finally, I will show how the common framework resolves the puzzle and helps show what Aristotle thought was important about the syllogistic method (§5.4).

5.1 The Puzzle of *APr* I.31

Here I will summarize the account of Plato's method of division developed in chapters 1-3, introduce Aristotle's syllogistic method and, in doing so, explain the puzzle of Aristotle's critical comparison.

5.1.1 Division

The "What is *F*?" question lies at the core of many Platonic dialogues. In these dialogues, the character Socrates searches for the essence of some philosophically important kind such as virtue (*Meno*), piety (*Euthyphro*), or knowledge (*Theaetetus*). Most of these dialogues, however, end in aporia, puzzlement, about the answer to the "What is *F*?" question. And this is tragic, because the character Socrates often commits himself to the priority of knowledge-what: without it, no other knowledge about that thing is possible (*Meno* 71b3-4). We are left with a skeptical conclusion: there is no systematic way to get knowledge-what (which I will henceforth call knowledge of the nature or essence), but all our other knowledge depends on it, so knowledge is impossible.

Plato, in several of his late dialogues (especially the *Phaedrus*, *Sophist*, *States*man, and Philebus), develops the method of division in part to solve this longstanding Socratic problem. This is a procedure for creating classifications by dividing more general kinds into increasingly specific ones. The most important use of the method for our purposes (although possibly not its only use) is to seek the nature or essence of a target natural kind. In the *Statesman*, for example, to find out what statesmanship *is* (i.e., the nature of statesmanship) the interlocutors begin by agreeing that it is a kind of *knowledge*. Then they distinguish kinds of knowledge in stages, first dividing knowledge into practical and theoretical, then dividing theoretical knowledge in turn, until they arrive at statesmanship. Plato recognized that, unguided, the method of division may not reach the target essence. To combat this problem, he describes norms for division that would apply in any domain whatsoever, for example, to divide a genus into the smallest possible number of species (Statesman 287c). According to Plato, applying these norms systematically when dividing makes it more likely that the method does not return anything but the essence of the target kind.¹

The method of division promises a way to systematically search for essence by situating the target kind in relation to other kinds in the same domain. By

¹I will defend this controversial claim below, §5.2.2.

understanding how statesmanship is fundamentally similar to and different from other kinds of knowledge, one can hone in on the thing itself. By setting it down as a kind of theoretical knowledge, for example, Plato can distinguish statesmanship from all the manual arts in one go. Instead of focusing on the target kind all by itself, as Socrates seems to do in the aporetic dialogues, the method of division provides a holistic way of searching for essence if one can "carve nature at its joints" (*Phaedrus* 265e), one will find out about the essences of a number of related kinds at once. These related kinds are all basic to the ontology—by coming to know how they are similar to and different from one another, one can discover their essences.

5.1.2 Syllogistic

Aristotle describes his syllogistic method in *Prior Analytics* I.27-30. In the preceding chapters, he defined the syllogism (his technical term for a valid argument) and analyzed under what conditions a syllogism comes about (*APr* I.1-22). In the chapters on the syllogistic method, he uses meta-logical results proven in I.23-26 to show that there is a method for discovering syllogisms with the desired conclusions by sorting any set of premises into six different lists. The end result is an algorithm that searches these lists and returns a syllogism with the desired conclusion. Depending on the logical form of the conclusion, only a subset of the lists will be relevant.

To see how the method works, let's work through an example. Suppose we want to demonstrate that no human is an insect, or, in the language of I.31, establish scientifically that insect belongs to no human. The first thing we need to do is sort *true* premises into various lists depending on their form. The relevant lists are:

- 1. Propositions of the form "No human is X" with different values of X: fish, horse, etc.
- 2. Propositions of the form "Every human is X": mammal, animal, etc.
- 3. Propositions of the form "No insect is X": mammal, horse, etc.
- 4. Propositions of the form "Every insect is X": perceiver, invertebrate, animal, etc.²

Aristotle's procedure looks for a pair of premises with the same term substituted for X either from lists 1 and 4 or from lists 2 and 3. If there is such a pair, you can construct a sound argument for the conclusion. In the example, we could use the propositions:

²The two other lists, not relevant for this example, would be propositions of the form "Every X is a human" and "Every X is an insect". Note that Aristotle describes these lists as lists of *terms* for which the corresponding proposition is true. However, it is equivalent to work with lists of propositions and the point is easier to understand.

- 1. Every human is a mammal. (list 2)
- 2. No insect is a mammal. (list 3)
- 3. No human is an insect. (desired conclusion)

While this is a trivial case, the method is quite powerful. It is a highly tractable procedure that always results in a syllogism with the desired conclusion. Depending on the form of the conclusion that you want to draw, Aristotle's procedure says which lists you need to look into. Finding pairs of premises will automatically allow you to draw the desired conclusion.

Aristotle thinks that this method is quite powerful:

The method is one and the same for all things, both concerning philosophy as well as any skill or learning whatever. $(APr I.30 46a3-4)^3$

The method can be applied in any argumentative context. While the arguments that one gives on a particular occasion differ depending on the subject matter (physics, biology) and intent (scientific demonstration, dialectically effective argument), the principles behind these arguments (what makes them *valid*) and the ways of finding them are the same. In what follows, when I use the term "syllogistic", I am referring to this method, although the term is used in the contemporary literature to refer more generally to Aristotle's logical theory.⁴

The basic idea, then, is that any scientist, philosopher, or technician inquiring into any subject matter with the aim of getting truth ought to first discover true premises of these particular kinds and then produce syllogisms for the conclusions desired. Demonstrations are syllogisms the possession of which gives us knowledge, since they have true, explanatory premises. This means that the syllogistic method plays an important role in our pursuit of knowledge.

5.1.3 The Puzzle

These two methods seem incredibly different. First, they have very different goals. The goal of the method of division is an account of the essence of a target natural kind, which Aristotle and later philosophers would call a "definition". The goal of the syllogistic is a deductive argument for a given conclusion. That conclusion need not state the *essence* of anything at all—in the example above, "not being an insect" is not a part of human nature, even if it is necessarily true. Most strikingly, division has as its goal something that must be *true*, which is neither necessary for the conclusion of a syllogism nor for a syllogism itself (which is not even a candidate for truth or falsity). While we want our syllogisms to have true premises (and hence a true conclusion) in demonstrative contexts, in dialectical contexts, we

³ Η μέν οῦν ὁδὸς κατὰ πάντων ἡ αὐτὴ καὶ περὶ φιλοσοφίαν καὶ περὶ τέχνην ὁποιανοῦν καὶ μάθημα·

⁴For the phrase *"syllogistike methodos"* see Alexander *In Topicorum* 2, 1-2 Wallies (1891).

only want to have premises in accordance with belief. So a perfectly good use of the syllogistic method might conclude a falsehood from falsehoods.

The methods also take very different routes. The method of division begins with a genus and progressively narrows down until one reaches the target kind. The syllogistic method consists of two steps, neither of which is anything like this. The first step organizes premises into six lists and takes premises for a given deduction from two of these lists. The second constructs a deductive argument for the conclusion with those premises. In neither case does the user of the syllogistic method "narrow down". Nor does the divider ever seek premises for a deductively valid argument or give such an argument.

Most striking, perhaps, is the problem that division and syllogistic seem to yield very different sorts of claims.⁵ The method of division investigates definitions, which are a kind of identity claim. Syllogistic, by contrast, investigates quantificational claims. These claims differ significantly in their syntactic structure, but also in their modal status. Definitions require a very strong connection between the definiens and definiendum, much stronger than even the modal propositions of Aristotle's syllogistic.⁶

Aristotle's comparison of the two methods in *Prior Analytics* I.31 seems so farfetched that, instead of clarifying their connection, it reinforces the impression that they are fundamentally different. He introduces his discussion by immediately finding fault with the fact that division does not do what the syllogistic does:

It is easy to see that division of genera is a small part of the aforementioned method. For division is a kind of weak syllogism. For, 1) it asks for what it ought to show and 2) it deduces something higher up. This first point eluded all those using division and they tried to persuade us how a demonstration concerning essence and the what it is can come about, with the result that they neither understood what in particular⁷ it is possible that those dividing⁸ deduce, nor that it was possible only as we said. (*APr* 46a31-39)⁹

In this passage, Aristotle both announces the substance of his criticism and describes how the users of division got in such trouble. The line of thought Aristotle attributes to the dividers is:

1. Divisions are demonstrations of essence.

⁵See Moravscik (2004) for a development of this worry.

⁶For example, it is necessary that all and only triangles have angles summing to two right angles, but that is not the definition of a triangle.

⁷Reading ő τι instead of őτι.

 $^{^{8}}$ Reading διαιρουμένους with Waitz over διαιρουμένοις (Ross) διαιρουμένοι (Cherniss).

⁹Ότι δ΄ ἡ διὰ τῶν γενῶν διαίρεσις μικρόν τι μόριόν ἐστι τῆς εἰρημένης μεθόδου, ῥἀδιον ἰδεῖν· ἔστι γὰρ ἡ διαίρεσις οἶον ἀσθενὴς συλλογισμός· ὃ μὲν γὰρ δεῖ δεῖξαι αἰτεῖται, συλλογίζεται δ΄ ἀεί τι τῶν ἄνωθεν. πρῶτον δ΄ αὐτὸ τοῦτο ἐλελήθει τοὺς χρωμένους αὐτῆ πάντας, καὶ πείθειν ἐπεχείρουν ὡς ὄντος δυνατοῦ περὶ οὐσίας ἀπόδειξιν γενέσθαι καὶ τοῦ τί ἐστιν. ὥστ΄ οὕτε ὅ τι ἐνδέχεται συλλογίσασθαι διαιρουμένους ξυνίεσαν, οὕτε ὅτι οὕτως ἐνεδέχετο ὥσπερ εἰρήκαμεν.

2. So, divisions deduce essence.

While Plato does occasionally call divisions demonstrations (e.g., *Statesman* 273e, 277a, b), he isn't obviously using the term in Aristotle's sense as a kind of deductive argument. The inference relies on something *Aristotle* believes—namely that all demonstrations are deductions (*APr* I.1, *APo* I.2).

As for the criticisms, their content is still unclear. Aristotle here is just assuming on the basis of the word "demonstration" that the two methods try to deduce something, and then arguing that, where division fails, syllogistic succeeds. But this assumption seems totally unjustified. From the basic descriptions above, division is a method for finding essences, syllogistic a method for finding and giving valid arguments. However, Aristotle doesn't provide us with any reason to think that one of the methods is "part" of the other, or that a division is a sort of "weak" syllogism.

But perhaps it is a mistake to put too much weight on these polemical remarks. Aristotle has a number of precise complaints against division, first appearing in 1 and 2 above. Importantly, these precise complaints are meant to *explain* the sense in which division is a weak syllogism and a small part of the syllogistic method, as can be seen from how he links the first three sentences with "for". Understanding these precise claims, then, offers the surest route to understanding the polemics. Aristotle ends up making four distinct claims in the body of the chapter:

- 1. Divisions are not deductions of the definitions they seek, because the definition does not follow necessarily from the divider's assumptions. (46a31-46b25)
- 2. Division cannot refute (a refutation is a syllogism of the contradictory of the claim under discussion). (46b26)
- 3. Division cannot deduce features that are not definitional such as accidents, properties, or genera. (46b27-8)
- 4. Division is useless in solving open problems, such as whether the diagonal of a square is commensurable or not with its side. (46b28-35)

Aristotle only argues for 1 and 4, treating 2 and 3 as acknowledged by the practitioners of division, who only claim that their method establishes definitions.

His argument for 1 begins with a general description of the problem that division encounters. He had shown earlier (*APr* I.26) that every syllogism with a conclusion of the form "All A is B" has a middle term that is between A and B in generality (46a39-b3). However, if we look at an arbitrary division, this will not be the case:

For: let A stand for animal, B for mortal, Γ for immortal, and Δ for human, whose account it is necessary to understand. Then [the divider] assumes all animal is either mortal or immortal: All this (what would

be A) is either B or Γ . Again the one dividing always sets down that a human is an animal, so that he assumes A belongs to Δ . On the one hand, there is a syllogism that all Δ will be either B or Γ , so that necessarily human is mortal or immortal, but it is not necessary that it is a mortal animal, but postulated. But this was what was necessary to deduce. (*APr* 46b3-12)¹⁰

While we can conclude something trivial like "every human is either mortal or immortal" on the basis of a division, there is clearly no way to argue deductively for the desired conclusion that every human is mortal. The only thing that the divider can do to get closer to her goal is to ask her interlocutor to agree to the claim that every human is mortal (46a33-34). By asking for this, the divider "begs the question" in the technical Aristotelian sense by assuming the very claim under discussion. By showing that division begs the question for an arbitrary case, Aristotel is entitled to conclude in general that division cannot produce a syllogism of its target. This is the sense in which division is a weak syllogism it cannot deduce its goal, but only a triviality.

Furthermore, this account of why division is a weak syllogism also explains, as we expected it would, why the method of division is "a small part" of the syllogistic method. Division is a weak syllogism because it can only deduce a triviality. The syllogistic can deduce this triviality, but also much else. So it is straightforward to think that division is a small part of the syllogistic method.

Aristotle also argues for 4 on the basis of an example. Suppose that we want to know whether the diagonal of a square is commensurable or incommensurable with a side. The divider assumes that the diagonal is a length and then divides length into commensurable or incommensurable. But without having proven the theorem, the divider is stuck: where should she put the "diagonal"? Division seems to offer no guidance on this.

These criticisms, like the general claims about division made at the outset of *APr* I.31, which claim that division is a defective syllogism, strongly suggest that Aristotle thinks division was intended to do what the syllogistic does (produce valid arguments), but give no hint as to why anyone should expect this. These methods seem to be, on their face, far too different to make such a connection.

5.1.4 **Previous resolutions of the puzzle**

Previous commentators have failed to motivate Aristotle's critical comparison. They have taken three different approaches towards resolving this puzzle.

¹⁰ἔστω γὰρ ζῷον ἐφ΄ οὕ Α, τὸ δὲ ϑνητὸν ἐφ΄ οῦ Β, καὶ ἀϑάνατον ἐφ΄ οῦ Γ, ὁ δ΄ ἄνϑρωπος, οῦ τὸν λόγον δεῖ λαβεῖν, ἑφ΄ οῦ τὸ Δ. ἄπαν δὴ ζῷον λαμβάνει ἢ ϑνητὸν ἢ ἀϑάνατον· τοῦτο δ΄ ἐστίν, ὃ ἂν ῆ Α, ἄπαν εἶναι ἢ Β ἢ Γ. πάλιν τὸν ἄνϑρωπον ἀεὶ διαιρούμενος τίϑεται ζῷον εῖναι, ὥστε κατὰ τοῦ Δ τὸ Α λαμβάνει ὑπάρχειν. ὁ μὲν οῦν συλλογισμός ἐστιν ὅτι τὸ Δ ἢ Β ἢ Γ ἅπαν ἔσται, ὥστε τὸν ἄνϑρωπον ἢ ϑνητὸν μὲν ἢ ἀϑάνατον ἀναγκαῖον εἶναι, ζῷον ϑνητὸν δὲ οὐκ ἀναγκαῖον, ἀλλ΄ αἰτεῖται· τοῦτο δ΄ ῆν ὃ ἔδει συλλογίσασϑαι.

- **Confusion:** At least one party to the debate is simply confused about the relationship between the methods.¹¹ If it is Aristotle, he is guilty of trying to force other methods to do what his method does. If it is someone else, they wrongly took their methods to be deductive, and Aristotle is merely pointing this out.
- **Discovery:** Both division and Aristotle's method at I.27-29 are aimed at *finding* premises for deduction. Because they share this project, the comparison is justified. Aristotle's point is that his method is better than division because it is far more general.¹²
- **Knowledge:** Both division and the syllogistic method aim at acquiring knowledge. Aristotle is arguing that, because divisions are question-begging, they cannot produce knowledge.¹³

Confusion can make sense of the existence of *APr* I.31, but is overly uncharitable. There is little evidence that any party was confused. Plato lacked Aristotle's technical language to talk about deduction, so its unclear what would even constitute evidence one way or the other. The same is true for other members of the Academy: there is no fragment or report in which someone claims divisions are deductions of a definition. There is a text that does seem to bear on the question. Sextus Empiricus reports Xenocrates as supplementing a division with a proof ($\dot{\alpha}\pi\delta\delta\epsilon\iota\xi\iota\varsigma$) of that division (*Against the Ethicists* I.4-5), suggesting that he at least distinguished the two. In the *Posterior Analytics*, Aristotle himself thinks that there is "no absurdity" about division making clear a definition without demonstrating or deducing it (*APo* 91b33-4) and calls division "useful" in the hunt for essence (*APo* 96b25-6). Thus some versions of this reading attribute not only confusion in Aristotle, but also inconsistency with his other discussions of division. We should therefore accept **Confusion** only if all else fails.

Discovery suffers from the problem that it does not have much to do with Aristotle's arguments against division. Aristotle here argues that divisions are not syllogisms of their target claims—he does not argue that they cannot be used *to find* premises for syllogisms. The comparanda of Aristotle's comparison simply do not fit with this interpretation.

Knowledge is, very broadly speaking, the right track. The two methods seem to play important roles in Plato's and Aristotle's respective conceptions of scientific knowledge, so it would be no surprise if the different views about division resulted from different conceptions of science. But this on its own is too vague to justify Aristotle in making the comparison, since after all, they could play quite

¹¹Various commentators have pointed the finger at Plato (Cherniss (1944), p. 28), Aristotle (Ebert and Norman (2007), p. 797), or anonymous members of the Academy (Striker (2009), p. 209 and perhaps also Philoponus, in *APr* 307.1-10 Wallies (1905)). According to Shorey (1924); Striker (2009), Plato himself never thought of division was demonstrative.

¹²Crubellier (2014), pp. 24, 298.

¹³Smith (1989), p. 160.

different roles in the acquisition of knowledge. According to **Knowledge**, the gap is filled by the requirement that knowledge cannot be acquired from question-begging arguments. The reason why, it seems, is that division doesn't give the inquirer anything *beyond what she started with*.

This diagnosis, however, is still in tension with Aristotle's claim that there is no absurdity if division "produces knowledge" (γνωρίζειν ποιεί) while begging the question (APo 91b28-35). This point could be finessed by saying that the sort of knowledge produced by division is of a different, less demanding sort of knowledge (γνῶσις) than that provided by syllogistic (ἐπιστήμη). Perhaps Aristotle's point is that what syllogisms get you is more intellectually rewarding than what division gets you. The scientific understanding that comes with possessing a demonstration is just a much more significant achievement than the knowledge one has from division. This would be adequate if there were demonstrative knowledge of the thing that division aims at: definitions, for then we could see how the syllogistic does something that division does not. But Aristotle denies that there are demonstrations of these kinds of definitions. Such a reply, in the end, seems to undermine Aristotle's justification for comparing them, since division and syllogistic are, after all, aimed at acquiring different things. Division will be aimed at getting yvwork of essences, while demonstrative syllogisms will aim at getting scientific understanding of other propositions. So, while Knowledge was right to look at the respective conceptions of science, it still failed to justify the comparison.

In what follows, I will reconsider the Platonic texts concerning division and use them to provide a new interpretation of Aristotle's comparison. This interpretation will make sense of both Aristotle's critical remarks about division in *APr* I.31 (unlike **Discovery**) and be consistent with his positive remarks about division as a method (unlike **Knowledge** and **Confusion**). First, I will make clear how both methods are intended to be rigorous and general ways of obtaining scientific knowledge concerning the part-whole relations among kinds, thereby justifying Aristotle's comparison. Then I will argue that Aristotle's point in comparing the methods is to show the importance of valid arguments in this scientific enterprise. Instead of *replacing* division with syllogistic, Aristotle is showing that syllogistic is a necessary part of a scientist's repertoire that is not already covered by division.

In §5.2, I show that both methods have the properties of epistemic generality and rigor. I argue in §5.3 that the methods both investigate predications conceptualized in terms of part and whole.¹⁴

¹⁴Previous scholars, arguing for division's influence on the syllogistic, have vaguely gestured at each of these, but here I am shedding their psychologistic and historicist tendencies and focusing on the more formal connections. See Maier (1896) (esp. II.2, pp. 56-85) on the first, Lutoslawski (1897), p. 464 on the second, and Solmsen (1929; 1941) on the third. It was the dominant position in the 19th and early 20th centuries that Platonic division influenced Aristotle's logic (see also Prantl (1853), p. 203; Jowett (1892), p. 192; Ross (1923), p. 32, although he would later change his position; de Strycker (1932a; 1932b); Bochenski (1951); Pellegrin (1981); Mignucci (2000)). Kneale and Kneale (1962) have some affinities with this group, especially Solmsen. For criticism of the influence claim, see Shorey (1924; 1933); Ross (1939; 1949); Cherniss (1944); Kapp (1942; 1975); Moravscik (2004)).

5.2 Division and Syllogistic as scientific methods

5.2.1 Epistemic Generality

Both Platonic division and Aristotelian syllogistic were intended to have a certain kind of generality that has historically been associated with logic. The sort of generality that I have in mind is that the theory is meant to apply to any sort of science one might engage in, whatever the subject matter.¹⁵ Moreover, division and syllogistic are supposed to be general ways of coming to know about any subject matter whatsoever through reasoning.

Plato claimed the method of division had this very special kind of generality. For instance, in the *Philebus*, when Socrates introduces the method of division (albeit with some unusual language, calling kinds "ones" that are discovered along the way from the genus and calling a division complete when the inquirer "knows how many the original one is"), he makes a strong statement about how widely his method is meant to apply:

Since this is the structure of things, we have to assume every time that there is always one form for every one of them, and we must search for it, as we will indeed find it there. And once we have grasped it, we must look for two, as the case would have it, or if not, for three or some other number. And we must treat every one of those further unities in the same way, until it is not only established of the original unit that it is one, many and unlimited, but also how many kinds it is. For we must not grant the form of the unlimited to the plurality before we know the exact number of every plurality that lies between the unlimited and the one. Only then is it permitted to release each kind of unity into the unlimited and let it go. *The gods, as I have said, have left us this legacy of how to inquire and learn and teach one another.* (16c-e, emphases mine)¹⁶

So far, we just have heard that this method is a divine way to inquire, learn, and teach. But soon after this, Socrates strengthens the claim:

So at the same time they [the first music theorists] made us realize that one should investigate about any one and many in this way. For when

While I am sympathetic to the influence view, here I will only be concerned with the connections between the methods.

¹⁵See, for example, MacFarlane (2002) for a defense of the centrality of this aspect of logic in Kant and Frege.

¹⁶δεϊν οὖν ἡμᾶς τούτων οὕτω διακεκοσμημένων ἀεὶ μίαν ἰδέαν περὶ παντὸς ἑκάστοτε θεμένους ζητεῖν – εὑρήσειν γὰρ ἐνοῦσαν – ἐὰν οὖν μεταλάβωμεν, μετὰ μίαν δύο, εἴ πως εἰσί, σκοπεῖν, εἰ δὲ μή, τρεῖς ἤ τινα ἄλλον ἀριθμόν, καὶ τῶν ἐν ἐκείνων ἕκαστον πάλιν ὡσαύτως, μέχριπερ ἂν τὸ κατ' ἀρχὰς ἐν μὴ ὅτι ἐν καὶ πολλὰ καὶ ἄπειρά ἐστι μόνον ἴδῃ τις, ἀλλὰ καὶ ὁπόσα· τὴν δὲ τοῦ ἀπείρου ἰδέαν πρὸς τὸ πλῆθος μὴ προςφέρειν πρὶν ἄν τις τὸν ἀριθμὸν αὐτοῦ πάντα κατίδῃ τὸν μεταξὺ τοῦ ἀπείρου τε καὶ τοῦ ἑνός, τότε δ' ἤδῃ τὸ ἐκαστον τῶν πάντων εἰς τὸ ἀπειρον μεθέντα χαίρειν ἐᾶν. οἱ μὲν οὕν θεοί, ὅπερ εἶπον, οὕτως ἡμῖν παρέδοσαν σκοπεῖν καὶ μανθάνειν καὶ διδάσκειν ἀλλήλους.

you have mastered these things in this way, then you have acquired expertise there, and when you have grasped the unity of any of the other things there are, you have become wise about that. (17d-e)¹⁷

Plausibly, this is meant to suggest that the method of division really is the way to inquire (or learn or teach) in *any* domain and that success in the method means that you have acquired wisdom. Those who ignore it, on the other hand, are said to be eristics simply grouping things "in a chance way" (*Philebus* 16e). This gets some confirmation by the discussion in the *Phaedrus*, where division is the ability in any instruction "to cut up any kind according to its species along its natural joints" (265e). We see just how serious Plato is about generality in his uncontroversial applications of the method to domains as diverse as music theory, political philosophy, and phonology.¹⁸ The enormous range of application further motivates ascribing the generality thesis to Plato.

Aristotle considers his syllogistic to have a similar scope. Recall that in the introduction of the method in *APr* I.30, he claims:

The method is one and the same for all things, both concerning philosophy as well as any skill or learning whatever. (46a3-4)

Aristotle claims that his syllogistic method works in the same way *for all things*. An argument will be a syllogism regardless of the epistemic status or subject matter of the premises and conclusion, and our method of looking for syllogisms and deducing validly will also be the same. Here we see a very close analogy with what Plato is claiming above for division, mirrored too in the smorgasbord of example terms that Aristotle uses in his development of the method: science, line, medicine, unit, good, condition, wisdom, swan, snow. The similarity is clear when we look at how closely the passage above is paralleled by the end of I.31: "this way of investigation [division] is neither suitable for every inquiry, nor even useful in those very cases in which it appears to be most appropriate." It seems that Aristotle denies for division almost exactly what he affirmed about syllogistic.¹⁹

While Aristotle does not think that every syllogism produces knowledge, a special kind of syllogism, demonstration, will. A syllogism requires special kinds of premises in order to be demonstrative: true, primary, immediate, better known than the conclusion, prior to the conclusion, and explanatory of the conclusion (*APo* I.2). Coming to know, then, requires the scientist to do two things: to have at hand premises of this sort and to construct syllogisms of the propositions to be demonstrated from those premises. The syllogistic is one of the crucial elements

¹⁷ καὶ ἄμα ἐννοεῖν ὡς οὕτω δεῖ περὶ παντὸς ἑνὸς καὶ πολλῶν σκοπεῖν – ὅταν γὰρ αὐτά τε λάβῃς οὕτω, τότε ἐγένου σοφός, ὅταν τε ἄλλο τῶν ἕν ὁτιοῦν ταύτῃ σκοπούμενος ἕλῃς, οὕτως ἔμφρων περὶ τοῦτο γέγονας·

¹⁸See Lukas (1888) for more or less explicit examples in nine dialogues, although he does not include at least two uses at the end of Euthyphro and in the Cratylus.

¹⁹What limits the scope of division here is that it is only appropriate to inquiring with the aim of establishing a definition, but is not appropriate for finding, e.g., genera, demolishing a definition, or investigating into open questions (*APr* 46b26-37).

in Aristotle's theory of science as described in the *Posterior Analytics*. The theory of science there is completely general and meant to apply to any domain of objects whatsoever. Thus the syllogistic too must be general.

We have found that the project of division and syllogistic are, despite appearances, very close in this respect. Both are meant to play a central role in every scientific enterprise.

5.2.2 Rigor

Plato and Aristotle wanted these procedures to be not only general but also rigorous. While generality regards what subject matters the procedure can be applied to, rigor is a matter of how that procedure is applied. The current literature on rigor has focused on the case of mathematics²⁰ There is no agreed upon analysis of rigor available. However, I will assume here that a procedure is rigorous just in case it minimizes error in pursuit of its goal: applications of the method should result in errors as little as possible.²¹ For example, the sieve of Eratosthenes, an algorithm for finding prime numbers, is rigorous in this sense: following it guarantees that you get the right answer. While algorithms are an important class of rigorous procedures, rigor is not only manifested in algorithms. The user of a rigorous but non-algorithmic method may need some ingenuity, but that ingenuity is either sufficiently constrained by the method or there are parts of the method that prevent errors, perhaps because there is an algorithm for checking the answer that one has. The ancient method of geometrical analysis, for instance, does not work without input from the inquirer, but is rigorous in that the technique does not lead to errors. A non-rigorous method, by contrast, can attain its goal, but not in virtue of the method alone—it also requires luck or the inquirer's knack.

According to Plato, divisions should be done rigorously. In the *Philebus*, he gestures at the need for rigor by pointing out defective divisions:

The gods, as I have said, have left us this legacy of how to inquire and learn and teach one another. However, the wise guys of today make a One in a chance way and a Many faster and slower than they ought. After the one they go directly to the unlimited and the intermediates escape them, which are what determines whether our speaking with each other is dialectical or eristic. (16e-17a)²²

According to Socrates, dividing incorrectly results in an eristical account, while his preferred method leads to a dialectical account. The difference between the dialec-

²⁰e.g., in Burgess (2015); Paseau (2016).

²¹For some types of problem there might be maximally rigorous methods, which are *infallible*, never leading to error at all, but in most situations there will inevitably be residual error.

²²οί μέν οὖν θεοί, ὅπερ εἶπον, οǚτως ἡμῖν παρέδοσαν σκοπεῖν καὶ μανθάνειν καὶ διδάσκειν ἀλλήλους· οἱ δὲ νῦν τῶν ἀνθρώπων σοφοὶ ἐν μέν, ὅπως ἂν τύχωσι, καὶ πολλὰ θᾶττον καὶ βραδύτερον ποιοῦσι τοῦ δέοντος, μετὰ δὲ τὸ ἐν ἄπειρα εὐθύς, τὰ δὲ μέσα αὐτοὺς ἐκφεύγει – οἶς διακεχώρισται τό τε διαλεκτικῶς πάλιν καὶ τὸ ἐριστικῶς ἡμᾶς ποιεῖσθαι πρὸς ἀλλήλους τοὺς λόγους.

tical and eristical accounts is not that the former must be true and the latter false. Rather, Socrates objects to the way that the eristic gets her account by dividing in a chance way or too quickly. He does not, however, say why this is problematic.

A passage in the *Statesman* is more explicit on this point. Young Socrates divided herd-rearing too quickly into rearing of human herds and beast herds. This is exactly the same kind of mistake mentioned in the *Philebus* passage. The Eleatic Visitor explains the problem:

Visitor: Ah, yes. You've made a very zealous and courageous division! However, we should try not to have this happen ever again! ... For it is most fine to separate off the object of inquiry from the others straightaway, should you do it correctly, just as a moment ago, when you thought that you had the division, you rushed the account, seeing that it was headed towards humans. But my friend, it is not safe to do such fine work. Instead, cutting through the intermediate stages is safer and one would more²³ encounter ideas. This makes all the difference in investigations. (*Statesman* 262a-b)²⁴

To illustrate the mistake, the Visitor gives two other divisions that reach the same end point but do so more quickly and more slowly and in which the fast division leads to a mistake. This suggests we should prefer that longer route because it is "safer."²⁵ The safety of the Visitor's method is meant to guard against a particular sort of error that the fast method could not: missing *ideas*, the explanatory natural kinds that should be in one's definitions. This constitutes evidence that Plato intends division to be rigorous in the sense that it minimized error.

Plato not only wants a rigorous method of division, he also proposes norms that are meant to prevent it, such as the recommendation to divide slowly or into the smallest number of subkinds as possible (*Statesman* 287b-c, *Philebus* 16d). Moreover, in one case, he explicitly defends the norm by arguing that it minimizes the possibility of error (*Statesman* 261e-264b). By preventing errors, applying these norms are concrete steps towards making division rigorous.²⁶

²³I am here following Rowe's translation in Cooper (1997) of mallon as "more". It could also be rendered "rather", which would strengthen the Visitor's point: on this reading he would be suggesting that his own method not only minimizes errors but is infallible. I am inclined toward the strong interpretation, but the point can be made even on the weak interpretation.

²⁴ΞΕ. Παντάπασί γε προθυμότατα καὶ ἀνδρειότατα διήρησαι· μὴ μέντοι τοῦτό γε εἰς αῦθις κατὰ δύναμιν πάσχωμεν. ...κάλλιστον μὲν γὰρ ἀπὸ τῶν ἄλλων εὐθὺς διαχωρίζειν τὸ ζητούμενον, ἂν ὀρθῶς ἔχῃ, καθάπερ ὀλίγον σὺ πρότερον οἰηθεὶς ἔχειν τὴν διαίρεσιν ἐπέσπευσας τὸν λόγον, ἰδὼν ἐπ' ἀνθρώπους πορευόμενον· ἀλλὰ γάρ, ὥ φίλε, λεπτουργεῖν οὐκ ἀσφαλές, διὰ μέσων δὲ ἀσφαλέστερον ἰέναι τέμνοντας, καὶ μᾶλλον ἰδέαις ἄν τις προστυγχάνοι. τοῦτο δὲ διαφέρει τὸ πᾶν πρὸς τὰς ζητήσεις.

²⁵Some, such as Gill (2012), think that the long route is too long. If that were so, we would have expected the Visitor to have corrected himself, since that is what he tends to do in this dialogue. He nowhere repudiates it, even though he makes other adjustments to this stretch of the division later on. For more discussion, see Ch. 3. Note also the parallel to "safer" causal explanations in *Phaedo* 100d. These explanations are safe precisely because they do not lead to errors.

²⁶For a more complete discussion of these norms and a defense of this account, see Ch. 3.

Rigor is manifested in Aristotle's syllogistic at two points in the method. Recall that the goal of his method is to produce a syllogism of a desired conclusion. As was mentioned above, the first stage of the method (discovering the syllogism) is algorithmic. If there is a set of premises contains a syllogism, Aristotle's procedure is guaranteed to find it and following the procedure will never lead to one giving an argument that is not a syllogism. This method is completely rigorous. Not only is the method for finding syllogisms rigorous, so too is the syllogism itself. Giving a valid argument with true premises infallibly leads you to truths. This is also important to Aristotle, since he criticizes some geometrical arguments on this point, claiming that they are not syllogisms unless certain axioms are made explicit (*APr* 41b13-22).²⁷ These two points together make the entire procedure described in *APr* I.31 rigorous, since that method consists in *both* the search for syllogisms *and* giving those syllogisms.

In both cases, we are demanding of the methods that they minimize the possibility of error. Because they are different methods it will turn out that rigor is manifested differently in each case, but they have a similar spirit.

5.3 Investigating Mereological Relations

Finally, the two methods are parts of a common project not only in *how* they investigate, but also in *what* they investigate: mereological relations between kinds.

To see this in Plato's case, it will be helpful to start with an example: *Skill*, a general kind, divided into *Production* and *Acquisition*. The relation between Skill and, e.g., *Production* is one of part to whole. This is suggested by the language of division itself: as a part is standardly defined as that into which a whole is divided. Plato also has the Eleatic Visitor state the claim dogmatically:

That whenever there is a species of something, it is necessarily also a part of whatever thing it is said to be a species of, but it is not at all necessary that a part is a species. You must always assert, Socrates, that this is what I say rather than the other way around. (*Statesman* 263b7-10)²⁸

Here the Visitor says that *A* being a species of *B* implies that *A* is a part of *B*, but not vice versa. Since division here and more generally in Plato is into kinds, this passage shows quite clearly that Plato thinks of the relationship between kinds in mereological terms. So, although definitions, the ultimate goals of division, may be identities, the method of division approaches definitions by first discovering mereological relations between kinds.

²⁷According to Malink (2015), this kind of rigor is one of the distinguishing marks of the treatment of syllogisms in the *Analytics* as opposed to the *Topics*.

^{28·}Ως είδος μέν ὅταν ἢ του, καὶ μέρος αὐτὸ ἀναγκαῖον εἶναι τοῦ πράγματος ὅτουπερ ἂν είδος λέγηται· μέρος δὲ είδος οὐδεμία ἀνάγκη. ταύτη με ἢ 'κείνη μᾶλλον, ὥ Σώκρατες, ἀεὶ φάθι λέγειν.

Mereology comes into Aristotle's method through his account of predication. The method, as he develops it, relies on a distinction between four kinds of definite propositions:

- Universal Affirmative: b belongs to (or is predicated of) every a. (Equivalent to saying "Every a is b")
- Universal Negative: b belongs to no a. ("No a is b")
- Particular Affirmative: b belongs to some a. ("Some a is b")
- Particular Negative: b does not belong to some a. ("Not every a is b")

The rules in Aristotle's method for deducing conclusions and finding premises for deduction are entirely determined by which of these categories the propositions fit into. This distinction is therefore crucial to his method.

We get a hint of the truth conditions of these propositions in the famous *dictum de omni et nullo*:

We use the expression "predicated of every" when nothing can be taken of which the other term cannot be said, and we use "predicated of none" likewise." $(APr \ 24b 28-30)^{29}$

The *dictum* gives the meaning of Aristotle's technical phrases "predicated of every" and "predicated of none". The basic idea is that "a is predicated of every b" means that for every *z*, if b is predicated of *z*, then a is also predicated of *z*. Similarly, "a is predicated of no b" means that for every *z*, if b is predicated of *z*, then a is *not* predicated of *z*. Because the *dictum* is used to justify the syllogisms of the first figure to which all the others are reduced, it carries enormous weight in understanding Aristotle's semantics. Moreover, while it directly explains the meanings for universal affirmative and universal negative propositions, it indirectly explains them for the particular propositions, which are each the *contradictory* of a universal proposition. (A particular affirmative proposition is the contradictory of a universal negative and a particular negative of a universal affirmative.)

I will now argue that the four types of proposition used in the syllogistic are defined by Aristotle in terms of a mereological predication relation. Because the terms in syllogistic propositions refer to *kinds*, this will show that it, like Platonic division, investigates mereological relations between kinds. The *dictum* shows that syllogistic propositions are undoubtedly defined in terms of predication, since one of Aristotle's ways of referring to the relation is talking about being "said of."³⁰ But is this a part-whole relation?

²⁹λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ἢ λαβεῖν καθ' οὕ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως. With Ross in excising "τοῦ ὑποκειμένου". Although it is present in all the manuscripts, it is absent in Alexander's citation and an otherwise unparalleled use of the term in the *Analytics*.

³⁰It is disputed whether they are defined in terms of a notion of predication different from universal affirmative. See Corcoran (1972, 1973); Barnes (2007), for the view that they are different, but

Two reasons have been suggested in the literature.³¹ To them, I add a third:

- 1. Aristotle's terminology for predication is steeped in mereological language. He marks his phrase for universal predication ("belongs to all") as equivalent to "is in as a whole" and his terms for different kinds of propositions "universal" (*katholou*) and "particular" (*kata meros/en merei*) are themselves derived from the language of whole (*holon*) and part (*meros*). If universal predication is a mereological relation, we would also expect it to be defined in terms of notions related to mereology.
- 2. When discussing a specific kind of predication, that between genus and species, Aristotle frequently refers to their relation as one of whole and part. (*Metaphysics* V.25, 26)
- 3. The minimal formal structure of the part-whole relation is sufficient for giving semantic clauses for all the syllogistic propositions and defining a consequence relation that is sound and complete for his deductive system.

This last claim, defended in the Appendix,³² is an advance on 1 and 2 above because it shows the *adequacy* of the mereological conception of predication in Aristotle's logical theory. Even if Aristotle is using the terminology metaphorically, or extending spatial and physical language to apply to logic, this last reason allows us to see what logical work such an association can do. We do not need anything beyond what is given in the *dictum* to understand the meaning of the syllogistic propositions, provided that the predication relation referred to there is a parthood relation. Furthermore, such a semantics can be used to straightforwardly define a notion of logical consequence that coincides with his deductive system. If conceiving of the meanings of the syllogistic propositions in mereological terms can do this kind of work, Aristotle's use of mereological language should be taken seriously.³³

Morison (2008); Malink (2013), following Michael Frede for the view that they are the same. While I will stay neutral with respect to the dispute between these two readings, the former reading is usually associated with a view of predication akin to Frege's, which takes predication to be a relation between two different syntactic types. This view has been discredited by Mignucci (2000); Malink (2009); Corkum (2015). See Crager (2015) for a version of the Corcoran/Barnes reading that does not use the Fregean conception of predication.

³¹See Mignucci (2000); Malink (2009); Corkum (2015). This was also the view of the ancient commentators: Alexander *in APr* 25.24 Wallies (1883), Philoponus *in APr* 47.2348.2, 73.223, 104.1116, 164.47 Wallies (1905).

³²This theorem extends the results in Malink (2013) to a more powerful deductive system, that of Corcoran (1973). While Malink treats only two-premise arguments and shows that the preorder semantics validates all and only Aristotle's syllogisms, Corcoran's deductive system generates deductive arguments with an arbitrary number of premises.

³³I do not want to suggest that this is the only way that Aristotle thinks of predication. Singular propositions stand out as the most likely exception. Aristotle never claims that an individual is a part of a universal, but see Mignucci (2000). He also sometimes uses form and matter to understand predication (e.g., *Metaphysics* Z.17, H.6) and this is not mereological either.

This shows that both Platonic division and Aristotelian syllogistic are in the business of investigating mereological claims between kinds.

5.4 Resolving the Puzzle

Let us return to our original puzzle: why did Aristotle compare division with the syllogistic when these are such different methods? In the previous sections I showed how these methods were both meant to contribute to general, rigorous scientific methods aimed at understanding mereological relations between kinds. This common project makes sense of Aristotle's comparison. Even though the goal and methods are very different, they were both intended to be general and rigorous ways of attaining scientific knowledge (§5.2). Despite the fact that the syllogistic yields quantificational claims and division definitions, both investigate the mereological relations between kinds (§5.3).

The problem remains, however, that Aristotle is treating division in the chapter as a putative demonstration and thus a syllogism, which despite this common project still seems uncharitable on his part. In what follows, I will argue that Aristotle's important points in the chapter do not rely on these claims. Rather, his strategy in the chapter is to show the limitations of division and the power of syllogistic within the common project—points that could be appreciated even bracketing the question of division's demonstrative character.

Doing so will show exactly how **Confusion** can be avoided. Aristotle assimilates division to syllogistic here *not* because he really thinks they are supposed to do the same thing, but instead because doing so brings out what is distinctive of his syllogistic. This bit of rhetoric is clever and not at all objectionable, so long as the features he discusses really are distinctive.

In I.31, Aristotle does not set out to show that only the syllogistic, and not division, is a necessary part of the scientific enterprise. This is important, because Aristotle seems to accept the use of division in any science:

But it is necessary, *whenever* one is dealing with *any* whole, to divide the genus into the indivisibles in species...(*APo* 96b15-16)³⁴

Divisions according to the differences are useful for going about in this way [i.e. investigating the essence]. $(APo \ 96b \ 25-26)^{35}$

Aristotle also wants division to be rigorous, since he argues that:

Further, only in this way [i.e. using the rules for division that he described] is it possible to leave nothing out in the what it is [the essence]. $(APo \ 96b \ 35-97a1)^{36}$

³⁴Χρὴ δέ, ὅταν ὅλον τι πραγματεύηταί τις, διελεῖν τὸ γένος εἰς τὰ ἄτομα τῷ εἴδει...

 $^{^{35}}$ αί δὲ διαιρέσεις αἱ χατὰ τὰς διαφορὰς χρήσιμοί εἰσιν εἰς τὸ οὕτω μετιέναι·

³⁶ Έτι πρός τό μηδέν παραλιπεῖν ἐν τῷ τί ἐστιν οὕτω μόνως ἐνδέχεται.

These texts strongly suggest that Aristotle does not reject division as Plato conceives of it.³⁷ What Aristotle wants to do in I.31 is highlight the most important feature of arguments produced by the syllogistic method: *validity*.³⁸ Let's return to his criticisms once more:

- 1. Divisions are not deductions of the definitions they seek, because the definition does not follow of necessity from the assumptions. (46a31-46b25)
- 2. Division cannot refute. (46b26)
- 3. Division cannot deduce features that are not definitional. (46b27-8)
- 4. Division is useless in solving open problems. (46b28-35)

Each of these corresponds to an innovative feature of the syllogistic:

- 1. The syllogistic leads to arguments whose conclusions follow from the premises *of necessity*.
- 2. The syllogistic allows one to establish or refute any claim that can be established or refuted given a premise set.
- 3. The syllogistic can be used in all sorts of problems, not just definitional ones.
- 4. The syllogistic can be used in situations of ignorance.

The most important of these features to Aristotle seems to be the first. He isolated a notion of *following of necessity* in his account of the syllogism and developed a method that leads to arguments that must have this property.

As was argued in §5.1.3, these precise claims are the route to understanding why division is like a "weak syllogism" and a small part of the syllogistic method. The syllogism is weak because it is unable to *force* its conclusion, the definition, on their interlocutor.³⁹ The exception to this claim, the trivial deductions of disjunctive conclusions, is reflected in the admission that division is "a small part" of the syllogistic method. These points about division hold whether or not it was intended to be deductive.

Aristotle is not claiming that there is a problem with division *per se* not leading to such arguments. However, the three applications he then goes on to discuss (refutation, non-definitional problems, and situations of ignorance) are all better served by syllogistic than division *because* it produces valid arguments. Valid arguments with known premises can be used to establish or refute any sort of claim

³⁷See Chapter 4 for a more complete defense of this claim.

³⁸Not every argument produced by the syllogistic method will be sound, since it is possible to choose one's premises *in accordance with opinion* when doing dialectic, but these may not be true.

³⁹The word for necessity (ἀνάγκη) that is a property of all syllogisms is etymologically related to the verb ἀναγκάζω, to force or compel.

whatsoever, unlike division, which can only plausibly be used to establish definitions. Many claims in science are not definitional, so it would be important to say something about them. In a situation of ignorance, a valid argument can allow the reasoner to put together pieces of knowledge that she already had to find out something she before did not. Division, even if it is illuminating, requires that the inquirer know at every point in the division where to put the target kind. Otherwise, she is stuck. Syllogisms, because they involve putting together multiple independent claims in new ways, can lead to new, surprising results.

More generally, valid arguments are important in both dialectical encounters and scientific demonstration. In dialectical encounters, it is useful to be able to force a conclusion on an interlocutor, for instance, to refute her. Using division, like using an inductive argument, cannot sway a recalcitrant opponent with the same force as a valid argument.

Part of what it is to know p scientifically, according to Aristotle, is to know that p is necessary. If this is the case and there is some plausible way to know that certain fundamental principles of a science are necessary, then valid argumentation will be of great use in coming to know derivative scientific truths, since the necessary consequences of a set of necessary truths are themselves necessary.

Note, however, that while syllogistic is useful in science, it also cannot be the whole story. It cannot be used to demonstrate these first principles, since those are precisely what must be taken for granted. One particularly important kind of indemonstrable principle, for Aristotle, is the definition (*APo* I.2). Here, it seems, is where Aristotle thinks division can be of service. Precisely because it is non-demonstrative, inquirers can use division to hunt for and establish the definition of a target kind (*APo* II.13).

This is where **Knowledge** went wrong. Recall that, according to **Knowledge**, division could not be a way of acquiring knowledge because it is not possible to use question-begging arguments for acquiring knowledge. On the view suggested here, division's power is, paradoxically, also its weakness. It is a bad way of acquiring certain kinds of knowledge because it is not a syllogism and thus not a demonstration. But knowledge cannot always be got through demonstrations (*APo* I.3). Indeed, Plato designed the method of division to hunt for definitions, one of the three kinds of indemonstrable principle in Aristotle's philosophy of science. Thus nothing in *APr* I.31 rules out division being of crucial importance in attaining such knowledge.

This positive role also explains why Aristotle, perhaps unfairly, stresses that his ancestors were wrong to call division a demonstration of the essence: because it is so important that division is not demonstrative *in his sense*. He sets up his predecessors as insensitive to the difference, so that when he makes the distinction between them, it shows up as a significant advance. This point is supported by *APo* II.5, where, immediately after repeating the *APr* I.31 criticisms of division, he claims that it would not be strange at all if division made the essence known in

another way, as induction does.⁴⁰

The interpretation of I.31 on offer gives Aristotle a clear, important point to be making in his critical comparison between division and the syllogistic. Through it, Aristotle also gives an unexpected explanation of the importance of a central logical notion: *validity*. It is equally important, however, to see how Aristotle situated this explanation. The method of division proves to be a helpful foil precisely because it shares so much of what else Aristotle thinks is important about his method: its generality, rigor, and yields claims about the mereological relations between kinds.

⁴⁰Aristotle's candor in this chapter strongly pushes against whatever might have motivated **Con-fusion**.

Chapter 6

The Influence of Platonic Division on Aristotelian Syllogistic

It is often remarked that Aristotle's rigorously developed logical theory, as presented especially in the first book of the *Prior Analytics*, is unprecedented in its scope and precision at that point in Greek philosophy. Indeed, it is a view that Aristotle himself encourages at the end of the *Sophistical Refutations*.

That our programme has been adequately completed is clear. But we must not omit to notice what has happened in regard to this inquiry. For in the case of all discoveries the results of previous labours that have been handed down from others have been advanced bit by bit by those who have taken them on, whereas the original discoveries generally make an advance that is small at first though much more useful than the development which later springs out of them. For it may be that in everything, as the saying is, 'the first start is the main part'; and for this reason it is the most difficult; for in proportion as it is most potent in its influence, so it is smallest in its compass and therefore most difficult to see—but when this is once discovered, it is easier to add and develop the remainder. This is in fact what has happened in regard to rhetorical speeches and to practically all the other arts; for those who discovered the beginnings of them advanced them in all only a little way, whereas the celebrities of to-day are the heirs (so to speak) of a long succession of men who have advanced them bit by bit, and so have developed them to their present form, Tisias coming next after the first founders, then Thrasymachus after Tisias, and Theodorus next to him, while several people have made their several contributions to it; and therefore it is not to be wondered at that the art has attained considerable dimensions. Of the present inquiry, on the other hand, it was not the case that part of the work had been thoroughly done before, while part had not. Nothing existed at all. (Sophistical Refutations 183b15-36)¹

 $^{^1}$ ὄτι μὲν οὖν ἔχει τέλος ἱχανῶς ἂ προειλόμεθα, φανερόν \cdot δεῖ δ΄ ἡμᾶς μὴ λεληθέναι τὸ συμβεβηχὸς περὶ

Although the passage in the *Sophistical Refutations* does not refer to the *Prior Analytics*, Aristotle here is discussing theories of the syllogism, broadly construed, and claiming that he possessed no predecessor for that. This claim, I think, is not very satisfying from a historical point of view. As historians, we want to know *how* a theory took the particular form that it did and calling it unprecedented without saying more is an admission that we have no such explanation.

In this chapter, I argue that Aristotle relies on the theory of division as developed in the late Platonic dialogues and the early Academy. Seeing a connection between division and the syllogistic is nothing new. Indeed, when Paul Shorey wrote "The Origin of the Syllogism" in 1924, it was *the prevailing view*. Now, however, opinion is quite reversed. Moravscik (2004), argues that there are no conceptual connections between division and syllogistic and that Aristotle did not take there to be such.

The original proponents of influence suggested, for instance, that Aristotle developed the syllogistic because of some "dissatisfaction" with division. But such a claim raises as many questions as it answers. Why does a theory of valid argument have anything to do with the failure of what seems to be a theory of inquiry? Why are they, as it were, even playing the same game? I want to remain neutral about whether the influence should be thought of as development within Aristotle's thought,² and instead look at several aspects of syllogistic inherited from division so that we can focus on the way in which Aristotle's theory is innovative, but *not* unprecedented. Division is a natural place to look as well since it is clearly Plato's greatest contribution to dialectic in the eyes of the tradition which follows him. There is extensive discussion and/or use of division in the Old Academy, the Lyceum, the Stoa, the Academic skeptics, Sextus Empiricus, Galen, the Middle-/Neo-Platonists, and later Peripatetics. Like Aristotle's syllogistic, division was embraced by thinkers who were strongly opposed to central Platonic doctrines. This commonality makes it, I think, a plausible place to look in Plato for connections with Aristotle's logic.³

ταύτην την πραγματείαν. τῶν γὰρ εὑρισχομένων ἀπάντων τὰ μὲν παρ' ἑτέρων ληφθέντα πρότερον πεπονημένα κατὰ μέρος ἐπιδέδωχεν ὑπὸ τῶν παραλαβόντων ὕστερον, τὰ δ' ἐξ ὑπαρχῆς εὑρισχόμενα μικρὰν τὸ πρῶτον ἐπίδοσιν λαμβάνειν εἴωθε, χρησιμωτέραν μέντοι πολλῷ τῆς ὕστερον ἐκ τούτων αὐξήσεως· μέγιστον γὰρ ἴσως ἀρχὴ παντός, ὥσπερ λέγεται. διὸ καὶ χαλεπώτατον· ὅσῳ γὰρ κράτιστον τῆ δυνάμει, τοσούτῳ μικρότατον ὃν τῷ μεγέθει χαλεπώτατόν ἐστιν ὀφθῆναι. ταύτης δ' εὑρημένης ῥặον τὸ προςτιθέναι καὶ συναύξειν τὸ λοιπόν ἐστιν· ὅπερ καὶ περὶ τοὺς ῥητορικοὺς λόγους συμβέβηκε, σχεδὸν δὲ καὶ περὶ τὰς ἄλλας ἁπάσας τέχνας. οἱ μὲν γὰρ τὰς ἀρχὰς εὑρόντες παντελῶς ἐπὶ μικρόν τι προήγαγον· οἱ δὲ νῦν εὐδοκιμοῦντες, παραλαβόντες παρὰ πολλῶν οἶον ἐκ διαδοχῆς κατὰ μέρος προαγαγόντων, οὕτως ηὐξήκασι, Τεισίας μὲν μετὰ τοὺς πρώτους, Θρασύμαχος δὲ μετὰ Τεισίαν, Θεόδωρος δὲ μετὰ τοῦτον, καὶ πολλοὶ πολλὰ συνενηνόχασι μέρη· διόπερ οὐδὲν θαυμαστὸν ἔχειν τι πλῆθος τὴν τέχνην. ταύτης δὲ τῆς πραγματείας οὐ τὸ μὲν ἦν τὸ δ' οὐκ ῆν προεξειργασμένον, ἀλλ' οὐδὲν παντελῶς ὑπῆρχεν.

²As in Jaeger (1934); Solmsen (1929); Mansion (1961).

³Am I suggesting that division was taken to be "ontologically innocent" in the same way that some thing of contemporary logic? In some sense, yes. The metaphysics underlying division has been hotly disputed: are the things divided separate forms or Aristotelian universals or Stoic concepts or...? This aspect of the Platonic view (which clearly does connect heady Platonic metaphysics

First, I will review the debate in the past century concerning the relationship between division and the syllogistic. Then I will argue that the similarities between division and the syllogistic discussed in the previous chapter provide good grounds for affirming the old prevailing view: Aristotle's theory of the syllogism was influenced by Platonic division. In conclusion, I will take a step back and argue that a conceptual and not merely historical connection between logic and scientific methodology has the resources to give a plausible account of logic that explains its normative significance.

6.1 The Question of Division and the Syllogistic

As I said, I am in some ways returning to a conception of the origin of the syllogism that has important precedent in earlier work, but has since largely been discarded, especially in the Anglophone world. My goal here is to set up the best developed version of this early view, that of Heinrich Maier in *Die Syllogistik des Aristoteles*,⁴ the final volume of which dealt with the discovery of the syllogism, and trace the 20th century scholarship that reacted to it.

According to Maier (1896, II.2, pp. 56-85), Aristotle developed his conception of the syllogism out of a critical comparison with division. In reaction to the radical challenges to reason from the fourth century Sophists, Plato had embarked on a search for method, culminating in the method of division as practiced in the *Sophist* and *Statesman*. In thinking through the problems for this method, Aristotle arrived at the syllogism. This, Maier thinks is best evidenced in the discussion of division in *Prior Analytics* I.31. Showing division's weakness is a sort of working out of Aristotle's own reasons for moving from it to the syllogism.⁵

The traditional view began to be upturned with Shorey (1924; 1933). As he points out, Maier offers no specific arguments for this claim beyond suggesting that *APr* I.31 fits nicely within the greater narrative as a backhanded acknowl-edgement of influence.⁶ Moreover, he points out that Maier's reading of *APr* I.31 is incredibly strained. It reads less like a bit of intellectual autobiography than a polemical comparison between two already completed theories. Instead, Shorey proposes that the theory of causes in Plato's *Phaedo* 100c ff as the true origin. There Plato presents causal chains such as Heat-Fire-Body. Ambitious (as opposed to safe) causal explanations are then cast as determined by a prior element in the

with division, e.g., *Philebus* 16) does not seem to have enjoyed the warm reception of the methodology.

⁴Others who held this view include Prantl (1853); Jowett (1892); Lutoslawski (1897); Ross (1923), although Ross would later join Shorey's ranks.

⁵Maier here is, in effect, historicizing and psychologizing Zabarella's interpretation and use of this chapter. In *De Methodiis* III, he paraphrases the argument of *APr* I.31 in order to show that division is not a method at all and that only the syllogism is.

⁶Lutoslawski (1897) p. 464 attributes the discovery of the middle term to the gift of the gods passage in the *Philebus*, but the notion of μ έσα there, as Shorey rightly suggests, is quite different than that in the syllogistic.

chain "belonging" to the later. This, according to Shorey, is exactly like Aristotle, for whom syllogisms are brought about by means of a chain of properties belonging to a subject. On Shorey's view, the transitivity of *causation* in the *Phaedo* is mirrored by the transitivity of universal predication in the syllogistic.

Emile de Strycker in a series of articles (1932ba; 1932ab) responding to Shorey defended the traditional interpretation by arguing that Plato did see division as demonstrative, that division contributes the part/whole relations in which Aristotle casts the syllogistic, and that there are in fact syllogisms present in Platonic divisions. Shorey, of course, freely admitted that Plato used the word "ἀπόδειξις" to describe divisions, but with a sufficiently different meaning. While he agrees that division does make use of the part/whole relations, this is not particularly distinctive of Platonic division according to Shorey. Finally, Shorey is also correct to note that the syllogisms de Strycker discusses are not presented *as such* in Plato's text, but are extracted by the interpreter. This provides us with no reason to think that Plato himself had already come up with the theory of the syllogism.

The outcome of this first debate between Maier, Shorey, and de Strycker was not clear-both sides claimed victory, along broadly national lines. Those on the continent⁷ still maintained the importance of division, while Cherniss (1944) and Ross (1939) would follow Shorey, not only in his negative point, but also embracing his claim that the syllogism originated in the Phaedo. It is not hard to see why. The continental camp did not provide very plausible specific arguments for influence, but Maier's broad strokes account of the development of logic up to Aristotle was very plausible and the use of the part/whole language also seems to directly link up with division. On the other hand, Shorey provided a connection between Plato and Aristotle that seems to be more closely grounded in the texts. However, his story has problems of its own. First of all, it is unclear how this could be extended to negative propositions or figures other than the first.⁸ Unlike Maier, who tries to situate Aristotle's syllogistic in the context of eristic and Platonic dialectic, Shorey's story does not have the same kind of narrative unity nor the faintest trace of an acknowledgement in Aristotle's text. This obviously is not decisive-the lines of intellectual history are never as tidy as we would desire.

This more or less set the tone for the debate between Ross and Solmsen. Solmsen saw Plato's influence on Aristotle's logic mediated by the theory of science in the *Posterior Analytics* from which the more general theory of the *Prior Analytics* grew.⁹ This earlier theory is more directly in contact with Platonic approaches in which the method of division loomed large and in which Aristotle was more di-

⁷Most notably, Solmsen (1929); Bochenski (1951); Pellegrin (1981).

⁸Ross goes beyond Shorey in arguing that the *Phaedo* passage does the same for Celarent that Shorey said it did for Barbara. I must admit with Solmsen (1941) that this strikes me as much less plausible.

⁹In his application of the developmental account in Jaeger (1934) to Aristotle's logic, Solmsen departs in significant ways from Maier, especially concerning the order of the logical writings. However, there are still important points of contact between them in relation to their attitudes towards division.

rectly concerned with mathematical demonstration. In particular, Solmsen thinks that Aristotle was, in the first draft of *APo* I, concerned to undermine Platonist "super-science" by arguing that the use of common principles is limited and most principles from each science come from that science.

Ross (1939)¹⁰ takes on Solmsen's views on division and his account more generally. He furthers the arguments of Shorey, adding "In particular, it is clear that syllogism has no connexion with the characteristic element of Platonic division, viz. the recognition of species mutually exclusive, and exhaustive of the genus; there is no 'either...or' in the syllogism as Aristotle conceives it."(26) On the other hand, he concedes to Solmsen that his notion of the *Eidoskette*, the chain of kinds stretching down from a genus to an infima species. is relevant to the birth of the syllogism, but correctly notes that Solmsen needs much more than this, since it seems equally present in the passages of the *Phaedo* mentioned by Shorey as well as "in the nature of things". Ross takes Aristotle to have turned a Platonic metaphysical principle (present in many places in the Platonic corpus) into a logical doctrine.¹¹

After this debate, we see two trends. On the one hand, we see new suggestions for the origin of the syllogism. Ernst Kapp (1942; 1975) argues for the importance of the dialectical tradition, as described in the *Topics* and *Sophistical Refutations* to the definition of the syllogism and the subsequent theory of the syllogism. Robin Smith (1978), picking up on under-discussed work of Einarson (1936ba; 1936ab), takes the presence of technical mathematical terms to point to a particularly mathematical origin. James Allen (1995) points to the importance of the rhetorical tradition. On the other, there is a new quietism. Authors are reluctant to commit themselves to the kinds of claims that were so boldly made before. While Aristotle seems to have had some inspiration from geometrical demonstration, not much more can be said. ¹²

With this multitude of origins available to us, the project of finding the origin of the syllogism should be transformed. Instead of seeking some single source of inspiration, we should instead try to explain various aspects of Aristotle's syllogistic theory. Different sources could explain different aspects of the theory. In what follows I want to isolate these aspects in both Platonic division and Aristotelian syllogistic and motivate Platonic influence specifically along these lines.

From the previous debates, we can see three very vague suggestions about these aspects. First, we have the mereological idea. First, there is the connection

¹⁰Reprinted seemingly without any changes in Ross (1949). References are to this work. Solmsen (1951) wryly comments that Ross must not have seen his critical discussion.

¹¹In his responses to Ross, Solmsen claims that it is a point of agreement between himself and Ross that the syllogism developed out of division. However, *he* is the one who seems to identify the *Eidosketten* with divisonal structures, which Ross explicitly denies. Because of this, his discussion really only concerned what he took to be the points of disagreement.

¹²For instance, Striker (2009); Lynn Rose and John Corcoran in Corcoran (1974); Kneale and Kneale (1962); Moravscik (2004), although the Kneales do have some points in common with Solmsen.

of the two to Plato and Aristotle's respective conceptions of science. Second, there is Lutoslawski's suggestion that something in the demand for "intermediates" in the divine method of the *Philebus* is connected to syllogistic. Finally, it seems like division and the syllogistic conceptualize the relationship between kinds in terms of mereological relations such as parthood and overlap.

It turns out that, suitably precisified, each of these ideas denote a real commonality between syllogistic and division. With that in hand, we can better understand Aristotle's criticism. In Ch. 5, I argued that both Plato's method of division and Aristotle's syllogistic method are intended to contribute towards a methodology for science that 1) applies in any domain and 2) is rigorous, as well as 3) yielding claims about part-whole (mereological) relations among kinds

The confluence of these similarities are striking enough that we should postulate Platonic influence on Aristotle's syllogistic. They point to Aristotelian engagement with both the structure and the purpose of division. Taking these similarities together, I do not see how any other proposed influence on the syllogistic discussed above can explain them as well as division. Shorey's *Phaedo* passage is unable to explain the generality, the more general dialectical and rhetorical tradition cannot explain the rigor, and the mathematical seems only to me to relate to the structure. That is not to say that these proposed origins cannot do much to explain other aspects of Aristotle's syllogistic. Perhaps the *Phaedo* does have something to do with Aristotle's conception of *aitia* in the *Posterior Analytics*. Dialectic and rhetoric seem to be relevant for understanding why Aristotle uses the language of "setting down" and "following of necessity" in his definition of the syllogism.

There is still, however, the possibility that these arose independently or in a more general fourth century context. In what follows, I will address this point for the different aspects. In the case of structures of division and the syllogistic, it seems to me particularly unlikely that they came up with the ideas independently. It is really quite peculiar to understand "A is B" or "All A is B" as some kind of parthood relation. Later traditions (notably the one following Leibniz), in fact, have given semantics for syllogistic which use *exactly the opposite* parthood relation under the heading of "conceptual containment".¹³ Even Aristotle is willing to talk of the opposite parthood relation when he says that the genus is in the species in *account*. (*Physics* IV.3, *Metaphysics* V.25, 26) Such a notion of parthood might not be useful in the *Analytics*, since there he wants to include non-essential predication in the scope of his theorizing. The later tradition, however, thought of *all truths* in this way. And if that is so much as coherent, we cannot just say that Aristotle used the one direction of the parthood relation because it was "in the nature of things" (*pace* Ross).

With respect to generality and rigor of method, it too seems to me unlikely

¹³See Anderson (2015) Ch 1; de Jong (2010); Rescher (1954) for discussion of the notion of concept containment and how this could be used to make sense of syllogistic. Shorey (1924) thinks that Aristotle's syllogistic is neither fully extensional (\approx division) nor intensional (\approx concept containment).

that these notions arose independently. There does not seem to be, *a priori*, any reason to suspect that there even *is* such a method. Indeed, given the great variety of scientific practices in the geometry, music, astronomy, and physics of Plato and Aristotle's time, it would be very surprising if there were a common core of practices to them all. But this is just what Plato and Aristotle are both saying. According to Plato, all inquiry, teaching, and learning come about through collection and division. For Aristotle, all scientific knowledge comes about by having demonstrations, particular kinds of syllogism.

Perhaps these can be explained by postulating a common ancestor in the dialectical practice formalized in the *Topics*. However, the *Topics* is even more clearly indebted to division than the *Analytics*. There are numerous references to division¹⁴ and *Topics* II.2 almost paraphrases the "gift from the gods" passage in *Philebus* 16-17 (noted by Cherniss). So, to the extent to which the generality is also a feature of the *Topics*, it seems likely that it and the *Analytics* inherited the project from Plato. On the other hand, Solmsen influentially denied that the *topoi* of the *Topics* are intended to be general on account of Aristotle's blasé attitude towards counterexamples, even though the art of dialectic *is* general. Moreover, it does not seem to me that the *Topics* has anything like the notion of rigor relevant to Plato and Aristotle. It was precisely was precisely this absence that motivated Malink to see rigor as crucial to differentiating the syllogistic in the *Topics* from the one in the *Analytics*.

Without either a common source or independent discovery, Platonic division is left standing as the only explanation of these features.

6.2 Logic and Method

If what came before was correct, there is an important historical connection between the first "logical" system and scientific method, since the method of division was devised by Plato to help us acquire knowledge of essences concerning anything whatsoever. Indeed, this was one of the respects in which I argued that it influenced Aristotle's syllogistic. But Aristotle's logic may not be representative of the discipline as a whole. Perhaps his reasons for taking syllogistic to be connected to the scientific method were idiosyncratic and there is no direct *conceptual* connection between logic and scientific method.

Perhaps this is right. Perhaps there is no conceptual connection. However, a brief glance at the history of logic shows that it is by no means idiosyncratic. A tight connection is assumed by thinkers from the Galen to Avicenna to Arnauld, but is nowhere more obvious than in the Renaissance logician Jacopo Zabarella, who *identifies* method with syllogistic in *De Methodiis* III.

I will not argue that there is a conceptual connection between logic and scientific methodology. Instead, I will ask: could the claim that there is such a concep-

¹⁴See Falcon (1996, 2002).

tual connection explain otherwise puzzling features of logic, such as its normativity?

First, I will get clearer about the sort of conceptual connection that I am proposing. It is not Zabarella's view, which identifies logic and methodology of science. This seems to me too strong. Rather, it is something that I believe is closer to the view expressed by Aristotle:

First we must state the subject of the enquiry and what it is about: the subject is demonstration, and it is about demonstrative knowledge. Next we must determine what a proposition is, what a term is, and what a deduction is (and what sort of deduction is perfect and what imperfect); and after that, what it is for one thing to be or not be in another as a whole, and what we mean by being predicated of every or of no. (*APr* 24a10-15)¹⁵

After he has defined all of these terms, he continues:

After these distinctions we now state by what means, when, and how every deduction is produced; subsequently we must speak of demonstration. Deduction should be discussed before demonstration, because deduction is the more general: a demonstration is a sort of deduction, but not every deduction is a demonstration. (25b26-31)¹⁶

On the view suggested by these programmatic passages, syllogistic extends more broadly than the study of demonstration, which is the study of what one must possess in order to have scientific knowledge. According to Aristotle, one must have a demonstration, a certain *kind* of syllogism. Moreover, the demonstrative syllogism and not the dialectical syllogism seems to play a privileged role, since only the former is discussed in the *Analytics* alongside the more general theory of syllogism. We might be led to suspect, indeed, that the whole inquiry is directed towards giving an account of demonstration.

This is not yet, however, to connect logic *as such* with scientific method., since (as Barnes (1981); Burnyeat (1981) have shown against the traditional view of people like Zabarella) demonstration is the object of knowledge and not a means of attaining it. What I want to argue is not inconsistent with this. Instead, the Aristotelian view I'm suggesting is that logic is connected to the *process* of acquiring scientific knowledge.

This isn't just because the object of knowledge is a special kind of deductive argument. It is because part of the means of attaining this argument is by the

¹⁵Πρῶτον εἰπεῖν περὶ τί καὶ τίνος ἐστὶν ἡ σκέψις, ὅτι περὶ ἀπόδειζιν καὶ ἐπιστήμης ἀποδεικτικῆς· εἶτα διορίσαι τί ἐστι πρότασις καὶ τί ὅρος καὶ τί συλλογισμός, καὶ ποῦος τέλειος καὶ ποῦος ἀτελής, μετὰ δὲ ταῦτα τί τὸ ἐν ὅλῳ εἶναι ἢ μὴ εἶναι τόδε τῷδε, καὶ τί λέγομεν τὸ κατὰ παντὸς ἢ μηδενὸς κατηγορεῖσθαι.

¹⁶Διωρισμένων δὲ τούτων λέγωμεν ἤδη διὰ τίνων καὶ πότε καὶ πῶς γίνεται πᾶς συλλογισμός· ὕστερον δὲ λεκτέον περὶ ἀποδείξεως. πρότερον δὲ περὶ συλλογισμοῦ λεκτέον ἢ περὶ ἀποδείξεως διὰ τὸ καθόλου μᾶλλον εἶναι τὸν συλλογισμόν· ἡ μὲν γὰρ ἀπόδειξις συλλογισμός τις, ὁ συλλογισμὸς δὲ οὐ πᾶς ἀπόδειξις.

use of distinctively logical tools. The capstone theorems of the first part of the *Prior Analytics,* the so-called Pons Asinorum, give a method for finding all the syllogisms generated by a given set of premises. With the right premises, Aristotle says, you will be able to find demonstrations. (*APr* I.30) So, while logical methods alone do not *suffice*—experience at least is needed for the subject-specific principles of a science—they are a necessary component. The procedures for proof-search in *APr* I.27-9 and praised in I.30 are then used in *APo* II.14-6, the chapters concerning the discovery of demonstrations.

We have arrived at the sought after conceptual connection: logical tools are a part of scientific method insofar as they are used in the discovery of demonstrations, a special kind of deductive argument. This is an interesting and, indeed, very controversial view. I believe that it was Aristotle's, but I am not at all sure that it is true. Here I will set aside worries about it and instead ask: what kind of light does this shed on otherwise puzzling features of logic?

I will focus on the question of the *normativity* of logic. The modern debate about the normativity of logic has primarily focused on whether logic is normative for:

- Reasoning, construed as belief revision.
- Thought as such.
- Public practices such as assertion.

I do not want to deny that logic is normative in any of these domains. But if I was right about the conceptual connection between logic and scientific methodology, a very different sort of conception of the normativity of logic emerges.

On this picture, logic is normative for *scientific practice* in particular. Logic binds us insofar as we aspire towards scientific knowledge. This is compatible with the other approaches, but has two important advantages over them. First, it does not face the pitfalls that the other approaches do.¹⁷ This is primarily because the problems for these other views come from deviant cases, far from normal scientific practice. By restricting the range of activities that logic is normative for, this conception of the normativity of logic is correspondingly weaker.

There might be a worry that it is now too weak. This view has the consequence, apparently, that those without scientific ambitions can thwart logic at will.

To a certain extent, though, this is a feature and not a bug. It seems like logic has been more helpful in the exact sciences than in everyday life. The most important early applications of Fregean logic have been in mathematics, where the discovery of contradictions (such as Russell's Paradox) or fundamental limitative results (such as Gödel's Incompleteness Theorems) have real upshots for the practice of the science. This has also been shown true in the physical sciences. As Davey

¹⁷For instance, the main objections to logic as normative for reasoning have been the Preface Paradox (Harman, 1986) and the possibility of an evidentialist error theory (Kolodny, 2007). In science, there is no preface paradox, and the evidentialist view Kolodny sketches does not seem to apply either.

(2014) has argued, "the discovery that a theory is logically inconsistent is typically followed by an attempt to identify and replace any unjustified assumptions in the theory." So if logic has its clearest normative force in science, that should count in favor of the sort of proposal that I am making.

So it is plausible that logic does play this kind of normative role for science. If there is the conceptual connection between logic and method that I suggested earlier, then this would be easily explained. The situations just described would be cases in which logical tools are used in the acquisition of scientific knowledge.

So the limitation of the normativity of logic to science has its advantages in terms of plausibility. It also brings out logic's importance. Although science is one human activity among many, it is a very special activity. It is, plausibly, our highest cognitive achievement. If logic can be shown to be normative for *that*, then, for all intents and purposes, the puzzle of its normative force has been resolved.

Suppose the hypothesized connection between logic and method is correct. Then, just as I argued in the Chapter 1 that formal methods should play an important role in studying the history of philosophy, history of philosophy can inform formal tools.

In this case, it is informative because 1) today's formal tools (such as Fregean logic) developed out of the historical tools and 2) the historical tools were originally thought up with a clear sense of their importance. We need not use the tools for the same purpose. The applications of logic in the study of electrical circuits, for instance, is far off Aristotle's radar. But in the debate about the demarcation of logic and its normativity, what has emerged is a puzzle about what logic could be for *at all*. If we are in search of *a* purpose, then we need look no further than the historical sources.

So, in a sense, Jonathan Barnes is right when he writes, at the start of *Truth, etc*

Most contemporary logicians have little interest in the history—or at least in the ancient history—of their subject. No doubt they suppose that their long-dead colleagues have little or nothing to teach them, and perhaps they prefer the present and the future to the past. If that is so, then it must be confessed that their supposition is quite true: no logician has anything to learn from a study of Aristotle; and the pages of this book make no contribution to logic or to philosophy. (2007, p. vii)

The tools of the contemporary logician far outstrip those of Aristotle and the Stoics. But what the ancients lacked in technical machinery, they made up with a profound appreciation of what that technical machinery is for.

Appendix A

Mereology in Aristotle's Assertoric Syllogistic

At the heart of Aristotle's syllogistic are four kinds of predication:

b A a	Universal Affirmative: b belongs to (or is predicated of) every a. (Equivalent to saying "Every a is b")
b E a	Universal Negative: b belongs to no a. ("No a is b")
b I a	Particular Affirmative: b belongs to some a. ("Some a is b")
b O a	Particular Negative: b does not belong to some a. ("Not every a is b")
In the <i>Prior Analytics</i> . Aristotle's investigations of logical relations between these	

In the *Prior Analytics*, Aristotle's investigations of logical relations between these kinds of propositions is primarily proof-theoretic. However, we get a hint of the semantics of these propositions in the famous *dictum de omni et nullo*:

We use the expression "predicated of every" when nothing can be taken of which the other term cannot be said, and we use "predicated of none" likewise." $(24b28-30)^1$

The *dictum* gives the meaning of Aristotle's technical phrases "predicated of every" and "predicated of none". The basic idea is that "a is predicated of every b" means that for every z, if b is predicated of z, then a is also predicated of z. Similarly, "a is predicated of no b" means that for every z, if b is predicated of z, then a is *not* predicated of z.² Because the *dictum* is used to justify the syllogisms

¹λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ἢ λαβεῖν καθ' οῦ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως. With Ross in excising "τοῦ ὑποκειμένου". Although it is present in all the manuscripts, it is absent in Alexander's citation and an otherwise unparalleled use of the term in the Analytics.

²It is disputed whether they are defined in terms of a notion of predication *different from* universal affirmative. See Corcoran (1973, 1972); Barnes (2007), for the view that they are different, but Malink (2013); Morison (2008), following Michael Frede for the view that they are the same. While I

of the first figure to which all the others are reduced, it carries enormous weight in understanding Aristotle's semantics. It is, as Morison (2015) says, "a governing principle in Aristotle's logic". Moreover, while it directly explains the meanings of universal affirmative and universal negative propositions, it indirectly explains the meanings of particular propositions, which are each the *contradictory* of a universal proposition.³

How are we supposed to understand this notion of predication?

My aim is to argue that the relation of predication defined or elucidated by the *dictum de omni et nullo* is a mereological relation between universals.

This position has been gaining in popularity in recent years. In the early days of mathematical reconstructions of Aristotle's logic, the schematic letters (called terms) were taken to denote non-empty sets of individuals and syllogistic propositions are about the extensional relations that hold between them.⁴ Despite its initial promise, the inadequacy of this account has been made clear, by Malink and others.⁵

In its place, the mereological interpretation of predication has been proposed as a viable alternative.⁶ While this conception does not suffer from the same problems as the set-theoretic interpretation, I will argue that the reasons so far given to support it are not compelling. Instead, I will give a new argument in support of the mereological interpretation. The core of this argument is a very general algebraic soundness and completeness result for Corcoran's deductive system RD, the standard natural deduction system used to study Aristotle's logic. In this proof, I show how RD is sound for the class of Preorders \mathbb{P} and complete for the class of Finite Boolean Algebras FBA. This result has the corollary that any class of models \mathbb{M} such that FBA $\subseteq \mathbb{M} \subseteq \mathbb{P}$ is also sound and complete for RD. In the final section, I argue 1) that we should interpret the predication relation as a preorder, since no

⁴See especially the pioneering completeness results of Corcoran (1972, 1973).

⁵Two areas of particular difficulty are some claims about conversion and Aristotle's modal syllogistic. See Malink (2009, 2013).

⁶See especially Mignucci (2000); Malink (2009); Corkum (2015) as well as Malink (2013) for an extension of this basic account to Aristotle's modal syllogistic.

will stay neutral with respect to the dispute between these two readings, the former reading is usually associated with a view of predication akin to Frege's, which takes predication to be a relation between two different syntactic types. This view has been discredited by Mignucci (2000); Malink (2009); Corkum (2015). See Crager (2015) for a version of the Corcoran/Barnes reading that does not use the Fregean conception of predication. In this chapter, I will not take a stand on whether the two kinds of predication are different. Everything in the semantics will be neutral between those two interpretations.

³The exact manner of the explanation is is a matter of dispute. Some, like Malink (2013), take the explanation to be through implicit definition, while Barnes (2007) takes it to be an explicit definition along Tarskian lines. Morison (2015) thinks Aristotle is not giving a definition at all but specifying a rule of inference that "characterizes" the meaning of the universal propositions. In what follows, I will follow Barnes/Malink, but the account I propose could be translated into a rule-based account along the lines Morison endorses without any serious difficulty. Instead of one rule, we would have two corresponding to the biconditional. This account, unlike Morison's, would provide both an introduction and an elimination rule for the universal propositions.

further structure is needed to capture Aristotle's validities and invalidities, 2) that preorders generally capture the formal structure of part-whole relations, and 3) that the mereological interpretation is thereby vindicated.

A.1 The Mereological Interpretation of Predication

In recent literature on Aristotle's notion of predication, there have been two arguments for a mereological interpretation.⁷

The first reason to think that Aristotle has a mereological conception of predication is because of the language he uses. Aristotle's terminology for predication is steeped in mereological language. He marks his phrase for universal predication ("belongs to all") as equivalent to "is in as a *whole*" and his terms for different kinds of propositions—"universal" (*katholou*) and "particular" (*kata meros/en merei*)—are themselves derived from the language of whole (*holon*) and part (*meros*). If universal predication is a mereological relation, we would also expect it to be defined in terms of notions related to mereology.

While the language is highly suggestive, it does not settle the case in favor of the mereological interpretation. In particular, the linguistic information, on its own, does not tell us that Aristotle isn't being metaphorical. Aristotle could be reaching for mereological language just because he feels it is the best approximation of what he has in mind. Nevertheless, we are not entitled to conclude that the mereology talk should be taken literally, for the same reason that we should not take Aristotle's use of the word " $\eth\lambda\eta$ " to mean that all matter is timber. What we want, and what the linguistic information does not yet give us, is a reason *why* Aristotle would want to conceive of predication mereologically. What *philosophical work* does it do? Without a good sense of its purpose, the metaphorical objection seems hard to resist.

One suggestion as to why Aristotle uses the language of part and whole comes from thinking about a specific kind of predication. When discussing the relation between genus and species, Aristotle explicitly says that they are related as whole to part. (*Metaphysics* V.25, 26)⁸ On this account, Aristotle inherits a mereological conception of the genus-species relation, presumably from Plato's method of division.⁹ In the method of division, a genus is *divided* into the species that are its parts. Because of its connection to the method, it seems like we can make good

⁷See Mignucci (2000); Malink (2009); Corkum (2015). This was also the view of the ancient commentators: Alexander *in APr* 25.2–4 Wallies, Philoponus *in APr*. 47.23–48.2, 73.22–3, 104.11–16, 164.4–7 Wallies.

⁸Malink (2013) also cites Aristotle's claim that medicine is a part of science (*APr* 2.15 64a17, 64b12–13), which is just a particular case of the general principle.

⁹In the *Statesman*, Plato is most explicit about this point: "That whenever there is a kind of something, it is necessarily also a part of whatever thing it is said to be a kind of, but it is not at all necessary that a part is a kind. You must always assert, Socrates, that this is what I say rather than the other way around." (263b7-10) For the connection to Aristotle's syllogistic, see Solmsen (1929); Mignucci (2000).

sense of the mereological language.¹⁰ Then, on this account, Aristotle *extends* the mereological conception from this special case of predication to predication in general. However, what is the evidence for this extension? It isn't clear that there is the same motivation for predication generally being a part-whole relation. To see this, we should turn to the passages in the *Metaphysics* in which the species/genus, part/whole connection is made.

Again, the results of any non-quantitative division of a kind are also called its parts; that is why people assert that species are parts of their genus. (*Met.* 1023b17-19, trans. Kirwin with modifications)¹¹

We call a whole both that of which no part is absent out of those of which we call it a whole naturally; and what contains its contents in such a manner that they are one thing, and this in two ways, either as *each* being one thing or as *making up* one thing. For what is universal and what is said to be as a whole, implying that it is a kind of whole, is universal as containing several things, by being predicated of each of them and by each severally being one thing; as for instance man, horse, god, because they are all animals. (1023b26-32)¹²

In the first passage, Aristotle seems to be saying that species are part of a genus *because* it is the product of a division. If this is the explanation, then why should we think that the broader, non-essential predication, can also be explained in the same way? For instance, does it motivate the claim that *Human* is a part of *Colored Thing*? In these cases, there simply does not seem to be a division that would do the job. The second passage is less clear. The language of predication could suggest that anything that a kind is predicated of is its part, in which case Aristotle's example at the end of the passage does not mean that only species are parts of genera. On the other hand, when Aristotle discusses a genus "containing" things elsewhere in his corpus, it is inevitably species, differences, or individual substances.¹³ Nowhere is it so much as hinted that containment includes accidental relations, as it must on this account. Therefore, the fact that Aristotle explicitly and literally claims the genus-species relation to be relation of whole to part does not support by itself the broader view that predication *in general* is a mereological relation. For Aristotle's

¹⁰Could this too be metaphorical? I find that less plausible, since Plato is insistent and explicit about the parthood language in a way that would be strange if we were to take him metaphorically. See n. 9.

¹¹ ἕτι εἰς ἂ τὸ εἶδος διαιρεθείη ἂν ἄνευ τοῦ ποσοῦ, καὶ ταῦτα μόρια λέγεται τούτου· διὸ τὰ εἶδη τοῦ γένους φασὶν εἶναι μόρια.

¹² Όλον λέγεται οὕ τε μηθὲν ἄπεστι μέρος ἐξ ῶν λέγεται ὅλον φύσει, καὶ τὸ περιέχον τὰ περιεχόμενα ὥστε ἕν τι εἶναι ἐκεῖνα· τοῦτο δὲ διχῶς· ἢ γὰρ ὡς ἕκαστον ἐν ἢ ὡς ἐκ τούτων τὸ ἕν. τὸ μὲν γὰρ καθόλου, καὶ τὸ ὅλως λεγόμενον ὡς ὅλον τι ὄν, οὕτως ἐστὶ καθόλου ὡς πολλὰ περιέχον τῷ κατηγορεῖσθαι καθ΄ ἑκάστου καὶ ἐν ἅπαντα εἶναι ὡς ἕκαστον, οἴον ἄνθρωπον ἴππον θεόν, διότι ἅπαντα ζῷα.

¹³*HA* 490b16, 504b13, 534b13, *PA* 644a13, *Pol* 1285a2, *Topics* 121b24-9, 1139b38, 144b12. This use of "containment" is also present in Plato *Ti*. 31a2–8, 33b2–7, and possibly *Soph*. 250b8. See Gill (2012), p. 206 ff for a different reading of the *Sophist* passage.

reasons for making the claim in the genus-species case do not carry over to nonessential predication. So, even if Aristotle *did* get the mereological language in this way, we have come no closer to understanding the work that it is supposed to do, since that does not obviously apply to the extended uses of the mereological relation.

In what follows, I will give a new piece of evidence, which also has a clear philosophical payoff. The minimal formal structure of the part-whole relation is sufficient for giving semantic clauses for all the syllogistic propositions and defining a consequence relation that is sound and complete for the standard proofsystem for studying Aristotle's logic.

This reason shows the *adequacy* of the mereological conception of predication in Aristotle's logical theory. Even if Aristotle is using the terminology metaphorically, or extending spatial and physical language to apply to logic, this reason allows us to see what logical work such an association can do. I show that we do not need anything beyond what is given in the *dictum* to understand the meaning of the syllogistic propositions, provided that the predication relation is preorder, the general formal structure of any mereological relation. If conceiving of the meanings of the syllogistic propositions in mereological terms can do this kind of work, Aristotle's use of mereological language should be taken seriously.¹⁴

A.2 Algebraic Semantics

Here we introduce the assertoric syllogistic, the fragment of Aristotle's syllogistic that only deals with non-modal propositions. Soundness will be shown for the class \mathbb{P} of Preorder models (domains with a reflexive and transitive relation) and completeness for the class \mathbb{FBA} of Finite Boolean Algebras.¹⁵

The language of assertoric syllogistic is defined by a finite set of terms: $T = \{a, b, ...\}$ and copulae $C = \{A, E, I, O\}$ such that, $\mathcal{L} = \{x\mathbf{Z}y \mid x, y \in T \land x \neq y \land \mathbf{Z} \in C\}$.

A model \mathcal{M} is a tuple, where $\langle D, \sqsubseteq \rangle$ where $D = \{A, B, \ldots\}$, and $\sqsubseteq \subseteq D \times D$. We write $B \sqsubseteq A$ instead of $\sqsubseteq (B, A)$. The denotation function $\llbracket t \rrbracket^{\mathcal{M}} : T \to D$. The semantic clauses for the formulae:

- $\mathcal{M} \models xAy \text{ iff } \forall Z (Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}} \to Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}});$
- $\mathcal{M} \models xEy$ iff $\neg \exists (Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}} \land Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}});$

¹⁴I do not want to suggest that this is the only way that Aristotle thinks of predication. Singular propositions stand out as the most likely exception. Aristotle never claims that an individual is a part of a universal, but see Mignucci (2000). He also sometimes uses form and matter to understand predication (e.g., *Metaphysics* Z.17, H.6) and this is not mereological either.

¹⁵As will become clear, it is a generalization of nearly every semantics present in the literature, including Corcoran (1973) and Martin (1997). The only exception is the incomparable approach of Andrade-Lotero and Dutilh Novaes (2010), who examine the first order models given by a translation from Aristotle's language.

- $\mathcal{M} \models xIy$ iff $\exists Z (Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}} \land Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}})$; and
- $\mathcal{M} \models x\mathbf{O}y \text{ iff } \forall Z (Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}} \to Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}}).$

We say that $\Gamma \models \psi$ if and only if for every \mathcal{M} such that for all $\varphi \in \Gamma$, $\mathcal{M} \models \varphi$, $\mathcal{M} \models \psi$.

The deductive system (Corcoran's RD) that we will be using is a natural deduction system consisting of the following rules (order does not matter):

(I)
$$\underline{xEy}$$
 (II) \underline{xAy} (III) \underline{xAy} (IV) \underline{xEy}
 \underline{yEx} (IV) \underline{xIy} \underline{yAz} \underline{yAz} \underline{yAz}
 \underline{xAz} \underline{xEz}

A sequence of sentences $\langle p_1, ..., p_n \rangle$ is a *direct deduction* of φ from Γ if and only if for all $i \leq n$ one of the following holds:

- 1. $p_i \in \Gamma$
- 2. $\exists j < i : p_i$ is obtained from p_i by (I) or (II).
- 3. $\exists j, k < i : p_i \text{ is obtained from } p_j \text{ and } p_k \text{ by (III) or (IV).}$

A sequence of sentences $\langle p_1, \ldots, p_n \rangle$ is an *indirect deduction* of φ from Γ if and only if $\exists j < n (p_j = c (p_n))$ (where $c : \mathcal{L} \to \mathcal{L}$ such that c (xAy) = xOy, c (xEy) = xIy, c (xIy) = xEy, c (xOy) = xAy) and for all $i \leq n$ one of the following holds:

- 1. $p_i \in \Gamma \cup \{c(\varphi)\}$
- 2. $\exists j < i : p_i$ is obtained from p_i by (I) or (II).
- 3. $\exists j, k < i : p_i$ is obtained from p_j and p_k by (III) or (IV).

We say that $\Gamma \vdash \varphi$ if and only if there is a direct or an indirect deduction of φ from Γ .¹⁶

A.2.1 Soundness

Theorem. Let \mathbb{P} be the class of Preorders, where \sqsubseteq is reflexive and transitive. With the semantic clauses provided above, if $\Gamma \vdash \varphi$, then $\Gamma \models \varphi$.

Proof. We proceed by first showing that (I)-(IV) above are satisfied in all models. We then show that, if a contradiction is proved in RD, then no model satisfies the premises so that $\Gamma \models \varphi$ is trivial, thus showing that indirect deductions are sound. *I*-conversion

Suppose $\mathcal{M} \models xIy$. Then $\exists Z (Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}} \land Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}})$. So, $\mathcal{M} \models yIx$.

¹⁶This deductive system does not perfectly capture Aristotle's notion of the syllogism: it validates explosion (which Aristotle denies in *APr* II.15), allows for conclusions to be in the premises, and superfluous premises (these last two being ruled out by the definition of the syllogism). Nevertheless, it is the best approximation currently at hand.

A-subalternation

Suppose $\mathcal{M} \models xAy$. Then $\forall Z (Z \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}} \to Z \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}})$, but since \sqsubseteq is reflexive, $\llbracket y \rrbracket^{\mathcal{M}} \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}}$ and hence $\mathcal{M} \models xIy$.

Barbara

Suppose $\mathcal{M} \models xAy$ and $\mathcal{M} \models yAz$. Then $\forall W (W \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}} \to W \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}})$ and $\forall W (W \sqsubseteq \llbracket z \rrbracket^{\mathcal{M}} \to W \sqsubseteq \llbracket y \rrbracket^{\mathcal{M}})$, but since \sqsubseteq is transitive, $\forall W (W \sqsubseteq \llbracket z \rrbracket^{\mathcal{M}} \to W \sqsubseteq \llbracket x \rrbracket^{\mathcal{M}})$ and hence $\mathcal{M} \models xAz$.

Celarent

Suppose $\mathcal{M} \models xEy$ and $\mathcal{M} \models yAz$. Then $\neg \exists W (W \sqsubseteq X \land W \sqsubseteq Y)$ and $\forall W (W \sqsubseteq [\![z]\!]^{\mathcal{M}} \rightarrow W \sqsubseteq [\![y]\!]^{\mathcal{M}})$. If there is a $W \sqsubseteq X$ and $W \sqsubseteq Z$, then by transitivity $W \sqsubseteq Y$, contradicting the assumption. Hence $\mathcal{M} \models xEz$.

Reductio

Suppose there is an indirect proof of φ from $\Gamma \cup \{c(\varphi)\}$ where the contradicting sentences in the proof are ψ and $c(\psi)$. By the semantic clauses, no model satisfies both ψ and $c(\psi)$. Since all models that satisfy $\Gamma \cup \{c(\varphi)\}$ satisfy both ψ and $c(\psi)$ because of the validity of rules (I)-(IV), no models satisfy $\Gamma \cup \{c(\varphi)\}$ and every model that satisfies Γ trivially also satisfies φ . Therefore, if $\Gamma \vdash \varphi$, then $\Gamma \models \varphi$.

A.2.2 Completeness

Now we show that if $\Gamma \models \varphi$, then $\Gamma \vdash \varphi$. In fact we will show something significantly stronger, by constructing a counter-model that is a Finite Boolean Algebra $\mathcal{M} = \langle D, \sqsubseteq, - \rangle$. The semantic clauses remain the same.

Theorem. Suppose that $\Gamma \nvDash \varphi$. There is a Finite Boolean Algebra model \mathcal{M} such that, for all $\gamma \in \Gamma \cup \{c(\varphi)\}, \mathcal{M} \vDash \gamma$ and thus, that $\Gamma \nvDash \varphi$.

Proof. First, extend $\Gamma \cup \{c(\varphi)\}$ by introducing witnesses for the existential statements. We do this by adjoining a new set of constants T' to the set of terms, giving rise to a new set of terms T^* and a new language \mathcal{L}^* . For each statement of the form $xIy \in \Gamma \cup \{c(\varphi)\}$, add the statements xAc and yAc, with a different $c \in T'$ resulting in a new theory Γ^* .

Proposition. *For any* $\psi \in \mathcal{L}$ *,* $\Gamma \cup \{c(\varphi)\} \vdash \psi \Leftrightarrow \Gamma^* \vdash \psi$ *.*

Proof. The left to right direction is trivial, as the deductive system is monotonic. For the right to left direction, consider some $\chi \in \Gamma^* \setminus (\Gamma \cup \{c(\varphi)\})$ is used in a derivation to prove some $\gamma \in \mathcal{L}$. This is only possible if it is used in conjunction with another proposition $\chi' \in \Gamma^* \setminus (\Gamma \cup \{c(\varphi)\})$ where χ and χ' share a term in T^* . For each χ , there is exactly one such formula by construction, so we have *aAc* and *bAc*, for *a*, *b* \in *T* and *c* \in *T*^{*}. The only formulae in \mathcal{L} that these can prove are: *aIb* and *bIa*. However, these are both derivable by $\Gamma \cup \{c(\varphi)\}$.

Corollary. $\Gamma \cup \{c(\varphi)\}$ and Γ^* are consistent.

Proof. A set of sentences $\Delta \subseteq \mathcal{L}$ is inconsistent iff for every $\psi \in \mathcal{L}$, $\Delta \vdash \psi$. By the initial hypothesis, $\Gamma \cup \{c(\varphi)\}$ is consistent (otherwise, φ could be derived by indirect deduction). Since, according to the Proposition, they prove all the same sentences, Γ^* is also consistent.

The counter model to be constructed will be the power set algebra of T^* , a Finite Boolean Algebra (a Boolean Algebra by construction and finite because T^* is finite) and hence a Preorder model. The denotation function of a term $t \in T$ will be defined as:

$$\llbracket t \rrbracket^{\mathcal{M}} = \{ s \in T^* : \Gamma^* \vdash tAs \lor s = t \}.$$

(Note that the denotation function always sends *t* to a non-empty subset of *T*^{*}.) Now we will show that every $\gamma \in \Gamma \cup \{c(\varphi)\}$ is satisfied in \mathcal{M} , proving it for each type of proposition:

- By the semantic clauses and definition of the interpretation function, $\mathcal{M} \models bAa \text{ iff } \forall Z (Z \subseteq \llbracket a \rrbracket^{\mathcal{M}} \to Z \subseteq \llbracket b \rrbracket^{\mathcal{M}}) \text{ iff } \llbracket a \rrbracket^{\mathcal{M}} \subseteq \llbracket b \rrbracket^{\mathcal{M}} \text{ iff } \forall s \in T^* ((\Gamma^* \vdash aAs \lor a = s) \to (\Gamma^* \vdash bAs \lor b = s)).$ Suppose that $bAa \in \Gamma \cup \{c(\varphi)\}$. If, for an arbitrary $s \in T^*$, $\Gamma^* \vdash aAs \lor a = s$, it will be the case that $\Gamma^* \vdash bAs$ either because of substitution or Barbara. Hence $\mathcal{M} \models bAa$.
- $\mathcal{M} \models bIa$ iff $\exists s \in T^* ((\Gamma^* \vdash aAs \lor a = s) \land (\Gamma^* \vdash bAs \lor b = s))$. Suppose $bIa \in \Gamma \cup \{c(\varphi)\}$. By the construction of Γ^* , for some $s \in T^*$, Γ^* contains bAs and aAs. Hence $\mathcal{M} \models bIa$.
- $\mathcal{M} \models bEa$ iff $\neg \exists s \in T^* ((\Gamma^* \vdash aAs \lor a = s) \land (\Gamma^* \vdash bAs \lor b = s))$. Suppose $bEa \in \Gamma \cup \{c(\varphi)\}$. Suppose for reductio that $\exists s \in T^* ((\Gamma^* \vdash aAs \lor a = s) \land (\Gamma^* \vdash bAs \lor b = s))$. Four cases are possible and in all of them a contradiction can be derived:
 - $\Gamma^* \vdash aAs$ and $\Gamma^* \vdash bAs$. Then $\Gamma^* \vdash aIb$. But in that case Γ^* is inconsistent, which is impossible given *Corollary*.
 - $\Gamma^* \vdash aAs$ and s = b. Then $\Gamma^* \vdash aAb$ and is inconsistent.
 - $\Gamma^* \vdash bAs$ and s = a. Same as 2.
 - s = a and s = b. But this is impossible because $aEa \notin \mathcal{L}$.
- $\mathcal{M} \models b\mathbf{O}a$ iff $\exists s \in T^* ((\Gamma^* \vdash aAs \lor s = a) \land \Gamma^* \nvDash bAs \land b \neq s)$. Suppose $b\mathbf{O}a \in \Gamma \cup \{c(\varphi)\}$. Because Γ^* is consistent, $\Gamma \nvDash bAa$ and because $a \neq b$, $\mathcal{M} \models b\mathbf{O}a$.

This completes the proof.

A.2.3 A Semantic Hierarchy

Recall that we showed that every Preorder model is sound for Aristotle's deductive system and that Aristotle's deductive system is complete with respect to Finite Boolean Algebras. Consider any class of models intermediate between Preorders and Boolean Algebras. The above result immediately gives soundness and completeness results to this class as well. We see then that Aristotle's language and deductive system cannot distinguish between these classes of models.¹⁷

A.3 The Mereological Interpretation Vindicated

In this section, I will argue that the technical result has bearing on how we ought to interpret Aristotle's notion of predication. The argument proceeds in two steps. First, I will argue that the above results motivate interpreting the predication relation as a preorder. Then, I will argue that the preorder captures the formal structure of the part-whole relations. Putting these two together, we have a good reason to interpret Aristotelian predication as a part-whole relation.

Recall that the result above immediately gives soundness and completeness results for a whole range of classes of models. Two of these classes deserve special mention: meet semi-lattices¹⁸ and Boolean Algebras. Corcoran (1972) used a settheoretic semantics equivalent to Boolean Algebras (by the Stone Representation Theorem) in proving his seminal completeness result. Martin (1997) argued for the use of meet semi-lattices. We have seen above, however, that it is possible to capture everything in Aristotle's assertoric syllogistic captured by Boolean Algebras and meet semi-lattices with just preorders, *with the very same semantic clauses*. There is nothing to be gained by positing all the additional structure in previous interpretations, since the result above shows that it can all be done with preorders. These other interpretations, therefore, go significantly beyond Aristotle's text. Since they all make the same predictions about the validities and invalidities, we should start with the weakest coherent reading of the text and only add structure when it is explicitly motivated.¹⁹

So we should understand Aristotle's predication relation in the assertoric syllo-

¹⁷Extending Aristotle's language, by including, say, identity or complex terms would be able to distinguish these classes of models, although this would not be particularly helpful in trying to interpret Aristotle's assertoric syllogistic, as he operates there with this relatively impoverished language.

¹⁸A preorder \leq is a meet semi-lattice if it is anti-symmetric ($\forall x, y ((x \leq y \land y \leq x) \rightarrow x = y)$) and has the greatest-lower bound property ($\forall x, y \exists z (z \leq x \land z \leq y \land \forall w ((w \leq x \land w \leq y) \rightarrow w \leq z))$).

¹⁹Indeed, Malink (2009) argues that, outside of the sections on the assertoric syllogistic, Aristotle makes claims inconsistent with both Corcoran's and Martin's interpretations. It might be objected that Corcoran (1972) and Martin (1997) are only giving formal models, not interpretations of Aristotle's text. While this is explicitly true of Andrade-Lotero and Dutilh Novaes (2010), Corcoran and Martin both use the terms "model" and "interpretation" interchangeably, except, of course, when talking about semantic models.

gistic as a preorder. But why should that be understood as a *mereological* relation? It is because the preorder captures the crucial formal structure of the part-whole relation. Recall that Preorders have two essential properties: transitivity and reflexivity. While it is easy to see how transitivity is part of the formal structure of the part-whole relation, the case is significantly harder with reflexivity. Aristotle's preferred use of the language of part and whole is generally proper: nothing is a part of itself. By contrast, any mereological relation used to model quantification must make use of improper parthood, since "A belongs to all A" is evidently true. Why would Aristotle have used the language of parthood when it seems so ill-suited to the task of modeling this reflexive aspect of quantification?

Indeed, Aristotle's terminology frequently has this problem. Another way he refers to universal affirmative predications is by saying that the predicate is "above" the subject. As we saw in Chapter 4, he is willing to use this terminology even in cases where the subject and predicate convert. So, although he uses language that suggests an irreflexive relation, it clearly is meant to indicate a reflexive one.

The answer comes in two stages. First, reflexive predications are not at the forefront of Aristotle's thinking in his account of predication. The only point where it gets explicitly discussed is *APr* II.15, where he is clearly assuming that "All A is A" is always true. But this text is the exception that proves the rule: while the syllogistic does in principle allow for such reflexive sentences, it is not designed to account for them. So the fact that the language of parthood does not fit well with it does not seem such a huge cost. Rather, reflexivity represents a sort of limit case of universal quantification.

Second, there do not seem to be any better relations at hand for expressing what Aristotle does want. It needs to generally be a kind of ordering, that is transitive, but not an equivalence relation. Perhaps he could have used the relation "part of or identical to" to model predication, but that is cumbersome. "Prior to" faces the same problems as "part". It seems to me that the language of parthood is the closest that Aristotle could have come describing preorders without modern technical terminology.

Corkum (2015), p. 804 argues that any genuine mereology must satisfy the principle of Weak Supplementation (where *PP* stands for Proper Part and *O* for Overlap):

$$\forall x, y \ (PPxy \rightarrow \exists z (PPzy \land \neg Ozx))$$

This principle states that whenever x is a proper part of y, there is a part of y "left over". I take no stand on our contemporary conception of parthood, but there is significant evidence from ancient mathematics that shows that the Greeks did not require Weak Supplementation to hold for the *meros-holon* relation. In Greek mathematics as early as Aristotle,²⁰ there are two technical uses of the term *meros*:

²⁰For its use in mathematics, see Euclid, *Elements* V Def 1, Heath ad loc. The argument here does

We call a part (1) that into which a quantity can in any way be divided; for that which is taken from a quantity qua quantity is always called a part of it, e.g. two is called in a sense a part of three.—(2) It means, of the parts in the first sense, only those which measure the whole; this is why two, though in one sense it is, in another is not, a part of three. (*Metaphysics* 1023b12-17)²¹

From the passage, we see the semantic range of the term is quite wide—it can apply to any sorts of *quantities*. According to either definition in Aristotle's text, the *meros-holon* relation need not obey Weak Supplementation. For simplicity, just consider numbers. On the broad definition, 2 is a proper part of 3. However, there is no number less than 3 that does not overlap with 2. On the narrow definition, 2 is a proper part of 4, but again, there are no numbers that 4 factors into that do not overlap with 2. This shows that, just because Aristotle conceptualized the predication relation in mereological terms, we do not thereby need to think that he was committed to Weak Supplementation for that relation.

So why did Aristotle use the mereological language in his account of predication? I submit that he was looking for a way to talk about preorders, but, without the resources of contemporary logic, was unable to formally describe the conditions of transitivity and reflexivity that he wanted to impose on the predication relation. So he reached for the most natural, general relation with the structural features of a preorder. This was the part-whole relation. He then used the language of parthood to describe the predication relation. Unlike previous accounts, this reason for the mereological interpretation makes the part-whole relation do important philosophical work for Aristotle. The soundness and completeness result makes the work explicit, showing how nothing more is necessary to account for the validities and invalidities of the deductive system.

In Aristotle's own practice, the work of the part-whole relation comes out differently and somewhat more indirectly. Central to his practice, as Morison has recently emphasized, is the proofs of first figure syllogisms by means of the *dictum de omni et nullo* and the other figures through these. Thus, at bottom, all that Aristotle has is the *dictum*, and hence the conception of predication that I have argued is grounded in a mereological conception of predication. But he does not have the tools to prove soundness and completeness, instead only supplying the semantic machinery to a small segment of his deductive system and using proof-theoretical tools to complete the other syllogisms.

not depend on any special features of numbers and could be given for any quantities: lines, shapes, harmonic intervals, etc. Weak Supplementation fails for all of them.

²¹Μέρος λέγεται ἕνα μὲν τρόπον εἰς ὃ διαιρεθείη ἂν τὸ ποσὸν ὁπωσοῦν (ἀεὶ γὰρ τὸ ἀφαιρούμενον τοῦ ποσοῦ ἢ ποσὸν μέρος λέγεται ἐχείνου, οἶον τῶν τριῶν τὰ δύο μέρος λέγεταί πως), ἄλλον δὲ τρόπον τὰ χαταμετροῦντα τῶν τοιούτων μόνον· διὸ τὰ δύο τῶν τριῶν ἔστι μὲν ὡς λέγεται μέρος, ἔστι δ' ὡς οὕ.

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